

Nordic Neutrino Physics

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NORDIC NEUTRINO PHYSICS

Lecture I & II:

- A bit of history: neutrinos in the Standard Model
- Neutrino masses: Majorana versus Dirac
- Neutrino oscillations in vacuum and in matter

Lecture III:

- Evidence for neutrino mass: review of experimental landscape
- The standard 3ν scenario
- A few outliers...

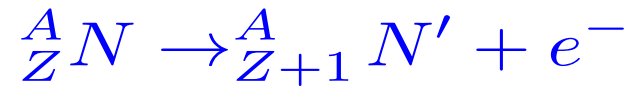
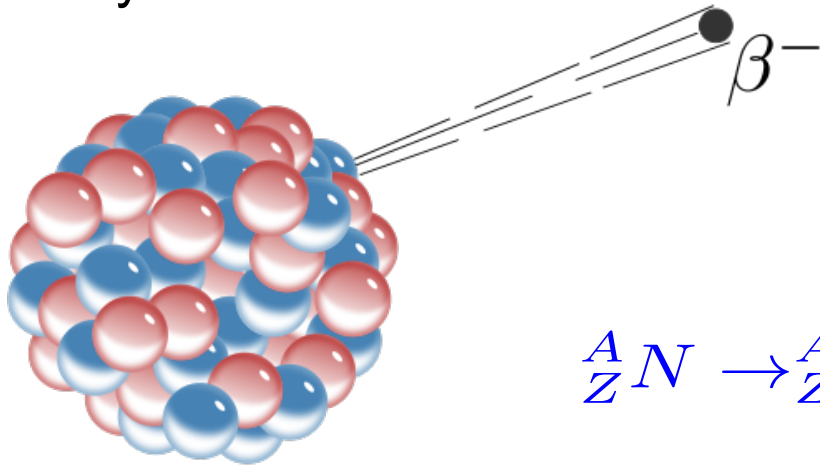
Lecture IV:

- Prospects in neutrino physics
- Leptogenesis & neutrinos in the cosmos
- Theory outlook

Neutrino: the phantom particle

1900 Radioactivity: Becquerel, M & P Curie, Rutherford...

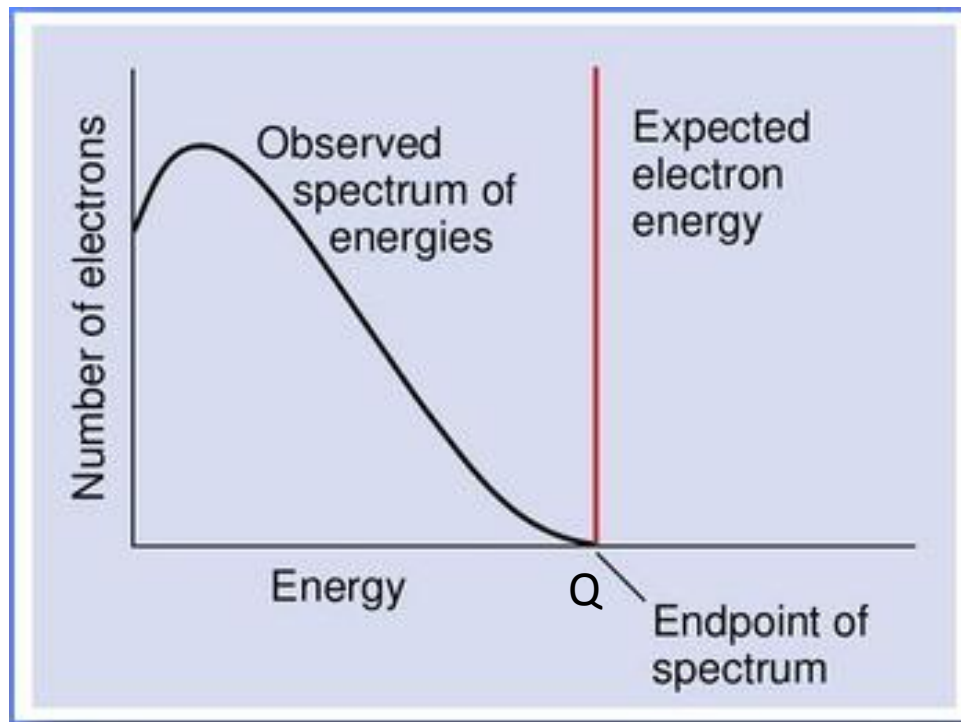
β decay



Energy conservation: $E_{\text{electron}} \simeq (M_N - M_{N'})c^2 = Q = \text{constante}$

1911/1914

Electron spectrum:

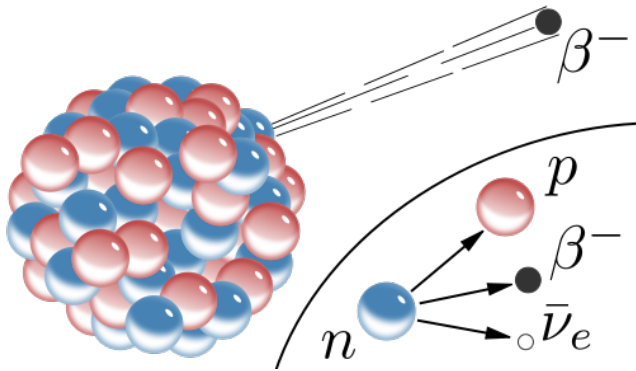


Meitner, Hahn
(Nobel 1944 only him!)



Chadwick (Nobel 1935)

1930



Pauli (Nobel 1945)

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li^6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin $1/2$ and obey the exclusion principle, and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...

Unfortunately, I cannot personally appear in Tübingen since I am indispensable here in Zürich because of a ball on the night from December 6 to 7...

1933: Solvay's conference

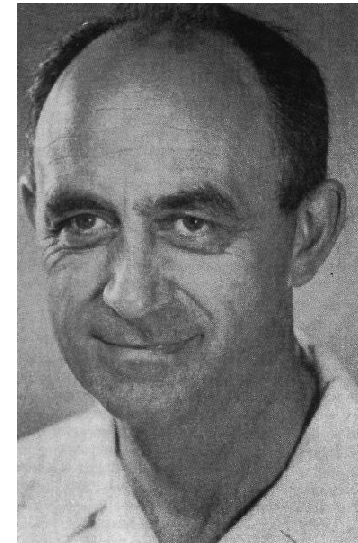
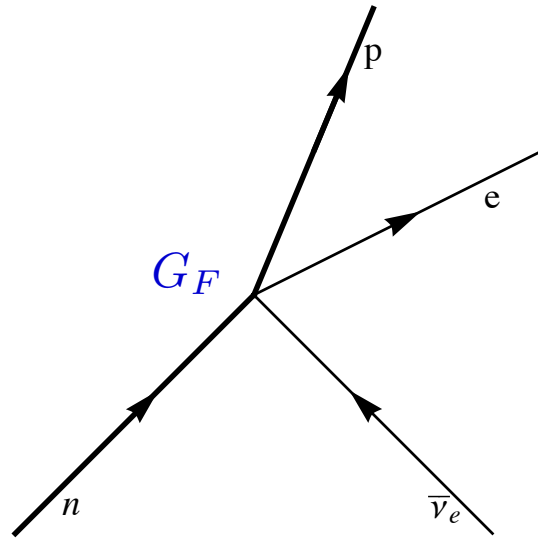
The neutron was discovered in 1932 by Chadwick ...



"... their mass can not be very much more than the electron mass. In order to distinguish them from heavy neutrons, mister Fermi has proposed to name them "neutrinos". It is possible that the proper mass of neutrinos be zero... It seems to me plausible that neutrinos have a spin 1/2... We know nothing about the interaction of neutrinos with the other particles of matter and with photons: the hypothesis that they have a magnetic moment seems to me not funded at all."

W. Pauli

1934: Theory of beta decay



E. Fermi
(Nobel 1938)

Nature did not publish his article: “contained speculations too remote from reality to be of interest to the reader...”

Bethe-Peierls (1934): compute the neutrino cross section using this theory

$$\sigma \simeq 10^{-44} \text{cm}^2, \quad E(\bar{\nu}) = 2 \text{ MeV}$$

“there is not practically possible way of detecting a neutrino”

How to detect them ?

$$\lambda \simeq \frac{1}{n\sigma}$$

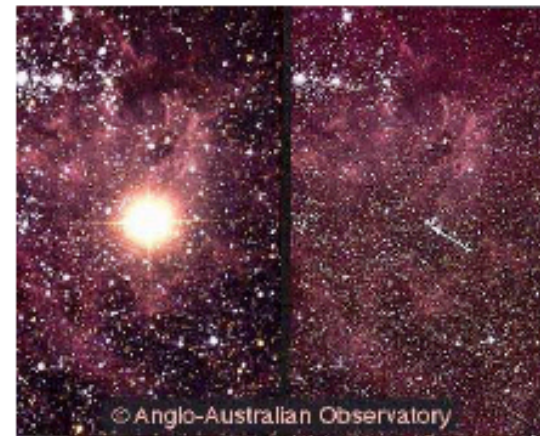
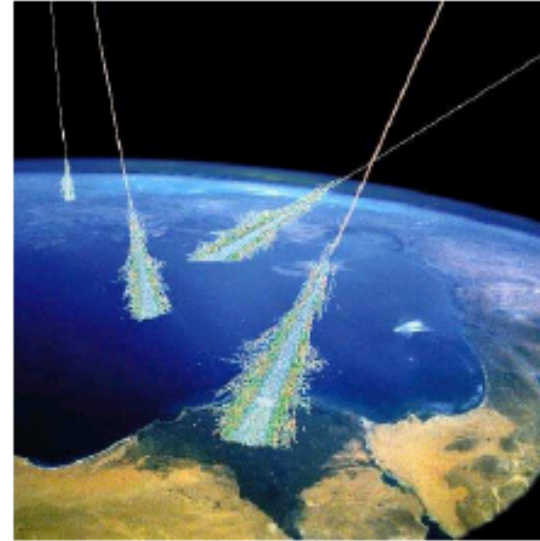
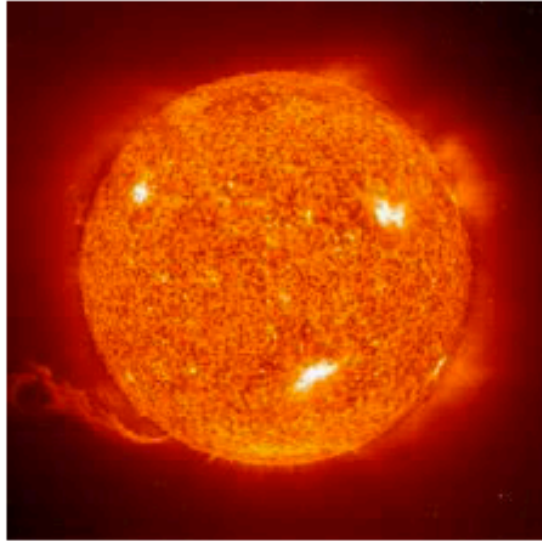
$$\lambda|_{\text{@water}} \simeq 1.5 \times 10^{21} \text{ cm} \simeq 1600 \text{ Light Years}$$

$$\lambda|_{\text{@interstellar}} \simeq 10^{44} \text{ cm} \simeq 10^{26} \text{ Light Years}$$

“I have done a terrible thing. I have postulated a particle that cannot be detected.” W. Pauli

“Not even wrong”

Revealing Pauli's dark matter was just a question of time and ingenuity...



How to detect them?

1946 Pontecorvo

Not so desperate...



Бруно Понтекорво

$$\begin{aligned} N_{CC} &= \Phi_\nu \times \sigma \times \text{Numero de blancos} \times \Delta T \\ &= \Phi_\nu (cm^{-2}s^{-1}) \times 10^{-44} cm^2 \times N_{\text{Avogadro}} \times \text{Detector mass (gr)} \times 10^5 s \times \# \text{dias} \end{aligned}$$

Needs a reaction where the final isotope is radioactive with a proper lifetime



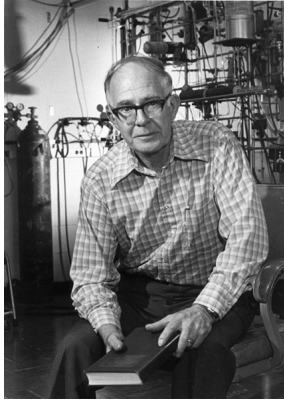
Argon can be separated: in ~ 35 days the inverse reaction takes place

Then the (by-then) recently invented nuclear reactors could be this source...

Reactors: $\sim 10^{20}$ /second!

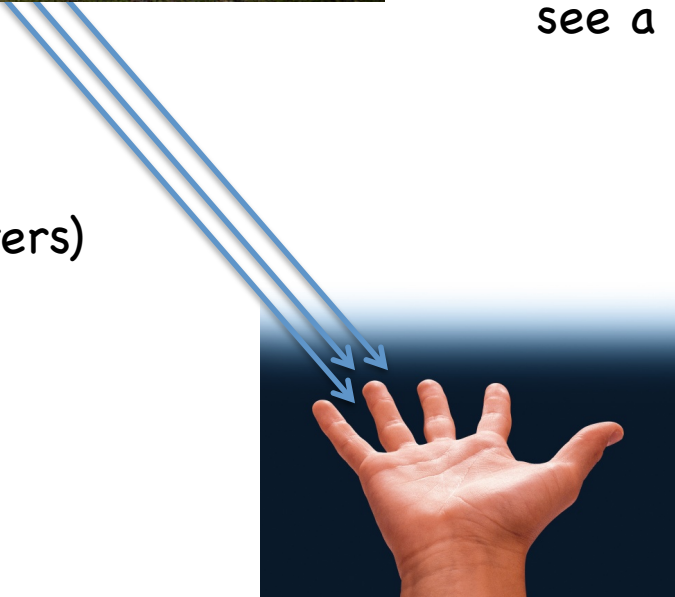


1955 Davies



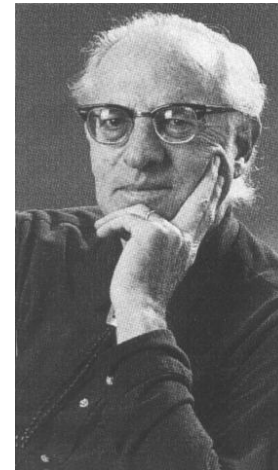
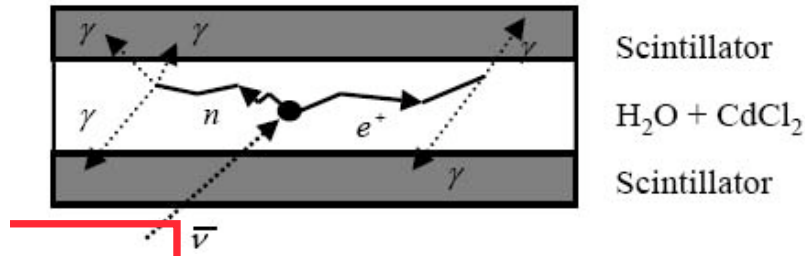
Built a 4000 liter detector, but did not see a thing...

(10^{11} /s@100 meters)

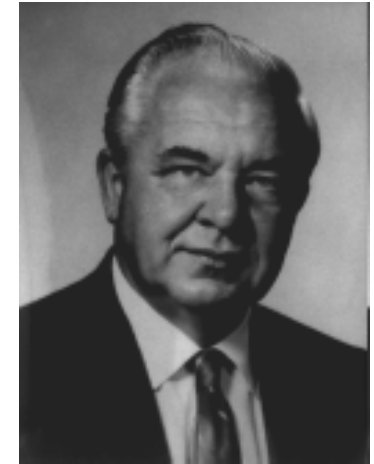


1956 (anti)neutrino detection

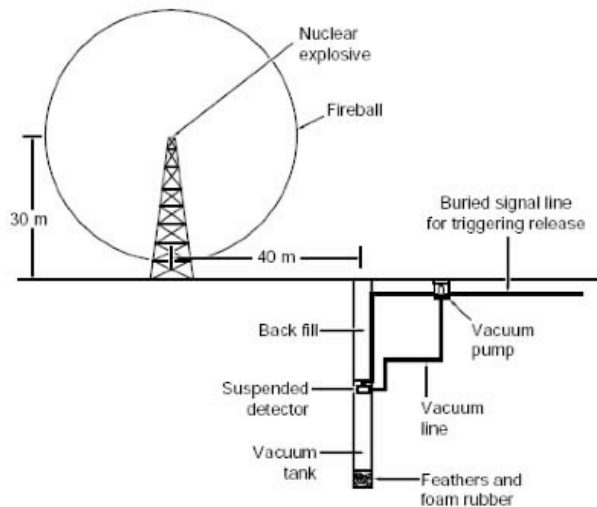
Poltergeist project



Reines Nobel 95



Cowan (died 74)



First idea: put the detector close to a nuclear explosion !

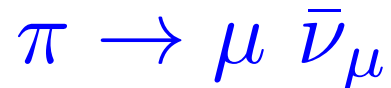
Finally use the reactor Savannah River to discover the anti-neutrino

The flavour of neutrinos

1937 μ discovered in cosmic rays

1947 Pontecorvo

Is a heavy version of the electron and not the nuclear agent (pion)



1959 Pontecorvo

The neutrino that accompanies the μ is different to that in beta decay

Neutrino cross section in Fermi theory grows with energy: he proposes the first experiment with a neutrino beam !



Neutrino Flavour

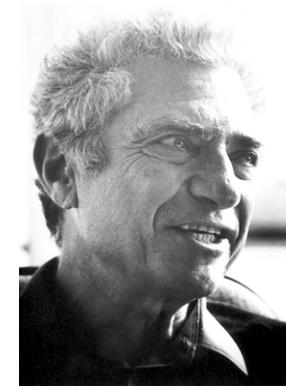
$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$



Lederman

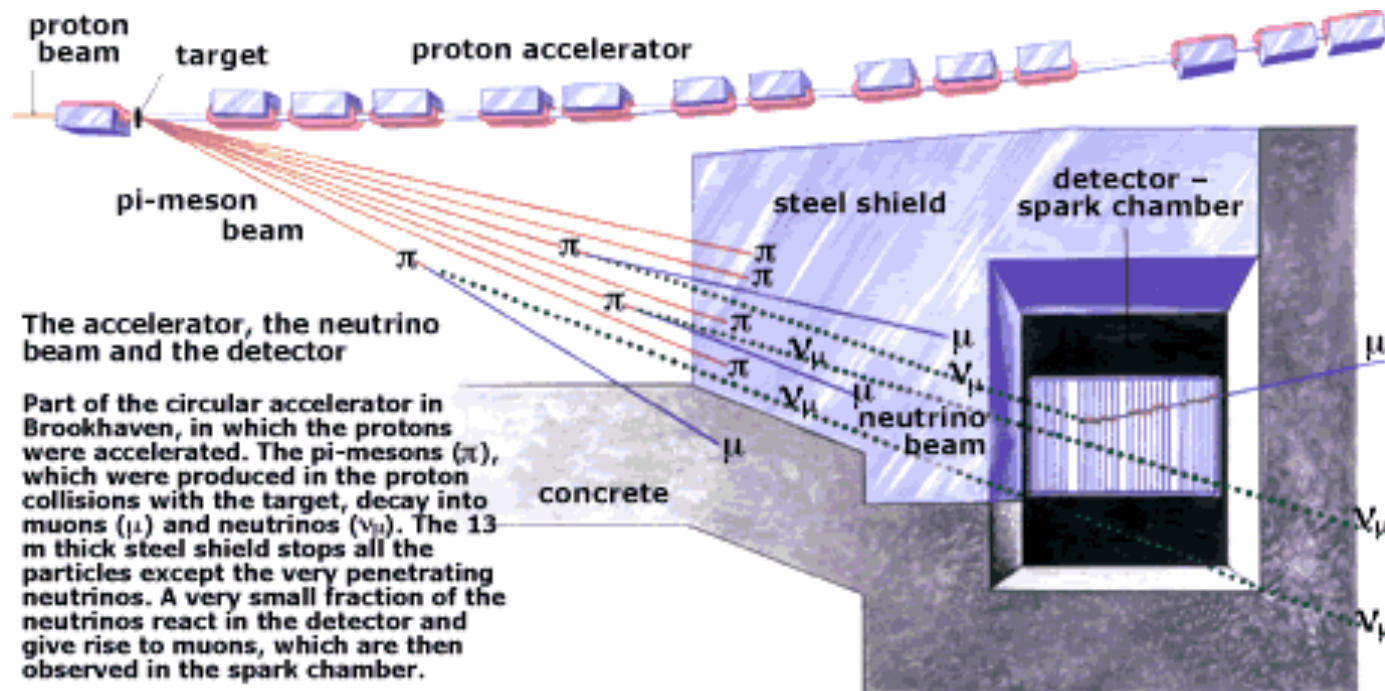


Schwartz



Steinberger

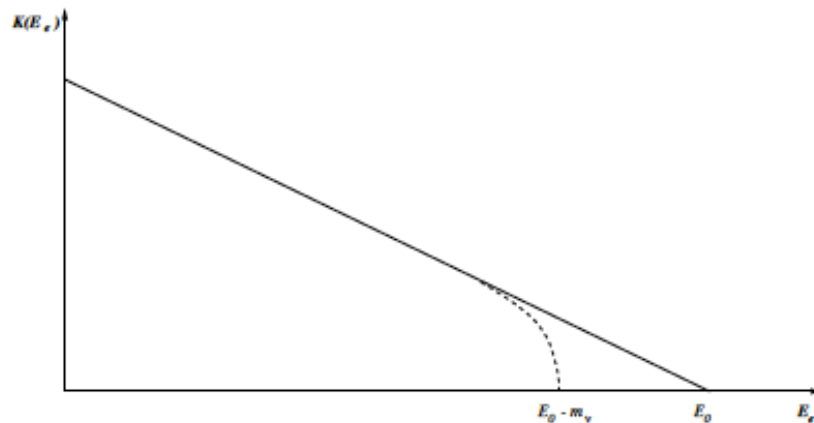
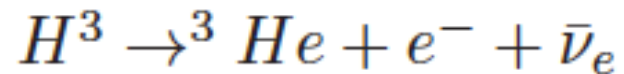
Nobel 1988



Based on a drawing in Scientific American, March 1963.

Kinematical effects of neutrino mass

Most stringent from Tritium beta-decay



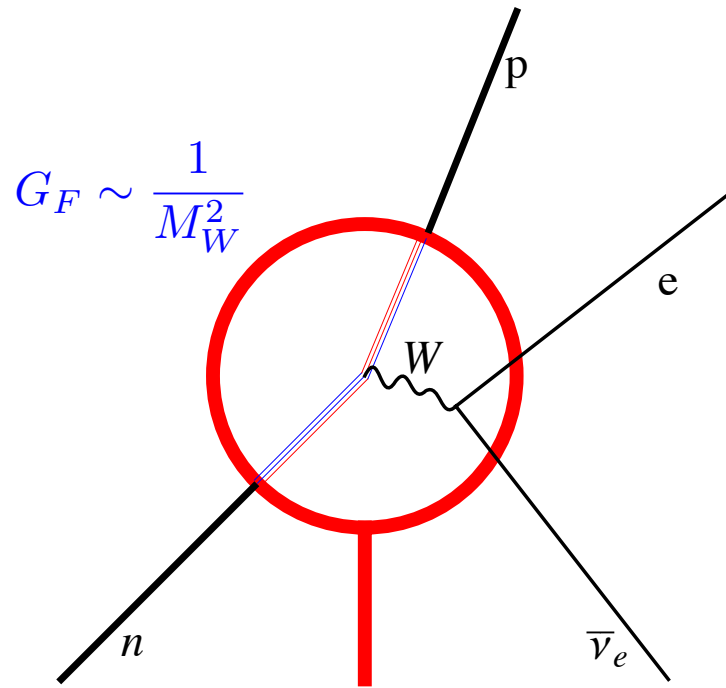
$$m_{\nu_e} < 2.2\text{eV (Mainz-Troitsk)}$$

$$m_{\nu_\mu} < 170\text{keV (PSI: } \pi^+ \rightarrow \mu^+ \nu_\mu)$$

$$m_{\nu_\tau} < 18.2\text{MeV (LEP: } \tau^- \rightarrow 5\pi \nu_\tau)$$

Standard Model neutrinos assumed massless

Neutrinos in the Standard Model



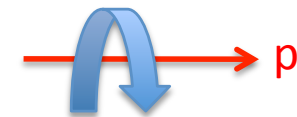
Charged currents: CC

$$SU(3) \times SU(2) \times U(1)_Y$$

$(1, 2)_{-\frac{1}{2}}$	$(3, 2)_{-\frac{1}{6}}$	$(1, 1)_{-1}$	$(3, 1)_{-\frac{2}{3}}$	$(3, 1)_{-\frac{1}{3}}$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$	e_R	u^i_R	d^i_R
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t^i_R	b^i_R

Left-handed

Right-handed



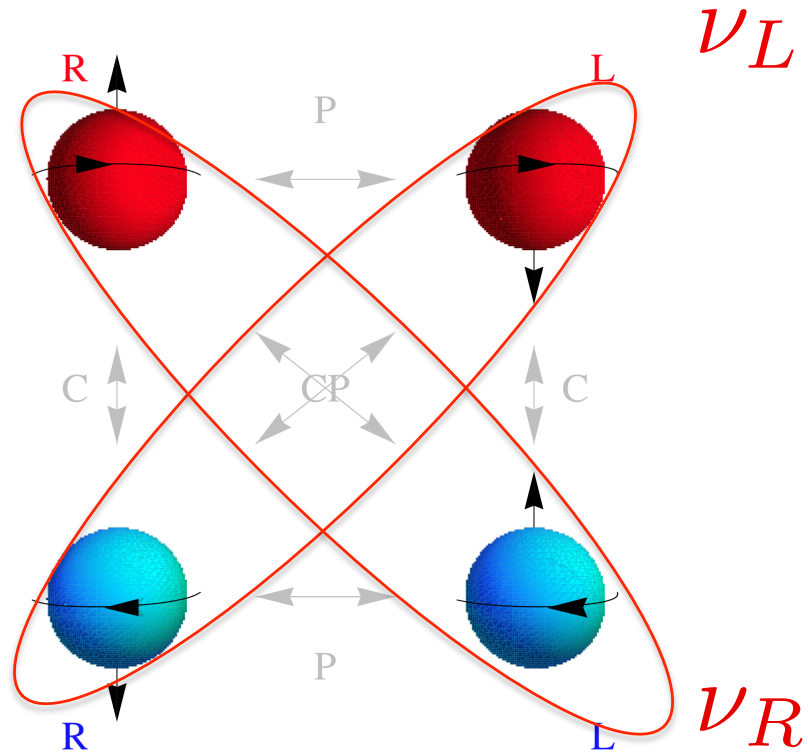
Weyl fermions

$$\nu = \frac{1 - \gamma_5}{2} \nu \simeq \frac{1}{2} \left(1 - \underbrace{\frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|}}_{\text{Helicity}} \right) \nu + \mathcal{O}(v/c)$$

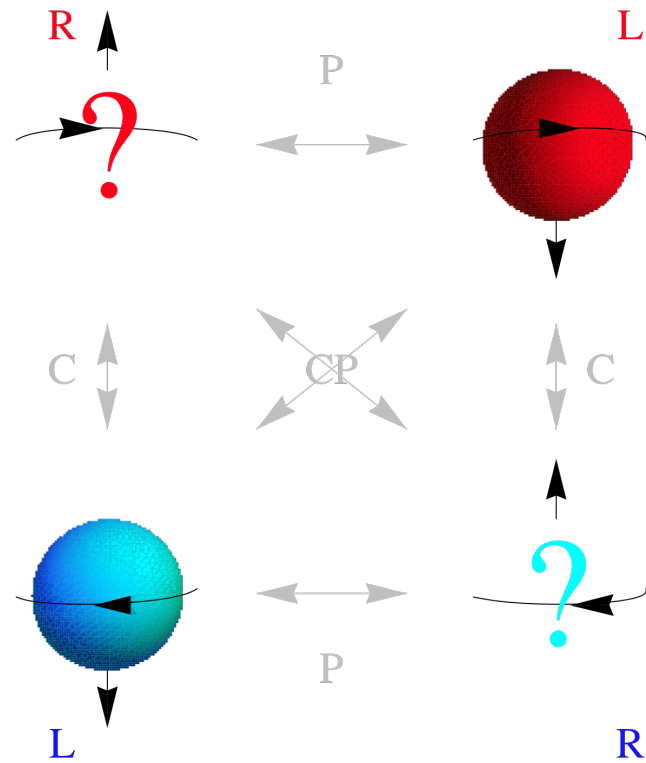
$$\Psi_{L/R} \equiv P_{L/R} \Psi$$

$$P_{L/R} \equiv \frac{1 \mp \gamma_5}{2}$$

Dirac fermion= 4-component spinor
 (Minimal spin $\frac{1}{2}$ + Parity)

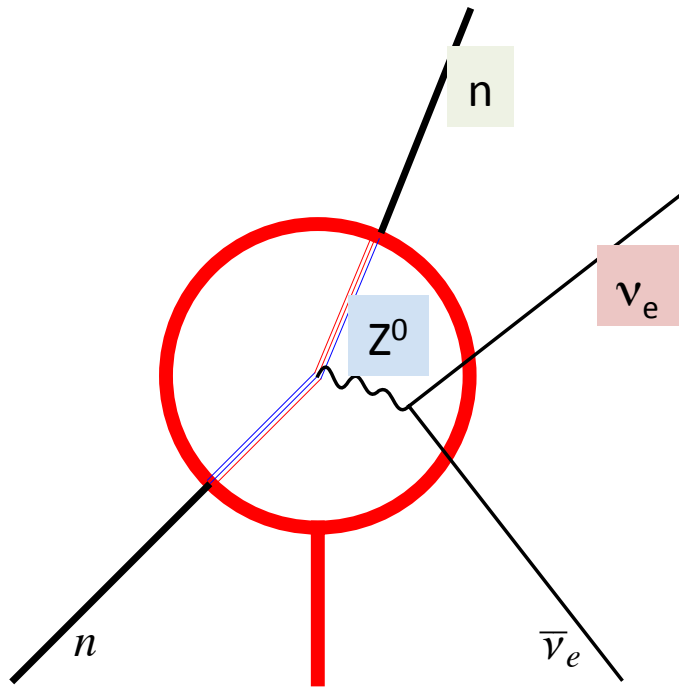


Weyl fermion= 2-component spinor
 (Minimal spin $\frac{1}{2}$)



Weyl fermion field = negative helicity particle + positive helicity anti-particle

Neutrinos in the Standard Model



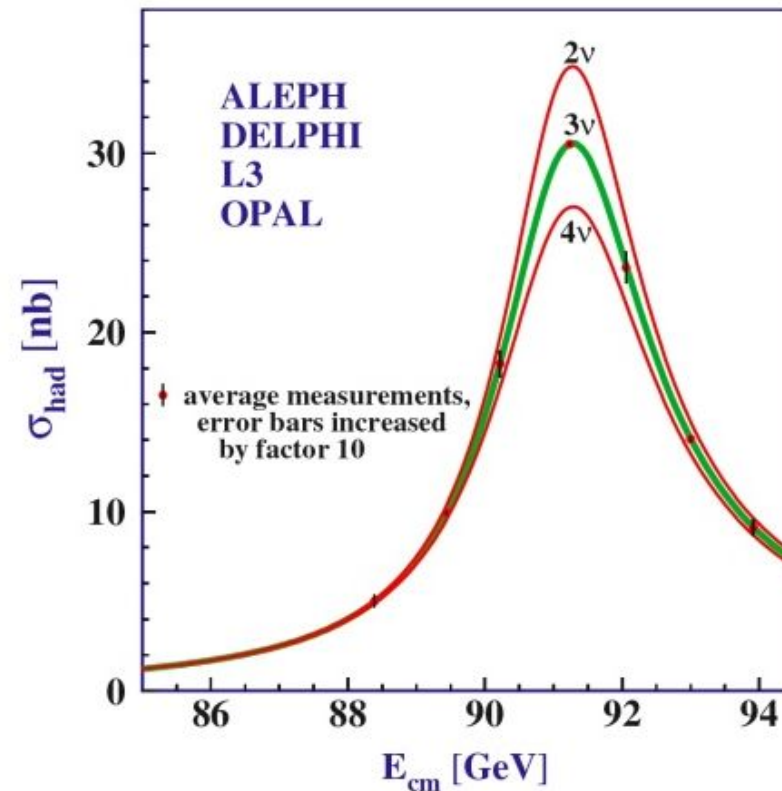
Neutral currents: NC

$$N_\nu = \frac{\Gamma_{\text{inv}}}{\Gamma_{\nu\bar{\nu}}} = 2.984 \pm 0.008$$

At LEP:

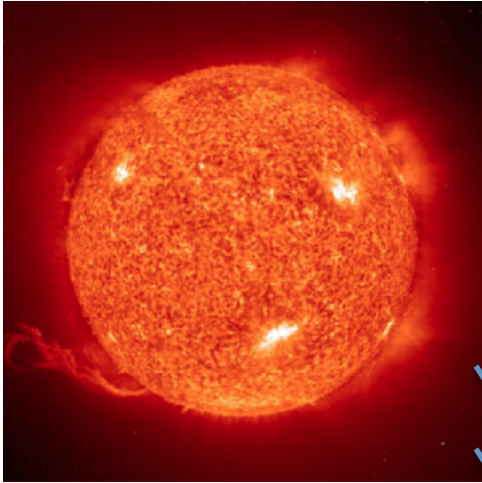
$$e^+e^- \rightarrow Z^0 \rightarrow f\bar{f}$$

Only three neutrinos were found

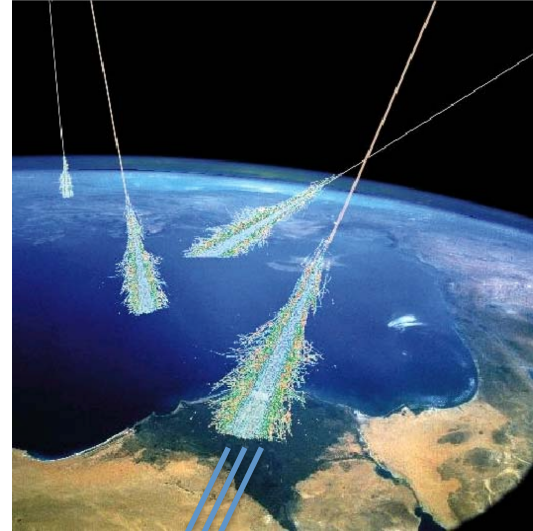


Ubiquitous Neutrinos

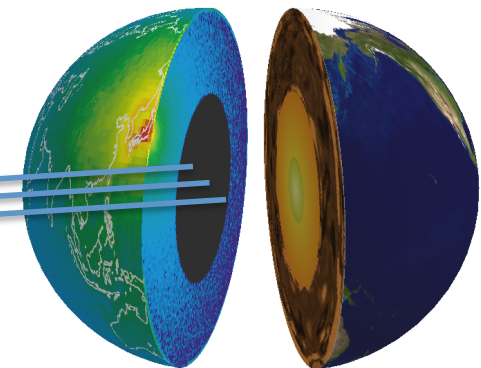
They are everywhere...



Sun: 5×10^{12} /second

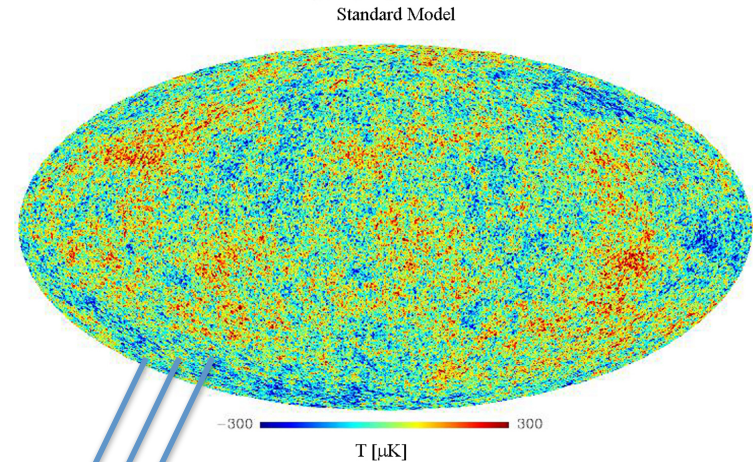


Atmosphere: ~ 20 /second



Earth: $\sim 10^9$ /second

Ubiquitous Neutrinos



Simulation showing the distribution on the sky of temperature fluctuations in the Cosmic Microwave Background with neutrinos as in the Standard Model.

