

NLO MONTE CARLO TOOLS FOR HIGGS PHYSICS AT THE LHC

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- Introduction to NLO Monte Carlo programs
- Higgs boson production in gluon fusion: H , H_j and H_{jj}
- H in VBF
- $t\bar{t}H$
- VH
- tH^\pm



Higgs boson production

- H in gluon fusion: MC@NLO, POWHEG BOX, POWHEG+SHERPA, POWHEG+HERWIG++, MC@NLO+SHERPA
- $H+1\text{jet}$: POWHEG+SHERPA, MC@NLO+SHERPA, POWHEG BOX
- $H+2\text{jet}$: POWHEG BOX
- H in VBF: POWHEG BOX, POWHEG+HERWIG++
- $t\bar{t}H$: POWHEG BOX + HELAS, aMC@NLO
- VH : POWHEG+HERWIG++, MC@NLO
- tH^\pm : MC@NLO, POWHEG BOX
- $H \rightarrow Q\bar{Q}$: POWHEG+HERWIG++

NLO vs Shower Monte Carlo

NLO

- ✓ accurate shapes at high p_T
- ✓ normalization accurate at NLO order
- ✓ reduced dependence on renormalization and factorization scales
- ✗ wrong shapes at small p_T
- ✗ description only at the parton level

SMC (LO + shower)

- ✗ bad description at high p_T
- ✗ normalization accurate only at LO
- ✓ correct Sudakov suppression at small p_T
- ✓ simulate events at the hadron level

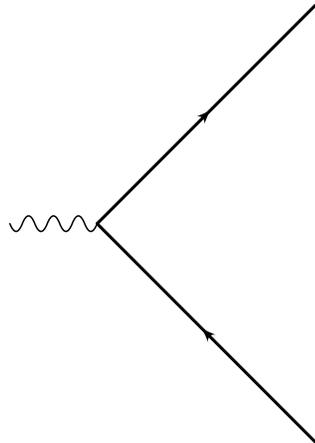
It is natural to try to merge the two approaches, keeping the good features of both

MC@NLO [Frixione and Webber, 2001] and POWHEG [Nason, 2004] do this in a consistent way

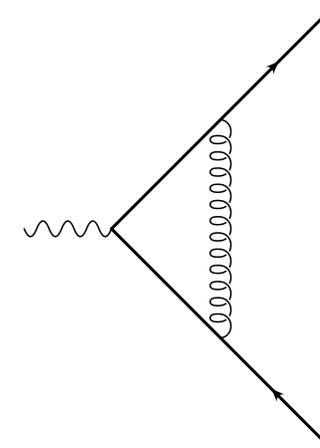
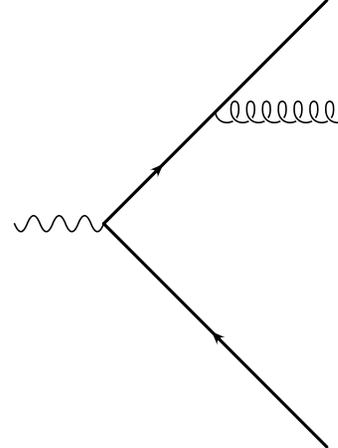
NLO + Parton Shower

The main problem in **merging** a **NLO** result and a **Parton Shower** is **not to double-count** radiation: the shower might produce some radiation **already present** at the NLO level (both at the **virtual** and at the **real** level).

LO:



NLO:



NLO differential cross section

The $(n + 1)$ -body phase space $d\Phi_{n+1}$ can be factorized in term of the n -body phase space Φ_n times the radiation phase space $d\Phi_r$: $d\Phi_{n+1} = d\Phi_n \times d\Phi_r$

$$d\sigma_{\text{NLO}} = d\Phi_n \left\{ B(\Phi_n) + V(\Phi_n) + [R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)] d\Phi_r \right\}$$

$$d\Phi_{n+1} = d\Phi_n \times d\Phi_r \qquad d\Phi_r \div dt dz d\varphi$$

$$V(\Phi_n) = V_b(\Phi_n) + \int d\Phi_r C(\Phi_n, \Phi_r) \quad \Leftarrow \text{finite}$$

$$d\sigma_{\text{SMC}} = B(\Phi_n) d\Phi_n \left\{ \Delta_{t_0} + \frac{\alpha_s}{2\pi} P(z) \frac{1}{t} \Delta_t d\Phi_r \right\}$$

$$\Delta_t = \exp \left[- \int d\Phi'_r \frac{\alpha_s}{2\pi} P(z') \frac{1}{t'} \theta(t' - t) \right] \quad \text{SMC Sudakov form factor}$$

The POWHEG differential cross section

$R = R_s + R_f$ with $R_s > 0$, $R_f > 0$, R_s singular in the infrared regions, R_f finite in collinear and soft limit. Define

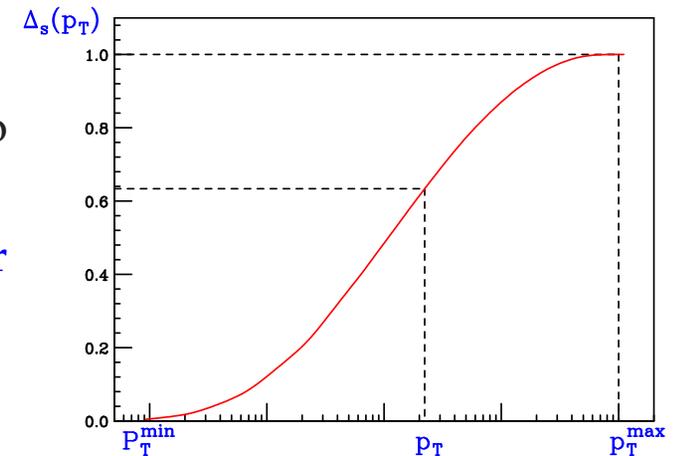
$$d\sigma = \bar{B}_s(\Phi_n) \underbrace{\left\{ \Delta_s(p_T^{\min}) + \Delta_s(p_T) \frac{R_s(\Phi_{n+1})}{B(\Phi_n)} d\Phi_r \right\}}_{1 \text{ by unitarity}} d\Phi_n + R_f(\Phi_{n+1}) d\Phi_{n+1}$$

$$\bar{B}_s(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_r [R_s(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)]$$

$$\Delta_s(p_T) = \exp \left[- \int d\Phi_r' \frac{R_s(\Phi_n, \Phi_r')}{B(\Phi_n)} \theta(p_T' - p_T) \right]$$

The expansion of $d\sigma$ up to the NLO level is exactly equal to $d\sigma_{\text{NLO}}$.

The part of the real cross section that is treated with the shower technique can be varied.



MC@NLO in the POWHEG language

Write the MC@NLO hardest jet cross section in the POWHEG language. Hardest emission can be written as [Nason 2004]

$$d\sigma = \underbrace{\bar{B}_{\text{HW}} d\Phi_n}_{\text{S event}} \underbrace{\left[\Delta_{\text{HW}}(p_T^{\min}) + \Delta_{\text{HW}}(p_T) \frac{R_{\text{HW}}(\Phi_{n+1})}{B(\Phi_n)} d\Phi_r \right]}_{\text{HERWIG event}} + \underbrace{\left[R(\Phi_{n+1}) - R_{\text{HW}}(\Phi_{n+1}) \right] d\Phi_{n+1}}_{\text{H event}}$$

$$\bar{B}_{\text{HW}}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int \left[R_{\text{HW}}(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r) \right] d\Phi_r$$

$$\Delta_{\text{HW}}(p_T) = \exp \left[- \int d\Phi'_r \frac{R_{\text{HW}}(\Phi_n, \Phi'_r)}{B(\Phi_n)} \theta(p'_T - p_T) \right]$$

Like POWHEG with $\begin{cases} R_s = R_{\text{HW}} \\ R_f = R - R_{\text{HW}} \end{cases} \iff \text{can be negative}$

This formula illustrates why MC@NLO and POWHEG are **equivalent at NLO!**

But differences can arise at **NNLO**.

The radiation cross section

$$\bar{B}_s(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_r [R_s(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)]$$

$$d\sigma = \bar{B}_s(\Phi_n) \left\{ \Delta_s(p_T^{\min}) + \Delta_s(p_T) \frac{R_s(\Phi_{n+1})}{B(\Phi_n)} d\Phi_r \right\} d\Phi_n + R_f(\Phi_{n+1}) d\Phi_{n+1}$$

The differential cross section describing the **hard radiation** is given by

$$\begin{aligned} d\sigma_{\text{rad}} &\approx \frac{\bar{B}_s(\Phi_n)}{B(\Phi_n)} R_s(\Phi_{n+1}) d\Phi_{n+1} + R_f(\Phi_{n+1}) d\Phi_{n+1} \\ &= \left\{ R_s(\Phi_{n+1}) + R_f(\Phi_{n+1}) + \left[\frac{\bar{B}_s(\Phi_n)}{B(\Phi_n)} - 1 \right] R_s(\Phi_{n+1}) \right\} d\Phi_{n+1} \\ &= R(\Phi_{n+1}) d\Phi_{n+1} + \mathcal{O}(\alpha_s) R_s(\Phi_{n+1}) \end{aligned}$$

- We expect differences at the **NNLO** level. While **formally** at NNLO, they may be large for particular processes (see i.e. Higgs boson production in gluon fusion).
- Notice that the $\bar{B}_s(\Phi_n)/B(\Phi_n)$ **also depends** on how the real contribution R has been **split** into R_s and R_f .

