Meaningful characterisation of perturbative theoretical uncertainties

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Outline

- Perturbative QCD
- Bayesian statistics
- Parton distribution functions (pdfs)
- Theoretical uncertainties in Bayesian statistics
  - Renormalization Scale (RS), μ_r
  - Factorization Scale (FS), μ_f
- Outlook

with Alberto Guffanti, Matteo Cacciari and Emanuele Bagnaschi

Perturbative QCD

Write an observable as an expansion in α_s

$$\sigma(Q) = \sum_{i=0}^{\infty} c^{(i)}(Q, \mu_r)\alpha_s^i(\mu_r),$$

where

$$\frac{d\alpha_s}{d\log\mu_r^2} = \beta(\alpha_s) = -\sum_{j=0}^{\infty}\beta_j\alpha_s^{j+2}.$$

Scale dependency of σ

$$0 = \frac{d\sigma}{d\log\mu_r^2} = \sum_{i=0}^{\infty}\left(\frac{dc^{(i)}}{d\log\mu_r^2}\alpha_s^i - ic^{(i)}\alpha_s^{i-1}\sum_{j=0}^{\infty}\beta_j\alpha_s^{j+2}\right)$$

$\Rightarrow$ recursive scale dependency of $c^{(i)}$

Approximation of scale dependency of the partial sum σ_k

$$\frac{d\sigma_k}{d\log\mu_r^2} \simeq \alpha_s^{k+1} k \beta_0 c^{(k)}$$

Approximation of Δ_k (through scale variation, $\mu_r = Q/2 \ldots 2Q$)

$$\Delta_k \simeq \left|\frac{d\sigma_k}{d\log\mu_r^2}\right|_{\mu=Q} (2\log(2Q) - 2\log(Q/2)) \simeq 3\alpha_s^{k+1} k \beta_0 c^{(k)}$$

Bayesian statistics

Definition of probability

- Frequentism: relative frequency of occurrence of an experiment's outcome
- Bayesianism: degree of belief (DoB), combination of a priori probability distribution and data incorporated in a likelihood function

Density function f, uncertain variable Δ_k, DoB that $\Delta_k \in [a, b]$

$$\mathbb{C}(\Delta_k \in [a, b]) \equiv \int_a^b f(\Delta_k)\, d\Delta_k$$

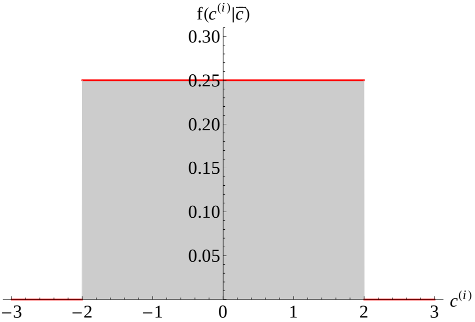
Conditional density: coefficients $c^{(0)} \ldots c^{(k)}$ are known,
$\Delta_k \simeq \alpha_s^{k+1} c^{(k+1)}$

$$f(c^{(k+1)}|c^{(0)}, \ldots, c^{(k)}) = \frac{f(c^{(k+1)}, c^{(0)}, \ldots, c^{(k)})}{f(c^{(0)}, \ldots, c^{(k)})}$$

- Expansion of observable

$$\sigma = \sum_{i=0}^{\infty} a^i c^{(i)}.$$

- Choice of a such that all $c^{(i)}$ of the same order
- $c^{(i)} \leq \bar{c}$ uniform distributed



Degree of belief (e.g. 68.3% or 95.5%)

$$\mathbb{C}\left(\Delta_k \in [-\delta_k, \delta_k]\,|c^{(0)}, \ldots, c^{(k)}\right) = \int_{-\delta_k}^{\delta_k} f\left(\Delta_k|c^{(0)}, \ldots, c^{(k)}\right) d\Delta_k$$

and we can derive an expression for the size of δ_k

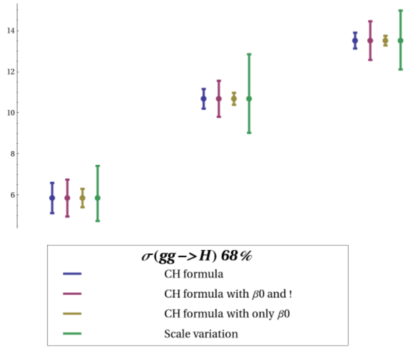
$$\delta_k^{(p)} = \begin{cases} a^{k+1}\max\left(c^{(0)}, \ldots, c^{(k)}\right) & \text{if } p\% \le \frac{k+1}{k+2} \\ a^{k+1}\max\left(c^{(0)}, \ldots, c^{(k)}\right)\left[(k+2)(1-p\%)\right]^{-1/(k+1)} & \text{if } p\% > \frac{k+1}{k+2} \end{cases}$$

where $p\% = p/100$ and $p = 1 \ldots 100$.

