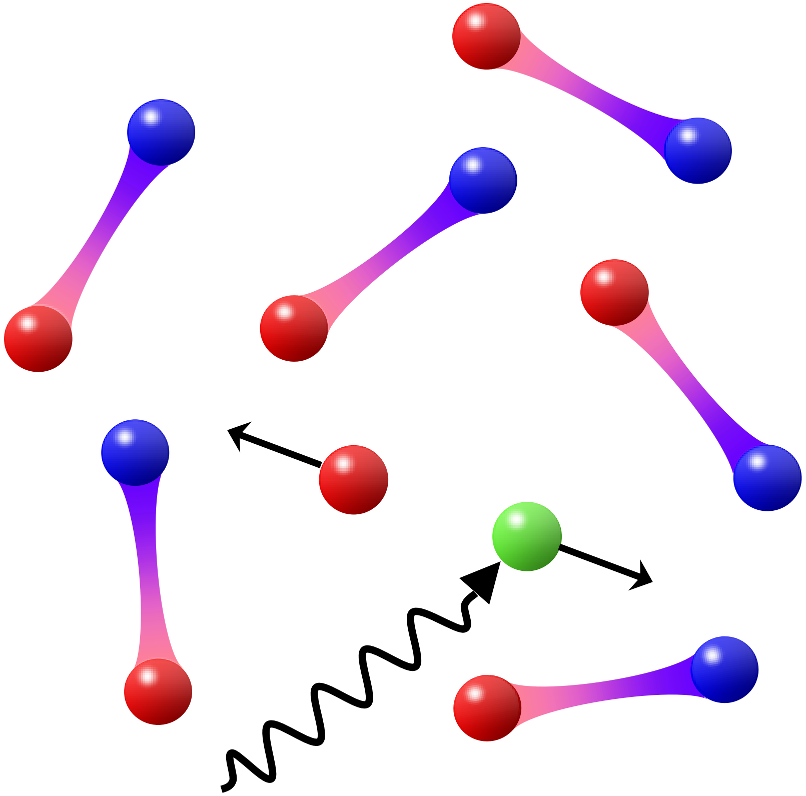
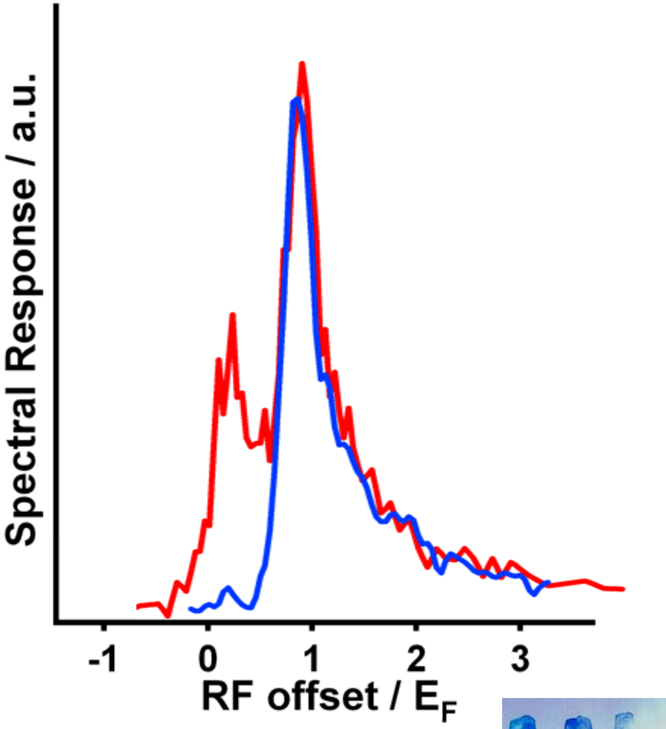
Coherent Quantum Gases - From Cold Atoms to Condensed Matter
Copenhagen, Jan. 15th/16th 2009

Radio Frequency Spectroscopy of Ultracold Fermi Gases (*1*)

Andre Schirotzek





Center for Ultracold Atoms at MIT and Harvard

Outline

Part 1: 1.) Introduction

- strongly interacting Fermions
- effect of density imbalance
- rf spectroscopy

2.) Experiments using rf spectroscopy:
Problems and solutions

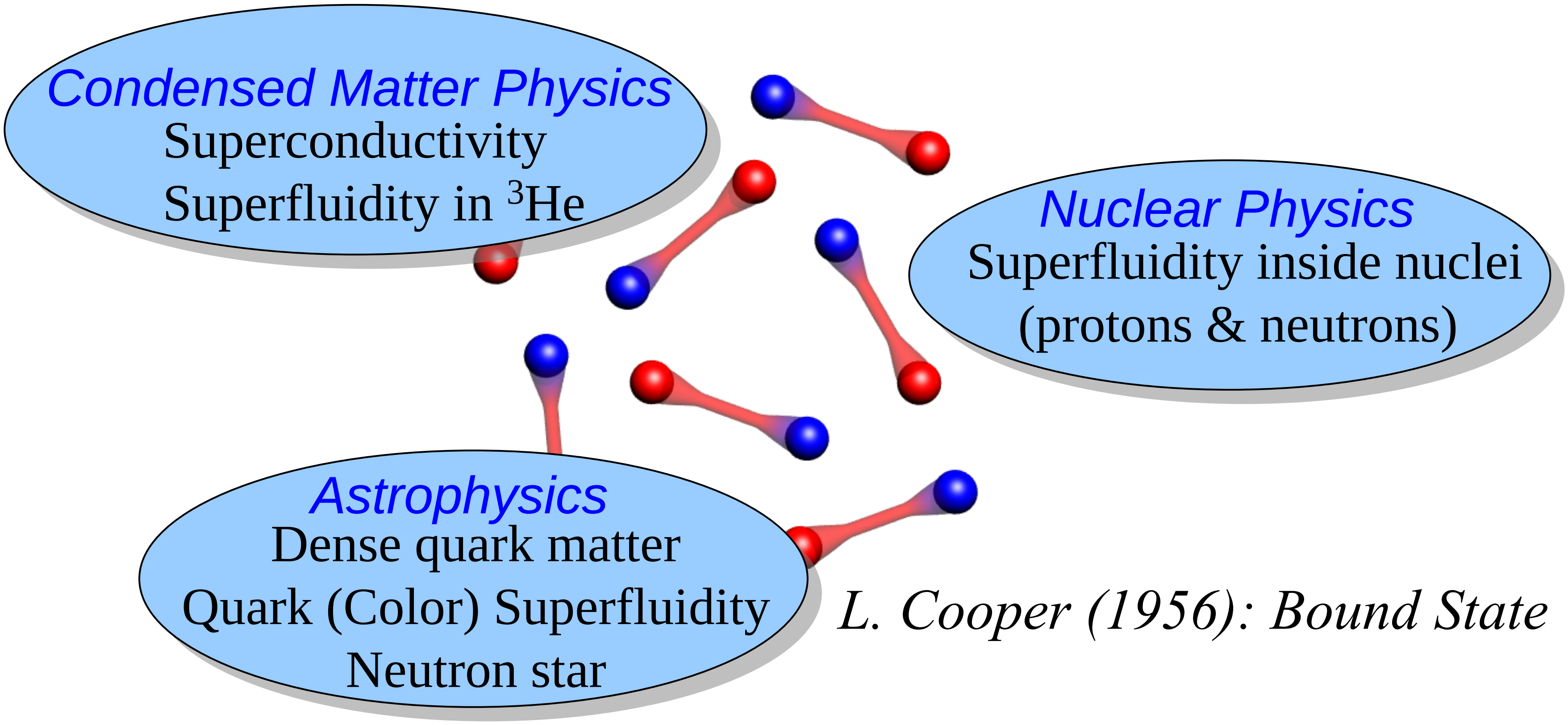
Part 2: 3.) Quantitative studies with rf spectroscopy

a) Quasiparticle spectroscopy and
determination of the superfluid gap

b) The 'N+1' body problem: Observation
of polarons in a highly imbalanced Fermi gas

Pairing and Superfluidity of Fermions

Interacting Fermi mixture



Condensed Matter Physics

Superconductivity

Superfluidity in ^{3}He

Nuclear Physics

Superfluidity inside nuclei

(protons & neutrons)

Astrophysics

Dense quark matter

Quark (Color) Superfluidity

Neutron star

L. Cooper (1956): Bound State

Formation of Pairs → Condensation → Superfluidity

Ultracold Atom System: Interacting Fermi Mixture

An Ideal Model System: A New Sample

Ultradilute $T_F = 0.1\sim10\ \mu K$

Ultracold $T/T_F < 0.05$

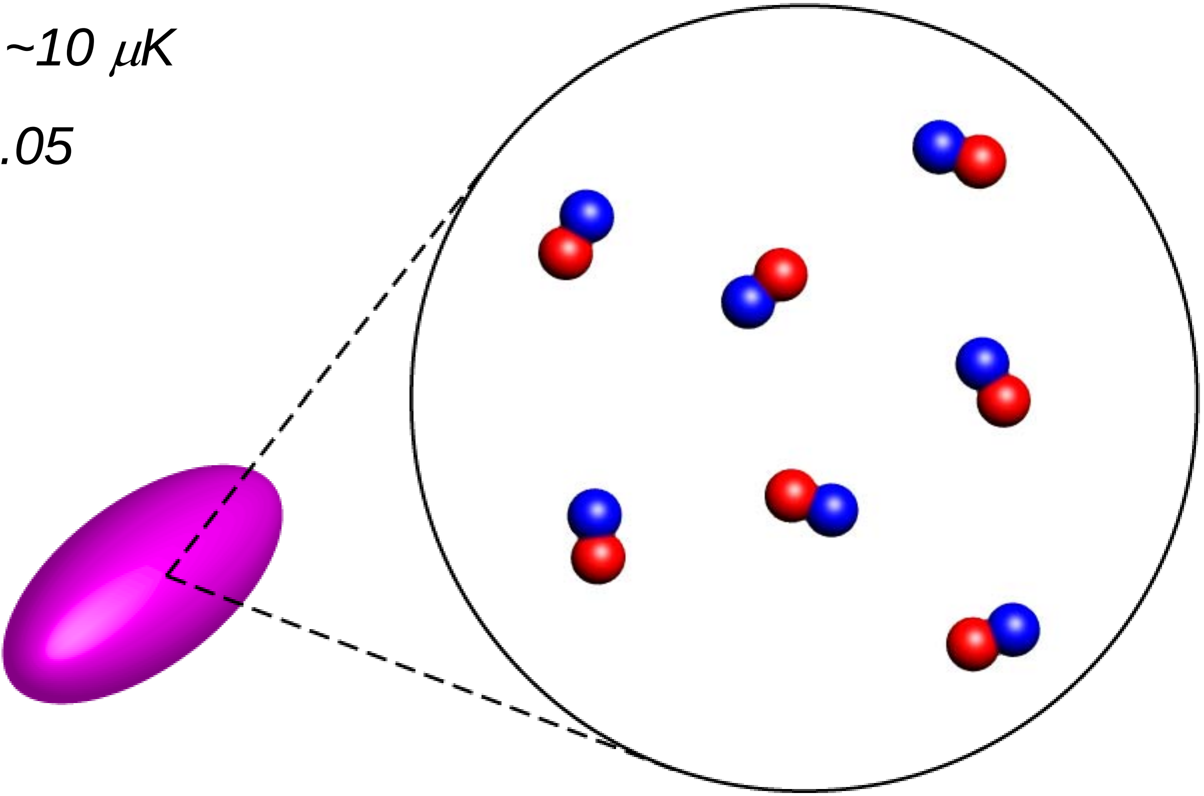
No impurities

Highly controllable:

Density

Temperature

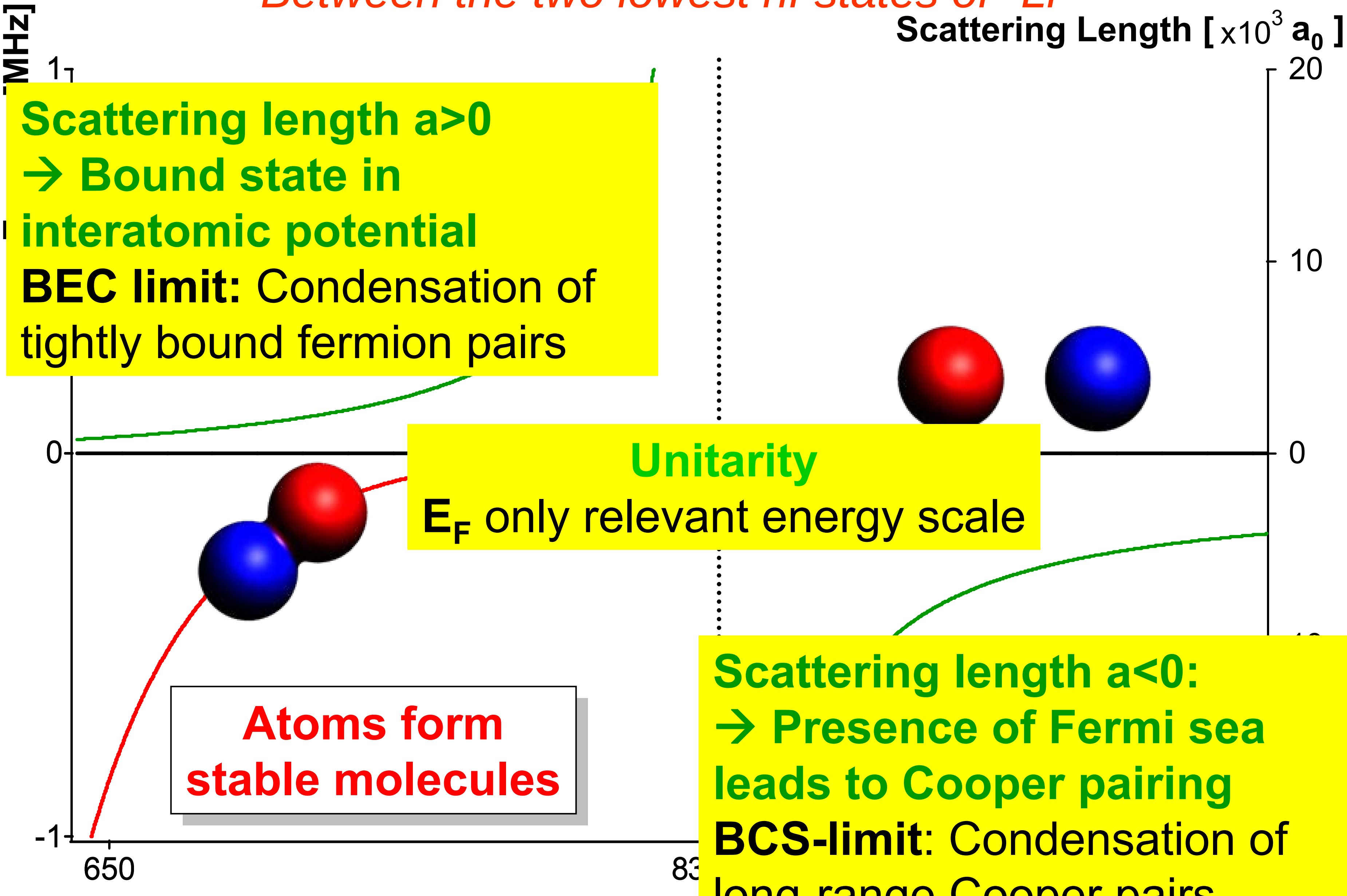
Dimensionality



Interaction strength

Tuning Interactions: Feshbach Resonances

Between the two lowest hf states of ^{6}Li



Scattering length a>0
→ Bound state in interatomic potential

BEC limit: Condensation of tightly bound fermion pairs

Unitarity

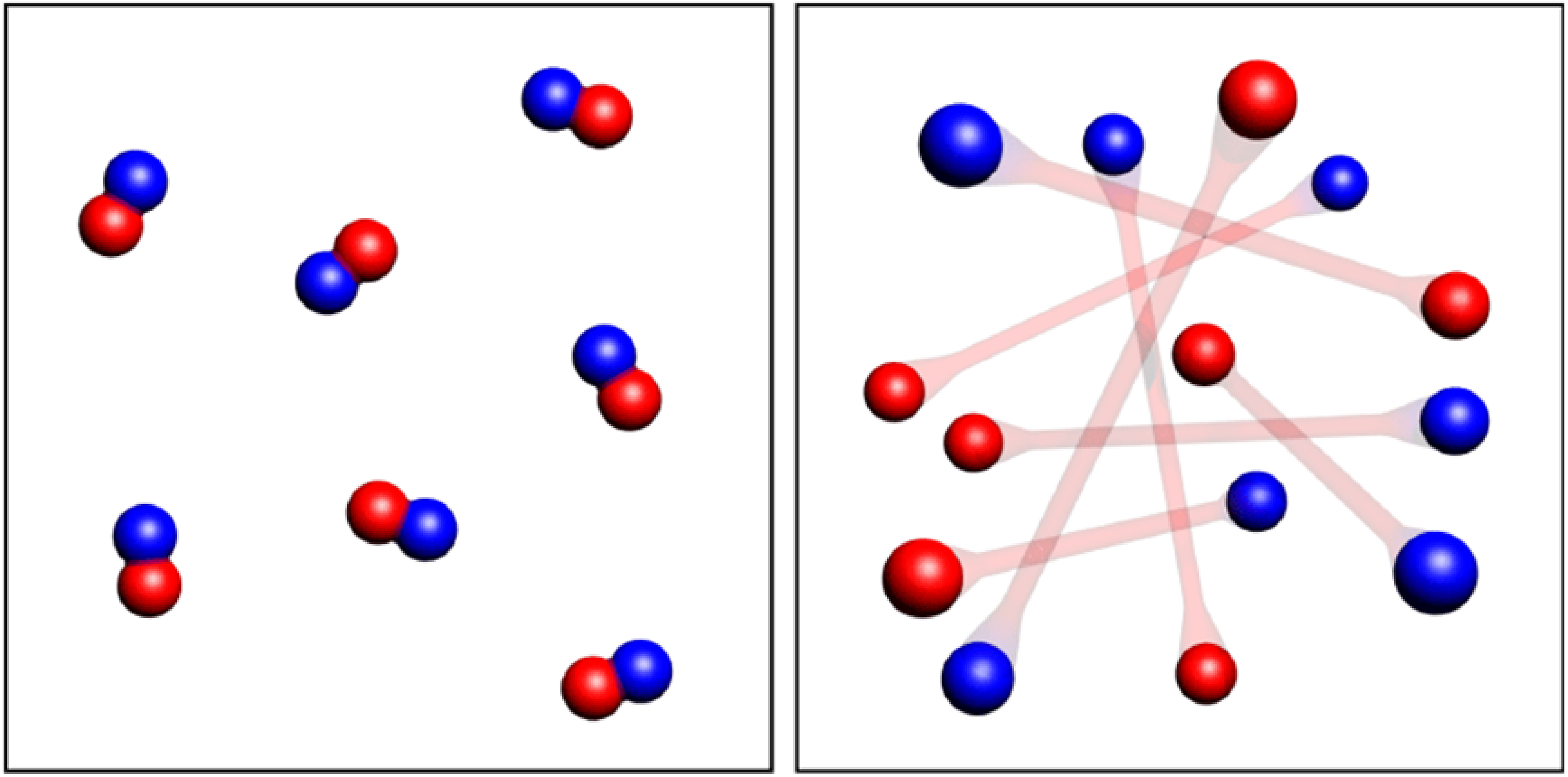
E_F only relevant energy scale

Atoms form stable molecules

Scattering length a<0:
→ Presence of Fermi sea leads to Cooper pairing

BCS-limit: Condensation of long-range Cooper pairs

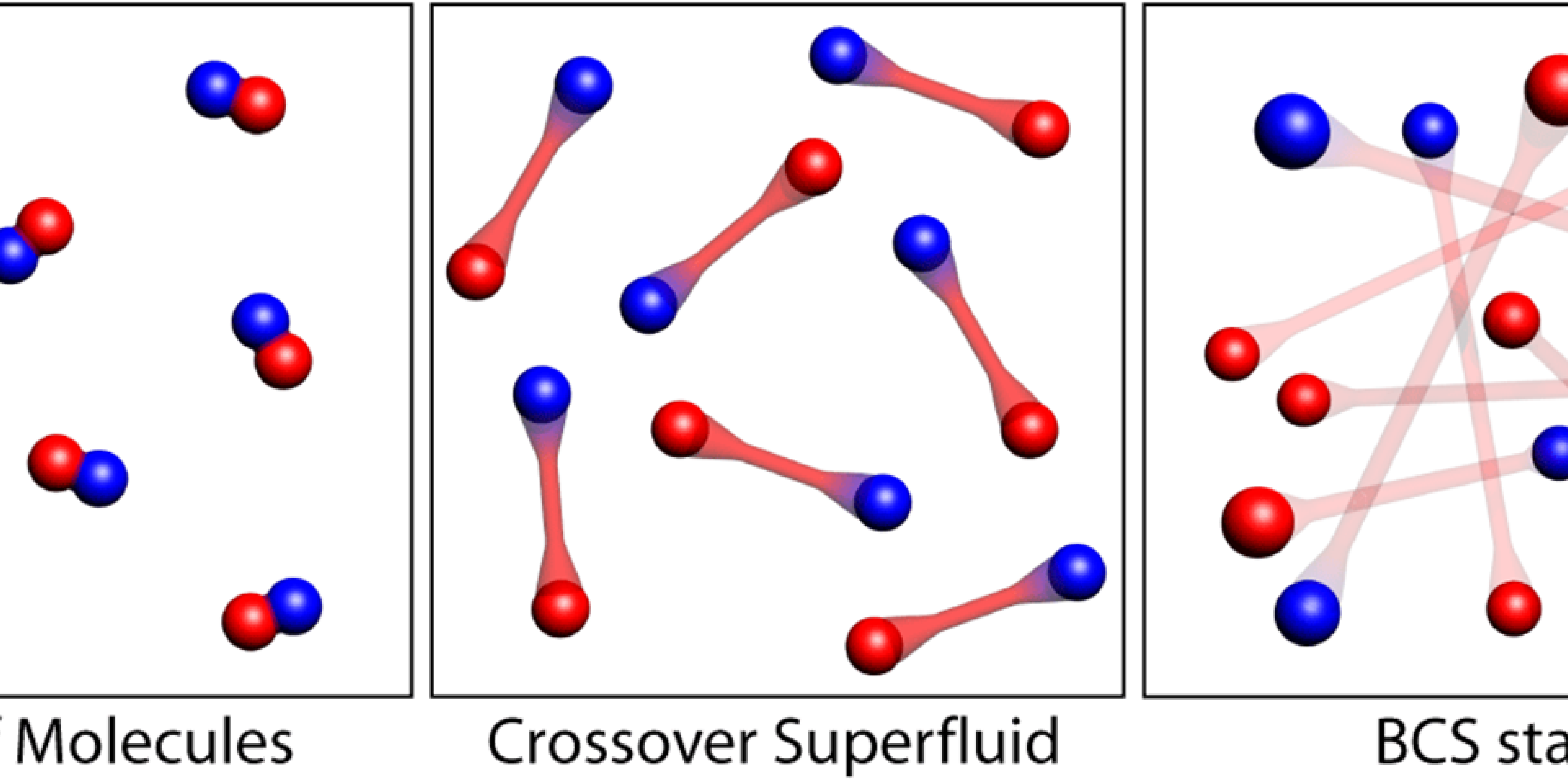
The BEC-BCS Crossover



BEC of Molecules

BCS state

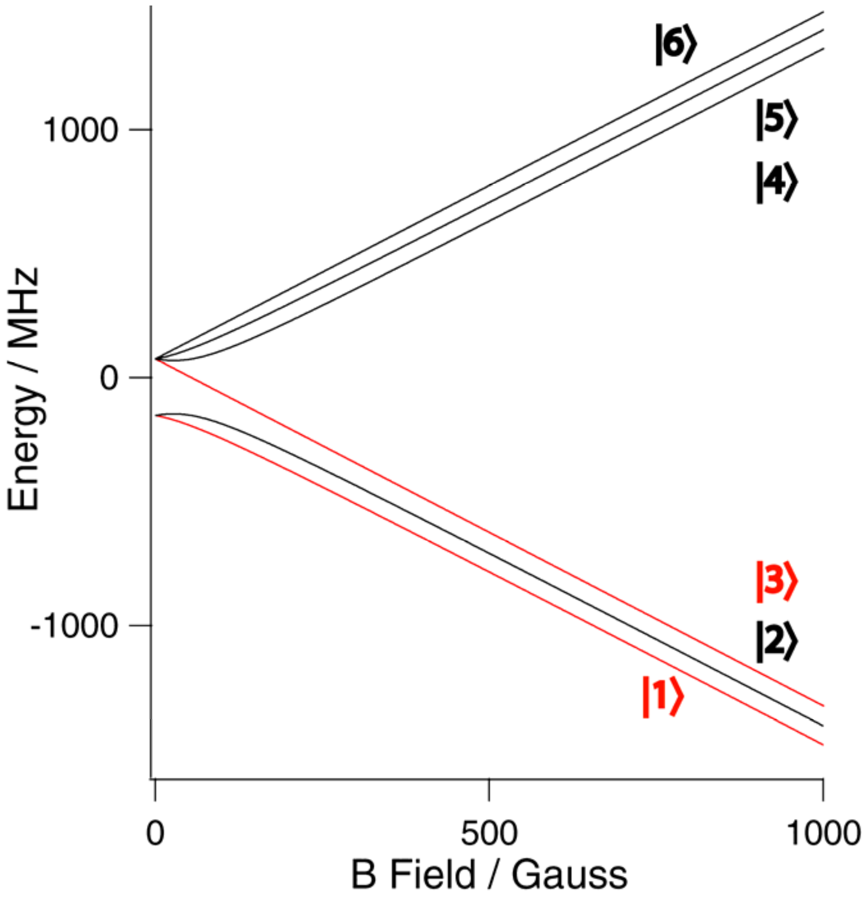
The BEC-BCS Crossover



The experiment

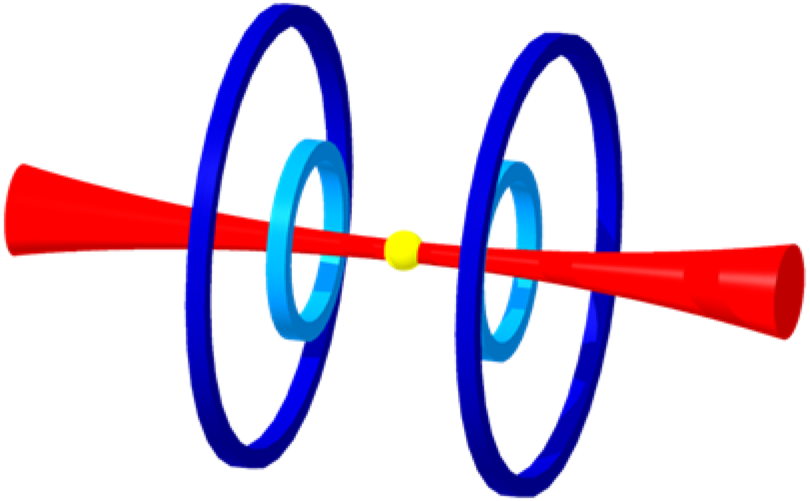
Preparation of an interacting Fermi system in ^{6}Li

^{6}Li - Atom: 6 hyperfine states



States |1> and |3> correspond to Pseudospin |↑> and |↓>

Optical trapping @ 1064 nm



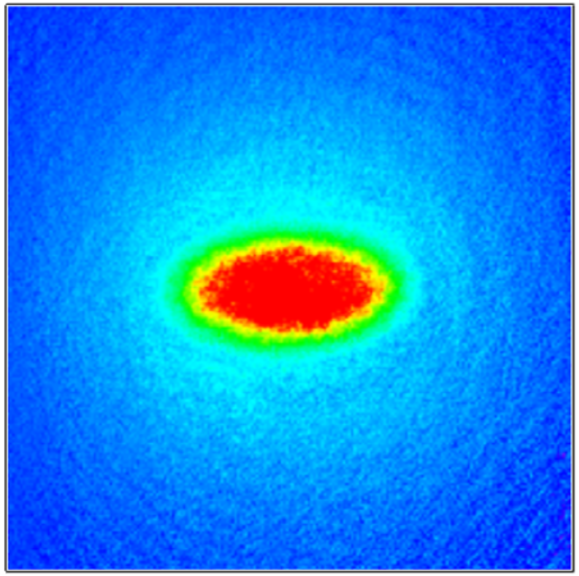
ν_{axial} = 22 Hz
ν_{radial}= 160 Hz
E_{trap} = 0.5 - 5 μK

Source of ultracold fermions

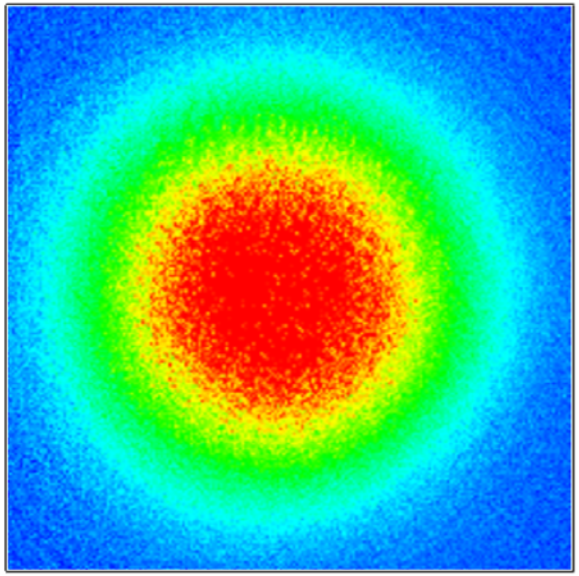
Cool fermionic lithium-6
using sodium as a refrigerator

10^7 atoms in BEC (w/o Li)