Sources of possible differences

$$d\sigma_{\text{rad}} \approx \frac{\bar{B}_s(\Phi_n)}{B(\Phi_n)} R_s(\Phi_{n+1}) d\Phi_{n+1} + R_f(\Phi_{n+1}) d\Phi_{n+1}$$

In an NLO+Parton Shower implementation, visible differences of the radiation cross section with respect to the fixed-order result will be present, due to

1. The $\Delta_s(p_T)$ factor, **dropped** in the NLO-accuracy derivation.
The Sudakov factor yields **resummation-improved** results at NLO.
It is **less than 1**: it always **reduces** the transverse-momentum spectrum of radiation with respect to the pure NLO result
2. The $\bar{B}_s(\Phi_n)/B(\Phi_n)$ factor, also dropped.
This factor spreads the K factor over the finite p_T region. The spreading of the K factor depends upon the R_s and R_f separation.
3. The choice of **scales** used in the process.

In summary

Experience in comparing MC@NLO and POWHEG results (various papers from the POWHEG BOX and from the Herwig++ collaborations) has shown that

- **all important differences** between MC@NLO and POWHEG can be tracked back to the rôle of the \bar{B}_s/B factor and to **scale-choice issues**.
- **Exponentiation** in $\Delta_s(p_T)$ does **not** seem to yield **important differences**. This is understood as due to the fact that the integral in

$$\Delta_s(p_T) = \exp \left[- \int d\Phi'_r \frac{R_s(\Phi_n, \Phi'_r)}{B(\Phi_n)} \theta(p'_T - p_T) \right]$$

is dominated by the region of **soft** p_T , where all R_s agree.

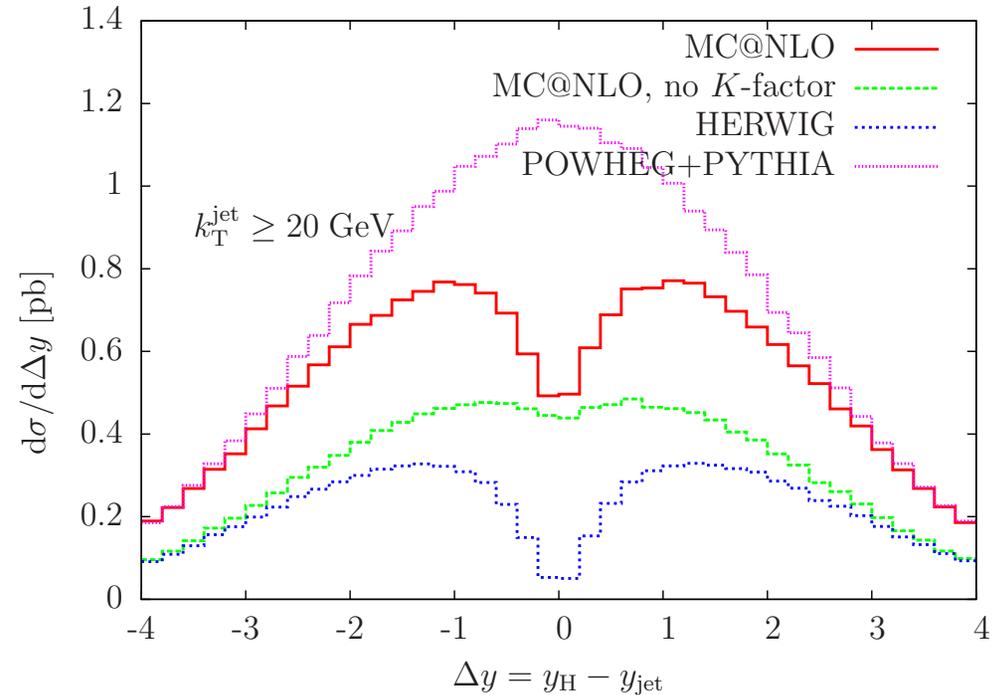
As seen before, at high p_T , the MC@NLO radiation cross section goes as

$$d\sigma \approx \underbrace{R(\Phi_{n+1})}_{\text{no dip}} d\Phi_{n+1} + \underbrace{\left(\frac{\bar{B}_{\text{HW}}(\Phi_n)}{B(\Phi_n)} - 1 \right)}_{\mathcal{O}(\alpha_s) \text{ but large for Higgs}} \underbrace{R_{\text{HW}}(\Phi_{n+1})}_{\text{pure HERWIG dip}} d\Phi_{n+1}$$

So: a **contribution** with a **dip** is added to the exact NLO result.

The contribution is $\mathcal{O}(\alpha_s R)$, i.e. **NNLO**

If we **replace** $\bar{B}_{\text{HW}} \rightarrow B$ in MC@NLO, the dip disappears



[Alioli, Nason, Re and C.O., 2008; Hamilton, Richardson and Tully, 2009; Nason and Webber, 2012]

Scale dependence

$$d\sigma = \bar{B}_s(\Phi_n, \mu_R) d\Phi_n \left\{ \Delta_s(\Phi_n, p_T^{min}) + \Delta_s(\Phi_n, p_T) \frac{R_s(\Phi_n, \Phi_r, \alpha_s(k_T))}{B(\Phi_n)} d\Phi_r \right\} \\ + R_f(\Phi_{n+1}, \alpha_s(\mu_R)) d\Phi_{n+1}$$

$$\bar{B}_s(\Phi_n, \mu_R) = B(\Phi_n) + V(\Phi_n, \alpha_s(\mu_R)) + \int d\Phi_r [R_s(\Phi_n, \Phi_r, \alpha_s(\mu_R)) - C(\Phi_n, \Phi_r, \alpha_s(\mu_R))]$$

$$\Delta_s(\Phi_n, p_T) = \exp \left[- \int d\Phi_r' \frac{R_s(\Phi_n, \Phi_r', \alpha_s(k_T))}{B(\Phi_n)} \theta(k_T(\Phi_n, \Phi_r') - p_T) \right]$$

- A scale variation in the curly braces $\{ \}$ is in practice never performed (in order not to spoil the NLL accuracy of the Sudakov form factor). The scale in the Sudakov has to go **exactly** to the **transverse momentum** of the radiation in the collinear **and** soft region.
- Scale dependence affects \bar{B}_s and R_f **differently**: \bar{B}_s is a quantity **integrated** over the radiation kinematics \implies milder scale dependence

Similar conclusions for the factorization scale μ_F

Scale dependence in $gg \rightarrow H$

$$d\sigma = \bar{B}_s(\Phi_n, \mu_R) d\Phi_n \left\{ \Delta_s(\Phi_n, p_T^{\min}) + \Delta_s(\Phi_n, p_T) \frac{R_s(\Phi_n, \Phi_r, \alpha_s(k_T))}{B(\Phi_n)} d\Phi_r \right\} \\ + R_f(\Phi_{n+1}, \alpha_s(\mu_R)) d\Phi_{n+1}$$

$$\bar{B}_s(\Phi_n, \mu_R) = B(\Phi_n) + V(\Phi_n, \alpha_s(\mu_R)) + \int d\Phi_r [R_s(\Phi_n, \Phi_r, \alpha_s(\mu_R)) - C(\Phi_n, \Phi_r, \alpha_s(\mu_R))]$$

- The \bar{B} prefactor is of order α_s^2 at the Born level, and it includes NLO corrections of order α_s^3 . Its scale variation must therefore be of order α_s^4 .
Therefore the relative scale variation $\delta\bar{B}/\bar{B}$ is of order α_s^2 .
- On the other hand, the R_f term (H in MC@NLO) is of order α_s^3 , and its scale variation is of order $\alpha_s^4 \implies$ its relative scale variation is of order α_s .

Thus, the **larger the contribution** to the transverse momentum distribution coming from R_s (or S in MC@NLO) events, the **smaller its relative scale** dependence will be.

