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On the General Kerr/CFT Correspondence in Arbitrary Dimensions

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Motivation

- A true understanding of the black hole entropy;
  - Strominger and Vafa '96, constructions in string theory, extremal black holes
  - Strominger '97, BTZ, neither string theory nor supersymmetry is necessary
  - Carlip '98, stretched horizon;
  - Guica, Hartman, Song and Strominger '08, the Kerr/CFT correspondence, extremal black holes;
  - Castro, Maloney and Strominger '10, Hidden Conformal Symmetry, non-extremal black holes;
  - Carlip '11, stretched horizon for Kerr/CFT
- Possible examples of gauge/gravity duality.

Objective

- Revisit the case for non-extremal black holes;
- Boundary conditions without using the intermediate stretched horizon;
- Thus study fluctuations on the horizon directly;
- A more direct evidence for the existence of a dual 2D conformal field theory.

Plan

- Kerr/CFT
- Black holes
- Boundary conditions and symmetries
- Non-extremal case
- Extremal case

Kerr/CFT correspondence

Guica, Hartman, Song and Strominger (2008).
Kerr metric,

$$ds^2 = \rho^2\Big(-\frac{\Delta}{v^2}dt^2 + \frac{dr^2}{\Delta} + d\theta^2\Big) + g(d\phi - wdt)^2 ,$$

$$v^2 = (r^2+a^2)^2 - a^2\sin\theta^2\Delta , \quad w = \frac{2Mar}{(r^2+a^2)^2 - a^2\sin\theta^2\Delta} ,$$

$$g = \frac{(r^2+a^2)^2 - a^2\sin\theta^2\Delta}{\rho^2}\sin\theta^2 ,$$

$$\Delta = (r^2+a^2) - 2Mr , \quad \rho^2 = r^2 + a^2\cos\theta^2 .$$

First law, $dM = TdS + \Omega dJ$,

$$T = \frac{r_0^2 - a^2}{4\pi r_0(r_0^2+a^2)} , \quad S = \pi(r_0^2+a^2) , \quad \Omega = \frac{a}{r_0^2+a^2} , \quad J = Ma .$$

NHEK

The Extremal limit $\Delta(r_0) = \Delta'(r_0) = 0\,, \implies r_0 = a = \sqrt{J} = M.$
Bardeen and Horowitz (1999)

$$ds^2 = r_0^2(1+\cos\theta^2)\Big[-(1+x^2)d\tau^2 + \frac{dx^2}{1+x^2} + d\theta^2 + \frac{4\sin\theta^2(d\phi + xdt)^2}{(1+\cos\theta^2)^2}\Big].$$

- $SL(2,R) \times U(1)$ isometry;
- Warped AdS_3 for fixed θ;

Asymptotic symmetries

$$g \sim \begin{pmatrix} \mathcal{O}(\frac{1}{x^2}) & & & \\ & \mathcal{O}(1) & & \\ & & \mathcal{O}(1) & \mathcal{O}(x) \\ & & \mathcal{O}(x) & \mathcal{O}(x^2) \end{pmatrix} .$$

"Equivalent" configurations: GHSS (2008)

$$\delta g \sim \begin{pmatrix} \mathcal{O}(\frac{1}{x^3}) & \mathcal{O}(\frac{1}{x^2}) & \mathcal{O}(\frac{1}{x}) & \mathcal{O}(\frac{1}{x^2}) \\ & \mathcal{O}(\frac{1}{x}) & \mathcal{O}(\frac{1}{x}) & \mathcal{O}(\frac{1}{x}) \\ & & \mathcal{O}(1) & \mathcal{O}(1) \\ & & & \mathcal{O}(x^2) \end{pmatrix} .$$

Entropy

The corresponding generators are

$$\xi_m = -e^{-im\phi}(imx\partial_x + \partial_\phi)\,.$$

The central charge is

$$c = 12r_0^2 = 12J\,.$$

Frolov-Thorne temperature, $T = \frac{1}{2\pi}$. Cardy formula

$$S = \frac{\pi^2}{3}cT = 2\pi J\,.$$

Simple examples

Schwarzschild,

$$ds^2 = -\Delta dt^2 + \frac{dr^2}{\Delta} + r^2 d\theta^2 + r^2 \sin\theta^2 d\phi^2 \,, \quad \Delta = 1 - \frac{2M}{r} \,.$$

