## Observational cosmology with X-ray luminous galaxy clusters

# Second Lecture: Cosmological Models and Cluster Abundance 

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## Cluster abundance and scaling relations

e.g. Mantz et al 08, 10a, 10b; Vikhlinin et al 09;

Rapetti et al. 09, 10; Schmidt et al 09

## Basic initial idea:

X-ray Cluster
Surveys; data: fluxes, z


## Theory: Growth of structure



- Simulated cosmologies to model the non-linear growth of structure.
- Even looking so apparently different can be conveniently related with the linear growth calculations through a fitting formula. (See e.g. Jenkins et al 2001, Tinker et al 2008, etc.)

Cole et al 2005

## Cluster abundance as a function of mass and redshift



## N -body simulations

Non-linear structure formation
Big clusters steep mass function; sensitive to the cosmological model; quintessence, selfinteracting, early, clustering dark energy as well as modified gravity

## Cluster abundance as a function of mass and redshift

## Linear theory

Sensitive to the cosmological model: quintessence, selfinteracting, early, clustering dark energy as well as modified gravity

## Cluster surveys



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## Cluster survey data



Low redshift (z<0.3)
$>$ BCS (Ebeling et al 98, 00) F $>4.4 \times 10^{-12} \mathrm{erg} \mathrm{s}^{-1} \mathrm{~cm}^{-2}$ ~33\% sky coverage
> REFLEX (Böhringer et al 04) F > $3.0 \times 10^{-12} \mathrm{erg} \mathrm{s}^{-1} \mathrm{~cm}^{-2}$ $\sim 33 \%$ sky coverage

## Intermediate redshifts ( $0.3<z<0.5$ )

$>$ Bright MACS (Ebeling et al 01, 10) $\mathrm{F}>2.0 \times 10^{-12} \mathrm{erg} \mathrm{s}^{-1} \mathrm{~cm}^{-2}$
$\sim 55 \%$ sky coverage
$\mathrm{L}>2.55 \times 10^{44} \mathrm{~h}_{70}{ }^{-2} \mathrm{erg} \mathrm{s}^{-1}$ (dashed line).
Cuts leave 78+126+34=238 massive clusters
All based on RASS detections. Continuous and all 100\% redshift complete.
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## Scaling relations data: $X$-ray follow-up for 94 clusters



Best fit for all the data (survey+follow-up+other data).


Both, power law, self-similar, constant log-normal scatter.

* Crucial: self-consistent and simultaneous analysis of survey+follow-up data, accounting for selection biases, degeneracies, covariances, and systematic uncertainties.
* Data does not require additional evolution beyond self-similar (see tests in Mantz et al 10b).
* Important cluster astrophysics conclusions (see Mantz et al 10b).

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# Gas mass fraction: calibration data Total mass proxy 



## New likelihood approach: simultaneous and self-consistent

- To properly account for selection biases [in a previous analysis of the mass function, Mantz et al 08 (M08), using an external data set to constrain the luminosity-mass relation, we restrict the data set of Reiprich \& Bohringer 02 to low redshift and high fluxes to minimize the effects of selection bias].
- M08, Vikhlinin et al 09a,b binned their detected clusters in redshift and mass with infinitesimally small bins taking the previous approach to its logical limit, but there was still no self-consistent fit for both scaling relations and cosmology.
- Generalization of M08 to allow a simultaneous and self-consistent fit using follow-up observations of flux-selected clusters over the whole redshift range of the data accounting for both Malmquist and Eddington biases.
- Likelihood can be derived from first principles beginning from a Bayesian regression model.
- General problem: counting sources as a function of their properties


## New likelihood approach: simultaneous and self-consistent

- A population function: <dN/dx> theoretical prediction of the distribution (i.e. number) of sources as a function of their properties.
- Population variables x (properties).
- Response variables $y$ obeying a stochastic scaling relation as a function of $x$.
- Stochastic scaling relation $\mathrm{P}(\mathrm{y} \mid \mathrm{x})$ : probability distribution of y given x .
- Observed values $\hat{x}$ and $\hat{y}$ (note that not all x and y need to be measured, except for those determining if a source belongs to the sample, i.e. if it is detected).
- Sampling distributions for the observations as a function of the population and response variables $P(\hat{x}, \hat{y} \mid x, y)$.
- A selection function $P(I \mid x, y, \hat{x}, \hat{y})$, where $/$ represents the inclusion in the sample, i.e. detection.


## New likelihood approach: simultaneous and self-consistent

- For our large sky coverage surveys of massive clusters we assume that the clustering of the sources is not important compared with the purely Poisson probability distribution of their occurrence (Hu \& Kravtsov 03; Holder 06).
- Binning derivation: We divide the observed space $(\hat{x}, \hat{y})$ into infinitesimal bins which contain at a maximum one detected source and the population function and scaling relations are assumed to be constant in each bin.

Expected number of detected sources

$$
\left\langle N_{\operatorname{det}, j}\right\rangle=\left(\Delta \hat{x}_{j} \Delta \hat{y}_{j}\right) \int \mathrm{d} x \int \mathrm{~d} y\left\langle\frac{\mathrm{~d} N}{\mathrm{~d} x}\right\rangle P(y \mid x) P\left(\hat{x}_{j}, \hat{y}_{j} \mid x, y\right) P\left(I \mid x, y, \hat{x}_{j}, \hat{y}_{j}\right)
$$

Likelihood (product of Poisson likelihoods)

$$
\underset{\mathcal{L}\left(\left\{N_{j}\right\}\right)=\prod_{j} \frac{\left\langle N_{\operatorname{det}, j}\right\rangle^{N_{j}} \mathrm{e}^{-\left\langle N_{\mathrm{det}, j}\right\rangle}}{N_{j}!} \underset{\uparrow}{\uparrow}=\mathrm{e}^{-\left\langle N_{\mathrm{det}}\right\rangle} \prod_{j: N_{j}=1}\left\langle N_{\mathrm{det}, j}\right\rangle}{N_{j} \in\{0,1\}}
$$

## New likelihood approach: simultaneous and self-consistent

- Regression derivation: for truncated data (with undetected sources) the total number of sources is the addition of detected plus undetected (missed) sources and is part of the model and must be marginalized over (Gelman et al 04; Kelly 07).

