X-ray phase-contrast and dark-field imaging

using scanning SAXS and
a grating interferometer

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High Contrast / Low Contrast

First x-ray image
made by Röntgen in 1896

Modern x-ray image
2011
X-Ray Phase-Contrast Imaging - Why?

Röntgen was also looking for refraction...

X-ray Imaging
**Imaging regimes**

Fresnel number:  
\[ F = \frac{a^2}{\lambda d} \]

- \( F << 1 \)  
  *Far-field regime (Fraunhofer)*  

- \( F \approx 1 \)  
  *Near-field regime (Fresnel)*  

- \( F >> 1 \)  
  *Contact regime*

**Three signals for imaging**

- Absorption – Standard x-ray image
- Refraction – Phase-contrast image
- Scattering – Dark-field image
Conventional X-Ray Radiography: Absorption Contrast

Refraction in a lens

Vacuum
n = 1

Light
n = 1.5

X-ray
n = 0.999999
Scattering

SAXS - Small Angle X-ray Scattering

Small features
wide angles

Large features
small angles

‘Wave-Optical’ X-ray Radiography: Phase Contrast

object
detector

phase shift
Visible light: Several contrast modalities

Zeiss microscope
www.zeiss.de

Full field vs pencil beam

Pencil Beam

Parallel Beam

Fan Beam

Cone Beam

1953 Nobel Prize
Frits Zernike
**STXM : Scanning Transmission X-ray Microscopy**

Displacement vector on detector: \( L \frac{\lambda}{2\pi} \left( \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y} \right) \)

**Figure courtesy: Jens Als-Nielsen**
SAXS: Small Angle X-ray Scattering

STXM / SAXS
**SAXS, Lymph nodes**

- **Histology**
  - Healthy
  - Invaded

- **Total scattered intensity**
  - Healthy
  - Invaded

- **Scattering asymmetry**
  - Healthy
  - Invaded

- **Scattering direction**
  - Healthy
  - Invaded

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**SAXS**

*Small Angle X-ray Scattering*

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**X-ray Imaging**
Coherence

Ability to interfere
Coherent x-ray scattering

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Abstract
This is a tutorial paper on the properties of partially coherent hard x-ray beams and their use in the structural analysis of condensed matter. The role of synchrotron radiation in the generation of coherent x-ray beams is highlighted and the requirements on the source properties are discussed. The techniques of...

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Coherence

Spatial

\[ L_T = \frac{\lambda R}{W} \]

Figure 1. Diffraction patterns from two narrow slits at distance \( d \), originating from the central part of the source (solid curve) and from the edge of the source at height \( w/2 \) (dashed curve). The slit distance \( d \) is such that the two patterns are in antiphase. This occurs for \( d = \lambda R/w \). Angles and distances are not to scale.
Coherence

Spectral

$$L_L = \frac{\lambda^2}{2\Delta\lambda}$$

Figure 2. Propagation of two waves with wavelengths $\lambda$ and $\lambda + \Delta\lambda$. The longitudinal coherence length $\xi_L$ is defined as the distance over which the phase difference of the two waves has become $\pi$.

Coherence

Intrinsic coherence of different sources

(a) Wiggler

(b) Undulator

Coolidge Tube

Filament

electrons

X-rays

anode

cathode

water

Log (Intensity)

Energy

Bremsstrahlung

K

M

K

Coolidge Tube

Rotating Anode

X-rays

Filament

electrons

cathode

anode

water

in

out
**SASE - Self Amplification by Spontaneous Emission**

![Diagram of SASE](image)

**Coherence**

![Coherence Diagram](image)

*Courtesy of Chang Chang, UC Berkeley and LBNL.*
Coherence

Figure 6. X-ray diffraction from a disordered medium with particle distance $d$. The object size is $a$. (a) Incoherent scattering, giving rise to a continuous diffraction ring; (b) coherent scattering, resulting in a speckled diffraction ring.
**History of phase contrast imaging**