5×10^7 Li atoms at $\frac{T}{T_F} < 0.3$

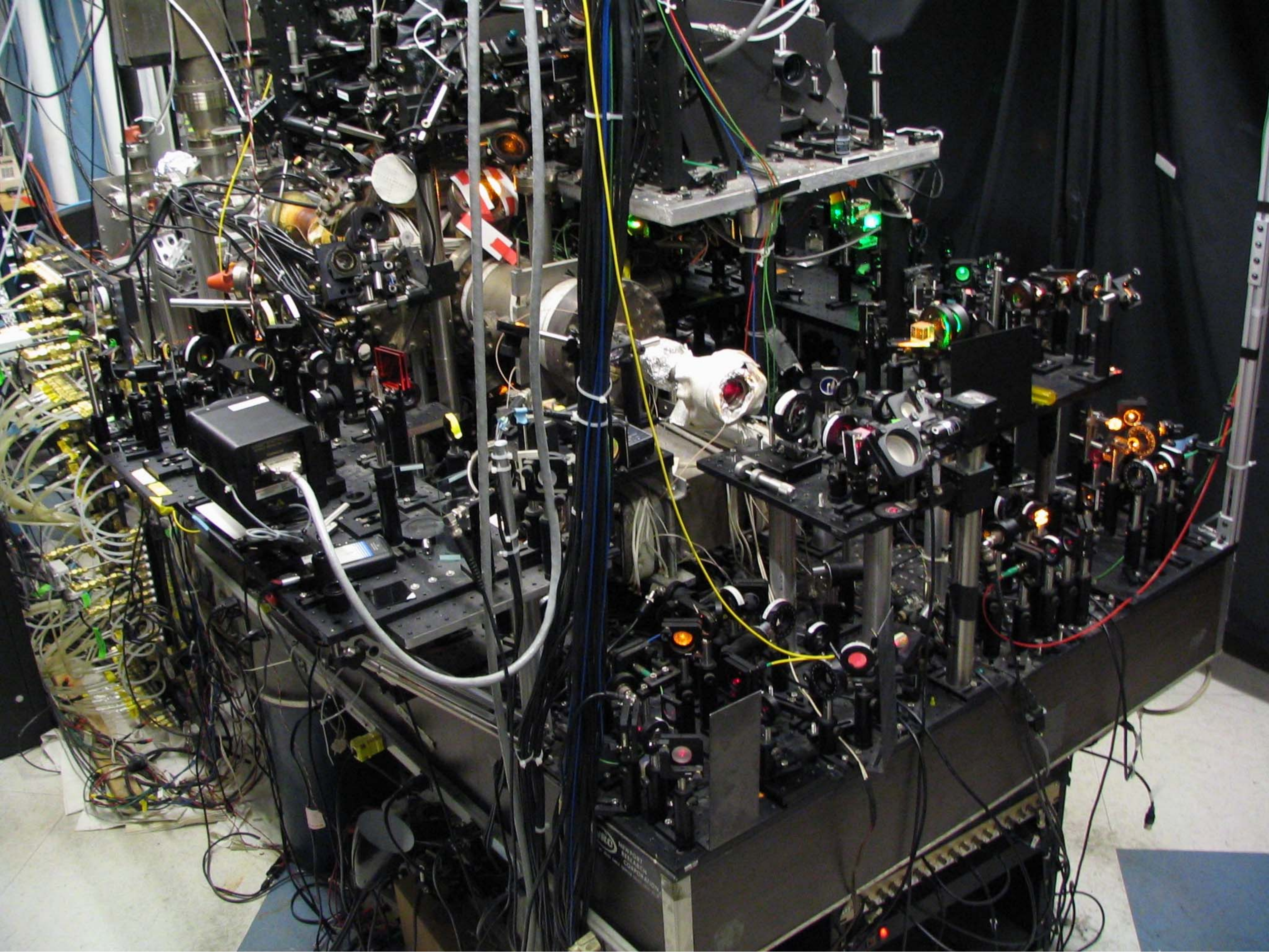


Bosons



Fermions

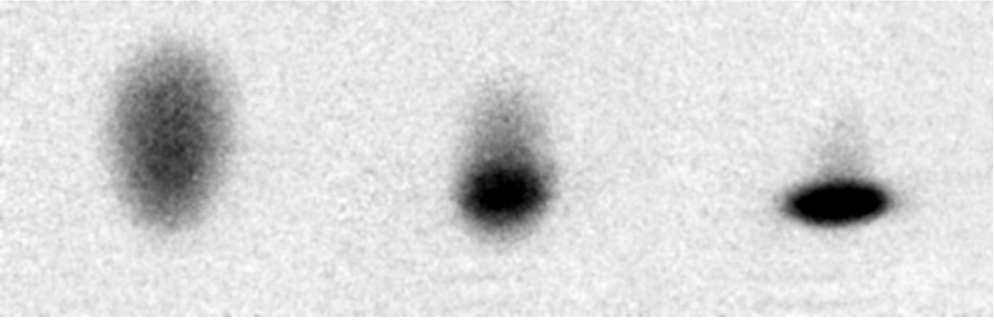
Z. Hadzibabic *et.al.*, PRL **91**, 160401 (2003)


NEWPORT RESEARCH CORPORATION

From Fermion Pair Condensation

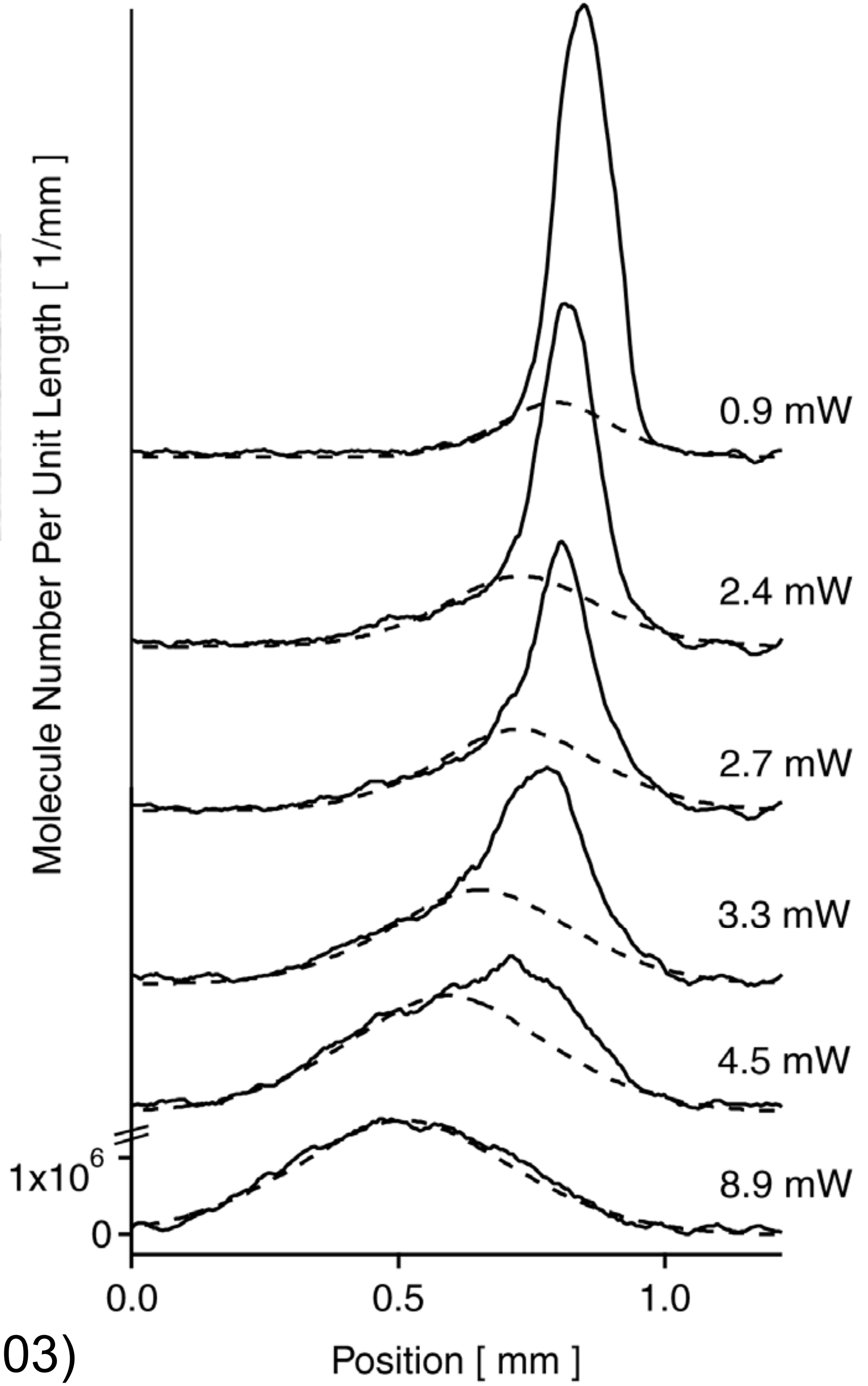
to High Temperature Superfluidity

BEC of Fermion Pairs (“Molecules”)



These days: Up to 10 million condensed molecules

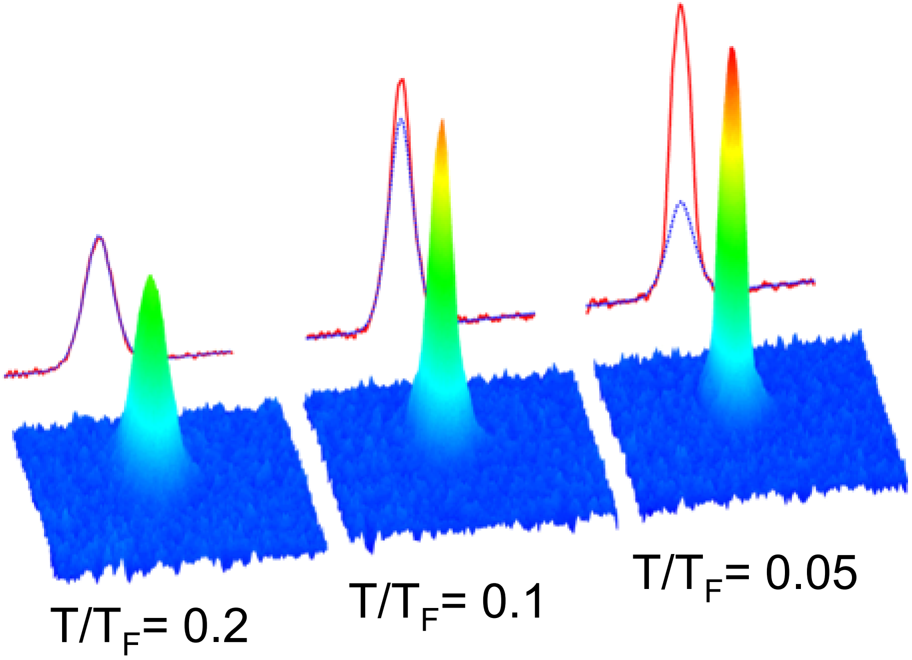
Boulder	Nov ‘03
Innsbruck	Nov ‘03, Jan ’04
MIT	Nov ’03
Paris	March ’04
Rice, Duke	

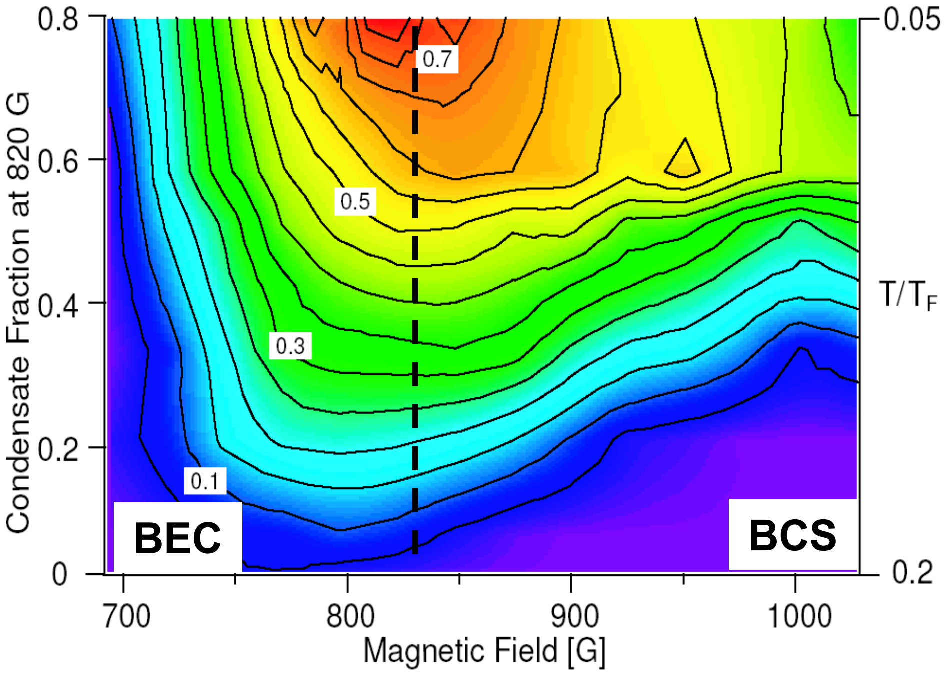


M.W. Zwierlein et al, PRL 91, 250401 (2003)

Observation of Pair Condensates

At 900 G

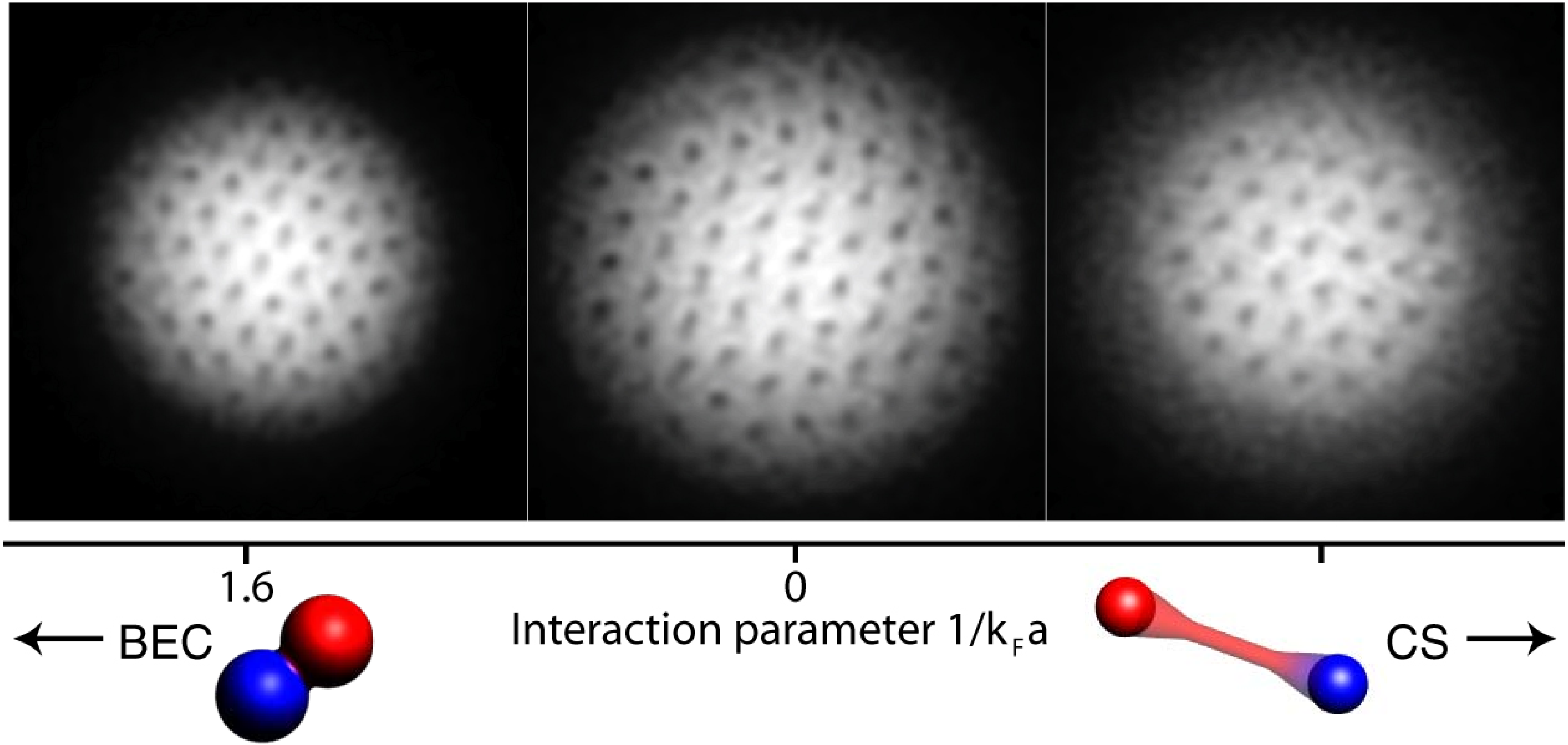



Condensate Fraction

C.A. Regal et al., PRL 92, 040403 (2004)
M.W. Zwierlein et al., PRL 92, 120403 (2004)

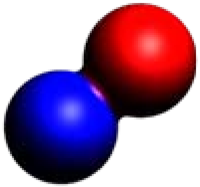
Vortex lattices in the BEC-BCS crossover

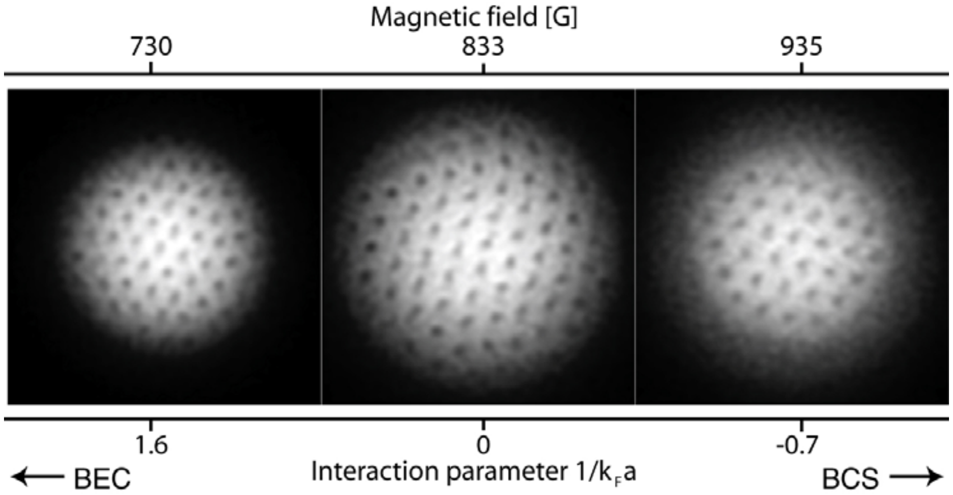
Establishes *superfluidity* and *phase coherence* in gases of fermionic atom pairs

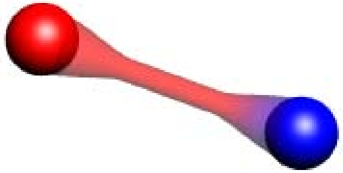


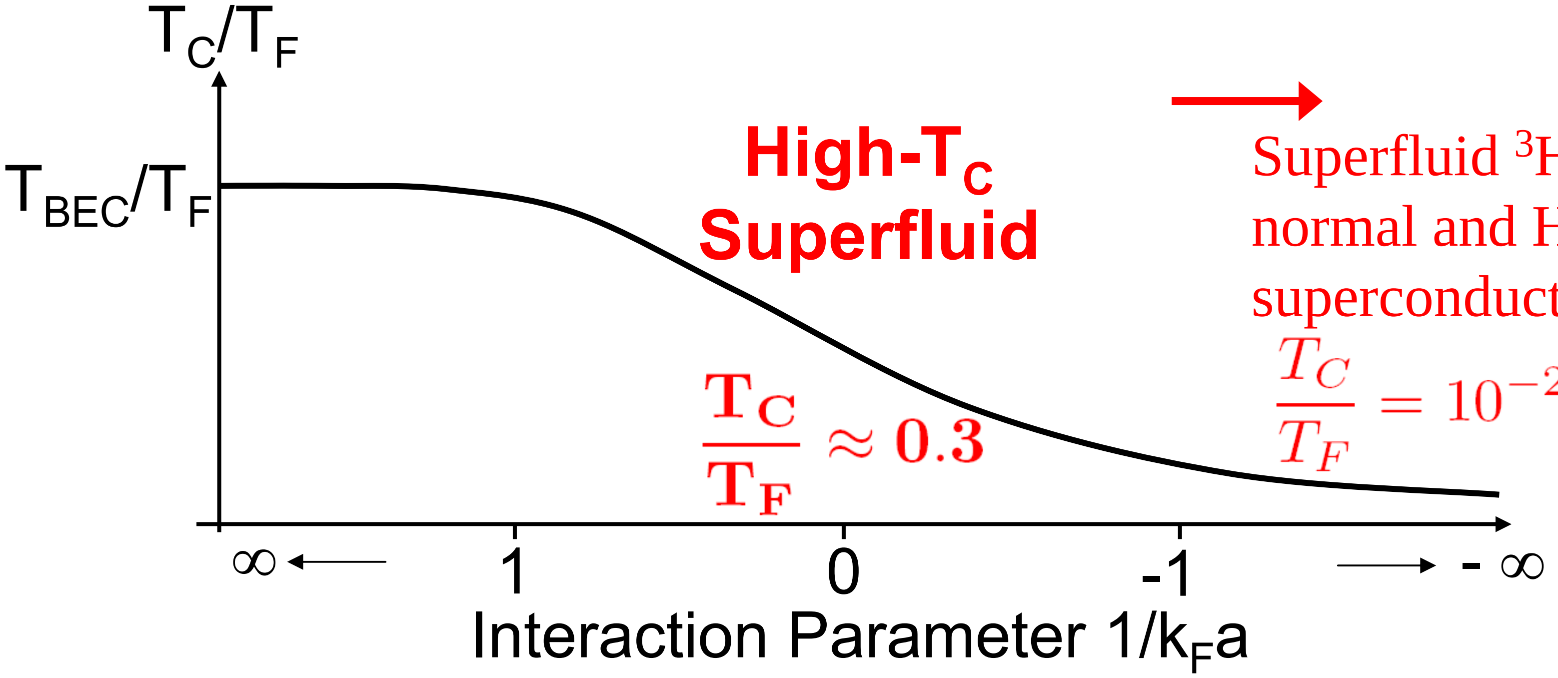
M.W. Zwierlein et al, Nature 435, 1047-1051 (2005)

High Temperature Superfluid at 100 nK?









High-T_C Superfluid

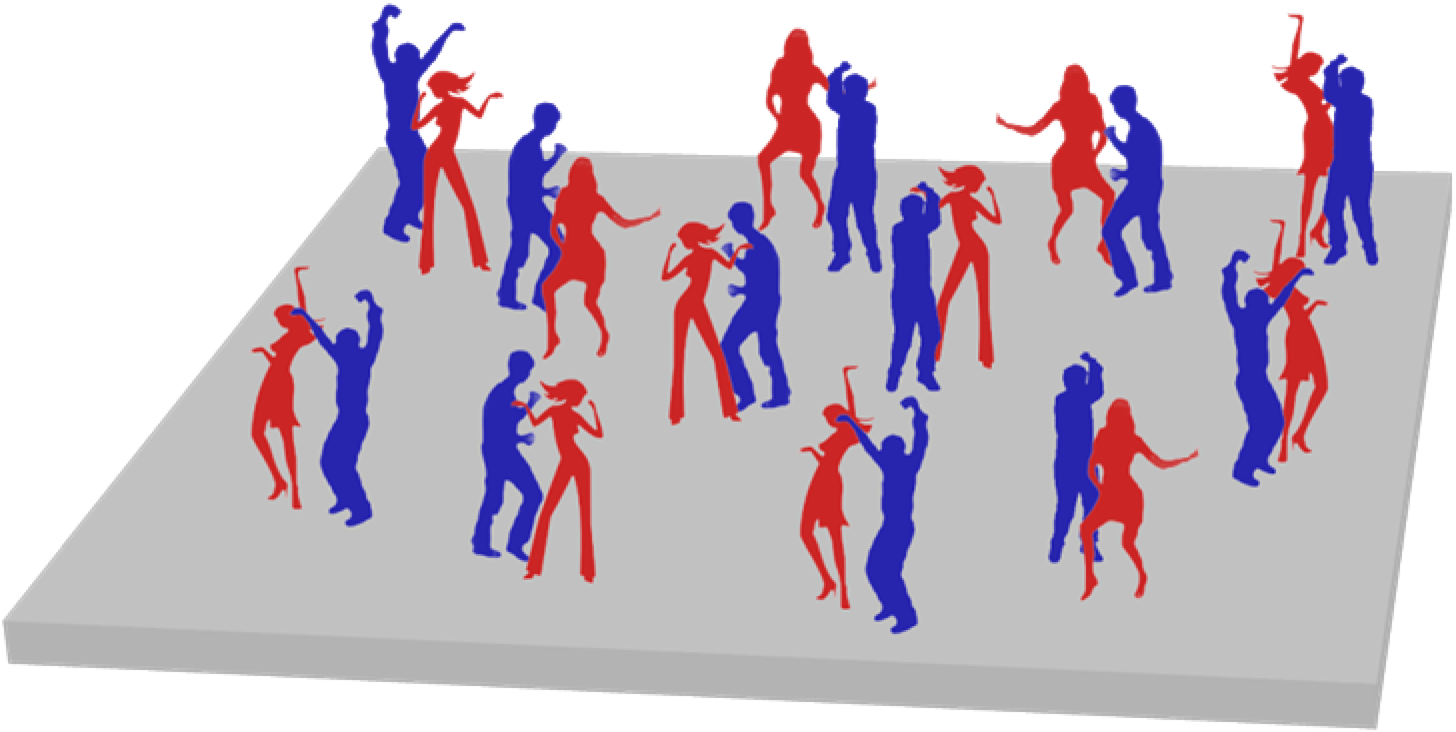
$$\frac{T_C}{T_F} \approx 0.3$$

→

Superfluid ^{3}He, normal and High-T_C superconductors

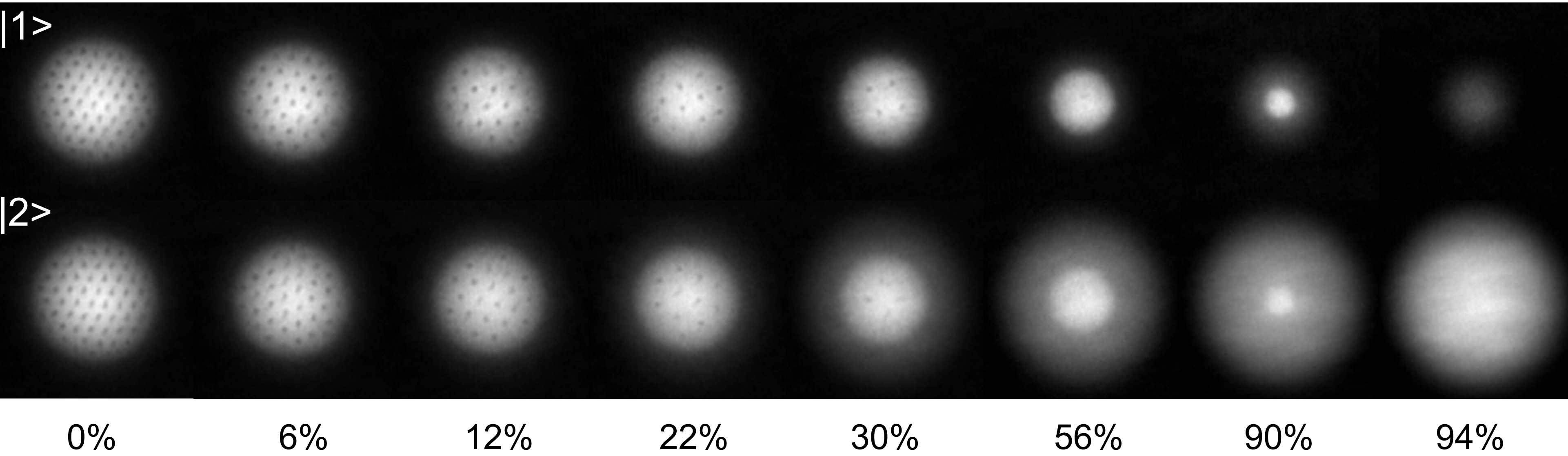
$$\frac{T_C}{T_F} = 10^{-2}..10^{-5}$$

Fermionic Superfluidity with Imbalanced Spin Populations



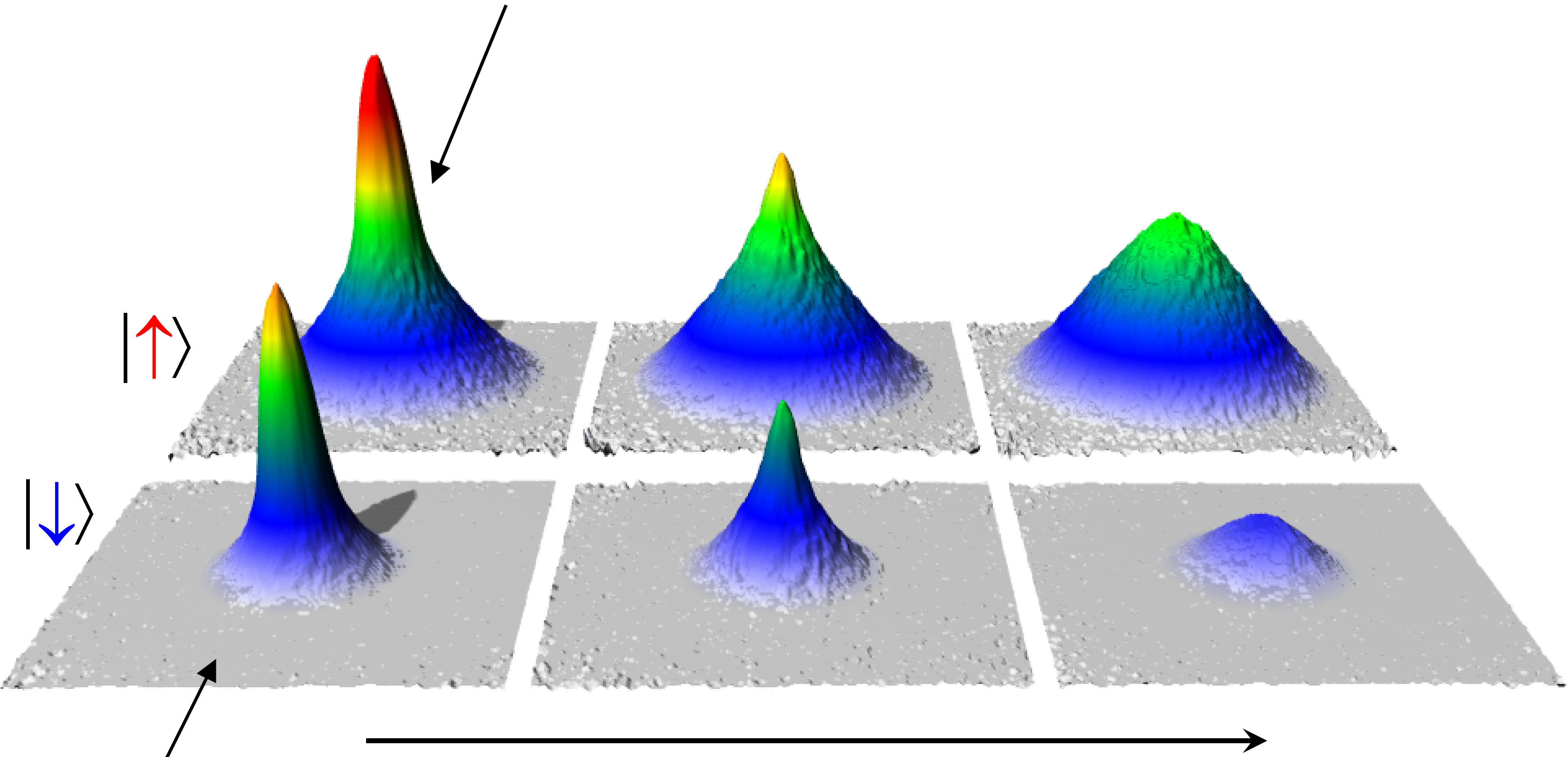
What if there are too many singles?