$$
\begin{aligned}
& \langle N\rangle=\int \mathrm{d} x\left\langle\frac{\mathrm{~d} N}{\mathrm{~d} x}\right\rangle \\
& \left\langle N_{\text {det }}\right\rangle=\int \mathrm{d} x\left\langle\frac{\mathrm{~d} N}{\mathrm{~d} x}\right\rangle \int \mathrm{d} y P(y \mid x) \int \mathrm{d} \hat{x} \int \mathrm{~d} \hat{y} P(\hat{x}, \hat{y} \mid x, y) P(I \mid x, y, \hat{x}, \hat{y}) \\
& \left\langle N_{\text {mis }}\right\rangle
\end{aligned}=\langle N\rangle-\left\langle N_{\text {det }}\right\rangle, ~ \$
$$

- Joint likelihood of the observations $(\hat{x}, \hat{y})$ and the total number of sources N is



## New likelihood approach: simultaneous and self-consistent

- Using the previous expressions we can calculate the probabilities of detecting a source with given properties and of missing a source

$$
\begin{aligned}
& P_{\mathrm{det}}\left(\hat{x}_{i}, \hat{y}_{i}, I\right)=\int \mathrm{d} x \int \mathrm{~d} y\left(\frac{\langle\mathrm{~d} N / \mathrm{d} x\rangle}{\langle N\rangle} P(y \mid x) P\left(\hat{x}_{i}, \hat{y}_{i} \mid x, y\right) P\left(I \mid x, y, \hat{x}_{i}, \hat{y}_{i}\right)=\frac{\left\langle\tilde{n}_{\text {det }, i}\right\rangle}{\langle N\rangle}\right. \\
& P(x) \text { Probability for a source to have properties } \mathrm{x} \\
& \left.P_{\text {mis }}(\bar{I})=\int \mathrm{d} x \int \mathrm{~d}\right\rangle \frac{\langle\mathrm{d} N / \mathrm{d} x\rangle}{\langle N\rangle} P(y \mid x) \int \mathrm{d} \hat{x} \int \mathrm{~d} \hat{y} P(\hat{x}, \hat{y} \mid x, y) P(\bar{I} \mid x, y, \hat{x}, \hat{y})=\frac{\left\langle N_{\mathrm{mis}}\right\rangle}{\langle N\rangle}
\end{aligned}
$$

Substituting these expressions we have

$$
\begin{array}{r}
\mathcal{L}(\hat{x}, \hat{y}, N)=\left[\frac{\langle N\rangle^{N}}{\langle N\rangle^{N_{\text {det }}}\langle N\rangle^{N_{\text {mis }}}}\right]\left[\frac{1}{N_{\text {det }}!}\right]\left[\frac{\left\langle N_{\text {mis }}\right\rangle^{N_{\text {mis }}} \mathrm{e}^{-\left\langle N_{\text {mis }}\right\rangle}}{N_{\text {mis }}!}\right] \mathrm{e}^{-\left\langle N_{\text {det }}\right\rangle} \prod_{i=1}^{N_{\text {det }}}\left\langle\tilde{n}_{\text {det }, i}\right\rangle \\
\left\langle\tilde{n}_{\text {det }, j}\right\rangle=\left\langle N_{\text {det }, j}\right\rangle /\left(\Delta \hat{x}_{j} \Delta \hat{y}_{j}\right)
\end{array}
$$

## Luminosity-mass scaling relation: selection biases



## Luminosity-mass scaling relation: selection biases



## Luminosity-mass scaling relation: selection biases



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For illustration purposes: Uniform distribution of simulated data and fictitious luminosity-mass relation (red line).

* The luminosity-mass relation has intrinsic scatter ( $\sim 40 \%$ ), which leads to Malmquist bias: brighter cluster are easier to find.
* For illustration purposes: fitting by eye (green line) only these data is wrong.


## Luminosity-mass scaling relation: selection biases



## Luminosity-mass scaling relation: selection biases



## Luminosity-mass scaling relation: selection biases



For illustration purposes: Exponential distribution of simulated data and fictitious luminosity-mass relation (red line).

* The luminosity-mass relation has intrinsic scatter (~40\%), which leads to Malmquist bias: brighter cluster are easier to find.
* The shape of the mass function leads to Eddington bias: much more low-mass clusters.
* For illustration purposes: fitting by eye (green line) only these data is wrong.


## Luminosity-mass scaling relation: selection biases

## Allen, Evrard, Mantz 11



For illustration purposes: Exponential distribution of simulated data and fictitious luminosity-mass relation (red line).

* The luminosity-mass relation has intrinsic scatter (~40\%), which leads to Malmquist bias: brighter cluster are easier to find.
* The shape of the mass function leads to Eddington bias: much more low-mass clusters


## X-ray luminosity-mass relation



Fitted with simple power law model, selfsimilar evolution and constant log-normal scatter $\sigma_{\mathrm{lm}}$

$$
\langle l(m)\rangle=\beta_{0}^{l m}+\beta_{1}^{l m} m
$$

Using the definitions

$$
\begin{aligned}
& l=\log _{10}\left(\frac{L_{500}}{E(z) 10^{40} \operatorname{erg~s}^{-1}}\right) \\
& m=\log _{10}\left(\frac{M_{500} E(z)}{10^{15} M_{\text {solar }}}\right)
\end{aligned}
$$

Current data do not require (i.e. acceptable fit) neither additional evolution beyond selfsimilar and constant scatter or asymmetric scatter (see details in Mantz et al 10b).

## X-ray luminosity-mass relation



For bolometric luminosities, the best fit using all the data (survey+follow-up+other cosmological data sets):
norm. $\quad \beta_{0}^{l m}=1.23 \pm 0.12$
slope $\quad \beta_{1}^{l m}=1.63 \pm 0.06$
scatter $\sigma_{l m}=0.185 \pm 0.019(\sim 40 \%)$

Slope steeper than the simple virial prediction: $\quad \beta_{1}^{l m}=1.33$

## Consistent with excess heating

Energy injection heats (e.g. AGN) the gas raising the temperature, decreasing the density and therefore the luminosity, being more important for less massive systems.

