De Sitter vacua in type IIB string theory/ F-theory by Kähler uplifting

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Outline:

1. Introduction: De Sitter vacua in Type IIB/ F-Theory
2. A sufficient condition for de Sitter vacua
3. Towards an explicit example: $\mathbb{CP}_{1,1,1,6,9}$
4. Conclusions & Outlook

1. Introduction: De Sitter vacua in Type IIB/ F-Theory

Introduction: De Sitter vacua in String Theory

Cosmology:

- Acceleration of the universe on large scales is observed.
- Simplest explanation: Small cosmological constant.

String Theory:

- Consistent quantum gravity and unification candidate.
- Can contain constructions leading to the MSSM.

$\Rightarrow$ **Can one construct (metastable) 4D de Sitter vacua with small cosmological constant?**

- Compactification to 4D yield moduli that have to be stabilized to obtain a vacuum.
- Focus on geometric (Kähler, complex structure) moduli and dilaton.

Introduction: IIB/ F-theory Compactifications

IIB on orientifolded Calabi-Yau 3-fold X:

- Spectrum: 3-form field strength $G_3 = F_3 - S\, H_3$, ...
- Moduli: $h^{1,1}(X)$ Kähler , $h^{2,1}(X)$ complex structure and dilaton.

F-theory on elliptically fibred Calabi-Yau 4-fold Z:

- Spectrum: 4-form field strength G_4, ...
- Moduli: $h^{1,1}(Z) - 1$ Kähler, $2h^{3,1}(Z)$ complex structure.

The theories are equivalent in the Sen limit $g_s \to 0$. [Sen '96]

The $D = 4$, $\mathcal{N} = 1$ effective supergravity Lagrangian of the compactification is

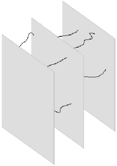
$$\mathcal{L} = K_{a\bar{b}} D_\mu \phi^a D^\mu \phi^{\bar{b}} - V + \ldots, \qquad \boxed{V = e^K \left(K^{a\bar{b}} D_a W \overline{D_b W} - 3|W|^2 \right)}$$

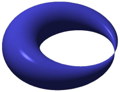
Introduction: Kähler moduli stabilization

Kähler moduli T_i are stabilized by the **interplay of non-perturbative effects:** [Kachru, Kallosh, Linde, Trivedi '03]

- D3-instantons.
- Gaugino condensation of stacks of N D7-branes wrapping a rigid Divisor.

$$W_{\text{n.p.}} = \sum_i A_i\, e^{-a_i T_i}, \quad a_i = \frac{2\pi}{N_i}$$





In F-theory, a_i is determined by the ADE singularity at the degeneration point of the fibred torus.

and α' corrections: [Becker, Becker, Haack, Louis '02]

$$K = -2\ln\left(\hat{\mathcal{V}}(T_i) + \alpha'^3 \hat{\xi}(S)\right), \quad \hat{\xi}(S) \propto \underbrace{(-\chi)}_{=2(h^{2,1}-h^{1,1})} \cdot (S+\bar{S})^{3/2}$$

Introduction: Complex structure moduli stabilization

- Complex structure moduli z^a are stabilized by **fluxes**:

$$\frac{1}{2\pi\alpha'}\int F_3 = 2\pi f \in 2\pi\mathbb{Z}, \qquad \frac{1}{2\pi\alpha'}\int H_3 = 2\pi h \in 2\pi\mathbb{Z}.$$

- For a symplectic basis $\{A^a, B_b\}$ for the $2h^{2,1}+2$ three cycles the period vector Π is defined as the integral over the holomorphic 3-form Ω:

$$\Pi = \left(z^a, \frac{\partial G}{\partial z^b}\right) = \left(\int_{A^a}\Omega\,,\int_{B_b}\Omega\right), \qquad \textbf{with prepotential G}.$$

- The Kähler and superpotential are then:

$$K = -\log\left(i\int_X \Omega\wedge\bar{\Omega}\right) = -\log\left(-i\,\Pi^\dagger\Sigma\Pi\right), \text{ with sympl. matrix } \Sigma.$$

$$W_0 = \frac{1}{2\pi}\int_X G_3\wedge\Omega = 2\pi\alpha'(f - S\,h)\cdot\Pi.$$ [Gukov, Vafa, Witten '00]

2. A sufficient condition for de Sitter vacua

Branches of de Sitter vacua

Manifold and flux choices $h^{1,1}, h^{2,1}, W_0, a_i, A_i$ determine what scenarios of moduli stabilization can be realized:

KKLT scenarios:[Giddings, Kachru, Polchinski '01][Kachru, Kallosh, Linde, Trivedi '03]

- Moduli stabilized supersymmetrically: $D_i W = 0 \Rightarrow$ AdS vacuum.
- Uplifting via $\overline{D3}$-brane $\Rightarrow$ explicit SUSY breaking.

