## Light stringy states

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## Introduction and Motivation

## Motivation

* D-brane compactifications provide a promising framework for model building
* They allow for large extra dimensions which imply a significantly lower string scale, even of just a few TeV.
* Scenarios of these kinds may explain the hierarchy problem, but also allow for stringy signatures that can be observed at LHC.

Antoniadis, Arkani-Hamed, Dimopoulos, Dvali

* There exists a class of amplitudes containing arbitrary number of gauge bosons and maximal two chiral fermions that exhibit a universal behaviour independently of the specifics of the compactification

Lust, Stieberger, Taylor, et. al.

* Due to their universal behaviour they have predictive power.
* The observed poles correspond to the exchanges of Regge excitations of the standard model gauge bosons, whose masses scale with the string mass $M_{s}$.
* Such poles might be observable at LHC if one has low string scale


## Motivation

* On the other hand there exist a tower of stringy excitations localized at the intersections of two stacks of D-branes.
* Their masses depend on the string mass $M_{s}$ and the intersection angle $\theta$ and thus can be significantly lighter than the Regge excitations of the gauge bosons.
* As we will see those light stringy states show up as poles in the scattering amplitudes containing four fermions.
* Such amplitudes are very model dependent, thus do not have the predictive power of the universal amplitudes.
* However the poles corresponding to light stringy states should be observed primary to Regge excitations for the universal amplitudes and thus may provide a first step towards evidence for string theory.


## Intersecting D6-branes

For the sake of calculability we assume two intersectng D6-branes on $T^{2} \times T^{2} \times T^{2}$, where the D6-branes wrap in each torus a one-cycle


* Supersymmetry translates into: $\theta_{1}+\theta_{2}+\theta_{3}=0 \bmod 2$
* We will take a closer look at the (massless and massive) states appearing at such an intersection.


## Quantization at angles

* Boundary conditions for strings between branes at angles:

$$
\begin{aligned}
& \partial_{\sigma} X^{p}(\tau, 0)=X^{p+1}(\tau, 0)=0 \\
& \partial_{\sigma} X^{p}(\tau, \pi)+\tan (\pi \theta) \partial_{\sigma} X^{p+1}(\tau, \pi)=0 \\
& X^{p+1}(\tau, \pi)-\tan (\pi \theta) X^{p}(\tau, \pi)=0
\end{aligned}
$$

* Mode expansion ( $\left.Z^{p}=X^{p}+i X^{p+1}\right)$ :


$$
\begin{aligned}
\partial Z^{I}(z) & =\sum_{n} \alpha_{n-\theta_{I}}^{I} z^{-n+\theta_{I}-1} & \partial \bar{Z}^{I}(z)=\sum_{n} \alpha_{n+\theta_{I}}^{I} z^{-n-\theta_{I}-1} \\
\Psi^{I}(z) & =\sum_{r \in \mathbb{Z}+\nu} \psi_{r-\theta_{I}}^{I} z^{-r-\frac{1}{2}+\theta_{I}} & \bar{\Psi}^{I}(z)=\sum_{r \in \mathbb{Z}+\nu} \psi_{r+\theta_{I}}^{I} \bar{z}^{-r-\frac{1}{2}-\theta_{I}}
\end{aligned}
$$

for $v=0,1 / 2$ for R and NS respectively.

* The commutator/anticommutators:

$$
\left[\alpha_{n \pm \theta}^{I}, \alpha_{m \mp \theta}^{I^{\prime}}\right]=(m \pm \theta) \delta_{n+m} \delta^{I I^{\prime}} \quad\left\{\psi_{m-\theta_{I}}^{I}, \psi_{n+\theta_{I}}^{I^{\prime}}\right\}=\delta_{m, n} \delta^{I I^{\prime}}
$$

## The vacuum

* Intersecting branes in 10D:


* NS sector (4D bosons):
- Positive angle

$$
\begin{array}{llll}
\alpha_{m-\theta_{I}}\left|\theta_{I}\right\rangle_{N S}=0 & m \geq 1 & \psi_{r-\theta_{I}}\left|\theta_{I}\right\rangle_{N S}=0 & r \geq 1 / 2 \\
\alpha_{m+\theta_{I}}\left|\theta_{I}\right\rangle_{N S}=0 & m \geq 0 & \psi_{r+\theta_{I}}\left|\theta_{I}\right\rangle_{N S}=0 & r \geq 1 / 2
\end{array}
$$

- Negative angle

$$
\begin{array}{llll}
\alpha_{m-\theta_{I}}\left|\theta_{I}\right\rangle_{N S}=0 & m \geq 0 & \psi_{r-\theta_{I}}\left|\theta_{I}\right\rangle_{N S}=0 & r \geq 1 / 2 \\
\alpha_{m+\theta_{I}}\left|\theta_{I}\right\rangle_{N S}=0 & m \geq 1 & \psi_{r+\theta_{I}}\left|\theta_{I}\right\rangle_{N S}=0 & r \geq 1 / 2
\end{array}
$$

* Recall: GSO-projection requires odd number of fermionic excitations.


## Lightest string states

* Recall the mass formula:

$$
M^{2}=M_{s}^{2} \sum_{I}\left(\sum_{m \in Z}: \alpha_{-m+\theta_{I}}^{I} \alpha_{m-\theta_{I}}^{I}:+\sum_{m \in Z}\left(m-\theta_{I}\right): \psi_{-m+\theta_{I}}^{I} \psi_{m-\theta_{I}}^{I}:+\epsilon_{0}^{I}\right)
$$

* Concrete setup: $\theta_{1}^{a b}<0, \theta_{2}^{a b}<0, \theta_{3}^{a b}<0$ with $\theta_{1}^{a b}+\theta_{2}^{a b}+\theta_{3}^{a b}=-2$ (SUSY).
* The lowest fermionic excitations of this configuration:

$$
\begin{aligned}
\psi_{-\frac{1}{2}-\theta_{I}^{a b}}\left|\theta_{1,2,3}^{a b}\right\rangle_{N S} & M^{2}=\frac{1}{2}\left(\theta_{I}^{a b}-\sum_{J \neq I} \theta_{J}^{a b}\right) M_{s}^{2}=\left(1+\theta_{I}^{a b}\right) M_{s}^{2} \\
\prod_{I} \psi_{-\frac{1}{2}-\theta_{I}^{a b}}\left|\theta_{1,2,3}^{a b}\right\rangle_{N S} & M^{2}=\left(1+\frac{1}{2}\left(\theta_{1}^{a b}+\theta_{2}^{a b}+\theta_{3}^{a b}\right)\right) M_{s}^{2}=0
\end{aligned}
$$

## Lightest string states

* Some additional light states (for small $\theta_{1}$ ):

$$
\begin{array}{rlr}
\alpha_{\theta_{1}} \prod_{I} \psi_{-\frac{1}{2}-\theta_{I}}\left|\theta_{1,2,3}^{a b}\right\rangle_{N S} & M^{2}=\left(1+\frac{1}{2} \sum_{I} \theta_{I}-\theta_{1}\right) M_{s}^{2}=-\theta_{1} M_{s}^{2} \\
\left(\alpha_{\theta_{1}}\right)^{2} \prod_{I} \psi_{-\frac{1}{2}-\theta_{I}}\left|\theta_{1,2,3}^{a b}\right\rangle_{N S} & M^{2}=\left(1+\frac{1}{2} \sum_{I} \theta_{I}-2 \theta_{1}\right) M_{s}^{2}=-2 \theta_{1} M_{s}^{2}
\end{array}
$$

* These scalars are potentially very light, depending on the intersection angles.
* If the string scale is low, and the angles are small, such states have very low masses.
* Additional states, such as Higher Spin states, but even $\theta_{I} \rightarrow 0$ massive.


