Finite temperature field theory and heavy ion collisions

Nordic Winter School on Particle Physics and Cosmology 2013, Skeikampen, Norway

Lecture 3: QCD plasma at weak coupling

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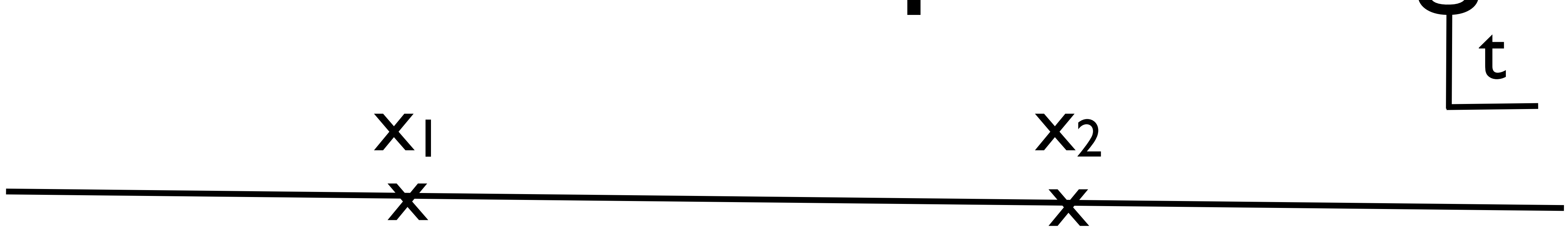
‘more fun with path integrals’



Q: Why is the contour *tilted downward* (slightly)? (in vacuum)

A1: Needed for convergence (infinitesimal Wick rotation)

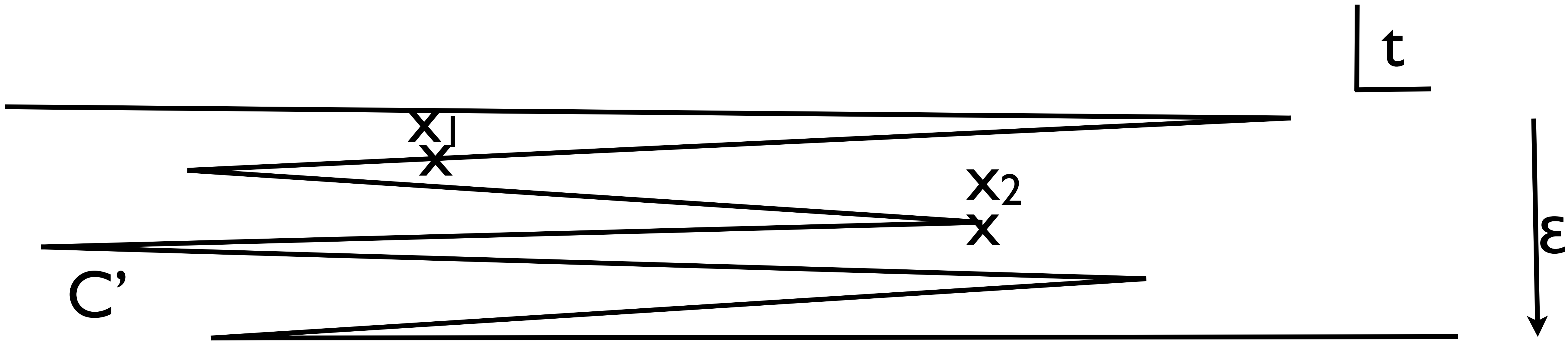
$$e^{i\int dt(\dot{\phi}^2(1+i\epsilon)-(\nabla\phi)^2(1-i\epsilon)-V(1-i\epsilon))}$$