LARGE Volume scenarios: [Balasubramanian, Berglund, Conlon, Quevedo'05]

- Volume can be tuned arbitrarily large $\Rightarrow$ good decoupling limit.
- Uplifting to de Sitter via D-terms $\Rightarrow$ strongly model dependent.

Kähler uplifting scenarios: [Balasubramanian, Berglund '05], [MR, Westphal '11]

- SUSY only broken spontaneously by F-term potential.
- No extra uplifting sector required $\Rightarrow$ de Sitter vacua directly in a model independent way.

A sufficient condition for de Sitter for Kähler uplifting

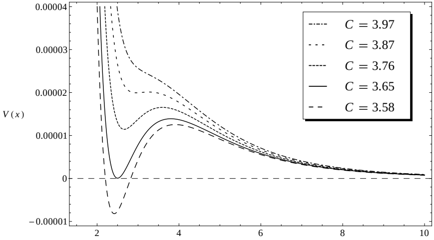
If a 3-fold is 'swiss-cheese', i.e. $\hat{\mathcal{V}}(T_i) = \gamma_1 \mathrm{Re}(T_1)^{3/2} - \sum_{i=2}^{h^{1,1}} \gamma_i \mathrm{Re}(T_i)^{3/2}$

one can perform a complete perturbative moduli stabilization in the limit

- $\hat{\mathcal{V}} \gg \hat{\xi}$ $\Rightarrow$ Large Volume $\hat{\mathcal{V}}$.
- $|W_0| \gg Ae^{-at}$ $\Rightarrow$ Non-perturbative effects are small.
- $D_i W(S, z^a) \simeq 0$ $\Rightarrow$ Supersymmetric stabilization to 0-th order.

Sufficient for a de Sitter vacuum: [MR, Westphal '11]

- $3.65 < \dfrac{27|W_0|\hat{\xi} a^{3/2}}{64\sqrt{2}\gamma A} < 3.89.$
- $V^{(c.s.)}_{z^a z^b} > 0$ to 0-th order.
- Need large gauge group $SU(N)$, typically $N \sim 30 - 100$.



3. Towards an explicit example: $\mathbb{CP}_{1,1,1,6,9}$

Moduli space of $\mathbb{CP}_{1,1,1,6,9}$ (I)

Consider Calabi-Yau 3-fold defined as degree 18 hypersurface in $\mathbb{CP}_{1,1,1,6,9}$: $(x_1, x_2, x_3, x_4, x_5) \sim (\lambda x_1, \lambda x_2, \lambda x_3, \lambda^6 x_4, \lambda^9 x_5)$, for example

$$x_1^{18} + x_2^{18} + x_3^{18} + x_4^3 + x_5^2 = 0$$

This 3-fold has $h^{1,1} = 2$ and $h^{2,1} = 272$.

Kähler moduli: [Denef, Douglas, Florea '04]

- Intersections: $\hat{\mathcal{V}} = \frac{\sqrt{2}}{18}\left(\mathrm{Re}(T_1)^{3/2} - \mathrm{Re}(T_2)^{3/2}\right) \Rightarrow$ 'swiss cheese'.
- Non-perturbative effects: $W_{\text{n.p.}} = A_1\, e^{-a_1 T_2} + A_2\, e^{-a_2 T_2}$.
  - $a_1 = 2\pi/30 \Rightarrow E_8$ singularity in the F-theory description.
  - $a_2 = 2\pi \Rightarrow$ D3-instanton.
  - $A_1, A_2 \simeq \mathcal{O}(1)$.

Moduli space of $\mathbb{CP}_{1,1,1,6,9}$ (II)

Complex structure moduli:

- The full $h^{2,1} = 272$ parameter prepotential $G(z)$ is not known. But, the moduli space allows a $\Gamma = \mathbb{Z}_6 \times \mathbb{Z}_{18}$ action, which fixes a 2 parameter subspace. [Greene, Plesser '89]
- Turn on flux only on the 2 invariant cylces $\Rightarrow$ $D_i W = 0$ for the 270 non-invariant cycles $\Rightarrow$ Effectively all 272 complex structure moduli stabilized.
- Prepotential can be obtained via mirror symmetry in the large complex structure limit: [Candelas, Font, Katz, Morrison '94]

$$G(z_1, z_2) = \frac{17}{4} z_1 + \frac{3}{2} z_2 + \frac{1}{2}\left(\frac{9}{2} z_1^2 + 3 z_1 z_2\right) + \frac{1}{6}\left(-9 z_1^3 - 9 z_1^2 z_2 - 3 z_1 z_2^2\right)$$

$$+ \xi + G_{\text{instanton}}(e^{-2\pi z_1}, e^{-2\pi z_2}), \quad \xi = \frac{\zeta(3)(h^{1,1} - h^{2,1})}{(2\pi i)^3} \simeq -1.30843\, i\,.$$