Fermionic Superfluidity with Imbalanced Spin Populations



The Clogston-Chandrasekhar limit

Majority component



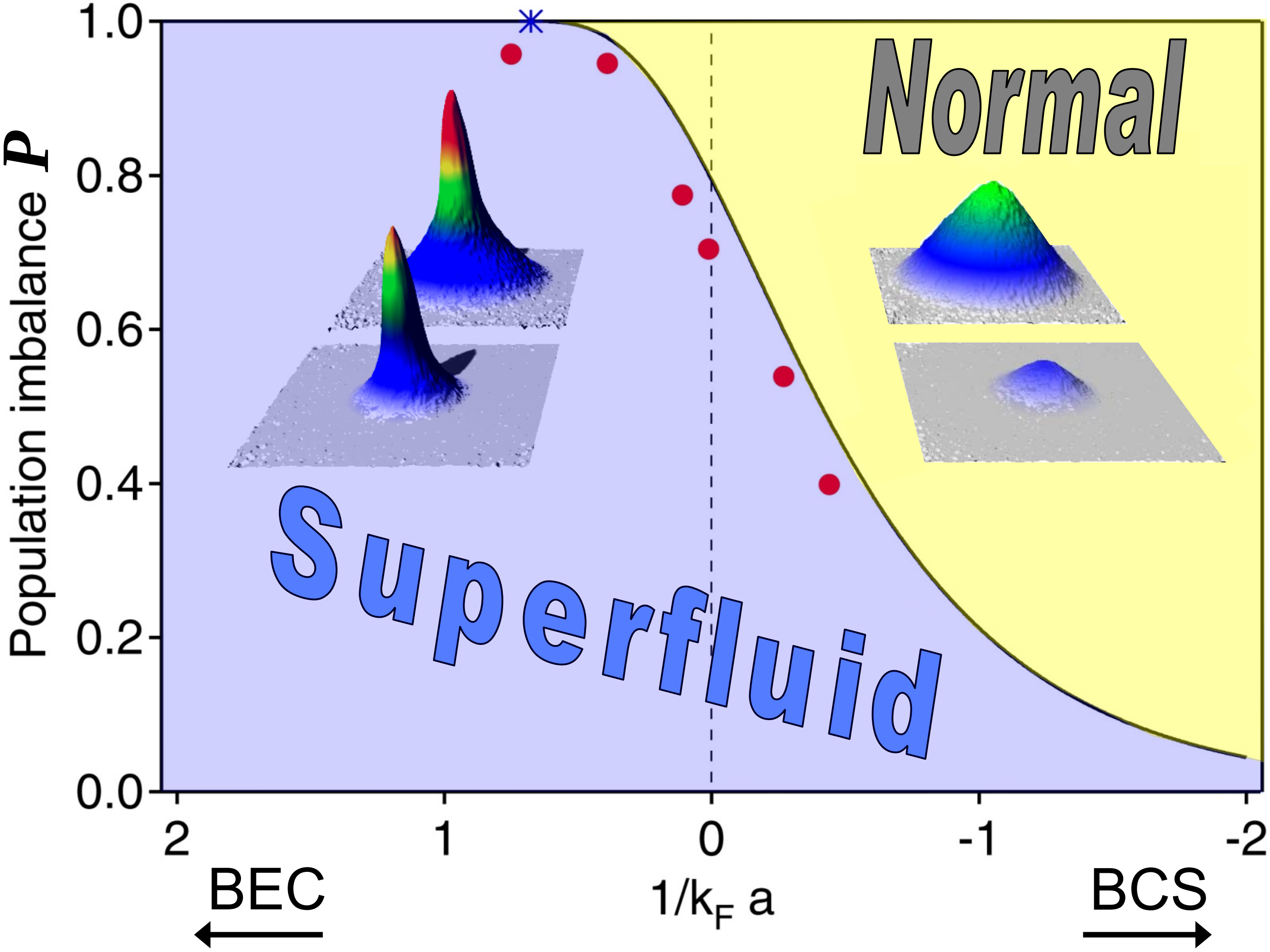
$|\uparrow\rangle$

$|\downarrow\rangle$

Increase population imbalance

Minority component

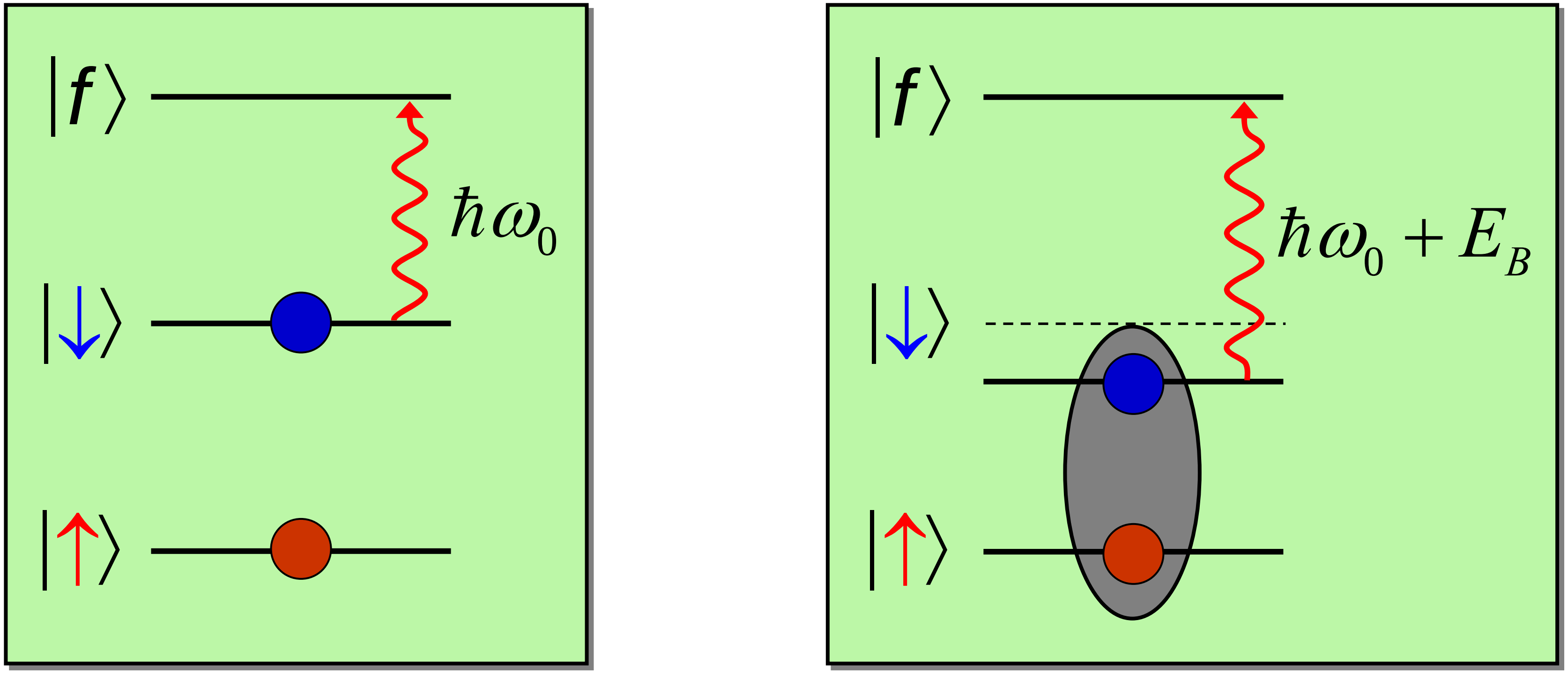
Phase Diagram for Unequal Mixtures



Breakdown: Critical polarization $P_c \propto$ Gap Δ

M.W. Zwierlein et al, Science 311, 492 (2006)

Radiofrequency spectroscopy



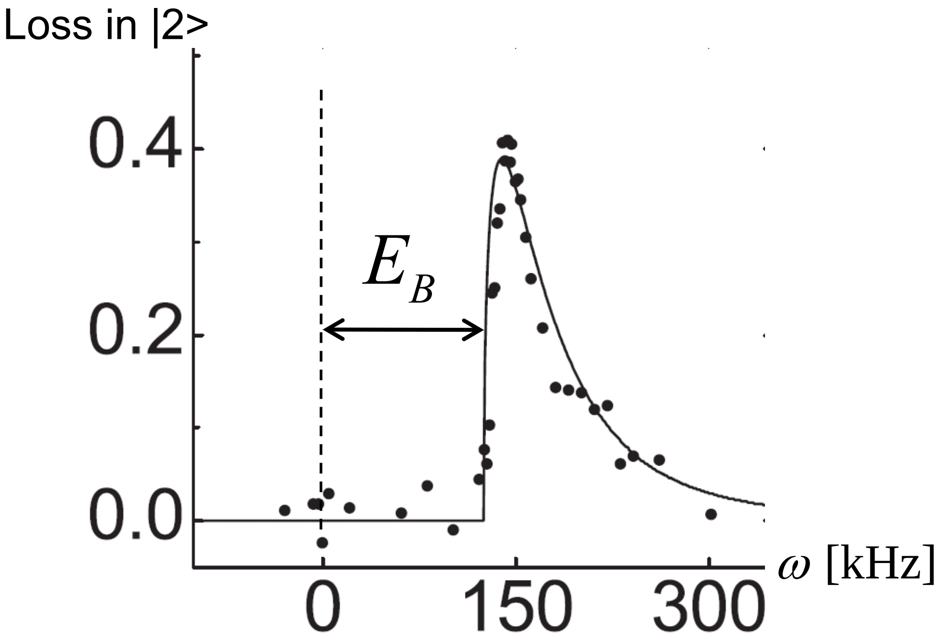
No interactions

Molecular Pairing

Photon energy = Zeeman + Binding + Kinetic energy

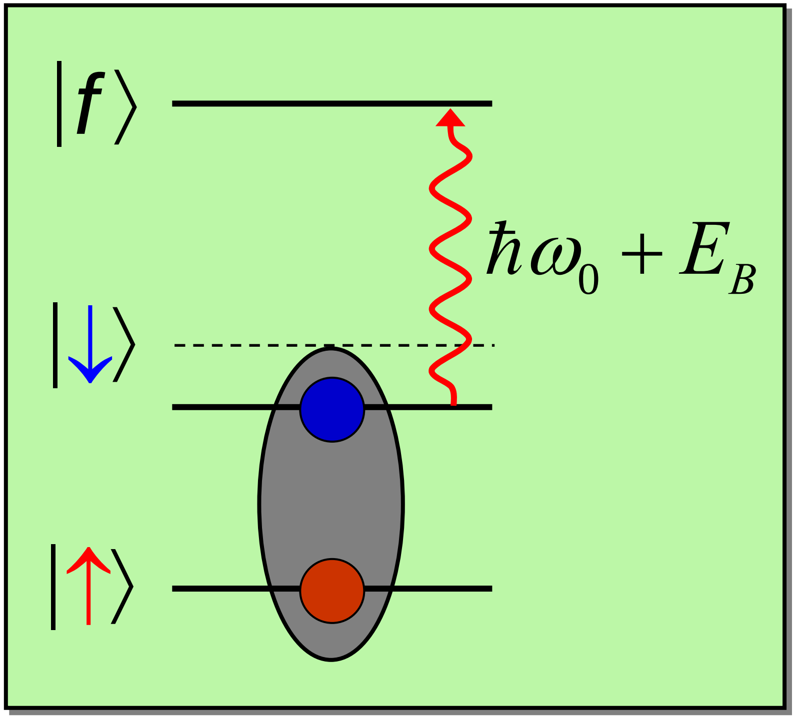
$$\hbar\omega = \hbar\omega_0 + E_B + 2\varepsilon_k$$

Radiofrequency spectroscopy



C.Chin et al. Science, **305**, 1128 (2004)

Molecular Pairing

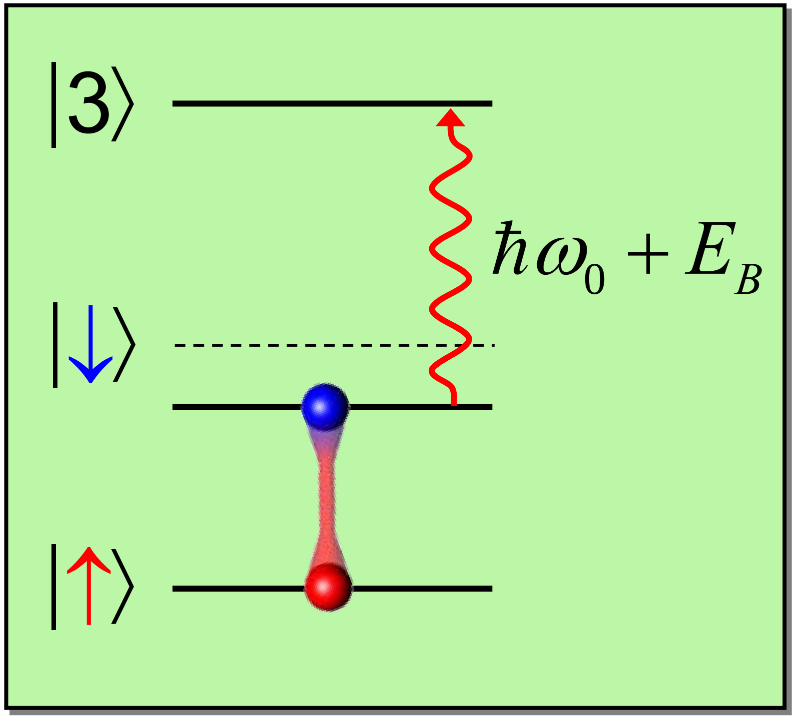


Photon energy = Zeeman + Binding + Kinetic energy

$$\hbar\omega = \hbar\omega_0 + E_B + 2\varepsilon_k$$

Radiofrequency spectroscopy

(in the BCS limit)



Binding energy per particle:

$$E_B \propto \frac{\Delta^2}{E_F}$$

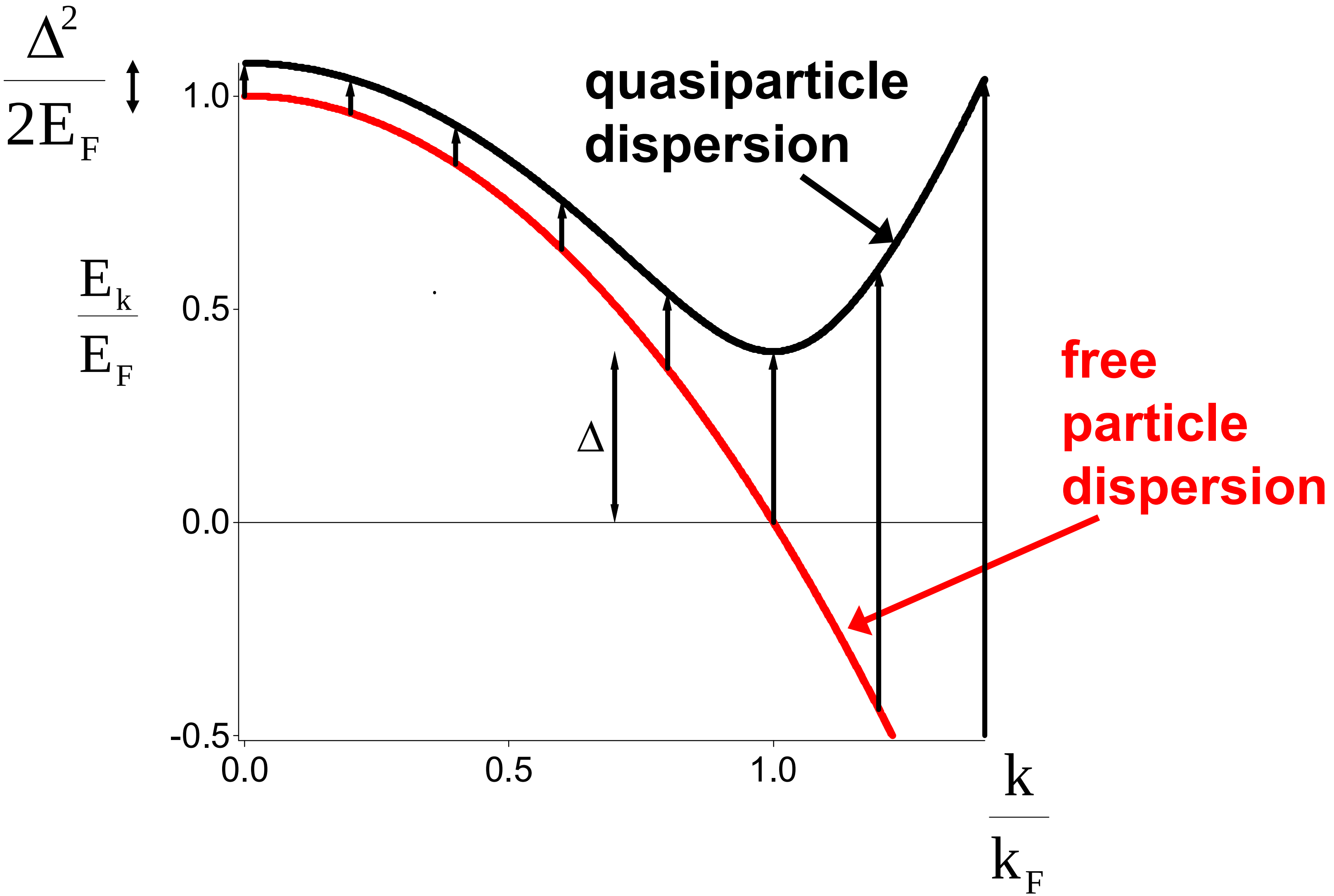
Photon energy = Zeeman + Quasiparticle + Kinetic energy

$$\hbar\omega = \hbar\omega_0 + E_k - \mu + \varepsilon_k$$

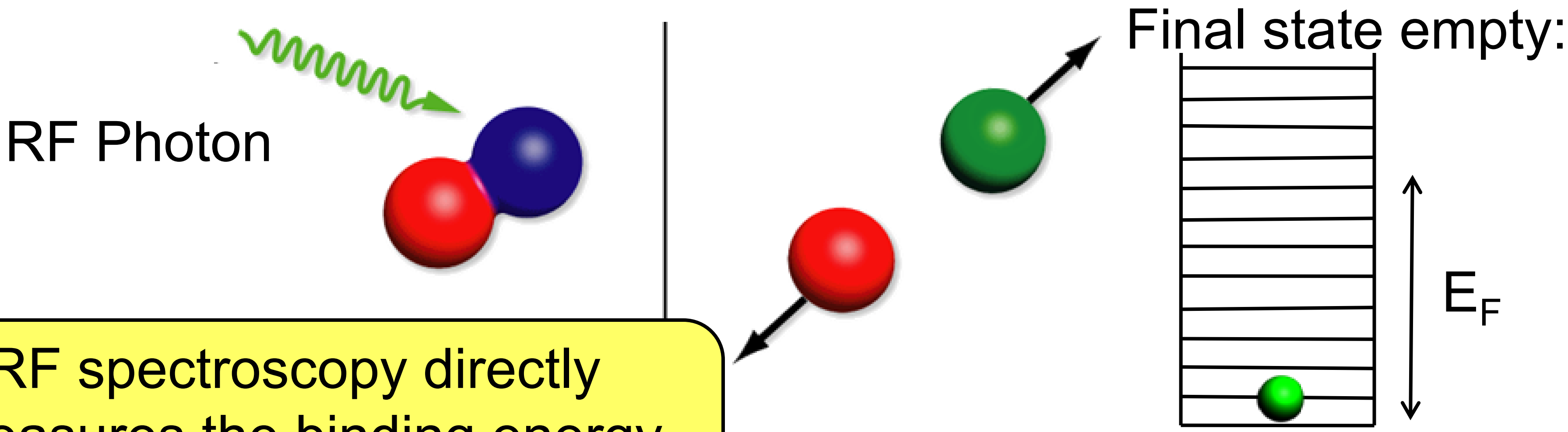
Onset at $$\hbar\omega = \hbar\omega_0 + \frac{\Delta^2}{2E_F}$$

RF Spectroscopy vs Tunneling

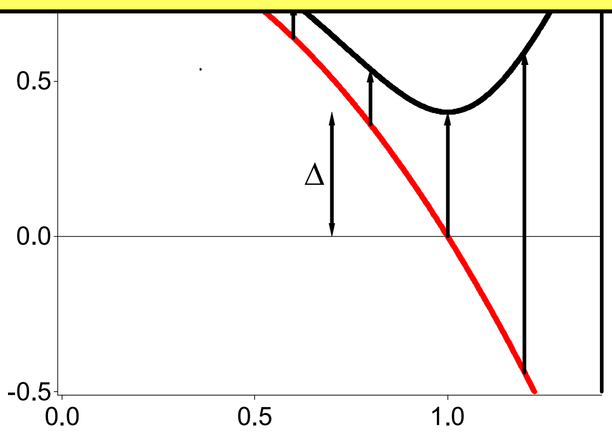
$$\hbar\omega = \hbar\omega_0 + E_k - (\mu - \varepsilon_k)$$

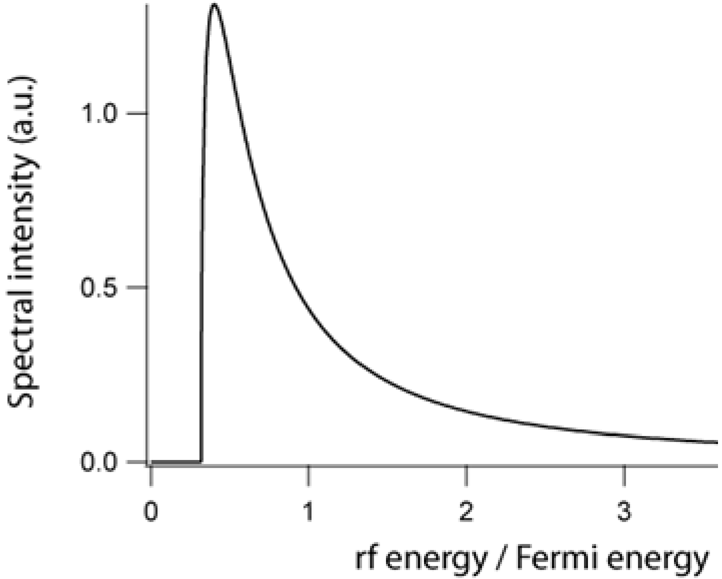


Connection to tunneling experiments



RF spectroscopy directly measures the binding energy of fermion pairs (not the gap)





BCS limit, onset at: $\hbar\omega - \hbar\omega_0 = \dfrac{\Delta^2}{2E_F}$

Connection to tunneling experiments

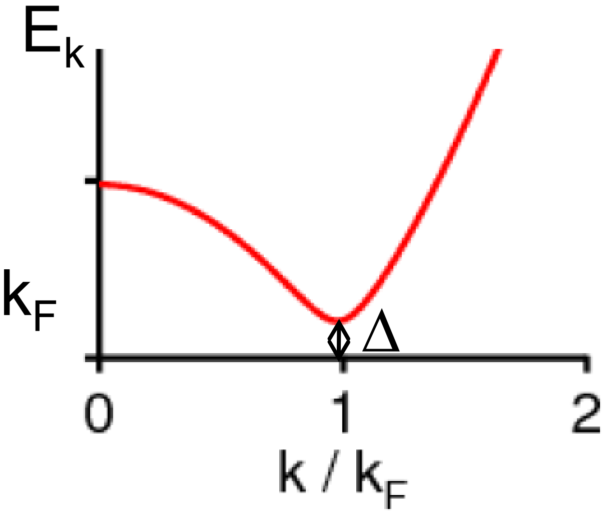
Final state (metal):

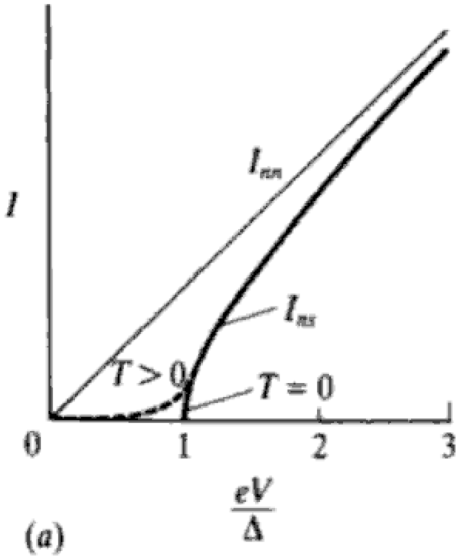
Superconductor

Insulator

Normal metal

Transfer at Fermi surface k=k$_F$





Onset at: $eV = \Delta$

Source: Tinkham,
Introduction to Superconductivity

Outline

Part 1: 1.) Introduction

- strongly interacting Fermions
- effect of density imbalance
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2.) Experiments using rf spectroscopy:
Problems and solutions

Part 2: 3.) Quantitative studies with rf spectroscopy

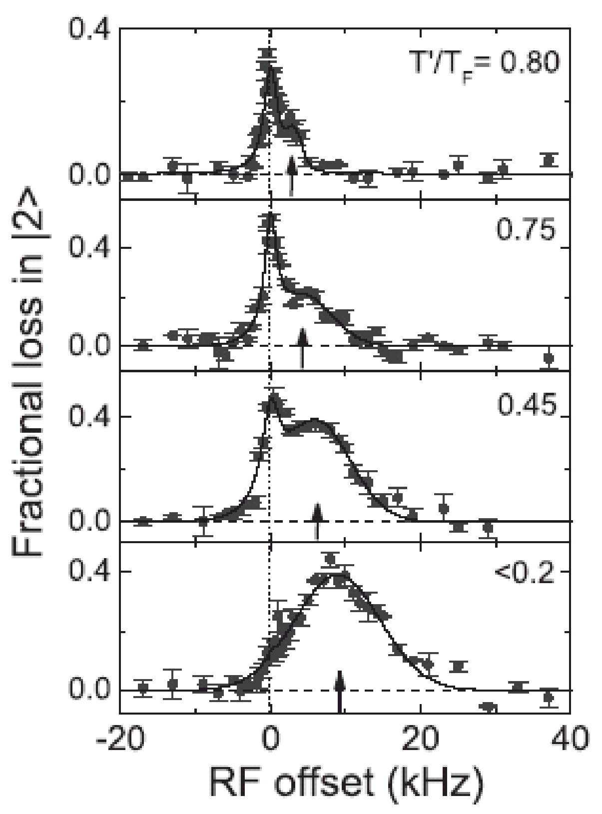
a) Quasiparticle spectroscopy and
determination of the superfluid gap

b) The ‘N+1’ body problem: Observation
of polarons in a highly imbalanced Fermi gas

Previous Experiments on RF Spectroscopy

First evidence a pairing gap, Grimm group:

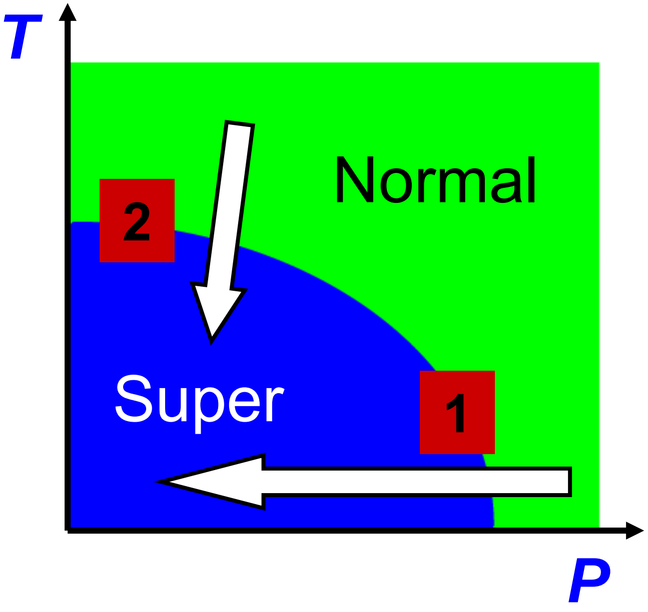
Equal Mixture

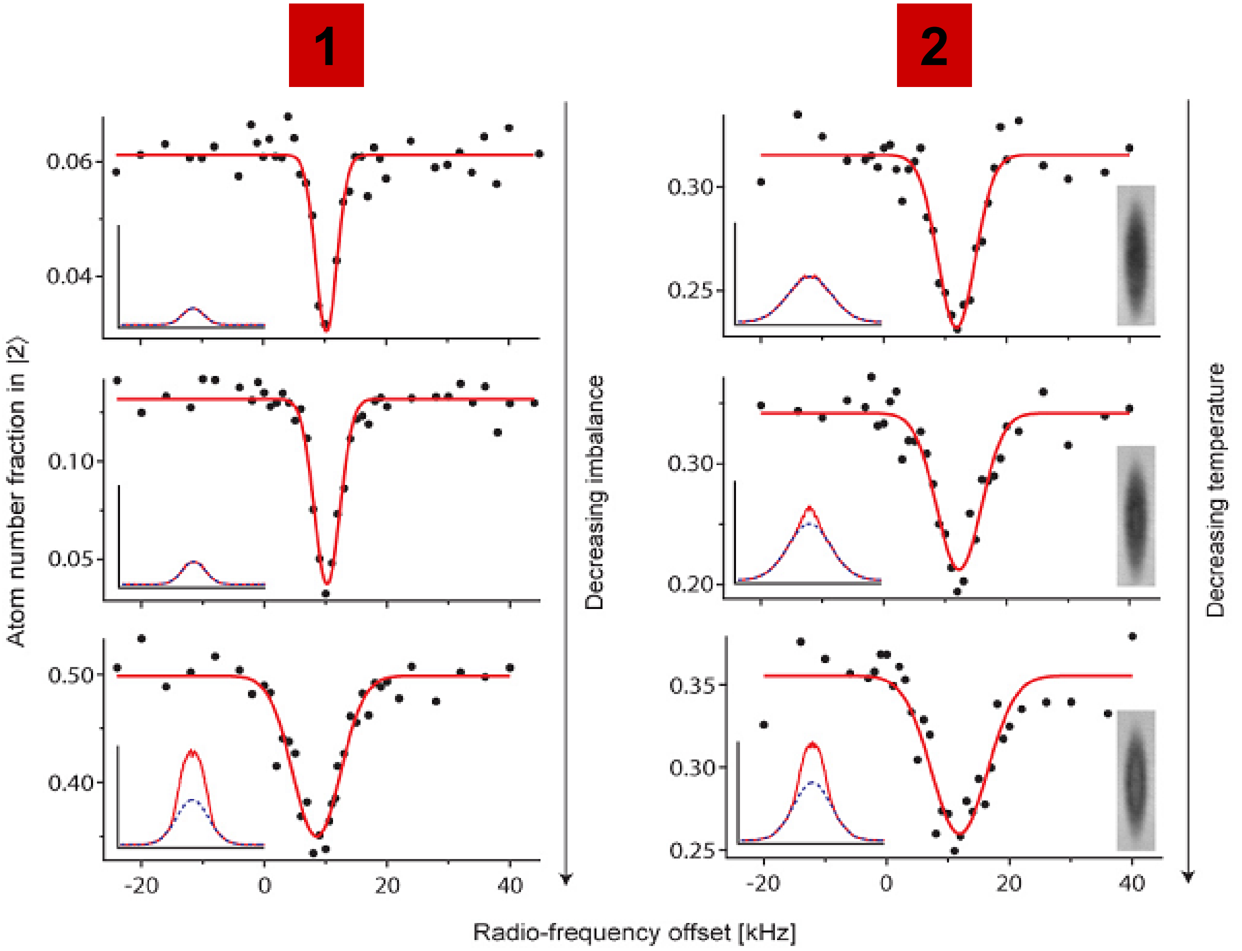


C. Chin et al., Science **305**, 1128 (2004)

Previous Experiments on RF Spectroscopy

Spectral gap in the normal phase, MIT group:





C.H. Schunck et al, Science 316, 815 (2007)

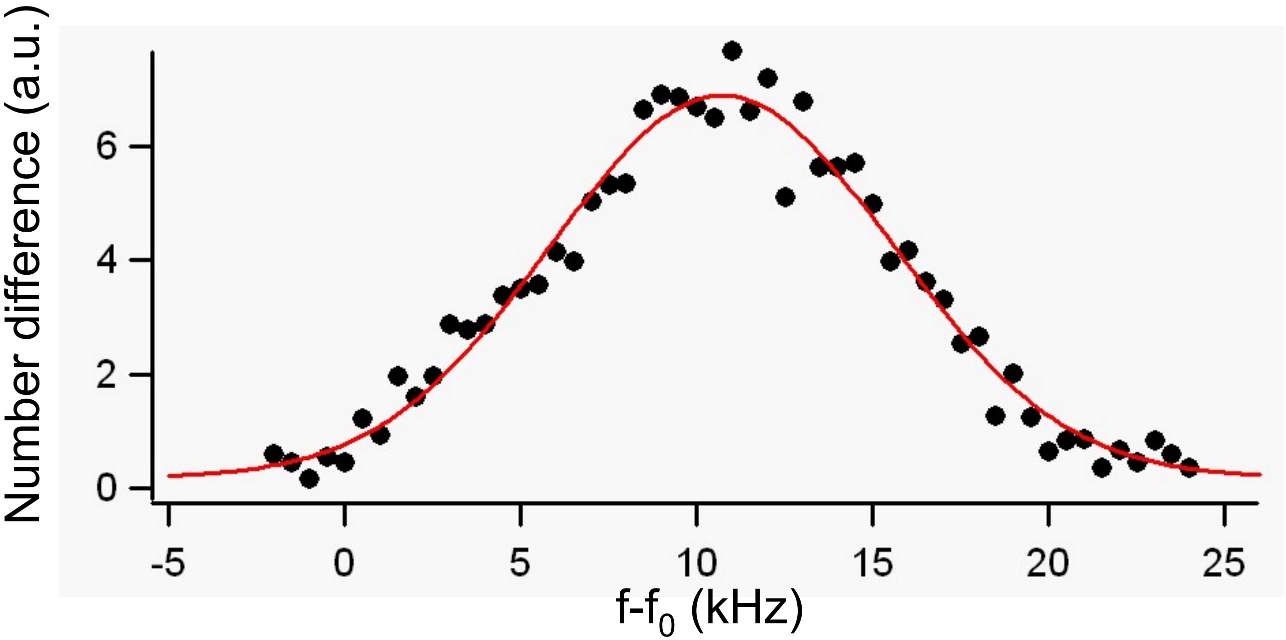
Problems of Previous RF experiments I

- Ultracold atom sample confined in a harmonic trap
- Inhomogeneous density profiles

→ Density broadening of the spectroscopic signal

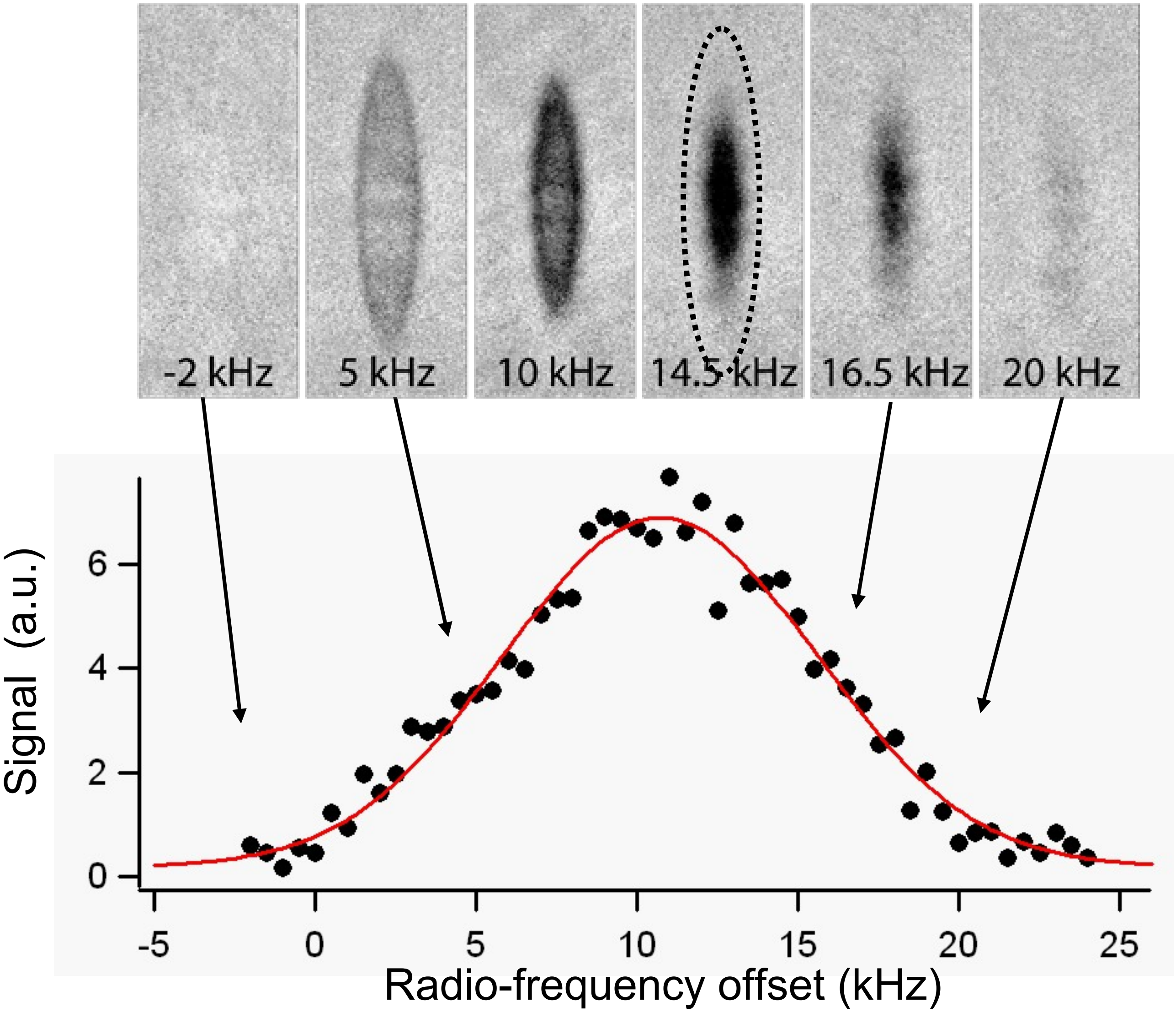
→ Difficult to compare with theoretical predictions

RF spectroscopy with equal mixtures



RF Spectroscopy with *in situ* Images

In situ phase contrast images measure the depletion region

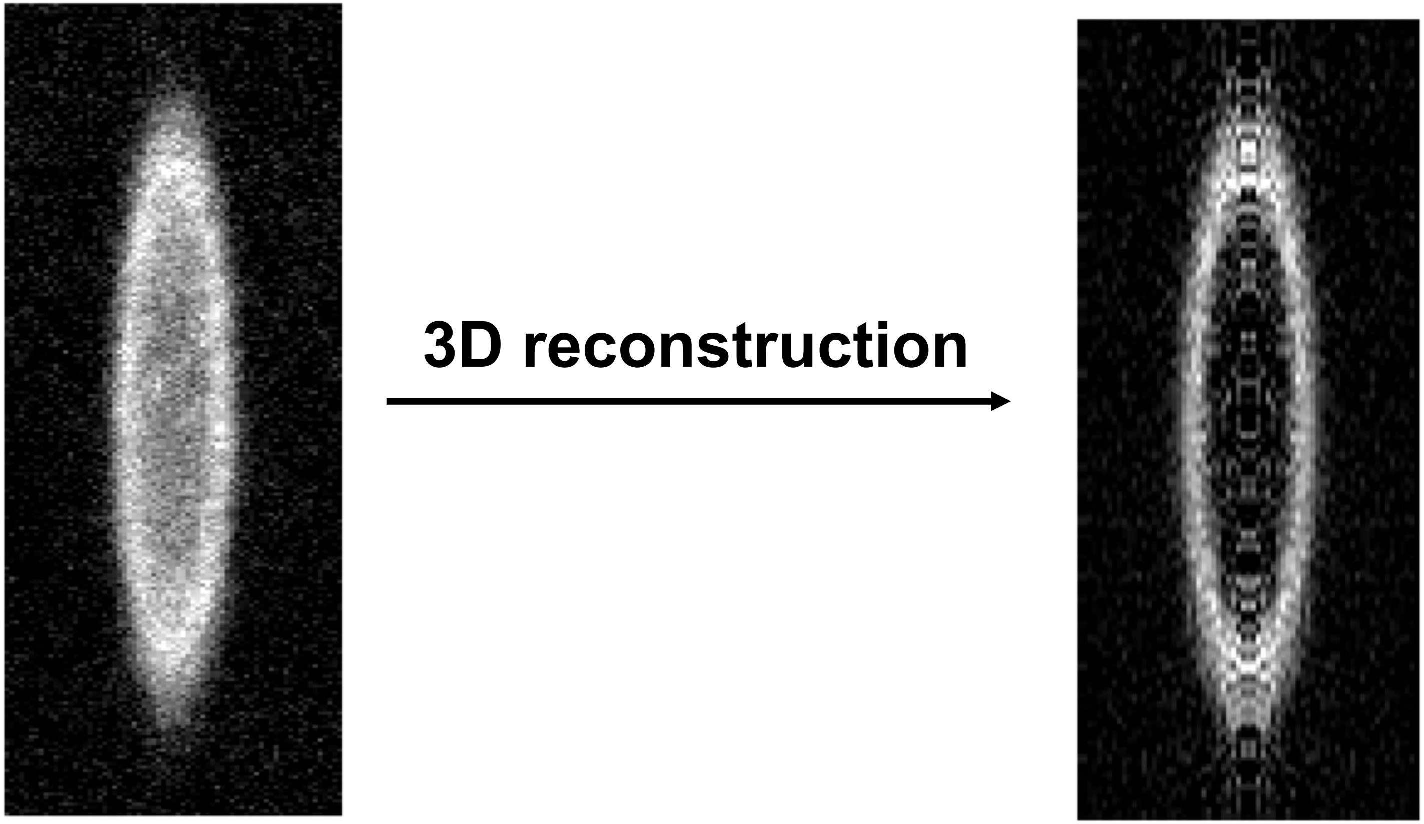


Y. Shin et al, PRL 99, 090403 (2007)

3d Density Reconstruction

Cylindrical symmetry, Inverse Abel transformation:

→ Tomographic RF spectroscopy:



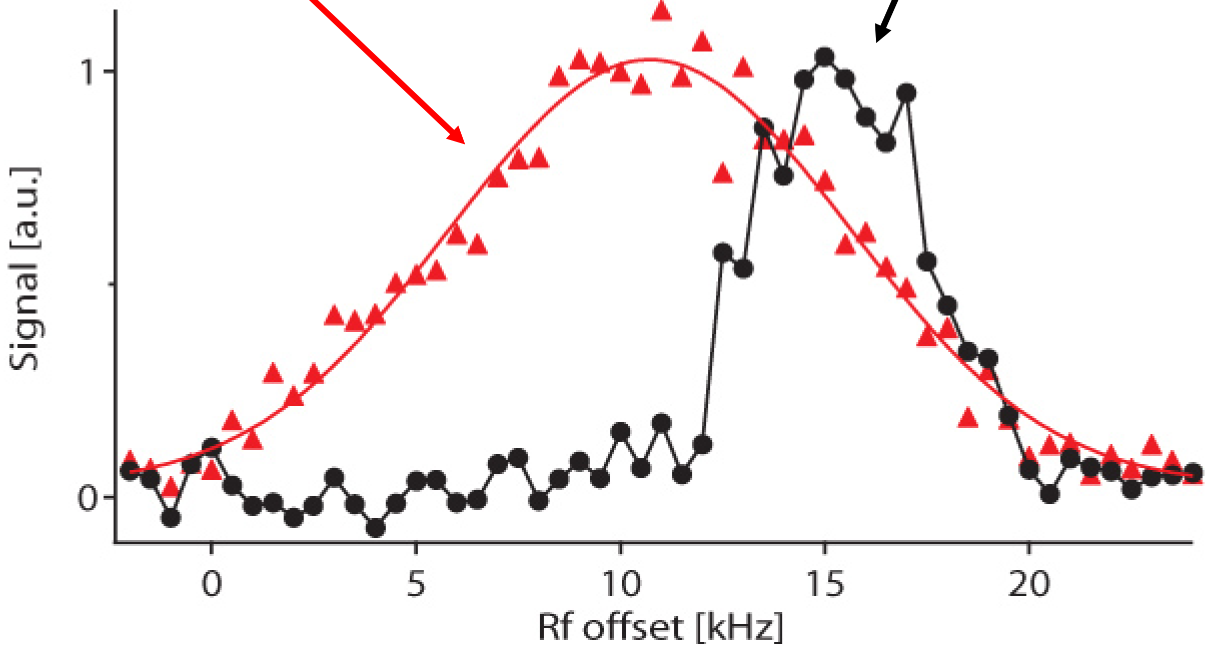
3D reconstruction

Spatially resolved RF Spectrum

Inhomogeneous spectrum

Homogeneous spectrum
at trap center

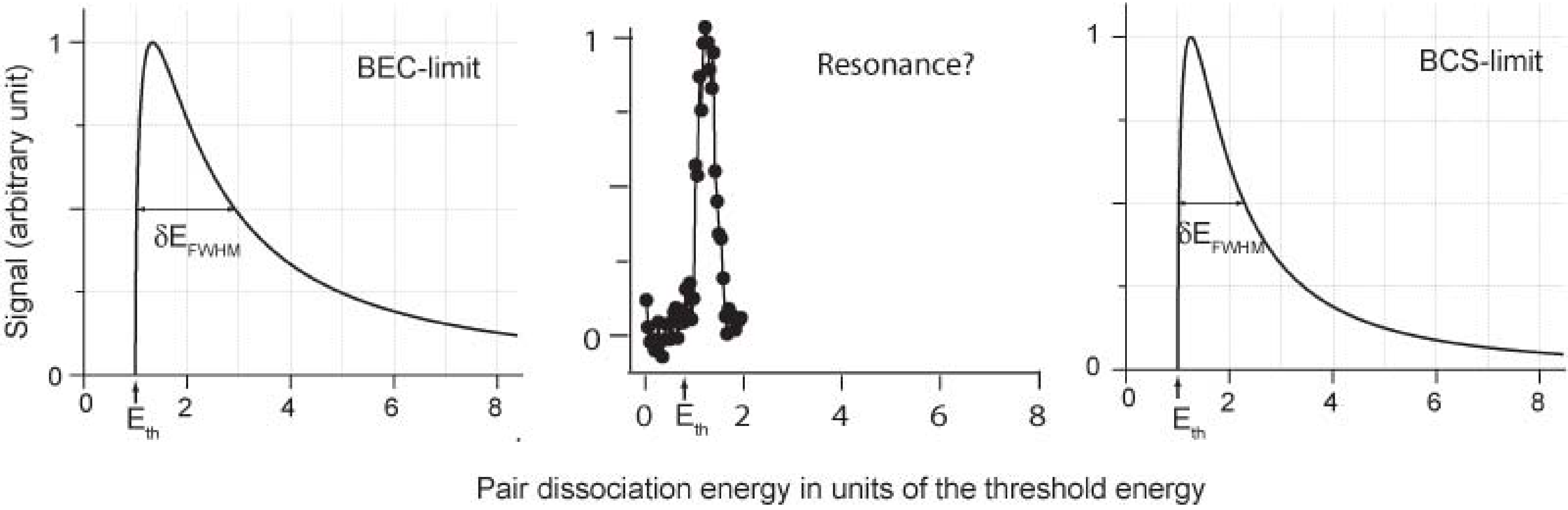
BUT: Still not the shape of a dissociation spectrum!

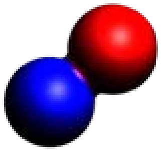


Y. Shin et al, PRL 99, 090403 (2007)

Problems of Previous RF experiments II

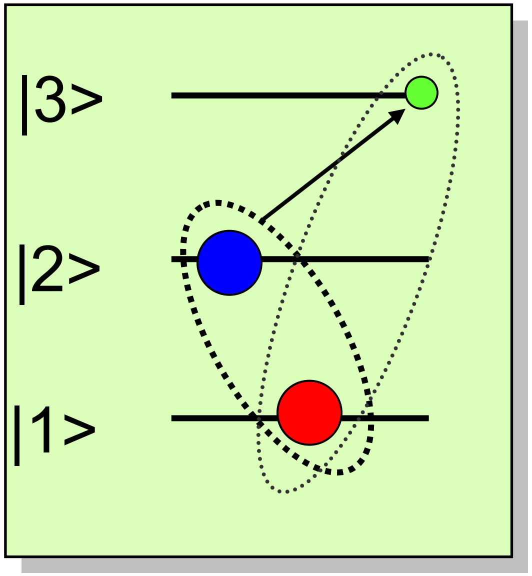
BEC **BCS**





C.H. Schunck et al, Nature 454, 739 (2008)

RF Spectroscopy: Final State Interactions



Final state interactions can strongly modify the spectrum

e.g. transition from 1-2 molecule to 1-3 molecule

How to change the final state interactions without changing the initial state?

3 candidates for high T_c Superfluids in ^{6}Li

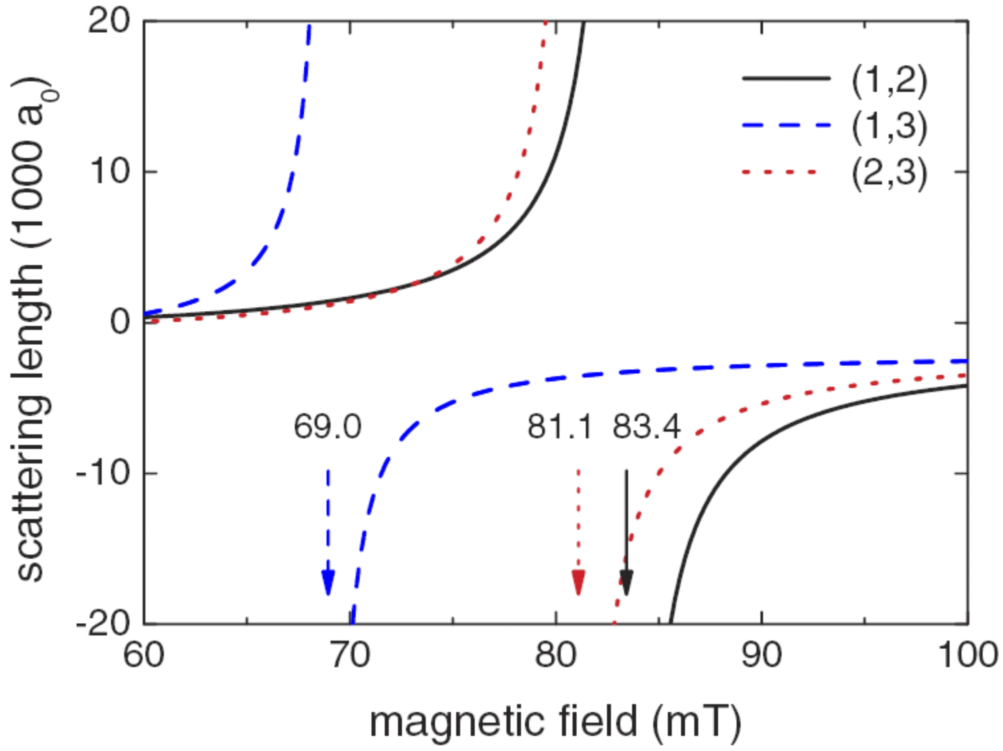


Figure: M. Bartenstein et al., PRL **94** 103201 (2005)

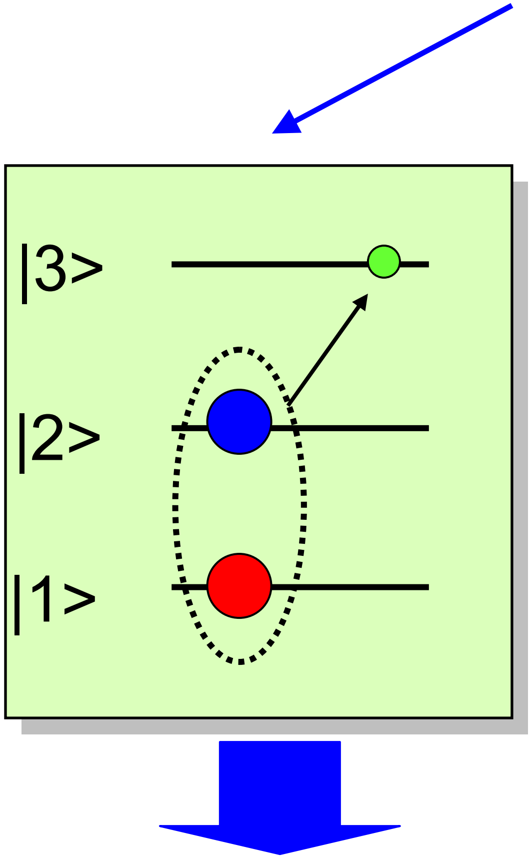
Mixture	Resonance
(1,2)	834 G
(2,3)	811 G
(1,3)	690 G

Lifetime of all mixtures exceeds 10 s

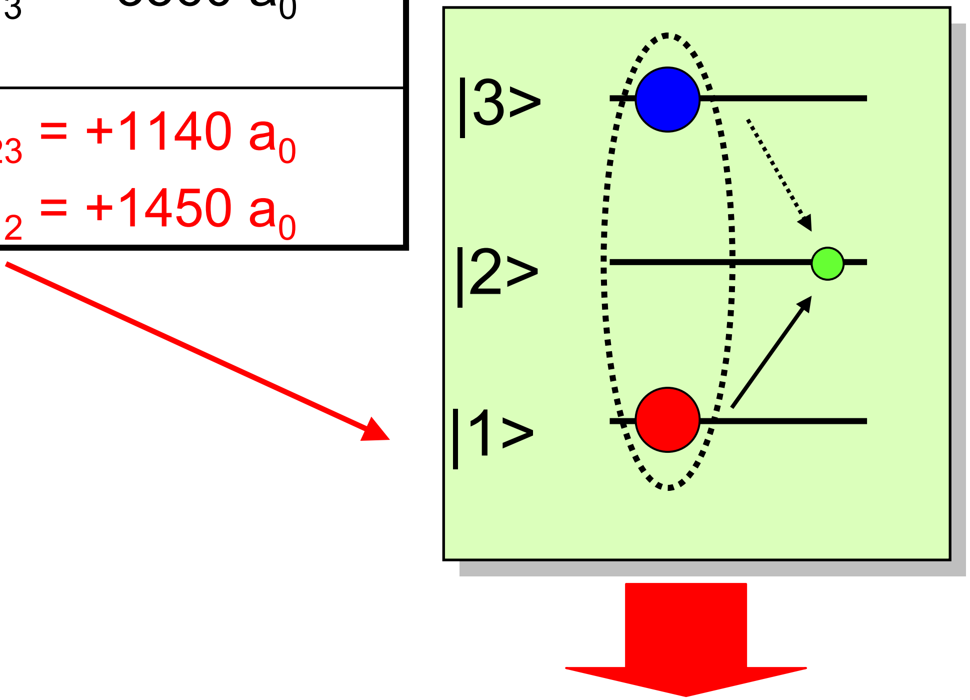
We observe fermion pair condensation in all mixtures: Two new high T_c superfluids

3 high T_c Superfluids for Rf Spectroscopy

Mixture	Final state scattering length at unitarity
(1,2)	$a_{13} = - 3290\ a_0$
(2,3)	$a_{13} = - 3560\ a_0$
(1,3)	$a_{23} = +1140\ a_0$ $a_{12} = +1450\ a_0$

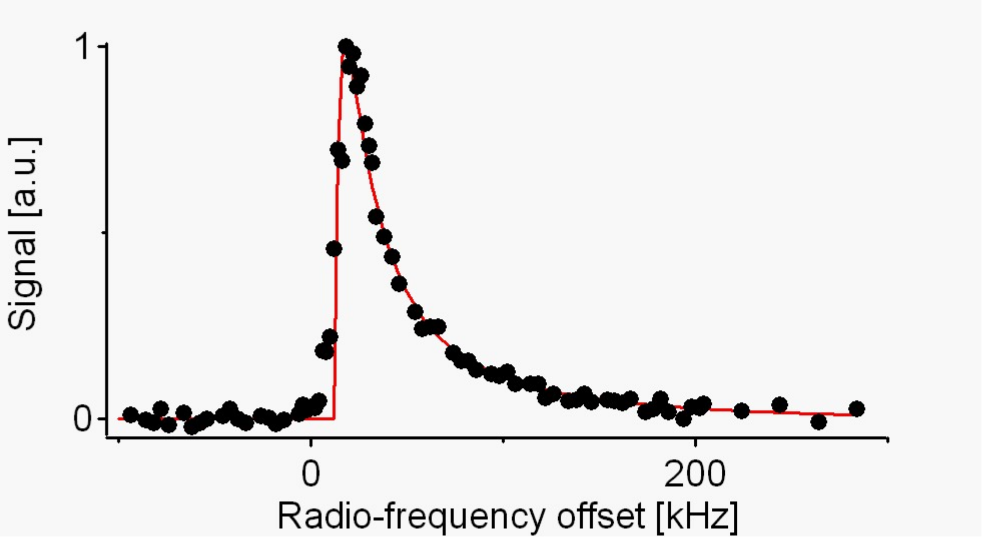


a_{final}: large and negative



a_{final}: smaller and positive

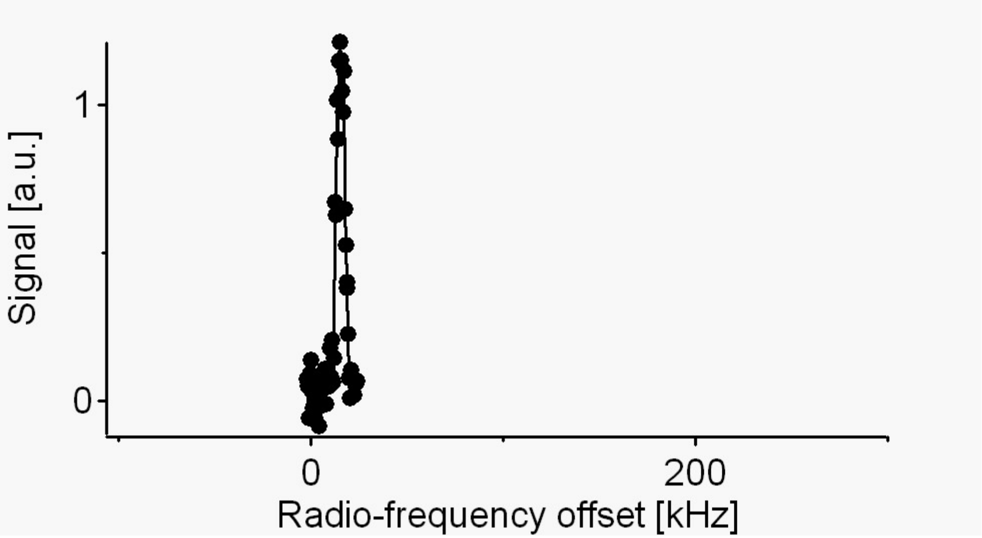
Dramatic effect of final state interactions