Big Bang: $\sim 2 \times 10^{12}$ /segundo

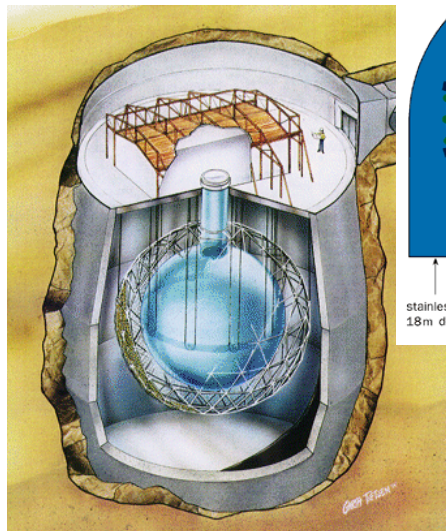
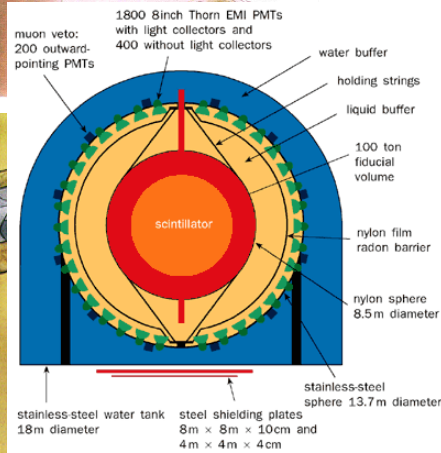
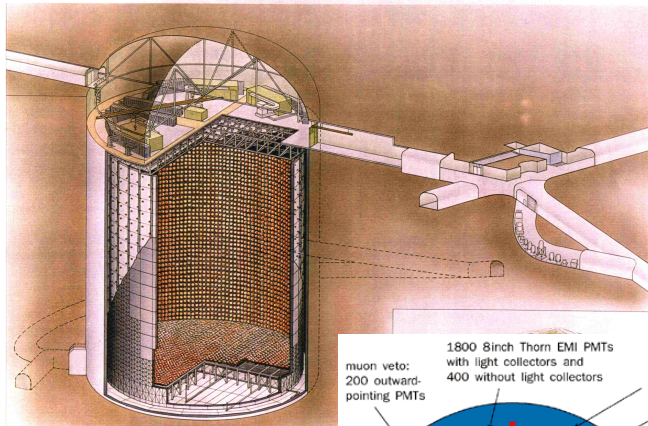
Supernova 1987: $\sim 10^{12}$ /second

@168000 Light years!
 10^8 farther from Earth



Using many of these sources, and others man-made, a decade of revolutionary neutrino experiments have demonstrated that **neutrinos are not quite standard, because they have a tiny mass & massive neutrinos require new dofs!**

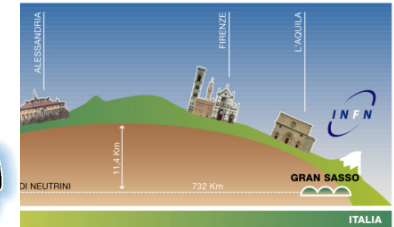
SuperKamiokande



SNO Borexino



MINOS, Opera



...and more

Massive fermions ?

Dirac fermion of mass m :

$$-\mathcal{L}_m^{\text{Dirac}} = m\bar{\psi}\psi = m(\overline{\psi_L + \psi_R})(\psi_L + \psi_R) = m(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L)$$

Majorana fermion of mass m (Weyl representation)

$$-\mathcal{L}_m^{\text{Majorana}} = \frac{m}{2}\overline{\psi^c}\psi + \frac{m}{2}\overline{\psi}\psi^c \equiv \frac{m}{2}\psi^T C\psi + \frac{m}{2}\bar{\psi}C\bar{\psi}^T,$$

$$\psi^c \equiv C\bar{\psi}^T = C\gamma_0\psi^* \quad C = i\gamma_2\gamma_0$$

✓ Non-zero for Weyl fermion: $\Psi = P_L\Psi \rightarrow \Psi^T C\Psi = \Psi_L^T i\sigma_2\Psi_L$

✓ Lorentz invariant

✓ Massive fermion: dispersion relation $E^2 - \mathbf{p}^2 = m^2$



Massive fermions & Weak Interactions ?

Dirac fermion of mass m :

$$-\mathcal{L}_m^{\text{Dirac}} = m\bar{\psi}\psi = m(\overline{\psi_L + \psi_R})(\psi_L + \psi_R) = m(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L)$$

Breaks SU(2) gauge invariance!

Majorana fermion of mass m (Weyl representation)

$$-\mathcal{L}_m^{\text{Majorana}} = \frac{m}{2}\bar{\psi}^c\psi + \frac{m}{2}\bar{\psi}\psi^c \equiv \frac{m}{2}\psi^T C\psi + \frac{m}{2}\bar{\psi}C\bar{\psi}^T,$$

No gauge/global symmetry of ψ possible!

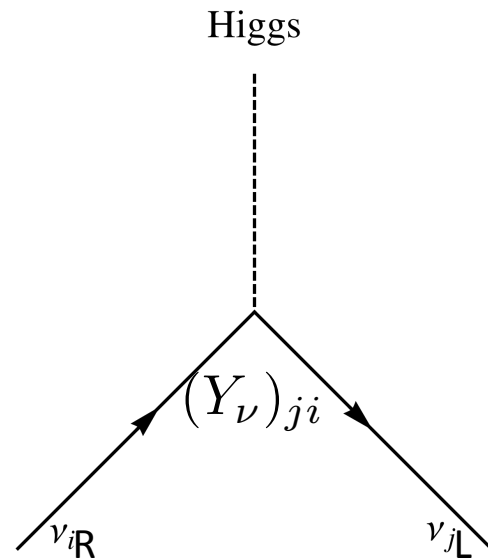
Spontaneous symmetry breaking can induce Dirac masses for all fermions but Majorana masses only for neutrinos !

Massive Dirac neutrinos & SSB ?

$$\tilde{\phi} \equiv \sigma_2 \phi^*, \quad \tilde{\phi} : (1, 2, -\frac{1}{2}), \quad \langle \tilde{\phi} \rangle = \begin{pmatrix} \frac{v}{2} \\ 0 \end{pmatrix}$$

Massive Dirac neutrino

$$-\mathcal{L}_m^{\text{Dirac}} = Y_\nu \underbrace{\bar{L}}_{(1,1,0)} \underbrace{\tilde{\phi}}_{(1,1,0)} \underbrace{\nu_R}_{(1,1,0)} + h.c. \rightarrow SSB \rightarrow Y_\nu \bar{\nu}_L \frac{v}{\sqrt{2}} \nu_R + h.c.$$



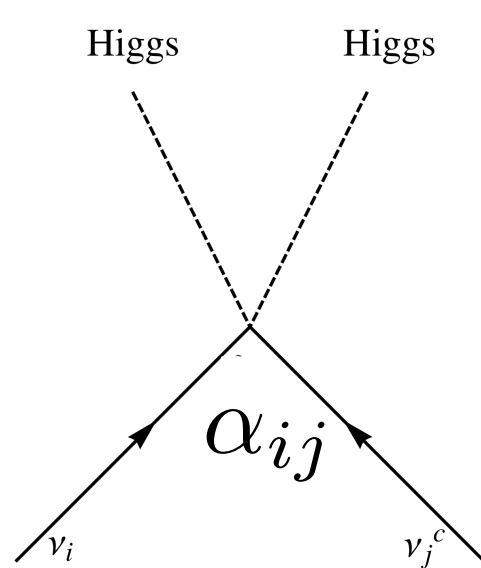
$$m_\nu = Y_\nu \frac{v}{\sqrt{2}}$$

Massive Majorana neutrinos & SSB ?

$$\tilde{\phi} \equiv \sigma_2 \phi^*, \quad \tilde{\phi} : (1, 2, -\frac{1}{2}), \quad \langle \tilde{\phi} \rangle = \begin{pmatrix} \frac{v}{2} \\ 0 \end{pmatrix}$$

Massive Majorana neutrino

$$-\mathcal{L}^{\text{Majorana}} = \alpha \bar{L} \tilde{\phi} C \tilde{\phi}^T \bar{L}^T + h.c. \rightarrow SSB \rightarrow \alpha \frac{v^2}{2} \bar{\nu}_L C \bar{\nu}_L^T + h.c.$$



$$m_\nu = \alpha \frac{v^2}{2}$$

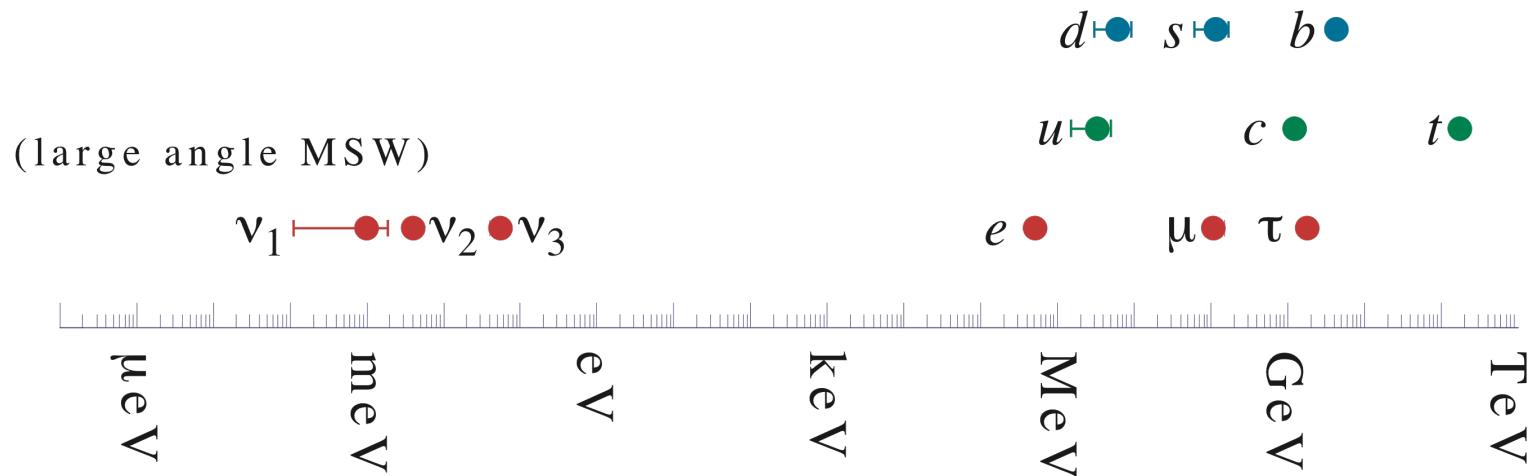
$$[\alpha] = -1$$

$$\alpha = \frac{Y_\nu}{\Lambda}$$

Implies the existence of a new physics scale unrelated to v !

Massive Majorana neutrinos & SSB ?

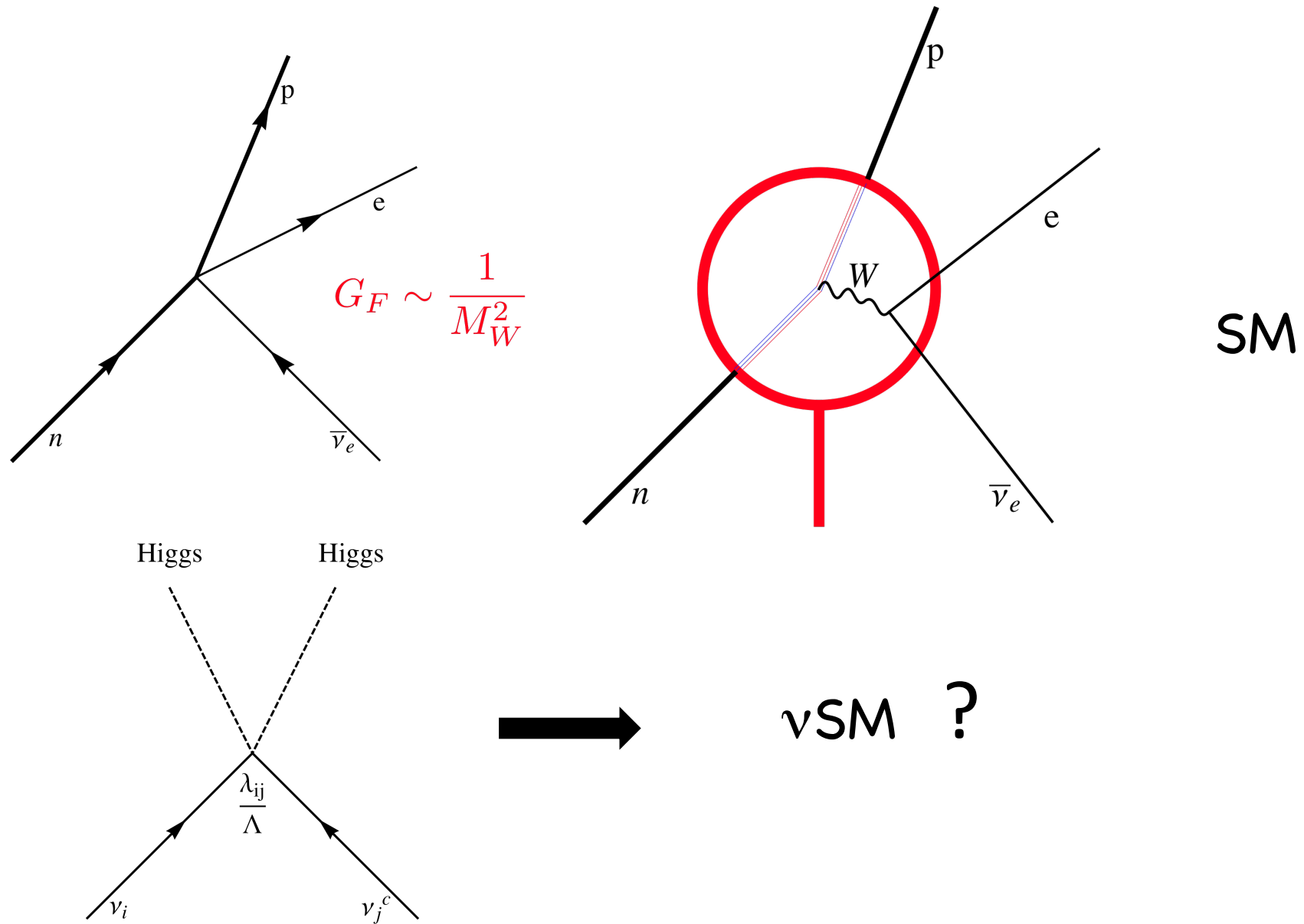
If $\Lambda \gg v$ natural explanation for the smallness of neutrino mass



$$m_f(\text{charged}) \sim Yv, \quad m_\nu \sim Y \frac{v^2}{\Lambda}$$

Lepton number is not conserved \rightarrow a new mechanism to explain the matter/antimatter asymmetry emerges

Majorana neutrinos imply a new Standard Model



Effective Theories of Neutrino Masses (model-independent)

If $\Lambda \gg v$ low-energy effects should be well described by an **effective field theory**:

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \frac{\alpha_i}{\Lambda} O_i^{d=5} + \sum_i \frac{\beta_i}{\Lambda^2} O_i^{d=6} + \dots$$

Weinberg; Buchmuller, Wyler;...

O_i^d built from SM fields satisfying the gauge symmetries

Only one with $d=5$: Weinberg's operator or neutrino masses !

$$O^{d=5} = \bar{L} \tilde{\Phi} C \tilde{\Phi}^T \bar{L}^T + h.c.$$

Generically $d=6$ operators: rich phenomenology

Neutrino masses & lepton mixing

Are generic complex matrices in flavour space

$$-\mathcal{L}_m^{lepton} = \bar{\nu}_{Li} \underbrace{(M_\nu)_{ij}}_{3 \times n_R} \nu_{Rj} + \bar{l}_{Li} \underbrace{(M_l)_{ij}}_{3 \times 3} l_{Rj} + h.c.$$

$$M_\nu = U_\nu^\dagger \text{Diag}(m_1, m_2, m_3) V_\nu, \quad M_l = U_l^\dagger \text{Diag}(m_e, m_\mu, m_\tau) V_l$$

In the mass eigenbasis

$$\mathcal{L}_{\text{gauge-lepton}} \supset -\frac{g}{\sqrt{2}} \bar{l}'_{Li} \underbrace{(U_l^\dagger U_\nu)_{ij}}_{U_{PMNS}} \gamma_\mu W_\mu^- \nu'_{Lj} + h.c.$$

Pontecorvo-Maki-Nakagawa-Sakata

$U_{PMNS}(\theta_{12}, \theta_{13}, \theta_{23}, \delta)$ unitary matrix analogous to CKM

Why only one phase ?

Counting physical parameters in lepton mixing (Dirac)

physical parameters = # parameters in Yukawas
 - # parameters in field redefinitions
 + # parameters of field redefinitions of exact symmetries

	Yukawas	Field. Red.	Symmetries	Physical
	Y_ν, Y_l	$U_L(n) \times U_{IR}(n) \times U_{\nu R}(n)$	$U(1)_L$	
Moduli	$2n^2$	$3(n^2-n)/2$	0	$n^2/2 + 3n/2$
Phases	$2n^2$	$3(n^2+n)/2$	1	$n^2/2 - 3n/2 + 1$

Moduli = $2n$ masses + $n(n-1)/2$ angles For $n=3$: 3 angles, 1 phase

Neutrino masses & lepton mixing

Are generic complex matrices in flavour space

$$-\mathcal{L}_m^{\text{lepton}} = \frac{1}{2} \bar{\nu}_{Li} (M_\nu)_{ij} \nu_{Lj}^c + \bar{l}_{Li} (M_l)_{ij} l_{Rj} + h.c.$$

$$M_\nu^T = M_\nu \rightarrow M_\nu = U_\nu^T \text{Diag}(m_1, m_2, m_3) U_\nu$$

In the mass eigenbasis

$$\mathcal{L}_{\text{gauge-lepton}} \supset -\frac{g}{\sqrt{2}} \bar{l}'_{Li} \underbrace{(U_l^\dagger U_\nu)_{ij}}_{U_{PMNS}} \gamma_\mu W_\mu^- \nu'_{Lj} + h.c.$$

$U_{PMNS}(\theta_{12}, \theta_{13}, \theta_{23}, \delta, \alpha_1, \alpha_2)$ depends on three phases

Counting physical parameters in lepton mixing (Majorana)

physical parameters = # parameters in Yukawas
 - # parameters in field redefinitions
 + # parameters of field redefinitions that are exact symmetries

	Yukawas	Field. Red.	Symmetries	Physical
	α_ν, Y_l	$U_L(n) \times U_{IR}(n)$	0	
Moduli	$n(n+1)/2 + n^2$	$n^2 - n$	0	$n^2/2 + 3n/2$
Phases	$n(n+1)/2 + n^2$	$n^2 + n$	0	$n(n-1)/2$

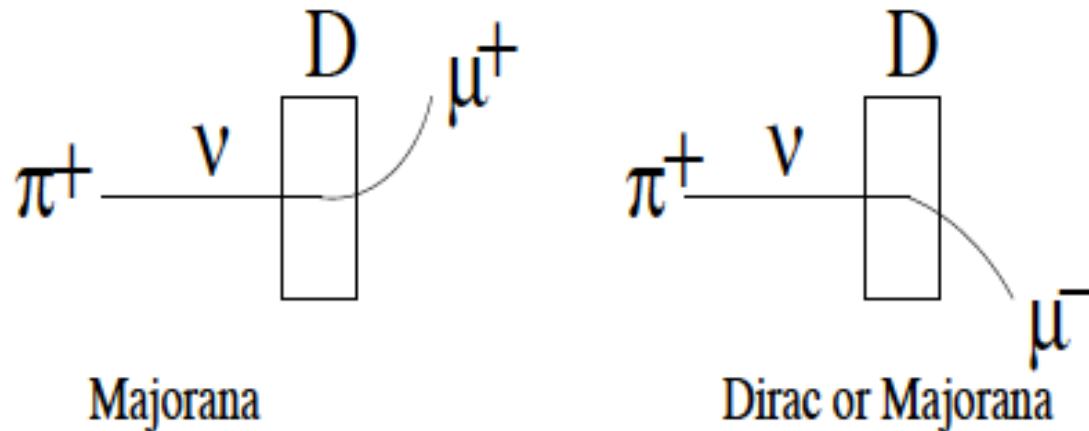
Moduli = $2n$ masses + $n(n-1)/2$ angles For $n=3$: 3 angles, 3 phases

Majorana versus Dirac

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix}}_{\text{Majorana phases}}$$

$c_{ij} \equiv \cos \theta_{ij} \quad s_{ij} \equiv \sin \theta_{ij}$

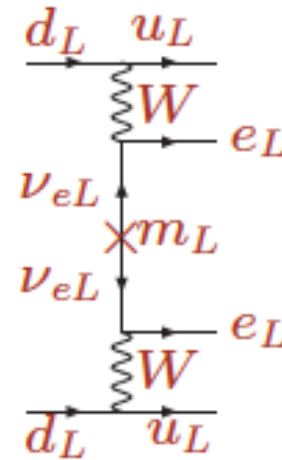
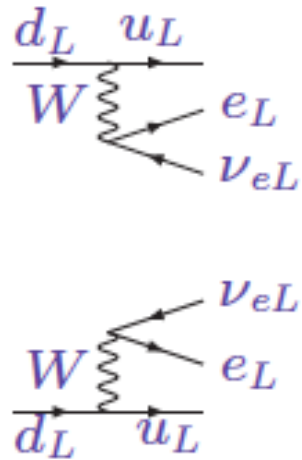
In principle clear experimental signatures



In practice these processes extremely rare are suppressed by $\left(\frac{m_\nu}{E}\right)^2$

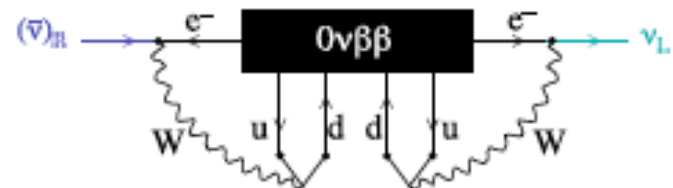
Neutrinoless double- β decay

Best hope is neutrinoless double- β decay



$$T_{2\beta 2\nu} \sim 10^{18} - 10^{21} \text{ years} \quad T_{2\beta 0\nu}^{-1} \sim \left(\frac{m_\nu}{E}\right)^2 10^9 T_{2\beta 2\nu}^{-1}$$

If neutrinos are Majorana this process must be there at some level and if this process is there neutrinos are Majorana



Neutrinoless double- β decay

$$T_{2\beta 0\nu}^{-1} \simeq \underbrace{G^{0\nu}}_{\text{Phase}} \underbrace{|M^{0\nu}|^2}_{\text{Nuclear M.E.}} \underbrace{\left| \sum_i (V_{MNS}^{ei})^2 m_i \right|^2}_{|m_{ee}|^2}$$

Present bounds:

Sarazin 2012

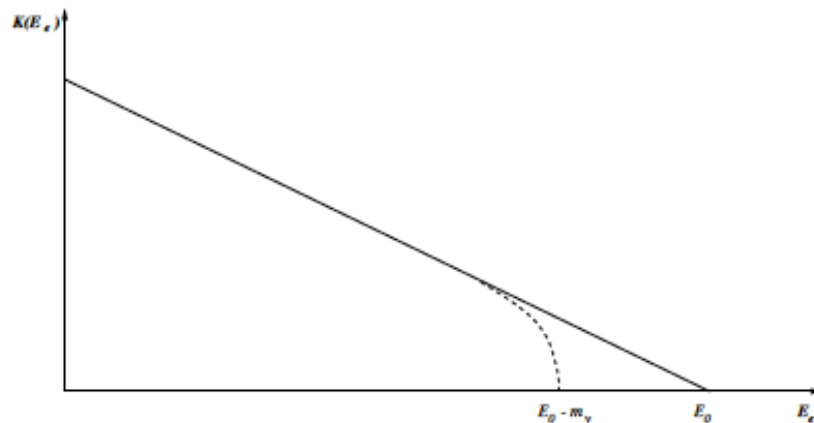
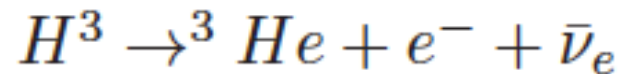
Isotope	$T_{1/2}^{2\nu}$ (yr)	Experiment	$T_{1/2}^{0\nu}$ (yr) (90% C.L.)	Experiment	$\langle m_{ee} \rangle$ (eV) Min. Max.
^{48}Ca	$4.2^{+2.1}_{-1.0} 10^{19}$	NEMO-3	$5.8 10^{22}$	CANDLES [111]	3.55 9.91
^{76}Ge	$1.5 \pm 0.1 10^{21}$	HDM	$1.9 10^{25}$	HDM [46]	0.21 0.53
^{82}Se	$9.0 \pm 0.7 10^{19}$	NEMO-3	$3.2 10^{23}$	NEMO-3 [40]	0.85 2.08
^{96}Zr	$2.0 \pm 0.3 10^{19}$	NEMO-3	$9.2 10^{21}$	NEMO-3 [35]	3.97 14.39
^{100}Mo	$7.1 \pm 0.4 10^{18}$	NEMO-3	$1.0 10^{24}$	NEMO-3 [40]	0.31 0.79
^{116}Cd	$3.0 \pm 0.2 10^{19}$	NEMO-3	$1.7 10^{23}$	SOLOTVINO [81]	1.22 2.30
^{130}Te	$0.7 \pm 0.1 10^{21}$	NEMO-3	$2.8 10^{24}$	CUORICINO [65]	0.27 0.57
^{136}Xe	$2.38 \pm 0.14 10^{21}$	Kamland	$5.7 10^{24}$	Kamland-Zen [93]	
^{150}Nd	$7.8 \pm 0.7 10^{18}$	NEMO-3	$1.8 10^{22}$	NEMO-3 [37]	2.35 8.65

^{136}Xe

EXO-Kamland 0.12 0.25

Kinematical effects of neutrino mass

Most stringent from Tritium beta-decay



$$m_{\nu_e} < 2.2\text{eV (Mainz-Troitsk)}$$

$$m_{\nu_\mu} < 170\text{keV (PSI: } \pi^+ \rightarrow \mu^+ \nu_\mu)$$

$$m_{\nu_\tau} < 18.2\text{MeV (LEP: } \tau^- \rightarrow 5\pi \nu_\tau)$$

Standard Model neutrinos assumed massless

Neutrino oscillations

1968 Pontecorvo

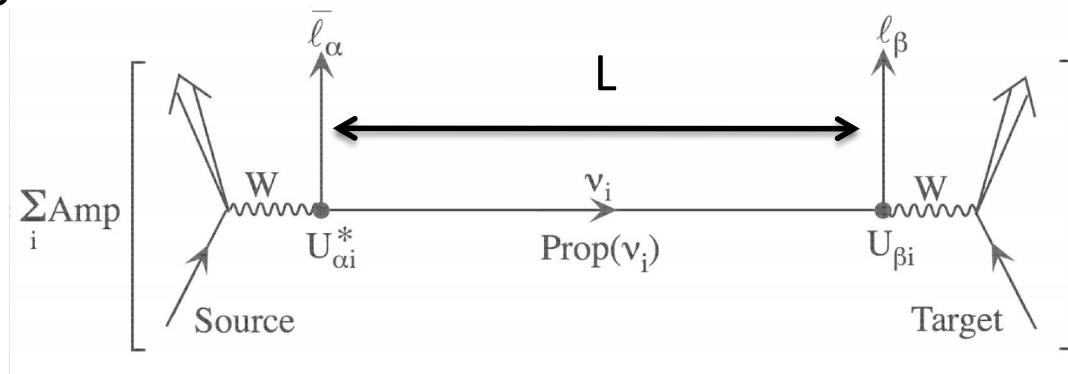
If neutrinos are massive

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS}(\theta_{12}, \theta_{23}, \theta_{13}, \delta, \dots) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



Бруно Понтекорво

A neutrino experiment is an interferometer in flavour space, because neutrinos are so weakly interacting that can keep coherence over very long distances !



ν_i travel at different velocities in vacuum: neutrino oscillations

Neutrino oscillations

Many ways to derive the oscillation probability master formula

Quantum mechanics with **neutrinos as plane waves**

Quantum mechanics with **neutrinos as wave packets**

Quantum Field Theory \leftrightarrow **neutrinos as intermediate states**

The basic ingredients:

- ✓ Uncertainty in momentum at production & detection (they must be better localized than baseline)
- ✓ Coherence of mass eigenstates over macroscopic distances

Neutrino oscillations in QM (plane waves)

$$|\nu_\alpha(t_0)\rangle = \sum_i U_{\alpha i}^* |\nu_i(\mathbf{p})\rangle, \quad \hat{H}|\nu_i(\mathbf{p})\rangle = E_i(\mathbf{p})|\nu_i(\mathbf{p})\rangle, \quad \mathbf{p}^2 + m_i^2 = E_i^2(\mathbf{p})$$

↓ time evolution

$$|\nu_\alpha(t)\rangle = \sum_i U_{\alpha i}^* e^{-iE_i(\mathbf{p})(t-t_0)} |\nu_i(\mathbf{p})\rangle$$

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta)(t) &= |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = \left| \sum_i U_{\beta i} U_{\alpha i}^* e^{-iE_i(t-t_0)} \right|^2 \\ &= \sum_{i,j} e^{-i(E_i - E_j)(t-t_0)} U_{\beta i} U_{\alpha i}^* U_{\beta j}^* U_{\alpha j} \end{aligned}$$

$$E_i(\mathbf{p}) - E_j(\mathbf{p}) \simeq \frac{1}{2} \frac{m_i^2 - m_j^2}{|\mathbf{p}|} + \mathcal{O}(m^4) \quad L \simeq t - t_0, \quad v_i \simeq c$$

$$P(\nu_\alpha \rightarrow \nu_\beta)(L) \simeq \sum_{i,j} e^{i \frac{\Delta m_{ji}^2 L}{2E}} U_{\beta i} U_{\alpha i}^* U_{\beta j}^* U_{\alpha j}$$

Neutrino oscillations in QM (plane waves)

Well founded criticism to this derivation

Why same p for the i -th states ?

Why plane waves if the neutrino source is localized ?

Why $\tau \leftrightarrow L$ conversion ?

Neutrino oscillations in QM (wavepackets)

Wave packet created at source @ $(t_0, \mathbf{x}_0) = (0, \mathbf{0})$

$$|\nu_\alpha(t, \mathbf{x})\rangle = \sum_i U_{\alpha i}^* \int_{\mathbf{p}} \underbrace{f_i^S(\mathbf{p} - \mathbf{Q}_i)}_{\text{Wave packet at source}} e^{-iE_i(\mathbf{p})t} e^{i\mathbf{p}\cdot\mathbf{x}} |\nu_i\rangle$$

$$E_i(\mathbf{p}) \equiv \sqrt{\mathbf{p}^2 + m_i^2}$$

For example: $f_i^S(\mathbf{p} - \mathbf{Q}_i) \simeq e^{-(\mathbf{p} - \mathbf{Q}_i)^2 / 2\sigma_S^2}$

$\sigma_S \leftrightarrow$ momentum uncertainty

$\mathbf{Q}_i \leftrightarrow$ average momentum of i -th wavepacket

Wave packet created at detector @ $(t_0, \mathbf{x}_0) = (t, \mathbf{L})$

$$|\nu_\beta(t, \mathbf{x})\rangle = \sum_j U_{\beta j}^* \int_{\mathbf{p}} f_j^D(\mathbf{p} - \mathbf{Q}'_j) e^{-iE_j(\mathbf{p})(t-T)} e^{i\mathbf{p}(\mathbf{x}-\mathbf{L})} |\nu_j\rangle$$

Neutrino oscillations in QM (wavepackets)

$$\begin{aligned}
 \mathcal{A}(\nu_\alpha \rightarrow \nu_\beta) &= \int_{\mathbf{x}} \langle \nu_\beta(t, \mathbf{x}) | \nu_\alpha(t, \mathbf{x}) \rangle \\
 &= \sum_i U_{\alpha i}^* U_{\beta i} \int_{\mathbf{p}} e^{iE_i(\mathbf{p})T} e^{-i\mathbf{p}\mathbf{L}} \underbrace{f_i^{D*}(\mathbf{p} - \mathbf{Q}'_i) f_i^S(\mathbf{p} - \mathbf{Q}_i)}_{\text{overlap}}
 \end{aligned}$$

For Gaussian wave packets overlap is also gaussian:

$$\begin{aligned}
 f_i^{D*} f_i^S &= f_i^{ov}(\mathbf{p} - \langle \mathbf{Q} \rangle_i) e^{-(\mathbf{Q}_i - \mathbf{Q}'_i)^2 / 4 / (\sigma_S^2 + \sigma_D^2)} \\
 \langle \mathbf{Q} \rangle_i &\equiv \left(\frac{\mathbf{Q}_i}{\sigma_S^2} + \frac{\mathbf{Q}'_i}{\sigma_D^2} \right) \sigma_{ov}^2 \\
 \sigma_{ov}^2 &\equiv \frac{1}{1/\sigma_S^2 + 1/\sigma_D^2}
 \end{aligned}$$

$$E_i(\mathbf{p}) \simeq E_i(\langle \mathbf{Q} \rangle_i) + \overbrace{\frac{\mathbf{V}_i}{\partial p_k}}_{\langle \mathbf{Q} \rangle_i} (p_k - \langle Q_k \rangle_i) + \mathcal{O}(p_k - \langle Q_k \rangle_i)^2$$

$$\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta) \propto \sum_i U_{\alpha i}^* U_{\beta i} e^{iE_i(\langle \mathbf{Q} \rangle_i)T} e^{-i\langle \mathbf{Q} \rangle_i \mathbf{L}} e^{-(\mathbf{Q}_i - \mathbf{Q}'_i)^2 / 4 / (\sigma_S^2 + \sigma_D^2)} e^{-(\mathbf{L} - \mathbf{v}_i T)^2 \sigma_{ov}^2 / 2}$$

Neutrino oscillations in QM (wavepackets)

$$\langle \mathbf{Q} \rangle_i \simeq \langle \mathbf{Q}' \rangle_i, \quad \mathbf{L} \parallel \langle \mathbf{Q} \rangle_i$$

$$P(\nu_\alpha \rightarrow \nu_\beta) \propto \int_{-\infty}^{\infty} dT |\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta)|^2$$

$$\propto \sum_{i,j} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* e^{i \frac{m_j^2 - m_i^2}{2E} L} \times e^{-L^2 / L_{coh}(i,j)^2} \times e^{-\left(\frac{\Delta_{ij} E \langle \mathbf{Q} \rangle}{2\sigma_{ov} \langle v \rangle}\right)^2},$$

$$L_{coh}^{-1}(i,j) \sim \sigma_{ov} \frac{|\mathbf{v}_i - \mathbf{v}_j|}{\sqrt{\mathbf{v}_i^2 + \mathbf{v}_j^2}} \simeq \frac{|m_j^2 - m_i^2| \sigma_{ov}}{2\langle Q \rangle \langle Q \rangle}$$

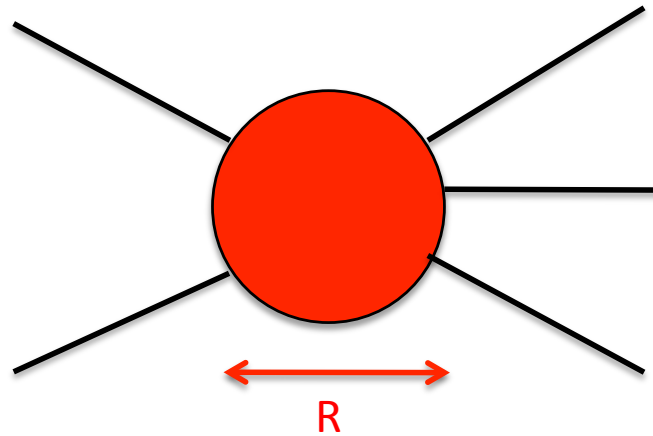
L L_{coh} coherence is lost

There must be sufficient uncertainty in production & detection so that wave packets include all mass eigenstates: $\Delta E \ll \sigma$

Problems: normalization is arbitrary, needs to be imposed a posteriori

$$\sum_{\beta} P(\nu_\alpha \rightarrow \nu_\beta) = 1$$

Neutrino oscillations in QFT



in-states

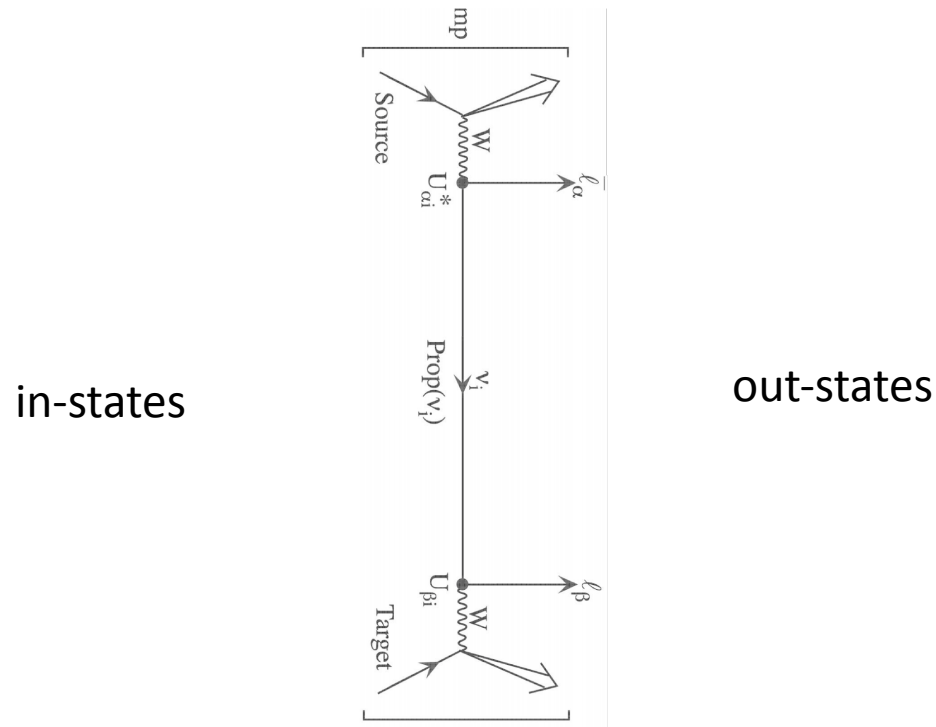
out-states

Idealization: asymptotic states are plane waves $R \ll$ Compton wavelength,
in reality in-states are wave packets

$$\mathcal{A} = \langle \text{out}; p'_1, \dots, p'_n | \text{in}; p_1, p_2 \rangle$$

Neutrino oscillations in QFT

Neutrinos are not the asymptotic states...



$$\mathcal{A} \sim \sum_i \mathcal{A}_S U_{\beta i}^* \frac{i}{\not{p} - m_i} U_{i\alpha} \mathcal{A}_D$$

Neutrino propagator: intermediate state

Neutrino oscillations in QFT

Example: neutrino beam from π decay at rest

in-states: pion + nucleus (or nucleon) in detector

out-states: μ from π decay + l + hadron jet from ν interaction

Necessary to adapt standard formalism:

1) macroscopic separation of Source and Detector L (eg. localized wave packets of in-states + static approximation)

2) oscillation probability from factorization:

decay \times propagation \times ν cross-section

$$\frac{dW(\pi n \rightarrow p\mu l_\beta)}{dtdp_\mu dp_p dp_l} = \int d|q| \underbrace{\frac{dW(\pi \rightarrow \mu\nu)}{L^2 dt d\Omega_\nu d|q| dp_\mu}}_{\text{Flux per unit neutrino momentum}} \times P(\nu_\mu \rightarrow \nu_\beta) \times \underbrace{\frac{1}{2|q|} \frac{dW(\nu n \rightarrow pl)}{dtdp_p dp_l}}_{\text{interaction probability per unit flux}}$$

Oscillation probability is indeed properly normalized!