Kerr-AdS,

$$\begin{aligned} ds^2 &= f\left(-\frac{\Delta}{v^2}dt^2 + \frac{dr^2}{\Delta}\right) + \frac{f\, d\theta^2}{\Delta_\theta} + g_{11}(d\phi^1 - w^1 dt)^2 \,, \\ \Delta &= (r^2 + a^2)(1 + g^2 r^2) - 2Mr \,, \quad w^1 = \frac{2Mar}{v^2} \,, \\ v^2 &= (r^2 + a^2)^2 - \frac{a^2 \sin^2\theta \Delta}{\Delta_\theta} \,. \end{aligned}$$

Horizon $\Delta(r_0) = 0$, angular velocity $\Omega = w^1(r_0)$,
and temperature $T = \frac{\Delta'(r_0)}{4\pi v(r_0)}$.

Less-simple examples - 1

Kerr-NUT-AdS in $D = 7$, Chen, Lu and Pope (2006)

$$\begin{aligned}
ds^2 \quad = \quad & \frac{(r^2+y^2)(r^2+z^2)\,dr^2}{X} + \frac{(r^2+y^2)(y^2-z^2)\,dy^2}{Y} + \frac{(r^2+z^2)(z^2-y^2)\,dz^2}{Z} \\
& - \frac{X}{(r^2+y^2)(r^2+z^2)}\Big(dt' + (y^2+z^2)d\psi_1 + y^2z^2d\psi_2\Big)^2 \\
& + \frac{Y}{(r^2+y^2)(z^2-y^2)}\Big(dt' + (z^2-r^2)d\psi_1 - r^2z^2d\psi_2\Big)^2 \\
& + \frac{Z}{(r^2+z^2)(y^2-z^2)}\Big(dt' + (y^2-r^2)d\psi_1 - r^2y^2d\psi_2\Big)^2 \\
& + \frac{C_3}{r^2y^2z^2}\Big(dt' + (y^2+z^2-r^2)d\psi_1 + (y^2z^2-r^2y^2-r^2z^2)d\psi_2 - r^2y^2z^2d\psi_3\Big)^2 ,
\end{aligned}$$

where

$$\begin{aligned}
X &= g^2r^6 + C_0r^4 + C_1r^2 + C_2 - 2M + \frac{C_3}{r^2} , \\
Y &= g^2y^6 - C_0y^4 + C_1y^2 - C_2 + 2L_1 + \frac{C_3}{y^2} , \\
Z &= g^2z^6 - C_0z^4 + C_1z^2 - C_2 + 2L_2 + \frac{C_3}{z^2} .
\end{aligned}$$

Less-simple examples - 2

Mei and Pope (2007)

$$
\begin{aligned}
ds^2 &= H_1^{2/3} H_3^{1/3} \Bigg\{ (x^2 - y^2) \left(\frac{dx^2}{X} - \frac{dy^2}{Y} \right) - \frac{x^2 X (dt + y^2 d\sigma)^2}{(x^2 - y^2) f H_1^2} \\
&\qquad + \frac{y^2 Y \left[dt + (x^2 + 2ms_1^2) d\sigma \right]^2}{(x^2 - y^2)(\gamma + y^2) H_1^2} \\
&\qquad - U \left(dt + y^2 d\sigma + \frac{(x^2 - y^2) f H_1 \left[ab d\sigma + (\gamma + y^2) d\chi \right]}{ab(x^2 - y^2) H_3 - 2ms_3 c_3 (\gamma + y^2)} \right)^2 \Bigg\} , \\
U &= \frac{\left[ab(x^2 - y^2) H_3 - 2ms_3 c_3 (\gamma + y^2) \right]^2}{(x^2 - y^2)^2 (\gamma + y^2) f H_1^2 H_3} , \\
X &= \frac{-2mx^2 + (\tilde{a}^2 + x^2)(\tilde{b}^2 + x^2)}{x^2} \\
&\quad + \frac{g^2 (\tilde{a}^2 + 2ms_1^2 + x^2)(\tilde{b}^2 + 2ms_1^2 + x^2)(2ms_3^2 + \gamma + x^2)}{x^2} , \\
Y &= \frac{(\tilde{a}^2 + y^2)(\tilde{b}^2 + y^2) \left[1 + g^2 (\gamma + y^2) \right]}{y^2} ,
\end{aligned}
$$

