Perturbing integrability: the Leigh-Strassler deformation of $\mathcal{N} = 4$ SYM

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Outline

The Leigh-Strassler deformation of $\mathcal{N} = 4$ SYM

action (in terms of vector superfield V, chiral superfields $\phi^i = (X, Y, Z)$):

$$S = \frac{1}{2g^2} \int d^4x \, d^2\theta \, \mathrm{tr}\left(W^\alpha W_\alpha\right) + S_{\mathrm{gf+FP}} + \int d^4x \, d^4\theta \, \mathrm{tr}\left(e^{-gV} \bar{\phi}_i \, e^{gV} \phi^i\right)$$
$$+ \int d^4x \, d^2\theta \, \mathcal{W} + \int d^4x \, d^2\bar{\theta} \, \overline{\mathcal{W}}$$

- chiral superfield strength: $W_\alpha = i \, \bar{\mathrm{D}}^2 \left(e^{-gV} \mathrm{D}_\alpha \, e^{gV} \right)$

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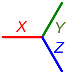
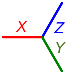
- chiral superfield strength: $W_\alpha = i\,\bar{\mathrm{D}}^2\left(\mathrm{e}^{-gV}\,\mathrm{D}_\alpha\,\mathrm{e}^{gV}\right)$
- $\mathcal{N}=4$ SYM superpotential:

$\mathcal{W} = ig\Big[$ 

$-$ 

$\Big]$

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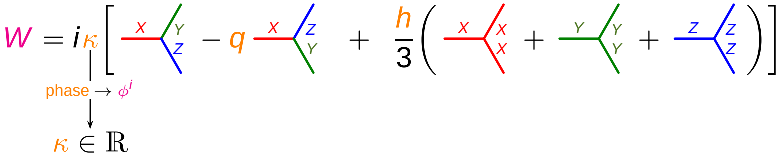
$$+ \int \mathrm{d}^4x\, \mathrm{d}^2\theta\, W + \int \mathrm{d}^4x\, \mathrm{d}^2\bar{\theta}\, \overline{W}$$

- chiral superfield strength: $W_\alpha = i\, \bar{\mathrm{D}}^2 \left(\mathrm{e}^{-gV}\, \mathrm{D}_\alpha\, \mathrm{e}^{gV}\right)$
- Leigh-Strassler superpotential:



conformal symmetry $\Rightarrow$ relation between the couplings:

$$2g^2 = \kappa^2(1 + |q|^2 + |h|^2) + \mathcal{O}(\kappa^8) + \mathcal{O}\left(\frac{1}{N^2}\right)$$

non-planar corr.
not considered here

The Leigh-Strassler deformation of $\mathcal{N} = 4$ SYM

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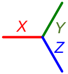
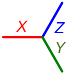
- chiral superfield strength: $W_\alpha = i\,\bar{\mathrm{D}}^2 \left(e^{-gV}\, \mathrm{D}_\alpha\, e^{gV} \right)$
- β-deformed ($q = e^{-2\pi i \beta}$) $\mathcal{N} = 4$ SYM superpotential:

$W = i\kappa\Big[$ 

$- q$ 

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conformal symmetry $\Rightarrow$ relation between the couplings:

$$2g^2 = \kappa^2(1 + |q|^2) + \mathcal{O}(\kappa^8) + \mathcal{O}\left(\tfrac{1}{N^2}\right)$$

$\uparrow$ absent if $|q| = 1$

The Leigh-Strassler deformation of $\mathcal{N} = 4$ SYM

action (in terms of vector superfield V, chiral superfields $\phi^i = (X, Y, Z)$):

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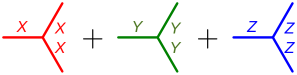
$$+ \int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathcal{W} + \int \mathrm{d}^4x\, \mathrm{d}^2\bar{\theta}\, \overline{\mathcal{W}}$$

- chiral superfield strength: $W_\alpha = i\,\bar{\mathrm{D}}^2 \left(\mathrm{e}^{-gV}\, \mathrm{D}_\alpha\, \mathrm{e}^{gV}\right)$
- cubic Leigh-Strassler ($\kappa \to 0$, $h \sim \frac{1}{\kappa}$) superpotential:

$$\mathcal{W} = i\kappa \qquad \frac{h}{3}\Bigg($$



$$\Bigg)$$

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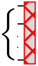
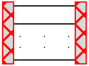
$$2g^2 = \kappa^2 \qquad |h|^2 + \mathcal{O}(\kappa^8|h|^8) + \mathcal{O}\left(\tfrac{1}{N^2}\right)$$

