Some Properties of Scattering amplitude

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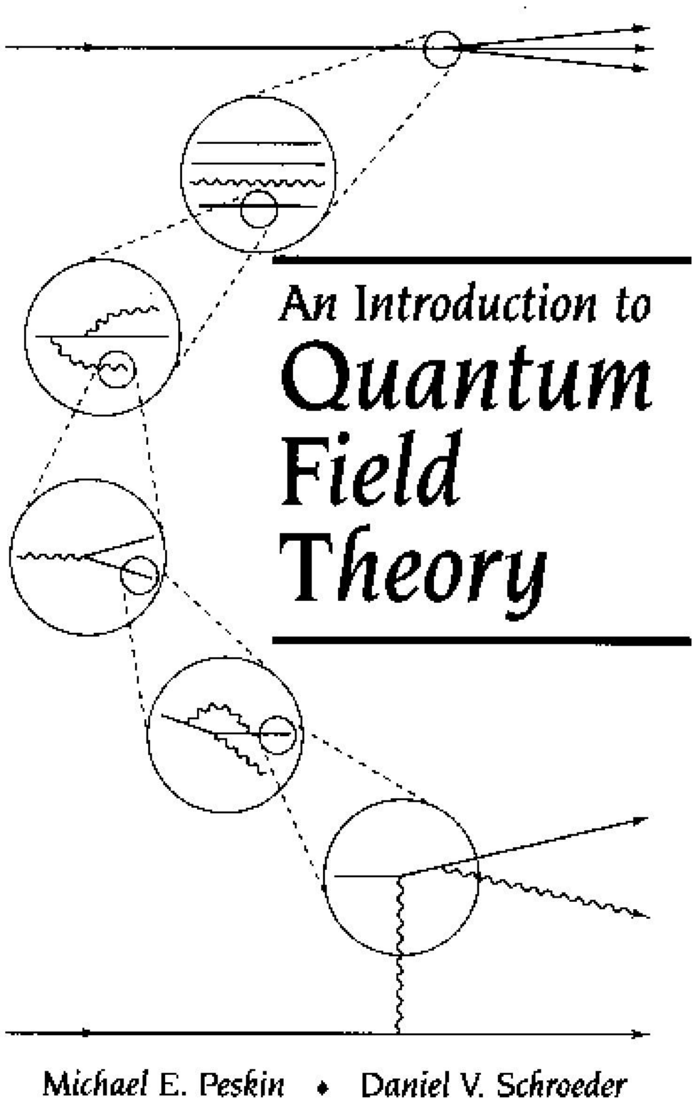
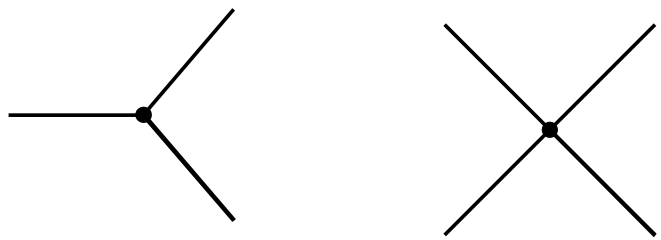


How to calculate amplitude?

Feynman Diagram!