Scale dependence in $gg \rightarrow H$

- $gg \rightarrow H$ at NLO+PS
- $m_H = 120$ GeV
- $0.5 < \mu_R/\mu_F < 2$ around central reference scale μ
- Comparison with HqT [Catani, Grazzini et al.]: NNLL + NNLO. “Switched” result, with resummation scale $Q = m_H/2$ and reference factorization and renormalization scale $\mu = m_H$, as recommended by the authors
- MSTW2008NNLO central pdf for all the curves. This pdf set is needed by HqT. Used for all the other programs, since we want to focus on the differences that have to do with the calculation, rather than the pdf

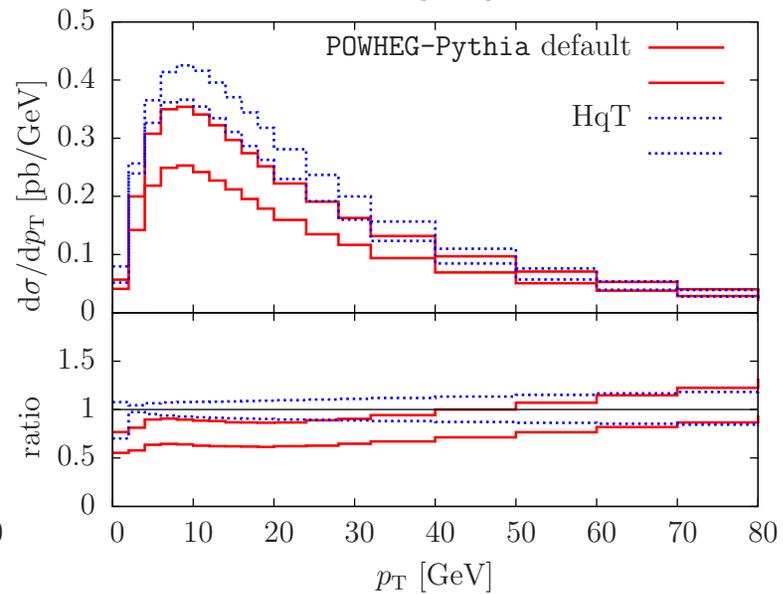
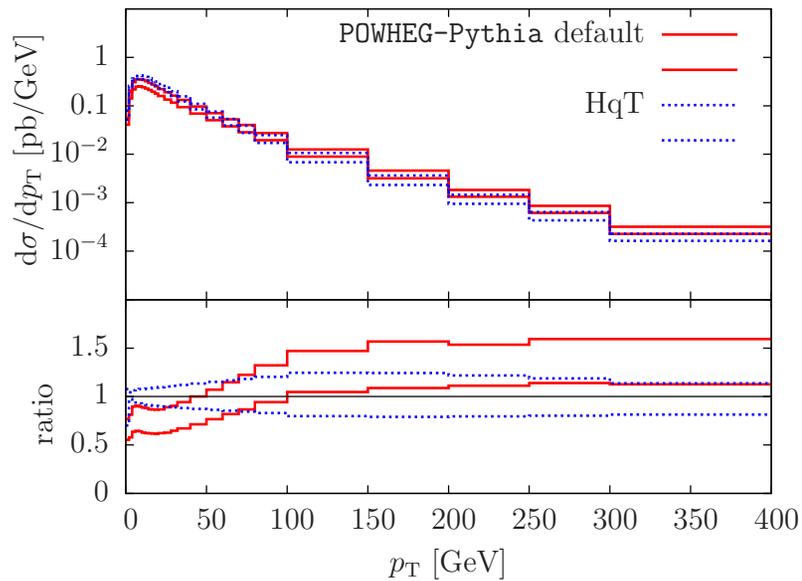
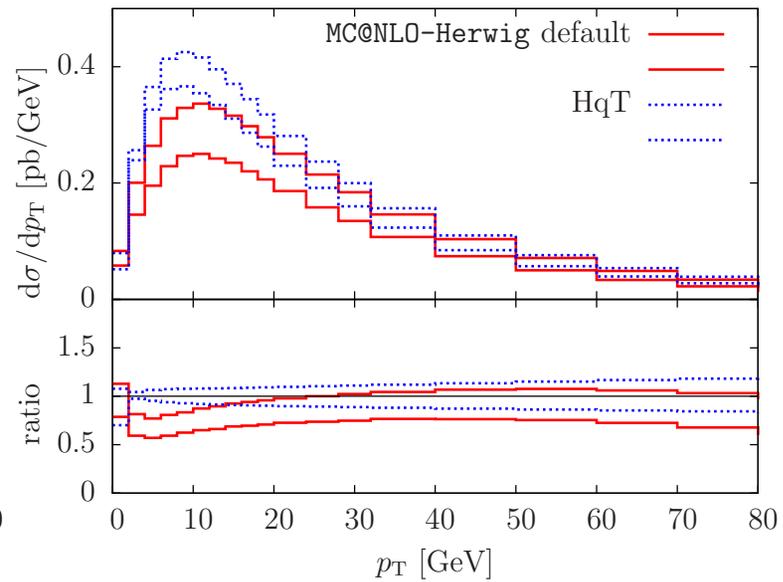
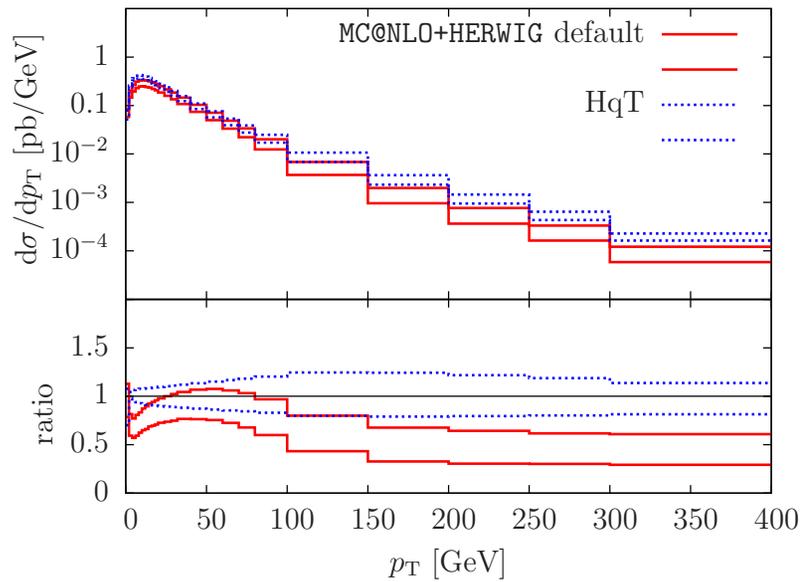
The R_s and R_f terms are chosen such that

$$R_s = \frac{h^2}{p_T^2 + h^2} R, \quad R_f = \frac{p_T^2}{p_T^2 + h^2} R, \quad R = R_s + R_f$$

If $h \rightarrow 0$, the NLO prediction is recovered, but the Sudakov region is dangerously squeezed and distorted.

If $h \rightarrow \infty$, $R_s = R$ and $R_f \rightarrow 0$ and the whole real contribution enters the Sudakov form factor. This is the default POWHEG BOX setting.

The NLO K factor, \bar{B}/B multiplies uniformly the whole transverse-momentum distribution