## Temperature-mass relation



Again, simple power law, self-similar, constant log-normal scatter. Best fit for all the data:
norm. $\quad \beta_{0}^{t m}=0.89 \pm 0.03$
slope $\quad \beta_{1}^{t m}=0.49 \pm 0.04$
scatter $\sigma_{t m}=0.055 \pm 0.008(\sim 15 \%)$

Slope shallower than the simple virial prediction: $\quad \beta_{1}^{t m}=0.67$

Consistent with excess heating
Energy injection heats (e.g. AGN) the gas raising the temperature, decreasing the density and therefore the luminosity, being more important for less massive systems.

## X-ray luminosity-mass relation



Core-included: scatter $\sim 40 \%$
Data consistent with self-similar evolution suggesting that excess heating occurred at $z>0.5$


Core-excised $\mathrm{r}<0.15 \mathrm{r}_{500}$.
Scatter undetected $<5 \%$.
$\beta_{1}^{l m}=1.30 \pm 0.05$ Consistent with the virial th.

Excess heating limited to the centers / effective mass-limited cluster sample could be possible January 6, 2013

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## Sampling model: follow-up observations

$$
M\left(r_{500}\right)=\frac{M_{\mathrm{gas}}\left(r_{500}\right)}{f_{\mathrm{gas}}\left(r_{500}\right)}=\frac{4 \pi}{3}(500) \rho_{\mathrm{cr}}(z) r_{500}^{3}
$$

$$
\begin{aligned}
& M_{\mathrm{gas}}(r) \propto \rho_{\mathrm{cr}}(z) r^{3} f_{\mathrm{gas}}(r) \propto r^{\eta_{g}} \\
& \eta_{g}=1.092 \pm 0.006 \\
& r_{500} \propto\left[f_{\mathrm{gas}}\left(r_{500}\right) H^{2}(z)\right]^{1 /\left(\eta_{g}-3\right)}
\end{aligned}
$$

$\mathrm{n}_{\mathrm{g}}$ logarithmic slope of the gas mass profiles at large radius; fit to the entire sample from 0.7-1.3r $\mathrm{F}_{500}$

$$
\frac{M^{\mathrm{ref}}(r)}{M(r)}=\frac{M_{\mathrm{gas}}^{\mathrm{ref}}(r) / f_{\text {gas }}^{\mathrm{ref}}(r)}{M_{\mathrm{gas}}(r) / f_{\mathrm{gas}}(r)} R_{\mathrm{NFW}}=\frac{d_{A}^{\mathrm{ref}}(z)^{2.5} f_{\mathrm{gas}}}{d_{A}(z)^{2.5} f_{\text {gas }}^{\mathrm{ref}}} R_{\mathrm{NFW}} \quad \begin{aligned}
& \text { We assume that the } \\
& \begin{array}{l}
\text { NFW profile is a good } \\
\text { approximation here }
\end{array}
\end{aligned}
$$

$$
L_{500}(r) \propto d_{\mathrm{L}}^{2}(z)\left(\frac{r_{500}}{d_{A}(z)}\right)^{\eta \mathrm{L}} \quad \mathrm{n}_{\mathrm{L}}=0.1135+-0.0005
$$

## Theory: linear and non-linear

$$
n(M, z)=\int_{0}^{M} f(\sigma) \frac{\bar{\rho}_{\mathrm{m}}}{M^{\prime}} \frac{\mathrm{d} \ln \sigma^{-1}}{\mathrm{~d} M^{\prime}} \mathrm{d} M^{\prime}
$$

Number density of galaxy clusters

$$
\sigma^{2}(M, z)=\frac{1}{2 \pi^{2}} \int_{0}^{\infty} k^{2} P(k, z)\left|W_{\mathrm{M}}(k)\right|^{2} \mathrm{~d} k \quad \begin{aligned}
& \text { Variance of the } \\
& \text { density fluctuations }
\end{aligned}
$$

$$
P(k, z) \propto k^{n_{\mathrm{s}}} T^{2}\left(k, z_{\mathrm{t}}\right) D(z)^{2}
$$

Linear power spectrum

$$
f(\sigma, z)=A\left[\left(\frac{\sigma}{b}\right)^{-a}+1\right] \mathrm{e}^{-c / \sigma^{2}}
$$

Fitting formula from N -body simulations (Tinker et al 08)

$$
x(z)=x_{0}(1+z)^{\varepsilon \alpha_{x}} \quad x \in\{A, a, b, c\}
$$

## Flat $\Lambda$ CDM

Mantz et al 10a



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## Agreement between cluster experiments

From Weinberg et al 12


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## Compilation of very different experiments:

cluster counts (optical, X-ray, SZ), N-point statistics in SZ maps, peculiar velocities, optical shear, CMB lensing


## Constraints on dark energy

## All data sets

1. Abundance of massive clusters (X-ray Luminosity Function, XLF) to measure cosmic expansion and growth of matter fluctuations with respect to the mean density.

$$
D(z) \equiv \frac{\delta(z)}{\delta\left(z_{\mathrm{t}}\right)}=\frac{\sigma(M, z)}{\sigma\left(M, z_{\mathrm{t}}\right)} \quad \delta=\left(\rho_{\mathrm{m}}-\bar{\rho}_{\mathrm{m}}\right) / \bar{\rho}_{\mathrm{m}}
$$

2. SNla, fgas, XLF, CMB, BAO to measure the cosmic expansion of the background density. We use three expansion histories well fitted by these data sets.

$$
\begin{aligned}
& E(a)=\left[\Omega_{\mathrm{m}} a^{-3}+\Omega_{\mathrm{de}} a^{-3(1+w)}+\Omega_{\mathrm{k}} a^{-2}\right]^{1 / 2} \\
& \begin{array}{ll}
\text { i) flat } \Lambda \mathrm{CDM} & \mathrm{w}=-1, \Omega_{\mathrm{k}}=0 \\
\text { ii) flat } w C D M & \mathrm{w} \text { constant, } \Omega_{\mathrm{k}}=0 \\
\text { iii) non-flat } \Lambda \mathrm{CDM} & \mathrm{w}=-1, \Omega_{\mathrm{k}} \text { constant }
\end{array}
\end{aligned}
$$