Neutrino Oscillation

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sum_{ij} U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} e^{-i \frac{(m_i^2 - m_j^2)L}{2E}}$$

$\alpha \neq \beta$ appearance probability

$\alpha = \beta$ disappearance or survival probability

$$L_{osc} \sim \frac{E}{m_i^2 - m_j^2}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \underbrace{2 \sum_{i < j} \text{Re}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*] + \sum_{i=j} |U_{\alpha i}|^2 |U_{\beta i}|^2}_{\delta_{\alpha\beta}}$$

CP-even

$$- 4 \sum_{i < j} \text{Re}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*] \sin^2 \left[\frac{\Delta m_{ji}^2 L}{4E} \right]$$

CP-odd

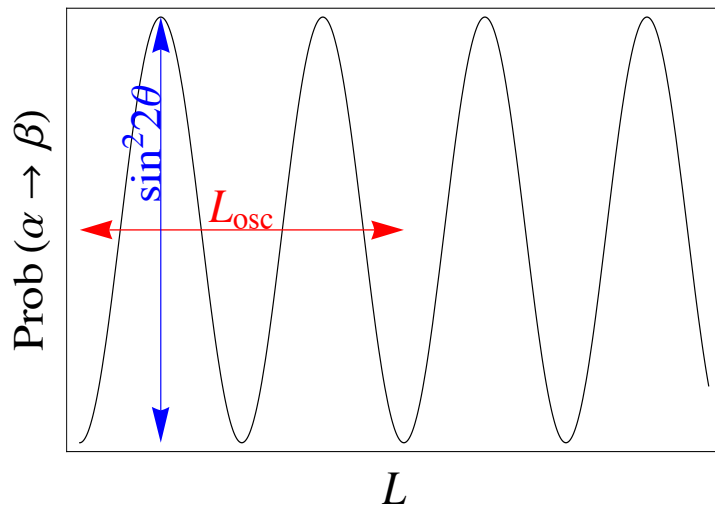
$$- 2 \sum_{i < j} \text{Im}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*] \sin \left[\frac{\Delta m_{ji}^2 L}{2E} \right]$$

Neutrino Oscillation: 2ν

Only one oscillation frequency,

$$U = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \sin^2 \left(1.27 \frac{\Delta m^2 (eV^2) L (km)}{E (GeV)} \right)$$



$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - P(\nu_\alpha \rightarrow \nu_\beta)$$

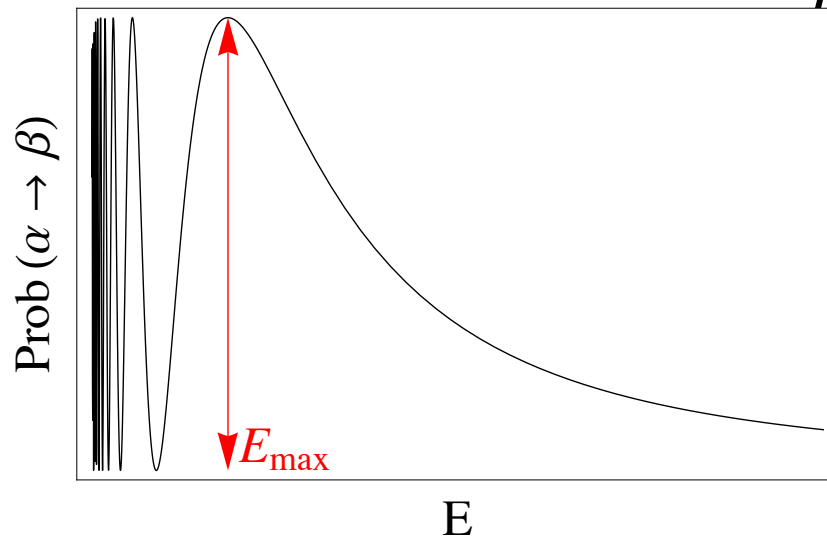
$$L_{osc} (km) = \frac{\pi}{1.27} \frac{E (GeV)}{\Delta m^2 (eV^2)}$$

Neutrino Oscillation: 2ν

Only one oscillation frequency, $U = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \sin^2 \left(1.27 \frac{\Delta m^2 (eV^2) L (km)}{E (GeV)} \right)$$

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - P(\nu_\alpha \rightarrow \nu_\beta)$$



$$E_{max} (GeV) = 1.27 \frac{\Delta m^2 (eV^2) L (km)}{\pi/2}$$

L, E dependence give Δm^2 amplitude of oscillation gives θ

Optimal experiment: $\frac{E}{L} \sim \Delta m^2$

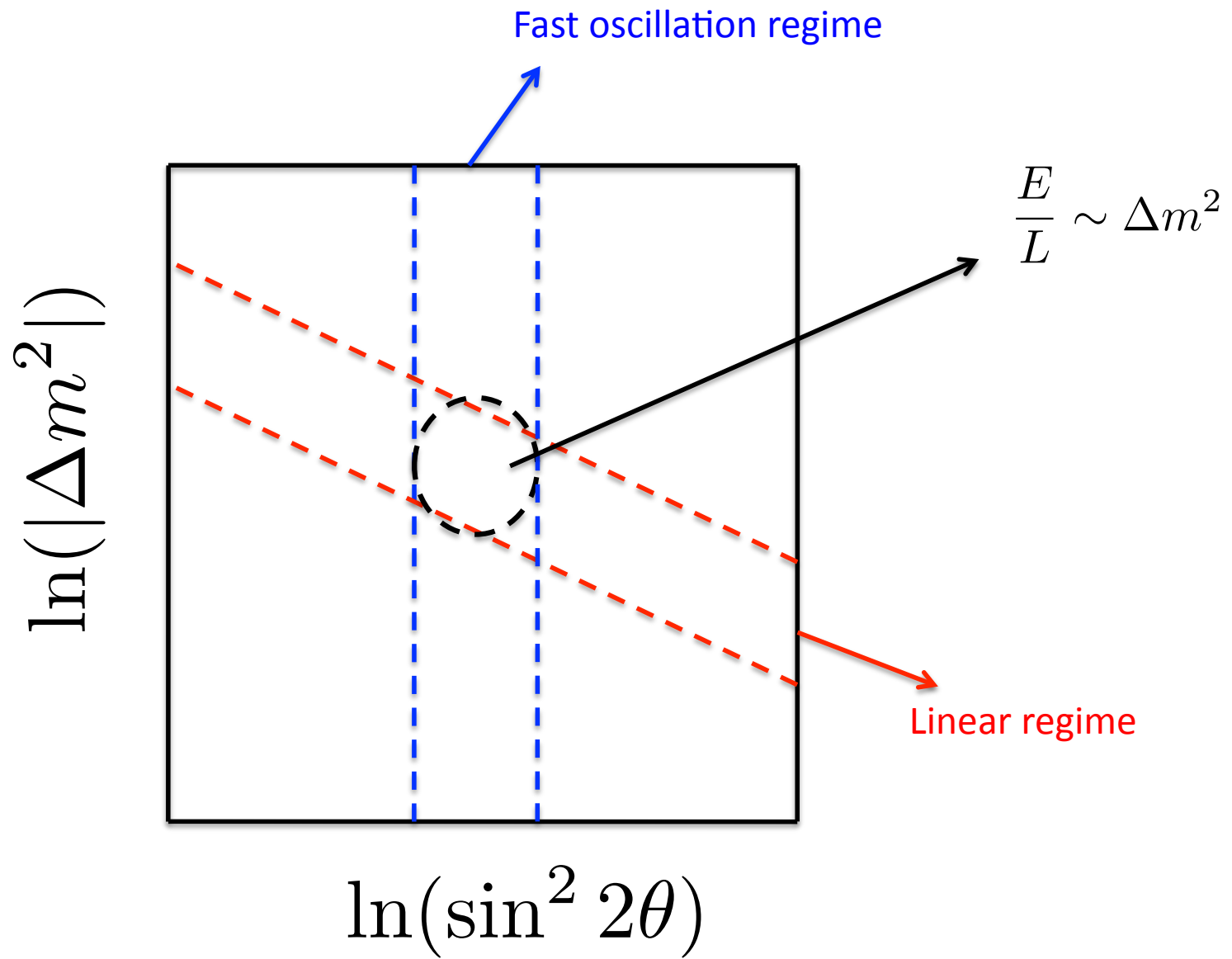
$\frac{E}{L} \gg \Delta m^2$ Oscillation suppressed

$$P(\nu_\alpha \rightarrow \nu_\beta) \propto \sin^2 2\theta (\Delta m^2)^2$$

$\frac{E}{L} \ll \Delta m^2$ Fast oscillation regime

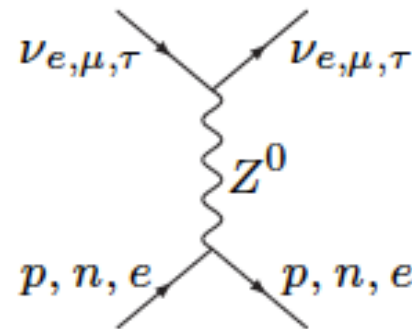
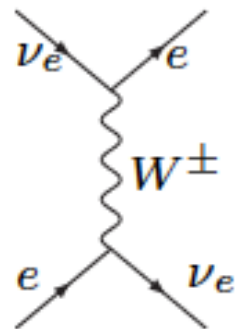
$$P(\nu_\alpha \rightarrow \nu_\beta) \simeq \sin^2 2\theta \left\langle \sin^2 \frac{\Delta m^2 L}{4E} \right\rangle \simeq \frac{1}{2} \sin^2 2\theta = |U_{\alpha 1}^* U_{\beta 1}|^2 + |U_{\alpha 2}^* U_{\beta 2}|^2$$

Equivalent to incoherent propagation: sensitivity to mass splitting is lost



Neutrino Oscillations in matter

Many neutrino oscillation experiments often involve neutrinos propagating in matter (Earth for atmospheric neutrinos or accelerator experiments, Sun for solar neutrinos, Supernova)



Wolfenstein

Coherent forward scattering can strongly affect the oscillation probability

$$\mathcal{H}_{CC} = \frac{G_F}{\sqrt{2}} [\bar{e}\gamma_\mu(1 - \gamma_5)\nu_e][\bar{\nu}_e\gamma^\mu(1 - \gamma_5)e] = \frac{G_F}{\sqrt{2}} [\bar{e}\gamma_\mu(1 - \gamma_5)e][\bar{\nu}_e\gamma^\mu(1 - \gamma_5)\nu_e]$$

$$\langle \bar{e}\gamma_\mu P_L e \rangle_{\text{unpol. medium}} = \delta_{\mu 0} \frac{N_e}{2}$$

Neutrino Oscillations in matter

$$\langle \mathcal{H}_{CC} + \mathcal{H}_{NC} \rangle_{\text{medium}} = \sqrt{2} G_F \bar{\nu} \gamma_0 \begin{pmatrix} N_e - \frac{N_n}{2} & & \\ & -\frac{N_n}{2} & \\ & & -\frac{N_n}{2} \end{pmatrix} \nu \equiv \bar{\nu} \gamma_0 V_m \nu$$

$$\mathcal{L} \simeq \bar{\nu} (i\partial - M_\nu - \gamma_0 V_m) \nu + \dots$$

$$\mathcal{O}(V_m^2, M_\nu^2 V_m) \quad E^2 - \mathbf{p}^2 = \pm 2 V_m E + M_\nu^2$$

$$\text{Earth:} \quad V_m \simeq 10^{-13} eV \rightarrow 2V_m E \simeq 10^{-4} eV^2 \left[\frac{E}{1\text{GeV}} \right]$$

$$\text{Sun:} \quad V_m \simeq 10^{-12} eV \rightarrow 2V_m E \simeq 10^{-6} eV^2 \left[\frac{E}{1\text{MeV}} \right]$$

Neutrino Oscillations in matter

In constant matter density: effective mixing angles and masses depend on energy

$$\begin{pmatrix} \tilde{m}_1^2 & 0 & 0 \\ 0 & \tilde{m}_2^2 & 0 \\ 0 & 0 & \tilde{m}_3^2 \end{pmatrix} = \tilde{U}_{\text{PMNS}}^\dagger \left(M_\nu^2 \pm 2E \begin{pmatrix} V_e & 0 & 0 \\ 0 & V_\mu & 0 \\ 0 & 0 & V_\tau \end{pmatrix} \right) \tilde{U}_{\text{PMNS}}$$

For two families (- neutrinos, + antineutrinos):

$$\sin^2 2\tilde{\theta} = \frac{(\Delta m^2 \sin 2\theta)^2}{(\Delta m^2 \cos 2\theta \pm 2\sqrt{2} G_F E N_e)^2 + (\Delta m^2 \sin 2\theta)^2}$$
$$\Delta\tilde{m}^2 = \sqrt{(\Delta m^2 \cos 2\theta \pm 2\sqrt{2} E G_F N_e)^2 + (\Delta m^2 \sin 2\theta)^2}$$

Resonance: $\Delta m^2 \cos 2\theta \pm 2\sqrt{2} G_F E N_e = 0$

$$\sin^2 2\tilde{\theta} = 1, \quad \Delta\tilde{m}^2 = \Delta m^2 \sin 2\theta$$

- Maximal mixing any $\theta \neq 0$
- Only for ν or $\bar{\nu}$, not both
- Only for one sign of Δm^2

MSW effect
Mikheyev, Smirnov '85

Neutrinos in variable matter

Solar neutrinos propagate in variable matter (the electron density varies exponentially as a function of radius)

$$N_e(r) \propto N_e(0)e^{-r/R}$$

If the variation is slow enough: **adiabatic approximation** (if a state is at $r=0$ in an eigenstate $\tilde{m}_i^2(0)$ it remains in the i -th eigenstate until it exits the sun)

$$P(\nu_e \rightarrow \nu_e) = \sum_i |\langle \nu_e | \tilde{\nu}_i(\infty) \rangle|^2 |\langle \tilde{\nu}_i(0) | \nu_e \rangle|^2$$

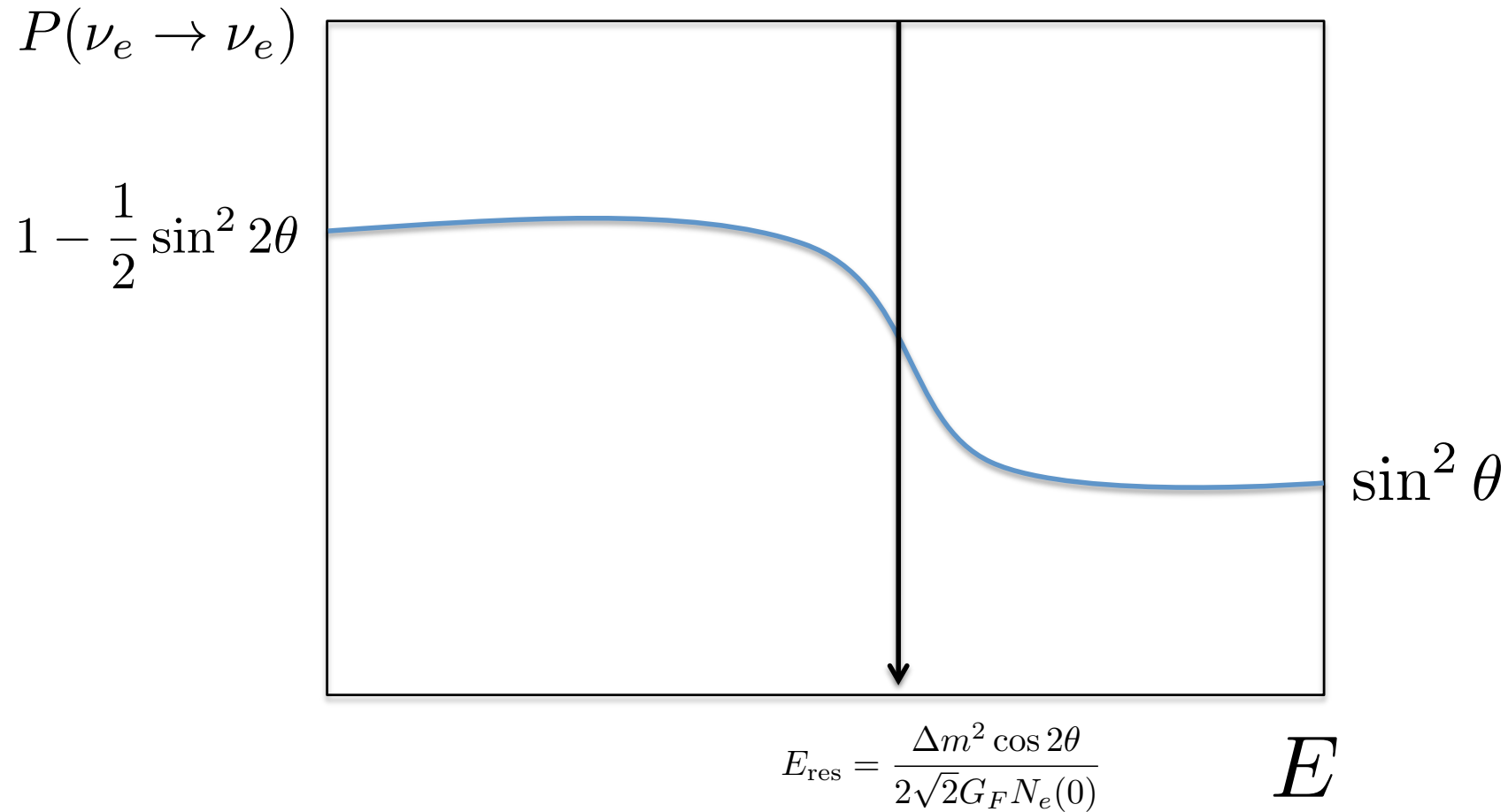
$$2\sqrt{2}G_F EN_e(0) \ll \Delta m^2 \cos 2\theta \rightarrow \tilde{\theta}(0) \simeq \theta$$

$$P(\nu_e \rightarrow \nu_e) \simeq 1 - \frac{1}{2} \sin^2 2\theta$$

$$2\sqrt{2}G_F EN_e(0) \gg \Delta m^2 \cos 2\theta \rightarrow \tilde{\theta}(0) \simeq \frac{\pi}{2}$$

$$P(\nu_e \rightarrow \nu_e) \simeq \sin^2 \theta$$

Solar neutrinos

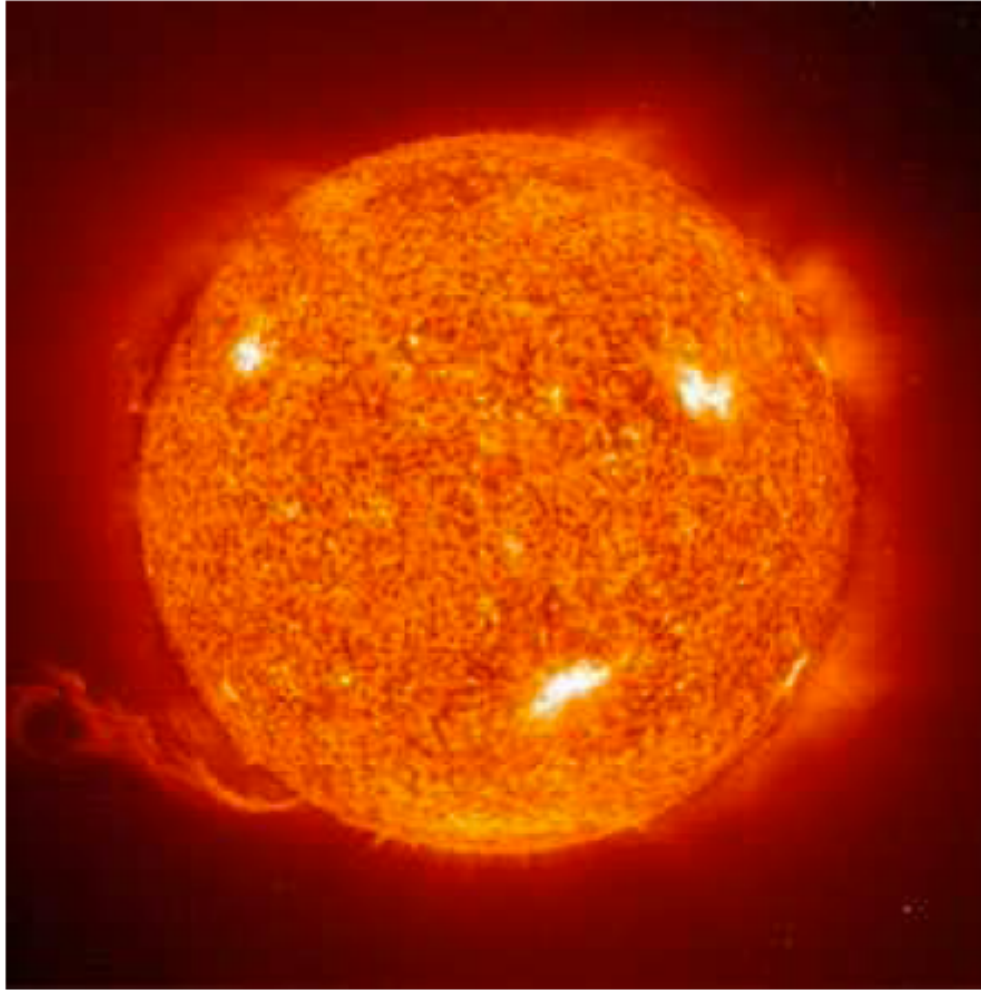


MSW resonance energy

Lecture III:

- Evidence for neutrino mass: experimental landscape
- The standard 3ν scenario
- A few outliers...

SOLAR Neutrinos



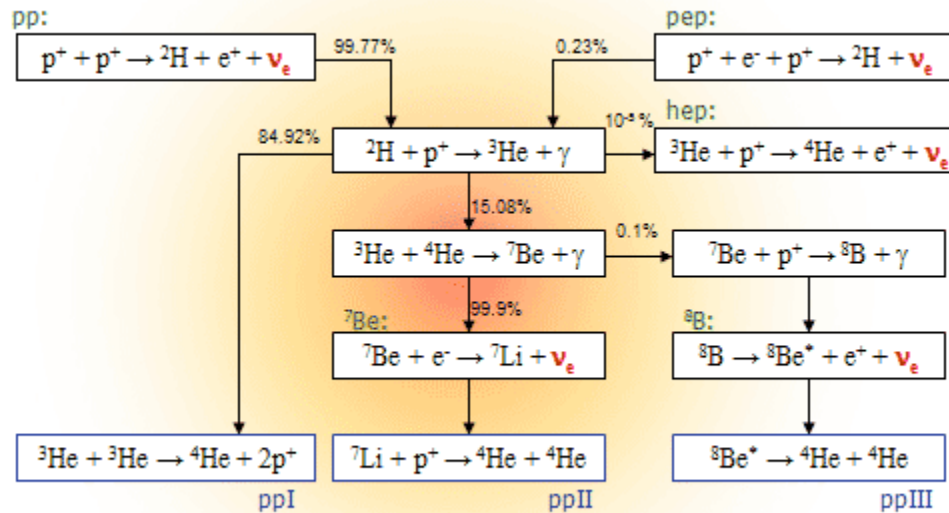
Stars shine neutrinos

1939 Bethe

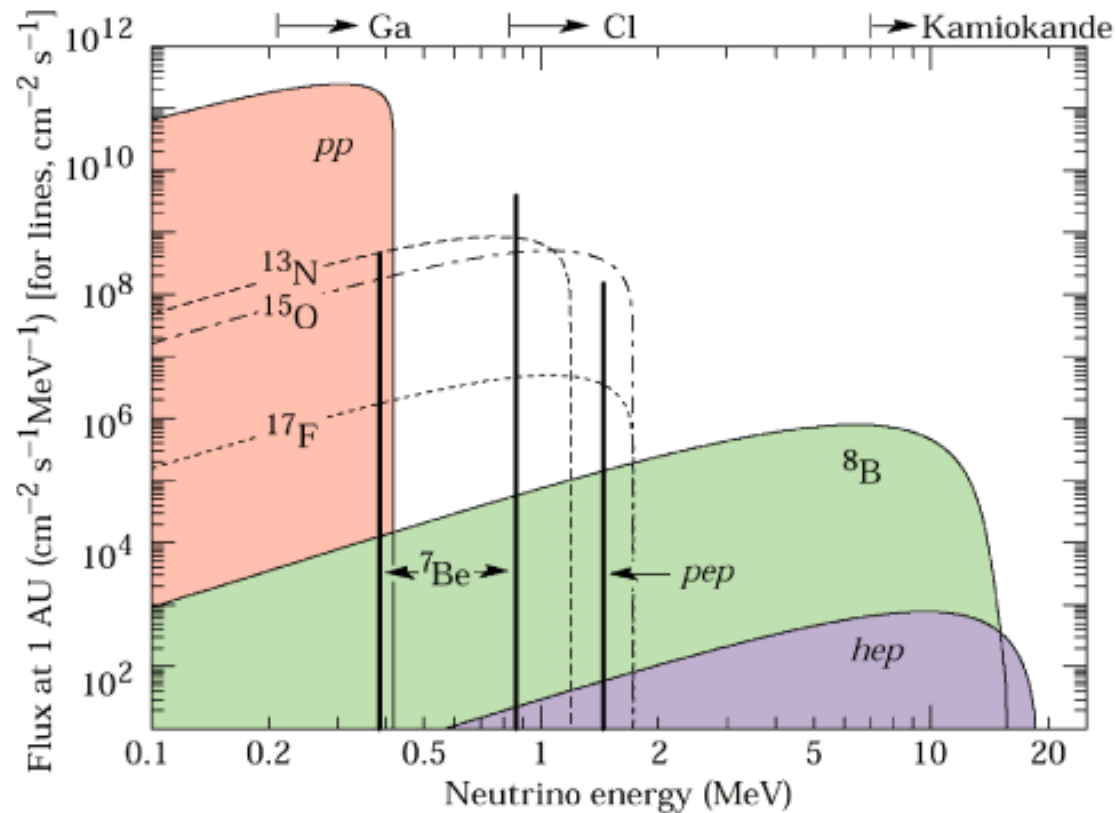
Stablishes the theory of stelar nucleosynthesis



Nobel 1967



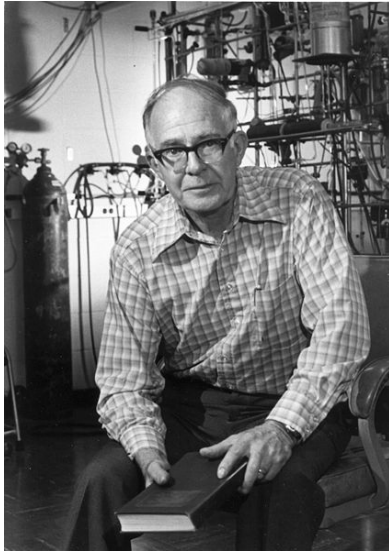
¿How many neutrinos from the Sun ?



Bahcall (died 2005)

1 ν each day in a olympic swimming pool (400000 liters of chlorine)...

The hero of the caves



Raymond Davies
Nobel 2002

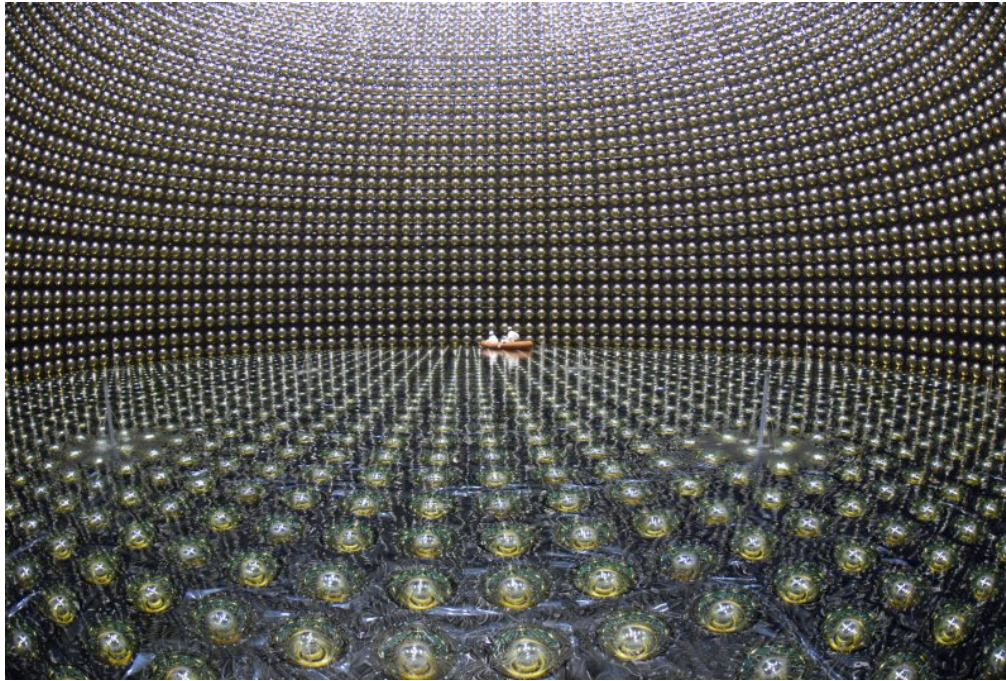
1966 he detects for the first time solar neutrinos in a pool of 400000 liters 1280m underground (Homestake mine)

Did not convince because he saw 0.4 of the expected....

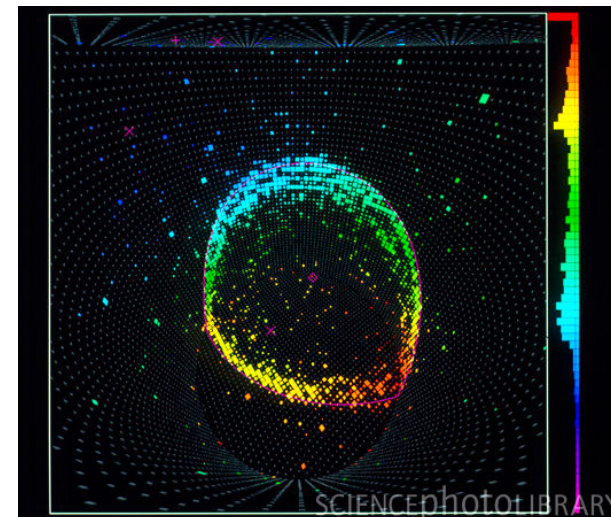
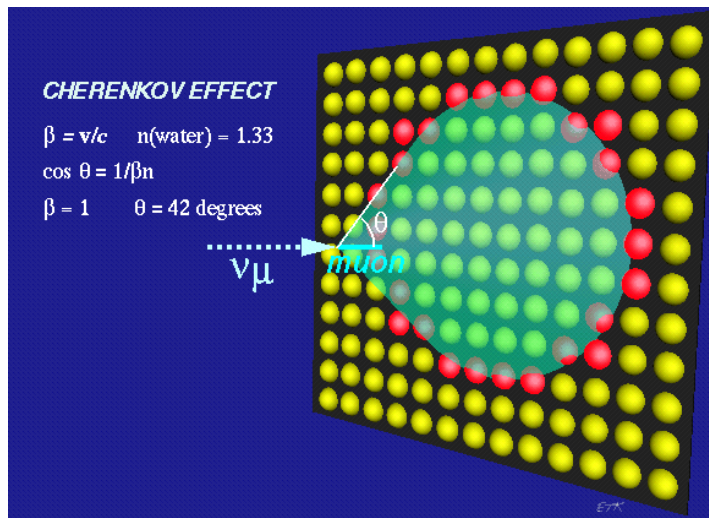
Problem in detector ? In solar model ? In neutrinos ?



Underground cathedrals of light



Koshiba (Nobel 2002)
and the anti-bulb

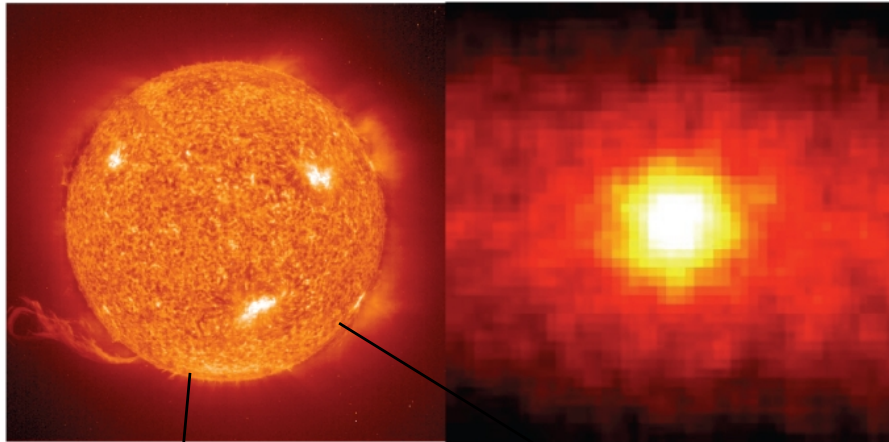


Allows to reconstruct velocity and direction

In reality they were looking for proton decay
and neutrinos were the background...