DEFAULT VALUES

POWHEG

$$\mu = m_H$$

$$h = \infty$$

MC@NLO

$$\mu = m_T = \sqrt{m_H^2 + p_T^2}$$

high p_T

$$\frac{\text{POWHEG}}{\text{MC@NLO}} \approx 3 = 2 \times 1.6$$

K fac ≈ 2

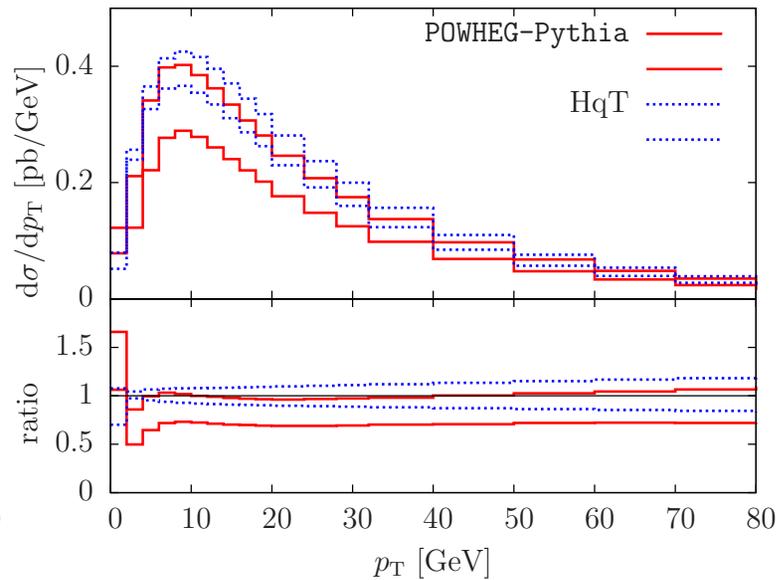
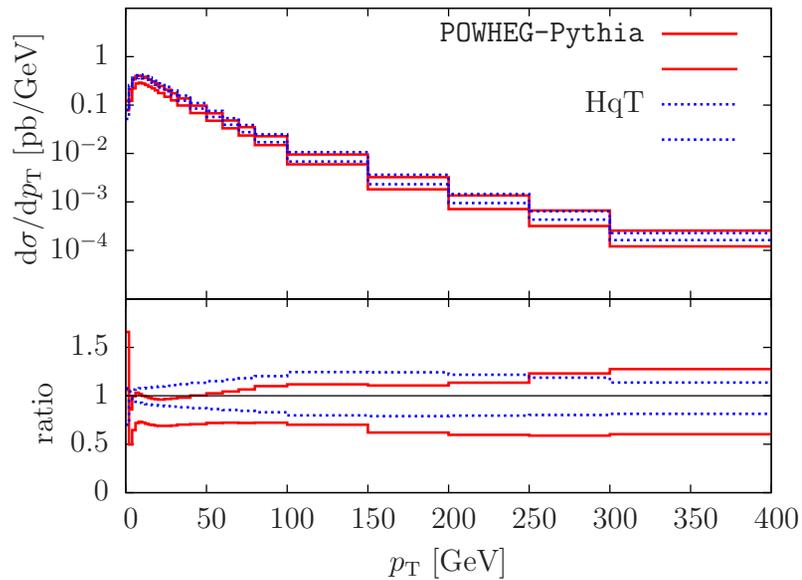
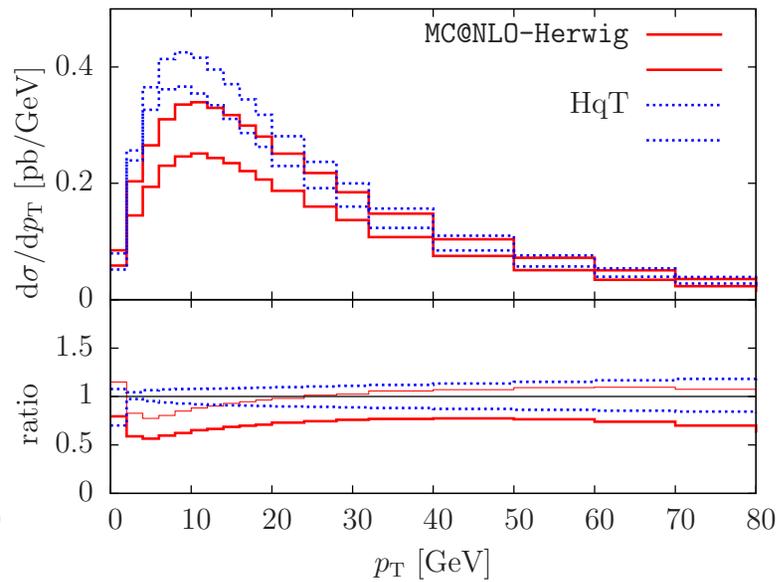
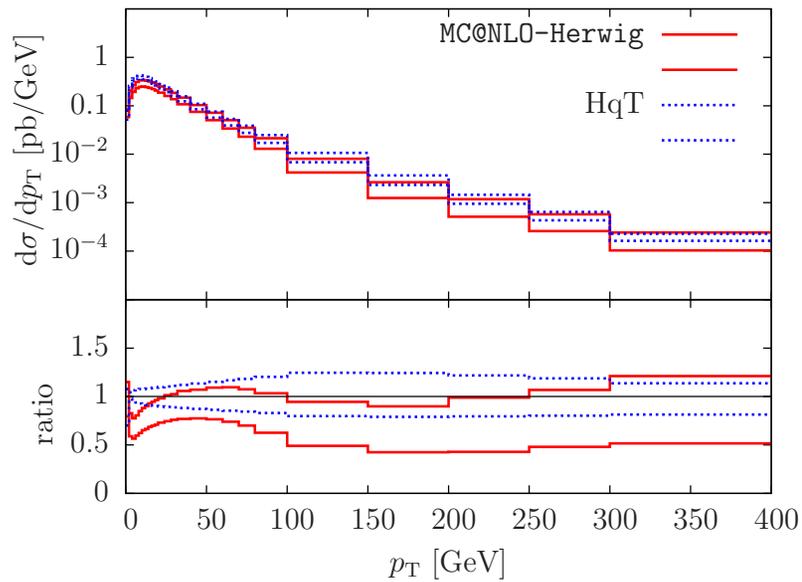
$$(\alpha_s(m_T)/\alpha_s(m_H))^3 \approx 1.6$$

in the last bin

narrow band at small p_T

larger band at large p_T

[YRHXS2; Nason and Webber, 2012]



BEST VALUES

POWHEG

$\mu = m_H$

$h = \mu/1.2$

MC@NLO

$\mu = m_H$

$h = 100$ GeV,

the **large- p_T** tail in

POWHEG and MC@NLO

are now very **similar**

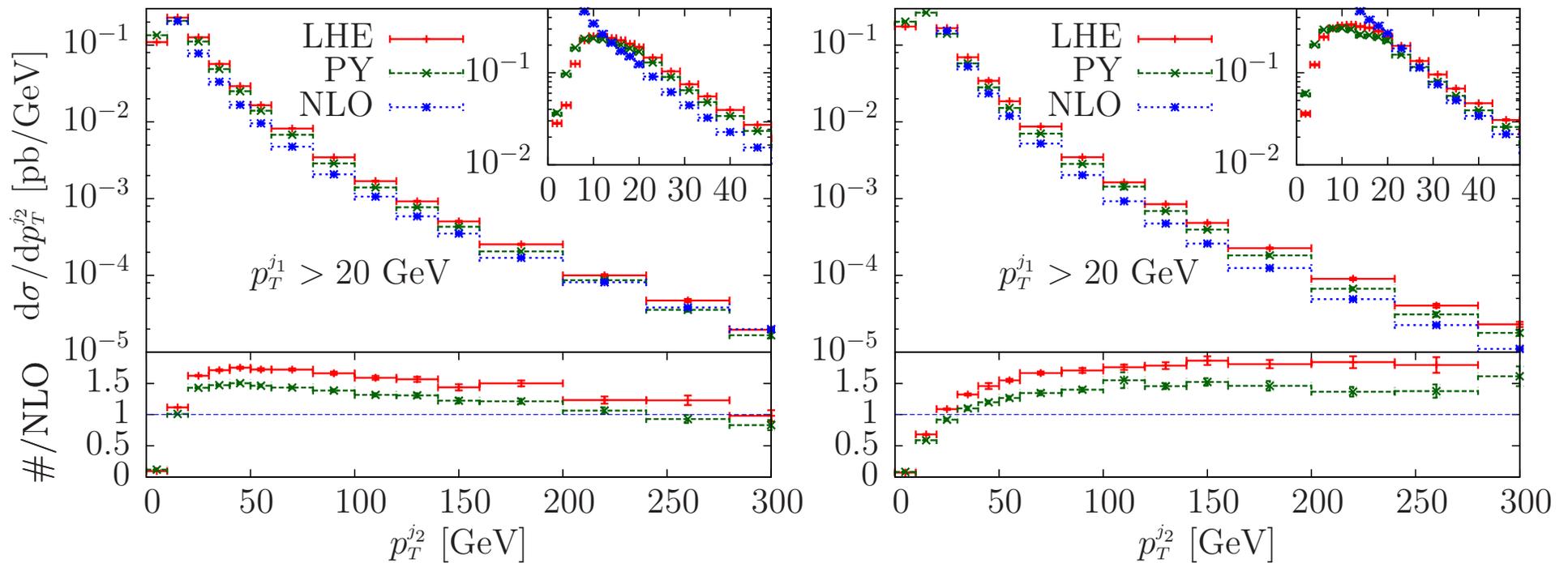
narrow band at **small p_T**

larger band at **large p_T**

Question: what is the most “appropriate” scale in the high- p_T region?

[YRHXS2; Nason and Webber, 2012]

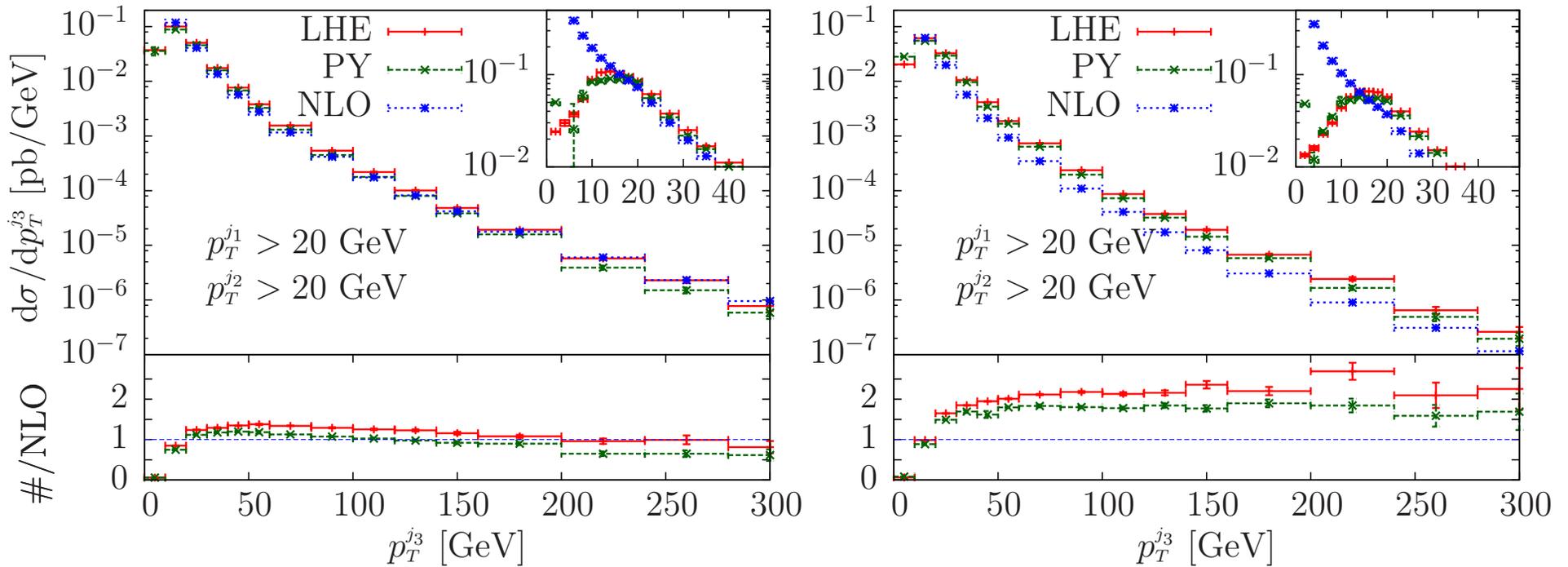
Higgs boson plus 1 jet production



- Diverging NLO, Sudakov suppression in LHE
- The trend of the \bar{B}_S/B factor very evident
- In LHE results, one power of α_s is evaluated at the p_T of the radiation