Common features

- Valid for all (known) stationary and axisymmetric solutions;
- Metric,

$$
\begin{aligned}
ds^2 &= f\left[-\frac{\Delta}{v^2}dt^2 + \frac{dr^2}{\Delta}\right] + h_{ij}d\theta^i d\theta^j \\
&\quad + g_{ab}(d\phi^a - w^a dt)(d\phi^b - w^b dt)\,,
\end{aligned}
$$

- Horizon, $\Delta(r_0) = 0$;
- Area, $S = \int_{Horizon} d\vec{\theta} d\vec{\phi} \sqrt{h\,g}$;
- Temperature, $T = \frac{\Delta'}{4\pi v}|_{r\to r_0}$;
- Angular velocity, $\Omega^a = w^a|_{r\to r_0}$;

cont.

- Mass ($\xi = \partial_t$) and angular momentum ($\xi = \partial_\phi$) can be defined through

$$\delta H_\xi = \int_{horizon} \delta \mathbf{Q}_\xi - i_\xi \mathbf{\Theta}_\delta \,.$$

For Einstein gravity plus a cosmological constant, $\delta \mathbf{Q}_\xi - i_\xi \mathbf{\Theta}_\delta = \frac{1}{16\pi}\sqrt{-g}\,(d^{n-2}x)_{\mu\nu} K^{\mu\nu}$,

$$K^{\mu\nu} = -\frac{\delta(\sqrt{-g}\,\xi^{\mu\nu})}{\sqrt{-g}} + \xi^\mu(\nabla_\rho h^{\nu\rho} - \nabla^\nu h) - \xi^\nu(\nabla_\rho h^{\mu\rho} - \nabla^\mu h)\,.$$

- The first law of black hole thermodynamics is satisfied,

$$dM = TdS + \Omega^a dJ_a + \Phi dQ + \cdots \,.$$

Note: All thermodynamical quantities can be defined using data from the horizon!

Fluctuations on the horizon

- Need to solve $\delta E_{\mu\nu} = 0$.
- One class of solutions, $\delta \tilde{g}_{\mu\nu} = \mathcal{L}_\xi \tilde{g}_{\mu\nu}$ $\Longrightarrow \quad \delta \tilde{E}_{\mu\nu} = \mathcal{L}_\xi \tilde{E}_{\mu\nu} = 0$.
- $\delta \tilde{g}_{\mu\nu} = \mathcal{L}_\xi \tilde{g}_{\mu\nu}$ paired with $\delta x^\mu = -\xi^\mu$ is diffeomorphism;
- But $\delta \tilde{g}_{\mu\nu} = \mathcal{L}_\xi \tilde{g}_{\mu\nu}$ leads to a new configuration if $\delta x^\mu = 0$;
- Equivalent to $\delta \tilde{g}_{\mu\nu} = 0$ but $\delta x^\mu = -\xi^\mu$.
- A coordinate system is a lattice of observers (clocks and rulers). $\delta x^\mu = -\xi^\mu$ can be understood as involuntary motion of observers, driven by quantum fluctuations of the spacetime.
- Expansion, $\xi^\mu = \sum_{k=0}^\infty \xi^\mu_{(k)} (r - r_0)^k \,, \quad \xi^r_{(0)} = \xi^i_{(0)} = 0$, where all the functions $\xi^\mu_{(k)}$ depend only on θ^i, ϕ^a and t.

One may do by only assuming analyticity, but harder to prove in general.

Boundary conditions

- The induced metric on the $r = r_0$ hypersurface is fixed, $\delta\tilde{g}_{ij} \approx \delta\tilde{g}_{iA} \approx \delta\tilde{g}_{AB} \approx 0$.
- The volume density is fixed, $\delta\sqrt{-\tilde{g}} \approx 0$.
- All (inverse) metric elements related to θ^i are fixed, $\delta\tilde{g}_{ir} \approx \delta\tilde{g}^{ir} \approx \delta\tilde{g}^{ij} \approx \delta\tilde{g}^{iA} \approx 0$.