## The vacuum

* Intersecting branes in 10D:


* R sector (4D fermions):
- Positive angle

$$
\begin{array}{llll}
\alpha_{m-\theta_{I}}\left|\theta_{I}\right\rangle_{R}=0 & m \geq 1 & \psi_{r-\theta_{I}}\left|\theta_{I}\right\rangle_{R}=0 & r \geq 1 \\
\alpha_{m+\theta_{I}}\left|\theta_{I}\right\rangle_{R}=0 & m \geq 0 & \psi_{r+\theta_{I}}\left|\theta_{I}\right\rangle_{R}=0 & r \geq 0
\end{array}
$$

- Negative angle

$$
\begin{array}{ll}
\alpha_{m-\theta_{I}}\left|\theta_{I}\right\rangle_{R}=0 & m \geq 0 \\
\alpha_{m+\theta_{I}}\left|\theta_{I}\right\rangle_{R}=0 & m \geq 1
\end{array}
$$

$$
\psi_{r-\theta_{I}}\left|\theta_{I}\right\rangle_{R}=0 \quad r \geq 0
$$

$$
\psi_{r+\theta_{I}}\left|\theta_{I}\right\rangle_{R}=0 \quad r \geq 1
$$

## Lightest string states

* Recall the mass formula:

$$
M^{2}=M_{s}^{2}\left(\sum_{I}\left(\sum_{m \in Z}: \alpha_{-m+\theta_{I}}^{I} \alpha_{m-\theta_{I}}^{I}:+\sum_{m \in Z}\left(m-\theta_{I}\right): \psi_{-m+\theta_{I}}^{I} \psi_{m-\theta_{I}}^{I}:\right)+\epsilon_{0}\right)
$$

* Concrete setup: $\theta_{1}^{a b}<0, \theta_{2}^{a b}<0, \theta_{3}^{a b}<0$ with $\theta_{1}^{a b}+\theta_{2}^{a b}+\theta_{3}^{a b}=-2$
* Massless state: the vaccum: $\left|\theta_{1,2,3}^{a b}\right\rangle_{R}$
* Light states: ( $\theta_{1}$ is small):

$$
\begin{array}{rc}
\alpha_{\theta_{1}}\left|\theta_{1,2,3}^{a b}\right\rangle_{R} & M^{2}=-\theta_{1} M_{s}^{2} \\
\left(\alpha_{\theta_{1}}\right)^{2}\left|\theta_{1,2,3}^{a b}\right\rangle_{R} & M^{2}=-2 \theta_{1} M_{s}^{2}
\end{array}
$$

* These states are the corresponding superpartners to the NS-scalars.
* Question: Can they be observed?



## The Amplitude

## Amplitude

* Consider three stacks of D-branes within a semi-realistic brane configuration:

* For the sake of concreteness we choose the setup

$$
\begin{array}{lll}
\theta_{a b}^{1}>0, & \theta_{a b}^{2}>0, & \theta_{a b}^{3}<0 \\
\theta_{b c}^{1}>0, & \theta_{b c}^{2}>0, & \theta_{b c}^{3}<0 \\
\theta_{c a}^{1}<0, & \theta_{c a}^{2}<0, & \theta_{c a}^{3}<0
\end{array} \quad \Longrightarrow \quad \begin{aligned}
& \theta_{a b}^{1}+\theta_{a b}^{2}+\theta_{a b}^{3}=0 \\
& \theta_{b c}^{1}+\theta_{b c}^{2}+\theta_{b c}^{3}=0 \\
& \theta_{c a}^{1}+\theta_{c a}^{2}+\theta_{c a}^{3}=-2
\end{aligned}
$$

* At the intersections live chiral fermions: $\psi, \bar{\psi}, \chi, \bar{\chi}$.


## Amplitude

* We want to compute the scattering amplitudes of four chiral fermions:

* The corresponding diagram contains different channels:

* Two difficulties: 1. Vertex operators

2. Bosonic Twist field correlator


## Vertex Operators

## Vertex Operators

* To each state there is a corresponding vertex operator, whose form crucially depends on the intersection angle
* challenging part is the internal part $\leadsto \rightarrow$ bosonic and fermionic twist fields
* Method: We determine the OPE's of $\partial Z, \partial \bar{Z}, \Psi$ and $\bar{\Psi}$ with vacuum and excitations of it
* Example: $\left|\theta_{I}\right\rangle_{N S} \sim s_{\theta_{I}} \sigma_{\theta_{I}}$

$$
\begin{aligned}
\partial Z^{I}(z)\left|\theta_{I}\right\rangle_{N S}=\sum_{n=-\infty}^{\infty} \alpha_{n-\theta_{I}}^{I} z^{-n+\theta_{I}-1}\left|\theta_{I}\right\rangle_{N S} & =\sum_{n=-\infty}^{0} \alpha_{n-\theta_{I}}^{I} z^{-n+\theta_{I}-1}\left|\theta_{I}\right\rangle_{N S} \\
\rightarrow z^{\theta_{I}-1} \alpha_{-\theta_{I}}^{I}\left|\theta_{I}\right\rangle_{N S} & =z^{\theta_{I}-1} \tau_{\theta_{I}}^{+}(0) \\
\partial \bar{Z}^{I}(z)\left|\theta_{I}\right\rangle_{N S}=\sum_{n=-\infty}^{\infty} \alpha_{n+\theta_{I}}^{I} z^{-n-\theta_{I}-1}\left|\theta_{I}\right\rangle_{N S} & =\sum_{n=-\infty}^{-1} \alpha_{n+\theta_{I}}^{I} z^{-n-\theta_{I}-1}|0\rangle_{N S} \\
\rightarrow z^{-\theta_{I}} \alpha_{-1+\theta_{I}}^{I}\left|\theta_{I}\right\rangle_{N S} & =z^{-\theta_{I}} \widetilde{\tau}_{\theta_{I}}^{+}(0)
\end{aligned}
$$

* analogous for fermionic degrees of freedom and higher excitations
* OPE's give us detailed knowledge of the conformal twist fields.