Crystal Interferometer, Bonse & Hart 1965  
Diffraction Enhanced Imaging, Davis 1995 and Ingal 1995  
Propagation Based Imaging, Snigirev 1995 and Wilkins 1996

Grating Interferometer:  
- Synchrotron  
- David 2002, Momose 2003  
- Weitkamp 2005  
- X-ray tube: Pfeiffer 2006  
- Darkfield: Pfeiffer 2008

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**Free space propagation**  
**Real Space**

Plane wave:  
Sphere wave:

Huygens-Fresnel principle:  
at any instant  
a wavefront can be considered as a collection of point sources
Free space propagation

\[ \Psi(x, y) = \iint q(X,Y)A \exp(i2\pi \frac{r}{\lambda})dXdY \]

\[ r = \sqrt{R^2 + (x-X)^2 + (y-Y)^2} = R + \frac{(x-X)^2}{2R} + \frac{(y-Y)^2}{2R} \]

Free space propagation

\[ \Psi(x, y) = \exp(ikR) \iint u_0(X,Y)A \exp \left( \frac{i\pi}{d\lambda} \left( (x-X)^2 + (y-Y)^2 \right) \right) dXdY \]

\[ \Psi(x, y) = q(x,y) * P_R(x,y) \quad P_R(x,y) = A \exp(ikd) \exp \left( \frac{i\pi}{d\lambda} (x^2 + y^2) \right) \]
Free space propagation

Fourier Space

Define Fourier transform as

\[ F(k) = \int f(x) \exp(-ikx) dx \]

Wavefront at \( z = 0 \)

\[ \tilde{q}(k_x, k_y) = \int q(x, y) \exp\left\{ -i(k_x x + k_y y) \right\} dx dy \]

\[ q(x, y) = \frac{1}{(2\pi)^2} \int \tilde{q}(k_x, k_y) \exp\left\{ i(k_x x + k_y y) \right\} dk_x dk_y \]

A superposition of planewaves

\[ q(x, y) = \frac{1}{(2\pi)^2} \int \tilde{q}(k_x, k_y) \exp\left\{ i(k_x x + k_y y) \right\} dk_x dk_y \]

\[ k_z = \sqrt{k_x^2 - k_i^2 - k_y^2} = k \sqrt{1 - \frac{k_i^2}{k^2}} = \frac{k}{\sqrt{k^2 - k_x^2}} \]

Propagation of a plane wave in free space is described by the phaseshift \( \exp(ikr) \)

\[ \Psi(x, y) = \frac{1}{2\pi} \int \tilde{q}(k_x, k_y) \exp\left\{ i(k_x x + k_y y) \right\} dk_x dk_y \]

Free space propagation

Fourier Space

\[ \Psi(x, y) = \frac{1}{2\pi} \int \tilde{q}(k_x, k_y) \exp\left\{ i k_z \right\} \exp\left\{ i(k_x x + k_y y) \right\} dk_x dk_y \]

When the angle \( \Theta \) between propagation direction and z-axis is small

\[ \Theta = |\sin(\Theta)| = \frac{\sqrt{k_x^2 + k_y^2}}{k} \]

is small

\[ k_z = \sqrt{k^2 - k_i^2 - k_y^2} = k \left( 1 - \frac{k_i^2}{2k^2} - \frac{k_y^2}{2k^2} \right) = \frac{k}{2k} (k_x^2 + k_y^2) \]
Free space propagation

**Fourier Space**

\[
\Psi(x,y) = \frac{1}{2\pi} \iiint \tilde{q}(k_x,k_y) \exp\left\{ i(k_x x + k_y y) \right\} \, dk_x dk_y \]