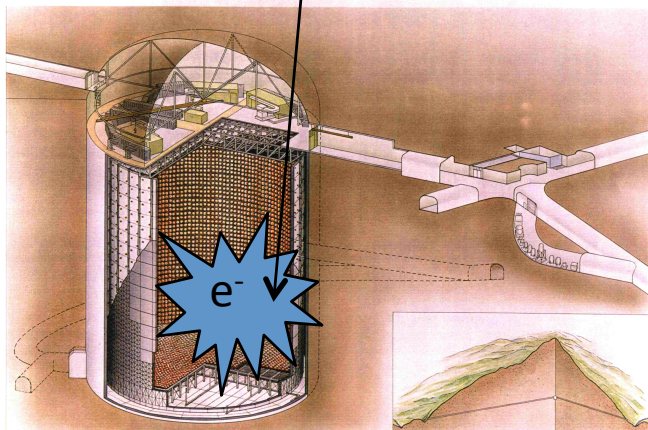
The decenium mirabilis of neutrino physics had started...

Solar Neutrinos

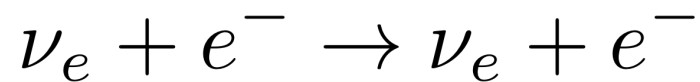


Neutrino-graphy of the sun

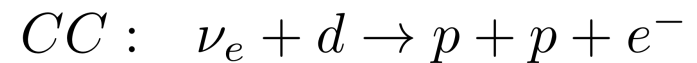
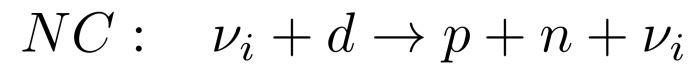
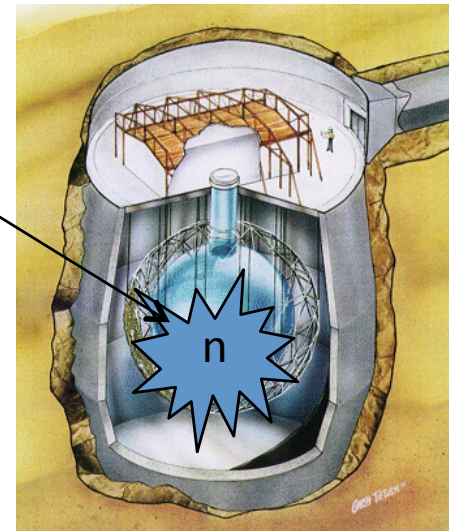
SuperKamiokande (22.5 kton!)



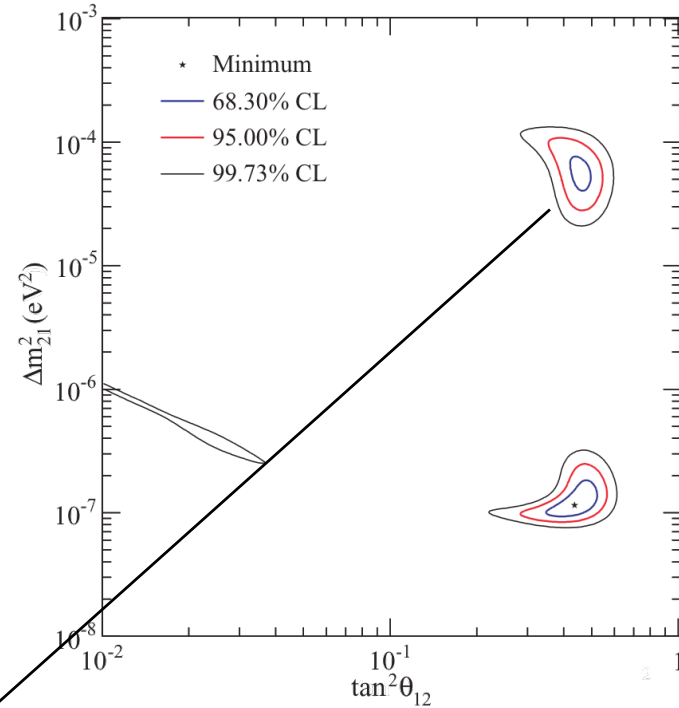
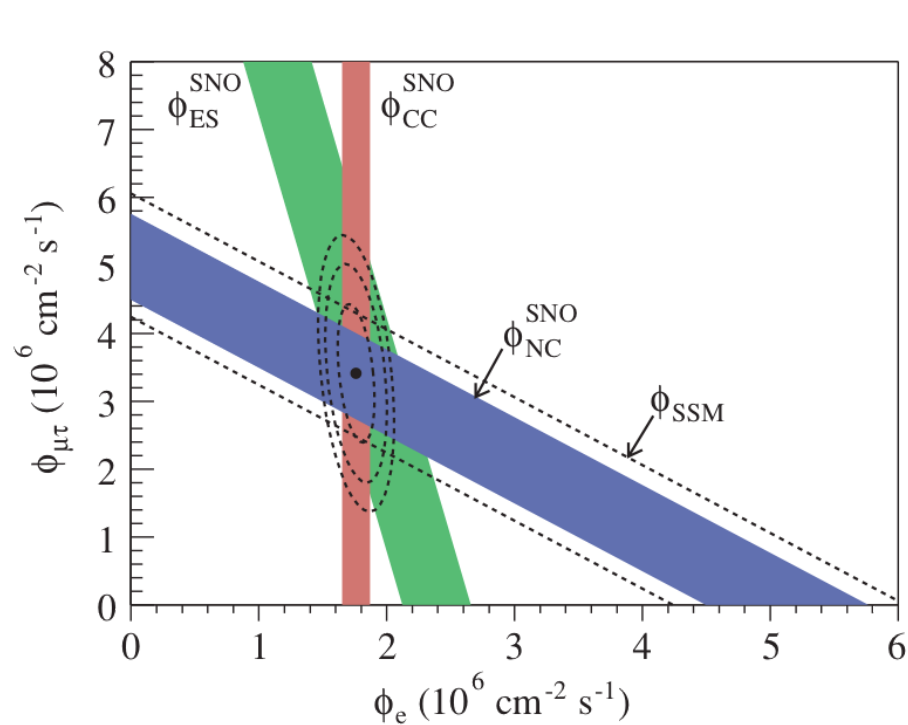
SUPERKAMIOKANDE INSTITUTE FOR COSMIC RAY RESEARCH UNIVERSITY OF TOKYO



SNO



Flavour of solar neutrinos



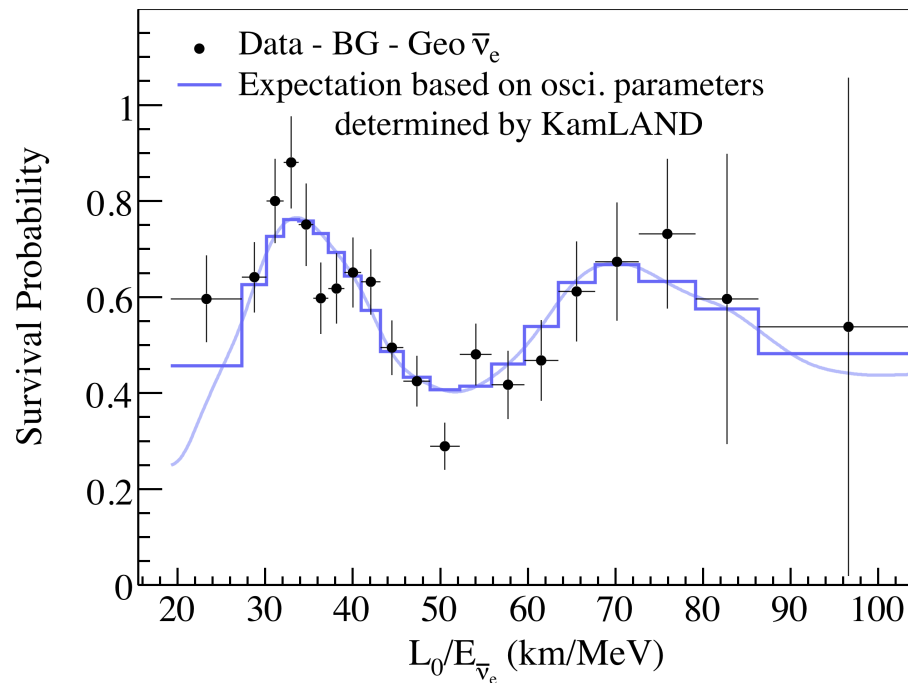
$$|\Delta m^2| \sim \frac{O(100 \text{ Km})}{O(\text{MeV})}$$

Can be tested in the Earth with Reines&Cowen experiment !

KamLAND: solar oscillation

$$\bar{\nu}_e \rightarrow \bar{\nu}_e$$

Reines&Cowan experiment ½ century afterwards
at 170 km from Japanese reactors (before the Big One)...



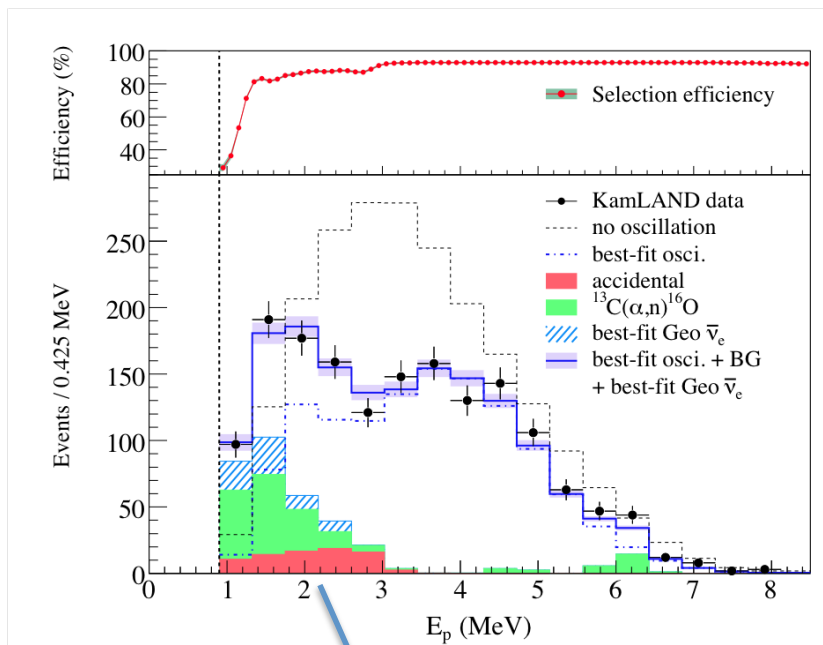
$$\Delta m_{\text{solar}}^2 \simeq 8 \times 10^{-5} \text{ eV}^2$$

Large mixing

KamLAND: solar oscillation

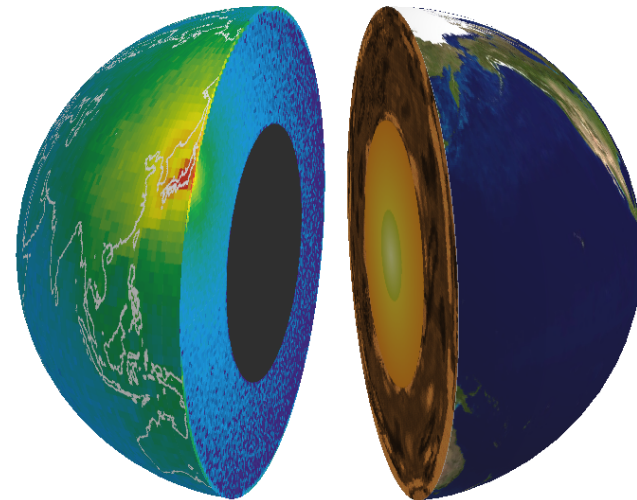
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Reines&Cowan experiment ½ century afterwards
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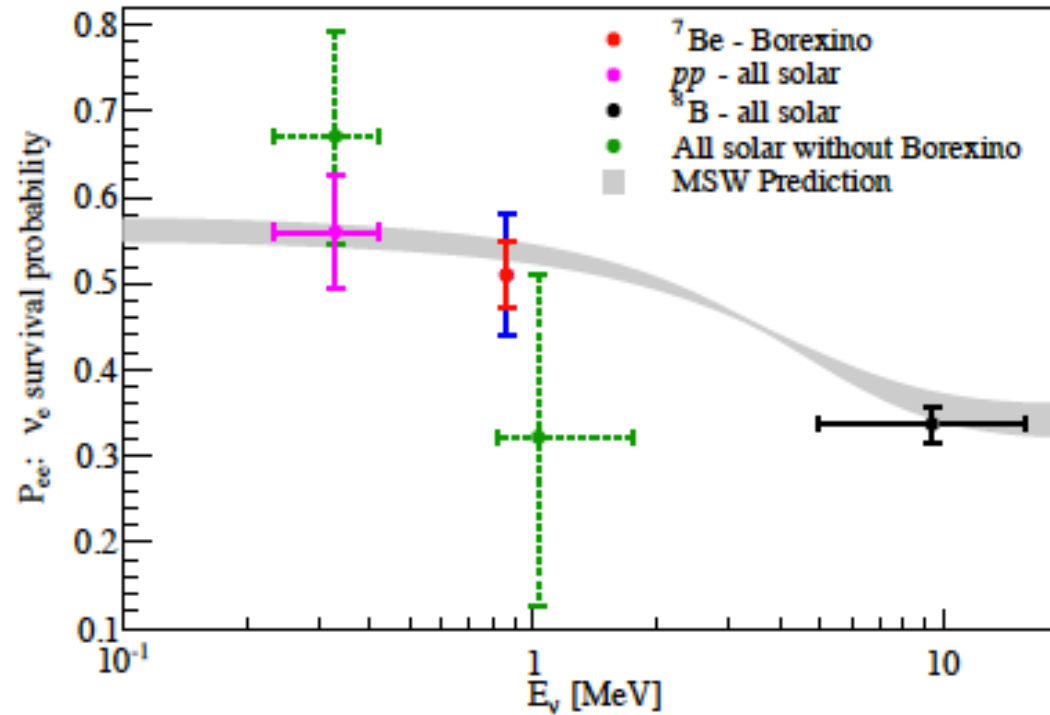
Geoneutrinos!!

$$\Delta m_{\text{solar}}^2 \simeq 8 \times 10^{-5} \text{ eV}^2$$



Start of Earth science with neutrinos !

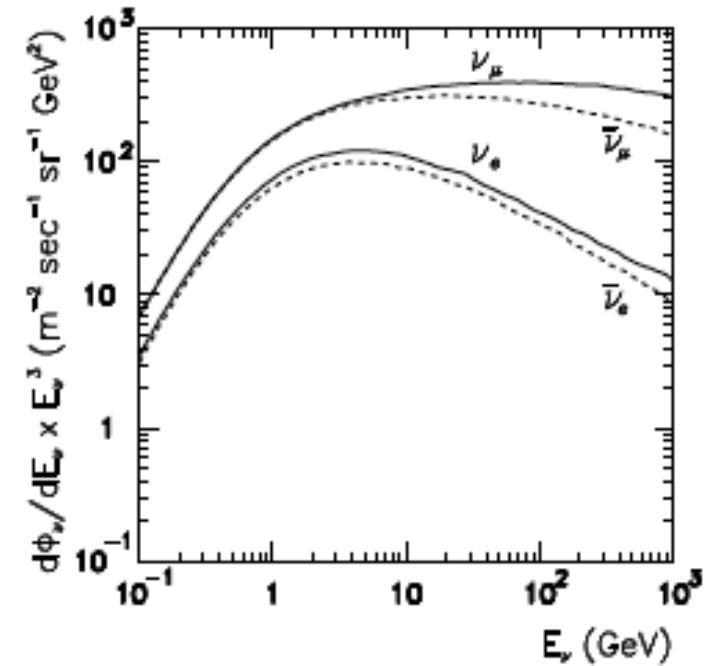
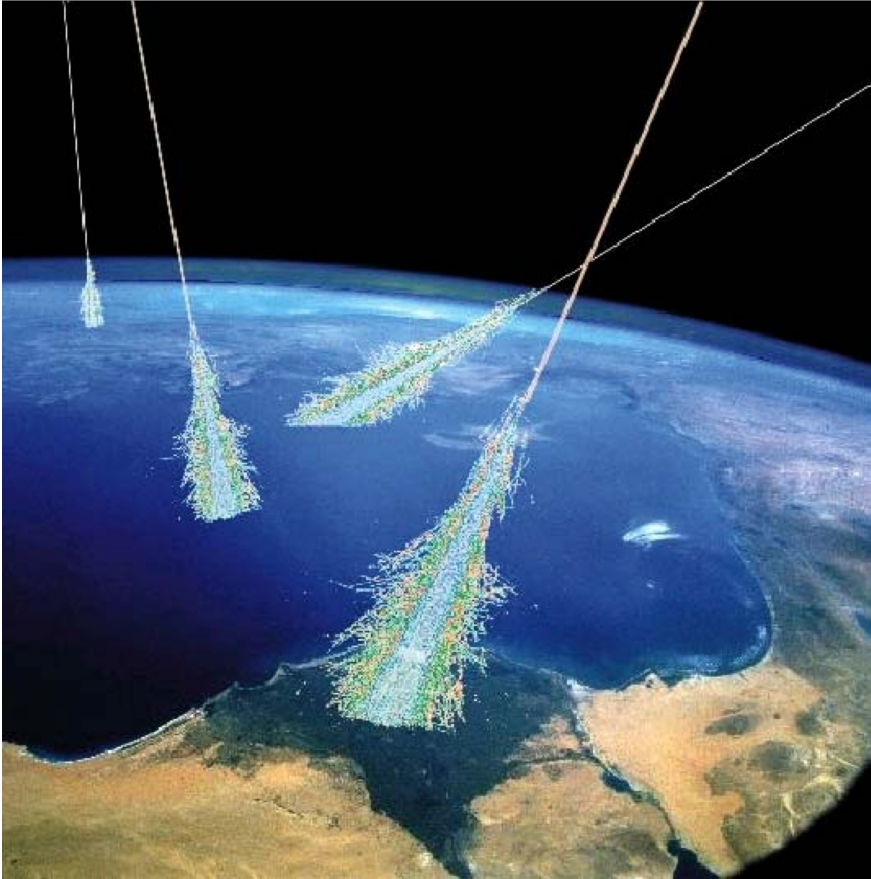
Borexino: precision era



Borexino 2011

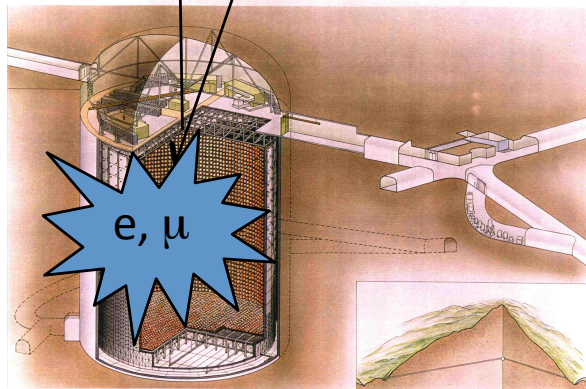
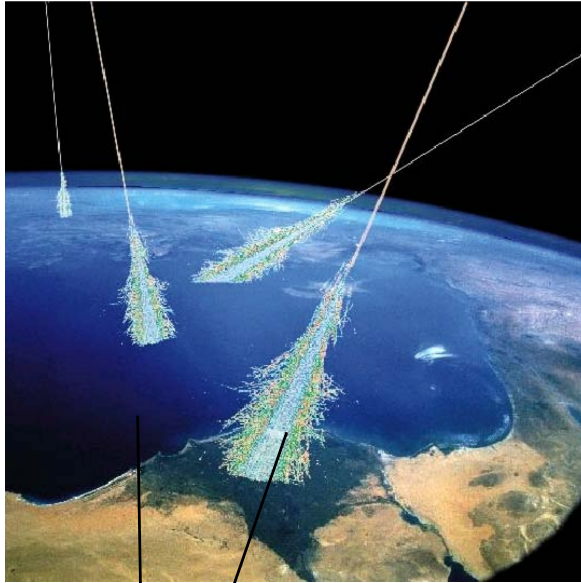
Solar neutrinos have become a tool to test solar models, to study the Earth...

Atmospheric Neutrinos

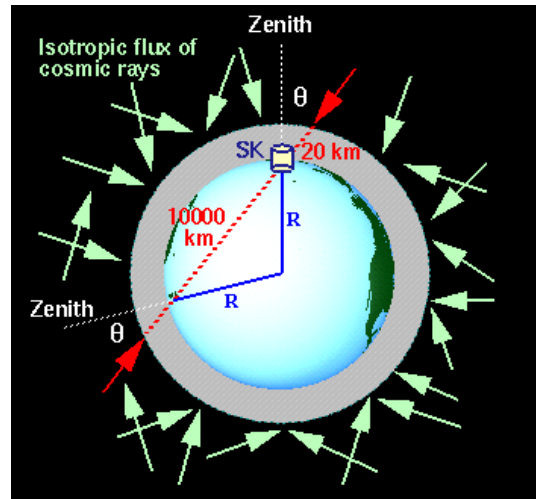
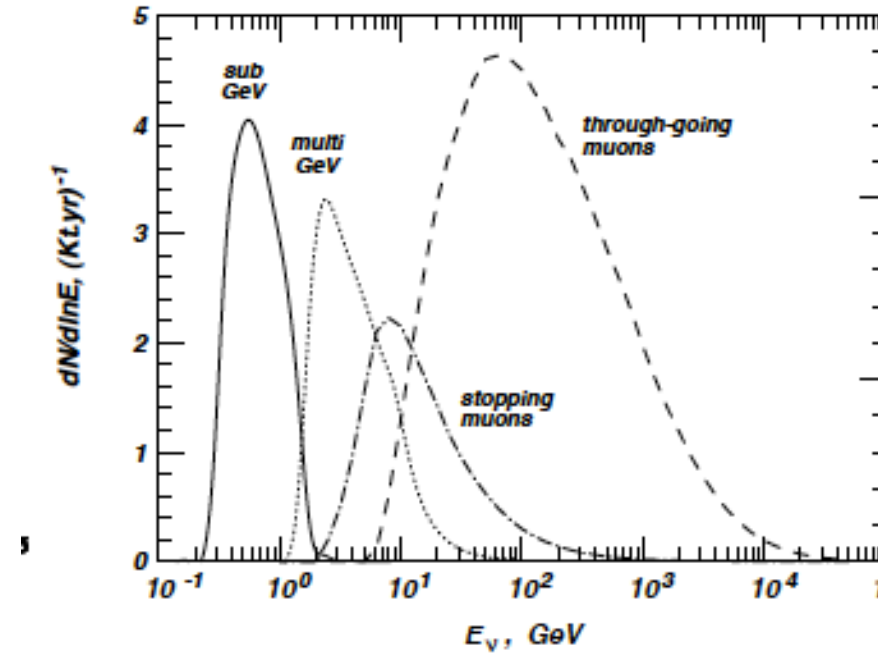


Produced in the atmosphere when primary cosmic rays collide with it, producing π , K

Atmospheric Neutrinos

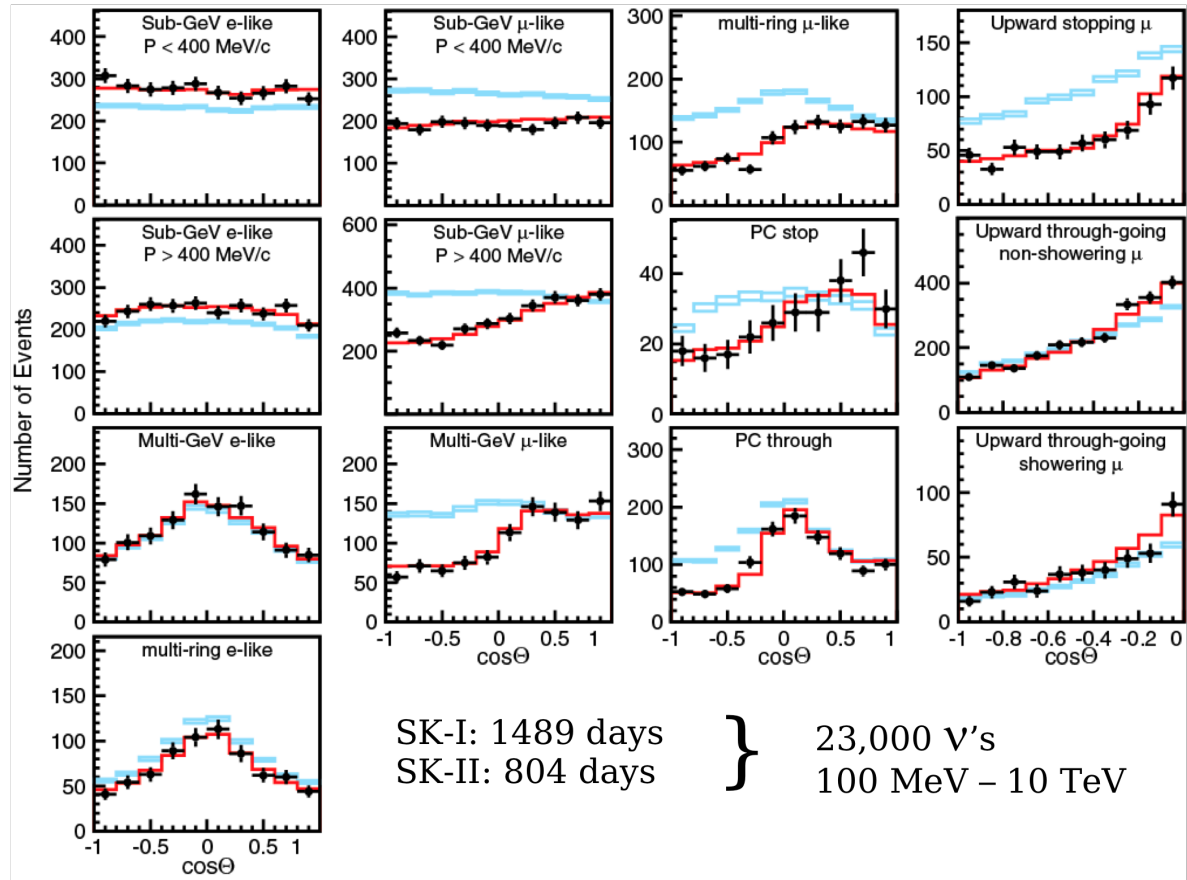


SUPERKAMIOKANDE (c) Kamioka Observatory, ICRR(Institute for Cosmic Ray Research), The University of Tokyo INSTITUTE FOR COSMIC RAY RESEARCH UNIVERSITY OF TOKYO

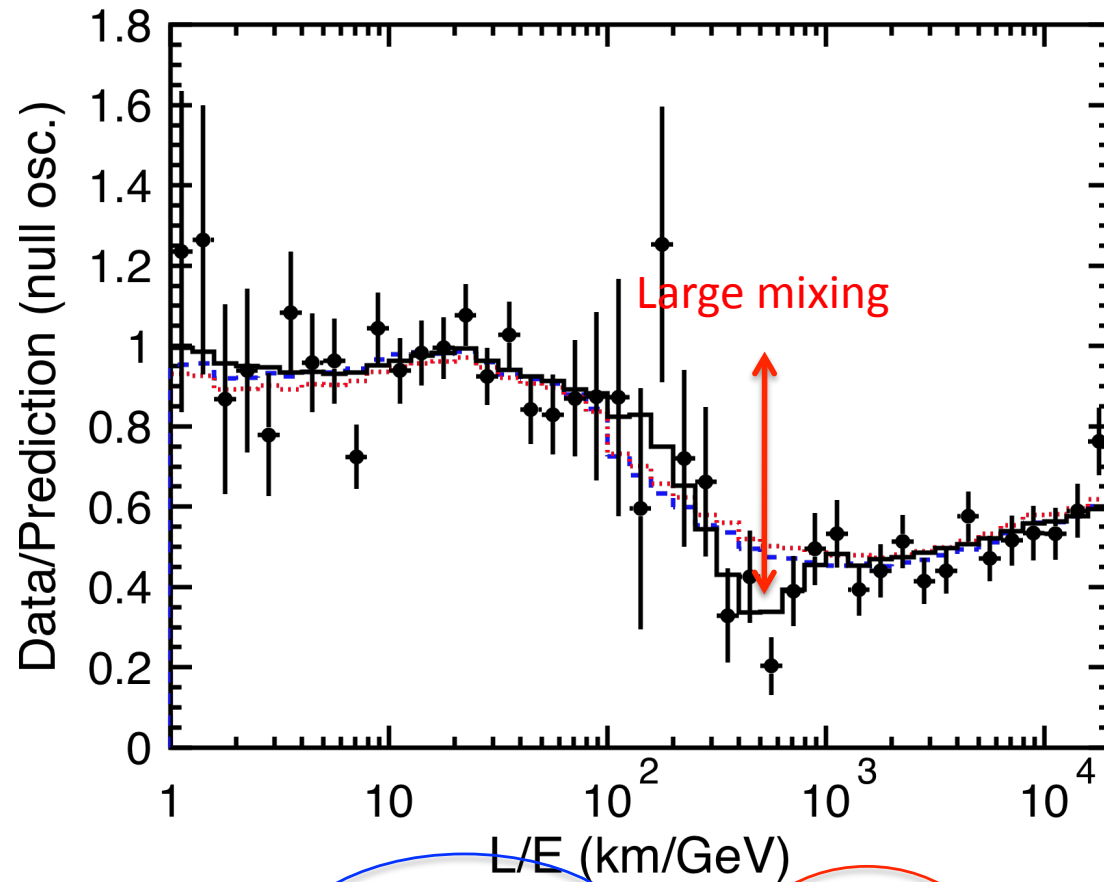


$$L = 10 - 10^4 \text{ Km}$$

Atmospheric Neutrinos



Atmospheric Oscillation



$$\Delta m_{\text{atm}}^2 = 2.5 \times 10^{-3} eV^2$$

$$|\Delta m^2| \sim \frac{O(1000 Km)}{O(GeV)} \sim \frac{O(1km)}{O(MeV)}$$

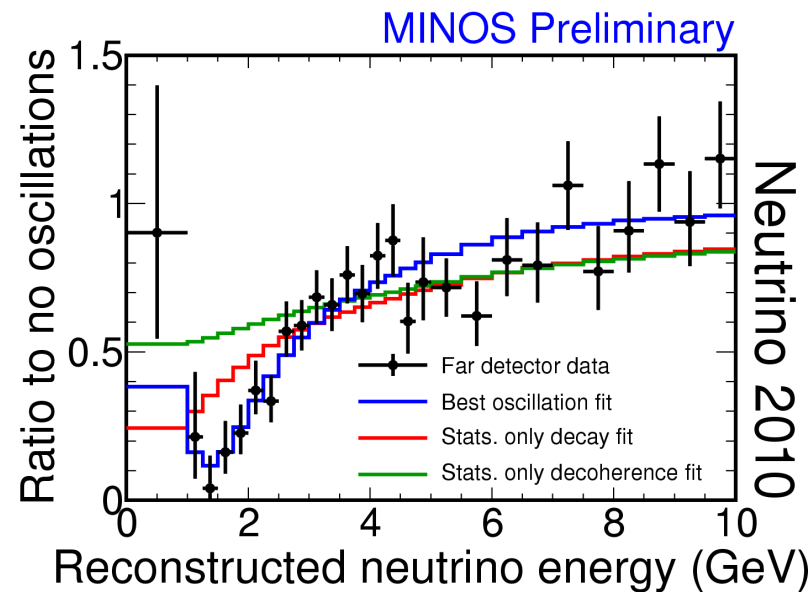
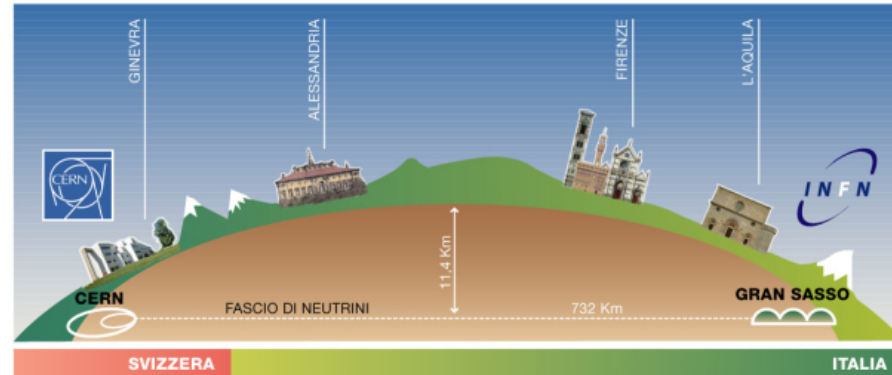
Reines&Cowan experiment at 1km!

Lederman&co experiment at 1000km!