## Vertex Operators

* For the NS-sector apply the following dictionary
positive angle $\theta$

$$
\begin{array}{ll}
|\theta\rangle_{N S} & : e^{i \theta H} \sigma_{\theta}^{+} \\
\alpha_{-\theta}|\theta\rangle_{N S} & : e^{i \theta H} \tau_{\theta}^{+} \\
\left(\alpha_{-\theta}\right)^{2}|\theta\rangle_{N S} & : e^{i \theta H} \omega_{\theta}^{+} \\
\psi_{-\frac{1}{2}+\theta}|\theta\rangle_{N S} & : e^{i(\theta-1) H} \sigma_{\theta}^{+} \\
\alpha_{-\theta} \psi_{-\frac{1}{2}+\theta}|\theta\rangle_{N S} & : e^{i(\theta-1) H} \tau_{\theta}^{+} \\
\left(\alpha_{-\theta}\right)^{2} \psi_{-\frac{1}{2}+\theta}|\theta\rangle_{N S} & : e^{i(\theta-1) H} \omega_{\theta}^{+}
\end{array}
$$

negative angle $\theta$

| $\|\theta\rangle_{N S}$ | $:$ | $e^{i \theta H} \sigma_{-\theta}^{-}$ |
| :--- | :--- | :--- |
| $\alpha_{\theta}\|\theta\rangle_{N S}$ | $:$ | $e^{i \theta H} \tau_{-\theta}^{-}$ |
| $\left(\alpha_{\theta}\right)^{2}\|\theta\rangle_{N S}$ | $:$ | $e^{i \theta H} \omega_{-\theta}^{-}$ |
| $\psi_{-\frac{1}{2}-\theta}\|\theta\rangle_{N S}$ | $:$ | $e^{i(\theta+1) H} \sigma_{-\theta}^{-}$ |
| $\alpha_{\theta} \psi_{-\frac{1}{2}-\theta}\|\theta\rangle_{N S}$ | $:$ | $e^{i(\theta+1) H} \tau_{-\theta}^{-}$ |
| $\left(\alpha_{\theta}\right)^{2} \psi_{-\frac{1}{2}-\theta}\|\theta\rangle_{N S}$ | $:$ | $e^{i(\theta+1) H} \omega_{-\theta}^{-}$ |

* For the R-sector apply the following dictionary
positive angle $\theta$

$$
|\theta\rangle_{R} \quad: \quad e^{i(\theta-1 / 2) H} \sigma_{\theta}^{+}
$$

negative angle $\theta$

$$
|\theta\rangle_{R} \quad: \quad e^{i(\theta+1 / 2) H} \sigma_{-\theta}^{-}
$$

## An example

* Consider $\theta_{1}^{a b}<0, \theta_{2}^{a b}<0, \theta_{3}^{a b}<0$ with $\theta_{1}^{a b}+\theta_{2}^{a b}+\theta_{3}^{a b}=-2$
* A massless state in the NS-sector and the corresponding VO:

$$
\prod_{I=1}^{3} \psi_{-1 / 2-\theta_{I}^{a b}}\left|\theta_{1,2,3}^{a b}\right\rangle_{N S}
$$

$$
V^{(-1)}=\Lambda_{a b} \Phi e^{-\varphi} \prod_{I=1}^{3} e^{i\left(\theta_{I}^{a b}+1\right) H_{I}} \sigma_{-\theta_{I}^{a b}}^{-} e^{i k X}
$$

Conformal dimension

$$
h=2-\frac{1}{2} \sum_{I} \theta_{I}^{a b}+\frac{k^{2}}{2}=1+\frac{k^{2}}{2}
$$

$$
U(1)_{W S}=\sum_{I=1}^{3}\left(\theta_{I}^{a b}+1\right)=1
$$

* A massless state in the R-sector and the corresponding VO:

$$
\left|\theta_{1,2,3}^{a b}\right\rangle_{R}: \quad \quad V^{-\frac{1}{2}}=\Lambda_{a b} \psi_{\alpha} S^{\alpha} e^{-\varphi / 2} \prod_{I=1}^{3} e^{i\left(\theta_{I}^{a b}+\frac{1}{2}\right) H_{I}} \sigma_{-\theta_{I}^{a b}}^{-} e^{i k X}
$$

Conformal dimension

$$
h=\frac{3}{8}+\frac{1}{4}+\frac{3}{8}+\frac{k^{2}}{2}=1+\frac{k^{2}}{2} \quad U(1)_{W S}=\sum_{I=1}^{3}\left(\theta_{I}^{a b}+\frac{1}{2}\right)=-\frac{1}{2}
$$

## Amplitude

*For this configuration, the amplitude is $\langle\bar{\psi} \psi \chi \bar{\chi}\rangle$

$$
\begin{array}{lll}
\theta_{a b}^{1}>0, & \theta_{a b}^{2}>0, & \theta_{a b}^{3}<0 \\
\theta_{b c}^{1}>0, & \theta_{b c}^{2}>0, & \theta_{b c}^{3}<0 \\
\theta_{c a}^{1}<0, & \theta_{c a}^{2}<0, & \theta_{c a}^{3}<0
\end{array}
$$


with the corresponding vertex operators:

$$
\begin{array}{ll}
a b: & V_{\psi}^{-\frac{1}{2}}=\Lambda_{a b} \psi^{\alpha} e^{-\varphi / 2} S_{\alpha} \prod_{I=1}^{2} \sigma_{\theta_{a b}^{I}}^{+} e^{i\left(\theta_{a b}^{I}-\frac{1}{2}\right) H_{I}} \sigma_{-\theta_{a b}^{3}}^{-} e^{i\left(\theta_{a b}^{3}+\frac{1}{2}\right) H_{3}} e^{i k X} \\
b a: & V_{\bar{\psi}}^{-\frac{1}{2}}=\Lambda_{b a} \bar{\psi}_{\dot{\alpha}} e^{-\varphi / 2} S^{\dot{\alpha}} \prod_{I=1}^{2} \sigma_{\theta_{a b}^{I}}^{-} e^{i\left(-\theta_{a b}^{I}+\frac{1}{2}\right) H_{I}} \sigma_{-\theta_{a b}^{3}}^{+} e^{i\left(-\theta_{a b}^{3}-\frac{1}{2}\right) H_{3}} e^{i k X} \\
b c: & V_{\chi}^{-\frac{1}{2}}=\Lambda_{b c} \chi^{\alpha} e^{-\varphi / 2} S_{\alpha} \prod_{I=1}^{2} \sigma_{\theta_{b c}^{I}}^{+} e^{i\left(\theta_{b c}^{I}-\frac{1}{2}\right) H_{I}} \sigma_{-\theta_{b c}^{3}}^{-} e^{i\left(\theta_{b c}^{3}+\frac{1}{2}\right) H_{3}} e^{i k X} \\
c b: & V_{\bar{\chi}}^{-\frac{1}{2}}=\Lambda_{c b} \bar{\chi}_{\dot{\alpha}} e^{-\varphi / 2} S^{\dot{\alpha}} \prod_{I=1}^{2} \sigma_{\theta_{b c}^{I}}^{-} e^{i\left(-\theta_{b c}^{I}+\frac{1}{2}\right) H_{I}} \sigma_{-\theta_{b c}^{3}}^{+} e^{i\left(-\theta_{b c}^{3}-\frac{1}{2}\right) H_{3}} e^{i k X}
\end{array}
$$