Reproduce renormalization scale variation

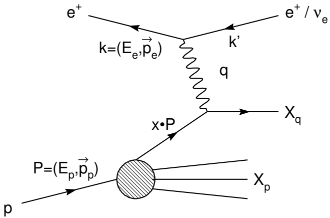
$$\delta_k^{(p)} = \begin{cases} a^{k+1}\max\left(c^{(0)},\ldots,c^{(k)}\right) & \text{if } p\% \leq \frac{k+1}{k+2} \\ a^{k+1}\max\left(c^{(0)},\ldots,c^{(k)}\right)\left[(k+2)(1-p\%)\right]^{-1/(k+1)} & \text{if } p\% > \frac{k+1}{k+2} \end{cases}$$

LO NLO NNLO



Choice of expansion parameter

- $\sigma = \sum_{i=0}^{\infty} \alpha_s^i c^{(i)}$
- $\sigma = \sum_{i=0}^{\infty} (i-1)!(\alpha_s\beta_0)^i d^{(i)}$
- $\sigma = \sum_{i=0}^{\infty} (\alpha_s\beta_0)^i d'^{(i)}$



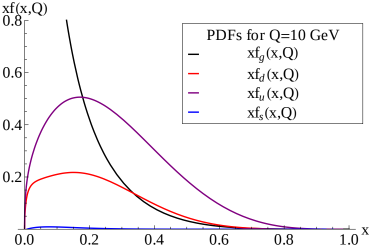
Parton distribution function

- $f_q(x)dx$: probability to find a quark q with momentum fraction between x and $x + dx$ in a proton/antiproton
- long distance part

Observable (e.g. structure fct.)

$$F_2(x, Q) = x \sum_{i=0}^{\infty} \alpha_s^i(\mu_r) \left[c_2^{(i)} \otimes f\right](x, Q, \mu_f)$$

- Expansion in α_s, convolution integral
- $c_2^{(i)}$ contain short distance, perturbative part



Factorization scale variation

Repeat same procedure as for renormalization scale variation ($\mu_f = \mu_r$)

$$0 = \frac{dF_2}{d\log\mu_f^2}$$
$$= x\sum_{i=0}^{\infty}\left(\alpha_s^{i-1}\frac{d\alpha_s}{d\log\mu_f^2}\left[c_2^{(i)}\otimes f\right] + \alpha_s^i\left[\frac{dc_2^{(i)}}{d\log\mu_f^2}\otimes f\right] + \alpha_s^i\left[c_2^{(i)}\otimes\frac{df}{d\log\mu_f^2}\right]\right)$$

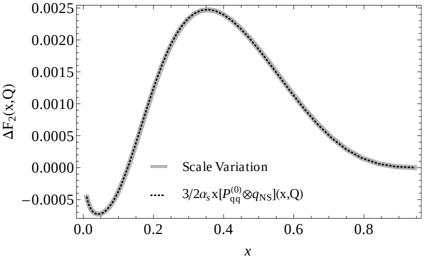
Factorization scale dependency of pdfs given by DGLAP evolution equation

$$\frac{df}{d\log\mu_f^2} = \sum_{i=0}^{\infty}\alpha_s^{i+1}\left[P_{qq}^{(i)}\otimes f\right](x,\mu_f),$$

where $P_{qq}^{(i)}$ are the splitting function and the convolution is given by

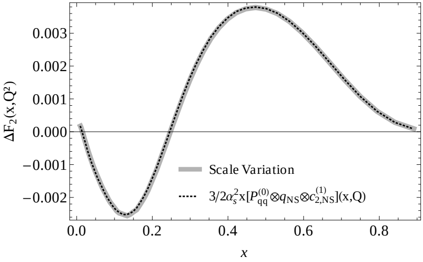
$$\left[P_{qq}^{(i)}\otimes f\right](x,\mu_f) \equiv \int_x^1 \frac{d\xi}{\xi}P^{(i)}(\xi)\otimes f(\frac{x}{\xi},\mu_f).$$

LO



- DoB $\sim$ 83%

NLO



- DoB $\sim$ 94%

Approximation

$$\Delta_k \simeq 3 \left| \frac{dF_2}{d \log \mu_f^2} \right|_{\mu_f = Q} \simeq 3x\alpha_s^{k+1} \left[P_{qq}^{(0)} \otimes c_2^{(k)} \otimes f \right] (x, Q)$$

Outlook

- Motivate scale variation from Bayesian statistics
- Expansion parameter for RS
- Extend the formalism to the FS
  - Finding the correct expansion parameter
  - Can we use $c^{(i)} \otimes f$ (FS) instead of just $c^{(i)}$ (RS)?
  - Check the formalism for more examples, e.g. top pair production
- Alternative to scale variation?