De Sitter vacua in type IIB string theory/ F-theory by Kähler uplifting

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1. Introduction: De Sitter vacua in Type IIB/ F-Theory
2. A sufficient condition for de Sitter vacua
3. Towards an explicit example: $\mathbb{CP}_{1,1,1,6,9}$
4. Conclusions & Outlook
1. Introduction: De Sitter vacua in Type IIB/ F-Theory
Introduction: De Sitter vacua in String Theory

Cosmology:
- Acceleration of the universe on large scales is observed.
- Simplest explanation: Small cosmological constant.

String Theory:
- Consistent quantum gravity and unification candidate.
- Can contain constructions leading to the MSSM.

⇒ Can one construct (metastable) 4D de Sitter vacua with small cosmological constant?

- Compactification to 4D yield moduli that have to be stabilized to obtain a vacuum.
- Focus on geometric (Kähler, complex structure) moduli and dilaton.
Introduction: IIB/ F-theory Compactifications

IIB on orientifolded Calabi-Yau 3-fold $X$:

- Spectrum: 3-form field strength $G_3 = F_3 - S H_3$, ...
- Moduli: $h^{1,1}(X)$ Kähler, $h^{2,1}(X)$ complex structure and dilaton.

F-theory on elliptically fibred Calabi-Yau 4-fold $Z$:

- Spectrum: 4-form field strength $G_4$, ...
- Moduli: $h^{1,1}(Z) - 1$ Kähler, $2h^{3,1}(Z)$ complex structure.

The theories are equivalent in the Sen limit $g_s \to 0$. [Sen '96]

The $D = 4$, $\mathcal{N} = 1$ effective supergravity Lagrangian of the compactification is

$$\mathcal{L} = K_{a\bar{b}} D_\mu \phi^a D^\mu \phi^{\bar{b}} - V + ... ,$$

$$V = e^K \left( K^{a\bar{b}} D_a W \overline{D_b W} - 3|W|^2 \right)$$

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Introduction: Kähler moduli stabilization

Kähler moduli $T_i$ are stabilized by the **interplay of non-perturbative effects**: [Kachru, Kallosh, Linde, Trivedi ’03]

- D3-instantons.
- Gaugino condensation of stacks of $N$ D7-branes wrapping a rigid Divisor.

$$W_{\text{n.p.}} = \sum_i A_i e^{-a_i T_i}, \quad a_i = \frac{2\pi}{N_i}$$

In F-theory, $a_i$ is determined by the ADE singularity at the degeneration point of the fibred torus.

**and $\alpha'$ corrections**: [Becker, Becker, Haack, Louis ’02]

$$K = -2 \ln \left( \hat{\mathcal{V}}(T_i) + \alpha'^3 \hat{\xi}(S) \right), \quad \hat{\xi}(S) \propto \left( -\chi \right) \cdot (S + \bar{S})^{3/2}$$

$$= 2(h^{2,1} - h^{1,1})$$

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6/18
Introduction: Complex structure moduli stabilization

- Complex structure moduli $z^a$ are stabilized by fluxes:
  \[ \frac{1}{2\pi\alpha'} \int F_3 = 2\pi f \in 2\pi\mathbb{Z}, \quad \frac{1}{2\pi\alpha'} \int H_3 = 2\pi h \in 2\pi\mathbb{Z}. \]

- For a symplectic basis $\{A^a, B^b\}$ for the $2h^{2,1} + 2$ three cycles the period vector $\Pi$ is defined as the integral over the holomorphic 3-form $\Omega$:
  \[ \Pi = \left( z^a, \frac{\partial G}{\partial z^b} \right) = \left( \int_{A^a} \Omega, \int_{B^b} \Omega \right), \quad \text{with prepotential } G. \]

- The Kähler and superpotential are then:
  \[ K = -\log \left( i \int_X \Omega \wedge \bar{\Omega} \right) = -\log \left( -i \Pi^\dagger \Sigma \Pi \right), \quad \text{with sympl. matrix } \Sigma. \]
  \[ W_0 = \frac{1}{2\pi} \int_X G_3 \wedge \Omega = 2\pi\alpha' (f - S h) \cdot \Pi. \quad \text{[Gukov, Vafa, Witten '00]} \]
2. A sufficient condition for de Sitter vacua
Branches of de Sitter vacua

Manifold and flux choices $h^{1,1}, h^{2,1}, W_0, a_i, A_i$ determine what scenarios of moduli stabilization can be realized:

**KKLT scenarios:** [Giddings, Kachru, Polchinski '01][Kachru, Kallosh, Linde, Trivedi '03]

- Moduli stabilized supersymmetrically: $D_i W = 0 \Rightarrow$ AdS vacuum.
- Uplifting via $\overline{D3}$-brane $\Rightarrow$ explicit SUSY breaking.