Lederman&co neutrinos oscillate with the atmospheric wave length

Pulsed neutrino beams to 700 km baselines

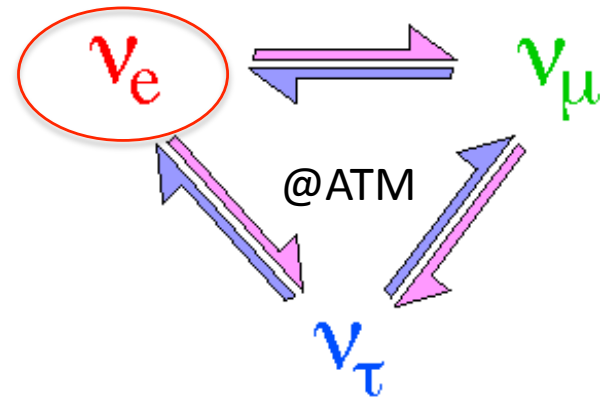
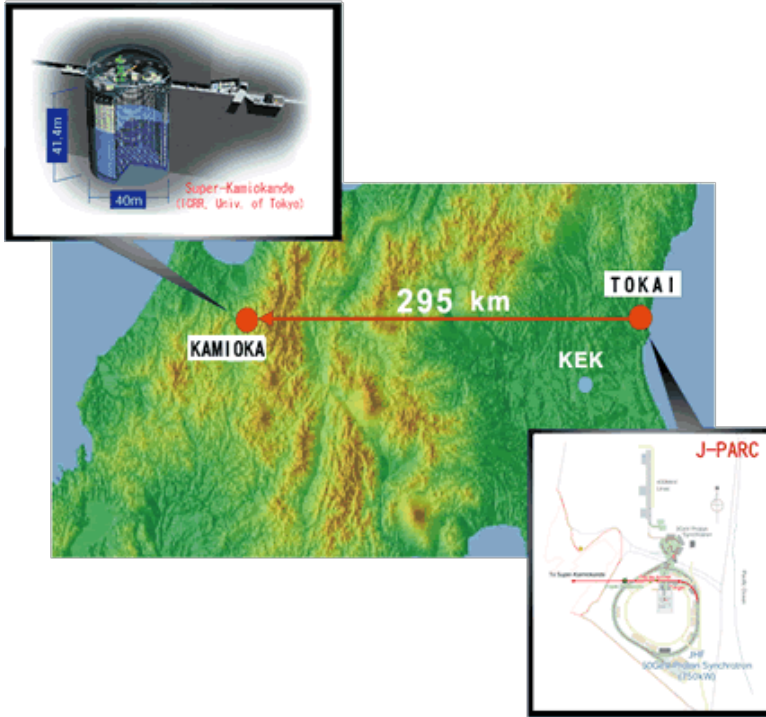
Opera



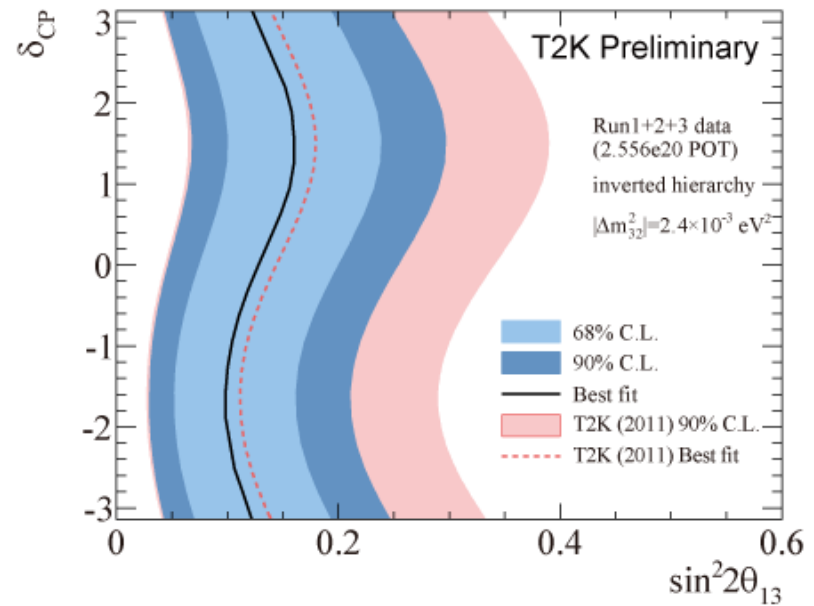
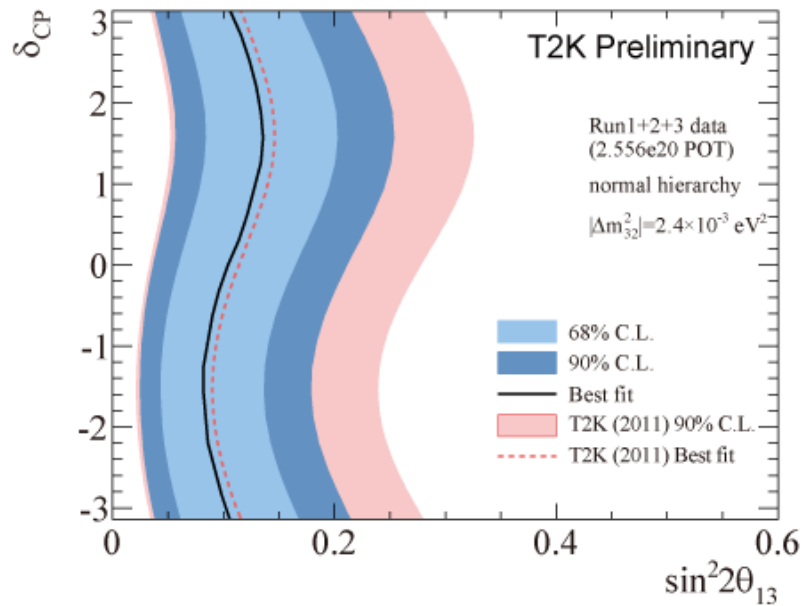
$$|\Delta m_{\text{atmos}}^2| \simeq 2.5 \times 10^{-3} \text{ eV}^2$$

$$\nu_{\mu} \rightarrow \nu_{\mu}$$

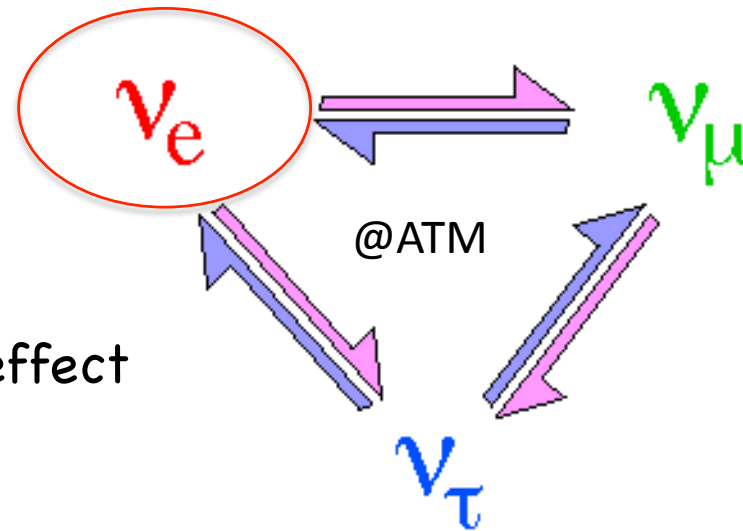
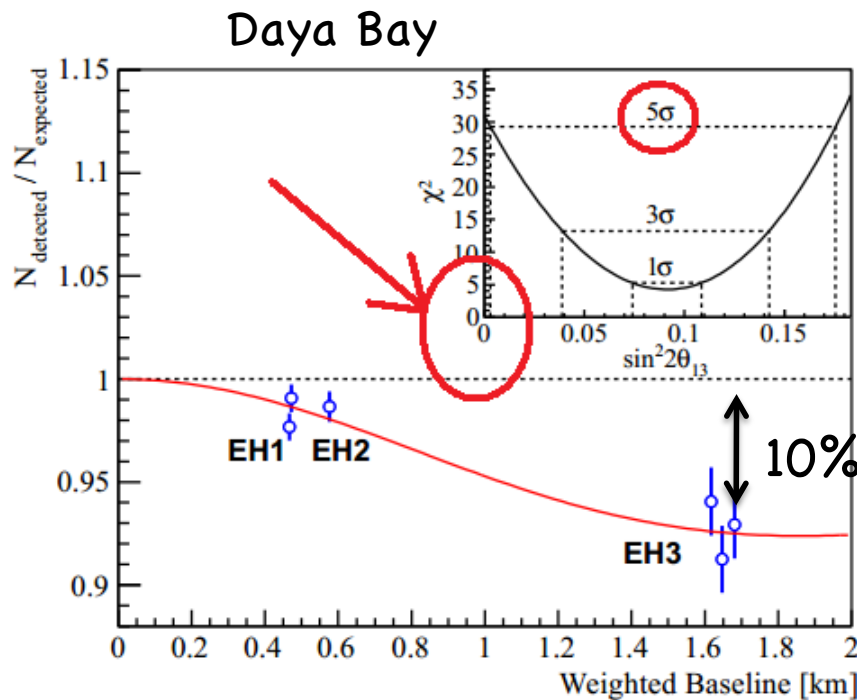
T2K



$$\nu_\mu \rightarrow \nu_e$$



Reines&Cowan (reactor) neutrinos oscillate with atmospheric wave length (only this year !)



$$\bar{\nu}_e \rightarrow \bar{\nu}_e$$

Two different wave lengths

2012 Double Chooz, Daya Bay, RENO

Modern copies of the influential experiment Chooz that barely missed the effect and set a limit

Standard 3ν scenario

$$\Delta m_{23}^2 = m_3^2 - m_2^2 \equiv \Delta m_{atm}^2$$

$$\Delta m_{12}^2 = m_2^2 - m_1^2 \equiv \Delta m_{sol}^2$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V_{MNS}(\theta_{12}, \theta_{13}, \theta_{23}, \delta) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Solar and atmospheric osc. decouple as 2x2 mixing phenomena:

- hierarchy $\frac{|\Delta m_{atm}^2|}{|\Delta m_{sol}^2|} > 10$
- small θ_{13}

$$E_\nu/L \sim \Delta m_{23}^2 \gg \Delta m_{12}^2$$

Chooz

$$P(\nu_e \rightarrow \nu_\mu) = s_{23}^2 \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{23}^2}{4E} L \right)$$

$$P(\nu_e \rightarrow \nu_\tau) = c_{23}^2 \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{23}^2}{4E} L \right)$$

$$P(\nu_\mu \rightarrow \nu_\tau) = c_{13}^4 \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m_{23}^2}{4E} L \right)$$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{23}^2}{4E} L \right) \approx 1 \rightarrow \theta_{13}=0$$

$$E_\nu/L \sim \Delta m_{23}^2 \gg \Delta m_{12}^2$$

Chooz

$$P(\nu_e \rightarrow \nu_\mu) = s_{23}^2 \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{23}^2 L}{4E} \right) \approx 0$$

$$P(\nu_e \rightarrow \nu_\tau) = c_{23}^2 \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{23}^2 L}{4E} \right) \approx 0$$

$$P(\nu_\mu \rightarrow \nu_\tau) = c_{13}^4 \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m_{23}^2 L}{4E} \right)$$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{23}^2 L}{4E} \right) \approx 1$$

Experiments in the atmospheric are described approximately by 2x2 mixing with

$$(\Delta m_{23}^2, \theta_{23}) = (\Delta m_{atm}^2, \theta_{atm})$$

$$E_\nu/L \sim \Delta m_{12}^2 \ll \Delta m_{23}^2$$

$$P(\nu_e \rightarrow \nu_e) = P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \simeq c_{13}^4 \left(1 - \sin^2 2\theta_{12} \sin^2 \left(\frac{\Delta m_{12}^2 L}{4E} \right) \right) + s_{13}^4$$

Experiments in the solar range are described approximately by 2x2 mixing with

$$(\Delta m_{12}^2, \theta_{12}) = (\Delta m_{\text{sol}}^2, \theta_{\text{sol}})$$

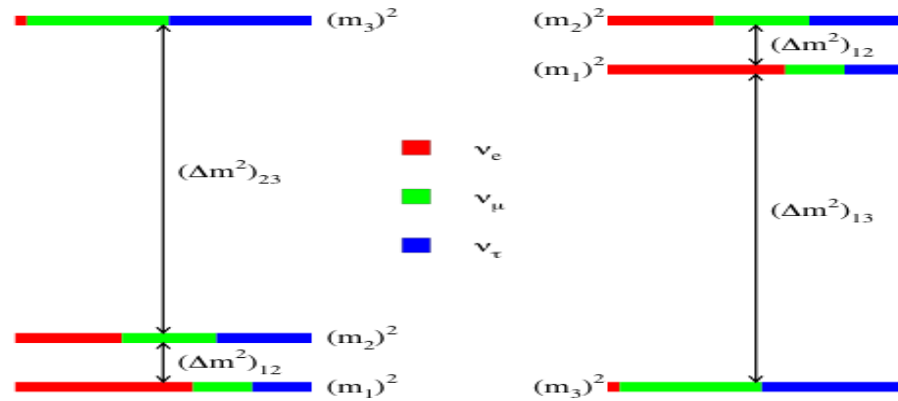
The measurement of $\theta_{13} \sim 9^\circ$ implies that corrections to these approximations are sizeable and need to be included in all analyses

Standard 3ν scenario

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS}(\theta_{12}, \theta_{23}, \theta_{13}, \delta, \dots) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

normal hierarchy

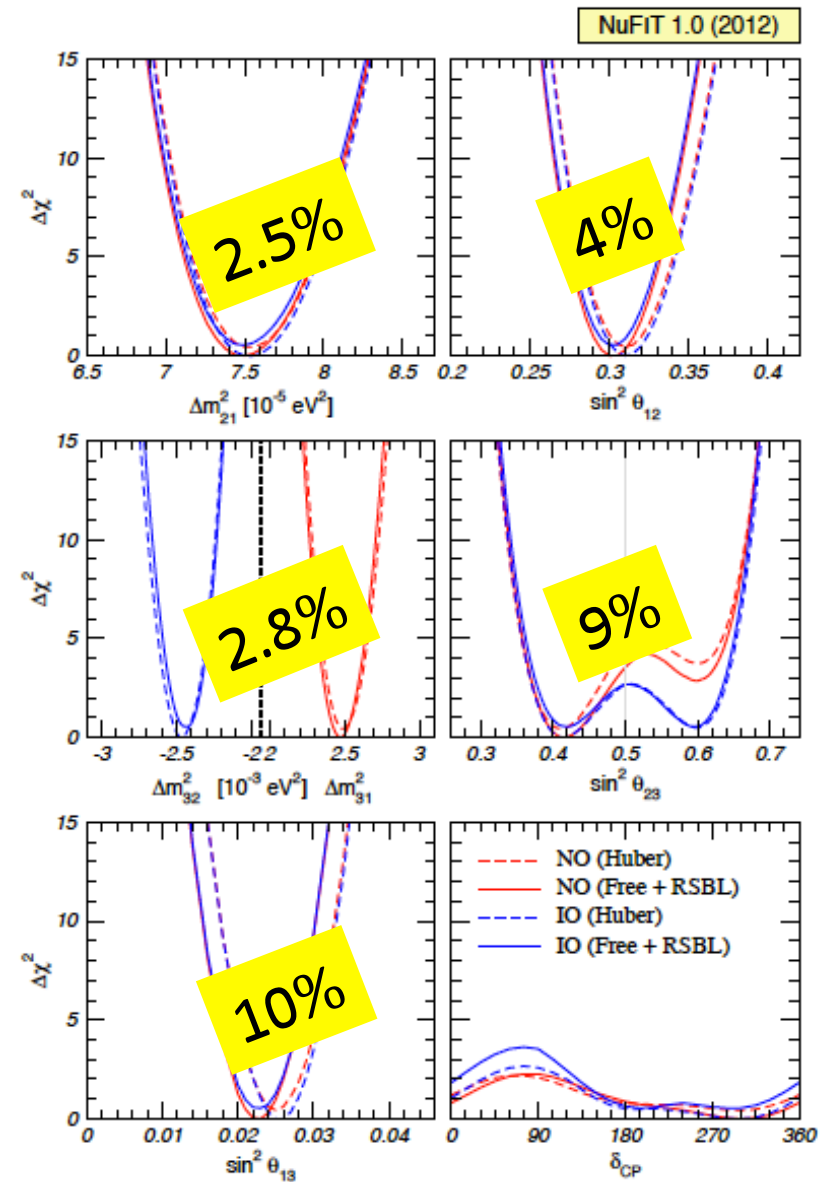
inverted hierarchy



■ ν_e
■ ν_μ
■ ν_τ

$$\Delta m_{13}^2 > 0$$

$$\Delta m_{13}^2 < 0$$



Gonzalez-Garcia et al 1209.3023

Outliers: LSND anomaly

$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

$$\nu_\mu \rightarrow \nu_e \text{ DIF } (28 \pm 6 / 10 \pm 2)$$

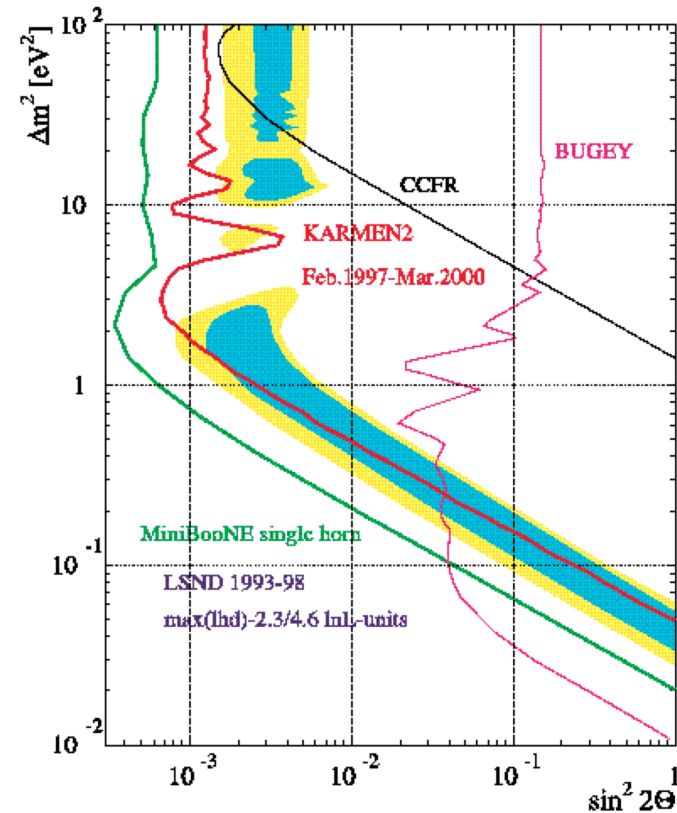
$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e \text{ DAR } (64 \pm 18 / 12 \pm 3)$$

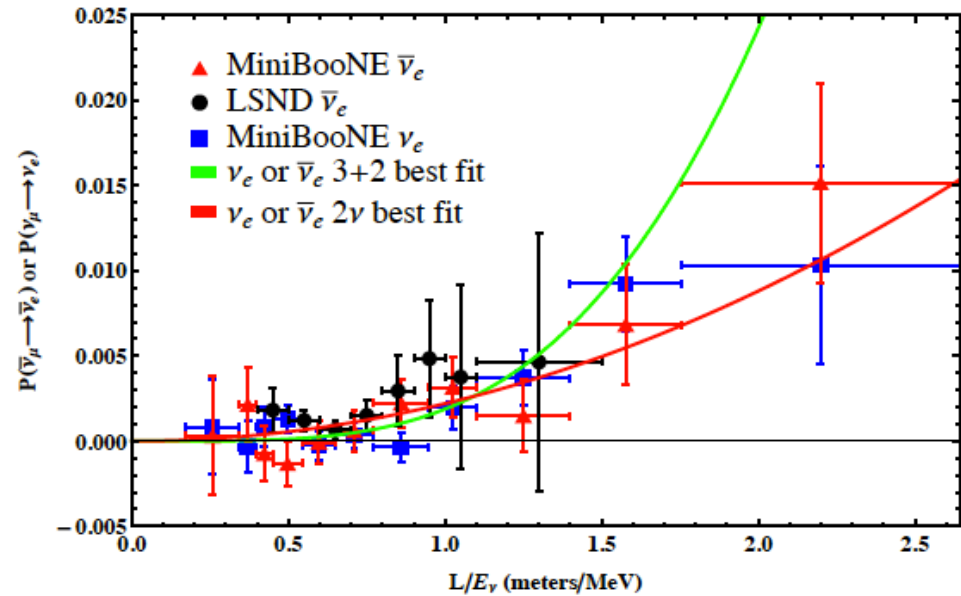
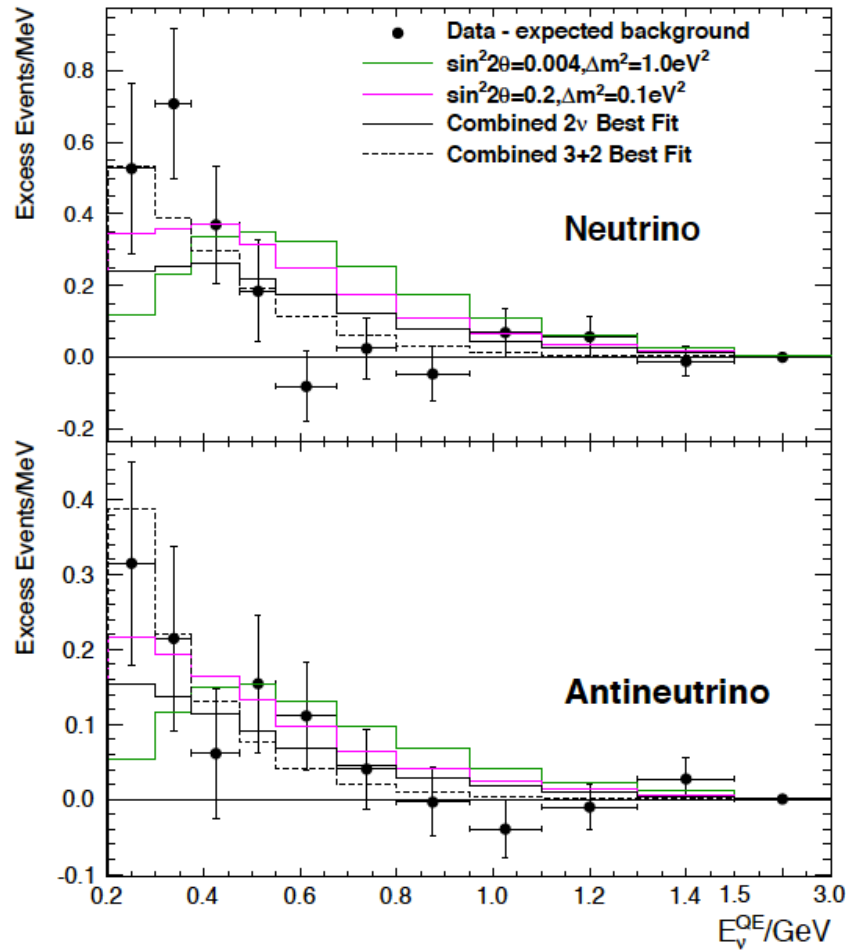
Appearance signal with very different

$$|\Delta m^2| \gg |\Delta m_{atm}^2|$$

LSND vs KARMEN

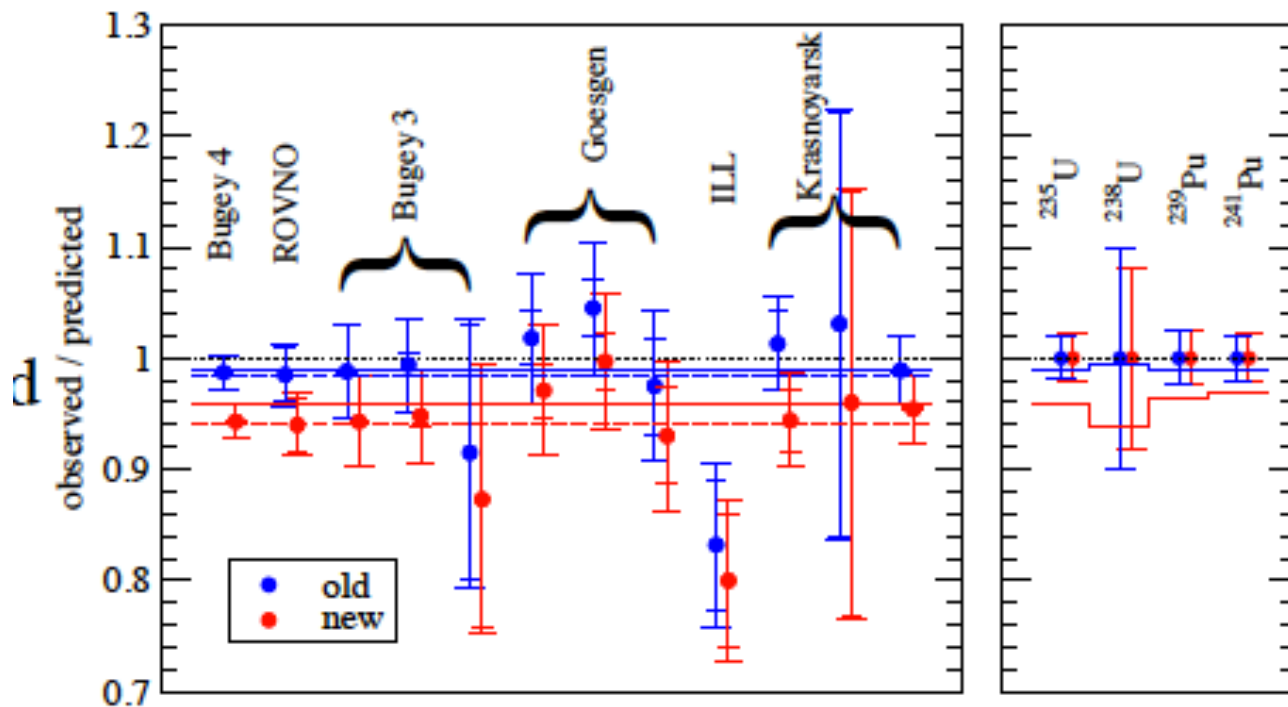


MINIBOONE



Extremely confusing situation!

Outliers: reactor anomaly



T. A. Mueller et al; P. Huber

Recent re-evaluation of reactor fluxes found to be 3% underestimated

+Gallium anomaly...

3+2 neutrino mixing model

Parametrized in terms of a general unitary 5x5 mixing matrix
(9 angles, >6 phases physical)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \\ \nu'_s \end{pmatrix} = U_{5 \times 5} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \nu_5 \end{pmatrix}$$

	Δm_{41}^2	$ U_{e4} $	$ U_{\mu 4} $	Δm_{51}^2	$ U_{e5} $	$ U_{\mu 5} $	δ/π	χ^2/dof
3+2	0.47	0.128	0.165	0.87	0.138	0.148	1.64	110.1/130
1+3+1	0.47	0.129	0.154	0.87	0.142	0.163	0.35	106.1/130

	3+1	3+2
χ_{\min}^2	100.2	91.6
NDF	104	100
GoF	59%	71%
Δm_{41}^2 [eV ²]	0.89	0.90
$ U_{e4} ^2$	0.025	0.017
$ U_{\mu 4} ^2$	0.023	0.018
Δm_{51}^2 [eV ²]		1.60
$ U_{e5} ^2$		0.017
$ U_{\mu 5} ^2$		0.0064
η		1.52 π
$\Delta\chi_{\text{PG}}^2$	24.1	22.2
NDF _{PG}	2	5
PGoF	6×10^{-6}	5×10^{-4}

Kopp, Maltoni, Schwetz (KMS) arXiv:1103.4570

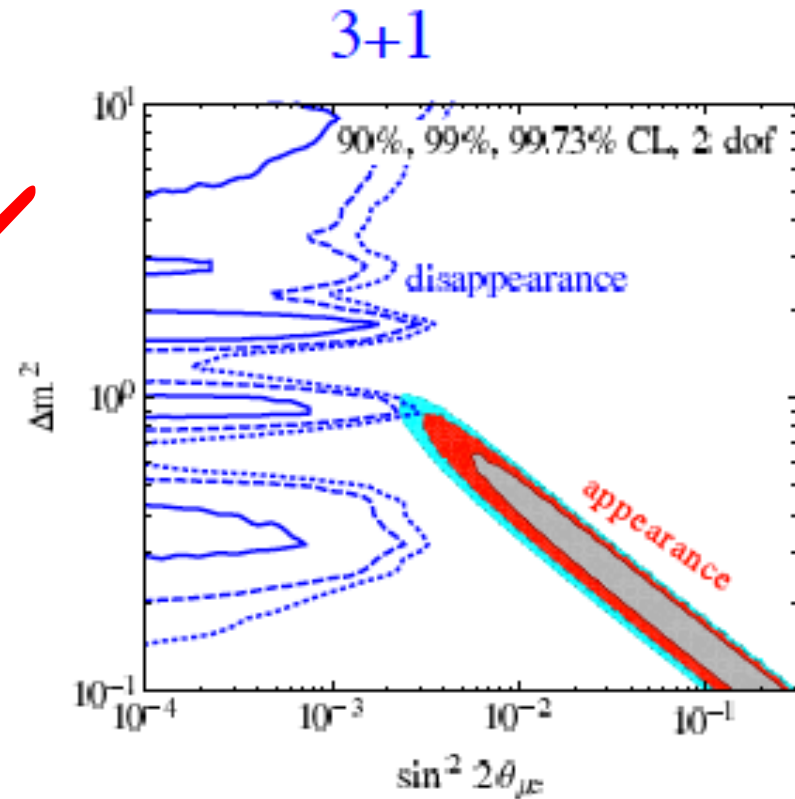
Giunti, Laveder, (GL) arXiv:1107.1452

Significant improvement over 3v scenario, but tension appearance/disappearance remains

$$P(\nu_e \rightarrow \nu_\mu) = O(|U_{ei}|^2 |U_{\mu i}|^2) \quad \checkmark$$

$$P(\nu_e \rightarrow \nu_e) = O(|U_{ei}|^2) \quad \checkmark$$

$$P(\nu_\mu \rightarrow \nu_\mu) = O(|U_{\mu i}|^2) \quad \times$$



T.Schwetz, talk ν 2012

Strong tension remains:

a convincing signal would be to find it in all the three...

We are still lacking an independent confirmation of LSND at the same level of confidence...

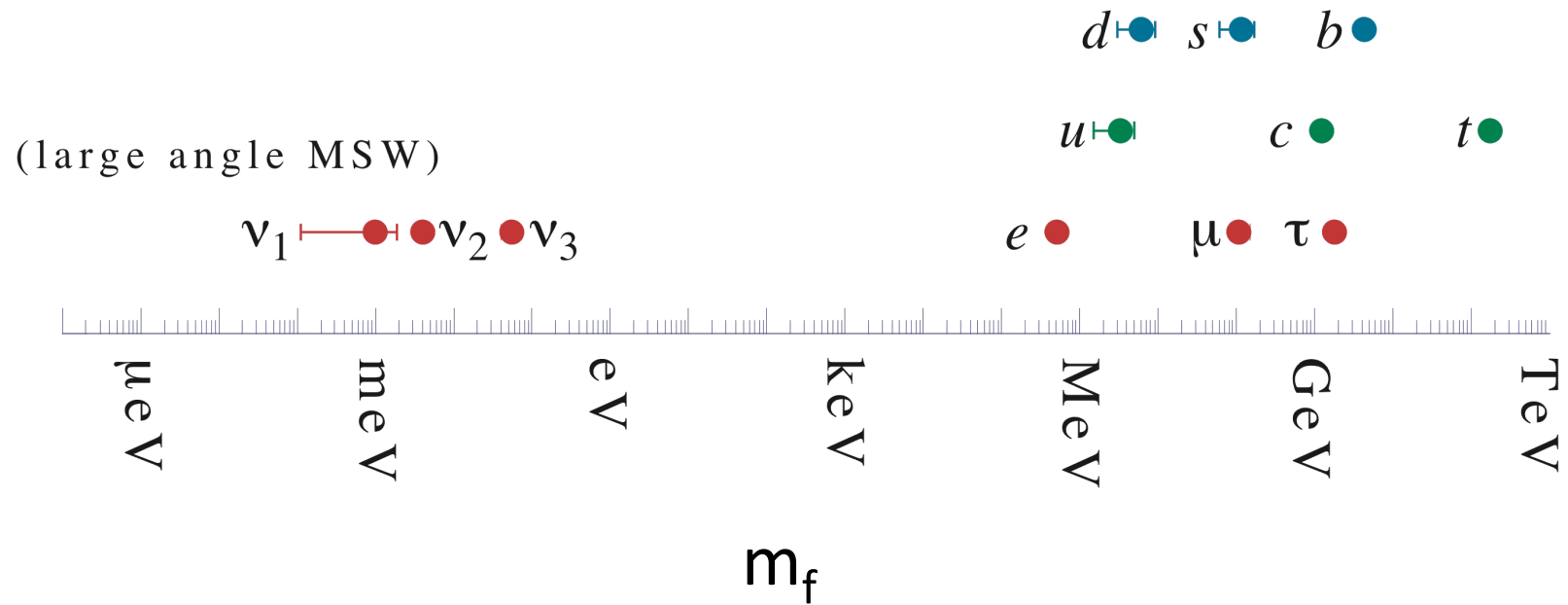


Lecture IV:

- Prospects in neutrino physics
- Leptogenesis & neutrinos in the cosmos
- Theory outlook
- Conclusions

Why are neutrinos so much lighter ?

Neutral vs charged hierarchy ?



Why so different mixing ?

CKM

$$|V|_{\text{CKM}} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.0065 & (3.51 \pm 0.15) \times 10^{-3} \\ 0.2252 \pm 0.00065 & 0.97344 \pm 0.00016 & (41.2_{-5}^{+1.1}) \times 10^{-3} \\ (8.67_{-0.31}^{+0.29}) \times 10^{-3} & (40.4_{-0.5}^{+1.1}) \times 10^{-3} & 0.999146_{-0.000046}^{+0.000021} \end{pmatrix}$$

PMNS

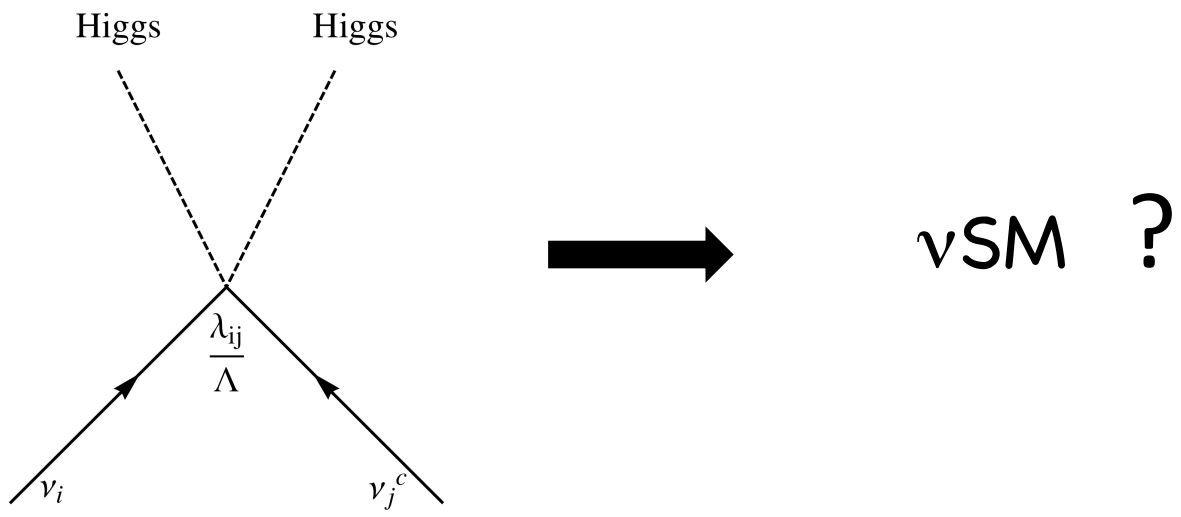
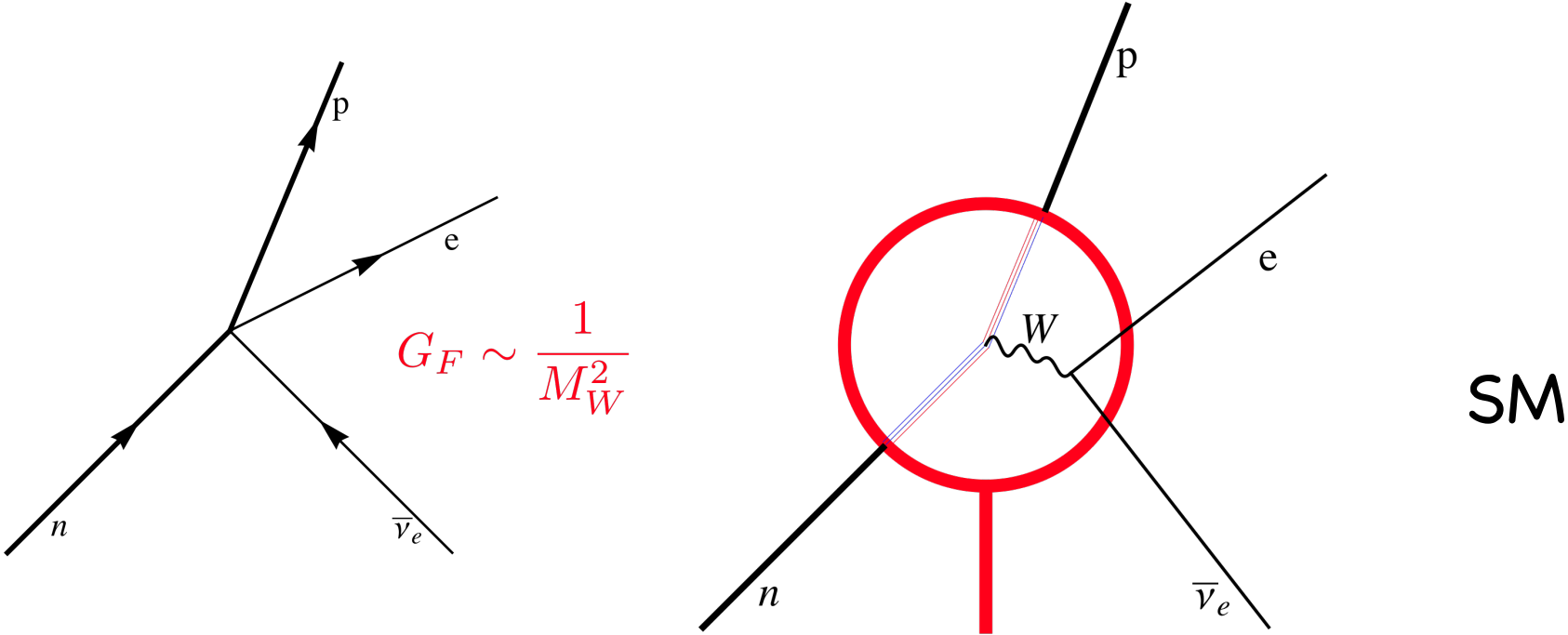
$$|U| = \begin{pmatrix} 0.795 \rightarrow 0.846 & 0.513 \rightarrow 0.585 & 0.126 \rightarrow 0.178 \\ 0.205 \rightarrow 0.543 & 0.416 \rightarrow 0.730 & 0.579 \rightarrow 0.808 \\ 0.215 \rightarrow 0.548 & 0.409 \rightarrow 0.725 & 0.567 \rightarrow 0.800 \end{pmatrix}$$

3σ

Gonzalez-Garcia, et al 1209.3023

PMNS no longer looks bi-maximal, tri-bimaximal, golden...no more than anarchy

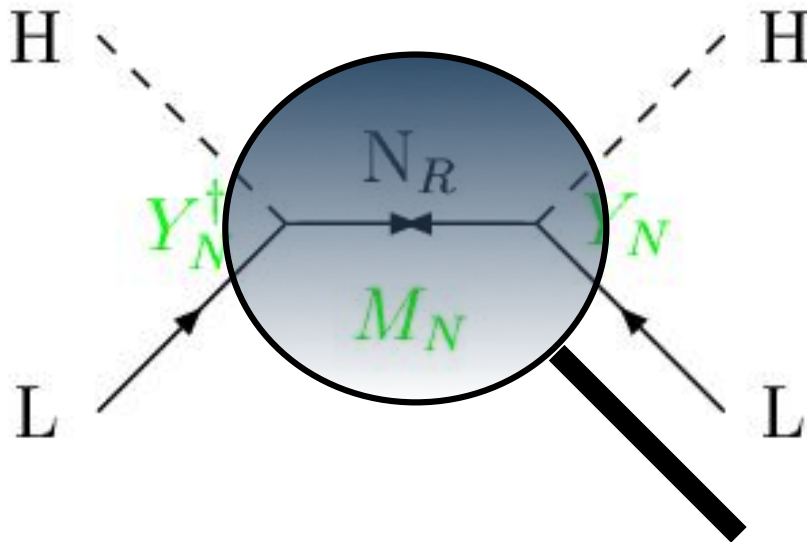
Neutrinos have tiny masses \rightarrow a new physics scale, what ?