propagator



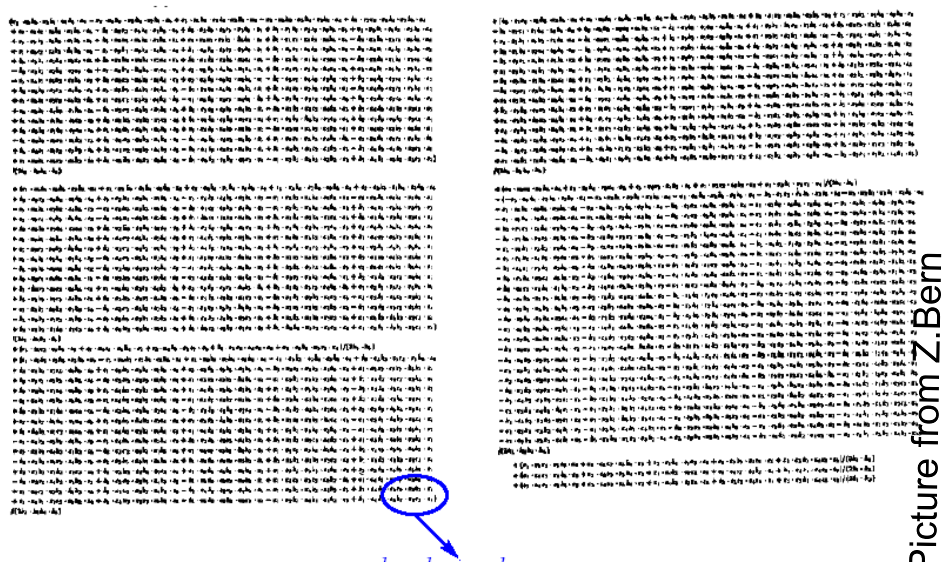
vertex

A PhD Student's Nightmare: Calculating Amplitude with Feynman Diagram.

5 gluon amplitude

$$A_5^{tree}(--+++) = \frac{\langle 1\ 2\rangle^4}{\langle 1\ 2\rangle\langle 2\ 3\rangle\langle 3\ 4\rangle\langle 4\ 5\rangle\langle 5\ 1\rangle}.$$

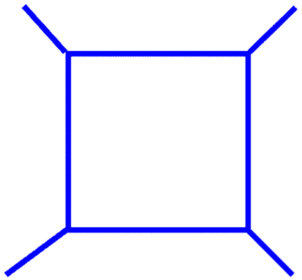
Result of a brute force calculation:



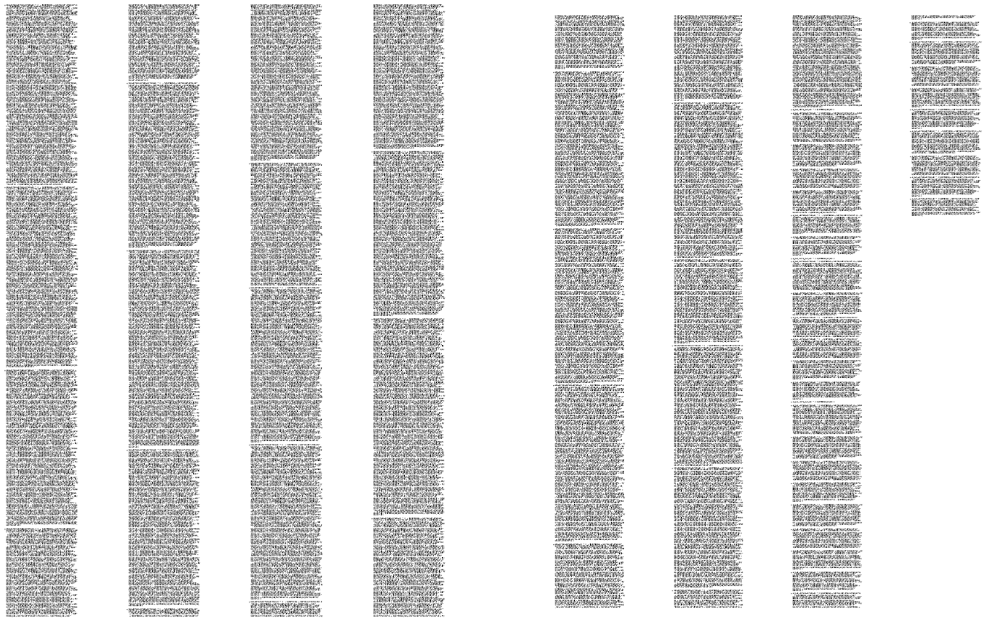
Picture from Z.Bern

6 gluon~10,000 diagrams 7 gluon~150,000 diagrams
8 gluon~3,000,000

A PhD Student's Nightmare:
Calculating Amplitude with Feynman Diagram.



$$\int \frac{d^{4-2\epsilon}\ell}{(2\pi)^{4-\epsilon}} \frac{\ell^\mu \, \ell^\nu \, \ell^\rho \, \ell^\lambda}{\ell^2 \, (\ell - k_1)^2 \, (\ell - k_1 - k_2)^2 \, (\ell + k_4)^2}$$



A PhD Student's Revenge:
Say NO to Feynman Diagram.