[Campbell, Ellis, Frederix, Nason, Williams, C.O., 2012]

Higgs boson plus 2 jet production

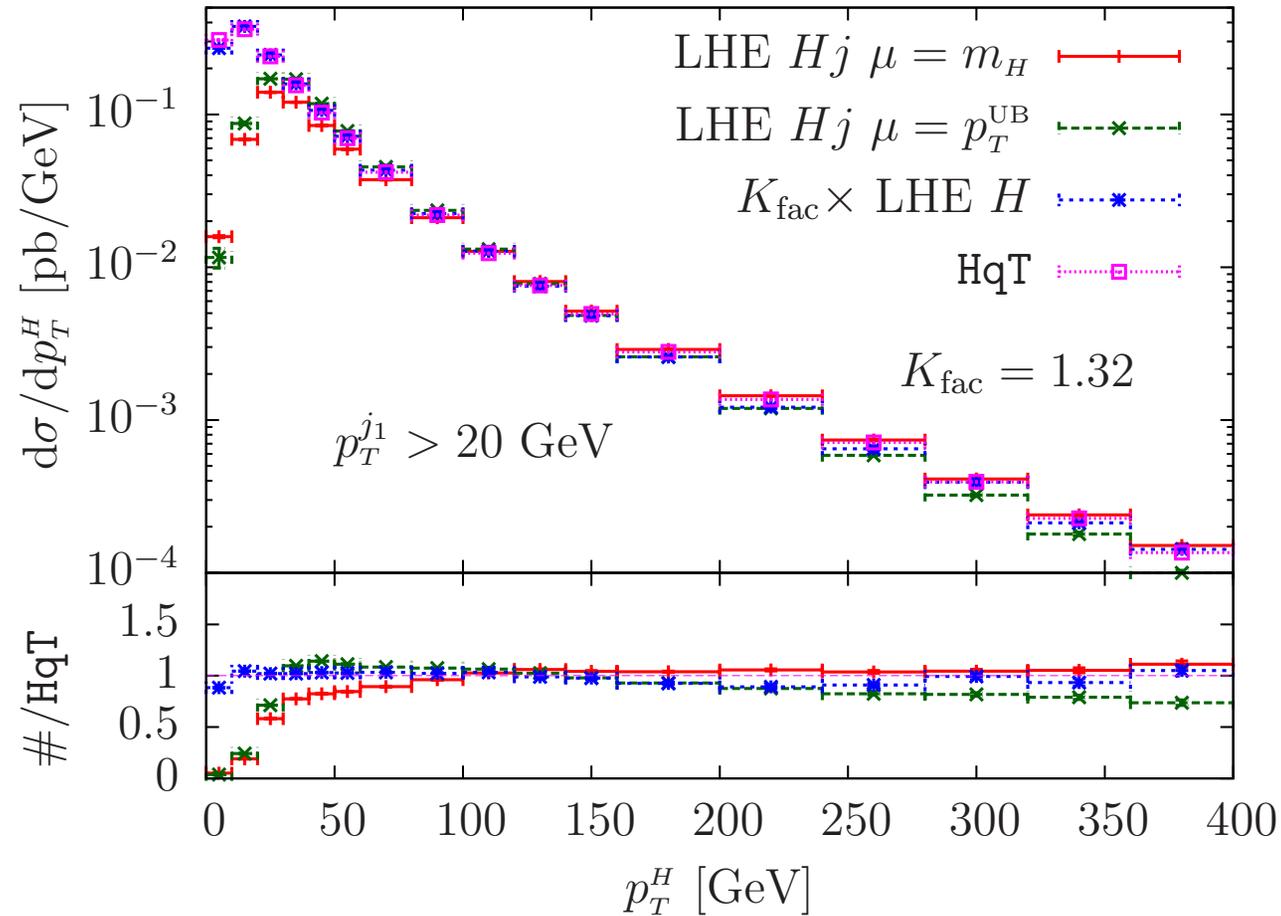


Left: $\mu_F = \mu_R = m_H$. **Right:** $\mu_F = \mu_R = \hat{H}_T = m_T^H + \sum_i p_{T_i}$ where $m_T^H = \sqrt{m_H^2 + (p_T^H)^2}$ and p_{T_i} are the final-state parton transverse momenta in the underlying-Born kinematics.

- The trend of the \overline{B}_s/B factor very evident
- K factor close to 1 for fixed scales
- These are two “extreme” scales. \hat{H}_T too big at large p_T

[Campbell, Ellis, Frederix, Nason, Williams, C.O., 2012]

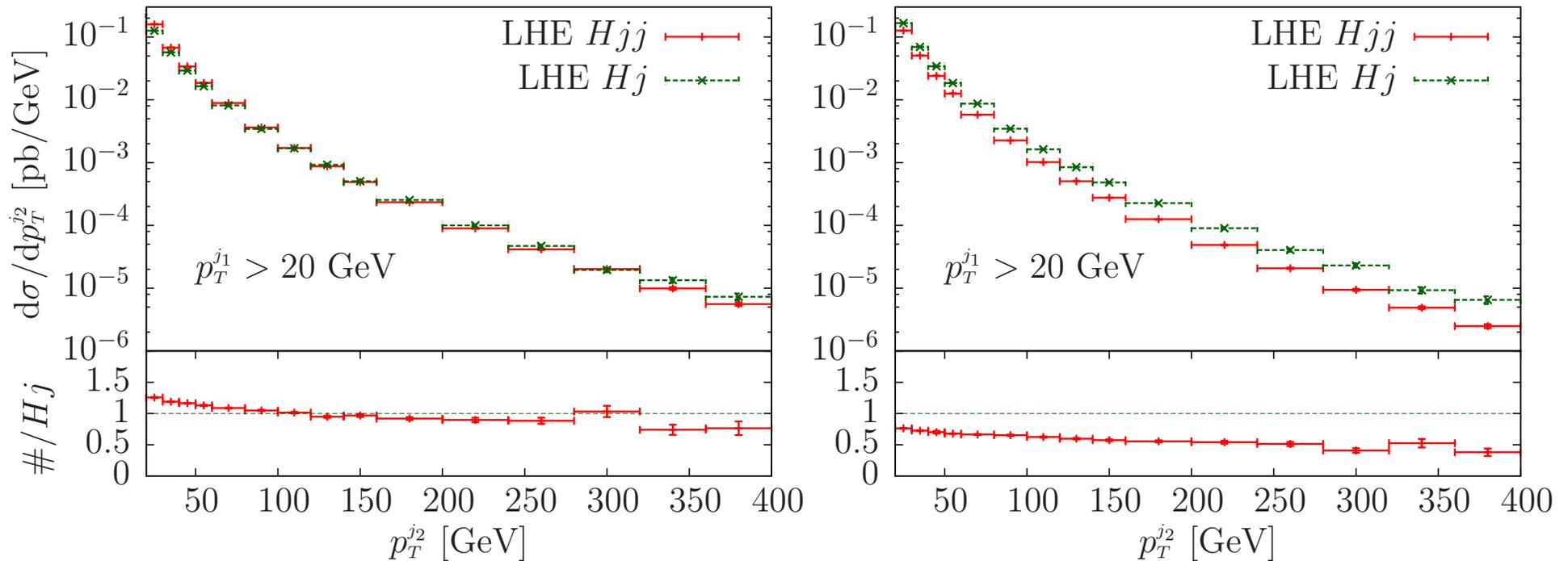
H and Hj comparison



- $\sigma_H^{\text{NLO}} = 10.85 \text{ pb}$, $\sigma_H^{\text{NNLO}} = 14.35 \text{ pb} \implies K = 1.32$
- *H* generator: **NLL** accuracy in the **low** p_T region but only **LO** at **high** p_T
- *Hj* generator: **NLO** accuracy only in the **high** p_T region. **No Sudakov resummation.**

[Campbell, Ellis, Frederix, Nason, Williams, C.O., 2012]