Solving the boundary conditions

Variation of the (inverse) metric elements,

$$
\begin{aligned}
\mathcal{L}_\xi \tilde{g}_{rr} &\approx \partial_r\left(\frac{f}{\Delta'}\xi^r_{(1)}\right) + \frac{\xi^i_{(1)}\partial_i f}{\Delta'} + \frac{f}{\Delta}\xi^r_{(1)}\,,\\
\mathcal{L}_\xi \tilde{g}_{rA} &\approx \frac{f}{\Delta'}\partial_A\xi^r_{(1)} + \tilde{g}_{AB}\xi^B_{(1)}\,,\\
\mathcal{L}_\xi \tilde{g}_{ri} &\approx \frac{f}{\Delta'}\partial_i\xi^r_{(1)} + q_{ij}\xi^j_{(1)}\,, \quad \mathcal{L}_\xi \tilde{g}^{ri} \approx 0\,,\\
\mathcal{L}_\xi \tilde{g}_{ij} &\approx \mathcal{L}_\xi \tilde{g}^{ij} \approx 0\,, \quad \mathcal{L}_\xi \tilde{g}_{ia} \approx g_{ab}D_i\xi^b_{(0)}\,,\\
\mathcal{L}_\xi \tilde{g}_{it} &\approx -w_a D_i\xi^a_{(0)}\,,\\
\mathcal{L}_\xi \tilde{g}^{iA} &\approx \frac{v^2}{f\Delta'}w^A w^B\partial_B\xi^i_{(1)} - q^{ij}\partial_j\xi^A_{(0)}\,,\\
\mathcal{L}_\xi \tilde{g}_{ab} &\approx g_{ac}D_b\xi^c_{(0)} + g_{bc}D_a\xi^c_{(0)}\,,\\
\mathcal{L}_\xi \tilde{g}_{at} &\approx g_{ab}D_t\xi^b_{(0)} - w_b D_a\xi^b_{(0)}\,, \quad \mathcal{L}_\xi \tilde{g}_{tt} \approx -2w_a D_t\xi^a_{(0)}\,,\\
\mathcal{L}_\xi\sqrt{-\tilde{g}} &\approx \sqrt{-\tilde{g}}\left(\xi^r_{(1)} + \partial_A\xi^A_{(0)}\right).
\end{aligned}
$$

cont.

They reduce to the following equations,

$$D_\mu \xi^a_{(0)} \equiv \partial_\mu \xi^a_{(0)} - w^a \partial_\mu \xi^t_{(0)} \approx 0\,, \quad \mu \neq r\,; \quad \xi^r_{(1)} = -\partial_A \xi^A_{(0)}\,,$$

$$\xi^i_{(1)} = -q^{ij} \frac{f}{\Delta'} \partial_j \xi^r_{(1)}\,, \quad \partial_i \xi^A_{(0)} = q_{ij} \frac{v^2}{f\Delta'} w^A w^B \partial_B \xi^j_{(1)}\,.$$

The first one $D_\mu \xi^a_{(0)} \approx 0$ is easily solved with

$$\xi^a_{(0)} = \Omega^a \xi^t_{(0)}\,, \quad \Longrightarrow \quad D_\mu \xi^a_{(0)} = (\Omega^a - w^a)\partial_\mu \xi^t_{(0)} \approx 0\,.$$

The other equations are then uniquely solved by

$$\xi^r_{(1)} = -\partial_A \xi^A_{(0)} = -\Omega^A \partial_A \xi^t_{(0)}\,, \quad \partial_i \xi^t_{(0)} = 0\,, \quad \xi^i_{(1)} = 0\,.$$

Result

They reduce to the following equations,

$$\tilde{g}_{\mu\nu} \sim \begin{pmatrix} \mathcal{O}(\frac{1}{\Delta}) & 0 & 0 \\ 0 & \mathcal{O}(1) & 0 \\ 0 & 0 & \mathcal{O}(1) \end{pmatrix},$$

$$\delta\tilde{g}_{\mu\nu} \sim \begin{pmatrix} \mathcal{O}(\frac{1}{\Delta}) & 0 & \mathcal{O}(\frac{1}{\Delta'}) \\ 0 & 0 & 0 \\ \mathcal{O}(\frac{1}{\Delta'}) & 0 & 0 \end{pmatrix}.$$