(1,3) mixture at unitarity (691 G)

$a_{23} = + 1140\ a_0$

200 kHz "tail" to higher frequencies



(1,2) mixture at unitarity (833 G)

$a_{13} = - \ 3290\ a_0$

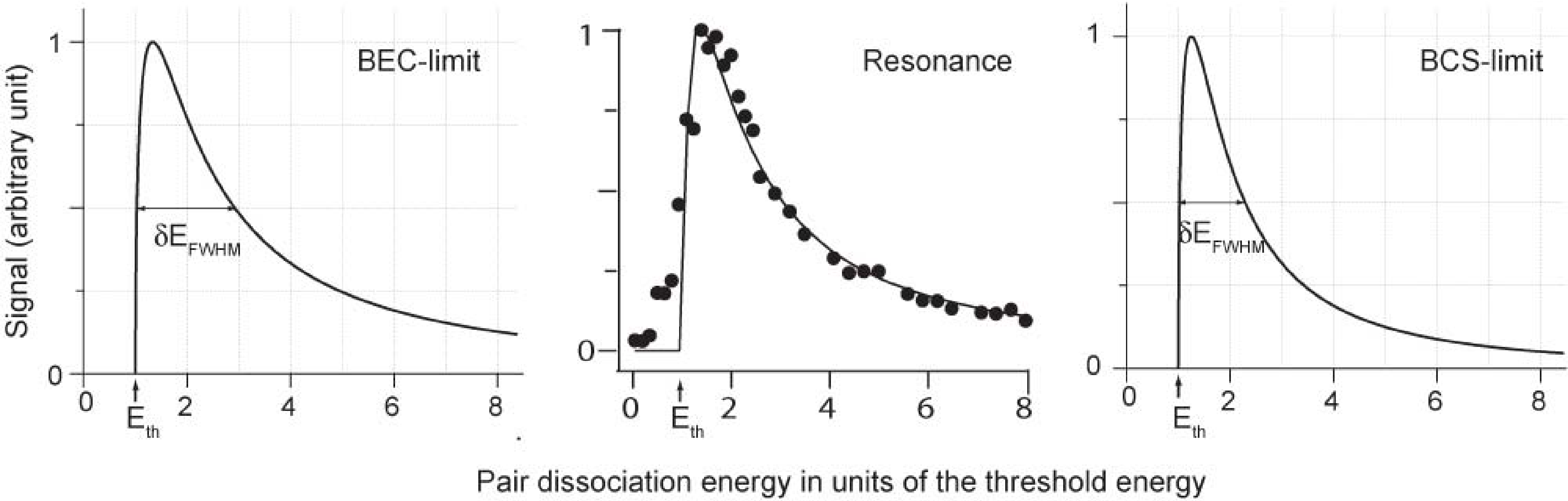
< 20 kHz linewidth

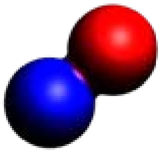
$\varepsilon_F \sim h{*}30$ kHz, $T/T_F \sim 0.1$

C.H. Schunck et al, Nature 454, 739 (2008)

Rf spectroscopy in the (1,3) mixture

BEC **Resonance** **BCS**

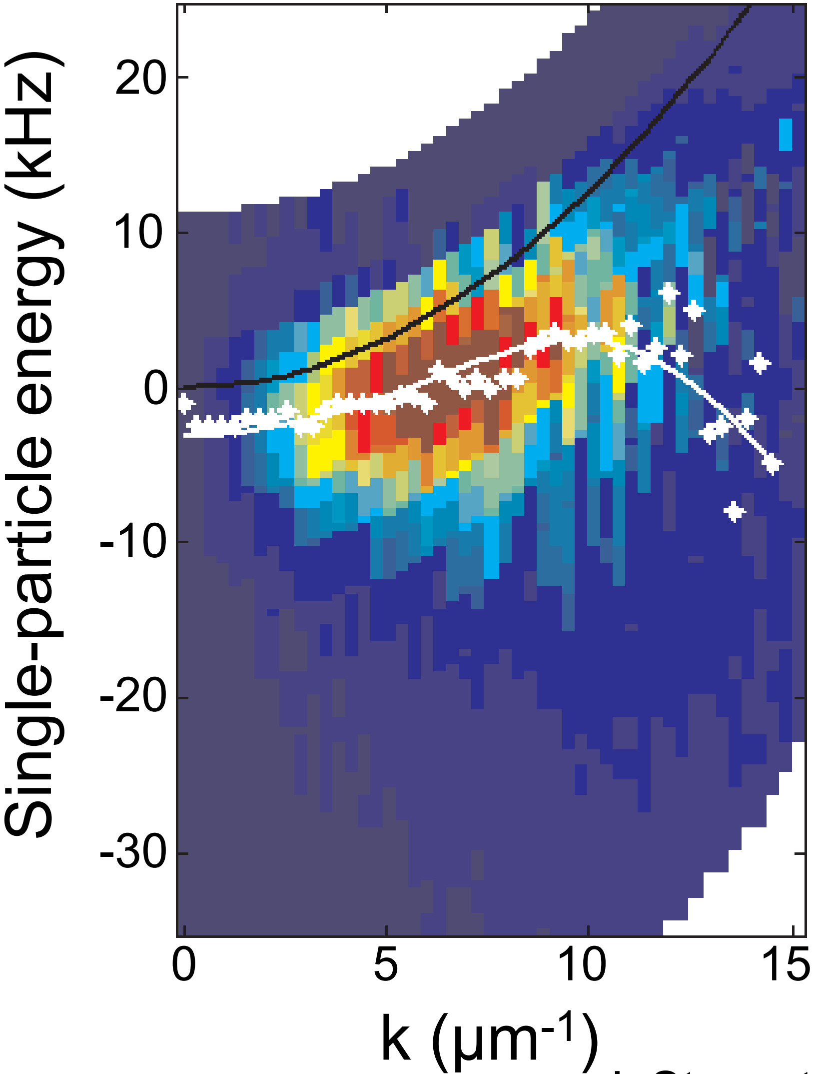




C.H. Schunck et al, Nature 454, 739 (2008)

Experiments on RF Spectroscopy

Photoemission Spectroscopy, extraction of Δ and μ, Jin group:

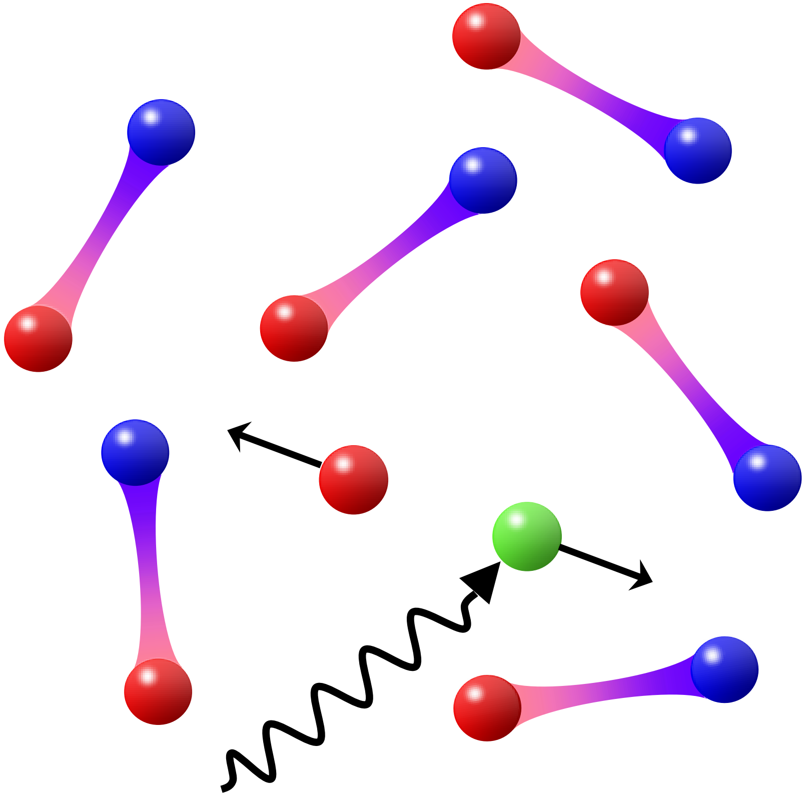


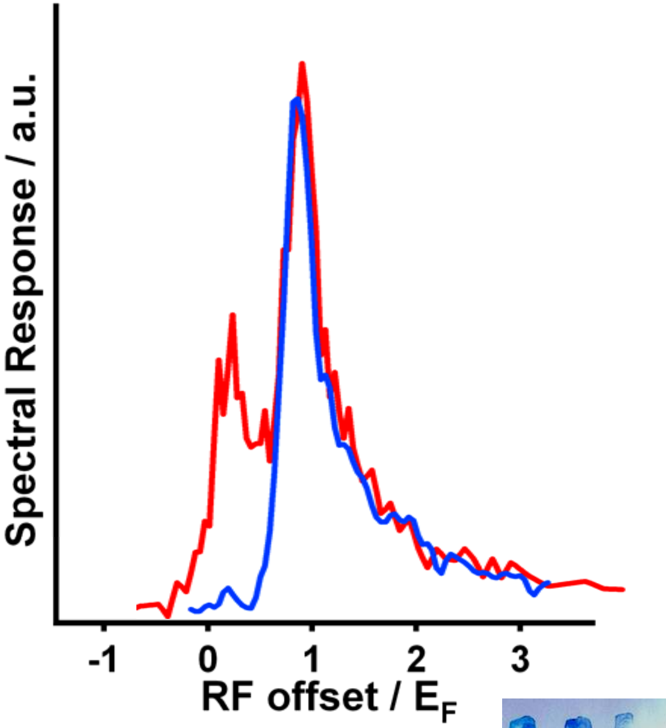
J. Stewart et al, Nature 454, 744 (2008)

Coherent Quantum Gases - From Cold Atoms to Condensed Matter
Copenhagen, Jan. 15th/16th 2009

Radio Frequency Spectroscopy of Ultracold Fermi Gases (2)

Andre Schirotzek





Center for Ultracold Atoms at MIT and Harvard

Outline

Part 1: 1.) Introduction

- strongly interacting Fermions
- effect of density imbalance
- rf spectroscopy

2.) Experiments using rf spectroscopy:
Problems and solutions

Part 2: 3.) Quantitative studies with rf spectroscopy

a) Quasiparticle spectroscopy and
determination of the superfluid gap

b) The ‘N+1’ body problem: Observation
of polarons in a highly imbalanced Fermi gas

Quantitative Studies

Address open questions, Superfluid state:

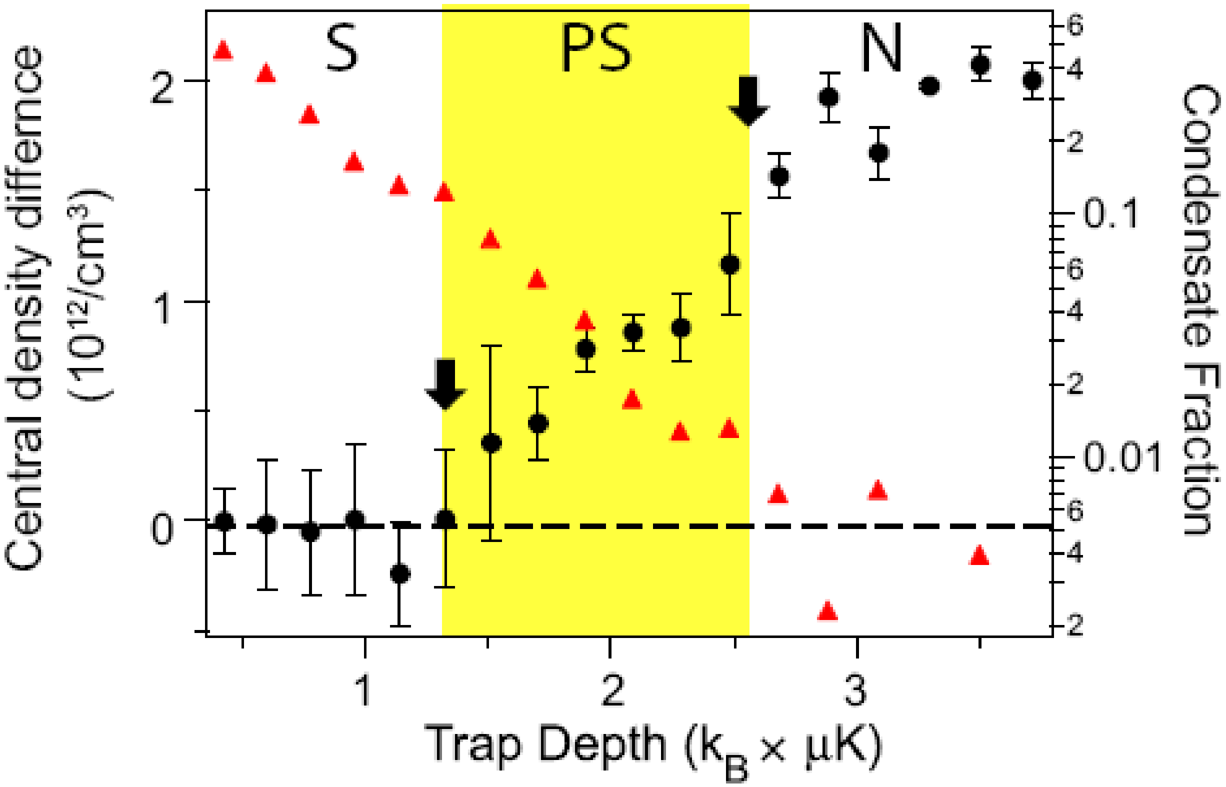
- Magnitude of Δ
  - → Presence of important Hartree energy

- *Local* double peak structure?
  - → evidence of quasiparticles

- “Pairing” in the normal phase?
  - → Need majority spectra

Motivation I: Quasiparticles?

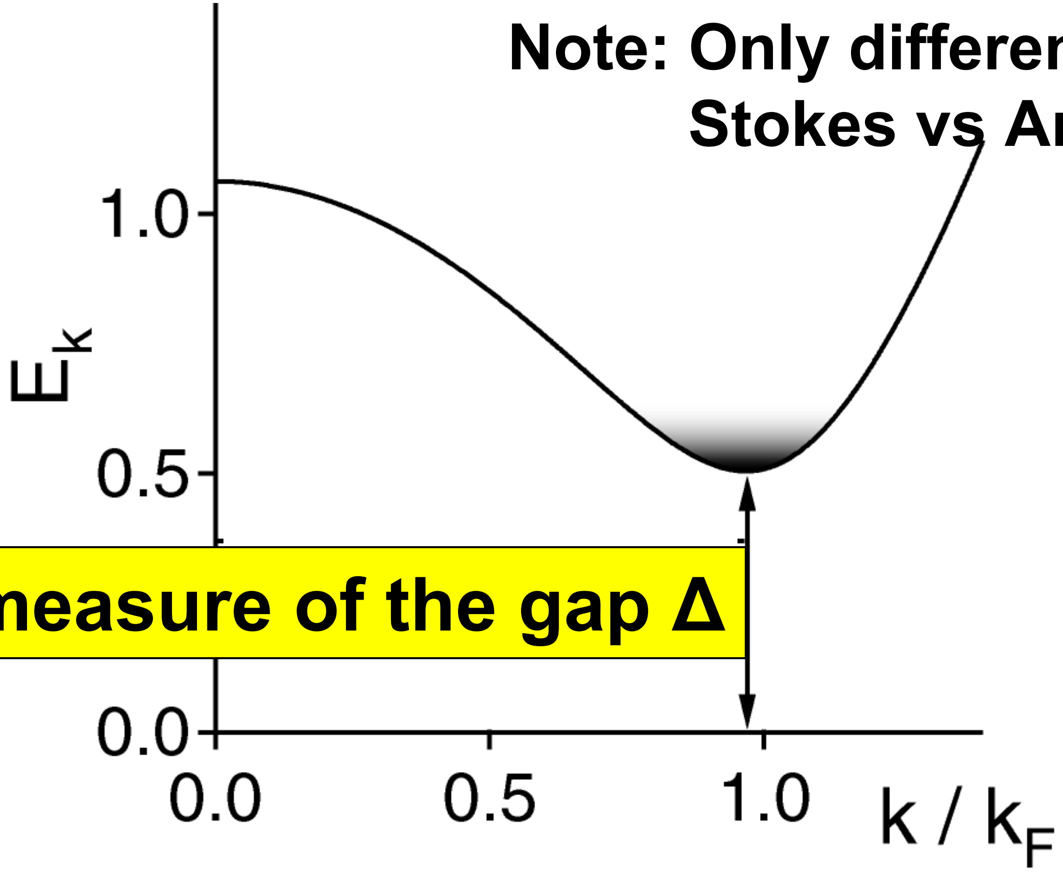
Polarized Superfluid
=
Finite Quasiparticle Occupation



Shin et al, PRL 97, 030401 (2006)

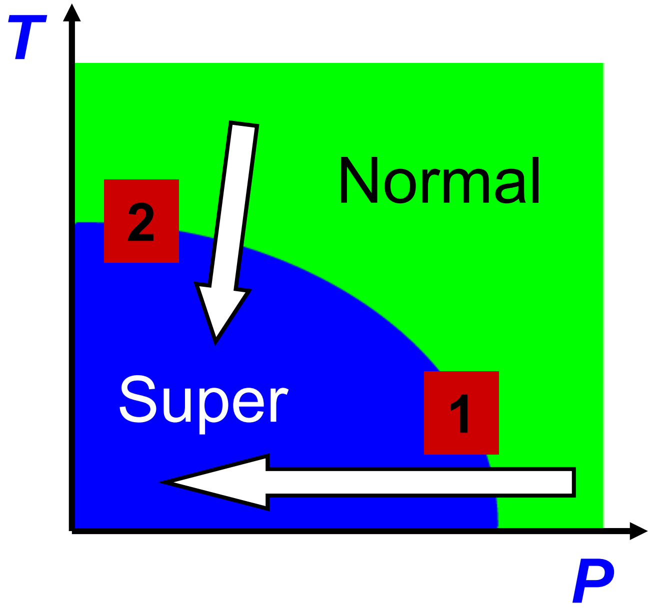
Motivation II: The Superfluid Gap

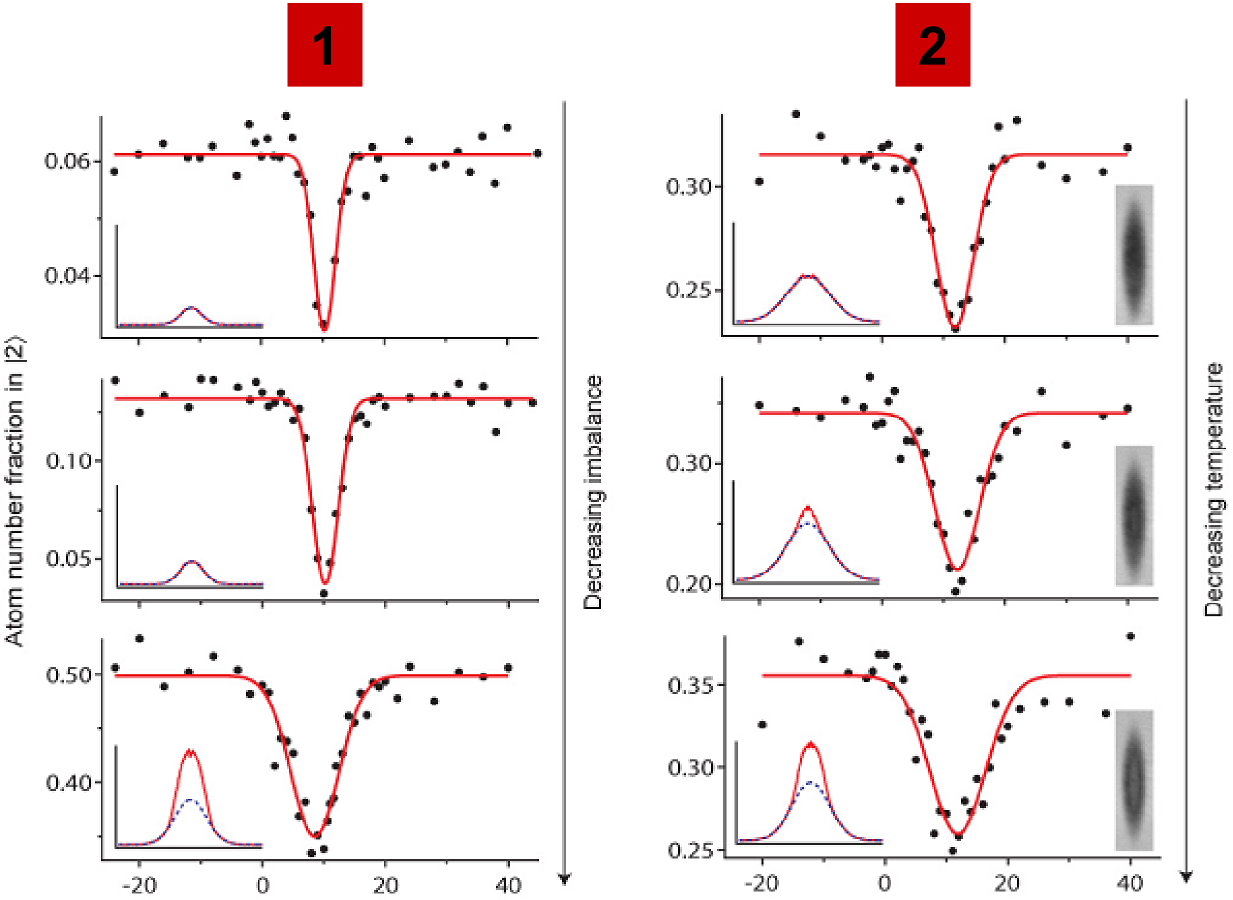
Note: Only difference to tunneling:
Stokes vs Anti-Stokes



Direct measure of the gap Δ

Motivation III: Minority Spectra

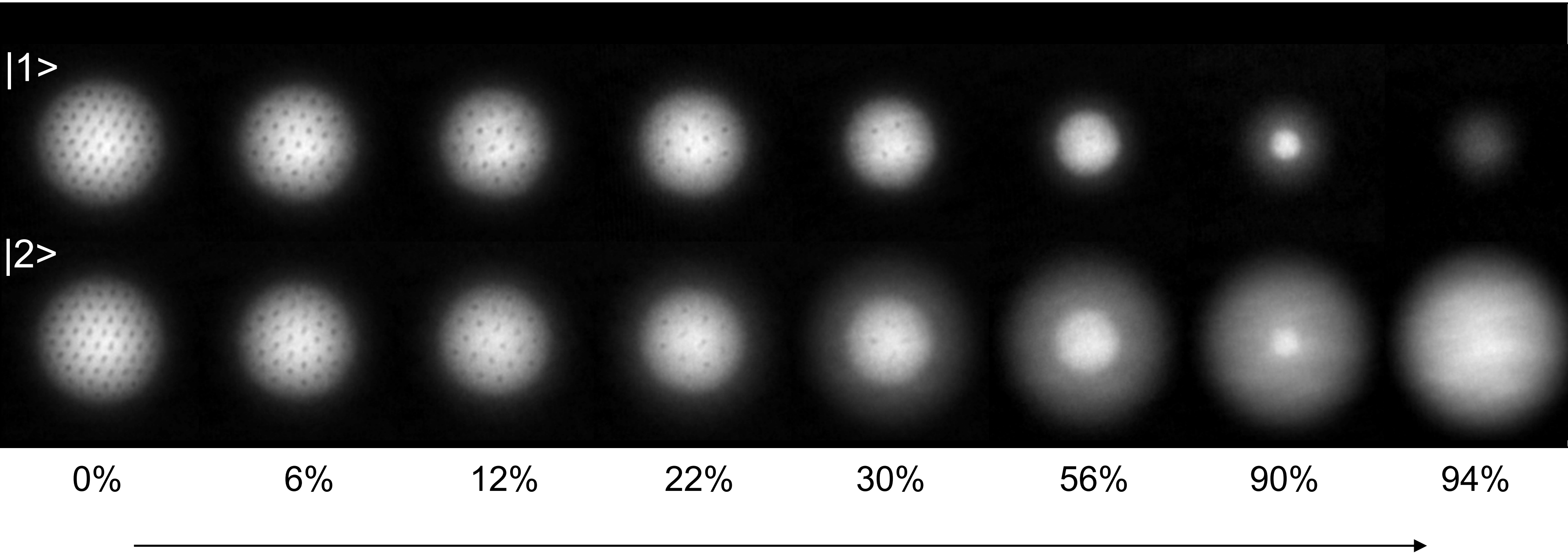




Minority Spectra do NOT reveal the onset of superfluidity

Full pairing of minority cloud?
Bosons which do not condense even at T=0 ?

Ingredient: Imbalanced Fermi Mixtures

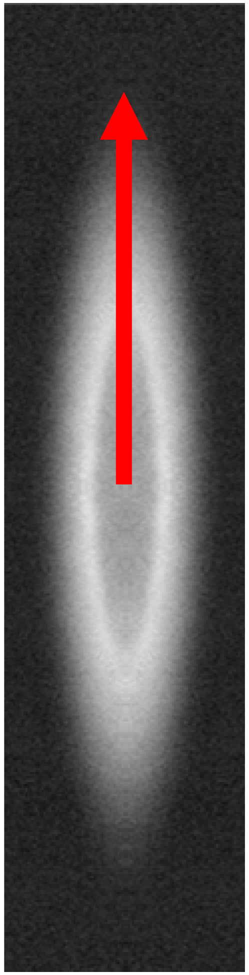


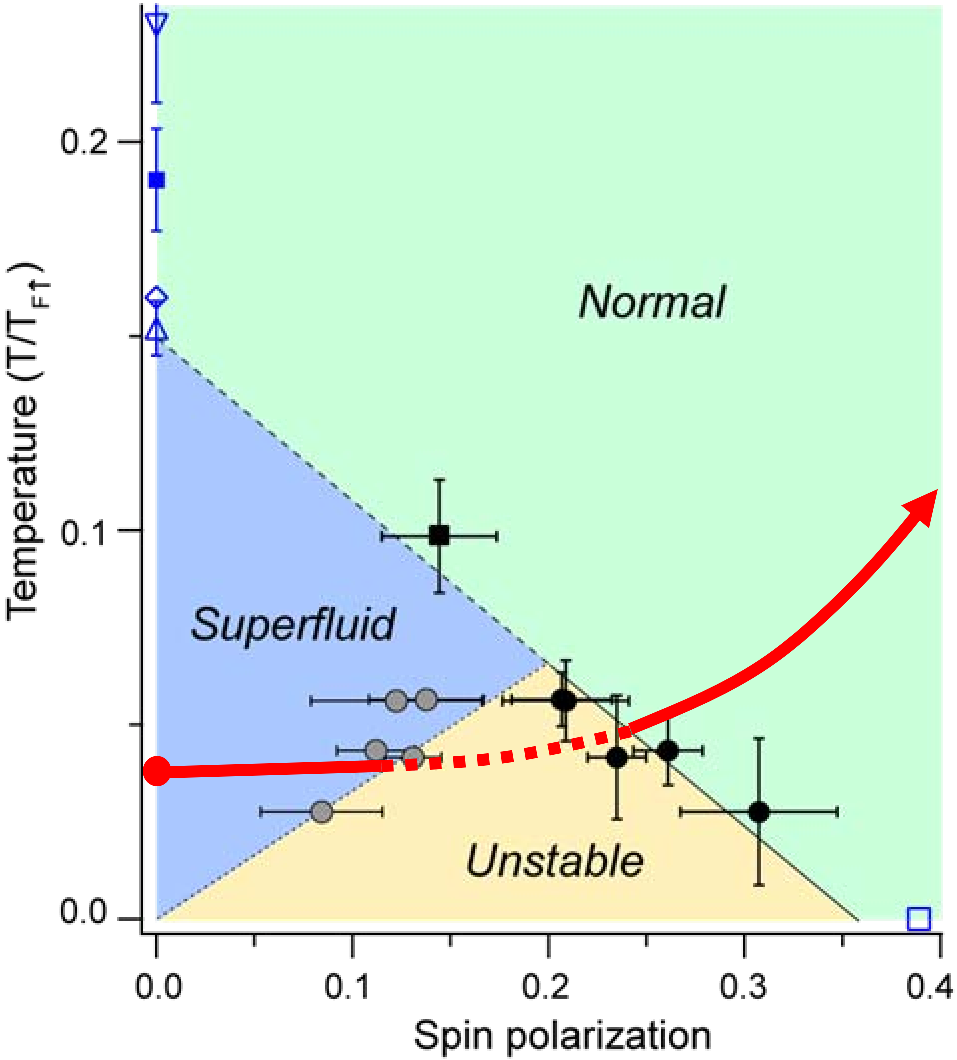
- System remains superfluid up to a critical imbalance

-Two (of several) ways to destroy Superfluidity:

  - Imbalance
  - Temperature

Phase Diagram of a Polarized Fermi Gas





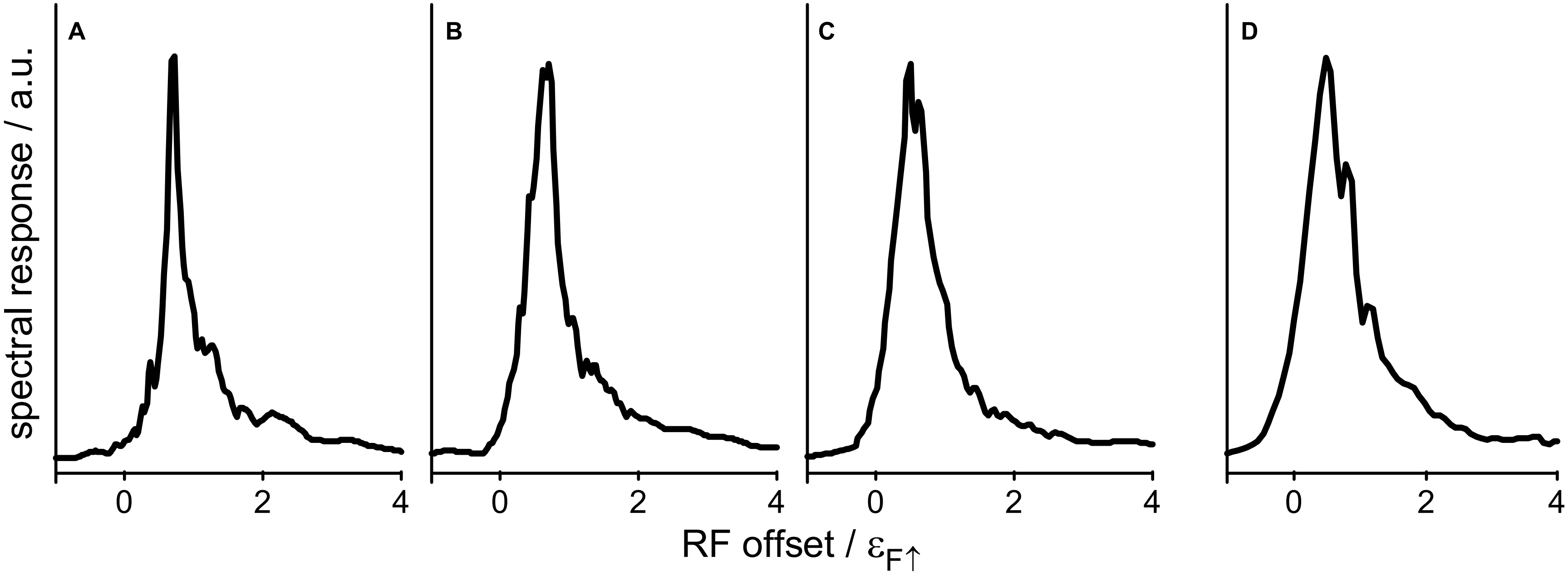
Spin polarization:

$$\frac{n_{\uparrow} - n_{\downarrow}}{n_{\uparrow} + n_{\downarrow}}$$

Y. Shin et al,Nature, **451**,689 (2008).

Quasiparticles in equal mixtures?