Hj and Hjj comparison



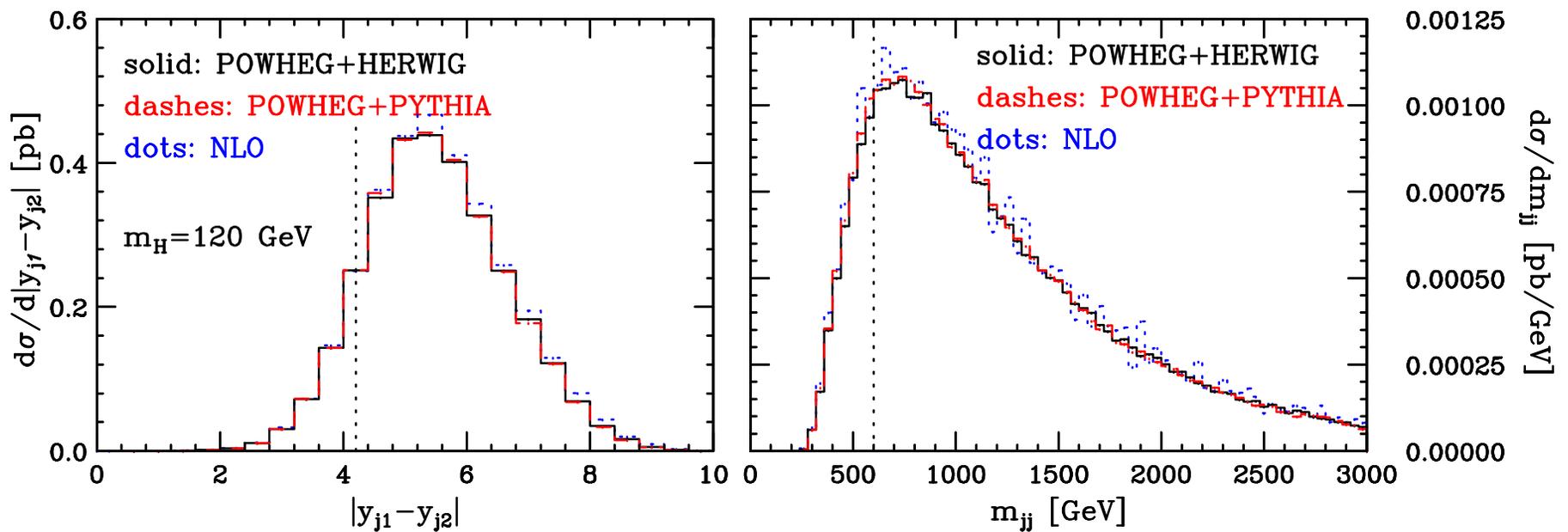
Left: $\mu_F = \mu_R = m_H$. **Right:** $\mu_F = \mu_R = p_T^{UB}$ for Hj and $\mu_F = \mu_R = \hat{H}_T$ for Hjj

- Hj generator: **NLL** accuracy in the **low** p_T region but only **LO** at **high** p_T
- Hjj generator: **NLO** accuracy only in the **high** p_T region. **No Sudakov resummation.**

Work in progress to merge the H , Hj and Hjj samples

[Campbell, Ellis, Frederix, Nason, Williams, C.O., 2012]

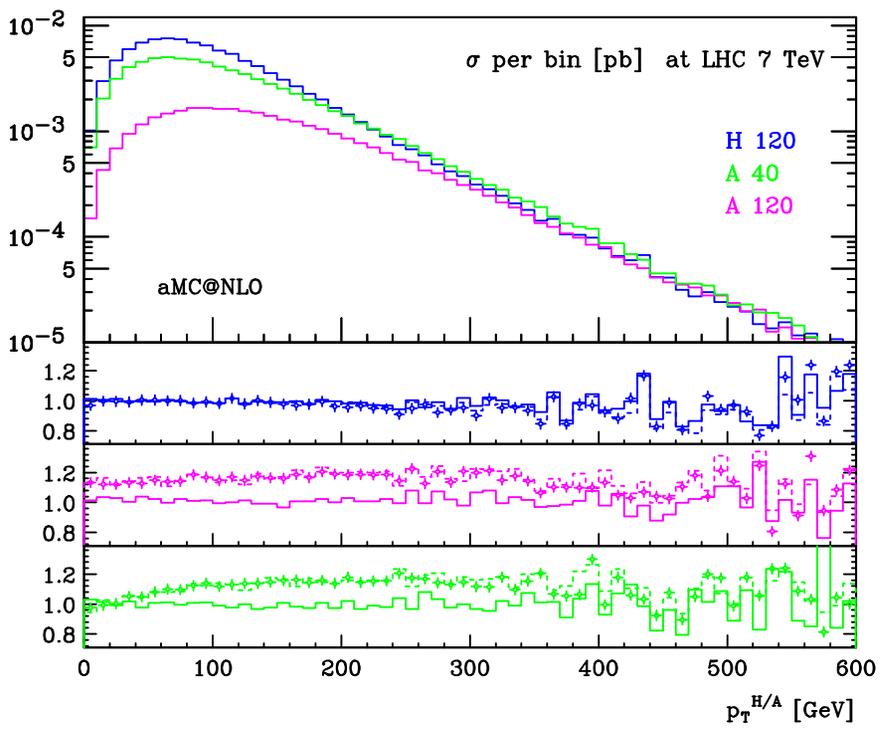
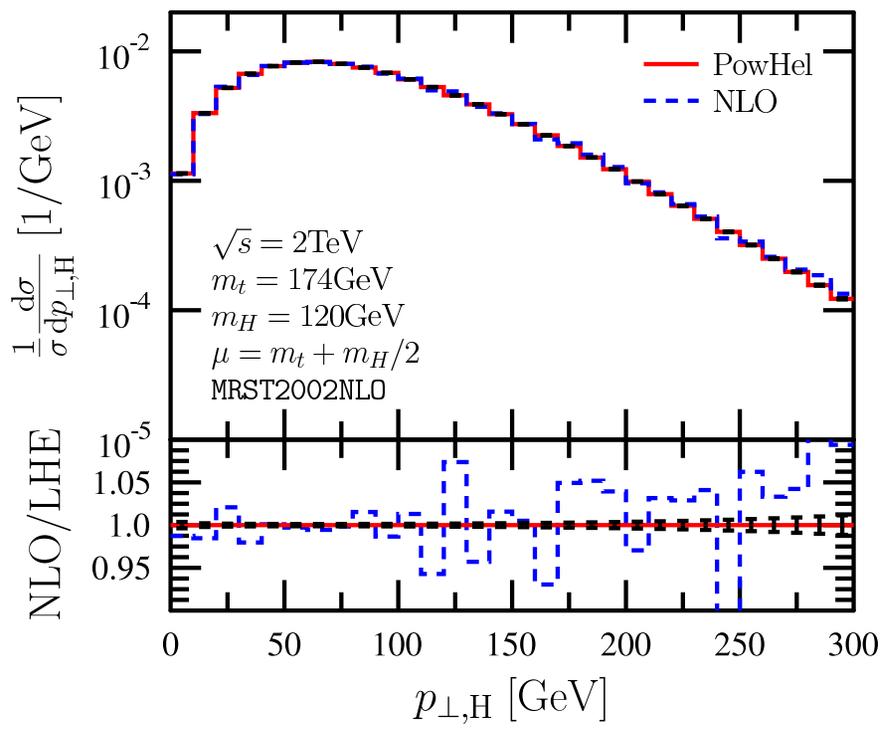
Higgs in vector-boson fusion



- Only t -channel vector-boson exchange diagrams: built having in mind **VBF cuts** [Nason and C.O, 2010].
- Nevertheless, the **cross section** is **finite** even with no cuts.

This is what has been used in the Yellow Report Higgs Cross Section 1 and 2.

$t\bar{t}H$

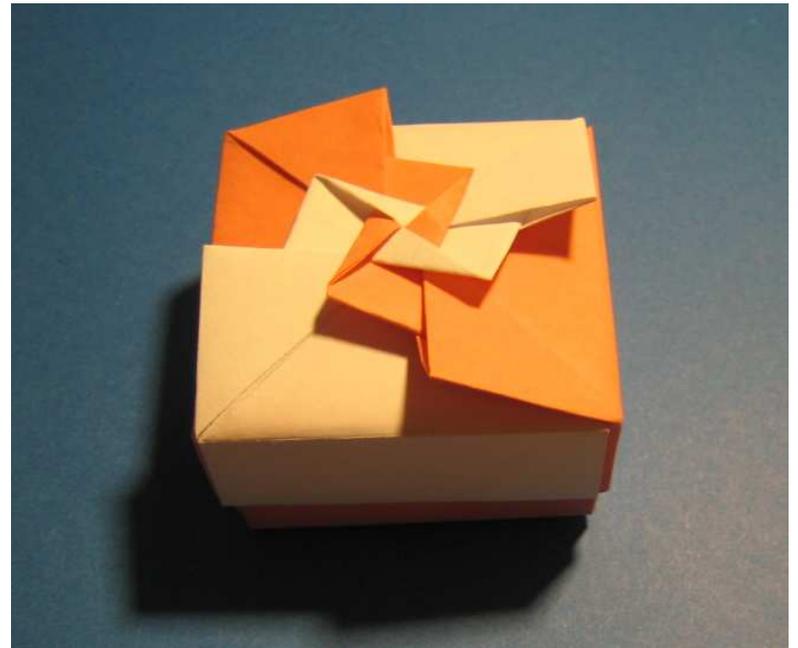


- POWHEG+HELAC [Garzelli, Kardos, Papadopoulos and Trocsanyi, 2011]
- aMC@NLO: scalar and pseudoscalar Higgs boson [Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli, 2011]