A2: Going to infinity, projects onto the vacuum

$$e^{-i\int_{-\infty}^{x} dtH}|\text{any state}\rangle \propto |0\rangle$$

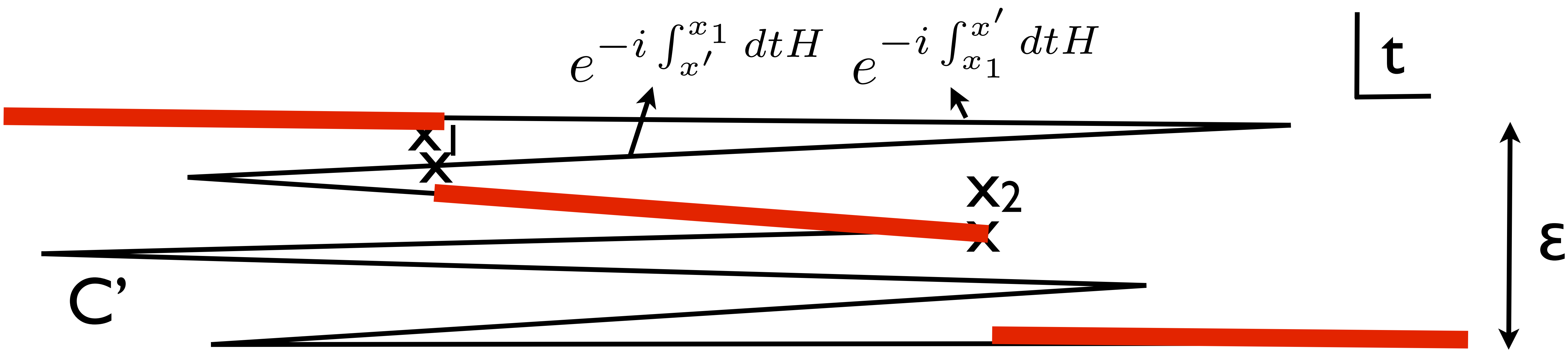
‘more fun with path integrals’

- What about more general contours?



Note: Always move downward the imaginary axis!

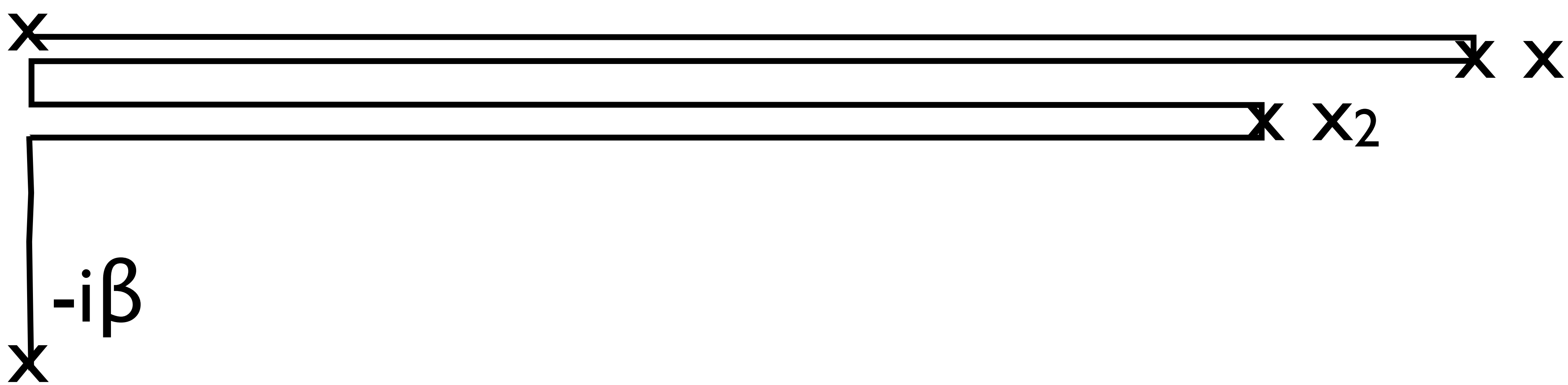
segments cancel out!



