Backreaction in Swiss Cheese models

Mikko Lavinto

University of Helsinki / Helsinki Institute of Physics

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Outline

- Introduction and motivation
- Model construction
- Results
- Summary

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- In cosmology we use (almost) always averaged Friedmann-Robertson-Walker metric.
- Is this correct?

Backreaction

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- Like Friedmann's equation, except an extra term

$$\mathcal{Q} \equiv \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2\right) - 2\langle\sigma^2\rangle$$

appears due to averaging

Swiss Cheese construction

- Lemaître-Tolman-Bondi metric:

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- $\rightarrow$ Darmois junction conditions

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- Expansion rate varies with r
- If the inside of an LTB region expands faster than outside, at some point the radial dust shells will overlap, generating a shell crossing singularity
- This combined with Darmois junction conditions restricts the models a lot

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- The spacetime matches smoothly to a FRW dust universe at the boundary.
- The areal radius at the matching surface is small compared to the spacetime curvature radius of the background FRW universe.

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- When $\frac{\partial R(t,r)}{\partial r} = 0$, there is a discontinuity in the extrinsic curvature.
- This induces a 2+1-dimensional thin singular shell on the surface
- Can we suspend our disbelief and construct a model with significant backreaction?

Not really

Having significant backreaction requires collapsing regions as

$$\mathcal{Q} \equiv \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2\right) - 2\langle\sigma^2\rangle$$

Collapsing regions require pressure to keep them from collapsing into singularities.

$$\Omega_{\mathcal{Q}} \lesssim 1\%$$

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- However, the junction conditions and having no pressure restrict backreaction to always be small in these models if the inhomogeneities are small.
- Even if we break one of the junction conditions, we still could not have large backreaction
- It appears likely that more complicated models are needed for estimation of backreaction.