## Amplitude



- Computing: $\mathcal{A}=\langle\bar{\psi}(0) \psi(x) \chi(1) \bar{\chi}(\infty)\rangle$
which takes the form:

$$
\begin{array}{rl}
\mathcal{A}= & \operatorname{Tr}\left(\Lambda_{b a} \Lambda_{a b} \Lambda_{b c} \Lambda_{c b}\right) \bar{\psi}_{\dot{\alpha}} \psi^{\alpha} \chi^{\beta} \bar{\chi}_{\dot{\beta}} \\
\int_{0}^{1} & d x\left\langle e^{-\varphi / 2(0)} e^{-\varphi / 2(x)} e^{-\varphi / 2(1)} e^{-\varphi / 2(\infty)}\right\rangle \\
& \times\left\langle S^{\dot{\alpha}}(0) S_{\alpha}(x) S_{\beta}(1) S^{\dot{\beta}}(\infty)\right\rangle\left\langle e^{i k_{1} X(0)} e^{i k_{2} X(x)} e^{i k_{3} X(1)} e^{i k_{4} X(\infty)}\right\rangle \\
& \times\left\langle\sigma_{-\theta_{a b}^{3}}^{+}(0) \sigma_{-\theta_{a b}^{3}}^{-}(x) \sigma_{-\theta_{b c}^{3}}^{-}(1) \sigma_{-\theta_{b c}^{3}}^{+}(\infty)\right\rangle \\
& \times \prod_{I=1}^{2}\left\langle\sigma_{\theta_{a b}^{I}}^{-}(0) \sigma_{\theta_{a b}^{I}}^{+}(x) \sigma_{\theta_{b c}^{I}}^{+}(0) \sigma_{\theta_{b c}^{I}}^{-}(\infty)\right\rangle \\
& \times\left\langle e^{i\left(-\theta_{a b}^{3}-\frac{1}{2}\right) H^{3}(0)} e^{i\left(\theta_{a b}^{3}+\frac{1}{2}\right) H^{3}(x)} e^{i\left(\theta_{b c}^{3}+\frac{1}{2}\right) H^{3}(1)} e^{i\left(-\theta_{b c}^{3}-\frac{1}{2}\right) H^{3}(\infty)}\right\rangle \\
& \times \prod_{I=1}^{2}\left\langle e^{i\left(-\theta_{a b}^{I}+\frac{1}{2}\right) H^{I}(0)} e^{i\left(\theta_{a b}^{I}-\frac{1}{2}\right) H^{I}(x)} e^{i\left(\theta_{b c}^{I}-\frac{1}{2}\right) H^{I}(1)} e^{i\left(-\theta_{b c}^{I}+\frac{1}{2}\right) H^{I}(\infty)}\right\rangle
\end{array}
$$

## Amplitude



* Computing: $\mathcal{A}=\langle\bar{\psi}(0) \psi(x) \chi(1) \bar{\chi}(\infty)\rangle$
which takes the form:

$$
\begin{aligned}
\mathcal{A}= & \operatorname{Tr}\left(\Lambda_{b a} \Lambda_{a b} \Lambda_{b c} \Lambda_{c b}\right) \bar{\psi}_{\dot{\alpha}} \psi^{\alpha} \chi^{\beta} \bar{\chi}_{\dot{\beta}} \quad[x(1-x)]^{-\frac{1}{4}} x_{\infty^{-\frac{3}{4}}} \\
& \int_{0}^{1} d x\left\langle e^{-\varphi / 2(0)} e^{-\varphi / 2(x)} e^{-\varphi / 2(1)} e^{-\varphi / 2(\infty)}\right\rangle
\end{aligned}
$$

$\epsilon_{\alpha \beta} \epsilon_{\dot{\alpha} \dot{\beta}}(1-x)^{-\frac{1}{2}} x_{\infty^{-\frac{1}{2}}} \times\left\langle S^{\dot{\alpha}}(0) S_{\alpha}(x) S_{\beta}(1) S^{\dot{\beta}}(\infty)\right\rangle\left\langle e^{i k_{1} X(0)} e^{i k_{2} X(x)} e^{i k_{3} X(1)} e^{i k_{4} X(\infty)}\right\rangle$

$$
\times\left\langle\sigma_{-\theta_{a b}^{3}}^{+}(0) \sigma_{-\theta_{a b}^{3}}^{-}(x) \sigma_{-\theta_{b c}^{3}}^{-}(1) \sigma_{-\theta_{b c}^{3}}^{+}(\infty)\right\rangle \quad x^{k_{1} \cdot k_{2}}(1-x)^{k_{2} \cdot k_{3}} x_{\infty}^{k_{4}\left(k_{1}+k_{2}+k_{3}\right)}
$$

$$
\times \prod_{I=1}^{2}\left\langle\sigma_{\theta_{a b}^{I}}^{-}(0) \sigma_{\theta_{a b}^{I}}^{+}(x) \sigma_{\theta_{b c}^{I}}^{+}(0) \sigma_{\theta_{b c}^{I}}^{-}(\infty)\right\rangle
$$

$$
\times\left\langle e^{i\left(-\theta_{a b}^{3}-\frac{1}{2}\right) H^{3}(0)} e^{i\left(\theta_{a b}^{3}+\frac{1}{2}\right) H^{3}(x)} e^{i\left(\theta_{b c}^{3}+\frac{1}{2}\right) H^{3}(1)} e^{i\left(-\theta_{b c}^{3}-\frac{1}{2}\right) H^{3}(\infty)}\right\rangle
$$

$$
\times \prod_{I=1}^{2}\left\langle e^{i\left(-\theta_{a b}^{I}+\frac{1}{2}\right) H^{I}(0)} e^{i\left(\theta_{a b}^{I}-\frac{1}{2}\right) H^{I}(x)} e^{i\left(\theta_{b c}^{I}-\frac{1}{2}\right) H^{I}(1)} e^{i\left(-\theta_{b c}^{I}+\frac{1}{2}\right) H^{I}(\infty)}\right\rangle
$$

$$
x^{\left(-\theta_{a b}^{3}-\frac{1}{2}\right)\left(\theta_{a b}^{3}+\frac{1}{2}\right)}(1-x)^{\left(\theta_{a b}^{3}+\frac{1}{2}\right)\left(\theta_{b c}^{3}+\frac{1}{2}\right)} x_{\infty}^{\left(-\theta_{b c}^{3}-\frac{1}{2}\right)\left(\left(-\theta_{a b}^{3}-\frac{1}{2}\right)+\left(\theta_{a b}^{3}+\frac{1}{2}\right)+\left(\theta_{b c}^{3}+\frac{1}{2}\right)\right)}
$$