$$P_{\alpha\dot{\alpha}} = \sigma^{\mu}_{\alpha\dot{\alpha}} P_{\mu} = \lambda_{\alpha}\widetilde{\lambda}_{\dot{\alpha}} = |p\rangle |p]$$

$$\langle i\ j\rangle = \epsilon^{\alpha\beta}\lambda_{i,\alpha}\lambda_{j,\beta} \qquad [i\ j] = \epsilon_{\dot{\alpha}\dot{\beta}}\widetilde{\lambda}_i^{\dot{\alpha}}\widetilde{\lambda}_j^{\dot{\beta}}$$

$$2k_i \cdot k_j = \langle i\ j\rangle\,[j\ i]$$

A PhD Student's Revenge: Say NO to Feynman Diagram.

...Color-decomposition

$$\mathcal{A}_n^{full}(\{k_i,\lambda_i,a_i\}) = g^{n-2}\sum_{\sigma\in S_n/Z_n}\mathrm{Tr}(T^{a_{\sigma_1}}\cdots T^{a_{\sigma_n}})A_n(k_{\sigma_1}^{\lambda_{\sigma_1}},\ldots,k_{\sigma_n}^{\lambda_{\sigma_n}})$$

A PhD Student's Revenge:
Say NO to Feynman Diagram.

$$A_n(1,2,\ldots,n) = A_n(2,3,\ldots,n,1)$$ Cyclic relation

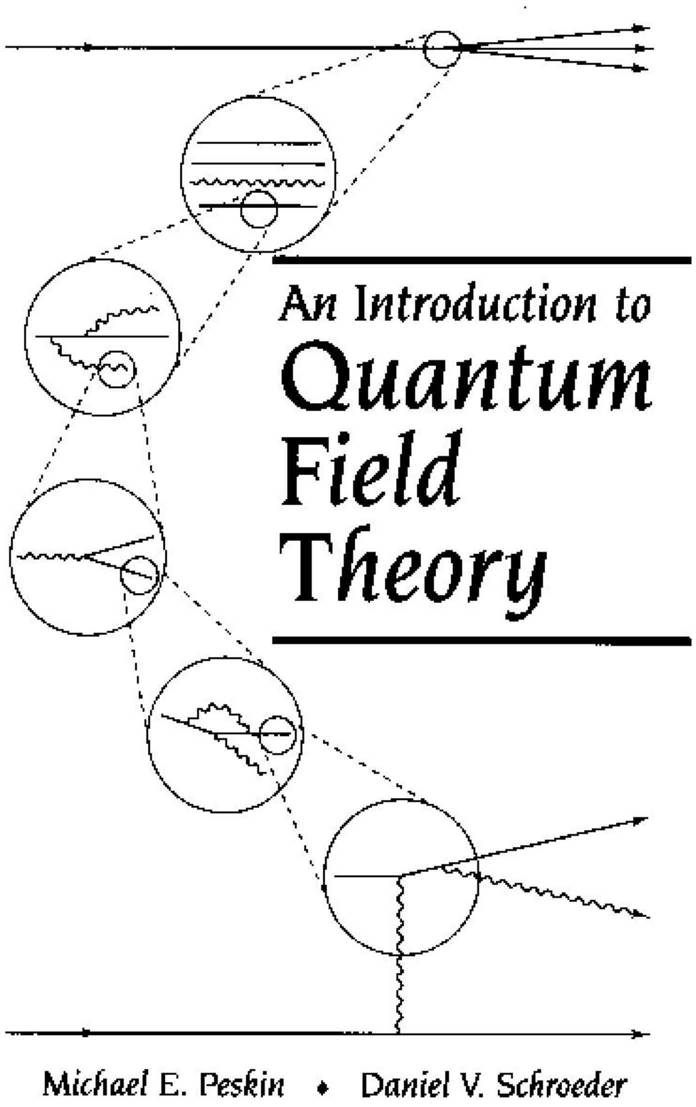
$$A_n(1,2,\ldots,n) = (-1)^n A_n(n,n-1,\ldots,1)$$ Reflection Relation

Kleiss-Kuijf (KK) relation(1989)

$$A_n(1,\{\alpha\},n,\{\beta\}) = (-1)^{n_\beta} \sum_{\sigma\in OP(\{\alpha\},\{\beta^T\})} A_n(1,\sigma,n)$$