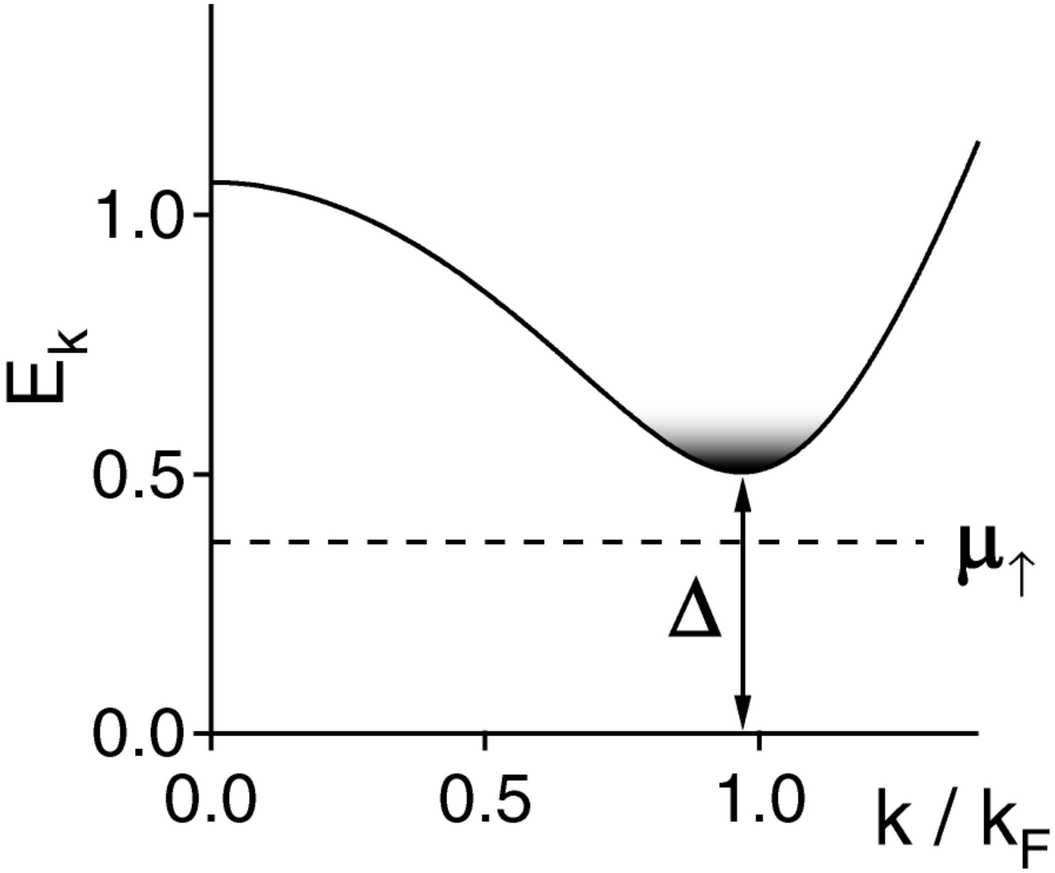
We were not able to resolve a local double peak structure between $0.1 < T / T_F < 0.55$



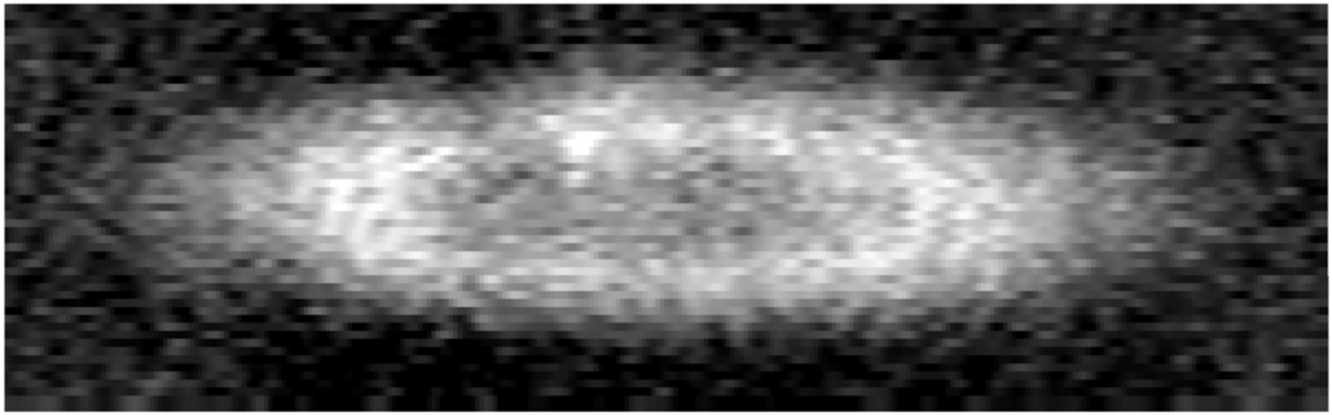
How can we create quasiparticles?

Solution: Low Temperature Quasiparticles

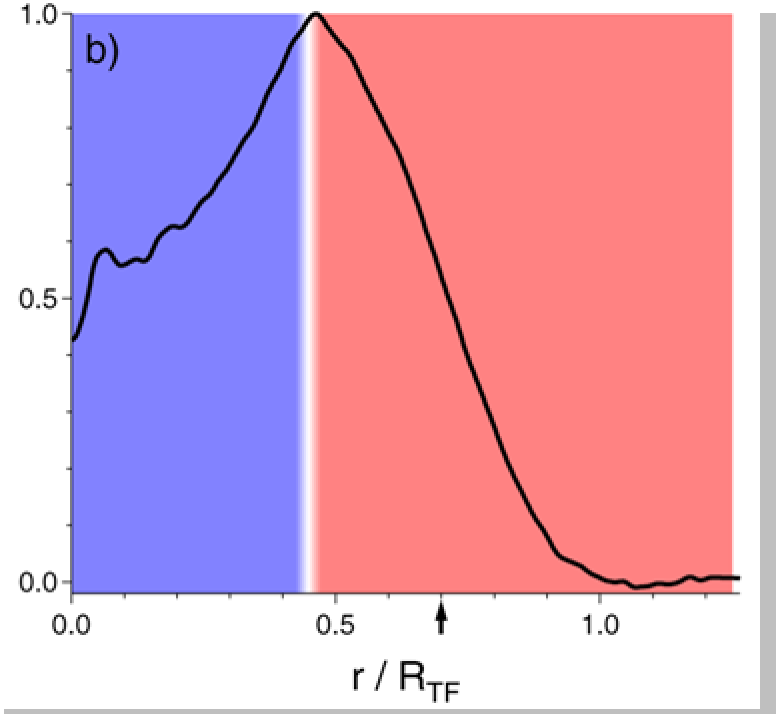
Generate quasiparticles at the minimum energy of the dispersion curve:



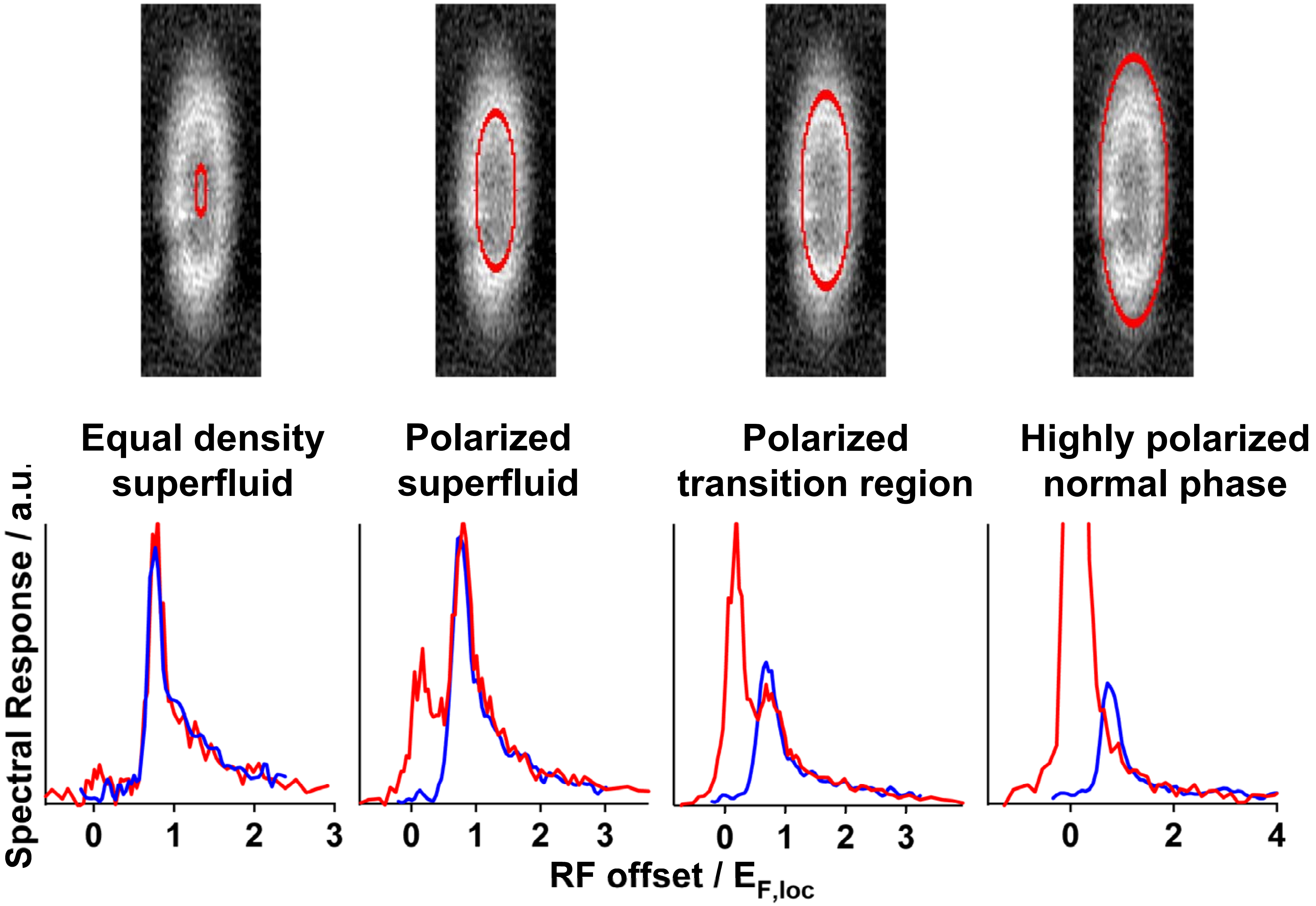
RF spectra for varying local imbalances



Elliptic Average



RF spectra for varying local imbalances



Equal density superfluid

Polarized superfluid

Polarized transition region

Highly polarized normal phase

Polarized Superfluid

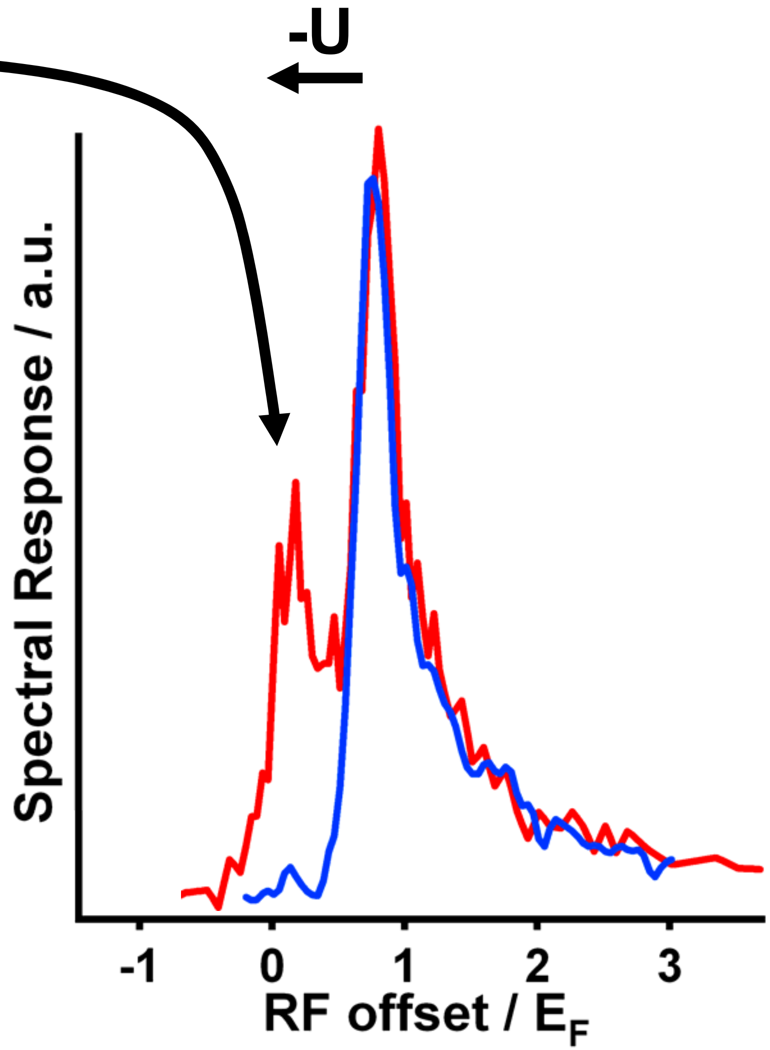
BCS Theory: Excess Fermions = Quasiparticles

→ local double peaks in RF spectrum

BUT:

Note that the quasiparticle peak is at $\omega_{RF} > 0$ and not at the expected position $-\Delta$

Hartree terms U introduce an overall shift of the RF spectrum

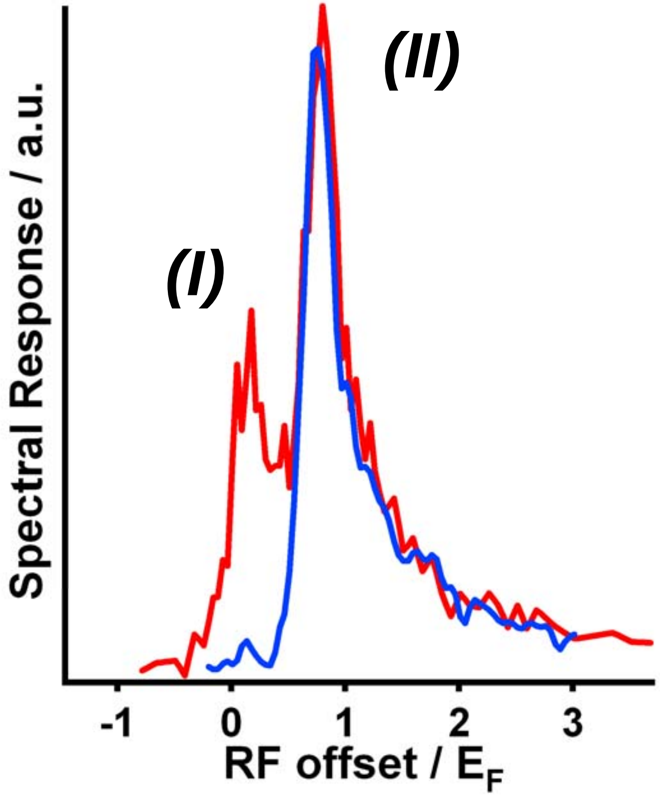


Two Peaks = Two Equations

(I) $$-\Delta - U \qquad + E_{final} = 0.15$$

(II) $$\frac{4}{3}\omega_{th}(\Delta, U, \mu) + E_{final} = 0.75$$

($\mu \approx 0.42$ from QMC or experiment)



Δ = 0.44(3) E_F

U = - 0.43(3) E_F

A. Schirotzek et al,
PRL 101, 140403
(2008)

Hartree Terms:
Where do they come from?

Hamiltonian in general difficult: $\mathrm{H} \propto c_1^\dagger c_2^\dagger c_3 c_4$

Hartree-Fock, replace 4-fermion-operator by best 2-fermion-operator you can find:

$$\mathrm{H} \propto \langle c_2^\dagger c_3 \rangle c_1^\dagger c_4 + \langle c_1^\dagger c_4 \rangle c_2^\dagger c_3$$

Hartree (direct)

$$\xcancel{- \langle c_1^\dagger c_3 \rangle c_2^\dagger c_4 - \langle c_2^\dagger c_4 \rangle c_1^\dagger c_3}$$

~~Fock (exchange)~~

$$+ \underbrace{\langle c_1^\dagger c_2^\dagger \rangle}_{\Delta^*} c_3 c_4 + \underbrace{\langle c_3 c_4 \rangle}_{\Delta} c_1^\dagger c_2^\dagger$$

Non-zero only in BCS

Hartree Terms:
Where do they come from?

BCS Hamiltonian: $V_{BCS} = -V_0 c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{k'\downarrow} c_{-k'\uparrow}$

Free Energy:

$$H - \mu N = \sum_{k\sigma} (\varepsilon_k - (\mu \boxed{- U})) c^\dagger_{k\sigma} c_{k\sigma}$$

$$- \sum_{k} (\Delta c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + \Delta^* c_{-k\downarrow} c_{k\uparrow})$$

Hartree Terms:
Where do they go?

Quasiparticle excitation spectrum:

$$E_k = \pm\sqrt{(\varepsilon_k - (\mu - U))^2 + |\Delta|^2}$$

Quasiparticle operator (Bogoliubov):

$$c^{\dagger}_{k\uparrow}|BCS\rangle \propto \gamma^{\dagger}_{k\uparrow}|BCS\rangle$$

→excess Fermion = Quasiparticle

Hartree Terms:
Where do they go?

RF spectrum (from FGR):

$$\Gamma(\omega) \propto \frac{\sqrt{\omega - \omega_{\mathrm{th}}}}{\omega^2} \times (\ldots)$$

Hartree Terms:
Where do they go?

Same functional form. Main effect:
Shifting the x-axis

$$\Gamma(\omega) \propto \frac{\sqrt{\omega' - \omega'_{th}}}{\omega'^2} \times (\ldots)$$

with:

$$\omega' = \omega + U$$

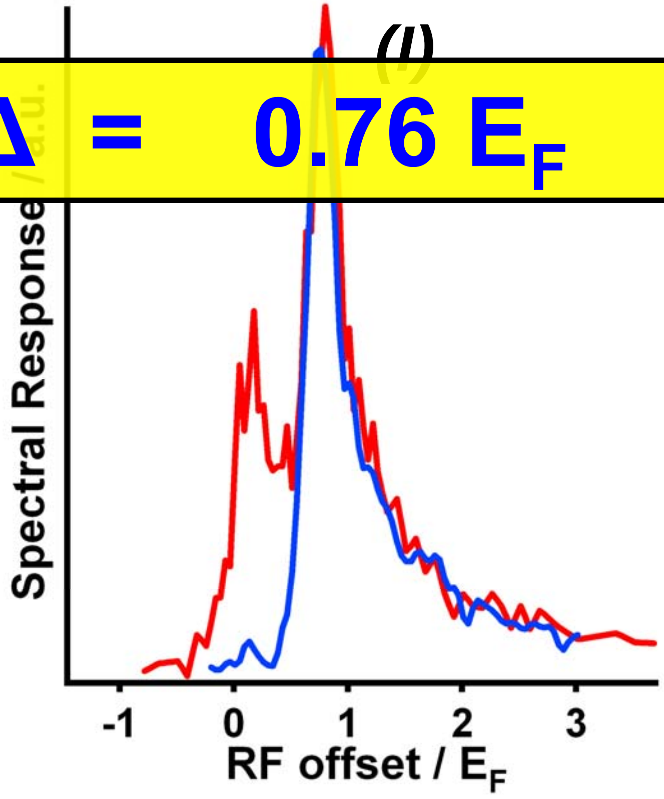
$$\mu' = \mu - U$$

The importance of Hartree Terms

(I)

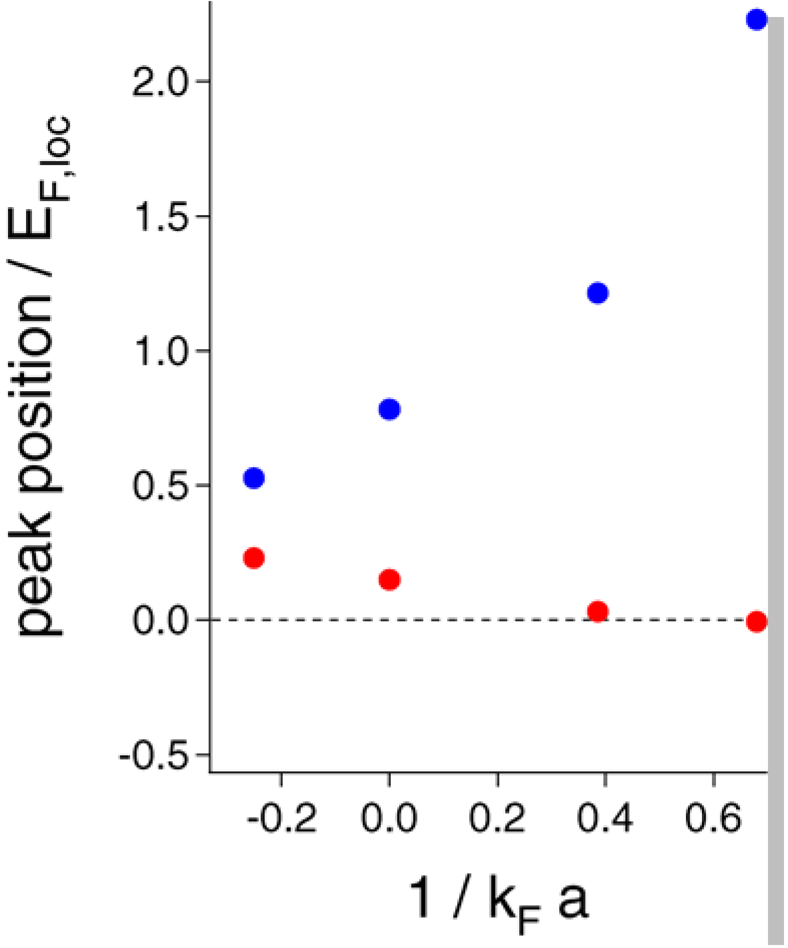
$$\frac{4}{3}\,\omega_{\mathrm{th}}(\mu,\Delta) + E_{\mathrm{final}} = 0.75$$

(again: μ ≈ 0.42)



$\Delta = 0.76\ E_F$

Peak Positions in the Polarized Superfluid



$1/k_F a$	Δ	U	E_{final}
−0.25	0.22	−0.22	0.22
0	**0.44**	**−0.43**	**0.16**
0.38	0.7	−0.59	0.14
0.68	0.99	−0.87	0.12

Conclusions, a)

- Comparison of Majority and Minority RF spectra reveal a change in pairing character

- Polarized Superfluid: Observation of Quasiparticles

- Position of Quasiparticle peak highlights importance of Hartree terms U in RF spectroscopy

- Double peak spectrum allows determination of Δ and U

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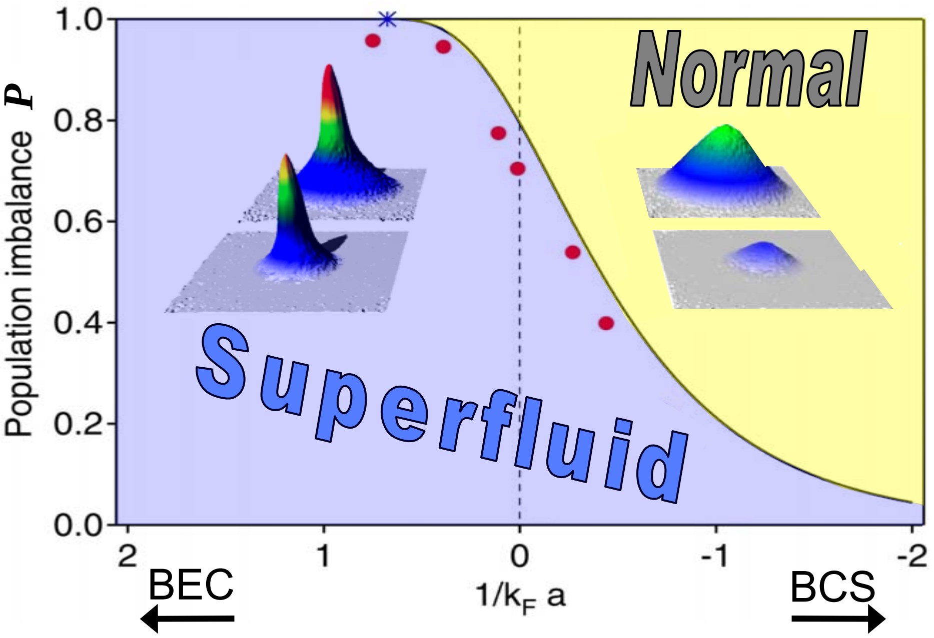
Quantitative Studies

Address open questions, Normal state:

- Magnitude of Fermi liquid parameters (μ, Z, m*, F)

- Critical interaction strength
Fermi liquid ←→ Bose liquid ?



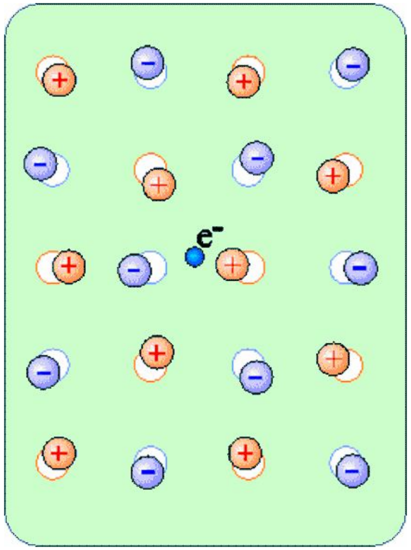
Swimming in the Fermi sea

What is the fate of a single impurity in a Fermi sea?

Crucial question for
- electron transport in lattices
- Kondo problem
(single magnetic impurity)
- mobility of ^{4}He in ^{3}He

…

Polarons: The “N+1”-body problem



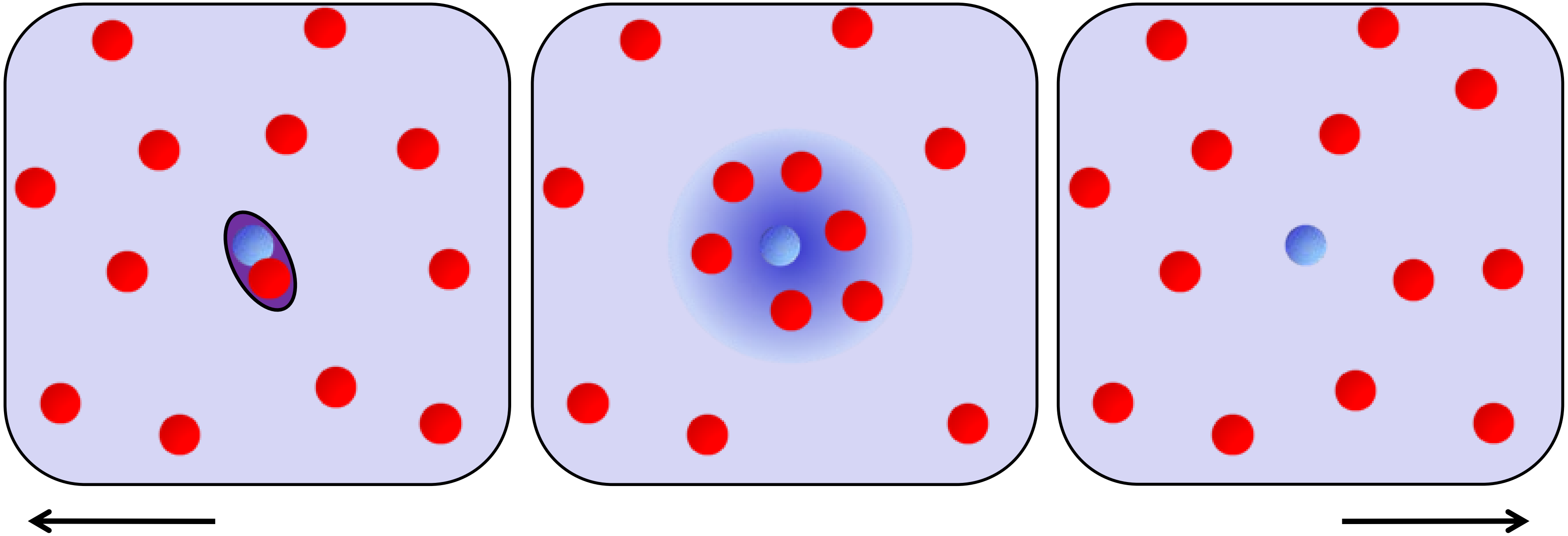
Polarons

- Historically: e$^-$ interacting with ion lattice
  L. D. Landau, Phys. Z. Sowjetunion 3 664 (1933).
- Quasi-particle:
  particle dressed by surrounding
- Long lifetime
- “Dressed” energy
- Quasiparticle residue or Weight *Z*
- Effective mass *m**
- Same question asked across all of physics for different types of particles, environments, and coupling between them (colossal magnetoresistance, the pseudo-gap phase of High-T_C superconductors, fullerenes, polymers, etc…)

Theory: Landau, Froehlich, Feynman, Anderson

> Every man, wherever he goes, is encompassed by a cloud of comforting convictions, which move with him like flies on a summer day.
> *(Bertrand Russell, 1919)*

Swimming in the Fermi Sea



Molecule

Polaron

Mean Field

strong attraction

$$\frac{\hbar^2}{ma^2}$$

?