This computes the same as before (for $x_1<x_2$)!

$$\langle 0|T[\phi(x_1)\phi(x_2)]|0\rangle = \frac{1}{Z}\int_{C'}[D\phi(x)]\phi(x_1)\phi(x_2)e^{iS[\phi]}$$

- A more useful contour:



$$G^{>}(x_1, x_2) := \frac{1}{Z} \mathrm{Tr}\left[e^{-\beta H}\phi(x_2)\phi(x_1)\right]$$

=Wightman function at finite T

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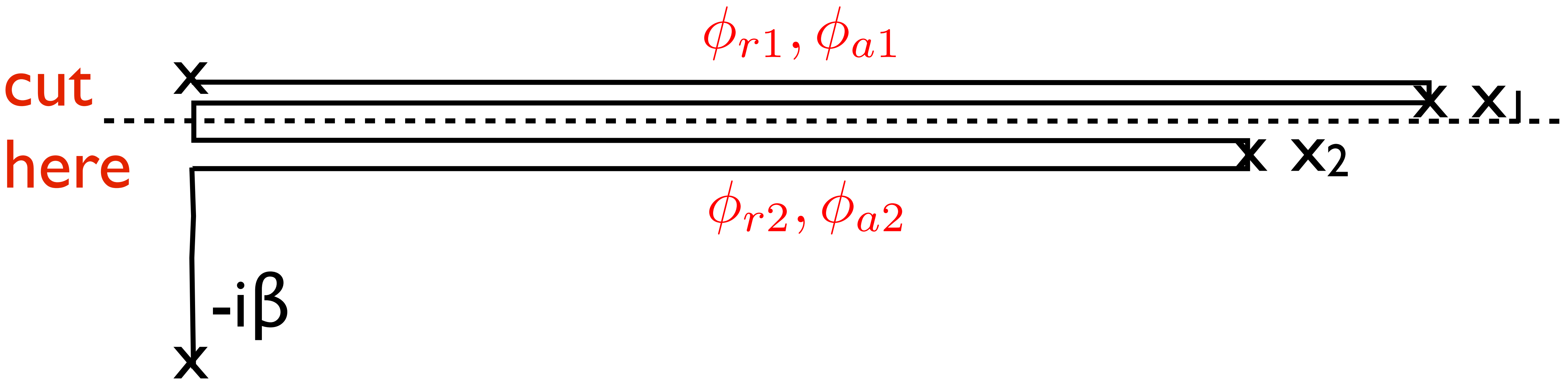

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=Wightman function

- Cross-correlators are simple:

$$G_{a1,a2} = G_{r1,a2} = G_{a1,r2} = 0, \quad G_{r1,r2} = G^{>}$$

- Optical theorem at finite T:

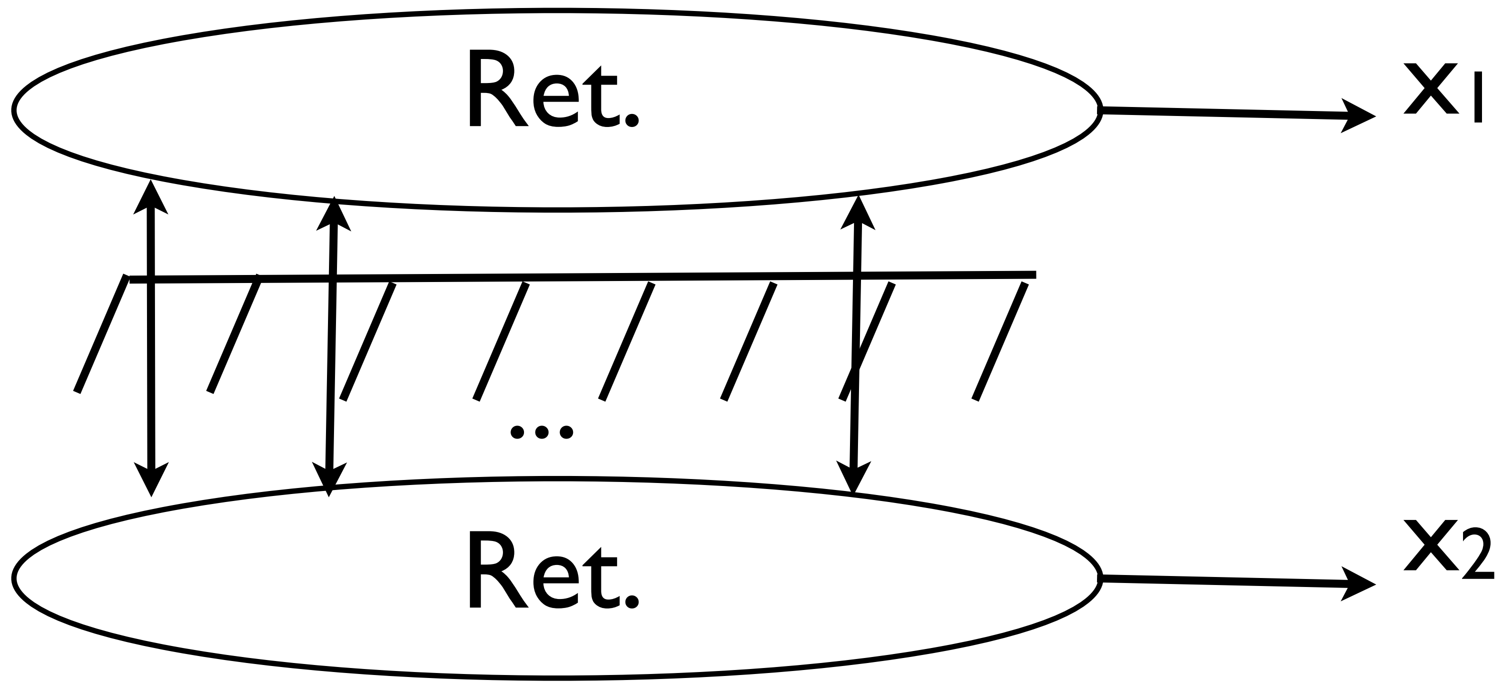
$$G^{>}(p) = \sum_n \int_{p_1,\ldots,p_n} G^{>}(p_1)\cdots G^{>}(p_n)$$

$$\times \left[G_{aaaaaar}(p_1,\ldots,p_n,p)G_{aaaaaaar}(-p_1,\ldots,-p_n,-p)\right]$$

on-shell particles

(retarded) matrix element squared

=



(Weldon '81)

Weakly coupled plasmas, I

- Consider the model (ϕ^4), at a finite T, $\lambda << 1$:

$$S = \int d^4x \left[-\frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{24}\phi^4 \right]$$

- Expectation:
  - number density $n \sim T^3$, typical energy $\sim T$
  - 2 to 2 cross-sections $\sigma \sim \lambda^2/T^2$
  - mean free path: $t \sim 1/\langle n\sigma \rangle$ $\sim 1/\lambda^2 T$

- A gas of nearly-free *quasi-particles*

Pressure

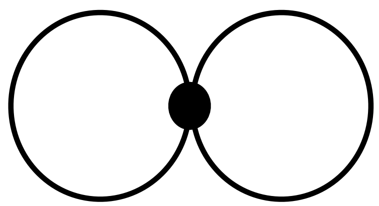
- Let's compute the pressure of this gas, order by order in λ
- First approximation: Stefan-Boltzman

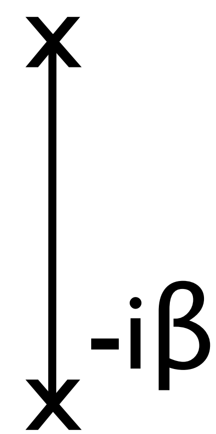
$$p_0 = \int \frac{d^3p}{(2\pi)^3}\frac{|p|}{3}n(p)$$

$$= \frac{1}{6\pi^2}\int_0^\infty \frac{p^3dp}{e^{\beta p^0}-1}$$

$$= \frac{\pi^2 T^4}{90} \qquad (\epsilon_0 = 3p_0)$$

- Leading correction:



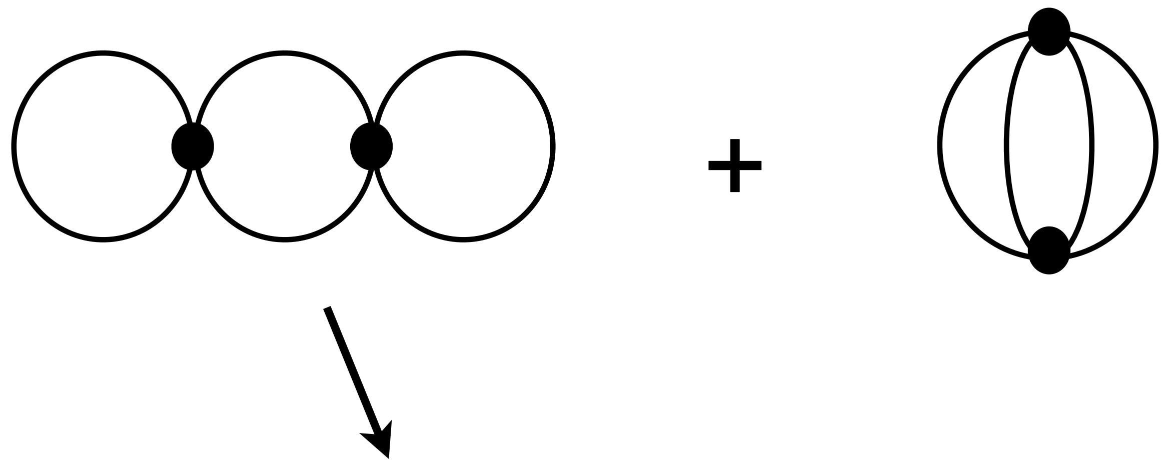
(use the Euclidean part of the contour for this: )

$$\delta p = -\frac{\lambda}{8}\langle\phi^2\rangle^2$$

$$= -\frac{\lambda T^4}{24 \cdot 48} \qquad (\langle\phi^2\rangle = \frac{T^2}{12})$$

Note: epsilon=3p is not yet violated at this order

- Next-to-leading correction:



$$\delta p = m^4 T \sum\!\!\!\!\!\!\int_p \frac{1}{(\omega_n^2 + \vec{p}^2)^2} \quad , \omega_n = 2\pi n T$$

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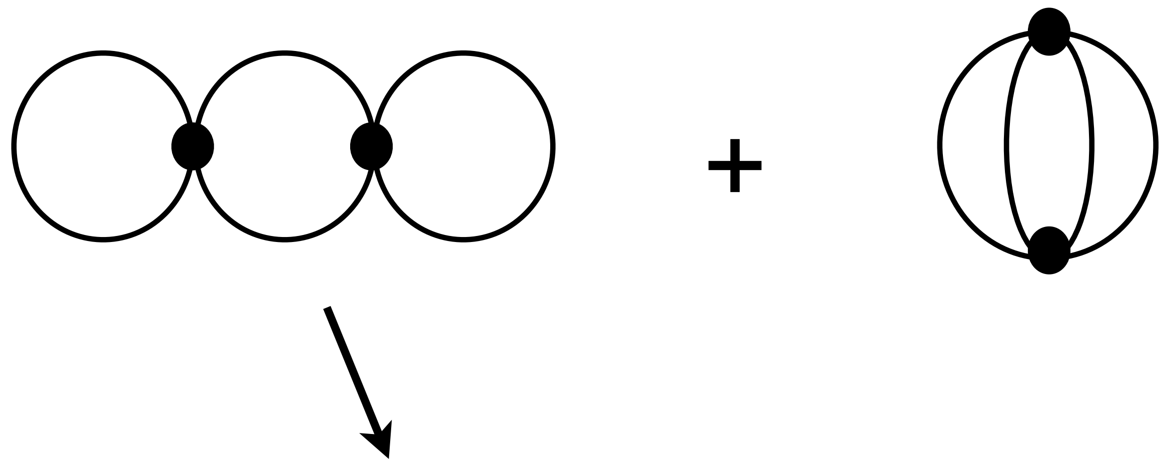

$$\delta p = m^4 T \sum\!\!\!\!\!\!\!\int_p \frac{1}{(\omega_n^2 + \vec{p}^2)^2}$$

n=0 mode is linearly IR divergent!