## Dark Energy results: flat wCDM



## Dark Energy results: flat wCDM



Green: SNIa (Kowalski et al 08, Union) Blue: CMB (WMAP5)
Red: cluster $\mathrm{f}_{\text {gas }}$ (Allen et al 08)
Brown: BAO (Percival et al 07)

XLF(survey+follow-up data): BCS +REFLEX+MACS (z<0.5) 238
clusters (Mantz et al 10a). Including systematics

$$
\begin{aligned}
& \Omega_{\mathrm{m}}=0.23+-0.04 \\
& \sigma_{8}=0.82+-0.05 \\
& \mathrm{w}=-1.01+-0.20
\end{aligned}
$$

## Dark Energy results: flat wCDM



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Gold: XLF $+\mathrm{f}_{\text {gas }}+$ WMAP5+SNIa+BAO
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& \mathrm{w}=-1.01+-0.20
\end{aligned}
$$

Good mass proxy at all z

## Dark Energy results: flat wCDM



Grey: XLF+WMAP5
Blue: CMB (WMAP5)
Gold: XLF $+\mathrm{f}_{\text {gas }}+$ WMAP5+SNIa+BAO

$$
\begin{aligned}
& \Omega_{\mathrm{m}}=0.272+-0.016 \\
& \sigma_{8}=0.79+-0.03 \\
& \mathrm{w}=-0.96+-0.06
\end{aligned}
$$

XLF(survey+follow-up data): BCS +REFLEX+MACS (z<0.5) 238
clusters (Mantz et al 10a). Including systematics

$$
\begin{aligned}
& \Omega_{\mathrm{m}}=0.23+-0.04 \\
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& w=-1.01+-0.20
\end{aligned}
$$

## Dark Energy results: flat wCDM

Allen, Evrard \& Mantz 11


Red: cluster $\mathrm{f}_{\mathrm{gas}}$ (Allen et al 08)

XLF(survey+follow-up data): BCS +REFLEX+MACS (z<0.5) 238 clusters (Mantz et al 10a). Including systematics

$$
\begin{aligned}
& \Omega_{\mathrm{m}}=0.23+-0.04 \\
& \sigma_{8}=0.82+-0.05 \\
& \mathrm{w}=-1.01+-0.20
\end{aligned}
$$

Both cluster experiments combined

## Dark Energy results: flat wCDM



Green: BAO
Blue: CMB (WMAP)
Red: Clusters
Gold: SNIa

$$
\begin{aligned}
& \Omega_{\mathrm{m}}=0.26+-0.08 \\
& \sigma_{8}=0.81+-0.04 \\
& \mathrm{w}=-1.14+-0.21
\end{aligned}
$$

## Beyond $\Lambda$ CDM: Neutrino properties

## Neutrinos and Cosmology

- Neutrino flavor oscillation experiments (solar, atmospheric, reactors) have conclusively shown that the neutrino mass eigenstates are non-degenerate (e.g. Fukuda et al 98, Ahn et al 03, 06, Sanchez et al 03, Aharmim et al 05, Beringer et al 12, etc.). However, measuring the absolute mass scale is still challenging.
- Three 'normal' neutrino species: $\mathrm{v}_{\mathrm{e}}, \mathrm{V}_{\mu}, \mathrm{V}_{\mathrm{T}}$. There are though some hints for possible additional, sterile neutrinos from oscillation data (Kopp et al 11, Huber 11, etc.). Relatively recent, CMB observations have also seemed to favor the presence of additional radiation at the time of decoupling over that from photons and the three 'normal' neutrino species.
- Recent constraints from laboratory experiments: lower bound on $M_{v}=\sum_{i} m_{i}$ (sum of the masses of the different species) of $\sim 0.056(0.095) \mathrm{eV} / \mathrm{c}^{2}$ for the normal (inverted) hierarchy; and an upper bound of $\sim 6 \mathrm{eV} / \mathrm{c}^{2}$ (from hereon $\mathrm{c}=1$ ). The Heidelberg-Moscow experiment has limited the mass of the electron neutrino to $<0.35 \mathrm{eV}$ (Klapdor-Kleingrothaus \& Krivosheina 06).


## Neutrinos and Cosmology

- Neutrinos play an important role in the early universe and therefore affect cosmological observations (review: Lesgourges \& Pastor 06).
- The primary cosmological effect of non-zero neutrino mass is to suppress the formation of cosmic structure on intermediate and small scales. CMB contains information on LSS at early times. The combination with probes today give good constraints on the absolute neutrino mass scale.
- Interference with dark energy and inflation physics. Combining experiments helps.
- Combined cosmological observations: $\Sigma_{\mathrm{i}} \mathrm{m}_{\mathrm{i}}<\sim 0.3-0.6 \mathrm{eV}$.
- Neutrino oscillation experiments favor a large mass for sterile neutrinos yielding a lower limit on their mass of 1 eV which is incompatible with cosmological observations. This can be alleviated with for example initial lepton asymmetry (Hannestad et al 12).


## Robust constraints on neutrino properties

$\Lambda C D M+\Sigma m_{v}$ : Breaking the degeneracy in the $\Sigma \mathrm{m}_{v}, \sigma_{8}$ plane


Note differences in scale between panels

## Robust constraints on neutrino properties

Basic: $\Lambda C D M+\Sigma m_{v}$

CMB+fgas+SNIa+BAO


CMB+fgas+SNIa+BAO+XLF


Mantz et al 10c

## Robust constraints on neutrino properties



## Breaking degeneracies with other data sets

Mantz et al 10c

$N_{\text {eff }}$ is free
Green contours: CMB+fgas

+ SNla+BAO (strong degeneracy).

Blue contours: adding $\mathrm{H}_{0}$ at the $5 \%$ level helps significantly with this degeneracy.