The correlator: $\left\langle\sigma_{1-\theta}^{+}(0) \sigma_{\theta}^{+}(x) \sigma_{1-\nu}^{+}(1) \sigma_{\nu}^{+}(\infty)\right\rangle$

## Recipe A

* Extend via the "doubling trick" the upper half plane to the whole complex plane.
* Quantum part is then computed by employing conformal field theory techniques (energy momentum tensor method) $\leadsto$ analogous to the closed string derivation:

$$
\begin{array}{r}
T(z) \Phi(w) \sim \frac{h_{\Phi} \Phi(w)}{(z-w)^{2}}+\frac{\partial_{w} \Phi(w)}{z-w}+\ldots \\
\lim _{z \rightarrow z_{2}}\left(\frac{\left\langle T(z) \sigma_{\alpha}\left(z_{1}\right) \sigma_{\beta}\left(z_{2}\right) \sigma_{\gamma}\left(z_{3}\right) \sigma_{\delta}\left(z_{4}\right)\right\rangle}{\left\langle\sigma_{\alpha}\left(z_{1}\right) \sigma_{\beta}\left(z_{2}\right) \sigma_{\gamma}\left(z_{3}\right) \sigma_{\delta}\left(z_{4}\right)\right\rangle}-\frac{h_{\sigma_{\beta}}}{\left(z-z_{2}\right)^{2}}\right) \\
=\partial_{z_{2}} \ln \left\langle\sigma_{\alpha}\left(z_{1}\right) \sigma_{\beta}\left(z_{2}\right) \sigma_{\gamma}\left(z_{3}\right) \sigma_{\delta}\left(z_{4}\right)\right\rangle
\end{array}
$$

* The classical part is given by the sum over all quadrangles connecting the four chiral fields $e^{-\sum \frac{A r e a}{2 \pi \alpha^{\prime}}}$.
* The final result is then given in the so-called Lagrangian Form.


## Recipe A

* One independent angle:

$$
\begin{aligned}
x_{\infty}^{\theta(1-\theta)}\left\langle\sigma_{1-\theta}(0)\right. & \left.\sigma_{\theta}(x) \sigma_{1-\theta}(1) \sigma_{\theta}(\infty)\right\rangle=\sqrt{\sin (\pi \theta)} \frac{[x(1-x)]^{-\theta(1-\theta)}}{\sqrt{F(x) F(1-x)}} \\
& \times \sum_{\tilde{p} \in \Lambda_{p}, \tilde{q} \in \Lambda_{q}^{*}} \exp \left[-\frac{\pi}{\alpha^{\prime}} \sin (\pi \theta) F(x) F(1-x)\left(\left(\frac{\tilde{p} L_{a}}{F(1-x)}\right)^{2}+\left(\frac{q L_{b}}{F(x)}\right)^{2}\right)\right]
\end{aligned}
$$

* Two independent angles:

$$
\begin{aligned}
& x_{\infty}^{\nu(1-\nu)}\left\langle\sigma_{1-\theta}(0) \sigma_{\theta}(x) \sigma_{1-\nu}(1) \sigma_{\nu}(\infty)\right\rangle=\frac{x^{-\theta(1-\theta)}(1-x)^{-\theta(1-\nu)}}{{ }_{2} F_{1}[\theta, 1-\nu, 1, x]} \sqrt{\frac{\sin (\pi \theta)}{t(x)}} \\
& \qquad \quad \times \sum_{\widetilde{p}, q} \exp \left[-\pi \frac{\sin (\pi \theta)}{t(x)} \frac{L_{a}^{2}}{\alpha^{\prime}} \widetilde{p}^{2}-\pi \frac{t(x)}{\sin (\pi \theta)} \frac{R_{1}^{2} R_{2}^{2}}{\alpha^{\prime} L_{a}^{2}} q^{2}\right] .
\end{aligned}
$$

with: $\quad t(x)=\frac{\sin (\pi \theta)}{2 \pi}\left(\frac{\Gamma(\theta) \Gamma(1-\nu)}{\Gamma(1+\theta-\nu)} \frac{{ }_{2} F_{1}[\theta, 1-\nu, 1+\theta-\nu ; 1-x]}{{ }_{2} F_{1}[\theta, 1-\nu, 1 ; x]}\right.$

$$
\left.+\frac{\Gamma(\nu) \Gamma(1-\theta)}{\Gamma(1+\nu-\theta)} \frac{{ }_{2} F_{1}[1-\theta, \nu, 1-\theta+\nu ; 1-x]}{{ }_{2} F_{1}[1-\theta, \nu, 1 ; x]}\right)
$$

## Recipe B

* A generic closed twisted-field correlator takes the form:

$$
\mathcal{A}_{\text {closed }}=|K(z)|^{2} \sum_{\vec{k}, \vec{v}} c_{\vec{k} \vec{v}} w(z)^{\frac{\alpha^{\prime} p_{L}^{2}}{4}} \bar{w}(\bar{z})^{\frac{\alpha^{\prime} p_{R}^{2}}{4}}
$$

where $w(z)$ is holomorphic, $\vec{k}, \vec{v} \in \Lambda^{*}$ and

$$
p_{L}^{2}=\left(\vec{k}+\frac{\vec{v}}{\alpha^{\prime}}\right)^{2} \quad p_{R}^{2}=\left(\vec{k}-\frac{\vec{v}}{\alpha^{\prime}}\right)^{2}
$$

* The open twisted-field correlator will look like (Hamiltonian Form):

$$
\mathcal{A}_{\text {open }}=K(x) \sum_{p, q} c_{p, q} w(x)^{\alpha^{\prime} p_{o p e n}^{2}}
$$

where for the simple case of a $D_{1}$-brane we have:

$$
p_{\text {open }}^{2}=\frac{1}{L^{2}} p^{2}+\frac{1}{\alpha^{\prime 2}} \frac{R_{1}^{2} R_{2}^{2}}{L^{2}} q^{2}
$$

* Our task is to bring the closed correlator to the above form and get the open one.


## Correlator: One angle

* The closed string result has the form:

$$
\begin{aligned}
\left|z_{\infty}\right|^{2 \theta(1-\theta)} & \left\langle\sigma_{1-\theta}(0) \sigma_{\theta}(z, \bar{z}) \sigma_{1-\theta}(1) \sigma_{\theta}(\infty)\right\rangle \\
& =\frac{\mathcal{C}}{V_{\Lambda}} \frac{|z(1-z)|^{-2 \theta(1-\theta)}}{\left.\left.\right|_{2} F_{1}[\theta, 1-\theta ; z]\right|^{2}} \times \sum_{k \in \Lambda^{*}, v_{1} \in \Lambda_{c}} \exp \left[-2 \pi i f_{23} \cdot k\right] w(z)^{\frac{1}{2}\left(k+\frac{v}{2}\right)^{2}} \bar{w}(\bar{z})^{\frac{1}{2}\left(k-\frac{v}{2}\right)^{2}}
\end{aligned}
$$

where $w(z)=\exp \left[\frac{i \pi \tau(z)}{\sin (\pi \theta)}\right]$ and $\tau(z)=\tau_{1}+i \tau_{2}=i \frac{{ }_{2} F_{1}[\theta, 1-\theta ; 1-z]}{{ }_{2} F_{1}[\theta, 1-\theta ; z]}$.