Energy

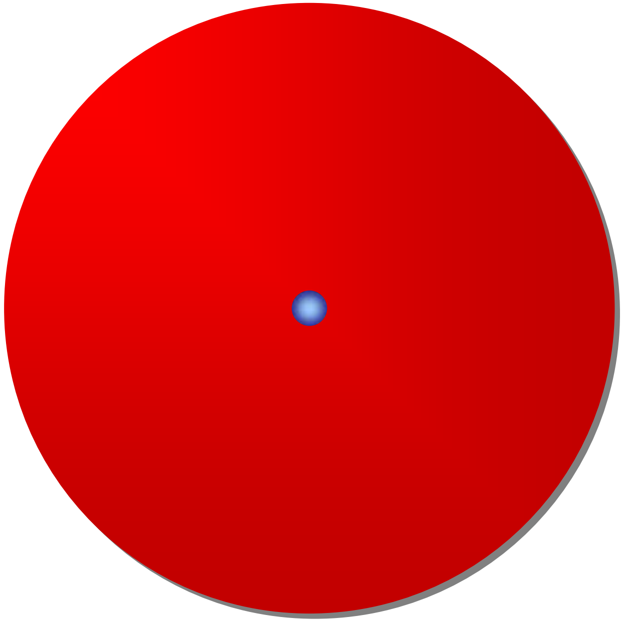
weak attraction

$$\frac{4\pi\hbar^2 a}{m} n_\uparrow$$

Theory: Chevy, Lobo, Giorgini, Stringari, Prokof'ev, Svistunov, Sachdev, Sheehy,
Radzihovsky, Lamacraft, Combescot, Sa de Melo

Swimming in the Fermi sea

A single $|\downarrow\rangle$ atom immersed in a $|\uparrow\rangle$ cloud
with unitarity limited interactions



Binding energy must be universal

$$\mu_{\downarrow} = \gamma\, E_{\mathrm{F}\uparrow}$$

$$\boxed{\gamma = -\,0.6}$$

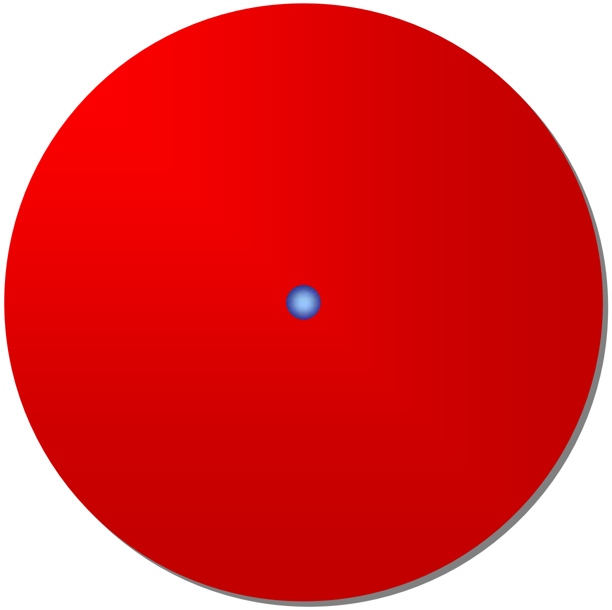
F. Chevy PRA **74**, 063628 (2006), Variational Cooper pair Ansatz

C. Lobo, A. Recati, S. Giorgini, S. Stringari, PRL **97**, 200403 (2006), Monte-Carlo

Swimming in the Fermi sea

Polaron:

$$|\Psi\rangle = \phi_0 |\mathbf{0}\rangle_\downarrow |FS\rangle_\uparrow$$



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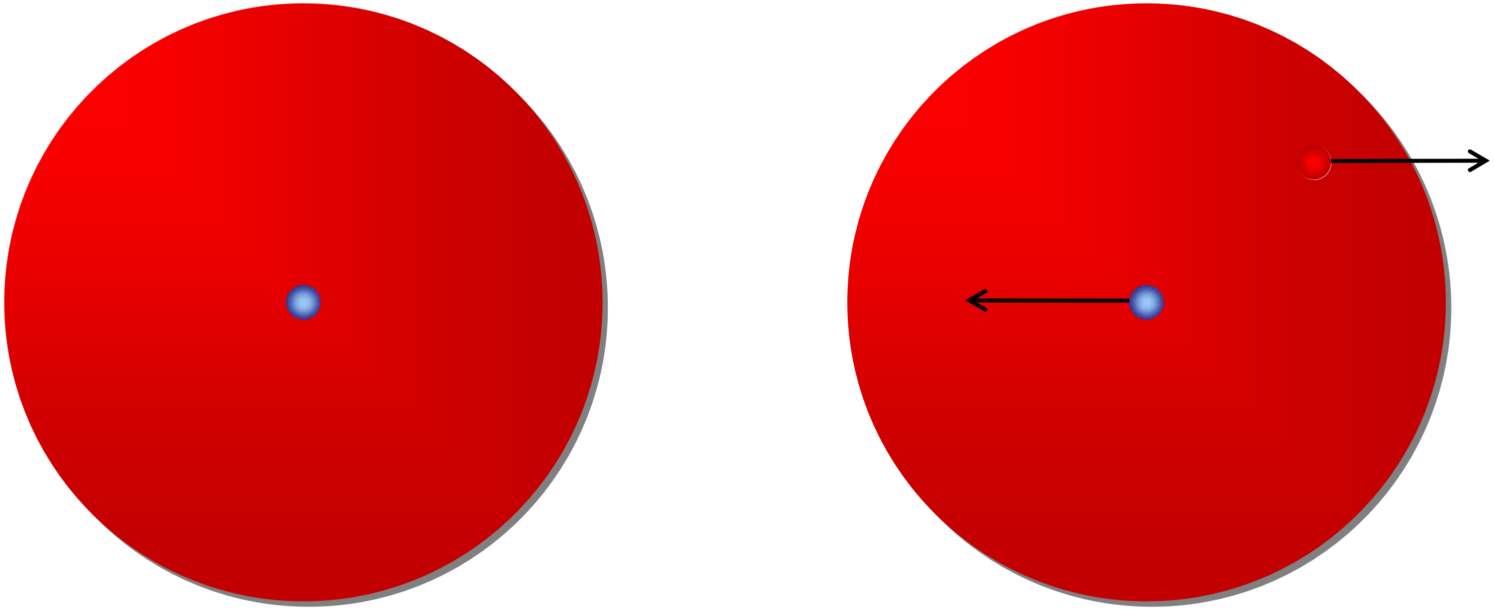
Swimming in the Fermi sea

Polaron:

$$|\Psi\rangle = \phi_0 |\mathbf{0}\rangle_{\downarrow} |FS\rangle_{\uparrow} + \sum_{q<k_F}^{k>k_F} \phi_{\mathbf{qk}} |\mathbf{q}-\mathbf{k}\rangle_{\downarrow} c^{\dagger}_{\mathbf{k}_{\uparrow}} c_{\mathbf{q}_{\uparrow}} |FS\rangle_{\uparrow}$$



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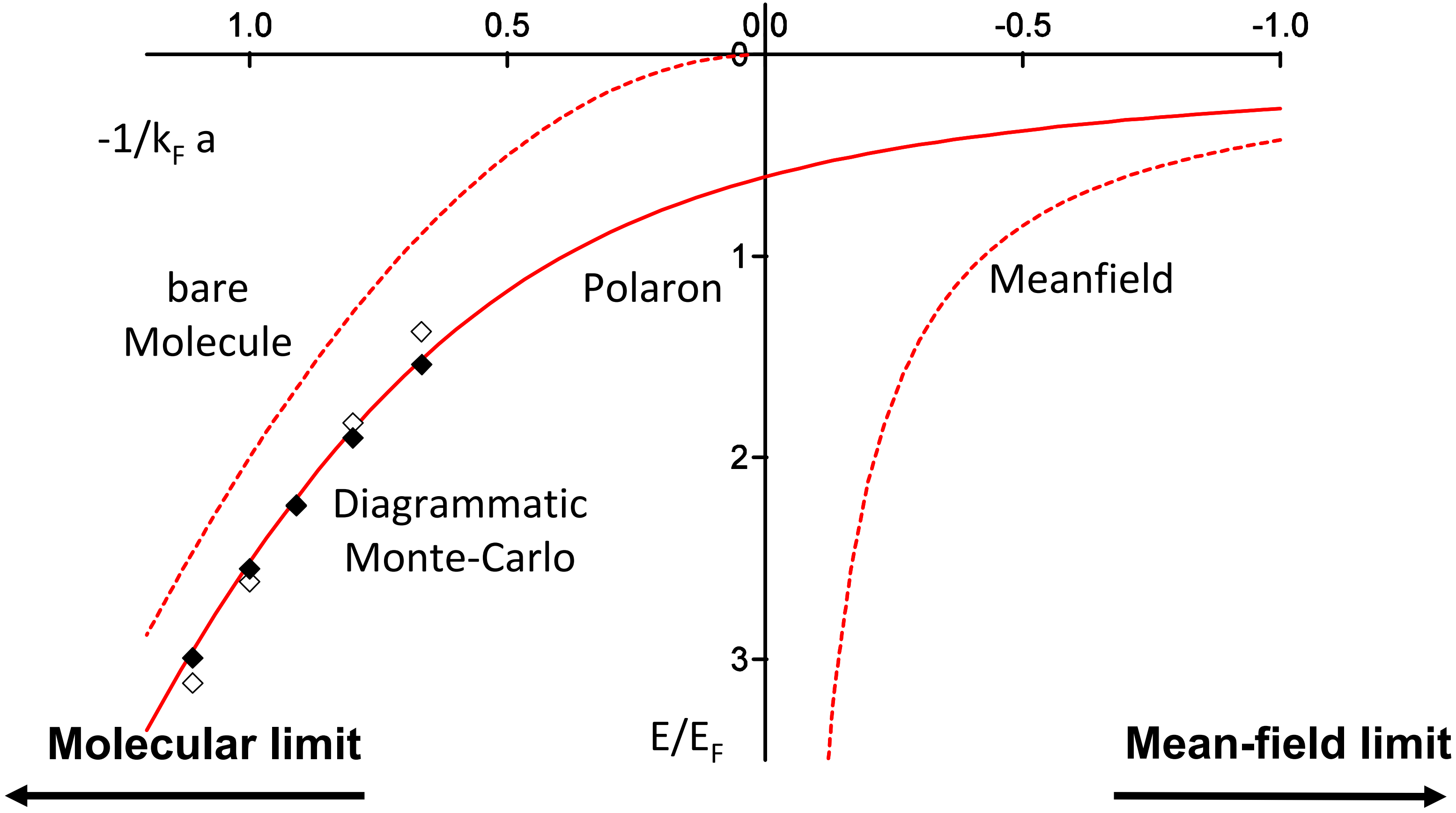
Polaron Energy

$$E=\frac{1}{\Omega}\sum_{q<k_F}\frac{1}{\dfrac{m}{4\pi\hbar^2 a}+\dfrac{1}{\Omega}\sum_{k>k_F}\left(\dfrac{1}{\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{q}}+\varepsilon_{\mathbf{q}-\mathbf{k}}-E}-\dfrac{1}{2\varepsilon_{\mathbf{k}}}\right)-\dfrac{1}{\Omega}\sum_{q<k_F}\dfrac{1}{2\varepsilon_{\mathbf{q}}}}$$



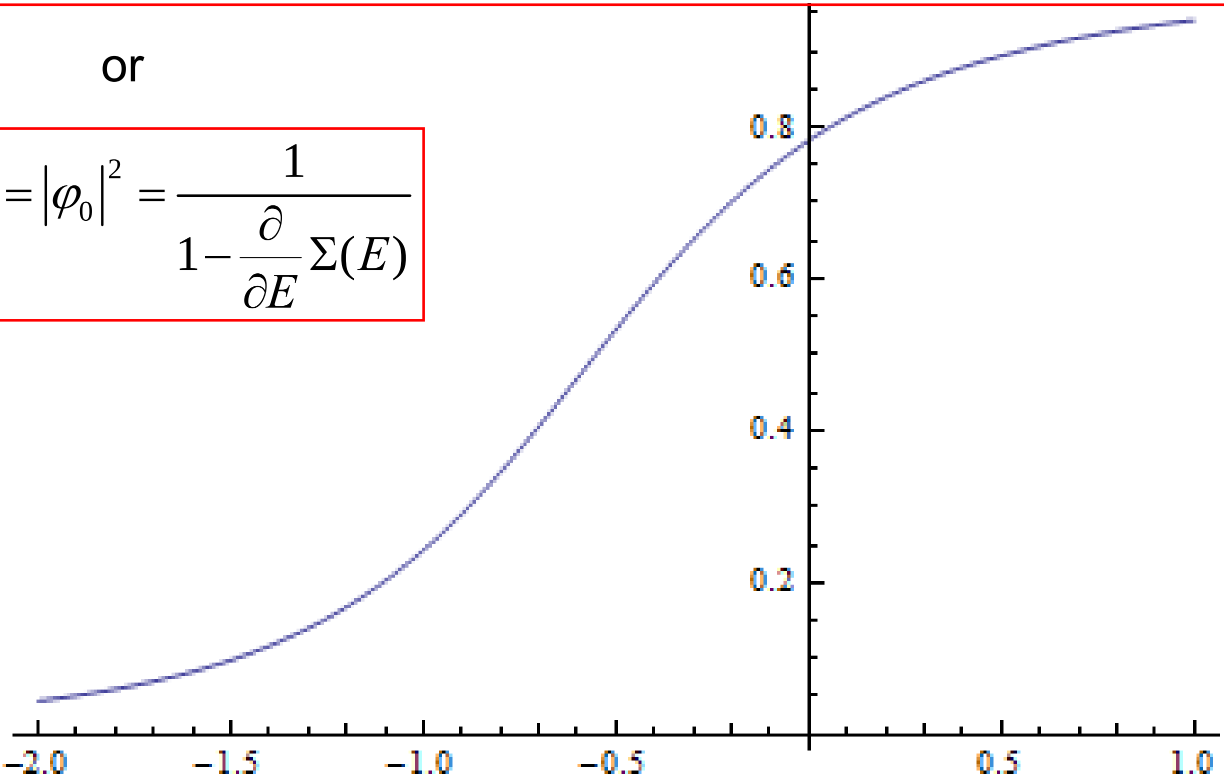
Diagrammatic Monte-Carlo: N. V. Prokof'ev and B. V. Svistunov, PRB 77, 125101 (2008)
Variational approach/T-Matrix: F. Chevy PRA **74**, 063628 (2006), R. Combescot and S. Giraud, PRL 101, 050404 (2008), R. Combescot et al.,PRL 98, 180402 (2007),
T-matrix/ladder approximation: P. Massignan, G. Bruun and H. Stoof, PRA 78, 031602 (2008)

Polaron Quasi-particle weight

$$Z = |\varphi_0|^2 = \frac{1}{1 - \frac{\partial}{\partial E}\frac{1}{\Omega}\sum_{q<k_F}\frac{1}{\frac{m}{4\pi\hbar^2 a} + \frac{1}{\Omega}\sum_{k>k_F}\left(\frac{1}{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{q}} + \varepsilon_{\mathbf{q}-\mathbf{k}} - E} - \frac{1}{2\varepsilon_{\mathbf{k}}}\right) - \frac{1}{\Omega}\sum_{q<k_F}\frac{1}{2\varepsilon_{\mathbf{q}}}}}$$

or

$$Z = |\varphi_0|^2 = \frac{1}{1 - \frac{\partial}{\partial E}\Sigma(E)}$$



$-1/k_F\, a$

F. Chevy PRA **74**, 063628 (2006), Variational Cooper pair Ansatz

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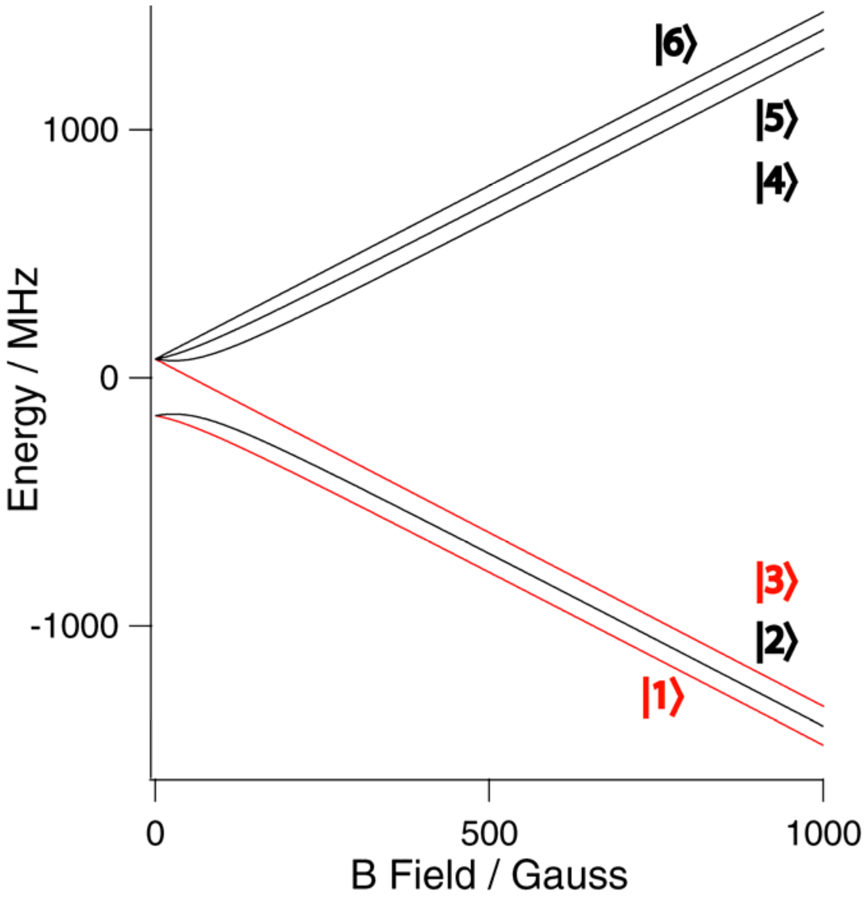
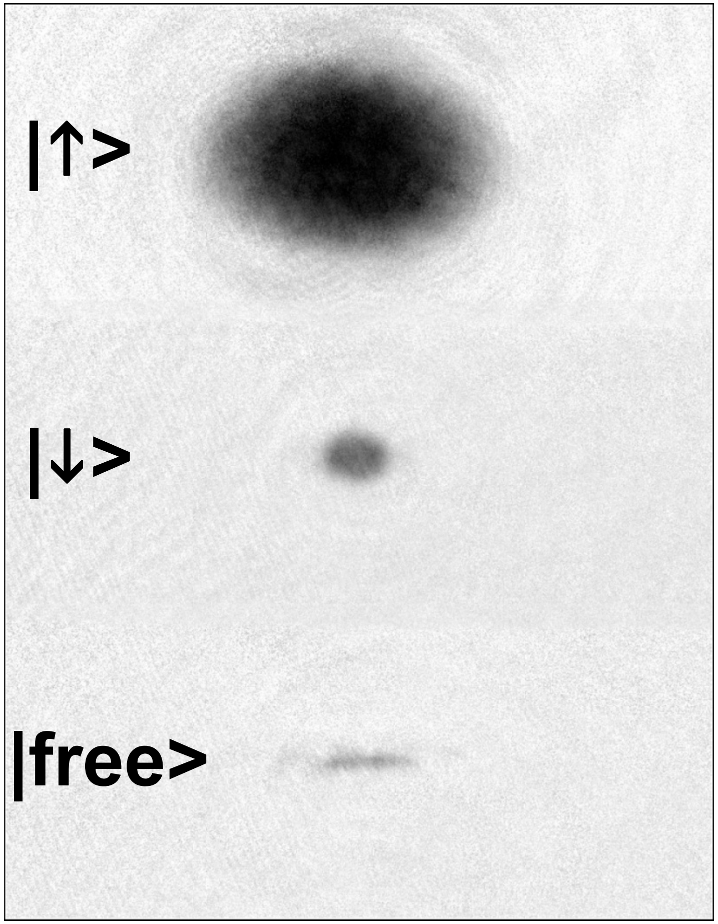
Experimental Realization

^{6}Li - Atom: 6 hyperfine states

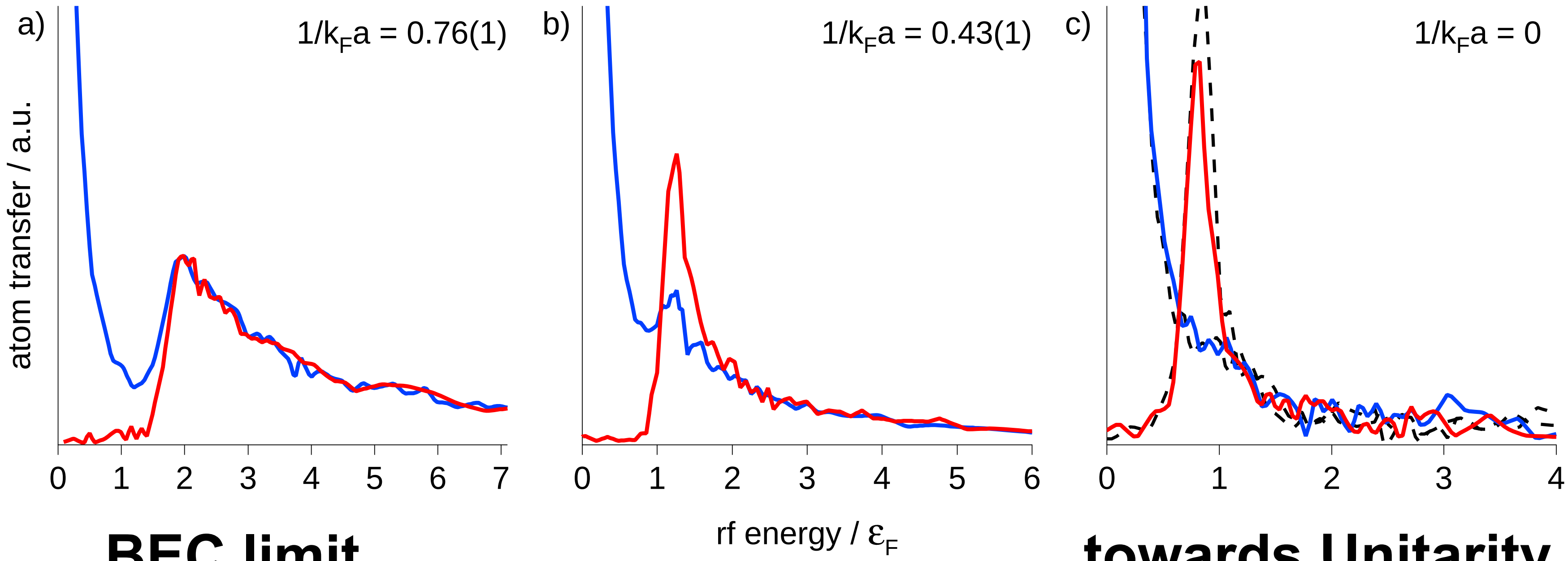


RF spectroscopy



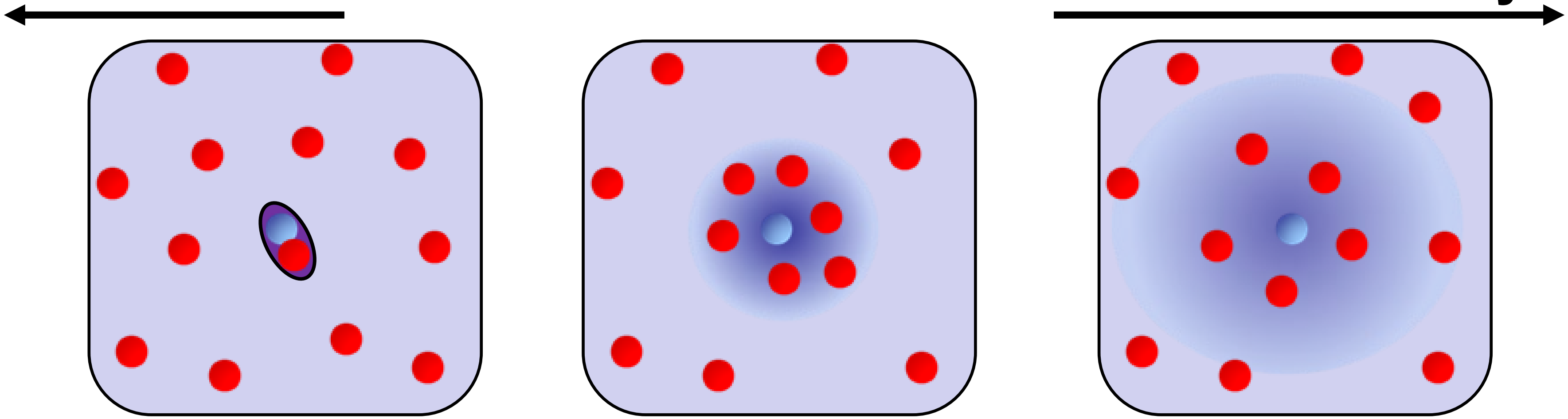
- Spatially resolved
- 3D reconstructed

RF Spectra with high density imbalance



BEC limit

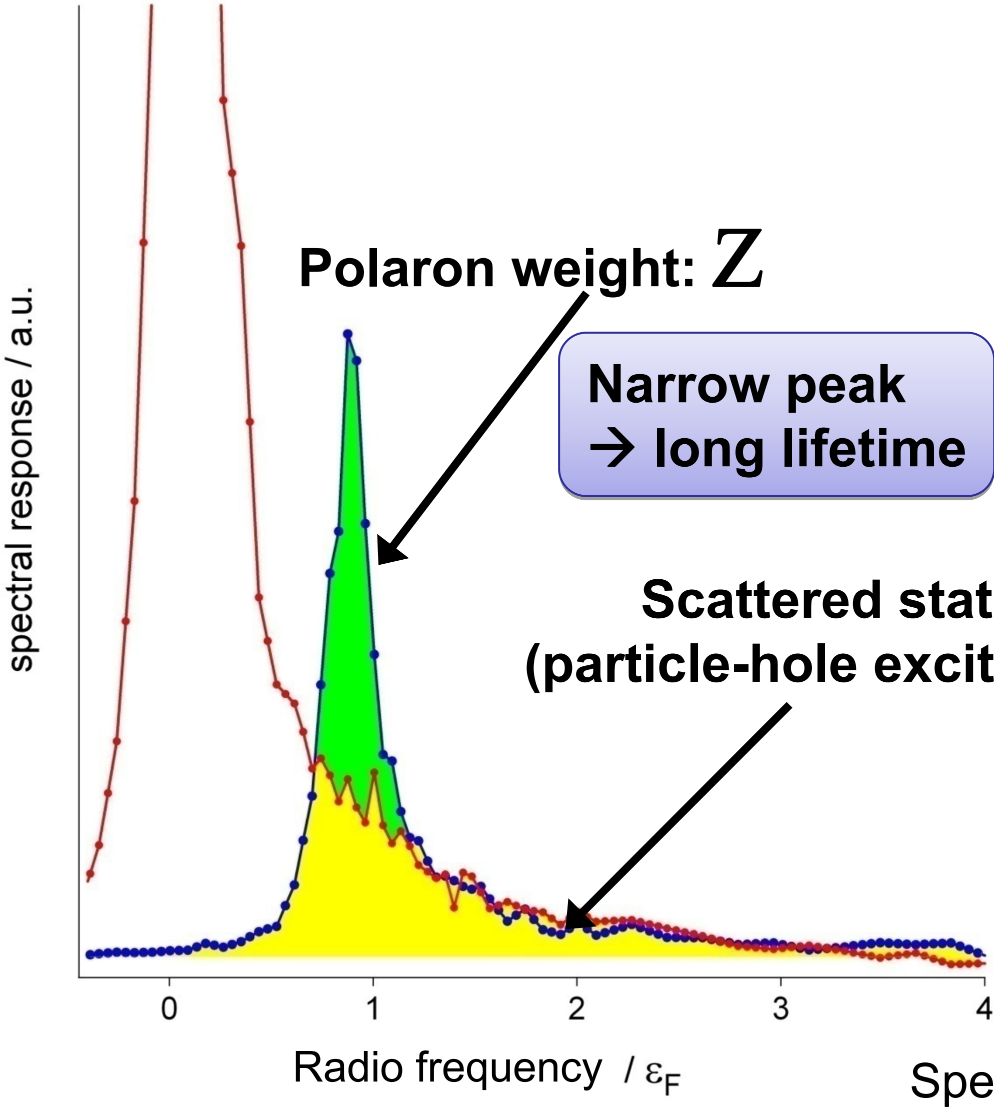
towards Unitarity



A. Schirotzek et al., in preparation

Interpretation of the spectra

Polaron =$\sqrt{Z}$ Free particle +$\sqrt{1-Z}$ scattered states



Polaron weight: Z

Narrow peak
→ long lifetime

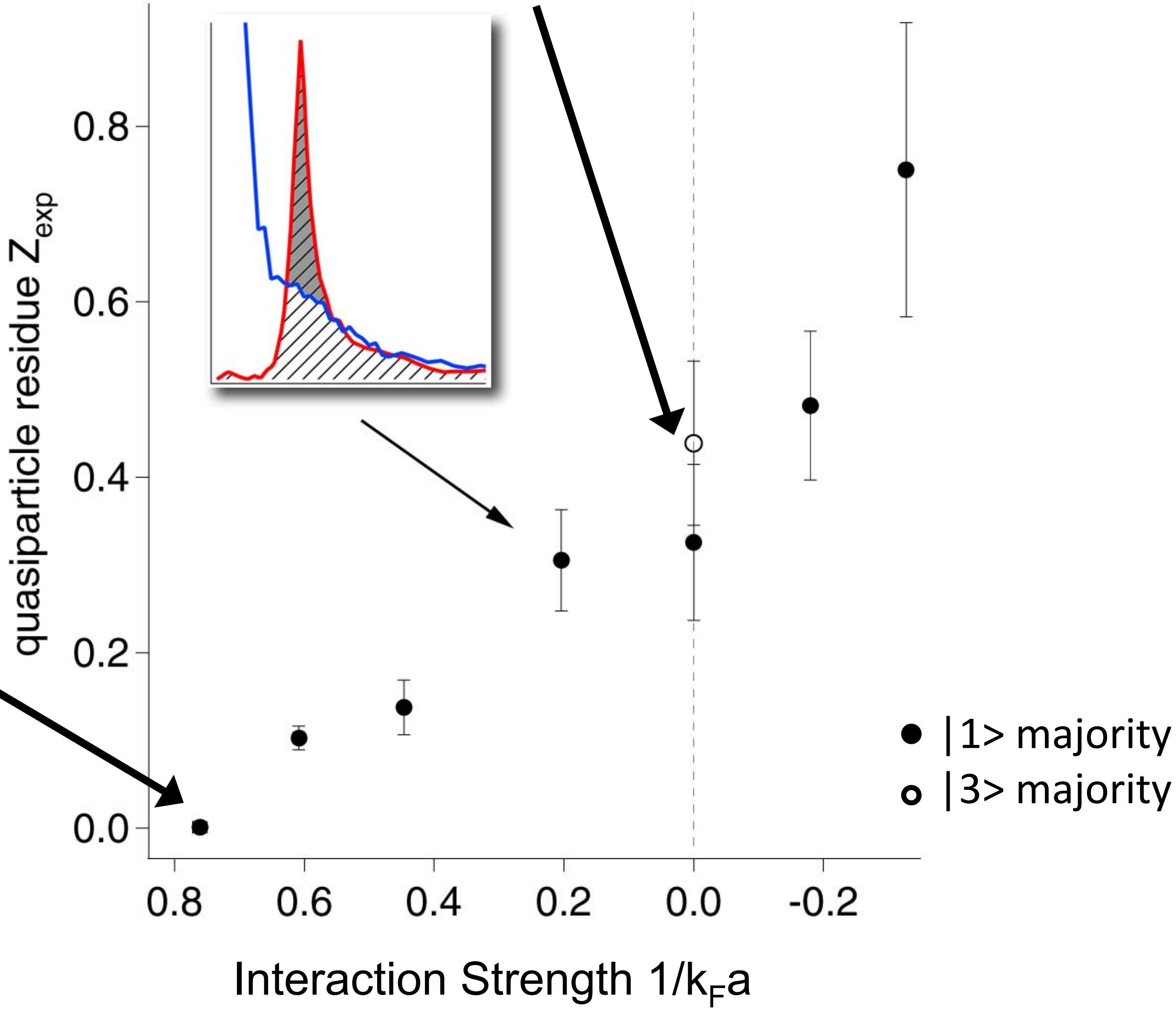
Scattered states
(particle-hole excitations)

