Perturbative Gravity and Gauge-Theory Relations

A complicated story made simple
by

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Scattering Amplitudes

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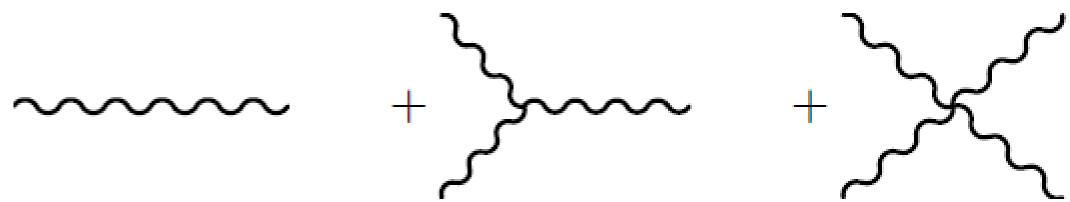

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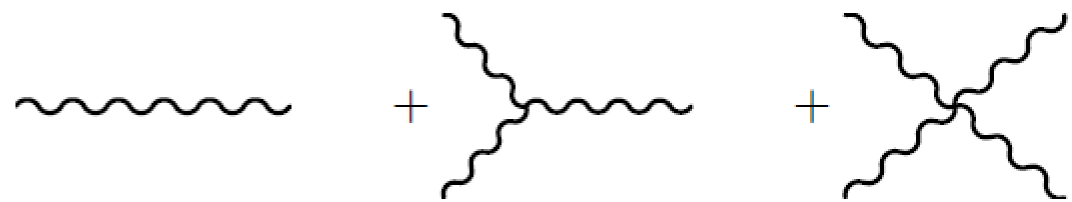

Einstein Gravity:

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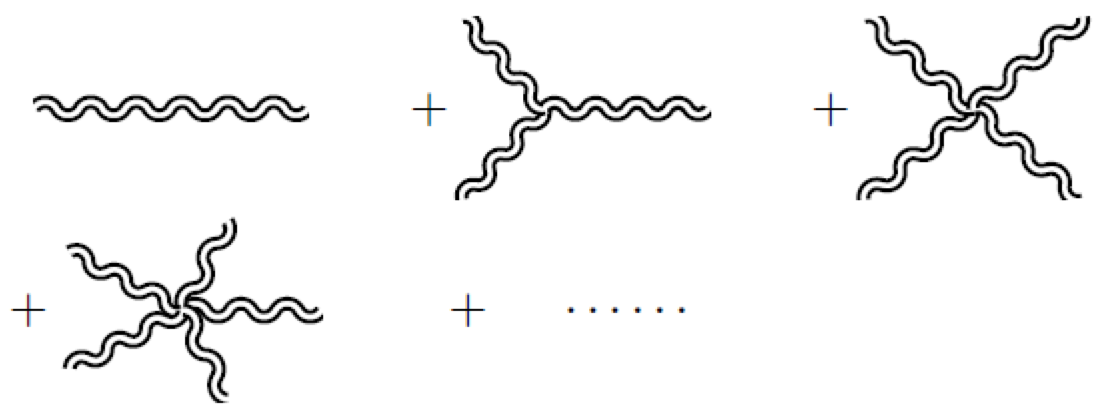

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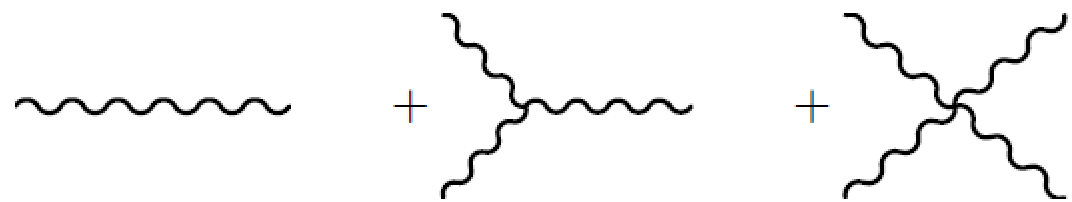