* The open string result has the form:

Dixon, Friedan, Martinec, Shenker
$x_{\infty}^{\theta(1-\theta)}\left\langle\sigma_{1-\theta}(0) \sigma_{\theta}(x) \sigma_{1-\theta}(1) \sigma_{\theta}(\infty)\right\rangle=\frac{L_{a}}{\alpha^{\prime}} \frac{[x(1-x)]^{-\theta(1-\theta)}}{{ }_{2} F_{1}[\theta, 1-\theta ; x]} \sum_{p, q} w(x)^{\left(\frac{\alpha^{\prime}}{L_{a}^{2}} p^{2}+\frac{1}{\alpha^{\prime}} \frac{R_{1}^{2} R_{2}^{2}}{L_{a}^{2}} q^{2}\right)}$
where $w(x)=\exp \left[-\frac{\pi t(x)}{\sin (\pi \theta)}\right]$ and $t(x)=\frac{1}{2 i}(\tau(x)-\bar{\tau}(x))=\frac{{ }_{2} F_{1}[\theta, 1-\theta ; 1-x]}{{ }_{2} F_{1}[\theta, 1-\theta ; x]}$.
after Poisson resummation the same result as via recipe A.

## Correlator: Two angles

* Following the same recipe, we get (original closed result is far too complicated to display here):


Burwick, Kaiser, Muller

$$
\begin{aligned}
& x_{\infty}^{\nu(1-\nu)}\left\langle\sigma_{1-\theta}(0) \sigma_{\theta}(x) \sigma_{1-\nu}(1) \sigma_{\nu}(\infty)\right\rangle= \\
& \quad \frac{\sqrt{\alpha^{\prime}}}{L_{a}} \frac{x^{-\theta(1-\theta)}(1-x)^{-\theta(1-\nu)}}{{ }_{2} F_{1}[\theta, 1-\nu, 1 ; x]} \sum_{p, q} w(x)\left(\frac{\alpha^{\prime}}{L_{a}^{2}} p^{2}+\frac{1}{\alpha^{\prime}} \frac{R_{1}^{2} R_{2}^{2}}{L_{a}^{2}} q^{2}\right)
\end{aligned}
$$

with the $w(x)=\exp \left[-\frac{\pi t(x)}{\sin (\pi \theta)}\right]$ and

$$
\begin{aligned}
t(x)= & \frac{\sin (\pi \theta)}{2 \pi}\left(\frac{\Gamma(\theta) \Gamma(1-\nu)}{\Gamma(1+\theta-\nu)} \frac{{ }_{2} F_{1}[\theta, 1-\nu, 1+\theta-\nu ; 1-x]}{{ }_{2} F_{1}[\theta, 1-\nu, 1 ; x]}\right. \\
& \left.+\frac{\Gamma(\nu) \Gamma(1-\theta)}{\Gamma(1+\nu-\theta)} \frac{{ }_{2} F_{1}[1-\theta, \nu, 1-\theta+\nu ; 1-x]}{{ }_{2} F_{1}[1-\theta, \nu, 1 ; x]}\right)
\end{aligned}
$$

* Again after Poisson resummation the same result as obtained via recipe A.


## Amplitude (Back)



- Computing: $\mathcal{A}=\langle\bar{\psi}(0) \psi(x) \chi(1) \bar{\chi}(\infty)\rangle$
which takes the form:

$$
\begin{aligned}
\mathcal{A}= & \operatorname{Tr}\left(\Lambda_{b a} \Lambda_{a b} \Lambda_{b c} \Lambda_{c b}\right) \bar{\psi}_{\dot{\alpha}} \psi^{\alpha} \chi^{\beta} \bar{\chi}_{\dot{\beta}} \quad[x(1-x)]^{-\frac{1}{4}} x_{\infty^{-\frac{3}{4}}} \\
& \int_{0}^{1} d x\left\langle e^{-\varphi / 2(0)} e^{-\varphi / 2(x)} e^{-\varphi / 2(1)} e^{-\varphi / 2(\infty)}\right\rangle
\end{aligned}
$$

$\epsilon_{\alpha \beta} \epsilon_{\dot{\alpha} \dot{\beta}}(1-x)^{-\frac{1}{2}} x_{\infty^{-\frac{1}{2}}} \times\left\langle S^{\dot{\alpha}}(0) S_{\alpha}(x) S_{\beta}(1) S^{\dot{\beta}}(\infty)\right\rangle\left\langle e^{i k_{1} X(0)} e^{i k_{2} X(x)} e^{i k_{3} X(1)} e^{i k_{4} X(\infty)}\right\rangle$

$$
\times\left\langle\sigma_{-\theta_{a b}^{3}}^{+}(0) \sigma_{-\theta_{a b}^{3}}^{-}(x) \sigma_{-\theta_{b c}^{3}}^{-}(1) \sigma_{-\theta_{b c}^{3}}^{+}(\infty)\right\rangle \quad x^{k_{1} \cdot k_{2}}(1-x)^{k_{2} \cdot k_{3}} x_{\infty}^{k_{4}\left(k_{1}+k_{2}+k_{3}\right)}
$$

$$
\begin{aligned}
& \times \prod_{I=1}^{2}\left\langle\sigma_{\theta_{a b}^{I}}^{-}(0) \sigma_{\theta_{a b}^{I}}^{+}(x) \sigma_{\theta_{b c}^{I}}^{+}(0) \sigma_{\theta_{b c}^{I}}^{-}(\infty)\right\rangle \\
& \times\left\langle e^{i\left(-\theta_{a b}^{3}-\frac{1}{2}\right) H^{3}(0)} e^{i\left(\theta_{a b}^{3}+\frac{1}{2}\right) H^{3}(x)} e^{i\left(\theta_{b c}^{3}+\frac{1}{2}\right) H^{3}(1)} e^{i\left(-\theta_{b c}^{3}-\frac{1}{2}\right) H^{3}(\infty)}\right\rangle \\
& \times \prod_{I=1}^{2}\left\langle e^{i\left(-\theta_{a b}^{I}+\frac{1}{2}\right) H^{I}(0)} e^{i\left(\theta_{a b}^{I}-\frac{1}{2}\right) H^{I}(x)} e^{i\left(\theta_{b c}^{I}-\frac{1}{2}\right) H^{I}(1)} e^{i\left(-\theta_{b c}^{I}+\frac{1}{2}\right) H^{I}(\infty)}\right\rangle
\end{aligned}
$$

$$
x^{\left(-\theta_{a b}^{3}-\frac{1}{2}\right)\left(\theta_{a b}^{3}+\frac{1}{2}\right)}(1-x)^{\left(\theta_{a b}^{3}+\frac{1}{2}\right)\left(\theta_{b c}^{3}+\frac{1}{2}\right)} x_{\infty}^{\left(-\theta_{b c}^{3}-\frac{1}{2}\right)\left(\left(-\theta_{a b}^{3}-\frac{1}{2}\right)+\left(\theta_{a b}^{3}+\frac{1}{2}\right)+\left(\theta_{b c}^{3}+\frac{1}{2}\right)\right)}
$$