The generators ($\rho \equiv r - r_0$),

$$\bar{a}_m \equiv \xi^\mu \partial_\mu = -e^{-im(\phi^{\bar{a}} - \tilde{\Omega}^{\bar{a}} t)} \Big\{ \Big[i\, m\, \rho + \mathcal{O}(\rho^2) \Big] \partial_r + \mathcal{O}(\rho^2) \partial_i + \Big[\frac{\Omega^A}{\Omega^{\bar{a}} - \tilde{\Omega}^{\bar{a}}} + \mathcal{O}(\rho) \Big] \partial_A \Big\}.$$

They satisfy the (centerless) Virasoro algebra,

$$i[\bar{a}_m\,,\, \bar{a}_n] = (m-n)\bar{a}_{m+n}\,.$$

Charge and central extension

- As before, one defines the charges through $(\xi = \bar{a}_m)$

$$\delta H_\xi = \int_{horizon} \delta \mathbf{Q}_\xi - i_\xi \mathbf{\Theta}_\delta \,.$$

- The is a central extension to the algebra. Central charge $c = 12i\Big(\text{the coefficient of } m^3 \text{ in } K[\mathcal{L}_{-m}, \mathcal{L}_m]\Big)$.

$$K[\mathcal{L}_{-m}, \mathcal{L}_m] = \delta H_\xi\big|_{(\delta \to \mathcal{L}_{\bar{a}_{-m}}, \xi \to \mathcal{L}_{\bar{a}_m})} \sim \int_{Horizon} (dx^{n-2})_{\mu\nu} \frac{\sqrt{-\tilde{g}}}{16\pi} K^{\mu\nu} \,. \quad (1)$$

Central charge

The key quantity to consider ($\tilde{h}_{\mu\nu} \equiv \mathcal{L}_{\bar{a}_n}\tilde{g}_{\mu\nu}$),

$$K^{tr} = -\frac{\tilde{h}}{2}\bar{a}^{tr}_m + \tilde{h}^{t\rho}\tilde{\nabla}_\rho \bar{a}^r_m - \tilde{h}^{r\rho}\tilde{\nabla}_\rho \bar{a}^t_m - (\tilde{\nabla}^t \tilde{h}^{r\rho} - \tilde{\nabla}^r \tilde{h}^{t\rho})\bar{a}_{m\rho} + \bar{a}^t_m(\tilde{\nabla}_\rho \tilde{h}^{r\rho} - \tilde{\nabla}^r \tilde{h}) - \bar{a}^r_m(\tilde{\nabla}_\rho \tilde{h}^{t\rho} - \tilde{\nabla}^t \tilde{h}) .$$

We calculate for the more general Virasoro generators,

$$\bar{a}_m = -e^{-im(\phi^{\bar{a}} - \hat{\Omega}^{\bar{a}} t)}\left\{\left[i\, m\, \rho + \mathcal{O}(\rho^2)\right]\partial_r + \mathcal{O}(\rho^2)\partial_i + \left[\chi^A + \mathcal{O}(\rho)\right]\partial_A\right\},$$

where $\hat{\Omega}^{\bar{a}}$ and χ^A are arbitrary, except for $\chi^{\bar{a}} = 1 + \hat{\Omega}^{\bar{a}}\chi^t$. For us, $\hat{\Omega}^{\bar{a}} = \tilde{\Omega}^{\bar{a}}$, $\quad \chi^{\bar{a}} = \frac{\Omega^{\bar{a}}}{\Omega^{\bar{a}} - \tilde{\Omega}^{\bar{a}}}$, $\quad \chi^t = \frac{1}{\Omega^{\bar{a}} - \tilde{\Omega}^{\bar{a}}}$.

cont.

In the non-extremal case, $T \propto \Delta'(r_0) \neq 0$ and

$$K^{tr} \approx -4im^3\left[\chi^t(\Omega^{\bar{a}} - \hat{\Omega}^{\bar{a}}) - \frac{1}{2}\right]\frac{v^2(\Omega^{\bar{a}} - \hat{\Omega}^{\bar{a}})}{f\Delta'} + \cdots ,$$

- The omitted terms are all finite and are linear in m;
- The subleading terms (those of $\mathcal{O}(\rho^2)$ for ∂_r and ∂_i, and those of $\mathcal{O}(\rho)$ for ∂_A) are not constrained in the generators. But they do not contribute to the central term either.

cont.