**LARGE Volume scenarios:** [Balasubramanian, Berglund, Conlon, Quevedo’05]

- Volume can be tuned arbitrarily large $\Rightarrow$ good decoupling limit.
- Uplifting to de Sitter via D-terms $\Rightarrow$ strongly model dependent.

**Kähler uplifting scenarios:** [Balasubramanian, Berglund ’05], [MR, Westphal ’11]

- SUSY only broken spontaneously by F-term potential.
- No extra uplifting sector required $\Rightarrow$ de Sitter vacua directly in a model independent way.
A sufficient condition for de Sitter for Kähler uplifting

If a 3-fold is ‘swiss-cheese’, i.e. \( \hat{V}(T_i) = \gamma_1 \text{Re}(T_1)^{3/2} - \sum_{i=2}^{h^{1,1}} \gamma_i \text{Re}(T_i)^{3/2} \)

one can perform a complete perturbative moduli stabilization in the limit

\[ \hat{V} \gg \hat{\xi} \Rightarrow \text{Large Volume } \hat{V}. \]

\[ |W_0| \gg A e^{-at} \Rightarrow \text{Non-perturbative effects are small.} \]

\[ D_i W(S, z^a) \approx 0 \Rightarrow \text{Supersymmetric stabilization to 0-th order.} \]

**Sufficient for a de Sitter vacuum:** [MR, Westphal ’11]

\[ 3.65 < \frac{27|W_0|\hat{\xi}a^{3/2}}{64\sqrt{2}\gamma A} < 3.89. \]

\[ V_{z^a z^b}^{(c.s.)} > 0 \text{ to 0-th order.} \]

\[ \text{Need large gauge group } SU(N), \text{ typically } N \sim 30 - 100. \]
3. Towards an explicit example: $\mathbb{CP}_{1,1,1,6,9}$
Moduli space of $\mathbb{CP}_{1,1,1,6,9}$ (I)

Consider Calabi-Yau 3-fold defined as degree 18 hypersurface in $\mathbb{CP}_{1,1,1,6,9}$: $(x_1, x_2, x_3, x_4, x_5) \sim (\lambda x_1, \lambda x_2, \lambda x_3, \lambda^6 x_4, \lambda^9 x_5)$, for example

$$x_1^{18} + x_2^{18} + x_3^{18} + x_4^3 + x_5^2 = 0$$

This 3-fold has $h^{1,1} = 2$ and $h^{2,1} = 272$.

Kähler moduli: [Denef, Douglas, Florea '04]

- Intersections: $\hat{V} = \sqrt{2}/18 \left( \text{Re}(T_1)^{3/2} - \text{Re}(T_2)^{3/2} \right) \Rightarrow \text{‘swiss cheese’}.$
- Non-perturbative effects: $W_{\text{n.p.}} = A_1 e^{-a_1 T_2} + A_2 e^{-a_2 T_2}$.
  - $a_1 = 2\pi/30 \Rightarrow E_8$ singularity in the F-theory description.
  - $a_2 = 2\pi \Rightarrow$ D3-instanton.
  - $A_1, A_2 \simeq O(1)$.
Moduli space of $\mathbb{CP}_{1,1,1,6,9}$ (II)

Complex structure moduli:

- The full $h^{2,1} = 272$ parameter prepotential $G(z)$ is not known. But, the moduli space allows a $\Gamma = \mathbb{Z}_6 \times \mathbb{Z}_{18}$ action, which fixes a 2 parameter subspace. [Greene, Plesser '89]

- Turn on flux only on the 2 invariant cycles $\Rightarrow D_i W = 0$ for the 270 non-invariant cycles $\Rightarrow$ Effectively all 272 complex structure moduli stabilized.