$$\sim m^4 T \int \frac{d^3\vec{p}}{\vec{p}^4}$$

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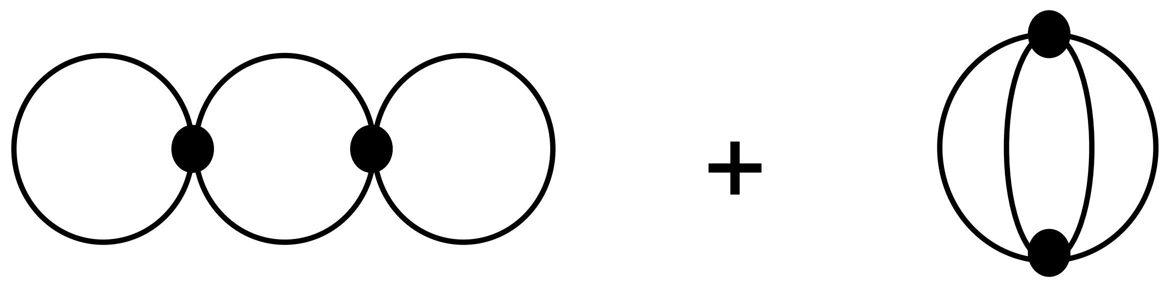


$$\delta p = m^4 T \sum\!\!\!\!\!\!\int_p \frac{1}{(\omega_n^2 + \vec{p}^2)^2}$$

n=0 mode is linearly IR divergent!

$$\sim m^4 T \int \frac{d^3\vec{p}}{\vec{p}^4}$$

Obviously trying to expand $1/(p^2+m^2)$

Resummation

- The solution is to recognize that we have a two-scale problem
  - T = hard scale
  - $\lambda^{1/2}T$ = soft scale ('screening')
- Trying to ignore the second scale triggers IR divergences
- In this case the effective theory at the scale $\lambda^{1/2}T$ is very simple: massive 3D ϕ^4.

- The correct NNLO correction to the pressure:
- What's the pressure of a free massive boson in 3d?

$$
\begin{aligned}
p_{3d} &= -\frac{1}{2}\log\det(p^2+m^2) \\
&:= -\frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\log(p^2+m^2) \\
&= -\frac{m^{3/2}}{2}\frac{1}{4\pi^2}\int_0^\infty u^{1/2}du\log(u+1) \\
&= +\frac{\lambda^{3/2}T^3}{24\cdot 24\pi\sqrt{6}}
\end{aligned}
$$

Series expansion for the pressure

$$p = \frac{\pi^2 T^4}{90} - \frac{\lambda T^4}{24\cdot 48} + \frac{\lambda^{3/2} T^4}{24\cdot 24\pi\sqrt{6}} + \mathcal{O}(\lambda^2 \log\lambda)$$

- Series proceeds in $\lambda^{1/2}$, to all orders
- Can be organized systematically using 3d effective theory ('dimensional reduction')
  (Kajantie, Laine, Rummukainen& Shaposhnikov ph/9508379)
- 3d theory is *fully* perturbative due to large mass $\sim\lambda^{1/2}T$

The QCD pressure

- QCD pressure is similar. Known to O(g^6log g):

$$\frac{p}{T^4} = \frac{\pi^2}{180}(4d_A + 7d_F) - \frac{d_A g^2}{144}(C_A + \frac{5}{2}T_F) + \frac{g^3 d_A}{12\pi}(\frac{1}{3}(C_A + T_F))^{3/2} + \ldots$$