## Other cosmological constraints on neutrinos



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Dotted orange: WMAP7
Solid orange: WiggleZ +WMAP7

Dotted black: WMAP7+BAO $+\mathrm{H}_{0}$

Solid black: WiggleZ + WMAP7+BAO+H ${ }_{0}$

Dashed grey: lower limit, oscillation experiments

Other vertical lines: 95\% confidence upper limits

## Effects of neutrinos on the CMB power spectrum



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Hinshaw et al 12

## Recent cosmological constraints on neutrinos


$N_{\text {eff }}=3.89 \pm 0.67(68 \% \mathrm{CL})$
$W M A P+e C M B ; Y_{\text {He }}$ fixed

$$
\begin{aligned}
& N_{\text {eff }}=3.26 \pm 0.35(68 \% \mathrm{CL}) \\
& W M A P+\mathrm{eCMB}+\mathrm{BAO}+H_{0} ; Y_{\mathrm{He}} \text { fixed }
\end{aligned}
$$

$N_{\text {eff }}=2.83 \pm 0.38$
$Y_{\mathrm{He}}=0.308_{-0.031}^{+0.032}$
(68\% CL)
$W M A P+\mathrm{eCMB}+\mathrm{BAO}+H_{0}$
$\mathrm{Y}_{\text {he }}$ primordial Helium abundance

No evidence for energy density of extra radiation species

## Inflation and primordial non-Gaussianities

## Measuring the primordial spectral index with CMB and BAO data



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## Comparing $\mathrm{CMB}+\mathrm{BAO}$ and $\mathrm{CMB}+\mathrm{H}_{0}$



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## Constraints on inflation models



Green contours:
WMAP+eCMB
Red contours:
WMAP+eCMB+BAO $+\mathrm{H}_{0}$

N is the number of $\mathrm{e}-$ folds between the end of inflation and the epoch at which the scale $\mathrm{k}=0.002$ $\mathrm{Mpc}^{-1}$ left the horizon during inflation.

## Constraints on primordial non-Gaussianities

$$
\begin{gathered}
f_{N L}^{\text {local }}=32 \pm 21(68 \% \mathrm{CL}) \text { Konaste etal 11 } \\
\text { The } 95 \% \text { limit is }-10<f_{N L}^{\text {local }}<74 \\
f_{N L}^{\text {equil }}=26 \pm 140(68 \% \mathrm{CL}) \\
\text { The } 95 \% \text { limit is }-214<f_{N L}^{\text {equil }}<266 \\
f_{N L}^{\text {orthog }}=-202 \pm 104(68 \% \mathrm{CL}) \\
\text { The } 95 \% \text { limit is }-410<f_{N L}^{\text {orthog }}<6 \\
-29<f_{N L}^{\text {local }}<70(95 \% \text { CL Slosar et al. 2008) } \\
-5<f_{N L}^{\text {local }}<59(95 \% \text { CL) wMAPT+LSS data }
\end{gathered}
$$

# Beyond $\Lambda$ CDM: Evolving dark energy w(z) 

## Models and probes of cosmic acceleration

- Some recent dark energy reviews:
- Copeland, Sami, Tsujikawa, 06, Int. J. Mod Phys D
- Frieman, Turner, Huterer, 08, Ann. Rev. Astr. \& Astrophys., 46, 385
- Weinberg, Mortonson, Eisenstein, Hirata, Riess, Rozo, 12, for Phys. Reports, arXiv:1201.2434
- Dark energy task forces and future dark energy missions:
- Albrecht, Bernstein, Cahn, Freedman, Hewitt, Hu, Huth, Kamionkowski, Kolb, Knox, Mather, Staggs, Suntzeff, 06, arXiv/0609591
- Albrecht, Amendola, Bernstein, Clowe, Eisenstein, Guzzo, Hirata, Huterer, Kirshner, Kolb, Nichol, 09, arXiv:0901.0721
- Amendola, et al (Euclid Satellite), 12, arXiv:1206.1225


## Constraints on $w_{0}, w_{\text {et }}$ marginalizing over $z_{t}$



Combined constraints (marginalized 68\%)
$\Omega_{\mathrm{m}}=0.299+0.029-0.027$
$w_{0}=-1.27+0.33-0.39$
$w_{e t}=-0.66+0.44-0.62$

WMAP1+CBI+ACBAR
SNIa: Riess et al 04
$\mathrm{f}_{\text {gas: }}$ Allen et al 04
marginalized over $0.05<z_{t}<1$

Two parameters:
$w=w_{0}+w_{1}(1-a)$ fix transition at $z_{t}=1$ between $w_{0}$ (present) and $w_{e t}=w_{0}+w_{1}$ (early times).
Three parameters (R05):
free transition $\mathrm{z}_{\mathrm{t}}$ between $\mathrm{w}_{0}$ and $\mathrm{w}_{\mathrm{e}}$ :
$w=\frac{w_{\mathrm{et}} z+w_{0} z_{\mathrm{t}}}{z+z_{\mathrm{t}}}=\frac{w_{\mathrm{et}}(1-a) a_{\mathrm{t}}+w_{0}\left(1-a_{\mathrm{t}}\right) a}{a\left(1-2 a_{\mathrm{t}}\right)+a_{\mathrm{t}}}$

## Constraints on $w_{0}, w_{\text {et }}$ marginalizing over $z_{t}$



Combined constraints (marginalized 68\%)
$\Omega_{\mathrm{m}}=0.254 \pm 0.022$
$w_{0}=-1.05+0.31-0.26$
$w_{\text {et }}=-0.83+0.48-0.43$
WMAP3+CBI+Boomerang+ACBAR
SNIa: Davis et al. 07
$\mathrm{f}_{\mathrm{gas}}$ : Allen et al. 08
marginalized over $0.05<z_{t}<1$

Three parameters (R05):
free transition $z_{t}$ between $w_{0}$ and $w_{\mathrm{et}}$ :
$w=\frac{w_{\mathrm{et}} z+w_{0} z_{\mathrm{t}}}{z+z_{\mathrm{t}}}=\frac{w_{\mathrm{et}}(1-a) a_{\mathrm{t}}+w_{0}\left(1-a_{\mathrm{t}}\right) a}{a\left(1-2 a_{\mathrm{t}}\right)+a_{\mathrm{t}}}$