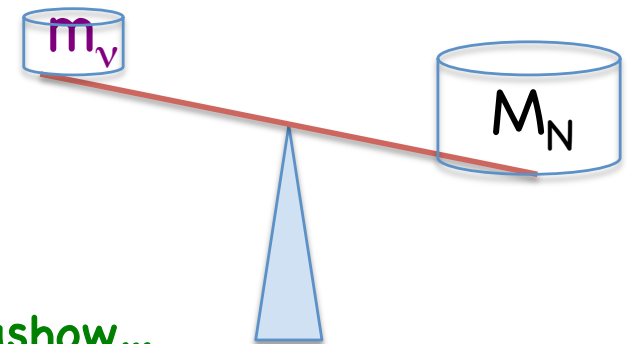
How does the ν scale relates to the EW scale ?

Example: Type I seesaw model (interchange heavy singlet fermions)

$$\mathcal{L} = \mathcal{L}_{SM} - \sum^{n_R} \bar{l}_L^\alpha Y^{\alpha i} \tilde{\Phi} \nu_R^i - \sum^{n_R} \frac{1}{2} \bar{\nu}_R^{ic} M_N^{ij} \nu_R^j + h.c.$$



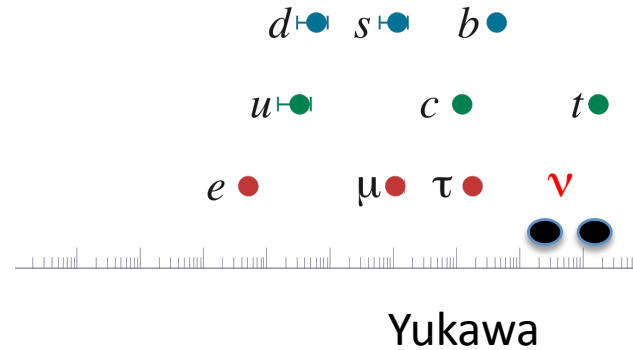
$$m_\nu = \frac{\alpha v^2}{\Lambda} \equiv Y_N^T \frac{v^2}{M_N} Y_N$$



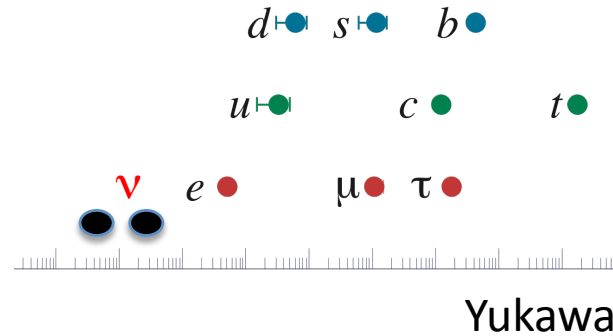
Minkowski; Gell-Mann, Ramond Slansky; Yanagida, Glashow...

Charged/neutral hierarchy in seesaw (I)

$\Lambda \leq \text{GUT}$



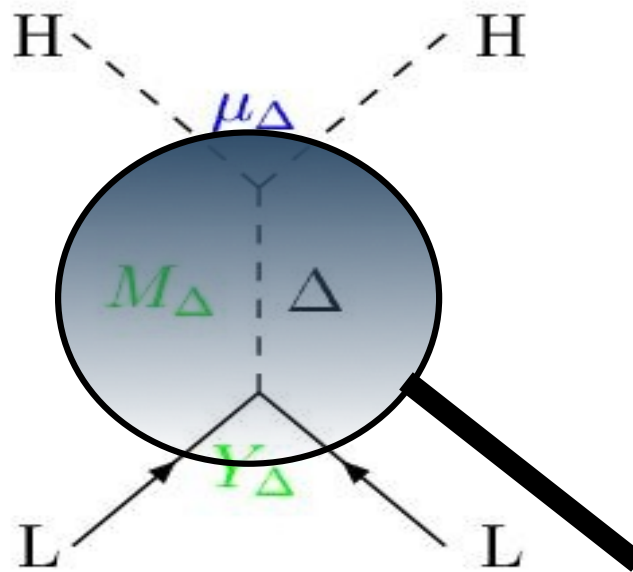
$\Lambda = \text{TeV}$



Minkowski; Gell-Mann, Ramond Slansky; Yanagida, Glashow...

New physics scale

Type II see-saw: interchange a heavy triplet scalar

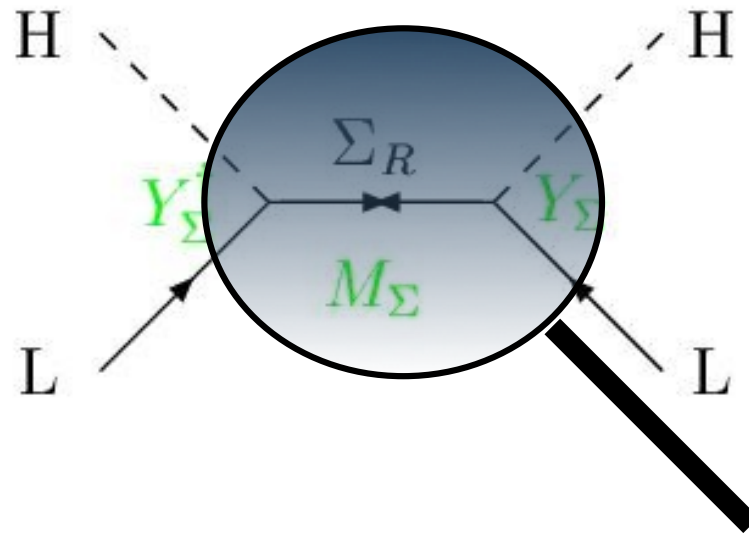


$$m_\nu = \frac{\alpha v^2}{\Lambda} \equiv Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Konetschny, Kummer; Cheng, Li; Lazarides, Shafi, Wetterich ...

New physics scale

Type III see-saw: interchange a heavy triplet fermion



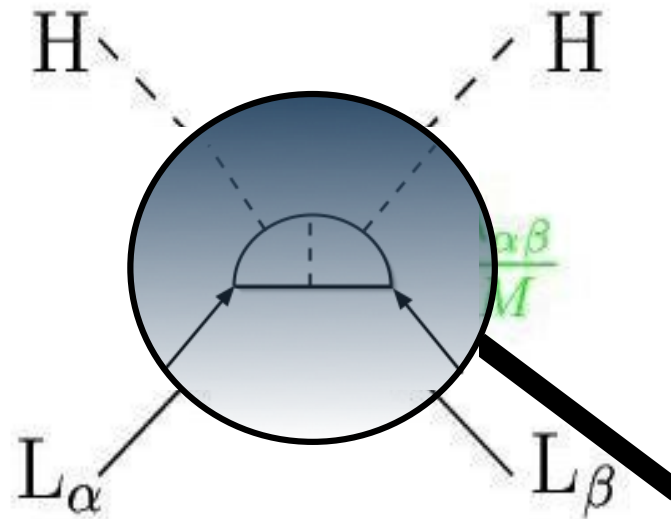
$$m_\nu = \frac{\alpha v^2}{\Lambda} \equiv Y_\Sigma^T \frac{v^2}{M_\Sigma} Y_\Sigma$$

Foot et al; Ma; Bajc, Senjanovic...

New physics scale

Also from loops !

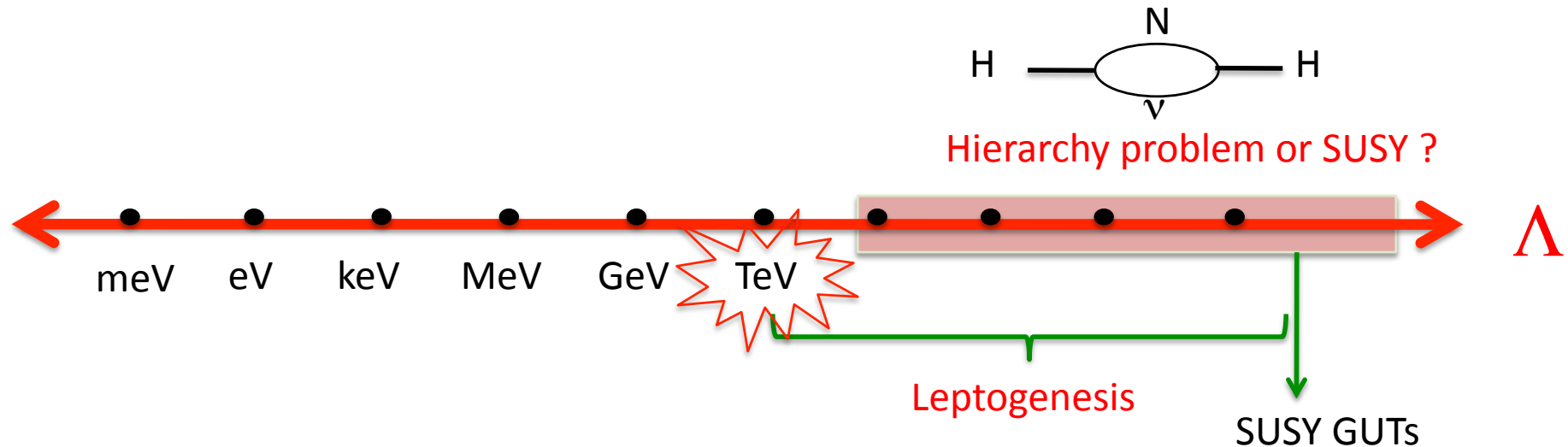
Zee-Babu



$$m_\nu \sim \mathcal{O} \left(\frac{1}{(16\pi^2)^2} \times \frac{\mu m_l^2}{M^2} \right)$$

Pinning down the New physics scale

The new scale is stable under radiative corrections due to Lepton Number Symmetry but the EW is not!



Robust predictions of high (and not so high) scale seesaw models:

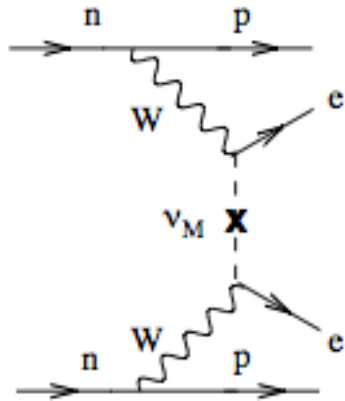
there is **neutrinoless double beta** decay at some level ($\Lambda > 100\text{MeV}$)

a matter-antimatter asymmetry if there is **CP violation** in the lepton sector !

there are other states out there at scale Λ !

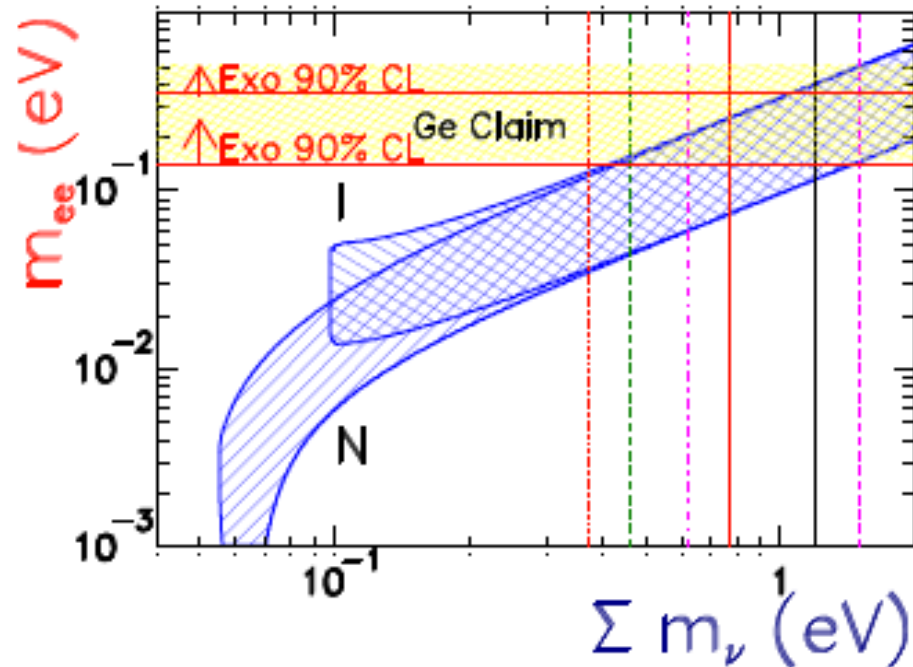
Lepton Number violation: Majorana nature

Plethora of experiments with different techniques/systematics: EXO, KAMLAND-ZEN, GERDA, CUORE, NEXT, SuperNEMO, LUCIFER...



$$m_{\beta\beta} \equiv |m_{ee}|$$

$$\Sigma \equiv \sum_i m_i$$



(Fogli et al (04))

Update Maltoni, Schwetz, Salvado, MCGG (95%)

$$|m_{ee}| = |c_{13}^2(m_1 c_{12}^2 + m_2 e^{i\alpha} s_{12}^2) + m_3 e^{i\beta} s_{13}^2|$$

Leptonic CP violation (in vacuum)

$$\begin{aligned}
 P_{\nu_e \nu_\mu (\bar{\nu}_e \bar{\nu}_\mu)} &= s_{23}^2 \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta_{23} L}{2} \right) \equiv P^{atmos} \\
 &+ c_{23}^2 \sin^2 2\theta_{12} \sin^2 \left(\frac{\Delta_{12} L}{2} \right) \equiv P^{solar} \\
 + \tilde{J} \cos \left(\pm\delta - \frac{\Delta_{23} L}{2} \right) \frac{\Delta_{12} L}{2} \sin \left(\frac{\Delta_{23} L}{2} \right) &\equiv P^{inter}
 \end{aligned}$$

$$\tilde{J} \equiv c_{13} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23}$$

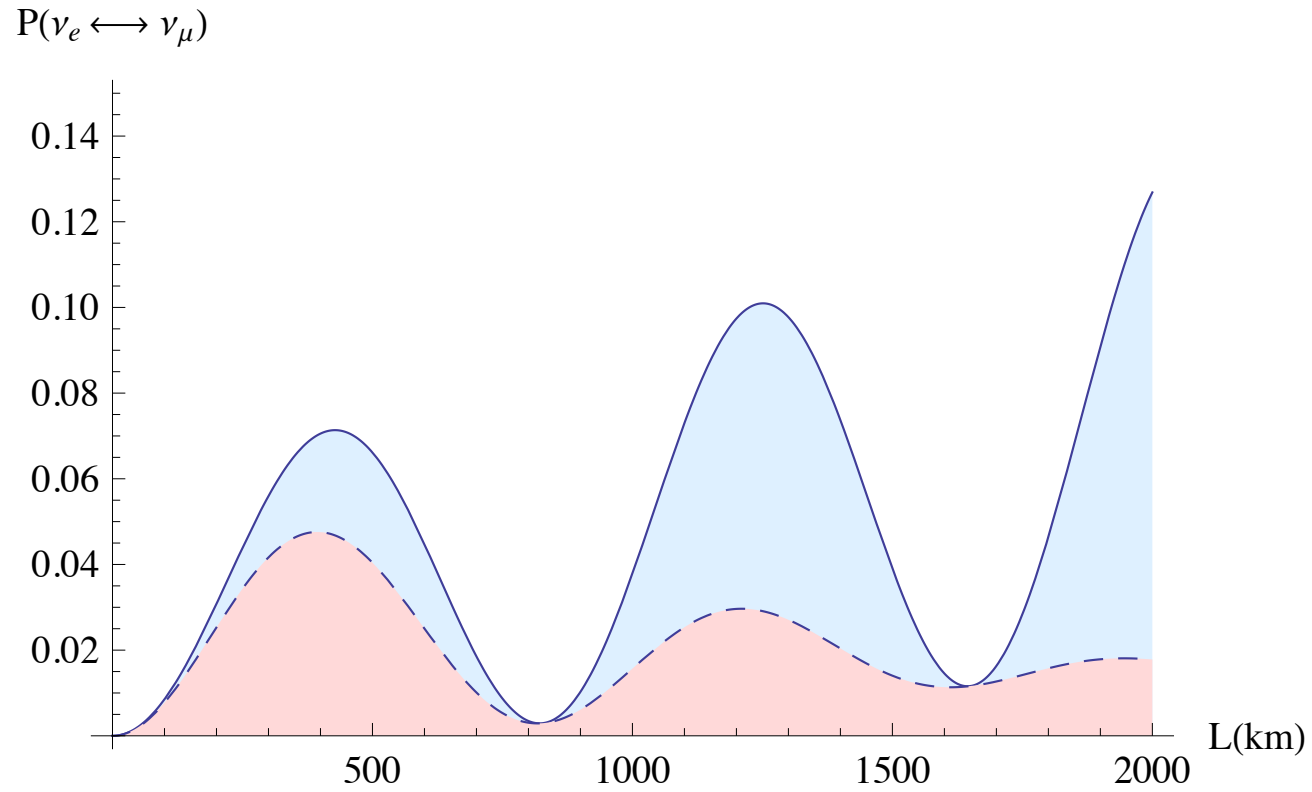
θ_{13} measurement

$$P^{atmos} \gg P^{solar} \rightarrow A_{\nu_e \nu_\mu (\nu_\tau)}^{CP,T} \sim \frac{\Delta_{12} L}{\sin 2\theta_{13}}$$

$$P^{solar} \gg P^{atmos} \rightarrow A_{\nu_e \nu_\mu (\nu_\tau)}^{CP,T} \sim \frac{\sin 2\theta_{13}}{\Delta_{12} L}$$

$$P^{solar} \simeq P^{atmos} \rightarrow A_{\nu_e \nu_\mu (\nu_\tau)}^{CP,T} = O(1)$$

$$P(\nu_e \rightarrow \nu_\mu) \text{ vs } P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$$



Golden Channel in matter

In matter:

$$\begin{aligned}
 P_{\nu_e \nu_\mu}(\bar{\nu}_e \bar{\nu}_\mu) = & \underbrace{s_{23}^2}_{\text{Octant dependence}} \sin^2 2\theta_{13} \underbrace{\left(\frac{\Delta_{13}}{B_\pm} \right)^2 \sin^2 \left(\frac{B_\pm L}{2} \right)}_{\text{Hierarchy dependence}} \\
 & + c_{23}^2 \sin^2 2\theta_{12} \left(\frac{\Delta_{12}}{A} \right)^2 \sin^2 \left(\frac{AL}{2} \right) \\
 & + \tilde{J} \frac{\Delta_{12}}{A} \sin\left(\frac{AL}{2}\right) \frac{\Delta_{13}}{B_\pm} \sin\left(\frac{B_\pm L}{2}\right) \cos\left(\pm\delta - \frac{\Delta_{13} L}{2}\right)
 \end{aligned}$$

$$\tilde{J} \equiv c_{13} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \quad B_\pm \equiv \sqrt{2}G_F n_e \pm \Delta_{13}$$

Cervera et al, hep-ph/0002108

Parameter degeneracies (eg. neutrino hierarchy, octant) compromise δ sensitivity

Burguet et al, hep-ph/0103258

Minakata, Nunokawa hep-ph/0108085

Barger, Marfatia, Whisnant hep-ph/0112119

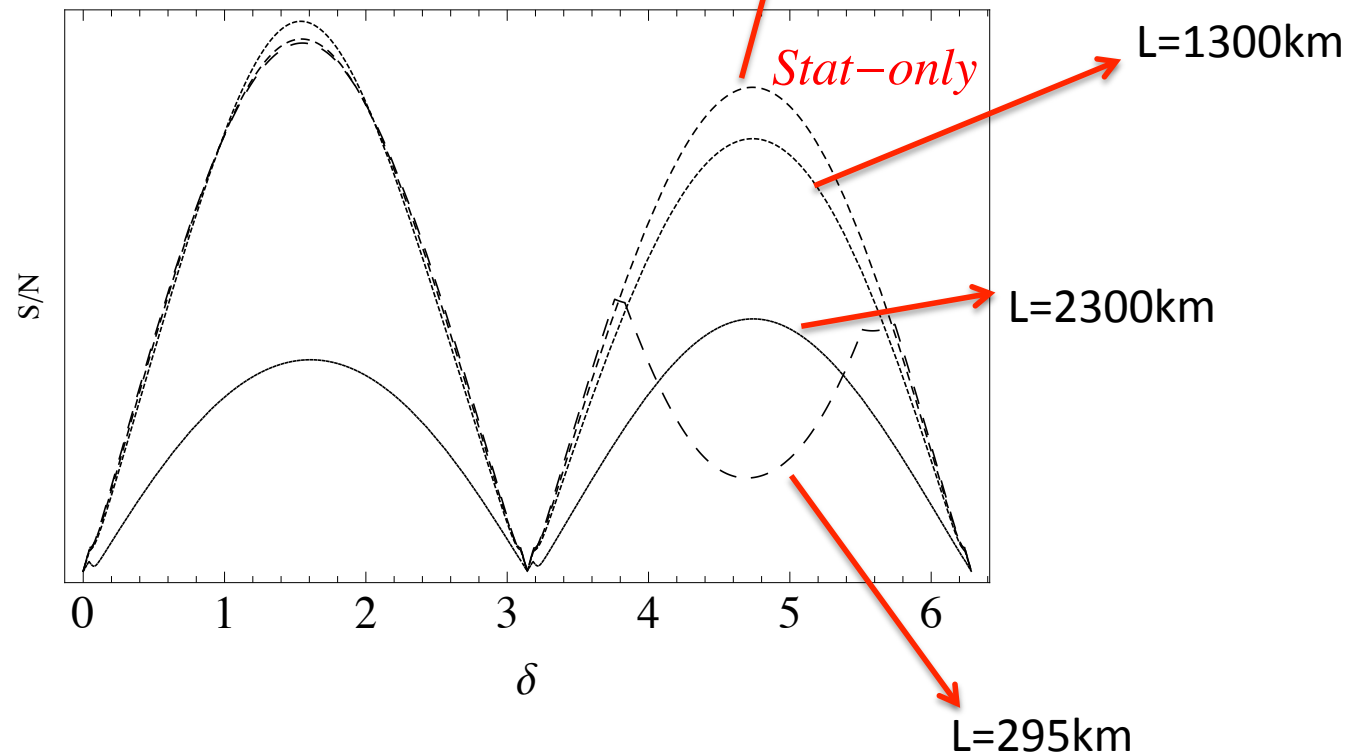
Optimization E, L

$$\begin{aligned} N_{events}(E, L) &= \Phi(E, L) \otimes \sigma(E) \otimes P_{osc}(E, L) \\ &= \frac{E^\alpha}{L^2} \times E^\beta \times P(E, L) \end{aligned}$$

Ignoring the hierarchy degeneracy: **S**ignal/**N**oise ($\delta \neq 0, \delta \neq \pi$) maximizes at

$$\frac{E}{L} = \frac{|\Delta m_{23}^2|}{2\pi}$$

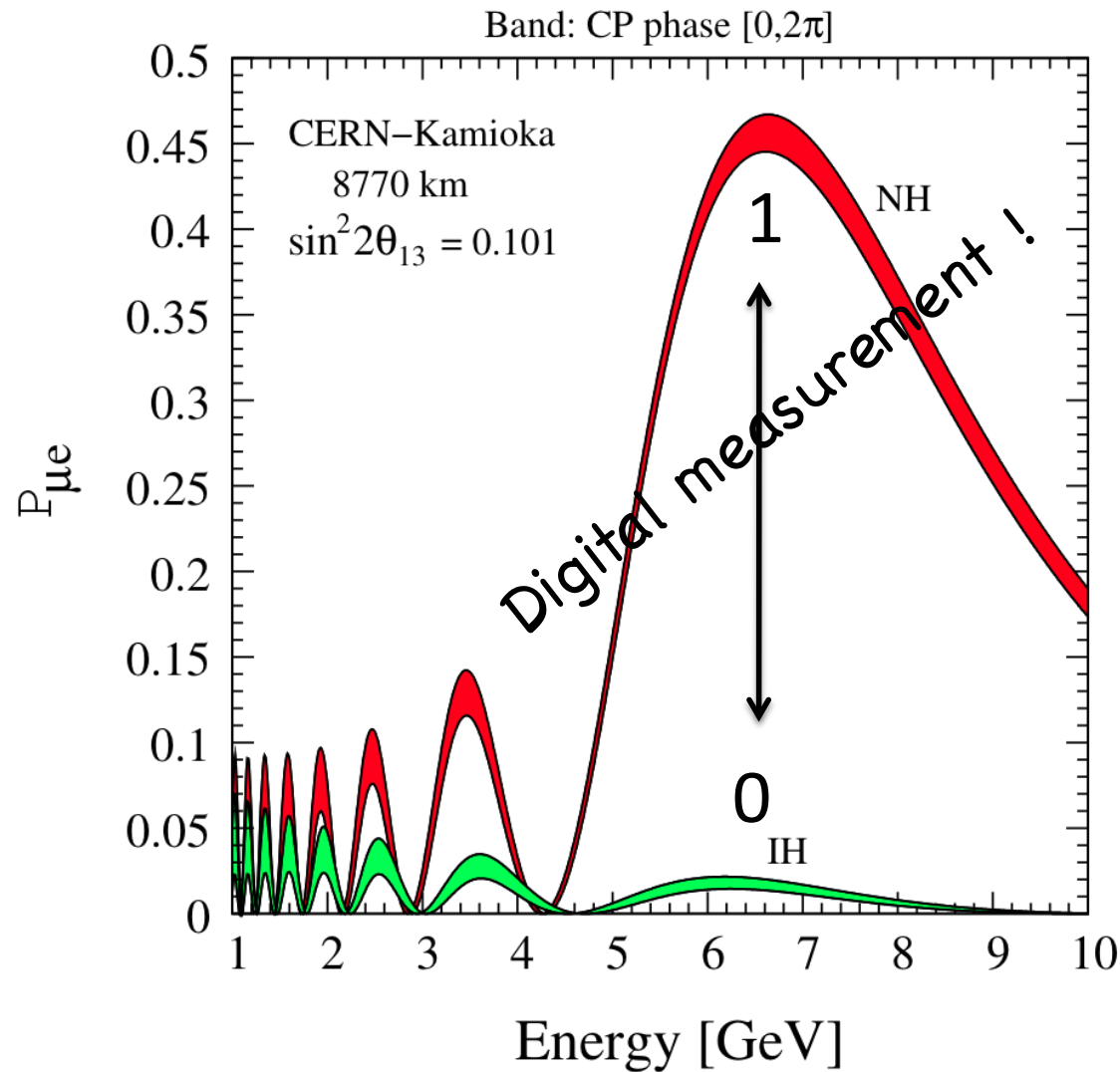
$$\text{@ } \frac{E}{L} = \frac{|\Delta m_{23}^2|}{2\pi}$$



Naive scaling of S/N assuming statistical errors dominate ...
 But systematics could change this conclusion...

To maximize sensitivity to CP violation don't go too far

Hierarchy through MSW



$$E_{\text{res}} \equiv \frac{\Delta m_{31}^2 \cos 2\theta_{13}}{2\sqrt{2}G_F n_e},$$

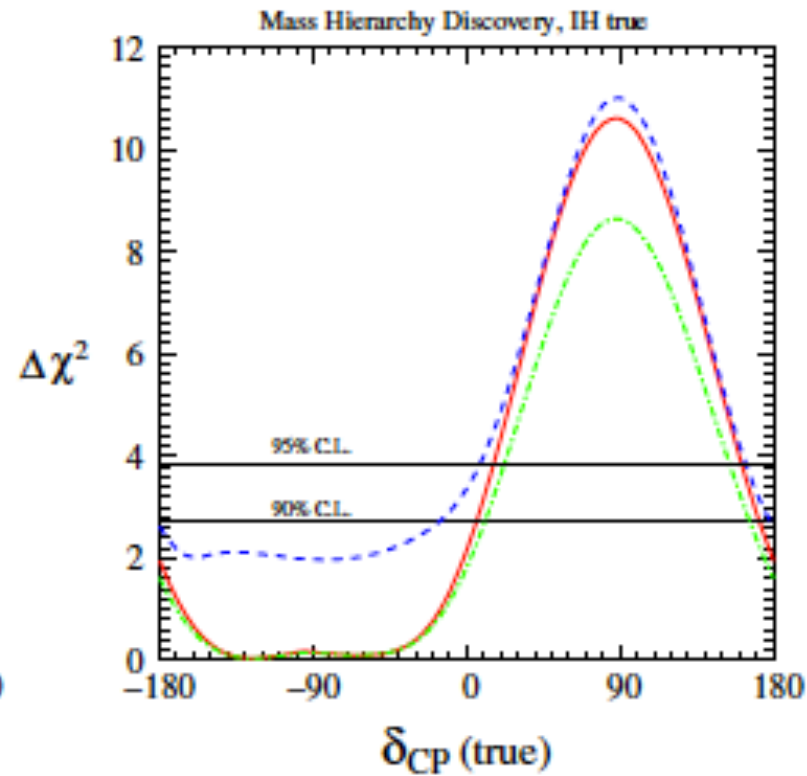
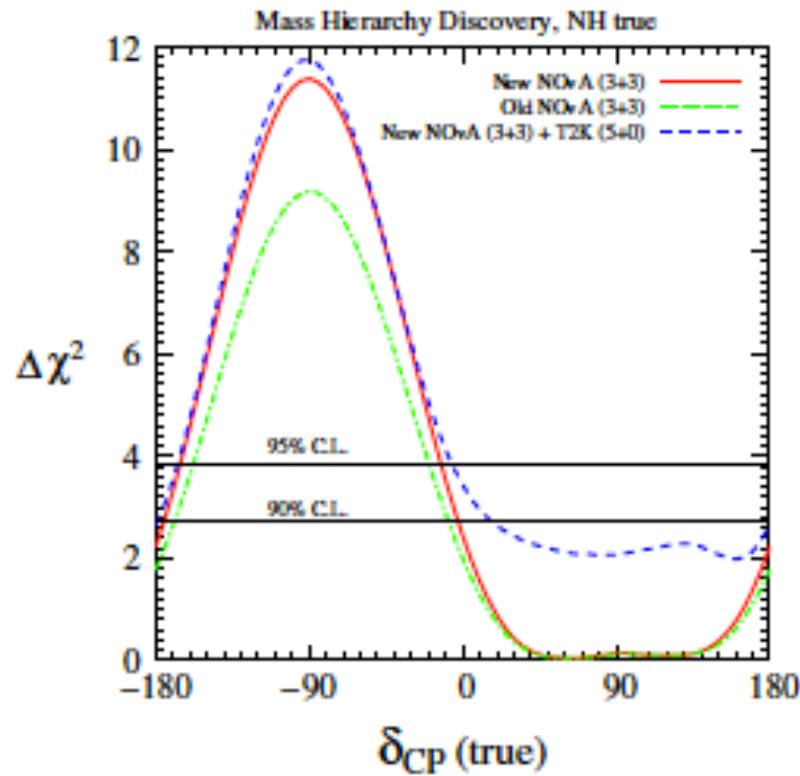
$$n_e(L)L|_{L_{\text{max}}} = \frac{\pi}{\sqrt{2}G_F \tan 2\theta_{13}}$$

Spectacular MSW effect at $O(6\text{GeV})$ and very long baselines: no need for spectral info nor two channels

Mikheev, Smirnov;Wolfenstein

First LBL experiments in < 10y : T2K, NOVA

T2K L=300 km
NOVA L=800km

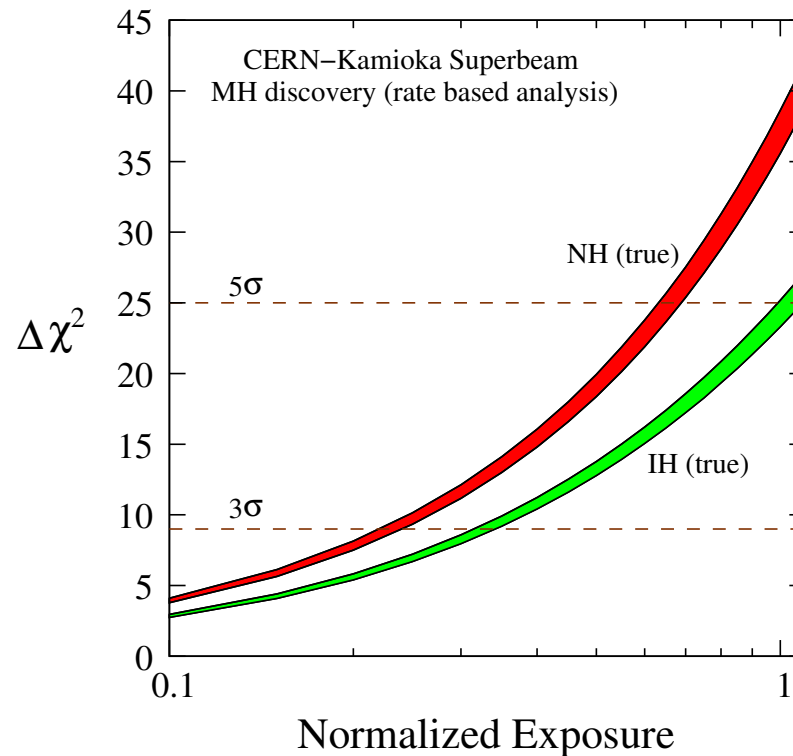


Agarwalla et al 1208.3644

With a new conventional beam (only ν) even with **existing** detectors, but shooting down !

One example: counting e-events at SuperK

4.5y of LBNO beam

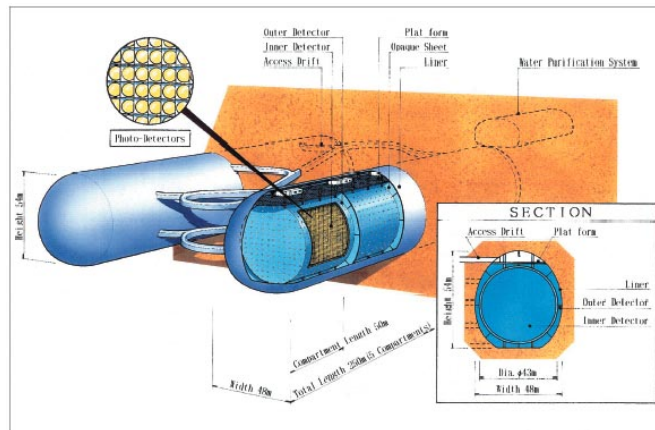


Hierarchy + CP in one go: a compromise...

