

Neutron Scattering: Theory, Instrumentation, and Simulation

Kim Lefmann

Niels Bohr Institute, University of Copenhagen

with contributions from Bente Lebech, Risø-DTU
and Lise Arleth, University of Copenhagen

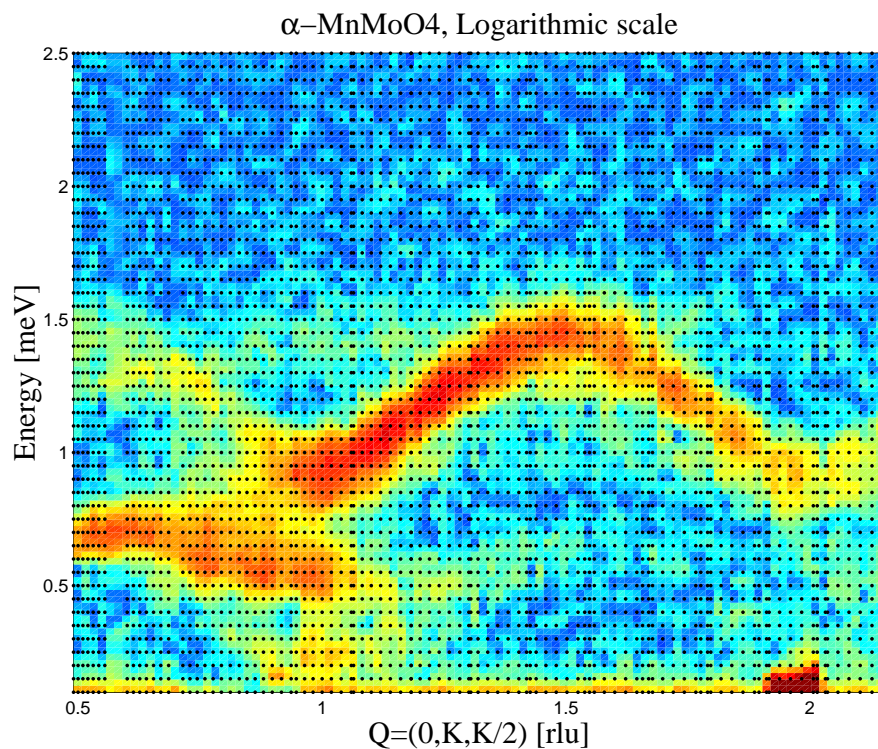


Image on front page: Inelastic neutron scattering data on a single crystal of the molecular magnet α -MoMnO₄, taken by the students at the Copenhagen neutron scattering course, 2005. From Ref. [1].

Foreword and acknowledgements

This text is written as lecture notes for a University course in Neutron Scattering, University of Copenhagen, autumn 2011. In contrast to most textbooks on this topic, these notes cover both theoretical and experimental aspects. Furthermore, the notes contain an introduction to ray-tracing simulations of neutron instruments.

The theoretical parts of these notes are largely inspired by the classical textbooks by Marshall and Lovesey[2], and Squires[3], but the material has been simplified to make it appropriate for beginners in this field and re-organised in (for me) a more streamlined way. Further, this text contains more recent topics like small-angle scattering and reflectivity. It is intended to make the notation consistent with Squires[3]. One major difference, though, is the use of q in stead of κ for the scattering vector, in order to make the notation compatible with current practice.

I am strongly indebted to Kurt N. Clausen for introducing me to the secrets of neutron scattering and for support far beyond the duties and timespan of a Ph.D. supervisor. Without him this work would have been utterly impossible.

A wholehearted thank you goes to Robert McGreevy for providing me the “five reasons” for neutron scattering. Even more important, he led the EU project SCANS, which inspired and funded much of the work related to computer simulations. He was also leading the previous EU NMI3 and ISIS-TS2 projects, both supporting the simulation work.

In the present EU projects (named NMI3 and NMI3-2), led by Helmut Schober and Mark Johnson (both from ILL), Pia Jensen, Linda Udby, and I develop simulation tools for the notes to create a virtual EU course for the teaching of neutron scattering and instrumentation. In this respect, the students at the Copenhagen course have acted as “guinea pigs”, helping us to find scores of mistakes and imprecisions. Thanks to all ~ 120 participants for good spirit and much discussion. Thanks in particular to Sara Eisenhardt, Jacob Larsen, Elisabeth Ulrikkeholm, Maria Thomsen, Pia Jensen, Johan Jacobsen, and Henrik Jacobsen for spotting many errors and for useful suggestions.

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