## Current constraints: evolving w



Combined constraints (marginalized 68\%)
$\Omega_{\mathrm{m}}=0.257+-0.016$
$w_{0}=-0.88+-0.21$
$w_{\text {et }}=-1.05+0.20-0.36$
WMAP5
SNIa: Kowalski et al. 08
$\mathrm{f}_{\text {gas }}$ : Allen et al. 08
BAO: Percival et al. 07
XLF: Mantz et al. 09a
marginalized over $0.05<z_{t}<1$
Three parameters (R05):
free transition $z_{t}$ between $w_{0}$ and $w_{\mathrm{et}}$ :
$w=\frac{w_{\mathrm{et}} z+w_{0} z_{\mathrm{t}}}{z+z_{\mathrm{t}}}=\frac{w_{\mathrm{et}}(1-a) a_{\mathrm{t}}+w_{0}\left(1-a_{\mathrm{t}}\right) a}{a\left(1-2 a_{\mathrm{t}}\right)+a_{\mathrm{t}}}$

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## Current constraints: evolving w



# Kinematical approaches to dark energy 

## Why kinematical approaches?

* Do not assume any particular gravity theory.
- Most of current cosmological analyses are dynamical, use the Friedmann equations (and General Relativity) employing $\Omega_{\mathrm{m}}$ and w as model parameters.
- Other dynamical approaches use modified gravity theories.
* Describe directly the expansion history of the Universe, a(t).
- We measure a late-time cosmic acceleration.
- It is important now to measure kinematically a transiton to a decelerating phase at earlier times.


## Constraints on the deceleration parameter

- Using for example $q(z)=q_{0}+z(d q / d z)$ as in Riess et al 04.
- Clusters (green contours) ; SNLS SNIa (blue contours) ; Gold SNIa sample (dashed contours); all combined (orange contours)
- Shapiro \& Turner 05 and Elgaroy \& Multamaki 06 also used other $q(z)$ parameterizations.
- However, the choice of a particular parameterization of $\mathrm{q}(\mathrm{z})$ is quite arbitrary.
- And in general does not have a direct meaningful physical interpretation.


## Our kinematical formalism: $\left(q_{0, j}, j\right)$ parameter space

Deceleration parameter
Jerk parameter

$$
\begin{gathered}
q(t)=-\frac{1}{H^{2}}\left(\frac{\ddot{a}}{a}\right) \quad q(a)=-\frac{1}{H}(a H)^{\prime} \quad j(t)=\frac{1}{H^{3}}\left(\frac{\dddot{a}}{a}\right) \quad j(a)=\frac{\left(a^{2} H^{2}\right)^{\prime \prime}}{2 H^{2}} \\
a^{2} V^{\prime \prime}(a)-2(a) \quad V(a)=0 \longleftarrow-\frac{a^{2} H^{2}}{2 H_{0}^{2}} \\
V(1)=-\frac{1}{2} \quad V^{\prime}(1)=-\frac{H_{0}^{\prime}}{H_{0}}-1=q_{0} \quad \begin{array}{l}
j(a)=1 \text { corresponds to } \\
\text { all } \Lambda \text { CDM models }
\end{array}
\end{gathered}
$$

For example, for constant j models we get

$$
\left.V(a)=-\frac{\sqrt{a}}{2}\left[\left(\frac{p-u}{2 p}\right) a^{p}+\left(\frac{p+u}{2 p}\right) a^{-p}\right] \quad p \equiv \frac{1}{2} \sqrt{1+8}(j) \quad u \equiv 2\left(q_{0}\right)+1 / 4\right)
$$

## Basic kinematical and dynamical models

Constant j model


$$
\begin{aligned}
& q_{0}=-0.81+-0.14 \\
& j=2.16+0.81-0.75
\end{aligned}
$$

Constant w model


$$
\begin{aligned}
\Omega_{\mathrm{m}} & =0.306+0.042-0.040 \\
\mathrm{w} & =-1.15+0.14-0.18
\end{aligned}
$$

Both models contain a simple representation of $\Lambda C D M(w=-1, j=1)$ and are consistent with it at the $1 \sigma$ level. This represents an additional support for the $\Lambda$ CDM paradigm.

## Hypothesis testing: How many model kinematical parameters are required?

$$
\begin{array}{cc}
\text { F-test } & \text { Bayesian Information Criterion }
\end{array} \begin{gathered}
\text { Bayesian Evidence } \\
F=\frac{\Delta \chi^{2}}{\chi_{v}^{2} \Delta m}
\end{gathered} \quad B I C=-2 \ln L+k \ln N \quad \begin{gathered}
E(M) \equiv P(D \mid M)= \\
2<\Delta B I C<6
\end{gathered} \quad \int d \theta P(D \mid \theta, M) P(\theta \mid M) .
$$

## Recent results on the kinematical model for various combinations of data sets



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## Alcock-Paczynski test data from WiggleZ data



$$
F(z)=(1+z) D_{\mathrm{A}}(z) H(z) / c
$$

$D_{A}(z)$ : angular diameter distance and $H(z)=H_{0} E(z)$ : Hubble parameter


Blake et al, 11
SNe+AP effect

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## Reconstruction of kinematical quantities



## Non-parametric reconstruction of the cosmic expansion history



## Beyond $\Lambda$ CDM: Gravity at large scales

## Testing GR on cosmic scales

1. From the evolution of the cluster abundance (XLF) we directly measure linear cosmic expansion and growth.
2. From a variety of measurements we find cosmic acceleration and face the cosmological constant problems.
3. We can either include a new energy component, dark energy, or modify the theory of gravity.
4. We test General Relativity (GR) for consistency.
5. GR has been very well tested from small to Solar system scales. Here we test modifications of GR at cosmological scales.

## Ingredients to test a given theory of gravity with cluster abundance data

1. Cosmic expansion model / mean matter density (theory).
2. Matter power spectrum / linear density perturbations (theory).
3. Halo mass function / nonlinear structure formation ( N -body simulations for $f(R)$ or DGP: e.g. Schmidt et al 2009, Schmidt 2009a/ b, Chan \& Scoccimarro 2009, Zhao, Li \& Koyama 2011).
4. Relation between the observed mass (e.g. "dynamical") and the true mass (e.g. "lensing") (Theory/N-body simulations: Schmidt 2010a).

## Consistency test of the growth rate of General Relativity

1. We use a phenomenological time-dependent parameterization of the growth rate and of the expansion history.
2. We assume the same scale-dependence as GR.
3. We test only for linear effects (not for non-linear effects). We use the "universal" dark matter halo mass function (Tinker et al 2008). Note that the relevant scales for the cluster abundance experiment are at the low end of the linear regime.
4. We match GR at early times and small scales.