Renormalization of composite operators

composite operator $\mathcal{O}_L = L$  of length L (with L scalar fields)

two-point functions of composite operators: tree level

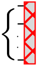
$$\big(\mathcal{O}_L^A(x),\ \mathbb{1}\ ,\mathcal{O}_L^B(y)\big) =$$



x y

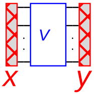
$$= \frac{\delta^{AB}}{(x-y)^{2\Delta}}\ , \qquad \Delta = L$$

Renormalization of composite operators

composite operator $\mathcal{O}_L = L$  of length L (with L scalar fields)

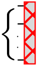
two-point functions of composite operators: with loop corrections

$$\left(\mathcal{O}_L^A(x), V_{2L}, \mathcal{O}_L^B(y)\right) =$$



$$= \frac{\delta^{AB}}{(x-y)^{2\Delta}}\,, \qquad \Delta = L + \gamma + \ldots$$

Renormalization of composite operators

composite operator $\mathcal{O}_L = L$  of length L (with L scalar fields)

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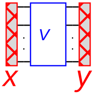
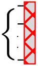


$$= \frac{\delta^{AB}}{(x-y)^{2\Delta}} \,, \qquad \Delta = L + \gamma + \ldots$$

renormalization of composite operators in a CFT in $D = 4 - 2\varepsilon$ dimensions

$$\mathcal{O}^A_{L,\mathrm{ren}} = \mathcal{Z}^A{}_B \mathcal{O}^B_{L,\mathrm{bare}} \,, \qquad \mathcal{D} = \mu \frac{\mathrm{d}}{\mathrm{d}\mu} \ln \mathcal{Z}(\lambda \mu^{2\varepsilon}) \,, \qquad \lambda = g^2 N$$

Renormalization of composite operators

composite operator $\mathcal{O}_L =$  of length L (with L scalar fields)

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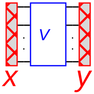
$$\left(\mathcal{O}_L^A(x), V_{2L}, \mathcal{O}_L^B(y)\right) =$$ 

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renormalization of composite operators in a CFT in $D = 4 - 2\varepsilon$ dimensions

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anomalous dimensions:

eigenvalues of the dilatation operator $\mathcal{D} = \sum_{k \geq 1} G^{2k} \mathcal{D}_k\,, \qquad G = \frac{\sqrt{\lambda}}{4\pi}$

$$\mathcal{D}\,\vec{\mathcal{O}}_L = \gamma\,\vec{\mathcal{O}}_L$$

Bethe ansatz in the flavour $SU(2)$ subsector

complex fields: $X = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$, $Y = \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4)$, $Z = \frac{1}{\sqrt{2}}(\phi_5 + i\phi_6)$

Y only as internal flavour in Feynman diagrams

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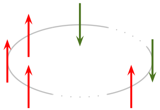
Y only as internal flavour in Feynman diagrams

map to integrable spin chain of length L [Minahan,Zarembo]

$\mathcal{O}_L = \mathrm{tr}(\underbrace{Y \dots Y}_{M}\underbrace{XXX \dots X}_{L-M}) \quad \leftrightarrow$ 

BPS operator $\mathrm{tr}(X \dots X)$ $\leftrightarrow$ ferromagnetic vaccum

impurities Y $\leftrightarrow$ spin excitations (magnons)

dilatation operator $\mathcal{D}$ $\leftrightarrow$ Hamiltonian H

anomalous dimensions γ $\leftrightarrow$ energies E

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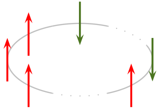