The central charge is

$$\begin{aligned} c^{\bar{a}} &= \frac{3}{\pi}\int_{horizon}(d^{D-2}x)_{tr}2\sqrt{-\tilde{g}}\left[\chi^t(\Omega^{\bar{a}}-\hat{\Omega}^{\bar{a}})-\frac{1}{2}\right]\frac{v^2(\Omega^{\bar{a}}-\hat{\Omega}^{\bar{a}})}{f\Delta'} \\ &= \frac{3}{\pi}\int_{horizon}(d^{D-2}x)_{tr}2\sqrt{qg}\left[\chi^t(\Omega^{\bar{a}}-\hat{\Omega}^{\bar{a}})-\frac{1}{2}\right]\frac{v_0(r_0)(\Omega^{\bar{a}}-\hat{\Omega}^{\bar{a}})}{\Delta'(r_0)} \\ &= \frac{3}{\pi^2}\left[\chi^t(\Omega^{\bar{a}}-\hat{\Omega}^{\bar{a}})-\frac{1}{2}\right]\cdot\frac{\Omega^{\bar{a}}-\hat{\Omega}^{\bar{a}}}{T}\cdot S\,. \end{aligned}$$

For our generators, $c^{\bar{a}} = \frac{3}{2\pi^2}\cdot\frac{\Omega^{\bar{a}}-\tilde{\Omega}^{\bar{a}}}{T}\cdot S$.
The Frolov-Thorne temperature for $(\phi^{\bar{a}}-\tilde{\Omega}^{\bar{a}}t)$ is $T^{\bar{a}} = \frac{T}{\tilde{\Omega}^{\bar{a}}-\Omega^{\bar{a}}}$.

Entropy

Central charge, $c^{\bar{a}} = \frac{3}{2\pi^2} \cdot \frac{\Omega^{\bar{a}} - \tilde{\Omega}^{\bar{a}}}{T} \cdot S$.
Temperature, $T^{\bar{a}} = \frac{T}{\tilde{\Omega}^{\bar{a}} - \Omega^{\bar{a}}}$.

- Both are non-negative unless $\tilde{\Omega}^{\bar{a}} = \Omega^{\bar{a}}$.
- Can be explained by the fact that the horizon is a frozen surface.

So $c^{\bar{a}}$ vanishes and $T^{\bar{a}}$ diverges, but the entropy is finite,

$$S^{\bar{a}} = \frac{\pi^2}{3} c^{\bar{a}} T^{\bar{a}} = \frac{S}{2}\,.$$

Missing by a factor of 2!

Extremal: need additional constraints

- Unconstrained subleading terms do contribute to the central term in the extremal case;
- Require that the Virasoro algebra is satisfied up to the subleading order.
- new generators: $\bar{a}_m = -e^{-im(\phi^{\bar{a}} - \hat{\Omega}^{\bar{a}} t)}(\cdots)$,

$$
\begin{aligned}
(\cdots) \quad = \quad & \Big\{ i\, m\, \rho + \Big[m u^r + \frac{i m^2}{2}(u^{\bar{a}} - \Omega^{\bar{a}} u^t) \Big] \rho^2 + \mathcal{O}(\rho^3) \Big\} \partial_r \\
& + \Big[m u^i \rho^2 + \mathcal{O}(\rho^3) \Big] \partial_i + \Big[\chi^A + m u^A \rho + \mathcal{O}(\rho^2) \Big] \partial_A \,,
\end{aligned}
$$

- u^r, u^i and u^A are free functions of θ^i. But their contribution cancels out.