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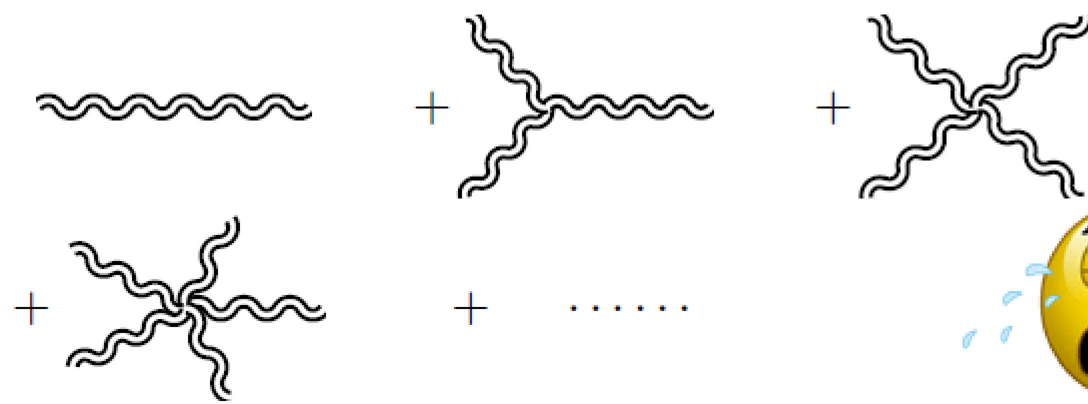
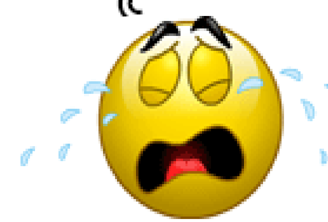
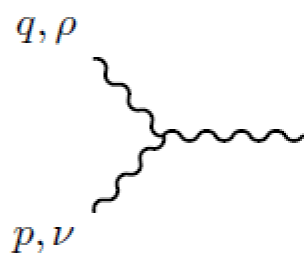


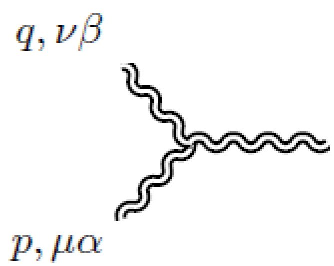


Three-Point Vertex

Off-shell:



$$k, \mu \;\sim\; \eta_{\nu\rho}(p-q)_\mu + \eta_{\rho\mu}(q-k)_\nu + \eta_{\mu\nu}(k-p)_\rho$$



$$k, \sigma\gamma \;\sim\; p\cdot q\, \eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma} + \text{approx. 100 more terms}$$

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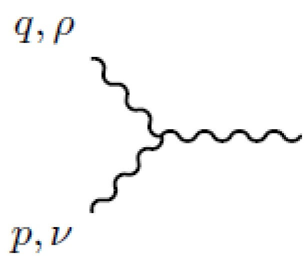

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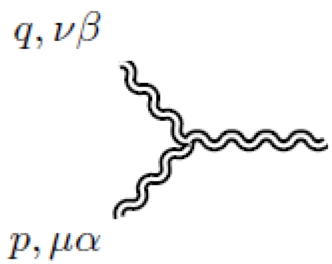

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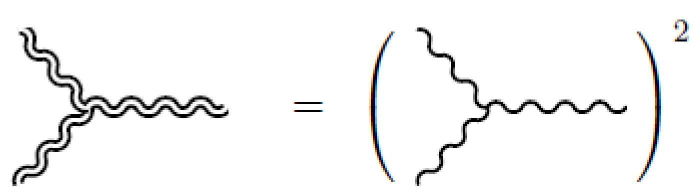
On-shell a “miracle” happens:



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and so on. What does this imply for the squaring relation at last slide

$$M_3(1,2,3) = A_3(1,2,3)^2 \xrightarrow[Rec.Rel.]{} ???$$

The KLT Relations

Using the three-point squaring relation we can show that

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$$M_n = (-1)^{n+1} \sum_{\sigma, \widetilde{\sigma} \in S_{n-3}} \widetilde{A}_n(n-1, n, \widetilde{\sigma}_{2,n-2}, 1) \mathcal{S}[\widetilde{\sigma}_{2,n-2} | \sigma_{2,n-2}]_{k_1} A_n(1, \sigma_{2,n-2}, n-1, n)$$

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This is the field theory version of a relation between closed and open strings first derived by Kawai, Lewellen and Tye in 1985.

Thank you for listening!