BPS operator $\mathrm{tr}(X \dots X)$	$\leftrightarrow$	ferromagnetic vaccum
impurities Y	$\leftrightarrow$	spin excitations (magnons)
dilatation operator $\mathcal{D}$	$\leftrightarrow$	Hamiltonian $H = H_{XXX_{1/2}} + \dots$
anomalous dimensions γ	$\leftrightarrow$	energies E

operator mixing problem solved by the asymptotic Bethe ansatz

$$\sum_{j=1}^{M} p_j = 0\,, \quad e^{ip_jL} = \prod_{k\neq j}^{M} \hat{S}(u_j,u_k) e^{2i\theta(u_j,u_k)}\,, \quad E = \sum_{j=1}^{M}\left(\sqrt{1+\frac{\lambda}{\pi^2}\sin^2\frac{p_j}{2}} - 1\right)$$

momentum conservation

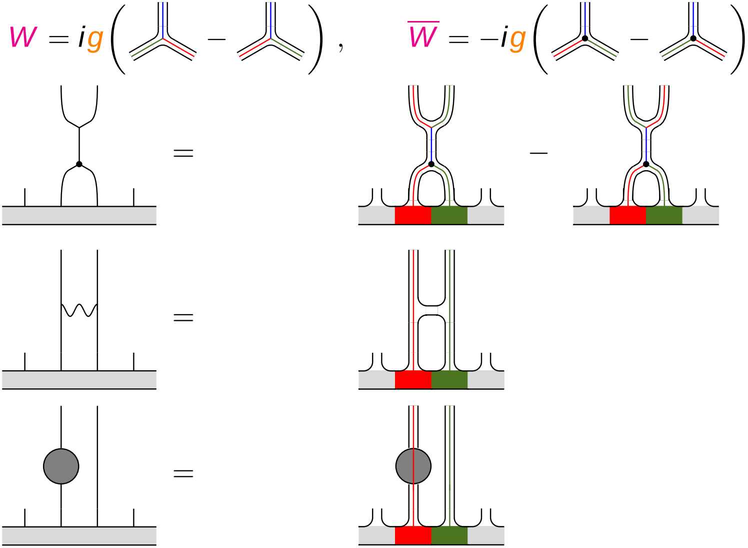
matrix part

dressing phase

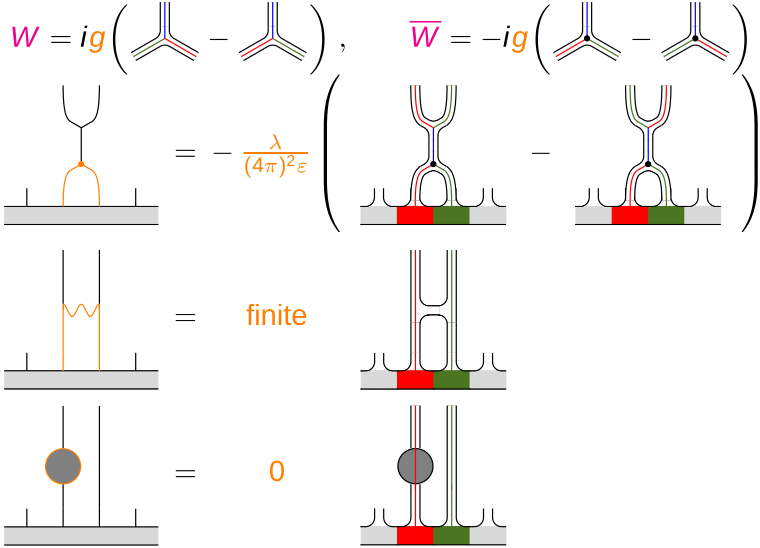
two-particle S-matrix

single magnon dispersion relation

$\mathcal{N} = 4$ SYM at one loop



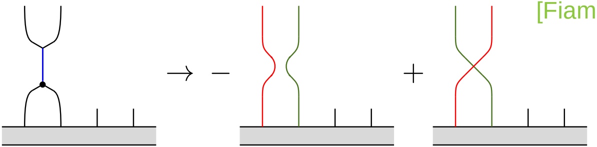
$\mathcal{N} = 4$ SYM at one loop



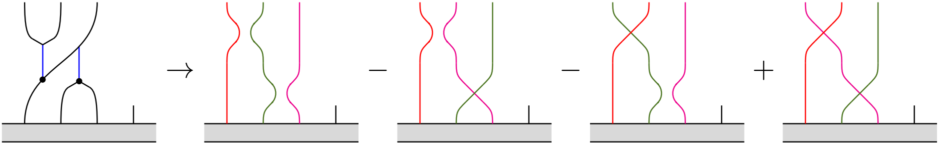
$$\mathcal{D}_1 = 2\left(\mathbb{1} - \sum_{i=1}^{L} \mathrm{P}_{i\,i+1}\right)$$