## Modeling linear, time-dependent departures from GR

$$
\begin{aligned}
& n(M, z)=\int_{0}^{M} f(\sigma) \frac{\bar{\rho}_{\mathrm{m}}}{M^{\prime}} \frac{d \ln \sigma^{-1}}{d M^{\prime}} d M^{\prime} \quad \begin{array}{l}
\text { Number density of } \\
\text { galaxy clusters }
\end{array} \\
& \sigma^{2}(M, z)=\frac{1}{2 \pi^{2}} \int_{0}^{\infty} k^{2} P(k, z)\left|W_{M}(k)\right|^{2} d k \\
& \text { Variance of the } \\
& \text { density fluctuations } \\
& P(k, z) \propto k^{n_{\mathrm{s}}} T^{2}\left(k, z_{\mathrm{t}}\right) D(z)^{2} \quad \text { Linear power spectrum } \\
& \text { General Relativity } \\
& \text { Phenomenological parameterization } \\
& \ddot{\delta}+2 \frac{\dot{a}}{a} \dot{\delta}=4 G \pi \overline{\rho_{\mathrm{m}}} \delta \quad \frac{d \delta}{d a}=\frac{\delta}{a} \Omega_{\mathrm{m}}(a)^{\gamma} \quad \text { GR } \gamma \sim 0.55 \\
& \text { Scale independent in the } \\
& \text { synchronous gauge } \\
& f(a) \equiv d \ln \delta / d \ln a=\Omega_{\mathrm{m}}(a)^{\gamma} \text { Growth rate }
\end{aligned}
$$

## Test of GR robust w.r.t evolution in the I-m relation

Rapetti et al 10


$$
\langle l(m)\rangle=\beta_{0}^{l m}+\beta_{1}^{l m} m+\beta_{2}^{l m} \log _{10}(1+z)
$$


$\sigma_{l m}(z)=\sigma_{l m}\left(1+\sigma_{l m}^{\prime} z\right)$

Current data do not require (i.e. acceptable fit) additional evolution beyond selfsimilar and constant scatter nor asymmetric scatter (Mantz et al 2010b).

## Investigating luminosity-mass evolution

Within the 238 flux-selected clusters we used pointed observations for


23 clusters ( $z<0.2$ ) from ROSAT 71 clusters ( $z>0.2$ ) from Chandra

Mass-luminosity and its intrinsic scatter

$$
\begin{aligned}
& \langle l(m)\rangle=\beta_{0}^{l m}+\beta_{1}^{l m} m+\beta_{2}^{l m} \log _{10}(1+z) \\
& \sigma_{l m}(z)=\sigma_{l m}\left(1+\sigma_{l m}^{\prime} z\right) \\
& l=\log _{10}\left(\frac{L_{500}}{E(z) 10^{4} e r g s^{-1}}\right) ; \quad m=\log _{10}\left(\frac{M_{500} E(z)}{10^{15} M_{\text {solar }}}\right)
\end{aligned}
$$

## flat $\Lambda C D M+$ growth index $\gamma$



XLF: BCS+REFLEX+MACS (z<0.5)
238 survey with 94 X-ray follow-up
CMB (WMAP5)
SNIa (Kowalski et al 2008, UNION) cluster $\mathrm{f}_{\text {gas }}$ (Allen et al 2008)

For General Relativity $\gamma \sim 0.55$

## Gold: Self-similar evolution and

 constant scatterBlue: Marginalizing over $\beta^{\mathrm{Im}}{ }_{2}$ and $\sigma^{\prime}{ }_{1 m}$ (only $\sim 20$ weaker: robust result on $\gamma$ ).

Remarkably these constraints are only a factor of $\sim 3$ weaker than those forecasted for JDEM/ WFIRST-type experiments (e.g. Thomas et al 2008, Linder 2009).

## flat wCDM + growth index $\gamma$



XLF: BCS+REFLEX+MACS (z<0.5)
238 survey with 94 X-ray follow-up
CMB (WMAP5)
SNla (Kowalski et al 2008, UNION)
cluster $\mathrm{f}_{\text {gas }}$ (Allen et al 2008)
For General Relativity $\gamma \sim 0.55$
Gold: Self-similar evolution and constant scatter

Simultaneous constraints on the expansion and growth histories of the Universe at late times:
Consistent with GR+ $\Lambda$ CDM

## Flat $\Lambda$ CDM + growth index $\gamma$



XLF: BCS+REFLEX+MACS (z<0.5) 238 survey with 94 X-ray follow-up CMB (WMAP5) SNIa (Kowalski et al 2008, UNION) cluster $\mathrm{f}_{\text {gas }}$ (Allen et al 2008)
For General Relativity $\gamma \sim 0.55$

## Gold: Self-similar evolution and

 constant scatterBlue: Marginalizing over $\beta^{\mathrm{Im}}{ }_{2}$ and $\sigma^{\prime}{ }_{\mathrm{Im}}$

$$
\gamma\left(\frac{\sigma_{8}}{0.8}\right)^{6.8}=0.55_{-0.10}^{+0.13}
$$

Tight correlation between $\sigma_{8}$ and $\gamma$ :

$$
\rho=-0.87
$$

# Redshift space distortions and Alcock-Paczynski effect 

e.g. Blake et al 11; Beutler et al 2012; Reid et al 12

## Anisotropic galaxy clustering: RSD and AP effect

Sources of anisotropy in the distribution of galaxies (2-point statistics) used to constrain the cosmological model:

- Redshift space distortions: due to velocity patterns of galaxies infalling into gravitational potential wells

$$
f \sigma_{8}(z) \text { fiz is the linear growth rate and } \sigma_{8}(z) \text { the variance in the }
$$

- Alcock-Paczynski distortion: between the tangential and radial dimensions of objects or patterns when the correct cosmological model is assumed to be isotropic

$$
F(z)=(1+z) D_{\mathrm{A}}(z) H(z) / \mathrm{C}
$$

## WiggleZ: two-dimensional power spectra



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## WiggleZ and 6dFGS constraints on RSD and AP effect

## Blake et al 11






Relative probability density


For WiggleZ (Blake et al 11):