- Prepotential can be obtained via mirror symmetry in the large complex structure limit: [Candelas, Font, Katz, Morrison '94]

\[
G(z_1, z_2) = \frac{17}{4} z_1 + \frac{3}{2} z_2 + \frac{1}{2} \left( \frac{9}{2} z_1^2 + 3z_1 z_2 \right) + \frac{1}{6} \left( -9z_1^3 - 9z_1^2 z_2 - 3z_1 z_2^2 \right) \\
+ \xi + G_{\text{instanton}}(e^{-2\pi z_1}, e^{-2\pi z_2}) \quad , \quad \xi = \frac{\zeta(3)(h^{1,1} - h^{2,1})}{(2\pi i)^3} \simeq -1.30843i
\]
Finding flux vacua

- The 3-fold fixes all free parameters except of the VEV’s $\langle T_1 \rangle, \langle T_2 \rangle, \langle S \rangle, \langle z_1 \rangle$ and $\langle z_2 \rangle$ and flux vectors $f = \{f_i\}, h = \{h_i\}, i = 1, \ldots, 6$.

- D3-Tadpole constraint:

$$L = \int_Z G_4 \wedge G_4 = \int_X F_3 \wedge H_3 = L_{\text{max}} - N_{D3}, \quad L_{\text{max}} = \frac{\chi(Z)}{24}.$$  

**Strategy:**

- Fix $\langle S \rangle, \langle z_1 \rangle$ and $\langle z_2 \rangle$ to rational value.

- Neglect $G_{\text{instanton}}$ and set $\xi$ to a rational value, such that:

$$0 = \{W_0, D_S W_0, D_{z_1} W_0, D_{z_2} W_0\} = A \cdot \{f_1, \ldots, f_6, h_1, \ldots, h_6\}, \text{ with } A \in \mathbb{Q}^{8 \times 12}.$$  

- Find basis of solutions $\{f_i\}, \{h_i\}$ with minimal tadpole $L$.

- Generate shift in $W_0, \langle S \rangle, \langle z_1 \rangle$ and $\langle z_2 \rangle$ for $\xi \in i\mathbb{R}, G_{\text{instanton}} \neq 0$. 

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Solutions and Kähler moduli stabilization

De Sitter vacuum can be constructed if:

- $L < L_{\text{max}}$,
- $\{W_0, g_s = \langle \text{Re} S \rangle^{-1}, A \}$ fullfills:
  
  $1.25 < |W_0| A g_s^{-3/2} < 1.34,$

- $V^{(c.s.)}_{z^a z^b} > 0.$

Explicit example:

- $\{f, h\} = \{0, -16, 56, -28, -12; 4, 0, 0, 0, -9, 10\}, \quad L = 408$

<table>
<thead>
<tr>
<th>$g_s$</th>
<th>$A$</th>
<th>$\langle T_1 \rangle$</th>
<th>$\langle T_2 \rangle$</th>
<th>$\langle z_1 \rangle$</th>
<th>$\langle z_2 \rangle$</th>
<th>$W_0$</th>
<th>$\hat{V}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>1.5</td>
<td>14.3</td>
<td>0.8</td>
<td>0.98</td>
<td>0.99</td>
<td>$-1.06$</td>
<td>4.2</td>
</tr>
</tbody>
</table>

TODO: Directly stabilize $V(T_1, T_2, S, z_1, z_2)$ numerically.
4. Conclusions & Outlook
Conclusions

- Sufficient condition for de Sitter with all moduli stabilized for all Calabi-Yau 3-folds of ‘swiss cheese’ type.

- Systematical understanding of de Sitter condition based on properties of the Calabi-Yau $\gamma$, $\xi$ and fluxes $W_0$ (F-Theory data).

- Well controlled spontaneous SUSY breaking by F-Terms only.

- Explicit flux vaccum has been constructed which can be used for Kähler uplifting.

- Small cosmological constant by tuning background fluxes.

[Bousso, Polchinski ’00]
Outlook

Statistics of $\mathbb{CP}_{1,1,1,6,9}$:

- How many flux vacua allow Kähler uplifting?
- Scan over $g_s, \langle z_1 \rangle, \langle z_2 \rangle$ and $\xi_{\text{rational}}$.

Kähler uplifting: $\hat{V} \simeq \gamma (6/a)^{3/2} \simeq \gamma N^{3/2} \Rightarrow \mathbb{CP}_{1,1,1,6,9}: \hat{V} \sim 4$.

⇒ Engineer 4-fold more suitable for Kähler uplifting:

- How much $N$ can be realized in F-theory on a compact 4-fold?
- How much survives in the Sen limit?
- Rigidity of divisors? [Witten '96]