Chiral functions

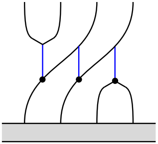
[Fiamberti, Santambrogio, C.S., Zanon]



$$\chi(1) = -\{\} + \{1\}$$



$$\chi(1,2) = \{\} - \{1\} - \{1\} + \{1,2\}$$



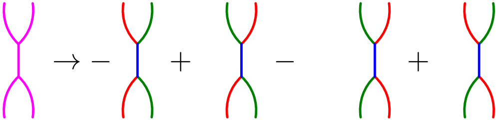
$$\{a_1,\dots,a_n\} = \sum_{i=1}^{L} \mathrm{P}_{i+a_1\; i+a_1+1}\cdots \mathrm{P}_{i+a_n\; i+a_n+1}$$

$$\chi(1,2,3) = -\{\} + 3\{1\} - 2\{1,2\} - \{1,3\} + \{1,2,3\}$$

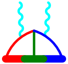
Action on chiral operators

$\mathcal{N} = 4$ SYM theory
flavour $SU(2)$ subsector, perturbatively closed
chiral operators built only from X, Y: $\quad \mathcal{O} = \mathrm{tr}(X \ldots XY \ldots YX \ldots X \ldots)$

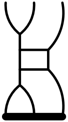
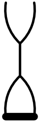
- action of the building block $-\frac{1}{g^2} W\overline{W}$:



$\Rightarrow$ ferromagnetic ground state ($\gamma = 0$): $\mathcal{O} = \mathrm{tr}(X \ldots X)$

- avoid two-loop mixing with $W_\alpha W_\beta$: 

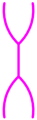
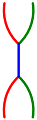
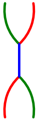
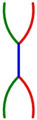
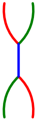
  flavour Y not included
  $\Rightarrow$ identity:

$\chi(1,2,1) =$  $=$  $=$ $\chi(1)$

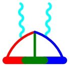
Action on chiral operators

β-deformed $\mathcal{N} = 4$ SYM theory
two-flavour subsector, perturbatively closed
chiral operators built only from X, Y: $\quad \mathcal{O} = \mathrm{tr}(X \dots X Y \dots Y X \dots X \dots)$

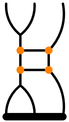
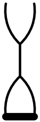
- action of the building block $-\frac{1}{g^2} W \overline{W}$:

 $\rightarrow -$  $+ q$  $- |q|^2$  $+ \bar{q}$ 

$\Rightarrow$ ferromagnetic ground state ($\gamma = 0$): $\mathcal{O} = \mathrm{tr}(X \dots X)$

- avoid two-loop mixing with $W_\alpha W_\beta$: 

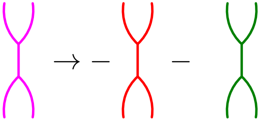
  flavour Y not included
  $\Rightarrow$ identity:

$\chi(1,2,1) =$  $= \frac{\kappa^4 |q|^2}{g^4}$  $= \frac{\kappa^4 |q|^2}{g^4} \chi(1)$

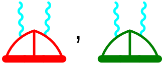
Action on chiral operators

cubic Leigh-Strassler deformation
no perturbatively closed subsector of
chiral operators built only from X, Y: $\quad \mathcal{O} = \mathrm{tr}(X \ldots X Y \ldots Y X \ldots X \ldots)$

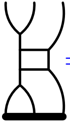
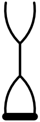
- action of the building block $-\frac{1}{g^2} W \overline{W}$:



$\Rightarrow$ anti-ferromagnetic ground state ($\gamma = 0$): $\mathcal{O} = \mathrm{tr}(XY \ldots XY)$

- avoid two-loop mixing with $W_\alpha W_\beta$: 

max. 2 neighbours have identical flavours: $\mathcal{O}$ is ‘three-string null’
$\Rightarrow$ no indentity, but simplifications:

$\chi(1,2,1) =$  $= 0 \neq$  $= \quad \chi(1) = -\frac{\kappa^2 |h|^2}{g^2} = -2 + \ldots$

Results to three loops

[C.S.]