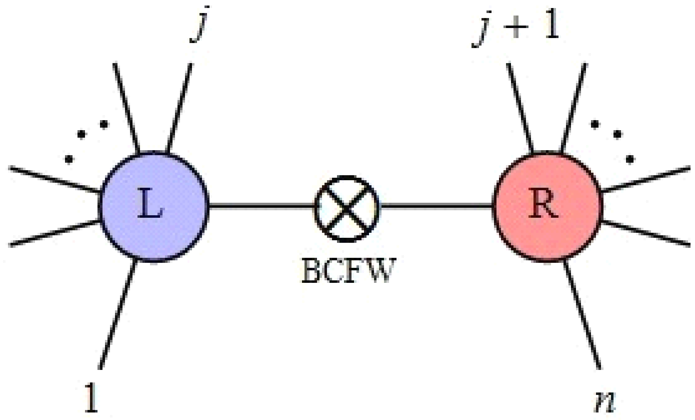
Bern-Carrasco-Johansson (BCJ) relation(2008)

$$A_n(1,2,\{\alpha\},3,\{\beta\}) = \sum_{\sigma_i\in POP(\{\alpha,\beta\})} A_n(1,2,3,\sigma_i)\mathcal{F}_i$$



How to calculate amplitude?

BCFW on-shell Recursion Relation!



Britto-Cachazo-Feng-Witten(BCFW)
On-shell Recursion Relation

...Shift of two momenta in $A_n(k_1, k_2, \ldots, k_n)$

$$\widetilde{\lambda}_i \to \widetilde{\lambda}_i + z\widetilde{\lambda}_j \quad \Rightarrow k_i(z) = \lambda_i\widetilde{\lambda}_i + z\lambda_i\widetilde{\lambda}_j$$

$$\lambda_j \to \lambda_j - z\lambda_i \quad \Rightarrow k_j(z) = \lambda_j\widetilde{\lambda}_j - z\lambda_i\widetilde{\lambda}_j$$

$$k_i^2(z) = 0 \ , \quad k_j^2(z) = 0$$ on-shell momenta

$$k_i(z) + k_j(z) = k_i + k_j$$ total momentum conservation

Britto-Cachazo-Feng-Witten(BCFW) On-shell Recursion Relation

...Complexified the amplitude

$$A_n(k_1, k_2, \ldots, k_n) \to A_n(\ldots, k_i(z), \ldots, k_j(z), \ldots) \equiv A_n(z)$$

...Contour integral

$$\frac{1}{2\pi i}\oint \frac{dz}{z} A(z) = C_\infty = A(0) + \sum_\alpha \mathrm{Res}_{z=z_\alpha} \frac{A(z)}{z}$$

Britto-Cachazo-Feng-Witten(BCFW) On-shell Recursion Relation

$$\frac{1}{2\pi i}\oint \frac{dz}{z}A(z) = C_\infty = A(0) + \sum_\alpha \mathrm{Res}_{z=z_\alpha}\frac{A(z)}{z}$$

(1) $A(z) \to 0$ when $z \to \infty$

(2) $A(z)$ a rational function of z

(3) $A(z)$ has single pole

Britto-Cachazo-Feng-Witten(BCFW) On-shell Recursion Relation

$$\frac{1}{2\pi i}\oint \frac{dz}{z}A(z) = C_\infty = A(0) + \sum_\alpha \text{Res}_{z=z_\alpha}\frac{A(z)}{z}$$

...Evaluate the Residue: Location of the poles

(1) Propagator K_α does not contain $k_i(z)$ and $k_j(z)$

(2) Propagator K_α contains both $k_i(z)$ and $k_j(z)$

(3) Propagator K_α contains one $k_i(z)$ or $k_j(z)$

$$K_\alpha(z) = zk' + p\ , \quad K_\alpha^2(z) = 0 \rightarrow z_\alpha$$

Britto-Cachazo-Feng-Witten(BCFW) On-shell Recursion Relation

$$\frac{1}{2\pi i}\oint \frac{dz}{z}A(z) = C_\infty = A(0) + \sum_\alpha \mathrm{Res}_{z=z_\alpha}\frac{A(z)}{z}$$

$$\Downarrow$$

$$A(0) = \sum_{\alpha,h} A^h_{n-m_\alpha+1}(z_\alpha)\frac{i}{K_\alpha^2}A^{-h}_{m_\alpha+1}(z_\alpha)$$