Conclusions

- There are several NLO+PS programs that describe the production of a Higgs boson in different channels
- Although they formally all agree at NLO, NNLO terms can be large for processes with large K factors.
- Differences among each other are well understood and have been studied in the past few years
- For processes in the POWHEG BOX, please visit the web page

<http://powhegbox.mib.infn.it>



Backup slides

Squeezing R_s

$$d\sigma = \bar{B}_s(\Phi_n) \left\{ \Delta_s(p_T^{min}) + \Delta_s(p_T) \frac{R_s(\Phi_{n+1})}{B(\Phi_n)} d\Phi_r \right\} d\Phi_n + R_f(\Phi_{n+1}) d\Phi_{n+1}$$

$$\bar{B}_s(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_r [R_s(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)]$$

$$\Delta_s(p_T) = \exp \left[- \int d\Phi'_r \frac{R_s(\Phi_n, \Phi'_r)}{B(\Phi_n)} \theta(p'_T - p_T) \right]$$

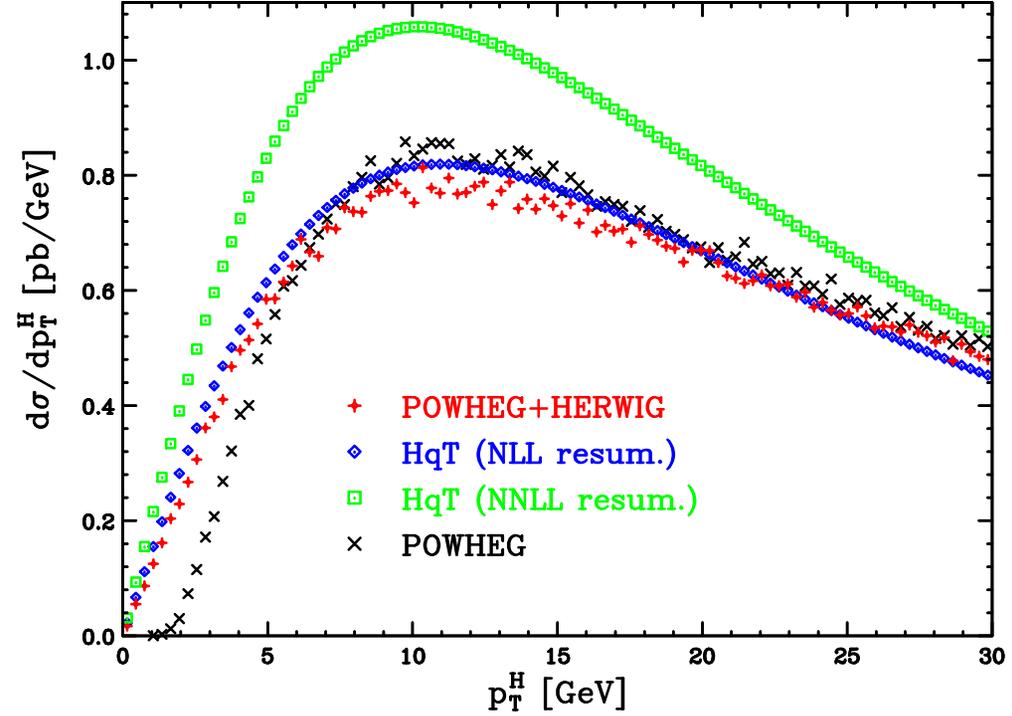
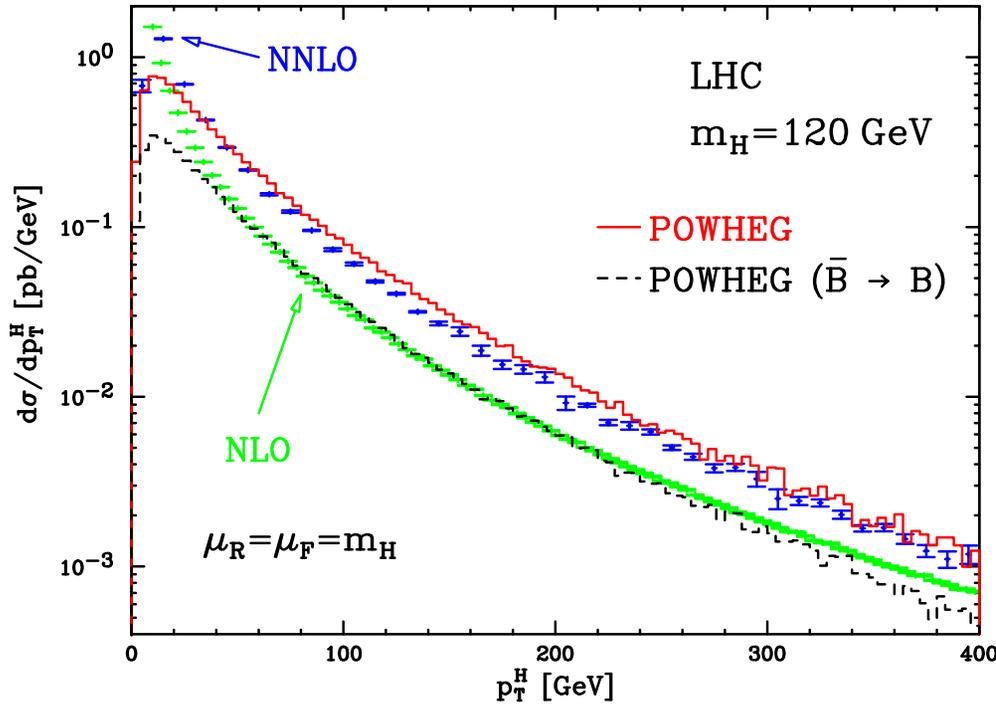
Notice: by **restricting R_s** more and more we recover **exactly the NLO result**, with no further higher order effects.

For example

$$R_s = \theta(k_{\text{cut}} - k_T) R \quad k_{\text{cut}} \rightarrow 0$$

But we **cannot** do this if we want an **NLO+PS** result: as $k_{\text{cut}} \rightarrow 0$ the **Sudakov region** becomes **squeezed and distorted**, even to a point when positivity is lost.

NNLO contributions: Higgs boson production



$$\bar{B}_s(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_r [R_s(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)]$$

$$d\sigma = \bar{B}_s(\Phi_n) \left\{ \Delta_s(\Phi_n, p_T^{\min}) + \Delta_s(\Phi_n, p_T) \frac{R_s(\Phi_{n+1})}{B(\Phi_n)} d\Phi_r \right\} d\Phi_n + R_f(\Phi_{n+1}) d\Phi_{n+1}$$

$$d\sigma_{\text{rad}} \approx \frac{\bar{B}_s(\Phi_n)}{B(\Phi_n)} R_s(\Phi_{n+1}) d\Phi_{n+1} + R_f(\Phi_{n+1}) d\Phi_{n+1}$$