dilatation operator $\mathcal{D} = \sum_k G^{2k} \mathcal{D}_k$ in $\mathcal{N} = 4$ SYM

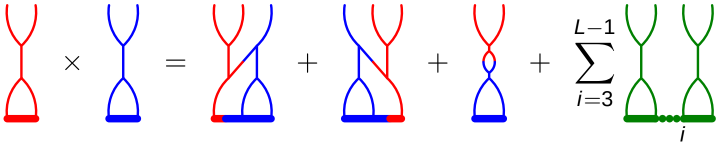
$$
\begin{aligned}
\mathcal{D}_1 =& -2\chi(1) \\
\mathcal{D}_2 =& -2[\chi(1,2)+\chi(2,1)]+4\chi(1) \\
\mathcal{D}_3 =& -4[\chi(1,2,3)+\chi(3,2,1)]+2[\chi(2,1,3)-\chi(1,3,2)]-4\chi(1,3) \\
& +16[\chi(1,2)+\chi(2,1)]-4[\chi(1,2,1)+\chi(2,1,2)]-16\chi(1)
\end{aligned}
$$

Results to three loops

[C.S.]

dilatation operator $\mathcal{D} = \sum_k G^{2k} \mathcal{D}_k$ in $\mathcal{N} = 4$ SYM

$$
\begin{aligned}
\mathcal{D}_1 &= -2\chi(1) \\
\mathcal{D}_2 &= -2[\chi(1,2) + \chi(2,1)] + 4\chi(1) \\
\mathcal{D}_3 &= -4[\chi(1,2,3) + \chi(3,2,1)] + 2[\chi(2,1,3) - \chi(1,3,2)] - 4\chi(1,3) \\
&\quad + 16[\chi(1,2) + \chi(2,1)] - 4[\chi(1,2,1) + \chi(2,1,2)] - 16\chi(1)
\end{aligned}
$$



$$
\chi(1) \times \chi(1) = \underbrace{\chi(1,2) + \chi(2,1) - 2\chi(1)}_{[\chi(1)\times\chi(1)]_{\mathrm{c}}} + \sum_{i=3}^{L-1} \chi(1,i)
$$

disconnected

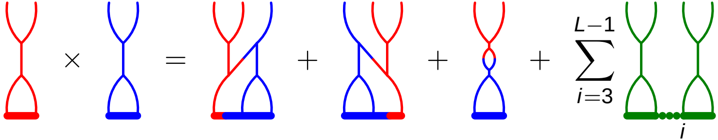
Results to three loops

dilatation operator $\mathcal{D} = \sum_k G^{2k} \mathcal{D}_k$ in $\mathcal{N} = 4$ SYM

$$\mathcal{D}_1 = -2\chi(1)$$

$$\mathcal{D}_2 = -2[\chi(1) \times \chi(1)]_\mathrm{c}$$

$$\mathcal{D}_3 = -4[[\chi(1) \times \chi(1)]_\mathrm{c} \times \chi(1)]_\mathrm{c} + 2[\chi(2,1,3) - \chi(1,3,2)] - 4\chi(1,3)$$



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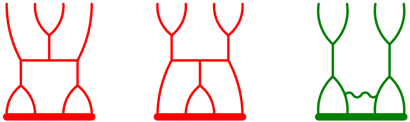

$\rightarrow$ 0 by similarity trafo

action on single magnon momentum eigenstate

$$\sum_n e^{inp} \underbrace{X \dots X}_{n-1} Y X \dots X$$

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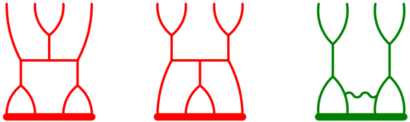
$$\chi(1) \to -4\sin^2 \tfrac{p}{2}$$



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action on single magnon momentum eigenstate

$$\sum_n e^{inp} \underbrace{X \dots X}_{n-1} Y X \dots X$$

$$E(p) = \sqrt{1 + 16 G^2 \sin^2 \tfrac{p}{2}} - 1 \qquad\qquad 0$$

magnon dipersion relation

only contributes to S-matrix

of the Bethe ansatz

[Beisert, Dippel, Staudacher]

Results to three loops

dilatation operator $\mathcal{D} = \sum_k G^{2k} \mathcal{D}_k$ in β-deformed $\mathcal{N} = 4$ SYM