Three concrete superbeam proposals (to be ready in 10y)

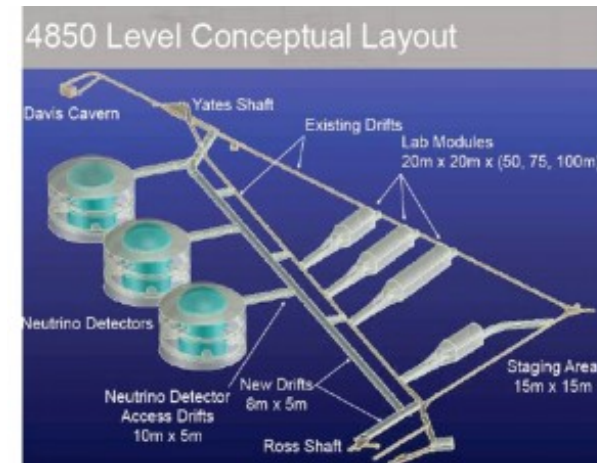
$p \rightarrow \text{Target} \rightarrow K, \pi \nu_\mu, \nu_e$

HyperK (Japan)



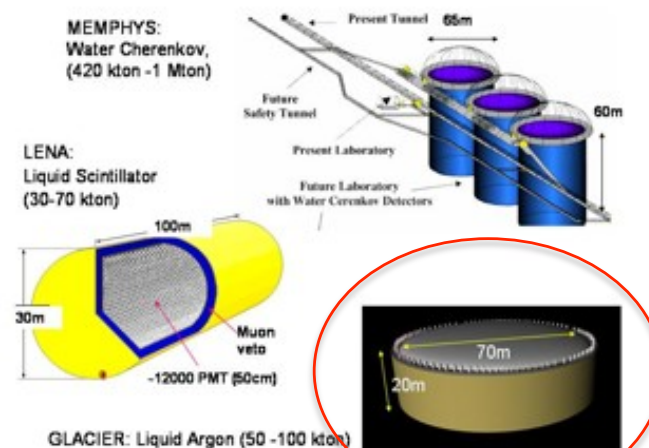
750MW, 560kton WC, Tokai-Kamioka (295km)

LBNE (USA)



800MW, 10kton-> 35kton LAr, Fermilab-Homestake(1300km)

LBNO (Europe)

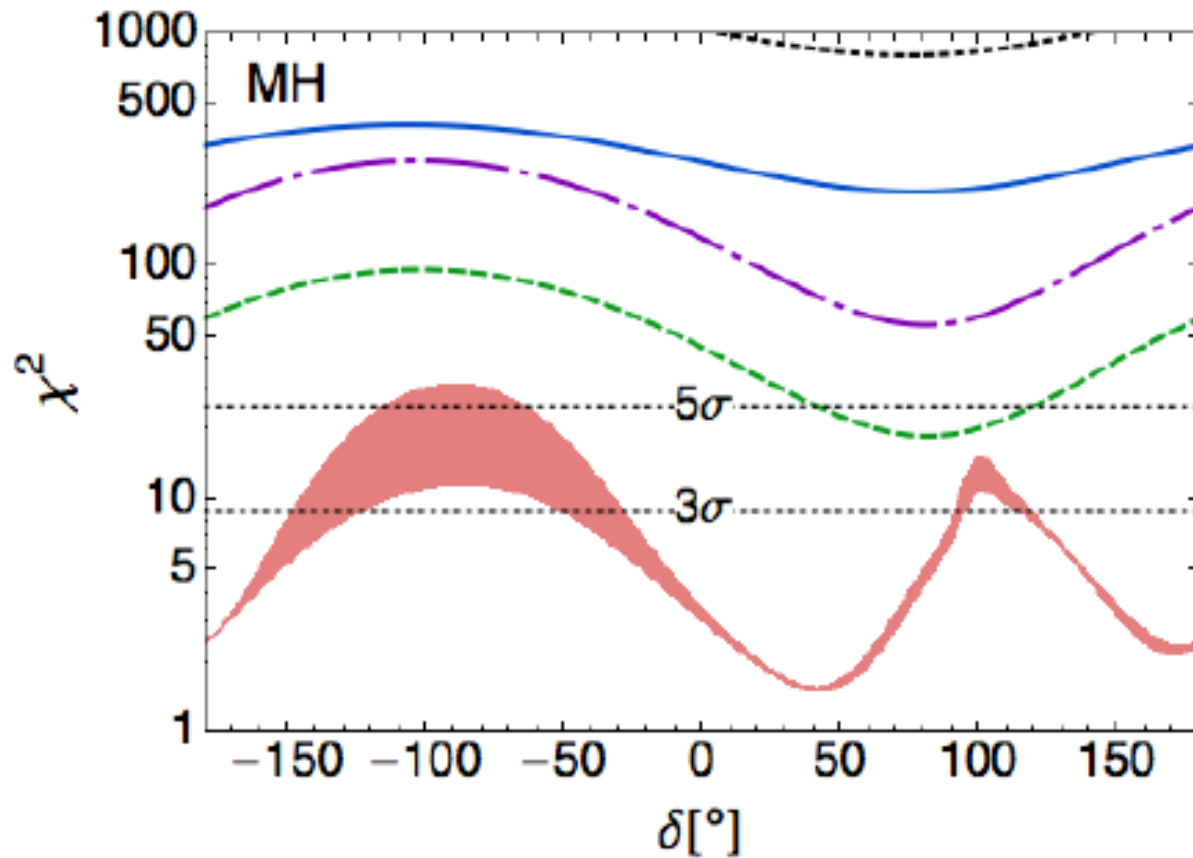


800MW, 20kton-> 100kton LAr, CERN-Pyhäsalmi (2300km)

In 20 years from now with conventional beams...

---- LBNO-100kt — LBNO-20kt
— LBNE-34kt - - - LBNE-10kt

■ T2HK

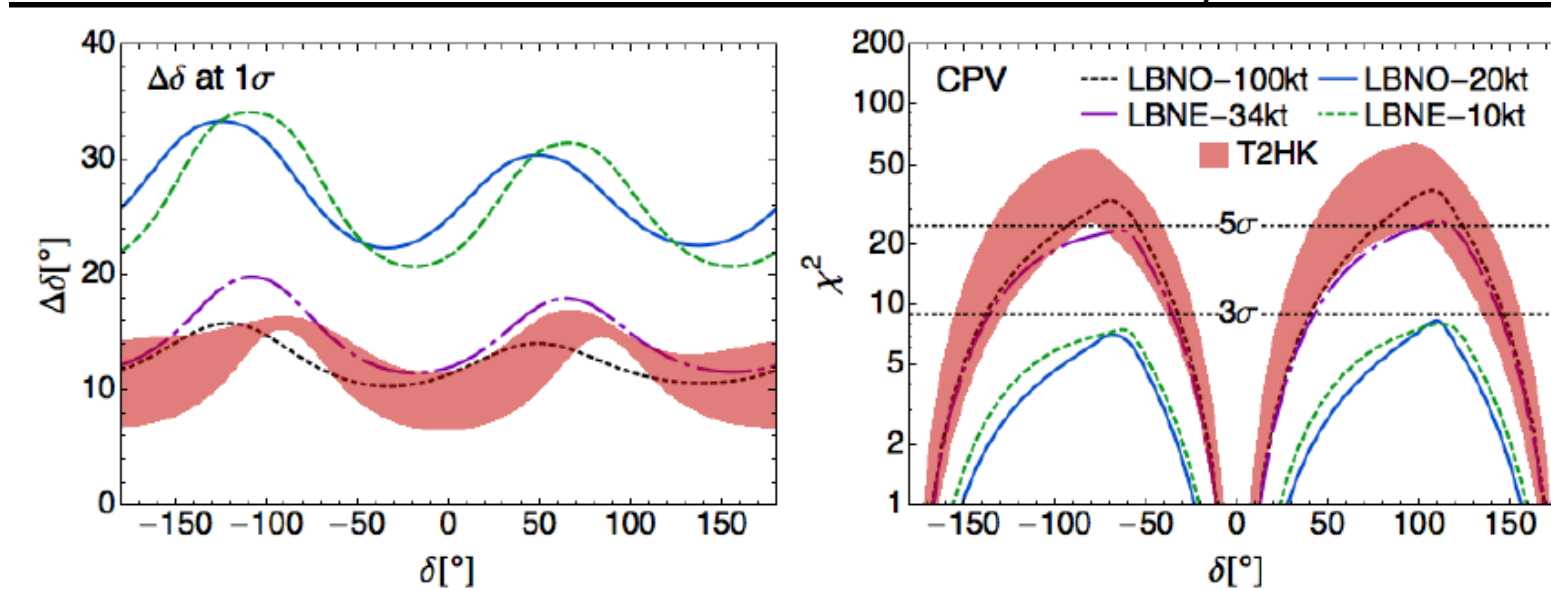


Compiled by P. Coloma

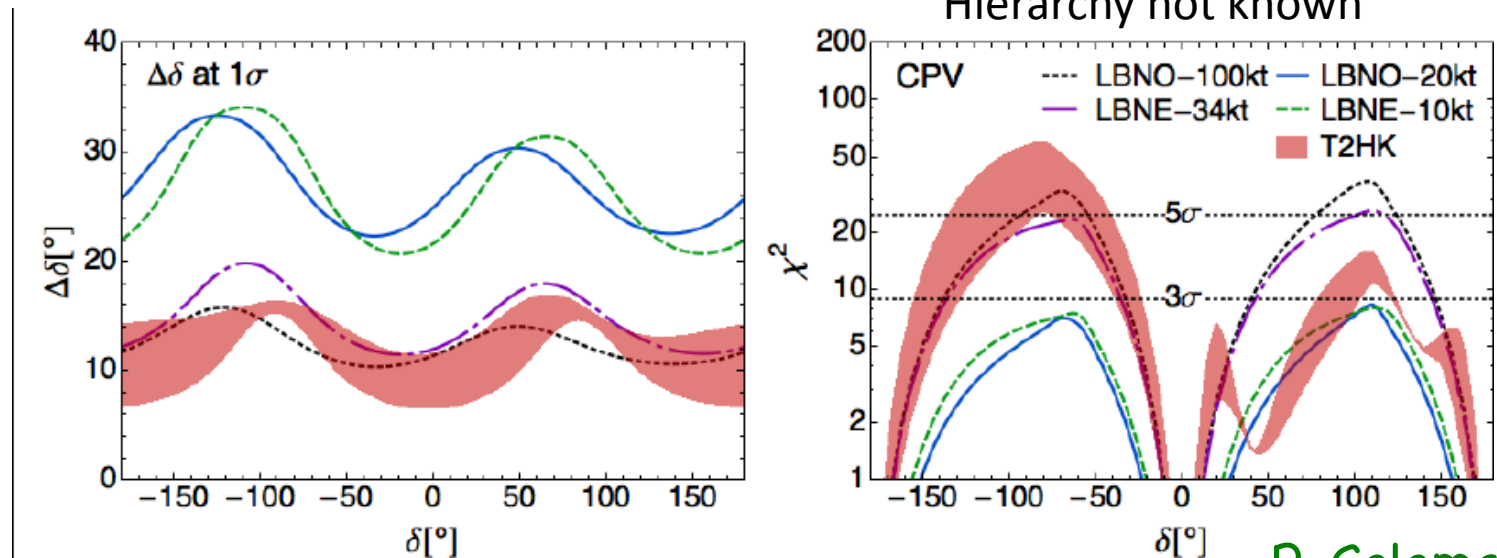
O(10kton) LAr can do the job easily

In 20 years from now with conventional beams...

Hierarchy known

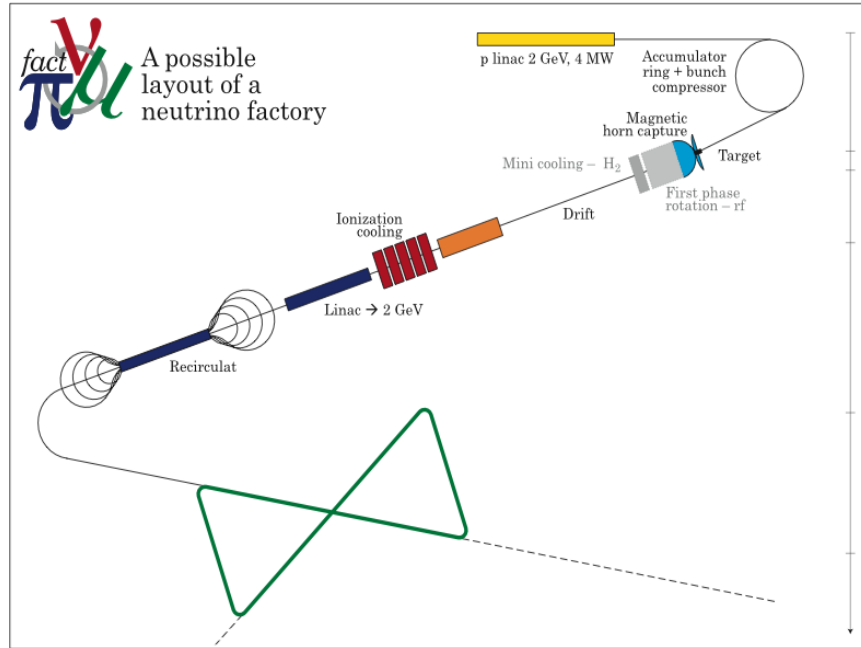


Hierarchy not known

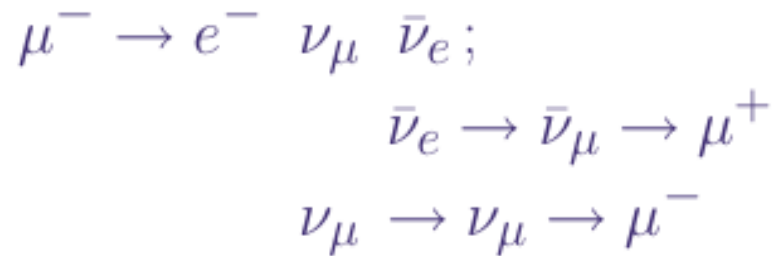


P. Coloma

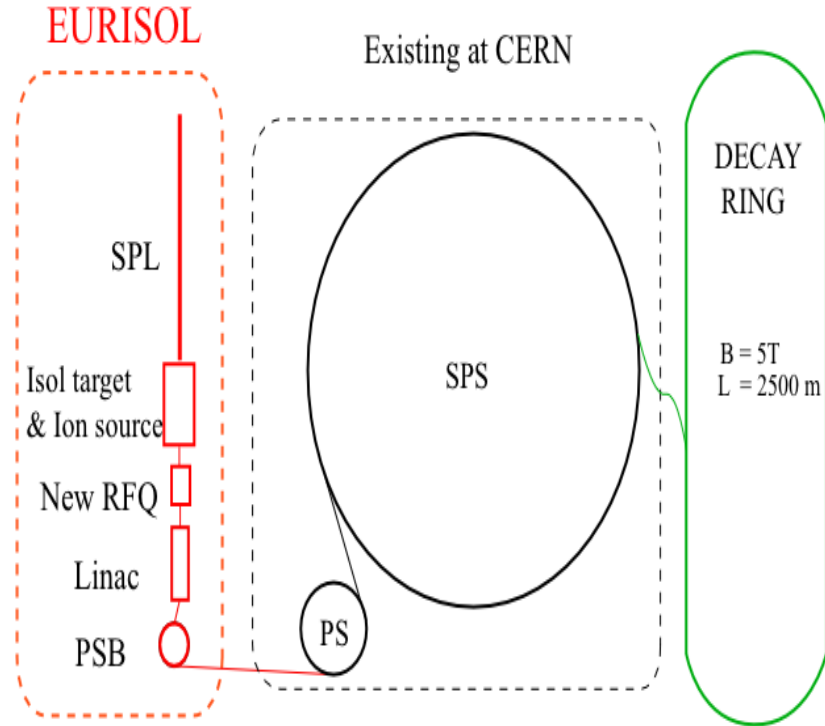
Neutrino factory



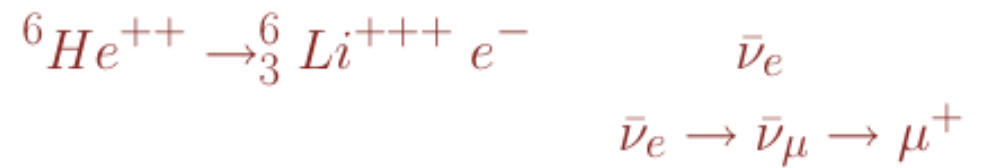
From μ decays



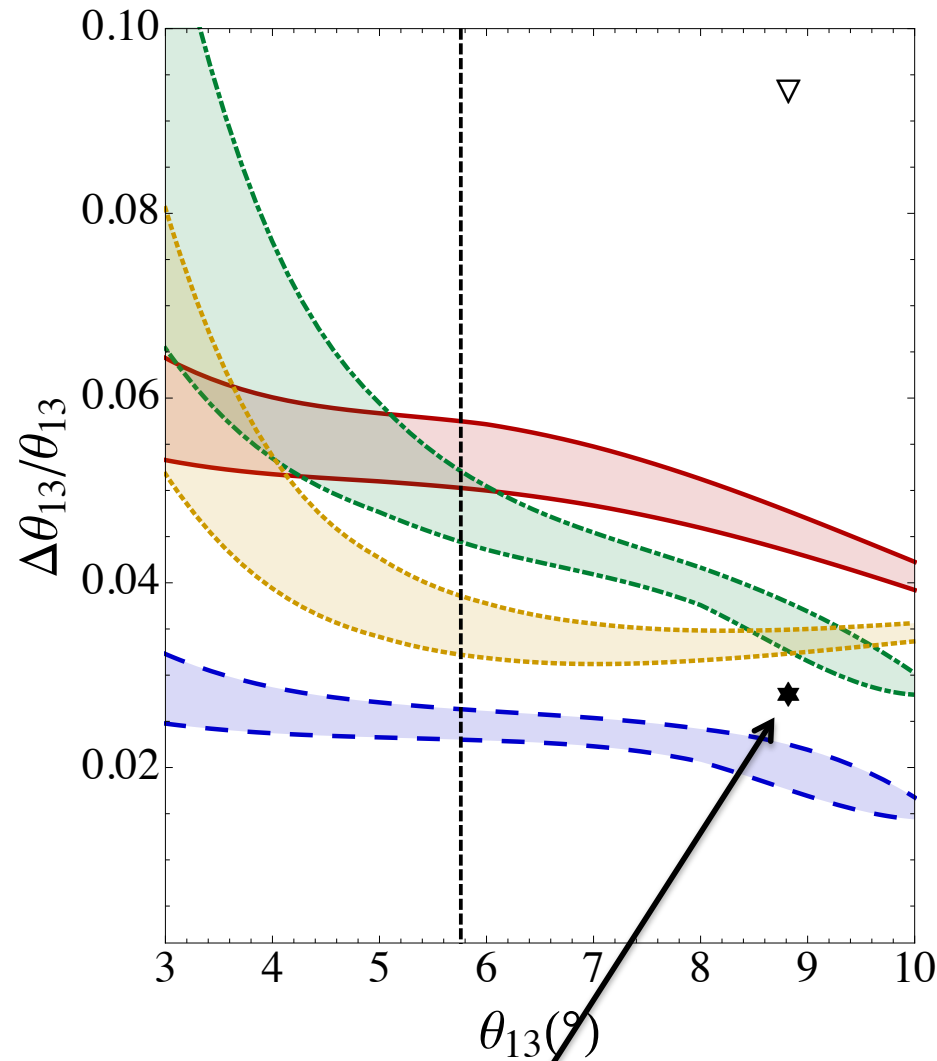
β beam



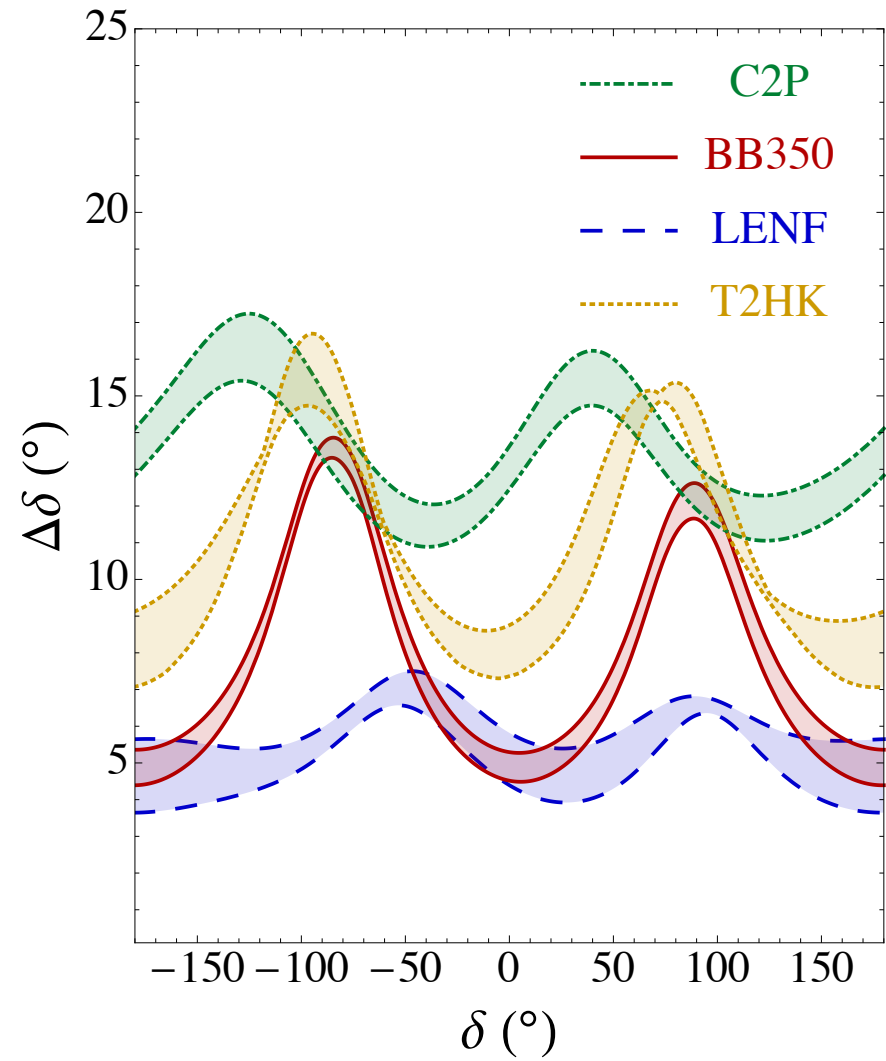
From radioactive ions



With better beams in XX years...



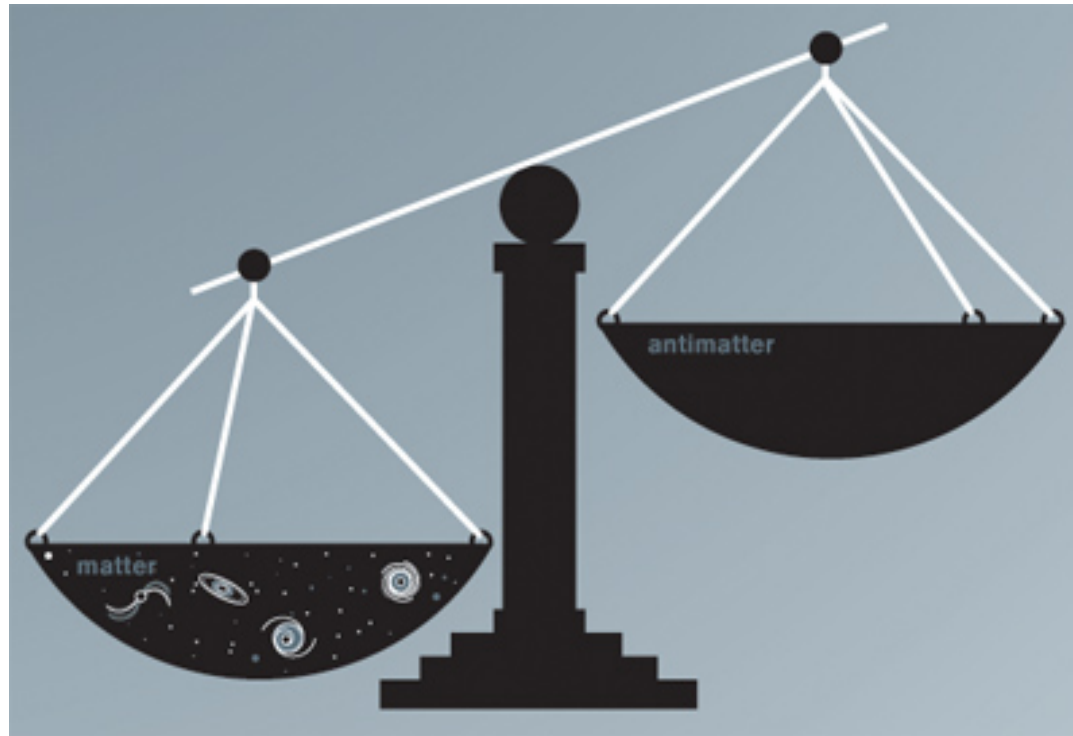
Daya Bay syst only!



Coloma, Donini, Fernandez-Martinez, PH 1203.5651

Baryon asymmetry

The Universe seems to be made of matter

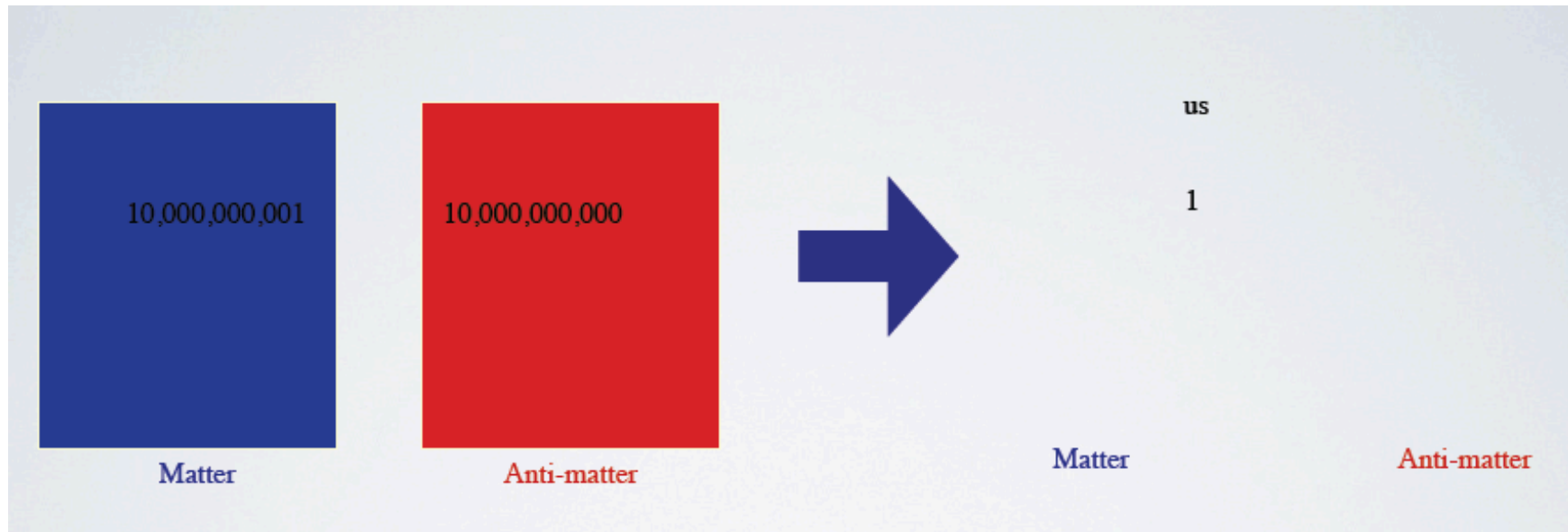


WMAP

$$\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = 6.21(16) \times 10^{-10}$$

Baryon asymmetry

In the early Universe this implies



WMAP

$$\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = 6.21(16) \times 10^{-10}$$

Baryon asymmetry

Can it arise from a symmetric initial condition with same matter & antimatter ?

Sakharov's necessary conditions for baryogenesis

- ✓ Baryon number violation (B+L violated in the Standard Model)
- ✓ C and CP violation (both violated in the SM)
- ✓ Deviation from thermal equilibrium (at least once: electroweak phase transition)

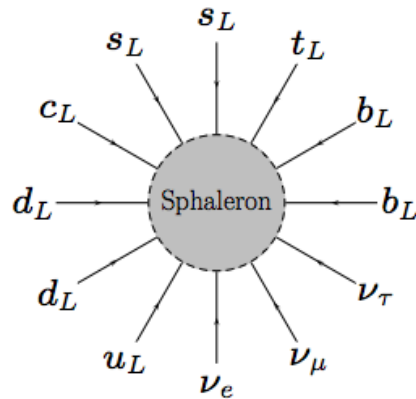
It does not seem to work in the SM with massless neutrinos ...

CP violation in quark sector far too small, EW phase transition too weak...

Baryon number violation

In the SM there is violation of $B+L$, preserve $B-L$

These processes are strongly suppressed at $T < T_{EW}$ and in equilibrium at $T > T_{EW}$



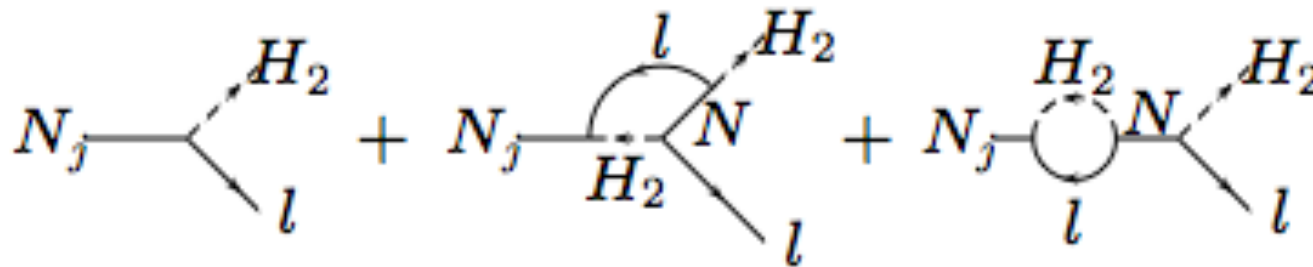
If $(B-L)$ is generated above T_{EW} sphalerons produce B

$$Y_B \simeq \frac{12}{37} Y_{B-L}$$

L, C and CP violation

New sources of CP violation and L violation in the neutrino sector can induce CP asymmetries in decays of heavy Majorana ν

Fukuyita, Yanagida



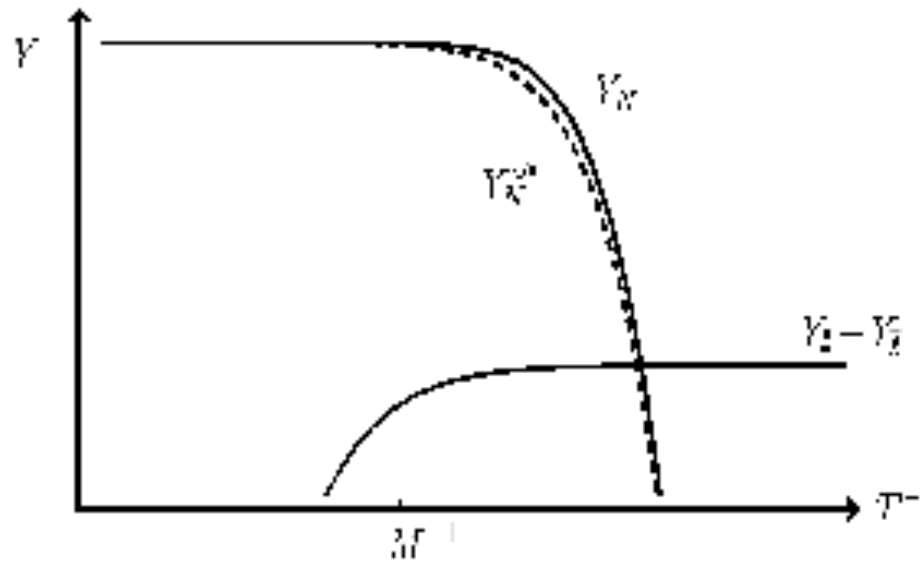
$$\epsilon_1 = \frac{\Gamma(N \rightarrow \Phi l) - \Gamma(N \rightarrow \Phi \bar{l})}{\Gamma(N \rightarrow \Phi l) + \Gamma(N \rightarrow \Phi \bar{l})}$$

Generic and robust feature of see-saw models

Out-of-equilibrium

The Majorana neutrinos decay out-of-equilibrium

$$\Gamma_N < \text{expansion rate} \Rightarrow N_N > N_N^{\text{thermal}}$$



Lepton asymmetry (seesaw I)

$$M_{2,3} \gg M_1$$

$$Y_B = 4 \times 10^{-3} \quad \underbrace{\quad}_{\epsilon_1} \quad \text{CP-asym eff. factor} \quad \underbrace{\quad}_{\kappa}$$

$$\epsilon_1 = -\frac{3}{16\pi} \sum_i \frac{\text{Im}[(\lambda_\nu^\dagger \lambda_\nu)_{i1}^2]}{(\lambda^\dagger \lambda)_{11}} \frac{M_1}{M_i} \longleftrightarrow m_\nu = \lambda_\nu^T \frac{1}{M} \lambda_\nu$$

Different combinations

Large enough asymmetry

$$|\epsilon_1| \leq \frac{8}{16\pi} \frac{M_1}{v^2} |\Delta m_{\text{atm}}^2|^{1/2}$$

$$M_1 \geq 10^9 \text{ GeV}$$

Davidson, Ibarra

Sufficiently large wash-out factor κ

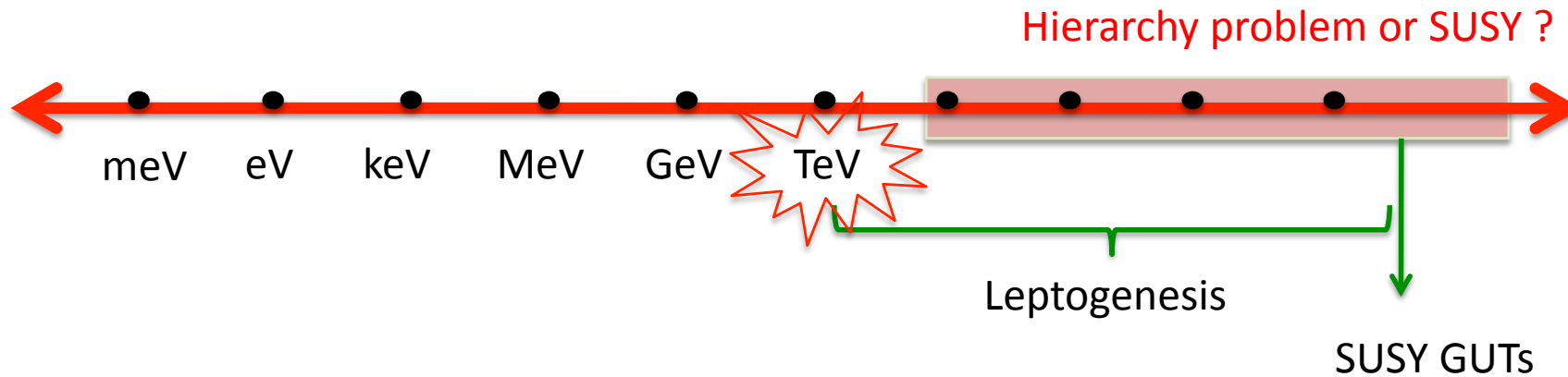
$$m_{\nu(\text{min})} < O(\text{eV})$$

Leptogenesis stew

But neutrino mass matrix does not provide the exact recipe for a precise prediction!

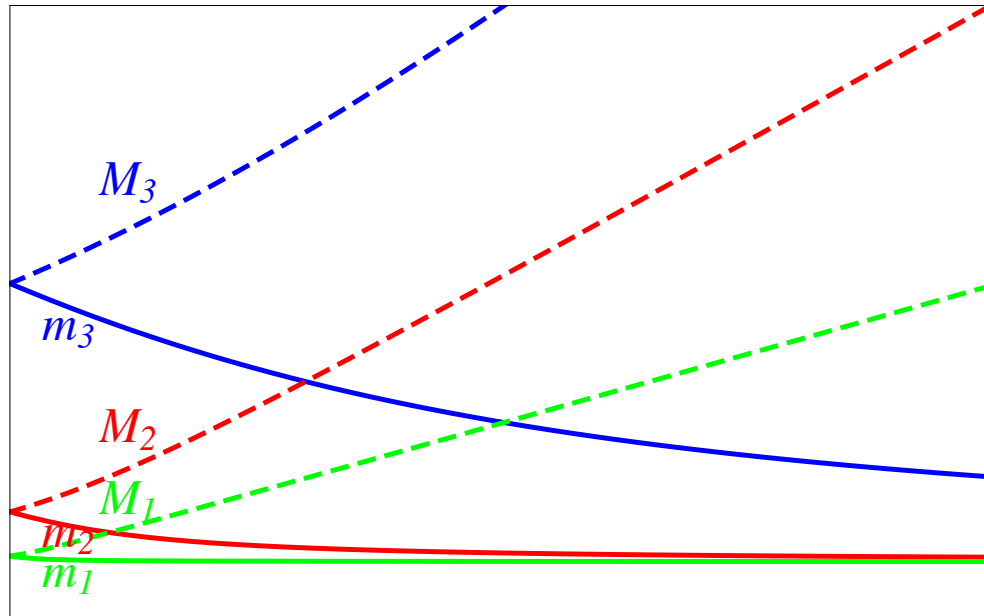


Pinning down the New physics scale



But could it be lower ? Yes...