## Amplitude (Back)



* Computing: $\mathcal{A}=\langle\bar{\psi}(0) \psi(x) \chi(1) \bar{\chi}(\infty)\rangle$
which takes the form:

$$
\begin{aligned}
\mathcal{A}= & \operatorname{Tr}\left(\Lambda_{b a} \Lambda_{a b} \Lambda_{b c} \Lambda_{c b}\right) \bar{\psi}_{\dot{\alpha}} \psi^{\alpha} \chi^{\beta} \bar{\chi}_{\dot{\beta}} \quad[x(1-x)]^{-\frac{1}{4}} x_{\infty^{-\frac{3}{4}}} \\
& \int_{0}^{1} d x\left\langle e^{-\varphi / 2(0)} e^{-\varphi / 2(x)} e^{-\varphi / 2(1)} e^{-\varphi / 2(\infty)}\right\rangle
\end{aligned}
$$

$\epsilon_{\alpha \beta} \epsilon_{\dot{\alpha} \dot{\beta}}(1-x)^{-\frac{1}{2}} x_{\infty^{-\frac{1}{2}}} \times\left\langle S^{\dot{\alpha}}(0) S_{\alpha}(x) S_{\beta}(1) S^{\dot{\beta}}(\infty)\right\rangle\left\langle e^{i k_{1} X(0)} e^{i k_{2} X(x)} e^{i k_{3} X(1)} e^{i k_{4} X(\infty)}\right\rangle$

$$
\times\left\langle\sigma_{-\theta_{a b}^{3}}^{+}(0) \sigma_{-\theta_{a b}^{3}}^{-}(x){\left.\left.\sigma_{-\theta_{b c}^{3}}^{-}(1) \sigma_{-\theta_{b c}^{3}}^{+}(\infty)\right\rangle \quad x^{k_{1} \cdot k_{2}}(1-x)^{k_{2} \cdot k_{3}} x_{\infty}^{k_{4}\left(k_{1}+k_{2}+k_{3}\right)}\right) .}^{2}\right.
$$

$$
\times \prod_{I=1}^{2}\left\langle\sigma_{\theta_{a b}^{I}}^{-}(0) \sigma_{\theta_{a b}^{I}}^{+}(x) \sigma_{\theta_{b c}^{I}}^{+}(0) \sigma_{\theta_{b c}^{I}}^{-}(\infty)\right\rangle
$$

$$
\times\left\langle e^{i\left(-\theta_{a b}^{3}-\frac{1}{2}\right) H^{3}(0)} e^{i\left(\theta_{a b}^{3}+\frac{1}{2}\right) H^{3}(x)} e^{i\left(\theta_{b c}^{3}+\frac{1}{2}\right) H^{3}(1)} e^{i\left(-\theta_{b c}^{3}-\frac{1}{2}\right) H^{3}(\infty)}\right\rangle
$$

$$
\times \prod_{I=1}^{2}\left\langle e^{i\left(-\theta_{a b}^{I}+\frac{1}{2}\right) H^{I}(0)} e^{i\left(\theta_{a b}^{I}-\frac{1}{2}\right) H^{I}(x)} e^{i\left(\theta_{b c}^{I}-\frac{1}{2}\right) H^{I}(1)} e^{i\left(-\theta_{b c}^{I}+\frac{1}{2}\right) H^{I}(\infty)}\right\rangle
$$

$$
x^{\left(-\theta_{a b}^{3}-\frac{1}{2}\right)\left(\theta_{a b}^{3}+\frac{1}{2}\right)}(1-x)^{\left(\theta_{a b}^{3}+\frac{1}{2}\right)\left(\theta_{b c}^{3}+\frac{1}{2}\right)} x_{\infty}^{\left(-\theta_{b c}^{3}-\frac{1}{2}\right)\left(\left(-\theta_{a b}^{3}-\frac{1}{2}\right)+\left(\theta_{a b}^{3}+\frac{1}{2}\right)+\left(\theta_{b c}^{3}+\frac{1}{2}\right)\right)}
$$

## Amplitude

* Combining all together we get:

$$
\begin{aligned}
\mathcal{A}=i g(\mathcal{C} & \operatorname{Tr}\left(\Lambda_{b a} \Lambda_{a b} \Lambda_{b c} \Lambda_{c b}\right) \bar{\psi} \cdot \bar{\chi} \psi \cdot \chi(2 \pi)^{4} \delta^{(4)}\left(\sum_{i}^{4} k_{i}\right) \\
& \times \int_{0}^{1} d x \frac{x^{-1+k_{1} \cdot k_{2}}(1-x)^{-\frac{3}{2}+k_{2} \cdot k_{3}} e^{-S_{c l}\left(\theta_{a b}^{1}, 1-\theta_{b c}^{1}\right)} e^{-S_{c l}\left(\theta_{a b}^{2}, 1-\theta_{b c}^{2}\right)} e^{-S_{c l}\left(1+\theta_{a b}^{3},-\theta_{b c}^{3}\right)}}{\left[I\left(\theta_{a b}^{1}, 1-\theta_{b c}^{1}, x\right) I\left(\theta_{a b}^{2}, 1-\theta_{b c}^{2}, x\right) I\left(1+\theta_{a b}^{3},-\theta_{b c}^{3}, x\right)\right]^{\frac{1}{2}}}
\end{aligned}
$$

where

$$
\begin{aligned}
& I(\theta, \nu, x)= \frac{1}{2 \pi}\left\{\frac{\Gamma(\theta) \Gamma(1-\nu)}{\Gamma(1+\theta-\nu)}{ }_{2} F_{1}[1-\theta, \nu, 1 ; x]_{2} F_{1}[\theta, 1-\nu, 1+\theta-\nu ; 1-x]\right. \\
&\left.\quad+\frac{\Gamma(\nu) \Gamma(1-\theta)}{\Gamma(1+\nu-\theta)}{ }_{2} F_{1}[\theta, 1-\nu, 1 ; x]_{2} F_{1}[1-\theta, \nu, 1-\theta+\nu ; 1-x]\right\} \\
& e^{-S_{c l}(\theta, \nu)}=\sum_{\widetilde{p}_{i}, q_{i}} \exp \left[-\pi \frac{\sin (\pi \theta)}{t(\theta, \nu, x)} \frac{L_{b^{i}}^{2}}{\alpha^{\prime}} \widetilde{p}_{i}^{2}-\pi \frac{t(\theta, \nu, x)}{\sin (\pi \theta)} \frac{R_{x_{i}}^{2} R_{y_{i}}^{2}}{\alpha^{\prime} L_{b^{i}}^{2}} q_{i}^{2}\right]
\end{aligned}
$$

* Finally, we need to normalize the amplitude.