Finding flux vacua

- The 3-fold fixes all free parameters except of the VEV's $\langle T_1 \rangle$, $\langle T_2 \rangle$, $\langle S \rangle$, $\langle z_1 \rangle$ and $\langle z_2 \rangle$ and flux vectors $f = \{f_i\}$, $h = \{h_i\}$, $i = 1, .., 6$.
- D3-Tadpole constraint:

$$L = \int_Z G_4 \wedge G_4 = \int_X F_3 \wedge H_3 = L_{\mathrm{max}} - N_{D3}, \quad L_{\mathrm{max}} = \frac{\chi(Z)}{24}.$$

Strategy:

- Fix $\langle S \rangle$, $\langle z_1 \rangle$ and $\langle z_2 \rangle$ to rational value.
- Neglect $G_{\mathrm{instanton}}$ and set ξ to a rational value, such that:

$$0 = \{W_0, D_S W_0, D_{z_1} W_0, D_{z_2} W_0\} = \mathbf{A} \cdot \{f_1, .., f_6, h_1, .., h_6\}, \text{ with } \mathbf{A} \in \mathbb{Q}^{8\times 12}.$$

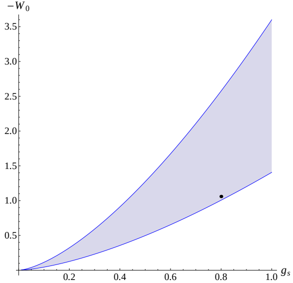
- Find basis of solutions $\{f_i\}$, $\{h_i\}$ with minimal tadpole L.
- Generate shift in W_0, $\langle S \rangle$, $\langle z_1 \rangle$ and $\langle z_2 \rangle$ for $\xi \in i\mathbb{R}$, $G_{\mathrm{instanton}} \neq 0$.

Solutions and Kähler moduli stabilization

De Sitter vaccum can be constructed if:

- $L < L_{\text{max}}$,
- $\{W_0, g_s = \langle \text{Re}\, S\rangle^{-1}, A\}$ fullfills:

  $1.25 < |W_0|\, A\, g_s^{-3/2} < 1.34,$
- $V^{(c.s.)}_{z^a z^b} > 0.$



Explicit example:

- $\{f, h\} = \{0, -16, 56, -28, -12; 4, 0, 0, 0, -9, 10\}, \quad L = 408$
-

g_s	A	$\langle T_1\rangle$	$\langle T_2\rangle$	$\langle z_1\rangle$	$\langle z_2\rangle$	W_0	$\hat{\mathcal{V}}$
0.8	1.5	14.3	0.8	0.98	0.99	-1.06	4.2

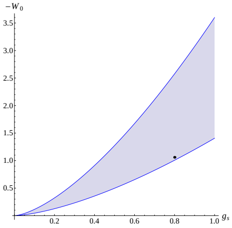
TODO: Directly stabilize $V(T_1, T_2, S, z_1, z_2)$ numerically.

4. Conclusions & Outlook

Conclusions

- Sufficient condition for de Sitter with all moduli stabilized for all Calabi-Yau 3-folds of 'swiss cheese' type.
- Systematical understanding of de Sitter condition based on properties of the Calabi-Yau γ, ξ and fluxes W_0 (F-Theory data).
- Well controlled spontaneous SUSY breaking by F-Terms only.
- Explicit flux vaccum has been constructed which can be used for Kähler uplifting.
- Small cosmological constant by tuning background fluxes.
  [Bousso, Polchinski '00]

Outlook



Statistics of $\mathbb{CP}_{1,1,1,6,9}$:

- How many flux vacua allow Kähler uplifting?
- Scan over g_s, $\langle z_1 \rangle$, $\langle z_2 \rangle$ and ξ_{rational}.

Kähler uplifting: $\hat{\mathcal{V}} \simeq \gamma(6/a)^{3/2} \simeq \gamma N^{3/2} \Rightarrow \mathbb{CP}_{1,1,1,6,9}$: $\hat{\mathcal{V}} \sim 4$.

$\Rightarrow$ **Engineer 4-fold more suitable for Kähler uplifting:**

- How much N can be realized in F-theory on a compact 4-fold?
- How much survives in the Sen limit?
- Rigidity of divisors? [Witten '96]