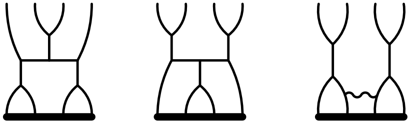
$$\mathcal{D}_1 = -2\chi(1)$$

$$\mathcal{D}_2 = -2[\chi(1) \times \chi(1)]_c$$

$$\mathcal{D}_3 = -4[[\chi(1) \times \chi(1)]_c \times \chi(1)]_c + 2[\chi(2,1,3) - \chi(1,3,2)] - 4\chi(1,3)$$

$$\chi(1) \to -4\sin^2 \pi\beta$$

$\beta \in \mathbb{R}$



$\to$ 0 by similarity trafo

action on single magnon excited state

$$\mathrm{tr}(Y X \dots X)$$

$$E(2\pi\beta) = \sqrt{1 + 16G^2 \sin^2 \pi\beta} - 1 + \mathcal{O}(G^8) \qquad 0$$

anomalous dimension

[Berenstein, Cherkis]

determine four-loop corr.

($\beta \notin \mathbb{R}$)-def.

Results to three loops

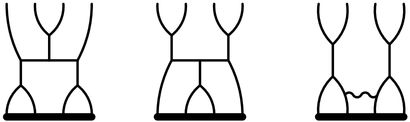
dilatation operator $\mathcal{D} = \sum_k G^{2k}\mathcal{D}_k$ in cubic Leigh-Strassler def.

$$\mathcal{D}_1 = -2\chi(1)$$

$$\mathcal{D}_2 = -2[\chi(1)\times\chi(1)]_{\mathrm{c}}$$

$$\mathcal{D}_3 = -4[[\chi(1)\times\chi(1)]_{\mathrm{c}}\times\chi(1)]_{\mathrm{c}} + 2[\chi(2,1,3)-\chi(1,3,2)] - 4\chi(1,3)$$

$\chi(1) \to -2$



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$$\mathrm{tr}(YYX\ldots YX)$$

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anomalous dimension
[Minahan]

determine four-loop corr.
cubic LS def.

Leigh-Strassler deformation of $\mathcal{N} = 4$ SYM

$q = 1$, $h = 0$ $\longrightarrow$ $\mathcal{N} = 4$ SYM

$q = e^{2\pi i \beta}$, $h = 0$ $\longrightarrow$ $|q| = 1 \longleftarrow \beta$-def. $\longrightarrow |q| \neq 1$

$\kappa \to 0$, $h \sim \frac{1}{\kappa}$ $\longrightarrow$ cubic LS-def.

integrability

no integrability
but: exceptional points
[Bundzik, Mansson], [Mansson], [Mansson, Zoubos]

$\boxed{\text{Leigh-Strassler deformation of } \mathcal{N} = 4 \text{ SYM}}$

$q = 1$, $h = 0$ $\quad$ $q = e^{2\pi i\beta}$, $h = 0$ $\quad$ $\kappa \to 0$, $h \sim \frac{1}{\kappa}$

$\mathcal{N} = 4$ SYM $\quad$ $|q| = 1 \longleftarrow \beta$-def. $\longrightarrow |q| \neq 1$ $\quad$ cubic LS-def.

conf. inv. $\Leftrightarrow$ finiteness

conf. inv. $\not\Leftrightarrow$ finiteness

$\mathcal{O}(\kappa^8)$ corr. in $g(\kappa, q, h)$

[Bork, Kazakov, Vartanov, Zhiboedov]
[Elmetti, Mauri, Penati, Santambrogio, Zanon]

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$(\mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3)|_{SU(2)} \xleftarrow{\text{test}} (\mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3) \xrightarrow{\text{prediction}} (\gamma_1, \gamma_2, \gamma_3)$

[Beisert, Kristjansen, Staudacher] $\quad$ [C.S.] $\quad$ [Minahan]

integrability $\quad$ Feynman diagram calc.

Leigh-Strassler deformation of $\mathcal{N}=4$ SYM

$q=1,\ h=0$ $\qquad$ $q=e^{2\pi i\beta},\ h=0$ $\qquad$ $\kappa\to 0,\ h\sim\frac{1}{\kappa}$

$\mathcal{N}=4$ SYM $\qquad$ $|q|=1 \longleftarrow \beta\text{-def.} \longrightarrow |q|\neq 1$ $\qquad$ cubic LS-def.