## Amplitude at the s-channel ( $x \rightarrow 0$ )

* At the limit $x \rightarrow 0$ the amplitude factorizes on the exchange of a gauge boson:

* that allows to normalize the amplitude ( $\left.p_{i}=q_{i}=0\right)$

$$
A_{4}\left(k_{1}, k_{2}, k_{3}, k_{4}\right)=\mathrm{i} \int \frac{\mathrm{~d}^{4} k \mathrm{~d}^{4} k^{\prime}}{(2 \pi)^{4}} \frac{\sum_{g} A_{\mu}^{g}\left(k_{1}, k_{2}, k\right) A^{g, \mu}\left(k_{3}, k_{4}, k^{\prime}\right) \delta^{(4)}\left(k-k^{\prime}\right)}{k^{2}-\mathrm{i} \epsilon}
$$

with

$$
A_{\mu}^{g}\left(k_{1}, k_{2}, k_{3}\right)=\mathrm{i} \sqrt{(2 \pi)^{4} \frac{\alpha^{\prime 3 / 2} g_{s}}{\prod_{i=1}^{3} 2 \pi L_{b_{i}}}}(2 \pi)^{4} \delta^{(4)}\left(\sum_{i=1}^{3} k_{i}\right) \bar{\psi} \sigma^{\mu} \psi \operatorname{Tr}\left(\Lambda_{b a} \Lambda_{a b} \Lambda_{b b}\right)
$$

* Comparing the two results we normalize the amplitude to: $\mathcal{C}=2 \pi$.


## Amplitude at the s-channel $(x \rightarrow 0)$

* At the limit $x \rightarrow 0$ there are other (higher order) poles corresponding to other massive exchanges:

* Other poles that arise from $p_{i} \neq 0 \neq q_{i}$, they correspond to KK and winding states exchanges.
* Additional poles from higher order poles of the "quantum part" corresponding to Regge excitations.
* Similar pole structure than the behavior of amplitudes containing at most two chiral fermions. Thus "universal behavior" dressed with poles arising from KK and winding states.


## Amplitude at the t-channel $(x \rightarrow 1)$

* In the limit $x \rightarrow 1$ the amplitude factorizes on the exchange of scalar particles

* In this limit the amplitude takes the form (SUSY preserved, ignore WS-instantons):

$$
\mathcal{A}=\bar{\psi} \cdot \bar{\chi} \psi \cdot \chi \int_{1-\epsilon}^{1} d x(1-x)^{-1+k_{2} \cdot k_{3}} \Gamma_{1-\theta_{a b}^{1}, 1-\theta_{b c}^{1}, \theta_{a b}^{1}+\theta_{b c}^{1}}^{-\frac{1}{2}} \Gamma_{1-\theta_{a b}^{2}, 1-\theta_{b c}^{2}, \theta_{a b}^{2}+\theta_{b c}^{2}}^{-\frac{1}{\downarrow}} \Gamma_{-\theta_{a b}^{3},-\theta_{b c}^{3}, 2+\theta_{a b}^{3}+\theta_{b c}^{3}}^{-\frac{1}{2}}
$$

massless scalar exchange

## Subdominant poles

* Assuming that $1-\theta_{a b}^{1}-\theta_{b c}^{1}=-\theta_{c a}^{1}$ is small, the amplitude becomes:

$$
\begin{aligned}
& \quad \mathcal{A}=\bar{\psi} \cdot \bar{\chi} \psi \cdot \chi \int_{1-\epsilon}^{1} d x(1-x)^{-1+k_{2} \cdot k_{3}} Y_{\psi \chi \phi}^{2}\left(1+c_{1}(1-x)^{2\left(1-\theta_{a b}^{1}-\theta_{b c}^{1}\right)}+\ldots\right) \\
& * \text { Thus we have the exchange of: } \\
& \quad \text { - a massless scalar } \Phi: \prod_{I} \psi_{-1 / 2-\theta_{c a}^{I}}\left|\theta_{1,2,3}^{c a}\right\rangle_{N S}
\end{aligned}
$$

- a massive scalar $\widetilde{\widetilde{\Phi}}:\left(\alpha_{\theta_{c a}^{1}}\right)^{2} \prod_{I} \psi_{-1 / 2-\theta_{c a}^{I}}\left|\theta_{1,2,3}^{c a}\right\rangle_{N S}$ with $M^{2}=-2 \theta_{c a}^{1} M_{s}^{2}$.
* Note that there is no coupling to the lightest massive field with mass $M^{2}=-\theta_{c a}^{1} M_{s}^{2}$.
* This is can be traced back to the fact that the two bosonic twist fields $\sigma_{\alpha}$ and $\sigma_{\beta}$ do not couple to the excited twist field $\tau_{\alpha+\beta}$, but only to an even excited twist field.

$$
\sigma_{\alpha}(w) \sigma_{\beta}(z) \sim C_{\sigma}(z-w)^{-\alpha \beta} \sigma_{\alpha+\beta}+C_{\rho}(z-w)^{-\alpha \beta+2-2 \alpha-2 \beta} \rho_{\alpha+\beta}
$$

## Further poles

* The exchange particle is a scalar field, thus the signatures induced by the tower $\left(\alpha_{\theta_{c a}^{1}}\right)^{m} \prod_{I} \psi_{-1 / 2-\theta_{c a}^{I}}|0\rangle$ resembles signatures of KK states in extra-dimensional theories.
* Above just the first sub-dominant poles, but there are many more poles.
* In case the fermions are too much separated in the internal manifold WS-instantons cannot be ignored; poles arising from them correspond to exchanges of KK and winding excitations.
* There are also integer poles, that correspond to exchange of Higher Spin states.
* Other poles that correspond to exchanges of massive scalar fields whose mass is nonvanishing even for vanishing intersection angles.
* Rich spectrum of signatures, but the first once to be observed correspond to lightest string states.


## Conclusions

* We have studied the spectrum of open strings localized at the intersections of D6branes.
* The masses of such states scale as $M^{2} \approx \theta M_{s}^{2}$ and can thus be parametrically smaller than the string scale if the relevant angle is small.
* We have considered scattering amplitudes that expose such light stringy states.


## Along the computation

Give a description to formulate the vertex operators for states localized at intersections.
Rederived the four bosonic twist field correlator with one and two independent angles.

* Investigated s- and t-channel and found poles corresponding to light stringy states.
* Assuming a scenario with a low string scale, these states may be observable at LHC.
* However further poles corresponding to KK and winding states, as well as Higher Spin states
$\leadsto \quad$ Rich spectrum of signatures