Low scale seesaw I models



$$\theta_{hl} \sim \frac{Yv}{M_N} \sim \sqrt{m_\nu/M_N}$$

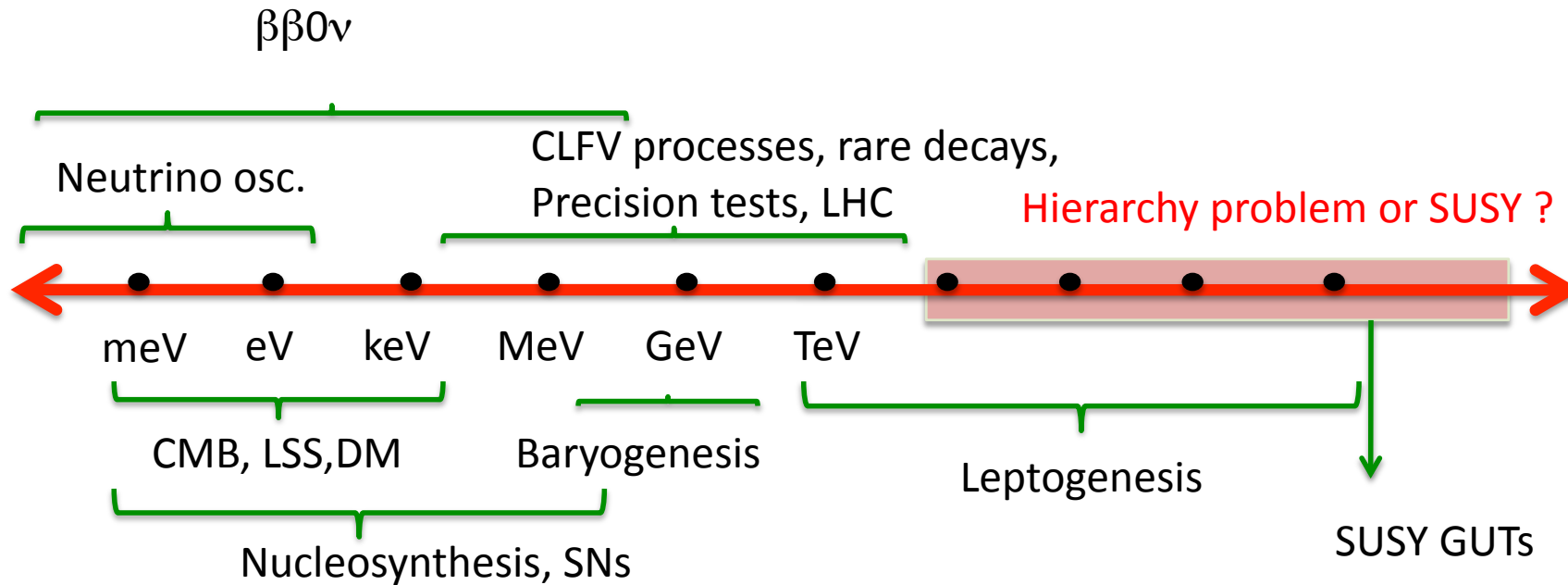
Dirac

M_N

Seesaw

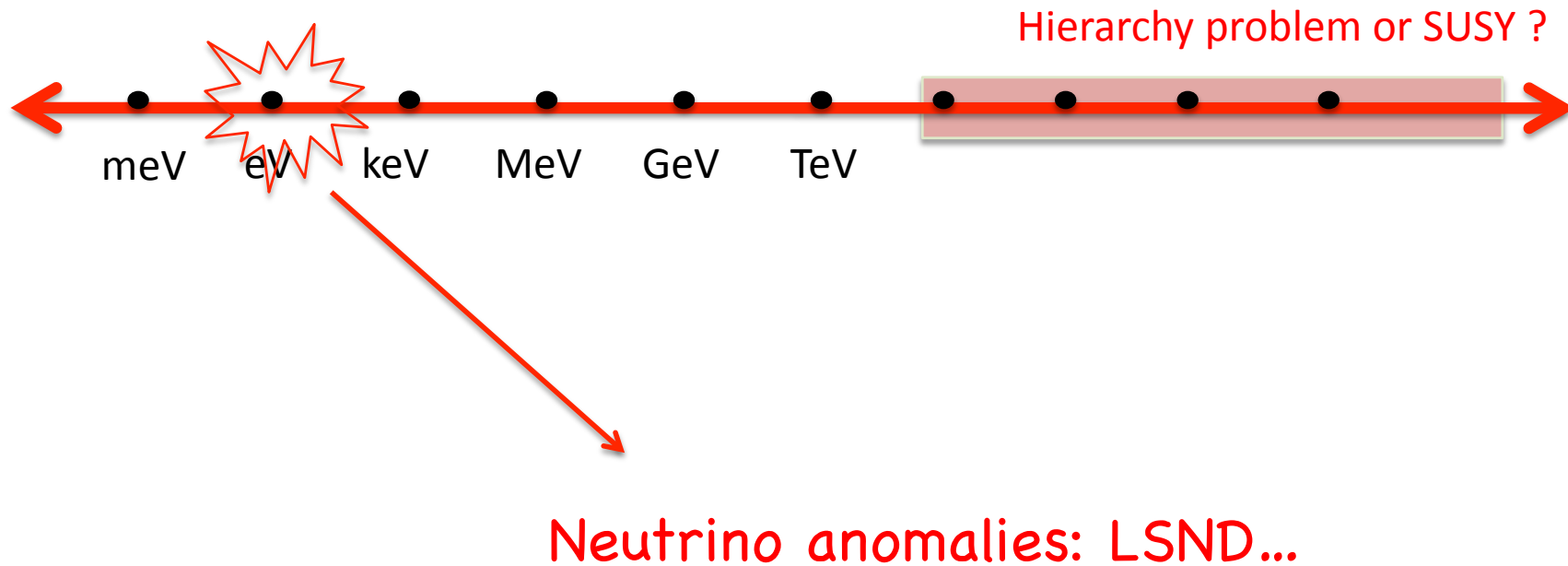
- More than three neutrinos → extra steriles
- Non unitarity in standard mixing
- Heavy states could be detected !

Pinning down the New physics scale

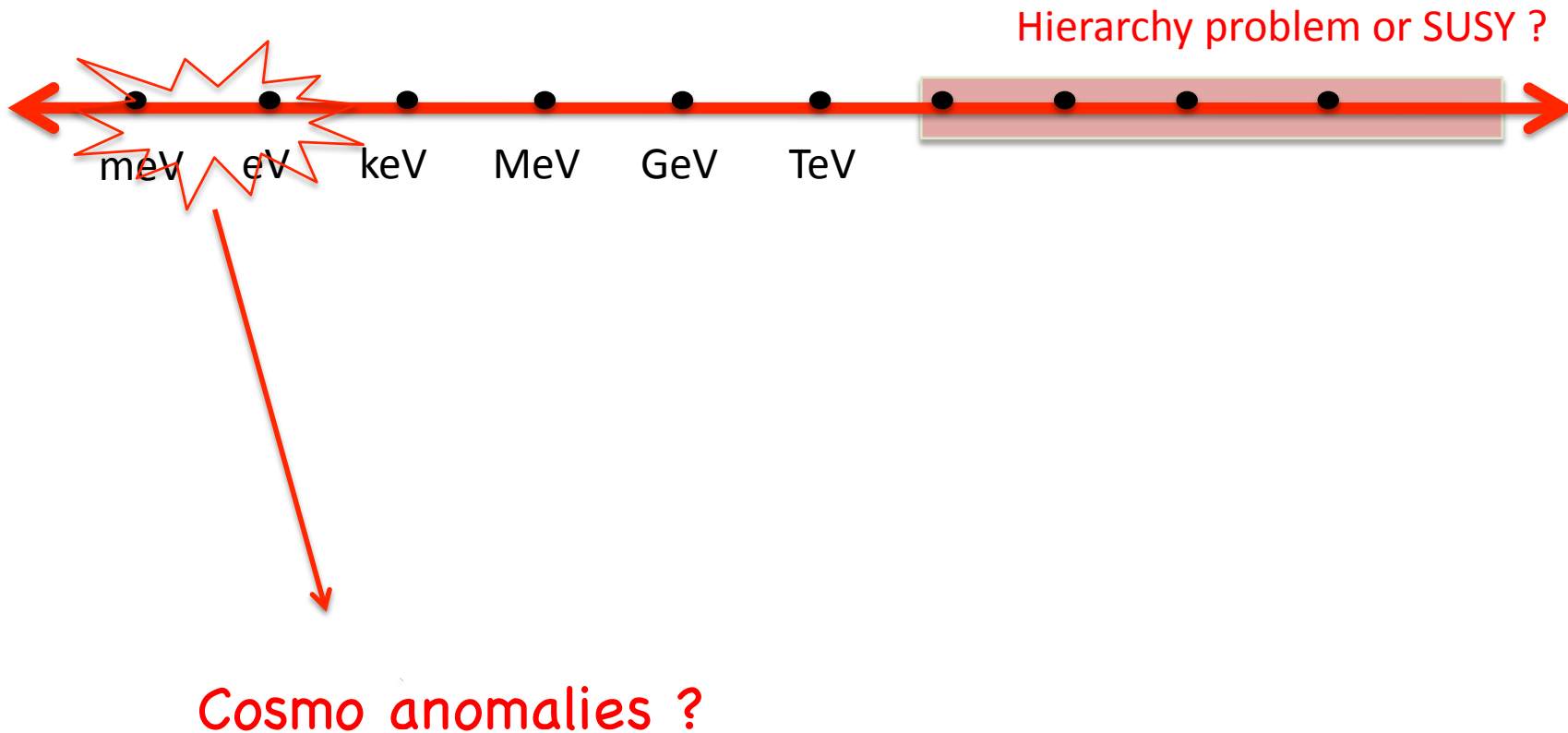


Light Sterile Neutrinos White Paper, Abazajian et al arXiv: 1204.5379 and refs. therein

Pinning down the New physics scale

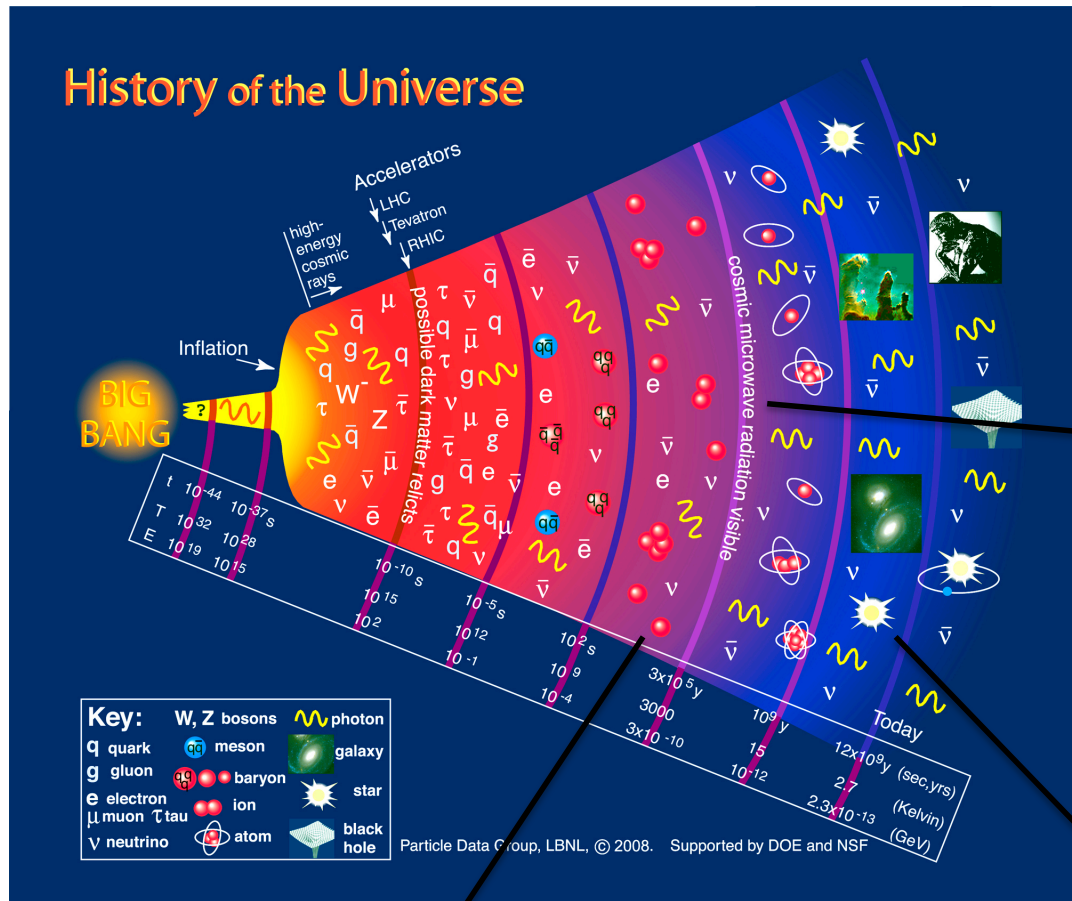


Pinning down the New physics scale



Cosmological neutrinos

Very hard to detect directly but they have left many traces in the history of the Universe

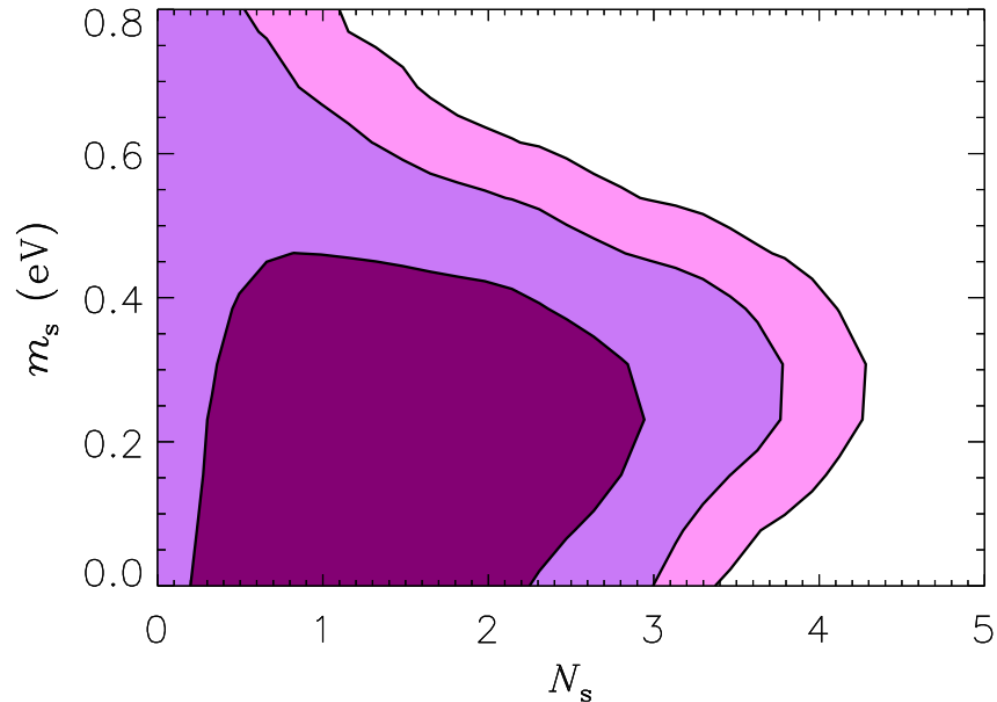


CMB \leftrightarrow N_ν

Nucleosynthesis \leftrightarrow N_ν

Galaxy distribution (LSS) \leftrightarrow $\sum_i m_i$

Cosmology



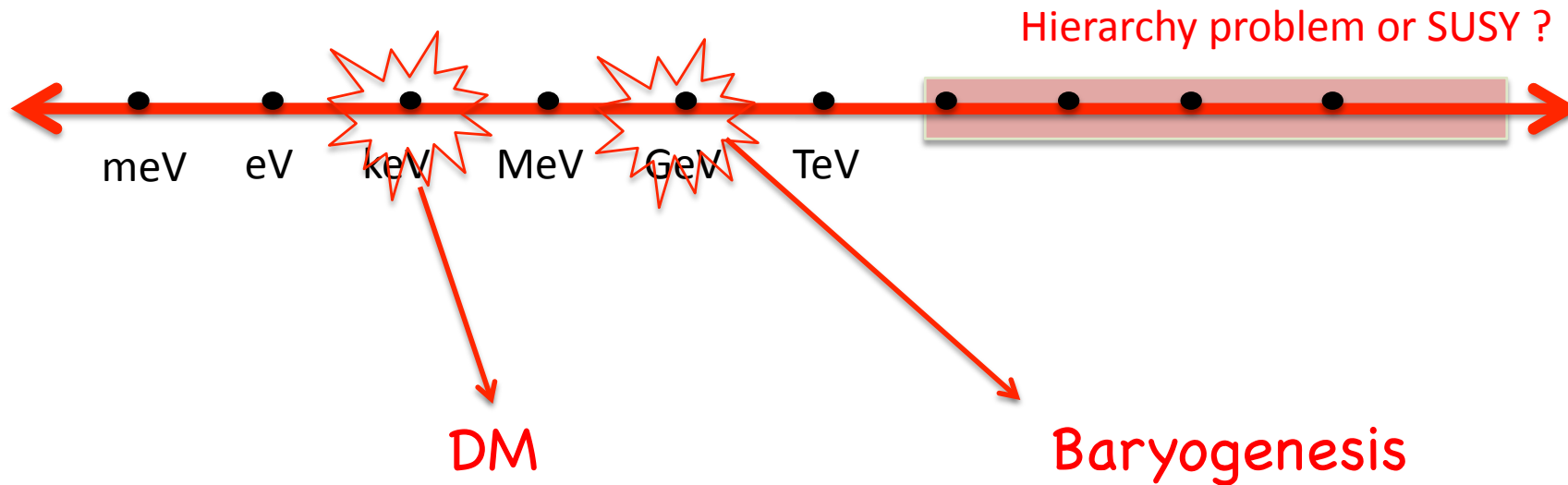
Hamann et al, ArXiv: 1006.5276

Sterile species favoured by **LSS** and **CMB**

Nucleosynthesis: $N_s = 0.68^{+0.80}_{-0.70}$ Izotov, Thuan

Many news to be expected in 2013 from Planck !

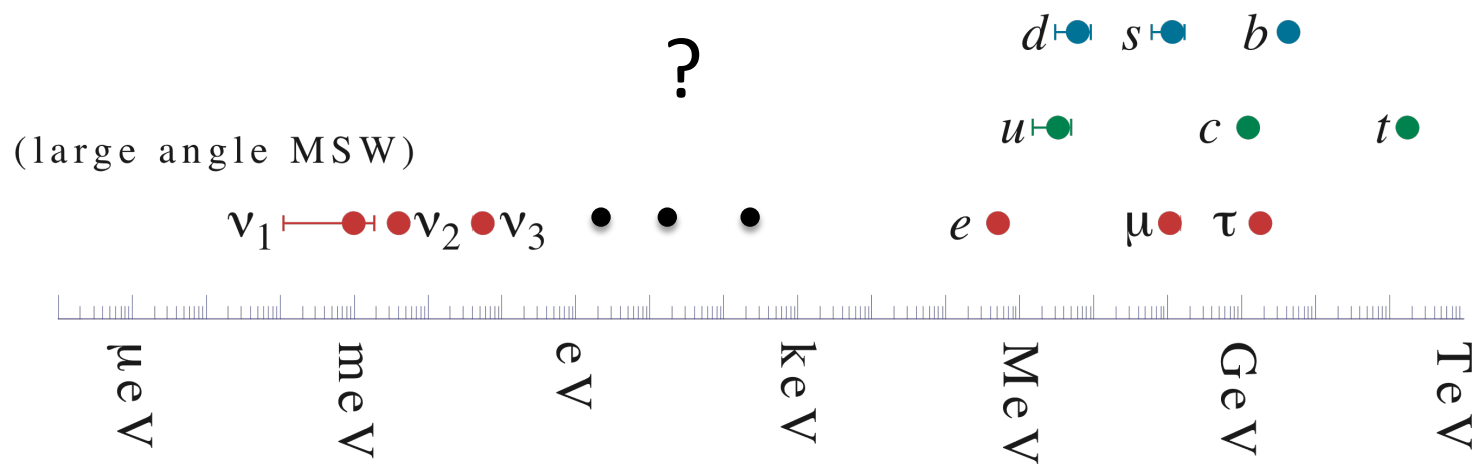
Pinning down the New physics scale



Even though there are typically more parameters than those in the neutrino mass, there are correlations...

Knowing as much as we can about what we can measure will make easier to search for and interpret the unexpected

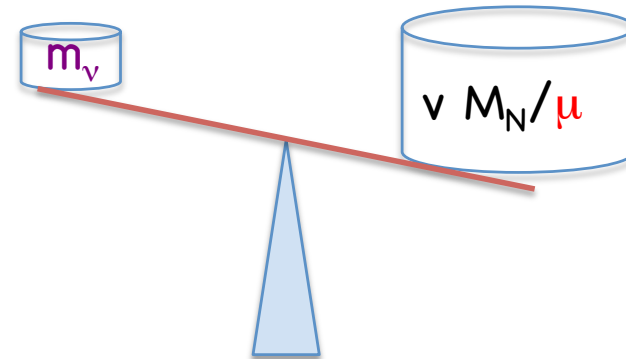
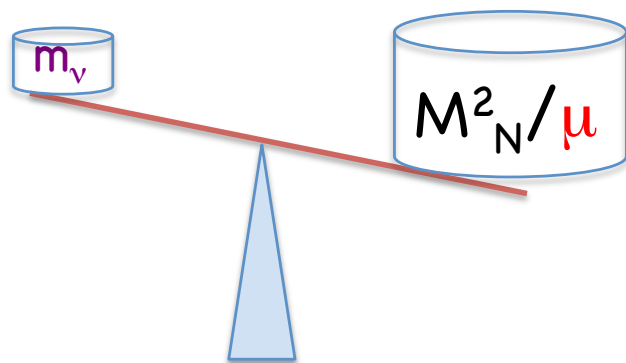
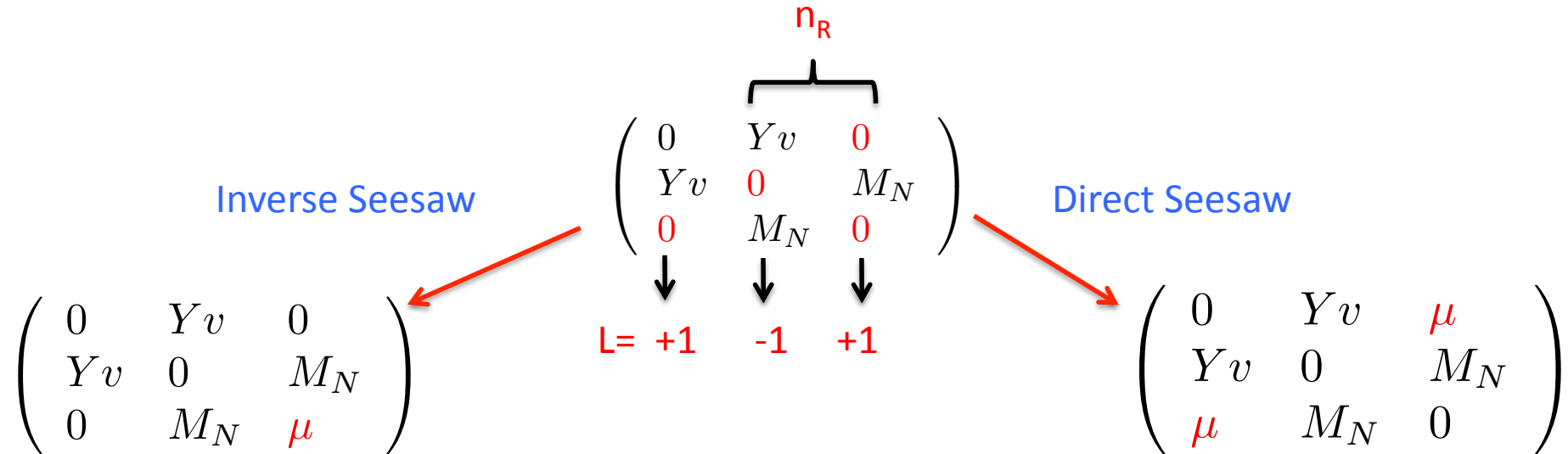
What about the hierarchy ?



Which hierarchy ?

Two scale see-saw models (approx) Lepton number

Wyler, Wolfenstein; Mohapatra, Valle;
Branco, Grimus, Lavoura, Malinsky, Romao,...



Y unsuppressed:

- > LFV effects large $\mu \rightarrow e \gamma$, etc
- > heavier spectrum M_N, Y, ν , at LHC

Kersten, Smirnov 07; Abada et al 07; Gavela, et al 09

Minimal Flavour type I seesaw model

Gavela, Hambye, Hernandez, PH 09

Only two extra Weyl fermions + SM

$$\frac{1}{2} \left(\nu_L \quad N_R^c \quad N_R'^c \right)^T C \begin{pmatrix} 0 & Y v & \epsilon Y' v \\ Y^T v & \mu' & \Lambda \\ \epsilon Y'^T v & \Lambda^T & \mu \end{pmatrix} \begin{pmatrix} \nu_l \\ N_R^c \\ N_R'^c \end{pmatrix}$$

Y, Y' complex vectors, Λ, μ, μ' numbers

$$c_{\alpha\beta}^{d=5} \equiv \epsilon \left(Y_N'^T \frac{1}{\Lambda^T} Y_N + Y_N^T \frac{1}{\Lambda} Y_N' \right)_{\alpha\beta} - \left(Y_N^T \frac{1}{\Lambda} \mu \frac{1}{\Lambda^T} Y_N \right)_{\alpha\beta},$$

$$c_{\alpha\beta}^{d=6} \equiv \left(Y_N^\dagger \frac{1}{\Lambda^\dagger \Lambda} Y_N \right)_{\alpha\beta} + \mathcal{O}(\epsilon).$$

Minimal type I seesaw model

Neutrino masses & mixings determine Y up to a global normalization (and Majorana phase)

$$Y_{Ni} = \frac{y}{2} \sqrt{\frac{m_3}{m_2 + m_3}} \left(U_{i3}^* + \sqrt{\frac{m_2}{m_3}} U_{i2}^* \right) \quad \text{Normal hierarchy}$$

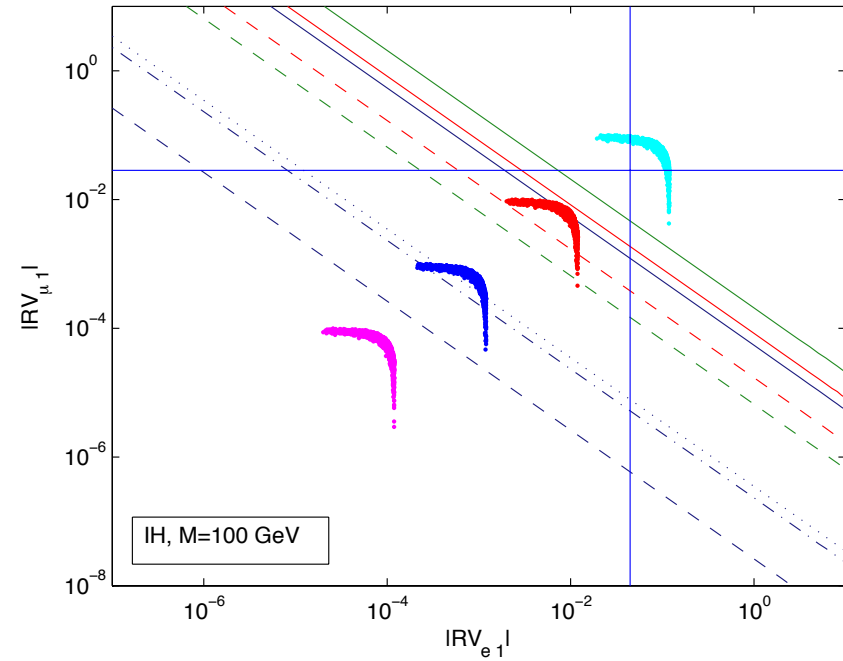
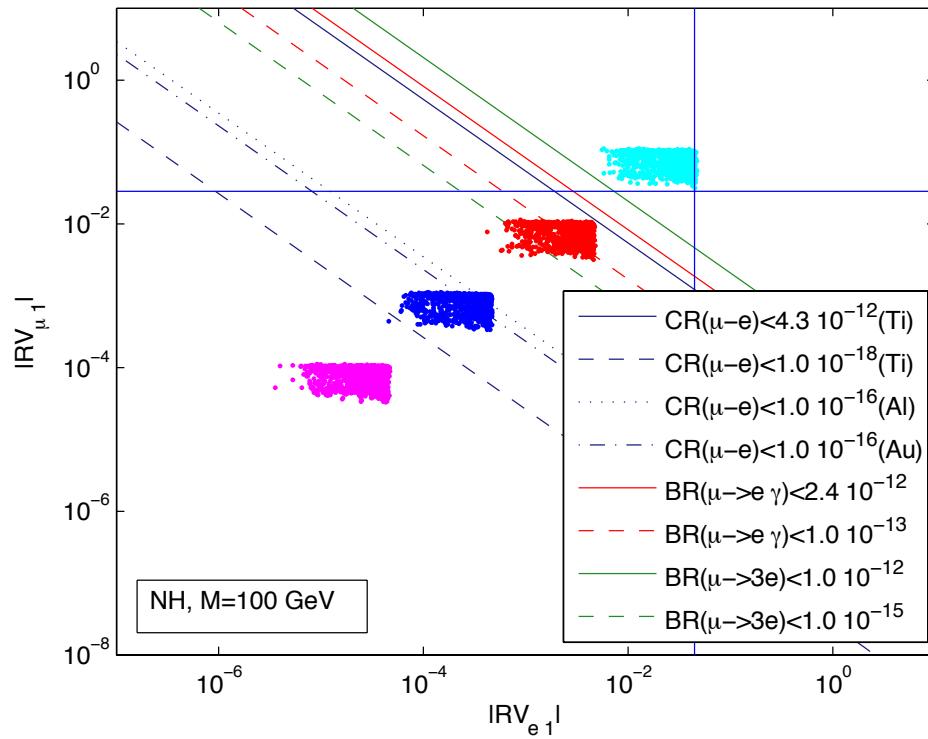
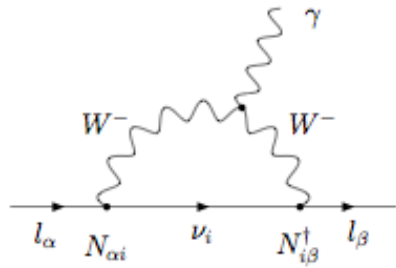
Inverted hierarchy

$$Y_{Ni} = \frac{y}{2} \sqrt{\frac{m_2}{m_1 + m_2}} \left(U_{i2}^* + \sqrt{\frac{m_1}{m_2}} U_{i1}^* \right)$$

$$B(l_i \rightarrow l_j \gamma) \sim |(Y_N)_i (Y_N)_j|^2$$

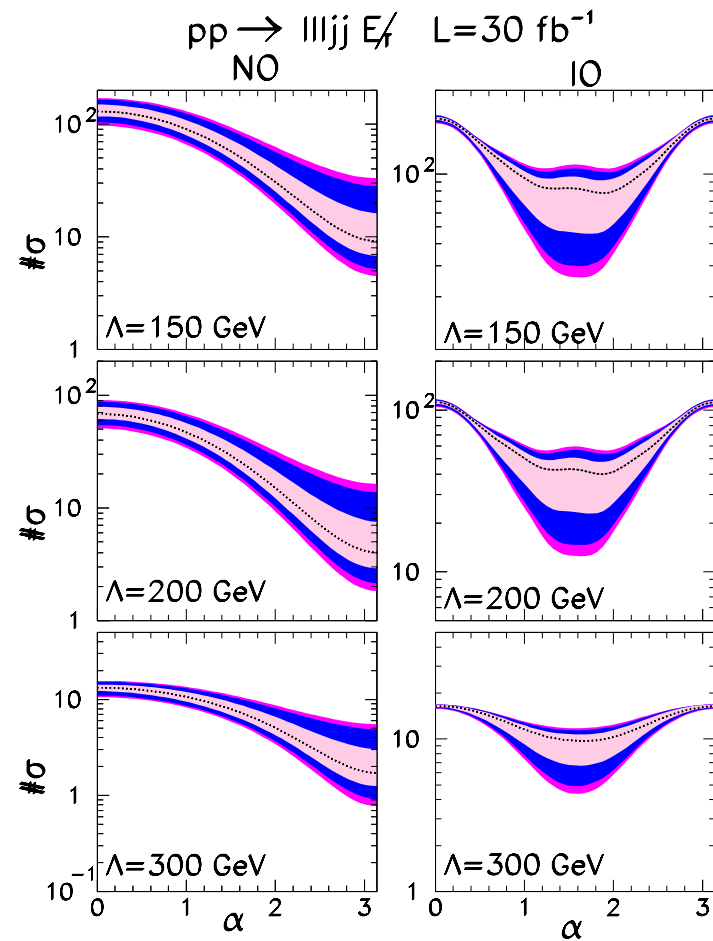
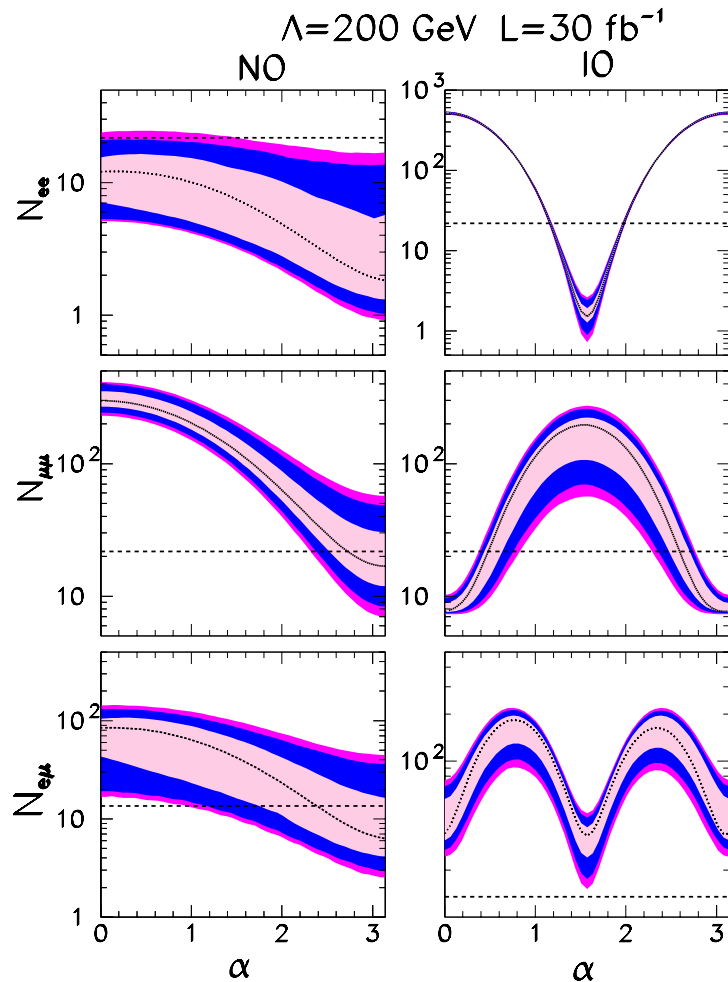
Also Raidal, Strumia, Turzynski 2004; Ibarra et al 2010

CLFV within reach



Dinh, Ibarra, Molinaro, Petcov 2012

Minimal Flavour See-saw III at LHC



Conclusions

What the SM does not explain...

Gauge Couplings	EWSB	Quark flavour	Lepton flavour (minimal)
$g_{\text{SU}(3)}, g_{\text{SU}(2)}, g_{\text{U}(1)}$	m_h, v	$\theta_{12}, \theta_{23}, \theta_{13}, \phi$ m_u, m_c, m_t m_d, m_s, m_b	$\theta_{12}, \theta_{23}, \theta_{13}, \delta$ m_1, m_2, m_3 $, m_e, m_\mu, m_\tau$

A more fundamental theory BSM should explain the origin of these parameters

- We still don't know what the $v\text{SM}$ is
- Neutrinos add at least as many parameters as quarks to the puzzle, but **with features that might hint to a new physics scale**

- The existence of a new physics scale in ν SM whether related or not to the EW scale would have clear implications for the hierarchy problem and EWSB
- The observation of neutrinoless double beta decay would be the discovery of such a new physics scale
- Predicting the matter-antimatter asymmetry in the Universe would be a major achievement of the ν SM

Two key ingredients: Leptonic CP violation
Lepton number violation

- Mass Hierarchy essential for reconstructing the underlying model of neutrino masses & predictions for other observables

- Sterile neutrinos are not particularly exotic but a **generic** prediction of seesaw models

It is of extreme importance to clarify neutrino anomalies and establish if there are other light sterile states since

Our predictions/constraints on

- 1) matter-antimatter asymmetry
- 2) large-scale structure
- 3) nucleosynthesis
- 4) supernova explosions
- 5) the dark matter content of the Universe
- 6) rate of neutrinoless double beta decay

....

would depend on it !

These elusive pieces of reality have brought many surprises, maybe they will continue with their tradition...