- We use a bivariate Gaussian likelihood on $\mathrm{fo}_{8}(\mathrm{z})$ and $\mathrm{F}(\mathrm{z})$ (good approximation):
$z=(0.22,0.41,0.60,0.78)$
$f \sigma_{8}(z)=(0.53 \pm 0.14,0.40 \pm 0.13,0.37 \pm 0.08$, $0.49 \pm 0.12)$
$F(z)=(0.28 \pm 0.04,0.44 \pm 0.07,0.68 \pm 0.06$, $0.97 \pm 0.12)$
$r=(0.83,0.94,0.89,0.84)$
For 6dFGS (Beutler et al 2012):
- We use a Gaussian likelihood on $\mathrm{fo}_{8}(z)$ only (since at low-z the AP effect is negligible):

$$
\mathrm{fo}_{8}(\mathrm{z}=0.067)=0.423 \pm 0.055
$$

## SDSS-III CMASS BOSS constraints



For CMASS BOSS (Reid et al 2012):

- We use either a bivariate (growth) or a trivariate (BAO) Gaussian likelihood on $\mathrm{fo}_{8}$ $(z), F(z)$ and $A(z)$ (good approximation):

$$
\begin{aligned}
& f \sigma_{8}(z=0.57)=0.43 \pm 0.07 \\
& F(z=0.57)=0.68 \pm 0.04 \\
& \mathrm{~A}(\mathrm{z}=0.57)=1.023 \pm 0.019 \\
& r_{\text {fof }}=0.87 \\
& r_{\text {ffA }}=-0.0086 \\
& r_{F A}=-0.080
\end{aligned}
$$

$$
A(z) \equiv\left(D_{\mathrm{v}} / r_{\mathrm{s}}\right) /\left(D_{\mathrm{v}} / r_{\mathrm{s}}\right)_{\text {fiducial }}
$$

$$
D_{V}(z)=\left[(1+z)^{2} D_{A}(z)^{2} c z / H(z)\right]^{1 / 3}
$$

Large scale distributions of galaxies: matter Percival et al 10, SDSS DR7 power spectrum


WiggleZ CosmoMC module: http://smp.uq.edu.au/wigglez-data
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# Combined constraints on growth and expansion: breaking degeneracies 

## Modeling the abundance of clusters and their scaling relations

$$
\begin{aligned}
& n(M, z)=\int_{0}^{M} \mathcal{F}(\sigma, z) \frac{\rho_{\mathrm{m}}}{M^{\prime}} \frac{d \ln \sigma^{-1}}{d M^{\prime}} d M^{\prime} \begin{array}{c}
\text { Number density of } \\
\text { dark matter halos }
\end{array} \\
& \mathcal{F}(\sigma, z)=A\left[\left(\frac{\sigma}{b}\right)^{-a}+1\right] e^{-c / \sigma^{2}} \begin{array}{l}
\text { Fitting formulae from } \\
\mathrm{N} \text {-body simulations }
\end{array} \\
& x(z)=x_{0}(1+z)^{\varepsilon \alpha_{\mathrm{x}}} \quad \begin{array}{l}
\text { x being A, a, b, or c } \\
\text { (Tinker et al 2008) }
\end{array} \\
& \langle\ell(m)\rangle=\beta_{0}^{\ell m}+\beta_{1}^{\ell m} m+\beta_{2}^{\ell m} \log _{10}(1+z) \quad \text { Luminosity-mass relation } \\
& \sigma_{\ell m}(z)=\sigma_{\ell m}\left(1+\sigma_{\ell m}^{\prime} z\right) \quad \text { Scatter in the luminosity-mass relation }
\end{aligned}
$$

(same expressions for the temperature-mass relation but changing | for t )

## Flat $\Lambda C D M+$ growth index $\gamma$


clusters ( $X L F+f_{\text {gas }}$ ): BCS+REFLEX
+MACS
CMB (ISW): WMAP
galaxies (RSD+AP): WiggleZ
+6dFGS+BOSS
Gold: clusters+CMB+galaxies

$$
\begin{aligned}
& (+\mathrm{BAO}+\mathrm{SNla+SHOES}) \\
& \gamma=0.576_{-0.059}^{+0.058} \\
& \sigma_{8}=0.789 \pm 0.019 \\
& \Omega_{m}=0.255 \pm 0.011 \\
& H_{0}=72.1 \pm 1.0
\end{aligned}
$$

## Flat $\Lambda$ CDM + growth index $\gamma$



## Flat wCDM + growth index $\gamma$ : growth plane



For General Relativity $\gamma \sim 0.55$

Magenta: clusters+galaxies
Purple: clusters+CMB
Turquoise: CMB+galaxies
Gold: clusters+CMB+galaxies

Platinum: clusters+CMB+galaxies
+BAO (Reid et al 12; Percival et al
10)+SNIa (Suzuki et al 12)
+SHOES (Riess et al 11)

## Flat wCDM + growth index $\gamma$ : expansion planes



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## Flat wCDM + growth index $\gamma$ : growth+expansion

Rapetti et al 12


For General Relativity $\gamma \sim 0.55$
For $\Lambda$ CDM w=-1

Gold: clusters+CMB+galaxies
Platinum: clusters+CMB+galaxies
+BAO+SNIa+SHOES

$$
\begin{aligned}
& \gamma=0.546_{-0.072}^{+0.071} \\
& \sigma_{8}=0.783_{-0.019}^{+0.020} \\
& w=-0.968 \pm 0.049 \\
& \Omega_{m}=0.256 \pm 0.011 \\
& H_{0}=71.5 \pm 1.3
\end{aligned}
$$

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## flat $+\Lambda C D M$ expansion history, $f(R)$ gravity model

Schmidt, Vikhlinin \& Hu et al 10


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## flat $+\Lambda$ CDM expansion history, $f(R)$ gravity model



Schmidt, Vikhlinin \& Hu et al 10

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## Cluster mass profiles in DGP and f(R)

Schmidt 10


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## Cluster mass profiles in DGP and f(R)

Schmidt 10



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## Summary/reminder of the take home messages for cosmological data analyses

- Deep understanding of astrophysical processes and objects
- Careful design of observations
- Justify and continuously revised assumptions
- Account properly for covariances between parameters, instrumental and astrophysical systematic uncertainties and biases
- Simultaneous fits of all the relevant astrophysical and cosmological parameters