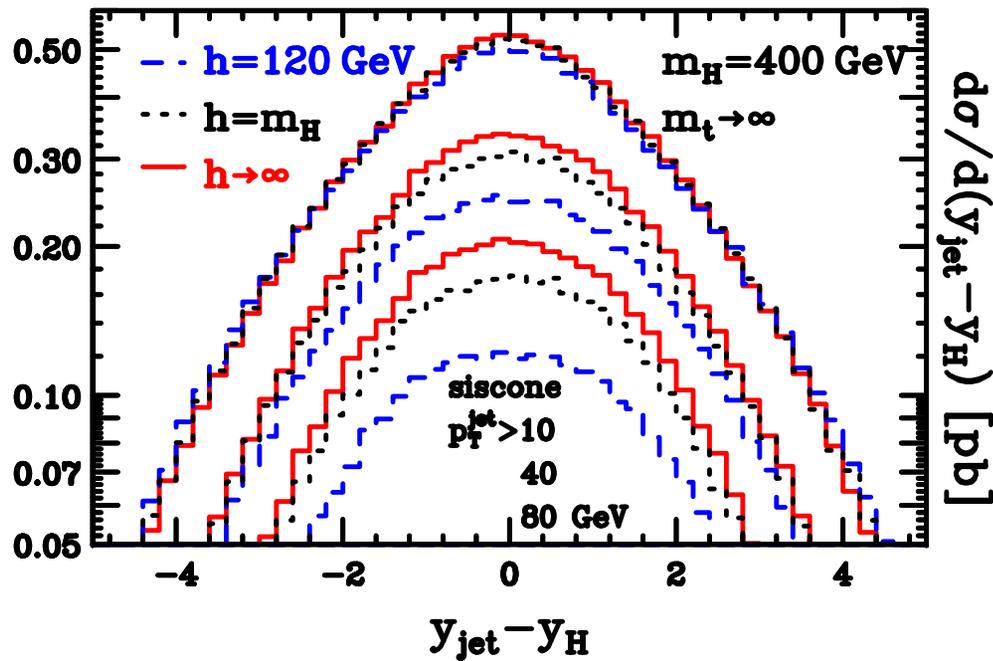
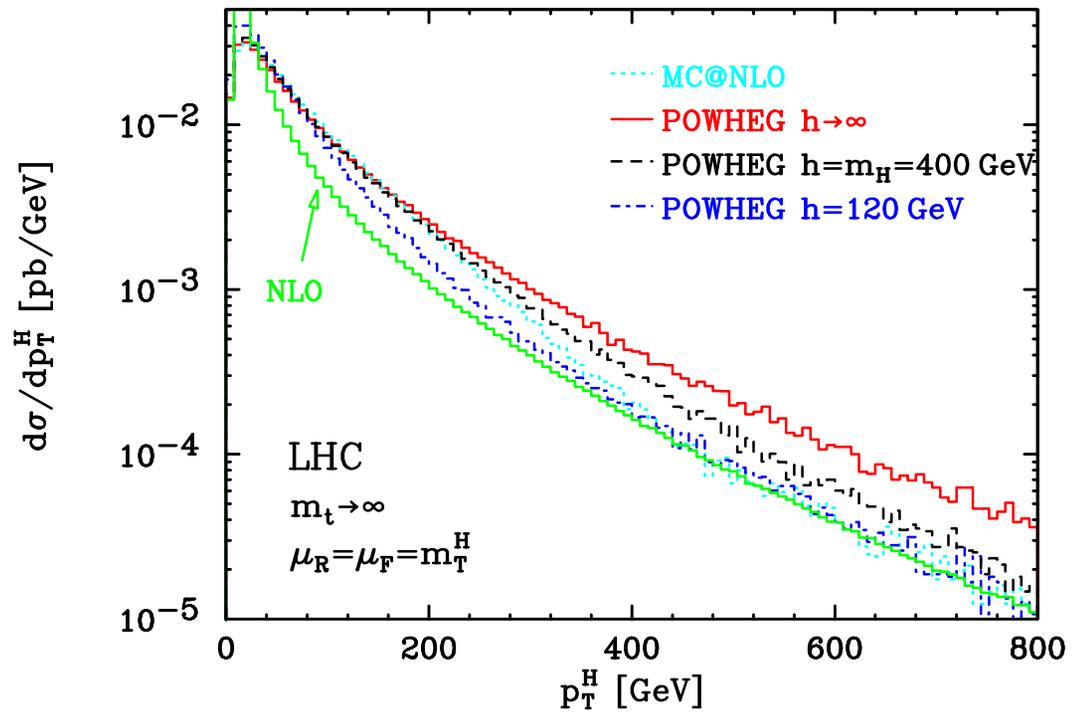
$$= \left\{ [1 + \mathcal{O}(\alpha_s)] R_s(\Phi_{n+1}) + R_f(\Phi_{n+1}) \right\} d\Phi_{n+1} = R(\Phi_{n+1}) d\Phi_{n+1} + \mathcal{O}(\alpha_s) R_s(\Phi_{n+1})$$

$$R_s = \frac{h^2}{p_T^2 + h^2} R$$

$$R_f = \frac{p_T^2}{p_T^2 + h^2} R$$

$$R = R_s + R_f$$

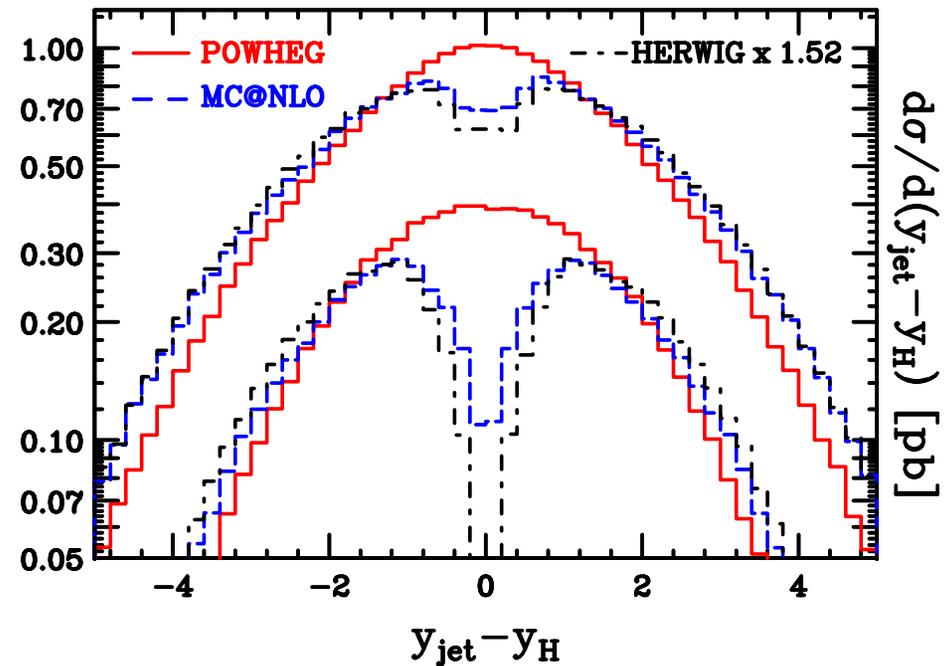
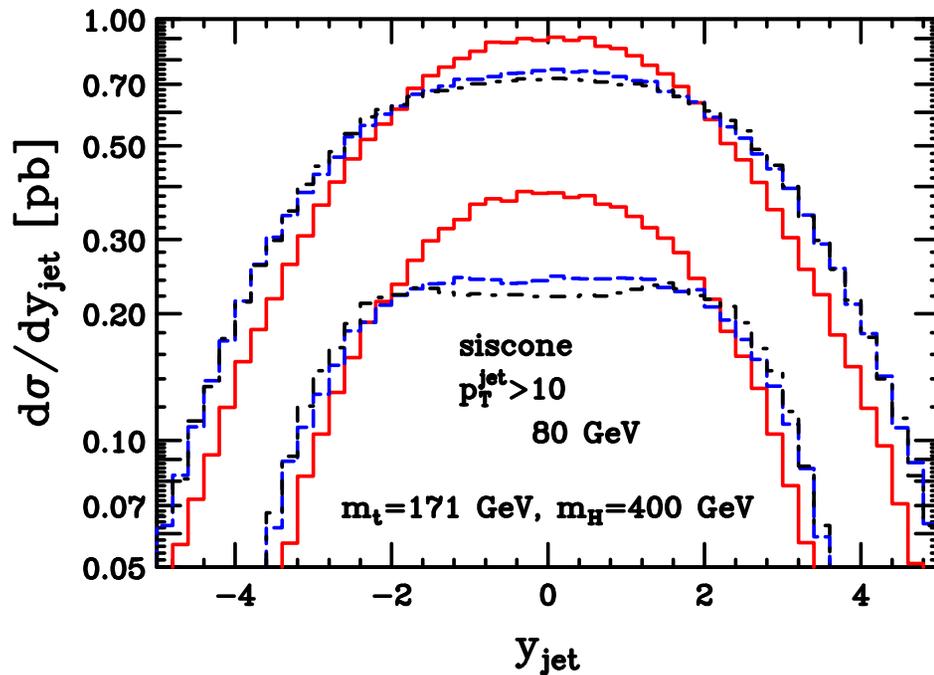
agrees with NLO at high p_T



No new features appear in **all** the other **distributions**

When $h \rightarrow 0$, we recover the pure **NLO** cross section

NNLO contributions: the dip in MC@NLO



- Why **MC@NLO** has a **dip** in the hardest jet rapidity?
ANSWER: because it is very **sensitive** to the **dead zone** in the HERWIG phase space
- Why **POWHEG** has **no dip**? Is that because of the hardest p_T spectrum?
ANSWER: NO, it does **not depend** on the hardest p_T spectrum. POWHEG generate by **itself** the **hardest radiation**.

Processes implemented in the POWHEG BOX

- heavy-quark pair production (Frixione, Nason, Ridolfi, 2007)
- Z/W (with decay) (Alioli, Nason, Re, C.O., 2008)
- Higgs boson in gluon fusion (Alioli, Nason, Re, C.O., 2008)
- single top (Alioli, Nason, Re, C.O., 2009) and tW (Re, 2010)
- Higgs boson in VBF (Nason, C.O., 2010)
- Z/W (with decay) + 1 jet (Alioli, Nason, Re, C.O., 2010)
- dijet (Alioli, Hamilton Nason, Re, C.O., 2010)
- $t\bar{t}$ + 1 jet (Kardos, Papadopoulos, Trocsanyi, 2011) also (Alioli, Moch, Uwer, 2011)
- $t\bar{t}H$, $t\bar{t}Z/\gamma$ (Garzelli, Kardos, Papadopoulos, Trocsanyi, 2011)
- W^+W^+ plus two jets (Melia, Nason, Rontsch, Zanderighi, 2011)
- W^+W^+ plus two jets via VBF (Jäger, Zanderighi, 2011)
- $Wb\bar{b}$ (with approximated decay) (Reina, C.O., 2011)
- diboson production (with decay), (Melia, Nason, Rontsch, Zanderighi, 2011)
- tH^- (Klasen, Kovaric, Nason, Weydert, in preparation)