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integrability $\qquad$ Feynman diagram calc.

[Beisert] $\qquad$ [Minahan, C.S.]

$\mathcal{D}_4|_{SU(2)} \xrightarrow{\text{generalization}} \mathcal{D}_4 \xrightarrow{\text{prediction}} \gamma_4$

$\mathcal{D}_4|_{SU(2)}$ contains $A\,\chi(\dots,a,\dots)$; $\mathcal{D}_4$ contains $(A-B)\,\chi(\dots,a,\dots) + B\,\chi(\dots,a,b,a,\dots)$

$A\,\chi(\dots,a,\dots) \xleftarrow{\text{projection}} (A-B)\,\chi(\dots,a,\dots) + B\,\chi(\dots,a,b,a,\dots)$

Leigh-Strassler deformation of $\mathcal{N}=4$ SYM

$q=1$, $h=0$ $\qquad$ $q=e^{2\pi i\beta}$, $h=0$ $\qquad$ $\kappa\to 0$, $h\sim\frac{1}{\kappa}$

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integrability $\qquad$ Feynman diagram calc.

[Minahan, C.S.]

$\mathcal{D}_4$

act. on single magnon comp. op.

$G=\frac{\sqrt{g^2N}}{4\pi}$ $\qquad$ $K=\frac{\sqrt{\kappa^2N}}{4\pi}$

$$\gamma_\beta=\sqrt{1+4K^2|1-q|^2}-1-32\zeta(3)G^4K^4(1-|q|^2)^2|1-q|^4+\dots$$

$$\gamma_{\text{c.LS}}=\sqrt{1+8\tilde{h}^2(G)}-1 \qquad \tilde{h}^2(G)=G^2-8\zeta(3)G^8+\dots$$

Four-loop result

$$
\begin{aligned}
\mathcal{D}_4|_{SU(2)} = & + 200 \quad \chi(1) \\
& - 150 \quad [\chi(1,2) + \chi(2,1)] \\
& + 8(10 + \epsilon_{3a}) \quad \chi(1,3) \\
& + 60[\chi(1,2,3) + \chi(3,2,1)] \\
& + 2(4 + \beta + 2\epsilon_{3a} - 2i\epsilon_{3b} + i\epsilon_{3c} - 2i\epsilon_{3d})\, \chi(1,3,2) \\
& + 2(4 + \beta + 2\epsilon_{3a} + 2i\epsilon_{3b} - i\epsilon_{3c} + 2i\epsilon_{3d})\, \chi(2,1,3) \\
& - 2(6 + \beta + 2\epsilon_{3a})\, \chi(2,1,3,2) \\
& - 4\, \chi(1,4) \\
& - 2(2 + 2i\epsilon_{3b} + i\epsilon_{3c})[\chi(1,2,4) + \chi(1,4,3)] \\
& - 2(2 - 2i\epsilon_{3b} - i\epsilon_{3c})[\chi(1,3,4) + \chi(2,1,4)] \\
& + 2(9 + 2\epsilon_{3a})[\chi(1,3,2,4) + \chi(2,1,4,3)] \\
& - 2(4 + \epsilon_{3a} + i\epsilon_{3b})[\chi(1,2,4,3) + \chi(1,4,3,2)] \\
& - 2(4 + \epsilon_{3a} - i\epsilon_{3b})[\chi(2,1,3,4) + \chi(3,2,1,4)] \\
& - 10[\chi(1,2,3,4) + \chi(4,3,2,1)]
\end{aligned}
$$

Four-loop result

[Minahan, C.S.]