...Three-point gluon amplitude

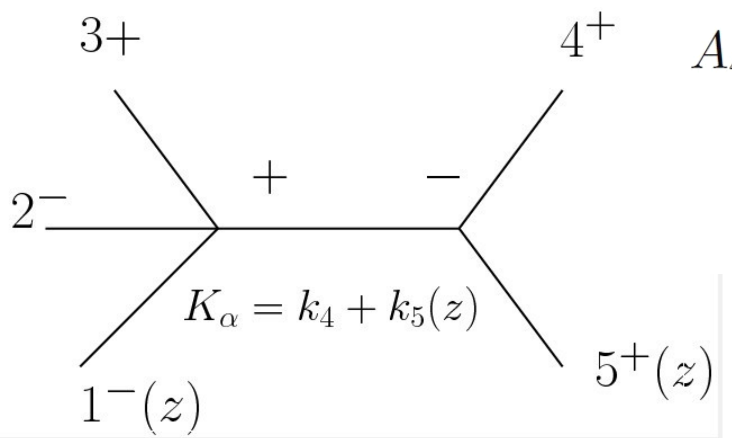
$$A_3(--+) = \frac{\langle 1\ 2\rangle^3}{\langle 2\ 3\rangle\langle 3\ 1\rangle}\,, \quad A_3(++-) = \frac{[1\ 2]^3}{[2\ 3]\,[3\ 1]}$$

Britto-Cachazo-Feng-Witten(BCFW) On-shell Recursion Relation

...Example $A_5(--+++)$

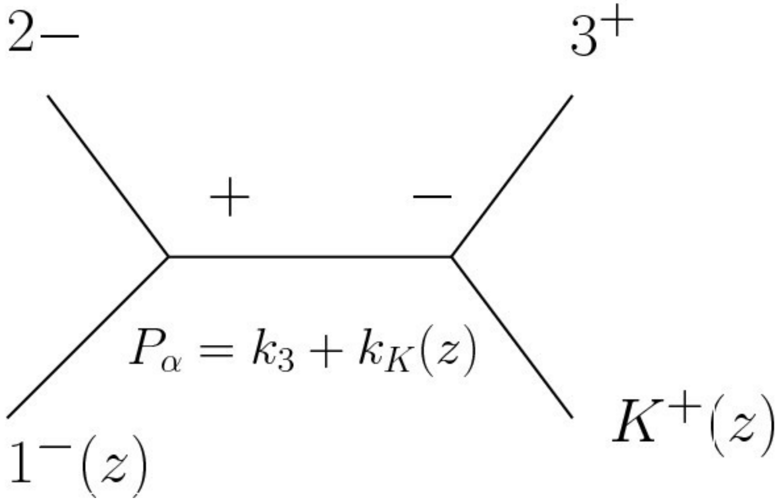
$$\widetilde{\lambda}_1 \to \widetilde{\lambda}_1 + z\widetilde{\lambda}_5 \quad \Rightarrow k_1(z) = \lambda_1\widetilde{\lambda}_1 + z\lambda_1\widetilde{\lambda}_5$$

$$\lambda_5 \to \lambda_5 - z\lambda_1 \quad \Rightarrow k_5(z) = \lambda_5\widetilde{\lambda}_5 - z\lambda_1\widetilde{\lambda}_5$$



$$A_4(1^-(z_\alpha), 2^-, 3^+, K^+(z_\alpha)) \times \frac{1}{(k_4+k_5)^2} A_3(K^-(z_\alpha), 4^+, 5^+(z_\alpha))$$

Britto-Cachazo-Feng-Witten(BCFW) On-shell Recursion Relation



$$A_4(1^-, 2^-, 3^+, K^+)$$

$$= A_3(1^-(z'_\alpha), 2^-, P^+(z'_\alpha)) \frac{1}{(k_1 + k_2)^2} A_3(P^-(z'_\alpha), 3^+, K^+(z'_\alpha))$$

~~Lagrangian~~
~~Wick Contraction~~
~~Feynman Rules~~
~~Feynman Diagram~~
~~……………~~

Finally The Freedom of PhD Students!

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