When the angle \( \Theta \) between propagation direction and z-axis is small

\[
\Theta = |\sin(\Theta)| = \frac{\sqrt{k_x^2 + k_y^2}}{k}
\]

is small

\[
\Psi(x,y) = \frac{1}{(2\pi)^2} \iiint \tilde{q}(k_x,k_y) \exp\left\{ ikd - id\left(\frac{k_x^2 + k_y^2}{2k}\right) \right\} \exp\left\{ i(k_x x + k_y y) \right\} \, dk_x dk_y
\]

\[
\Psi(x,y) = \frac{1}{(2\pi)^2} \iiint \tilde{\Psi}(k_x,k_y) \exp\left\{ i(k_x x + k_y y) \right\} \, dk_x dk_y
\]

\[
\tilde{\Psi}(k_x,k_y) = \tilde{q}(k_x,k_y) \exp\left\{ ikd - id\left(\frac{k_x^2 + k_y^2}{2k}\right) \right\} = \tilde{q}(k_x,k_y) \tilde{P}_d(k_x,k_y)
\]

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**Free space propagation**

**Fourier Space**

\[
\tilde{P}_d(k_x,k_y) = \exp\left\{ -id\left(\frac{k_x^2 + k_y^2}{2k}\right) \right\}
\]

\[
= \exp\left\{ -i\lambda d\left(\frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2}\right) \right\}
\]
Free space propagation

Fourier Space

\[ \tilde{\Psi}(f,g) = \tilde{q}(f,g) \tilde{P}_d(f,g) \]
\[ \tilde{P}_d(f,g) = \exp\left\{ ikd - i\pi\lambda \, d(f^2 + g^2) \right\} \]
\[ \Psi(x,y) = \iiint \tilde{q}(f,g) \tilde{P}_d(f,g) \exp\left\{ i(k_x x + k_y y) \right\} df dg \]

Simulated propagation

Figure: Timm Weltkamp
Applications

Propagation based phase contrast image

Figure 5. A mouse femur head imaged at 15 keV at different sample–detector distances. From [26].
Wavefront propagation. Phase grating = -dp x \cos(p_d x)
Phase-Contrast Imaging using X-Ray Optical Gratings

Momose et al | Optics Express | 2003
Weitkamp et al | Optics Express | 2005
Pfeiffer et al | Physical Review Letters | 2005

Franz Pfeiffer

**Talbot Effect**

In the Fourier transform of a periodic function, only $k = \frac{2 \pi n}{p}$ is non-zero

$$q(x) = \frac{1}{2\pi} \int \tilde{q}(k_x) \exp\{i(k_x x)\} dk_x = \sum_n \tilde{q}(2\pi n/p) \exp\{i(2\pi n/p) x\}$$

The fourier space propagator becomes

$$\tilde{P}_d(k_x) = \exp(ikd)\exp\{-id(\frac{n}{p})^2/2k\} = \exp(ikd)\exp\{-i\frac{\lambda d}{4\pi} \left(\frac{2\pi n}{p}\right)^2\}$$

i.e. apart from a phase factor, the wavefront repeats itself at distance

$$d_T = \frac{2P^2}{\lambda} \Rightarrow \tilde{P}_d\left(2\pi n/p\right) = \exp(ikd_T)\exp\{-i2\pi(n)^2\}$$
**Grating interferometer**

- **Reference grating**
- **Analyzer grating**

T. Weitkamp et al., Optics Express 13, 6296 (2005)

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**Differential Phase Imaging**

\[ \delta \Theta = \frac{\lambda}{2\pi} \frac{\partial \Phi}{\partial x} \]
An interferometer based on the Talbot effect

A.W. Lohmann and D.E. Silva
Optics communication 2 (1971), 413-415

X-ray phase contrast imaging with a grating interferometer

C. David, B. Nöhammer, H. H. Solak and E. Ziegler
Scattering

Extracting phase contrast image

\[ I(m, n, x_g) = \sum a_i(m, n) \cos(ikx_g + \phi_i(m, n)) \]

absorption  local scattering power  phase gradient

Chicken wing

Nature Materials, January 2008