Spectrum @ 680 G

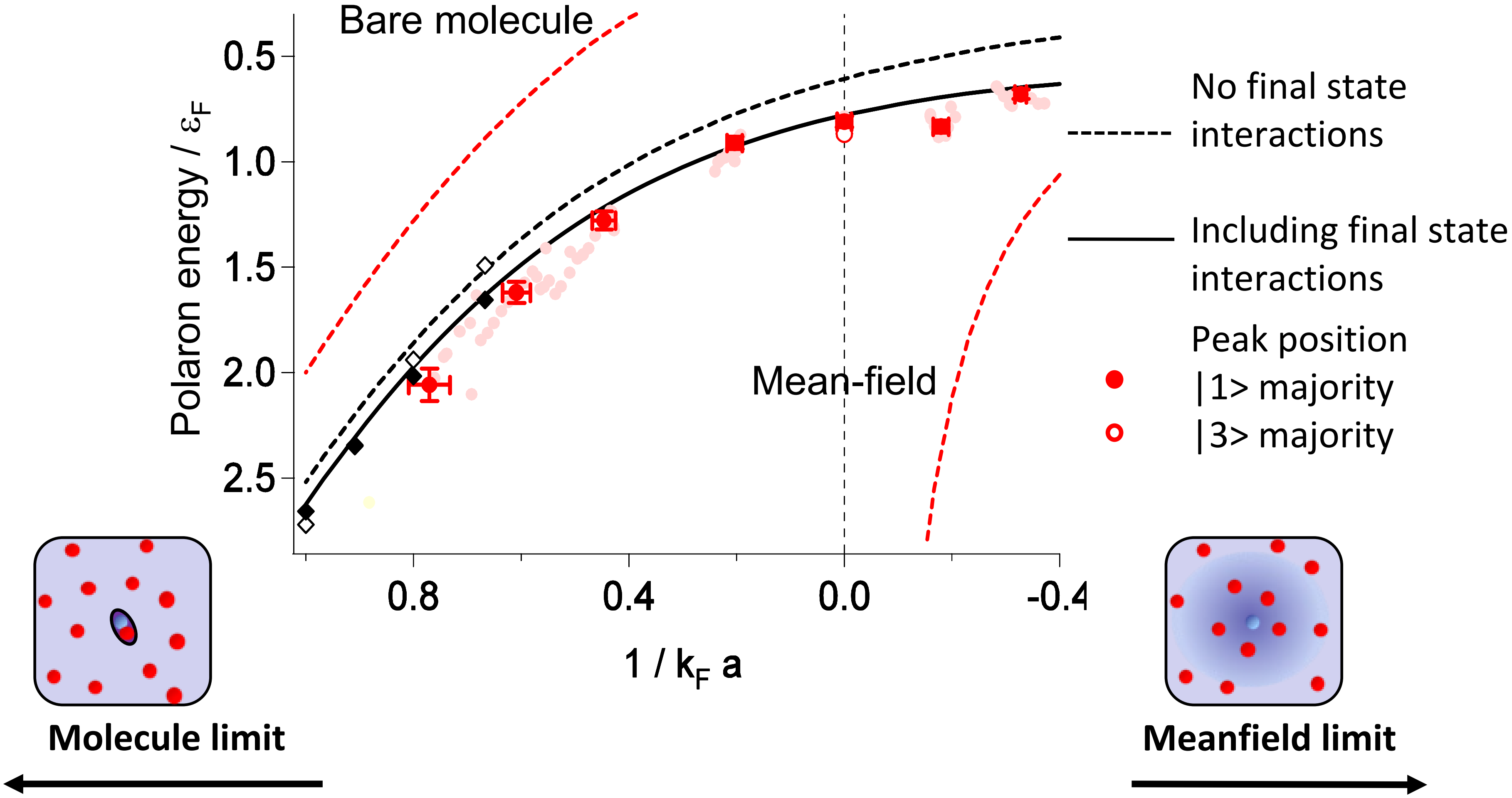
Z for various interaction strengths

- Polarons become molecules for a critical interaction strength: 0.76
- Breakdown of Fermi liquid
- Coincides with critical strength for always having a BEC

Polaron weight 40% on resonance



Polaron Energy vs Interaction Strength



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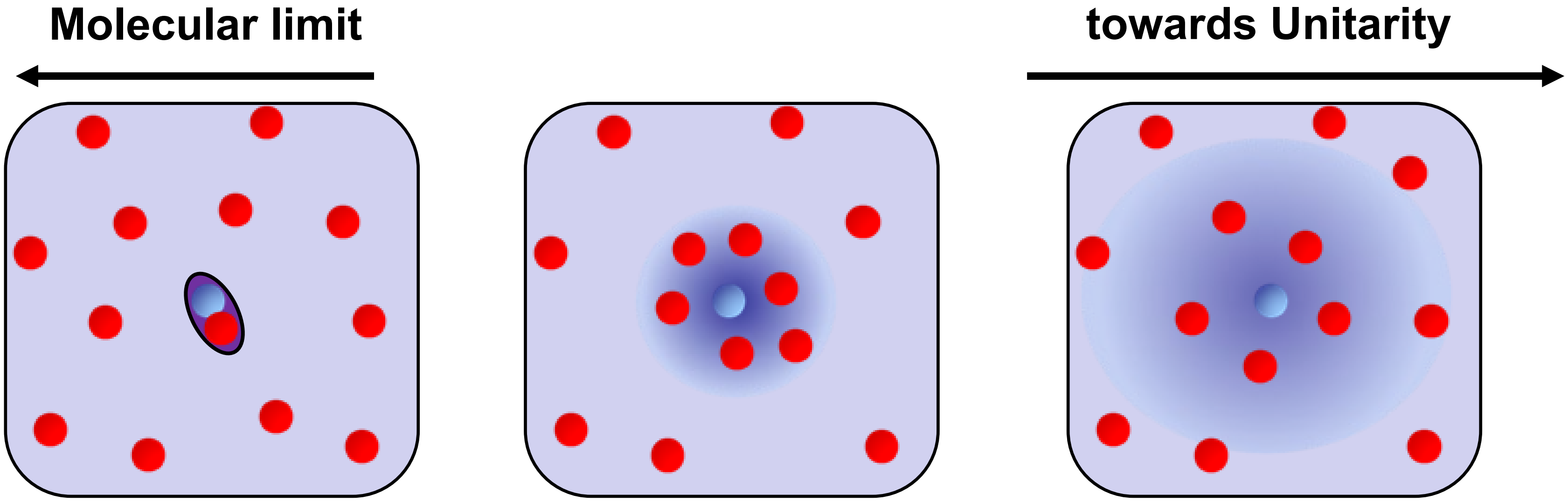
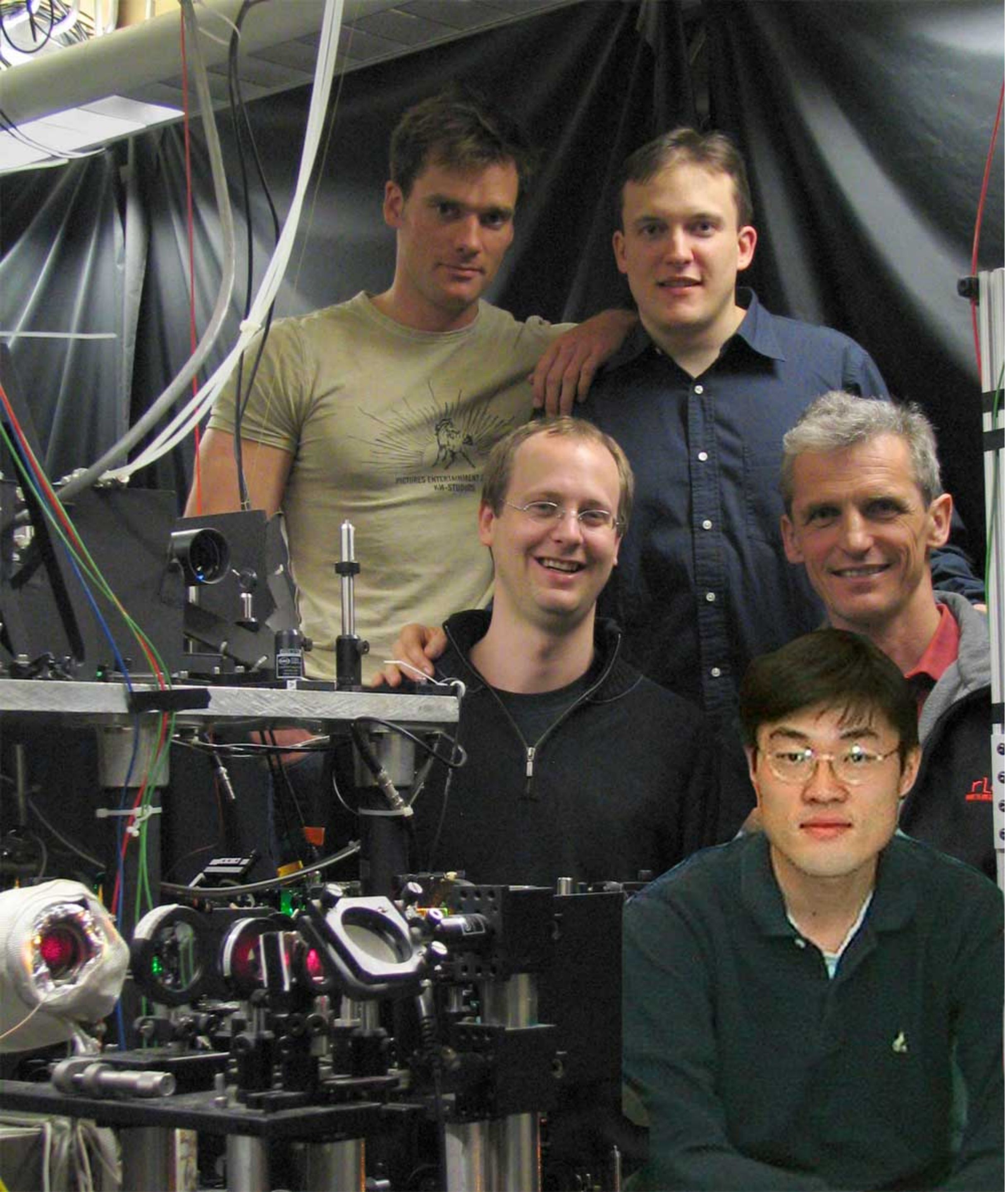
Conclusion

- Observation of Spin-Polarons in a new Fermi liquid with tunable interactions
- Benchmark test for many-body theories
- Very good agreement in the binding energy

- Determines the low-temperature phase diagram of imbalanced Fermi gases
- Future: Dynamic properties. Measure effective mass m*.





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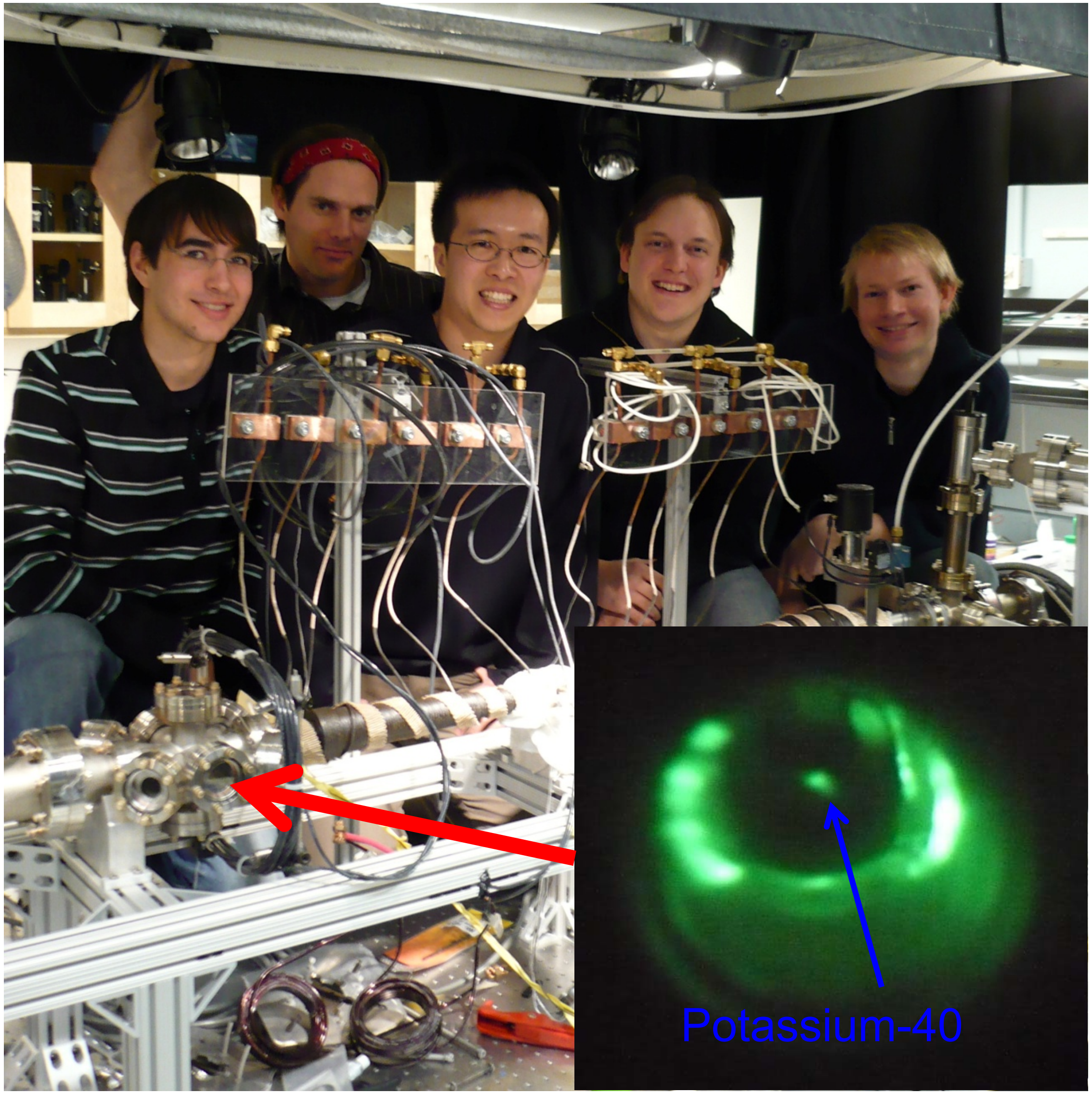
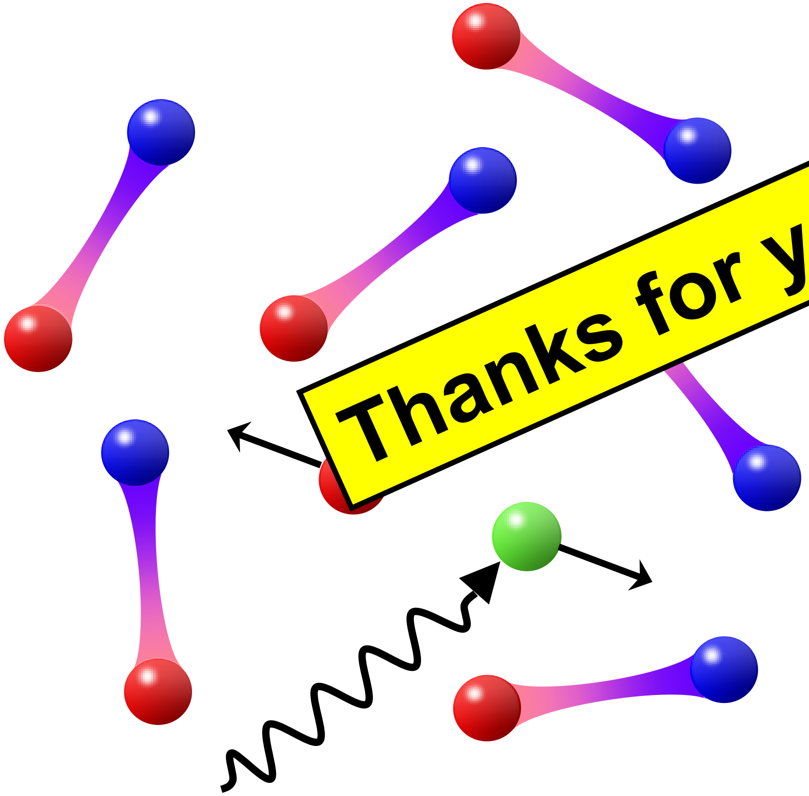


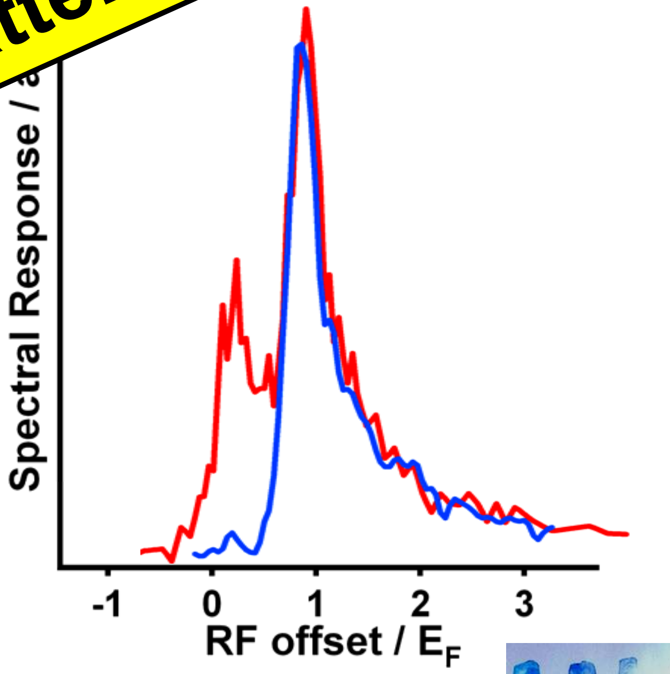
Coherent Quantum Gases - From Cold Atoms to Condensed Matter
Copenhagen, Jan. 15th/16th 2009

Radio Frequency Spectroscopy of Ultracold Fermi Gases

Andre Schirotzek

Thanks for your attention!





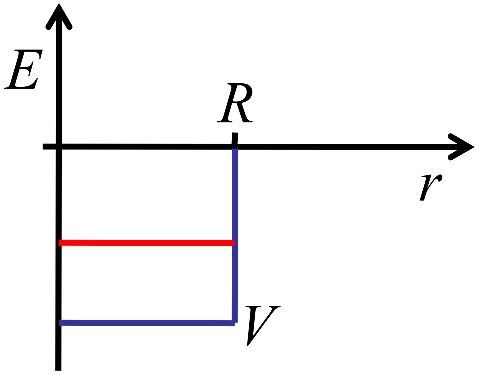
Center for Ultracold Atoms at MIT and Harvard

Realization in ultracold Fermi gases

Atomic Fermi Gases allow to freely choose

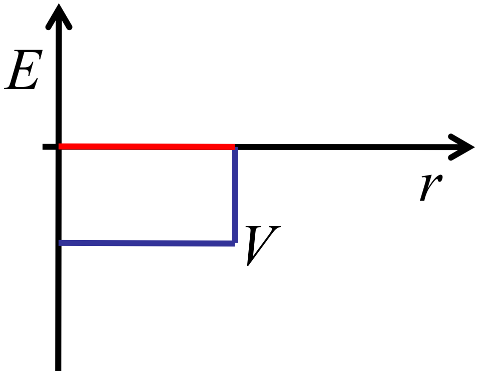
- Spin imbalance
- Interaction strength between spin up and spin down

Situation maps onto tunable square well ($k_F r_{eff} << 1$):



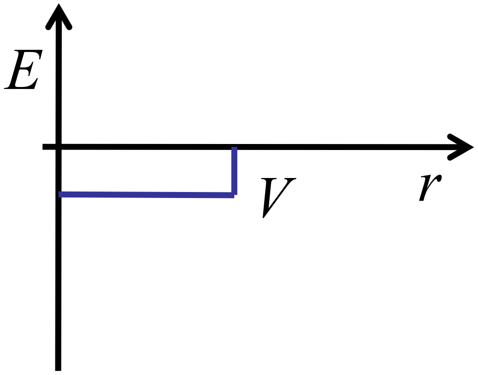
strong attraction

deep bound state



Resonance

bound state appears



weak attraction

no bound state

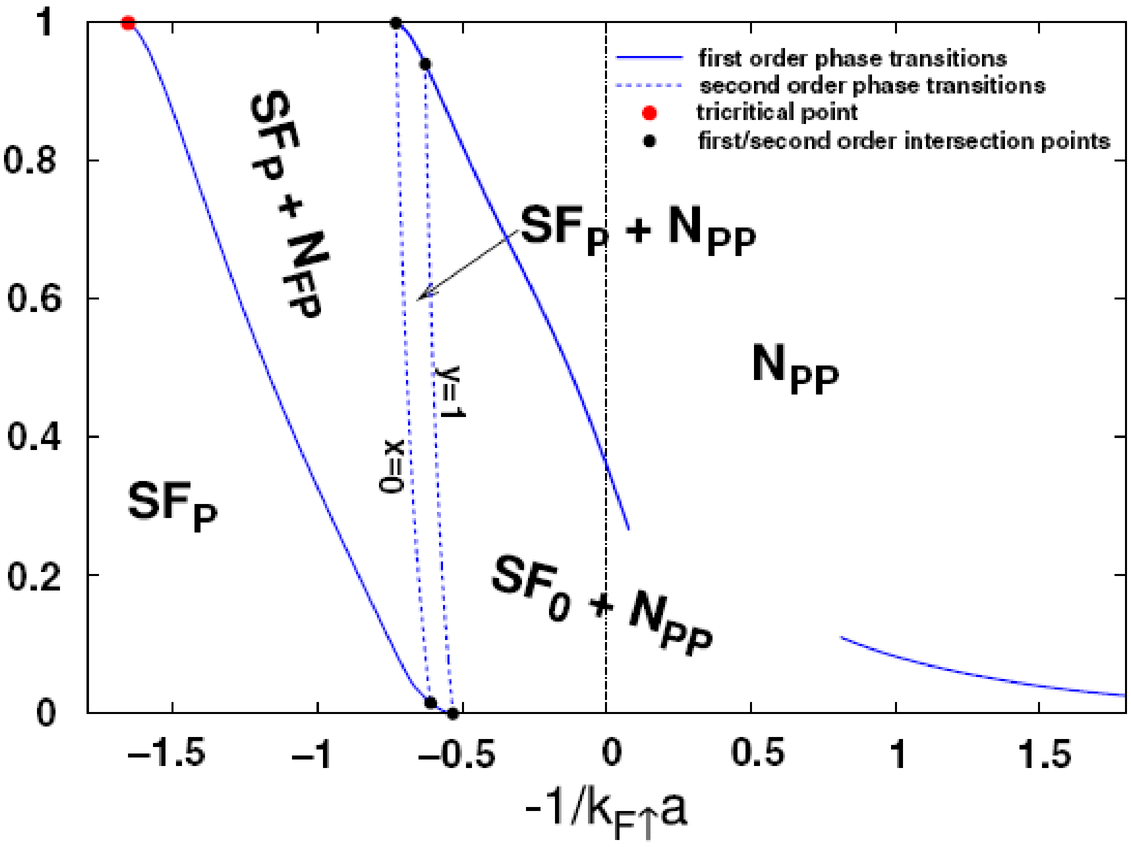
scattering length

$a > 0$

$a \to \pm\infty$

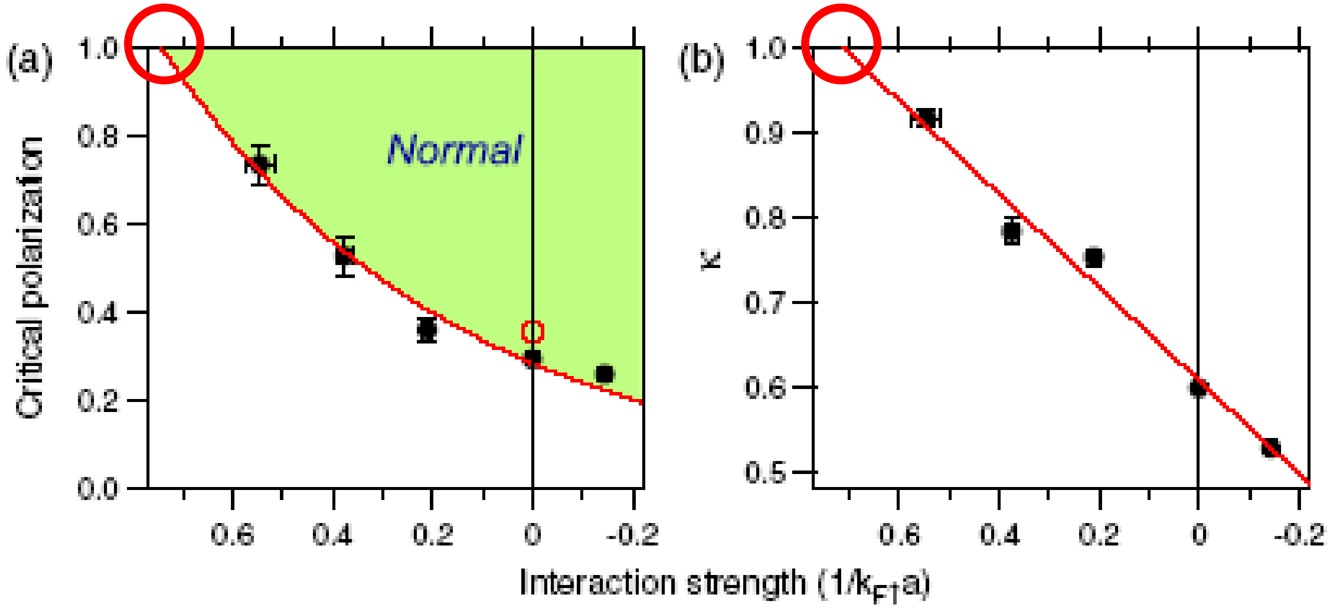
$a < 0$

Full Phase Diagram



S. Pilati and S. Giorgini, PRL 101, 030408 (2008)

Phase Transition from Density Distribution



Yong-il Shin et al., PRL 070404 (2008)