- 3d theory is 3d YM (g^2T) + massive scalar 'A_0' (gT)
- nonperturbative contribution at g^6: vacuum energy of 3d pure YM, which confines. Has been measured on the lattice. ('*Linde problem*')
- Convergence per se is not very good, but many good resummations have been proposed

QCD plasma at weak coupling: dynamics

- In a weakly coupled QCD, just like in phi[4], there is a screening scale

$$m_D \approx gT \qquad\qquad (m_D = \sqrt{1.5}gT \text{ in pQCD})$$

- Inverse mean free path for *large angle* scattering

$$\Gamma^{\rm hard} \sim \frac{g^4 T}{4\pi}$$

- Assume: $\Gamma^{\rm hard} \ll m_D$

‘rare scatterings within a Debye radius’

Hard thermal loops

- What's the description of the screening scale, e.g., the gT scale?
- Let's assume gT<<T (not very realistic, but makes the physics cleaner to understand)
- *Soft* classical YM field (gT) coupled to hard *point-like* (E~T) particle
- UV degrees of freedom best characterized by phase space distribution $n(\vec{p}, x)$

- Warm up: (relativistic) QED plasma.

$$v \cdot \partial_x n(\vec{p}, x) + e v^\mu F_{\mu i} \frac{\partial}{\partial p_i} n(\vec{p}, x) = 0 \qquad (v^\mu = (1, \vec{v}))$$

(Boltzmann-Vlasov equation)

- For typical EM field with ~1/gT coherence, deflection will be small: work to first order in F. For initially isotropic n, B-field drops out.

$$\delta n(\vec{p}, x) = \frac{1}{v \cdot \partial} e \vec{v} \cdot \vec{E} \ \times \ \frac{-\partial n(|p|)}{\partial |p|}$$

$$J^\mu_{\rm ind} = \int d^2\Omega_v v^\mu \frac{1}{v \cdot \partial} \vec{v} \cdot \vec{E} \times \left(e^2 \int |p|^2 d|p| \frac{-\partial n(|P|)}{\partial |p|} \right)$$

- Gauge-invariant extension to QCD is straightforward:

$$J^{\mu a}_{\rm ind} = \int d^2\Omega_v \frac{v^\mu}{v\cdot D} \vec{v}\cdot\vec{E}^a \times \left(g^2 \int |p|^2 d|p| \frac{-\partial n(|P|)}{\partial |p|} \right)$$

[retarded b.c.]

(<-> Wong equations)

- 'Hard thermal loop effective action'

[Braaten& Pisarski]