Central term

- General result: $K^{tr} \approx -m^3 \left(\frac{\Delta'}{\Delta^2} Z_1 + \frac{Z_2}{\Delta} \right) + \cdots$

$$
\begin{aligned}
Z_1 &= -\frac{2iv_0^2(r_0)(\Omega^{\bar{a}} - \hat{\Omega}^{\bar{a}})}{f(r_0,\theta^i)}\rho^2 + \Big[-\frac{2iv_0^2(r_0)w'^{\bar{a}}(r_0)}{f(r_0,\theta^i)} + (\Omega^{\bar{a}} - \hat{\Omega}^{\bar{a}})G_1(r_0,\theta^i) \Big]\rho^3 + \mathcal{O}(\rho^4)\,, \\
Z_2 &= \frac{4iv_0^2(r_0)(\Omega^{\bar{a}} - \hat{\Omega}^{\bar{a}})^2\chi^t}{f(r_0,\theta^i)}\rho + \Big[\frac{2iv_0^2(r_0)w'^{\bar{a}}(r_0)}{f(r_0,\theta^i)} + (\Omega^{\bar{a}} - \hat{\Omega}^{\bar{a}})G_2(r_0,\theta^i) \Big]\rho^2 + \mathcal{O}(\rho^3)\,.
\end{aligned}
$$

- All dependence on r only through Δ, Δ' and $\rho(= r - r_0)$.
- When $\Delta'(r_0) \neq 0$, one recovers the result in the non-extremal case.

cont.

In the extremal case $\Delta'(r_0) = 0$,

$$K^{tr}(\mathcal{L}_{-m}, \mathcal{L}_m) \approx \frac{4im^3 v_0^2(r_0) w_0'^{\bar{a}}(r_0)}{\Delta''(r_0) f(r_0, \theta^i)} (1 + G) + \cdots ,$$

$$G = \frac{\Omega^{\bar{a}} - \hat{\Omega}^{\bar{a}}}{w_0'^{\bar{a}}(r_0)} \Big\{ \frac{2}{\rho} \Big[1 - \chi^t (\Omega^{\bar{a}} - \hat{\Omega}^{\bar{a}}) \Big] + \frac{2\Delta'''(r_0)}{3\Delta''(r_0)} \Big[\chi^t (\Omega^{\bar{a}} - \hat{\Omega}^{\bar{a}}) - \frac{1}{2} \Big] - \frac{f(r_0, \theta^i)}{2iv_0^2(r_0)} (2G_1 + G_2) \Big\} ,$$

In our case, the central charge is

$$c^{\bar{a}} = -\frac{3}{\pi} \int_{horizon} (d^{D-2}x)_{tr} 2\sqrt{-\tilde{g}}\, \frac{v_0^2(r_0) w_0'^{\bar{a}}(r_0)}{\Delta''(r_0) f(r_0, \theta^i)} = \frac{3}{\pi^2} \frac{S}{\widetilde{T}^{\bar{a}}} .$$

Entropy

- The central charge, $c^{\bar{a}} = \frac{3}{\pi^2}\frac{S}{\widetilde{T}^{\bar{a}}}$,

$$\widetilde{T}^{\bar{a}} = -\frac{\Delta''(r_0)}{4\pi v_0(r_0) w'^{\bar{a}}(r_0)} \,.$$

- Temperature, $T^{\bar{a}} = \frac{T}{\tilde{\Omega}^{\bar{a}} - \Omega^{\bar{a}}}$;
- $T^{\bar{a}}$ is indefinite because $T = \Omega^{\bar{a}} - \tilde{\Omega}^{\bar{a}} = 0$;
- One choice is to identify $T^{\bar{a}}$ with $\widetilde{T}^{\bar{a}}$. In this case, Cardy's formula gives

$$S^{\bar{a}} = \frac{\pi^2}{3} c^{\bar{a}} T^{\bar{a}} = S \,.$$

In terms of the central charge, no smooth transition from the non-extremal case to the extremal case.

Summary

- Physically reasonable (and stringent) boundary conditions exist on the horizon;
- For these boundary conditions, there is no need to take the near horizon limit or to introduce an intermediate stretched horizon;
- One copy of Virasoro algebra is uniquely identified, for each of the azimuthal angles;
- The usual machinery leads to the full entropy for extremal black holes, and half the entropy for non-extremal ones;
- Better evidence for the existence of a dual 2D CFT;
- Works for any stationary and axisymmetric black hole in arbitrary dimensions.

Immediate outstanding problems

- Singular behaviors of the symmetry generators and the Frolov-Thorne temperature;
  - Intrinsic or artificial?
- The calculated entropy for non-extremal black holes is missing by a factor of 2;
  - A second copy of the Virasoro algebra?
  - Need to go beyond Cardy's formula for non-extremal black holes?