$$\mathcal{D}_4 \quad \text{applicable to} \quad \mathcal{O}(X, Y, Z)$$

$$\begin{aligned}
\mathcal{D}_4 = &+ 16(5+\zeta(3))\chi(1) + 4(15-2\zeta(3))[\chi(1,2,1)+\chi(2,1,2)] \\
&- 8(15+\zeta(3))[\chi(1,2)+\chi(2,1)] - 10[\chi(1,2,1,2)+\chi(2,1,2,1)] \\
&- (10 + i\epsilon_{3e} - i\epsilon_{3f})[\chi(2,1,2,3)+\chi(2,3,2,1)] \\
&- (10 - 8\zeta(3) - i\epsilon_{3e} + i\epsilon_{3f})[\chi(1,2,3,2)+\chi(3,2,1,2)] \\
&+ 8\left(\tfrac{23}{3} - \zeta(3)\right)\chi(1,3) \\
&+ \left(\tfrac{14}{3} + 8\zeta(3) + 2\epsilon_{3a} - 4i\epsilon_{3b} + i\epsilon_{3e}\right)[\chi(1,3,2,3)+\chi(3,1,2,1)] \\
&+ \left(\tfrac{14}{3} - 4\zeta(3) + 2\epsilon_{3a} + 4i\epsilon_{3b} - i\epsilon_{3e}\right)[\chi(1,2,1,3)+\chi(3,2,3,1)] \\
&+ 60[\chi(1,2,3)+\chi(3,2,1)] \\
&+ 2(4 + \beta + 2\epsilon_{3a} - 2i\epsilon_{3b} + i\epsilon_{3c} - 2i\epsilon_{3d})\,\chi(1,3,2) \\
&+ 2(4 + \beta + 2\epsilon_{3a} + 2i\epsilon_{3b} - i\epsilon_{3c} + 2i\epsilon_{3d})\,\chi(2,1,3) \\
&- 2(6 + \beta + 2\epsilon_{3a})\,\chi(2,1,3,2) \\
&- 4\,\chi(1,4) \\
&- 2(2 + 2i\epsilon_{3b} + i\epsilon_{3c})[\chi(1,2,4)+\chi(1,4,3)] \\
&- 2(2 - 2i\epsilon_{3b} - i\epsilon_{3c})[\chi(1,3,4)+\chi(2,1,4)] \\
&+ 2(9 + 2\epsilon_{3a})[\chi(1,3,2,4)+\chi(2,1,4,3)] \\
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&- 10[\chi(1,2,3,4)+\chi(4,3,2,1)]
\end{aligned}$$

Conclusions

Leigh-Strassler deformation encompasses (non) integrable cases:
integrability $+$ Feynman diagrams $\rightarrow \gamma$ of non-integrable deformations

possible in efficient language:
dilatation operator $\mathcal{D}$, $\mathcal{N} = 1$ superfields, chiral functions
$\Rightarrow$ direct Bethe ansatz interpretation:

$\mathcal{D} =$ dispersion terms $+$ scattering term at ≥ 3 loops

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...and to conclude:

for the organizers
Charlotte, Niels, Donovan, Costas
and hosts
Niels Bohr Institute, Niels Bohr International Academy

a big THANK YOU!

$$\boxed{\text{AdS/CFT correspondence}}$$

IIB in $\text{AdS}_5 \times \text{S}^5$

$\mathcal{N} = 4$ SYM theory

planar limit

energy E
(semicl.) strings

$\longleftarrow \lambda \gg 1 \longrightarrow \lambda \ll 1 \longrightarrow$

anom. dim. γ
comp. operators
$\mathcal{O}_L \propto \text{tr}\, \phi_{i_1} \ldots \phi_{i_L}$

$$E(\lambda) = \gamma(\lambda)$$

$\boxed{\text{AdS/CFT correspondence}}$

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integrable spin chain
(asymptot.) Bethe ansätze

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string $\qquad$ gauge

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Feynman graph calculations
in the flavour $SU(2)$ subsector:

1 loop: [Berenstein, Maldacena, Nastase]
2 loops: [Gross, Mikhailov, Roiban]
3 loop γ: [Kotikov, Lipatov, Onishchenko, Velizhanin] [Eden, Jarczak, Sokatchev]
3 loop $\mathcal{D}$: [C.S.]

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[Hofmann, Maldacena]
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transcendental terms

Feynman graph calculations
of $\mathcal{D}$ in chiral subsectors:

interpolating quiver, 3 loops: [Pomoni,C.S.]
Leigh-Strassler def., 4 loops: [Minahan,C.S.]
$\mathcal{N} = 6$ CS, 4 loops: [Minahan,Ohlsson Sax,C.S.]
[Leoni,Mauri,Minahan,Santambrogio,Ohlsson Sax,C.S.,Tartaglino-Mazzucchelli]