- $$m_D^2 = \frac{g^2T^2}{3}[C_A + n_F T_F]$$

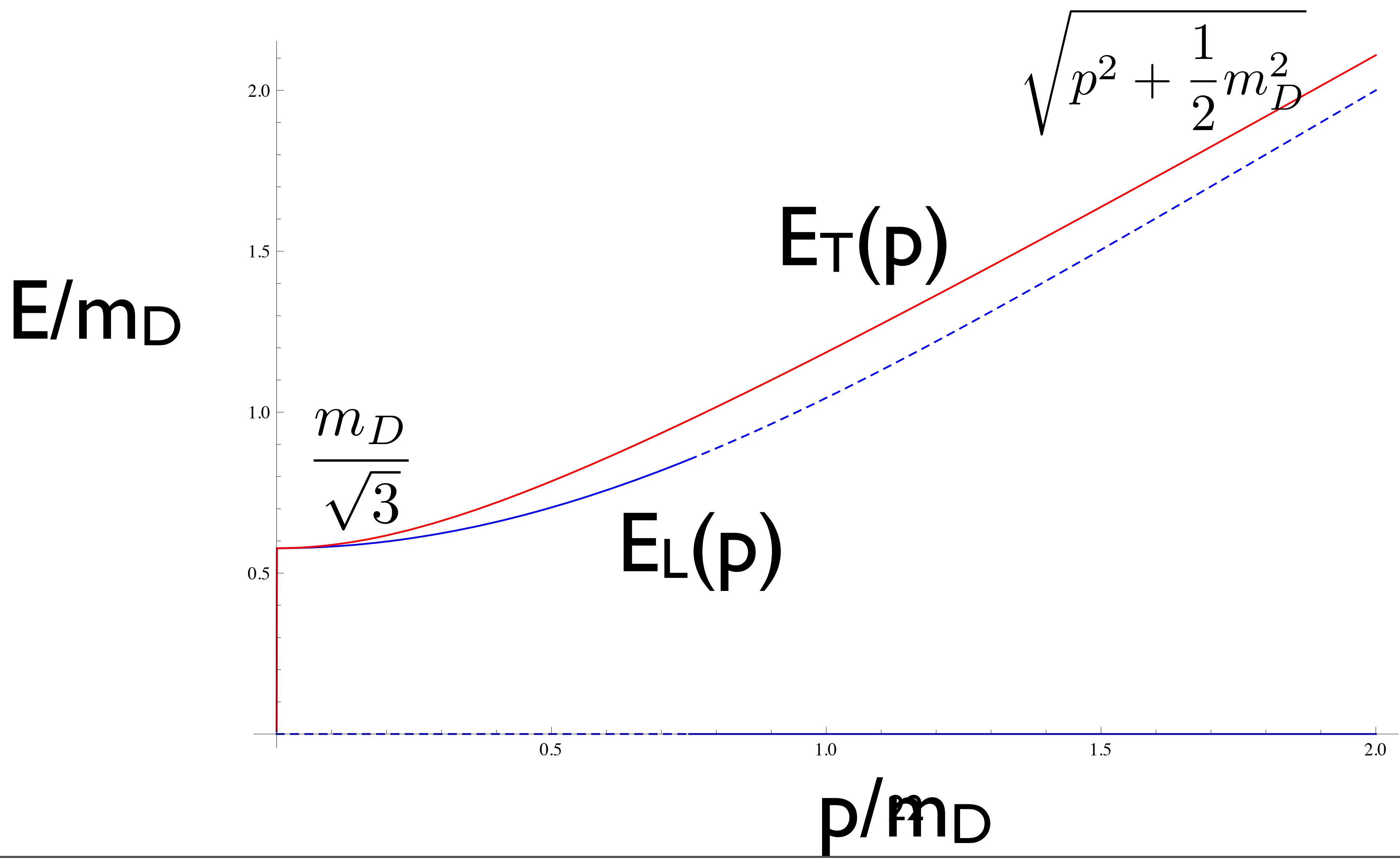
$$= 1.5 g^2 T^2$$ for n_F=3

- Gluon self-energy: linearize J[A]

$$\Pi^{00} = m_D^2 \int d^2\Omega_v \frac{\vec{v}\cdot\vec{p}}{v\cdot p} = m_D^2 \left[1 - \frac{1}{2}\frac{p^0}{p}\log\frac{p^0+p+i\epsilon}{p^0-p-i\epsilon}\right]$$

$$\Pi^{0i} = \ldots, \Pi^{ij} = \ldots$$

- Resummed propagator: *massive dispersion relation*



Plasma oscillations

- How to understand the longitudinal mode at low momentum? Take a homogeneous electric field

$$\frac{d\vec{E}}{dt} = -\vec{J} \qquad\qquad \vec{J} = m_D^2 \int d^2\Omega_v \vec{v} \frac{1}{\partial_t} \vec{v}\cdot\vec{E}$$

$$\text{or: } \frac{d}{dt}\vec{J} = \frac{m_D^2}{3}\vec{E}$$

- E oscillates with natural frequency $m_D/\sqrt{3}$

HTL summary

- Gauge-invariant gluon mass at finite T
- Extra gluon DOF comes collectively from medium
- Similar effective action for chiral (Weyl) fermions -- also acquire mass.
- Assumptions:
  -no rescattering within a Debye crossing *[maybe ok]*
  -gT<<T: gradient expansion *[questionable]*