Numerical Simulations of Relativistic Multifluids

I. Hawke

with thanks to Nils Andersson, Greg Comer, Cesar Lopez-Monsalvo and Lars Samuelsson

School of Mathematics,
University of Southampton, UK

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Outline

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2. Summary

Introduction

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Punchline:

Multifluid models may be useful or essential for representing physics such as superfluidity. However, it is computationally expensive and the additional instabilities are messy.

Introduction

- Multiple particle species will be important in NSs.
- For certain phenomena (e.g. superfluids, heat conduction) a single ideal fluid approximation is insufficient.
- Multiple fluids occupying the same volume may give a sufficient model;
  - Standard hydrodynamic model of superfluids is inherently multifluid;
  - Some causal heat conduction models can be derived from multifluid approaches.

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  - Standard hydrodynamic model of superfluids is inherently multifluid;
  - Some causal heat conduction models can be derived from multifluid approaches.

The key physics question is determining what the fluids represent.

That question is ignored here; instead look at numerical methods for a toy problem.

Carter Framework

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Carter introduced a Hamiltonian framework based on species number currents n^{ν}_{X} and a master function ("energy") $\Lambda \equiv \Lambda\left(n^{\alpha}_{X} n^{Y}_{\alpha}\right)$.

- Action principle varying Λ; clean coupling to GR.
- Conjugate momenta μ^{X}_{ν} need not be parallel to n^{ν}_{X}; *entrainment*.
- Covariant continuity and Euler equations follow:

$$\nabla_{\mu} n^{\mu}_{X} = \Gamma^{X},$$
$$2 n^{\mu}_{X} \nabla_{[\mu} \mu^{X}_{\nu]} = f^{X}_{\nu}.$$

- Typically assume inter-species forces balance, $\sum_{X} f^{X}_{\nu} = 0$.
- Even in simple case with no species production ($\Gamma^{X} = 0$) and no forces the fluids can still interact through the entrainment.

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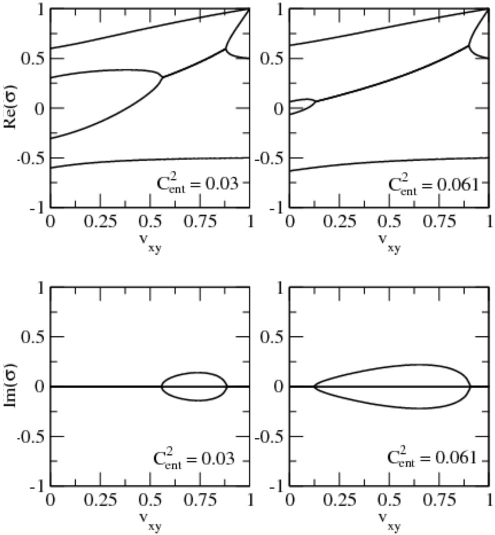
Two-stream instabilities

Linear analysis shows instabilities when the *relative flow* is large and entrainment occurs.

Plane parallel waves on flat backgrounds without boundaries are checked.

The dispersion relation predicts high frequency modes will explode.



Samuelsson et al, arXiv:0906.4002

Nonlinear code

As a first attempt we look at the simplest case:

- No species production or forces.
- "Generalized polytrope" EOS (Prix et al, PRD71 043005 (2005), Samuelsson et al, arXiv:0906.4002).
- Periodic boundaries, 1+1 or 2+1d, special relativity.
- Central differencing with Kreiss-Oliger dissipation.

The equations become

$$\partial_t n^t_X = -\partial_j n^j_X,$$

$$\partial_t \mu^X_i = \partial_i \mu^X_t + 2 \sum_j \frac{n^j_X}{n^t_X} \partial_{[i} \mu^X_{j]}.$$

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Implementation issues

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The evolved variables (n^t_X, μ^X_i) do not immediately contain all the necessary information. All other components of n^ν_X must be found from e.g. the master function.

Currently the code

1. guesses the value of the scalars $n^\alpha_X n^Y_\alpha$;
2. computes Λ and derivatives from these guesses;
3. uses the definition of μ^X_i to find n^i_X by solving a linear system;
4. uses the resulting approximation to n^ν_X to check the scalar guesses, implying a nonlinear root-find.

This "primitive variable recovery" requires over 90% of the run-time.

1d results

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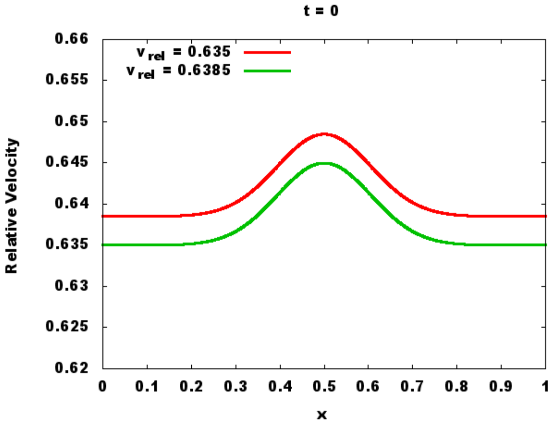

Near the instability limit the fully nonlinear evolutions reproduce the linear analysis. The instability cascades down from high frequency.

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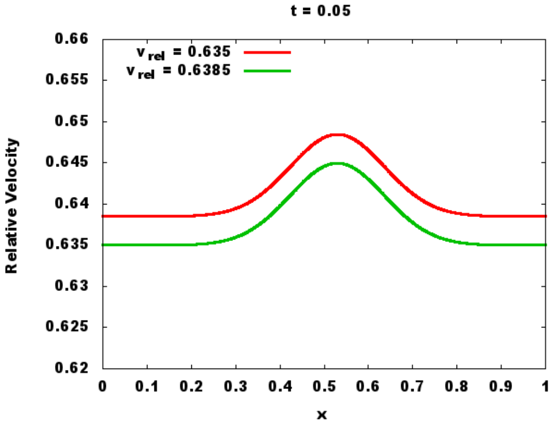

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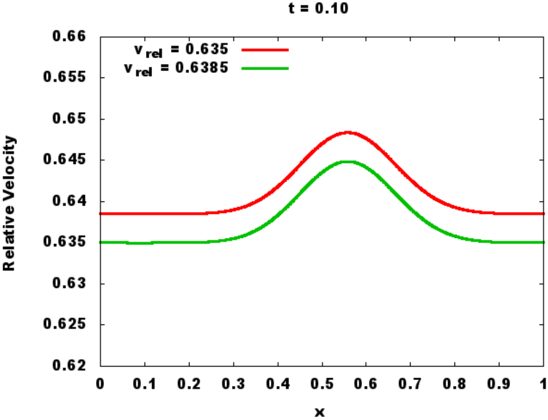

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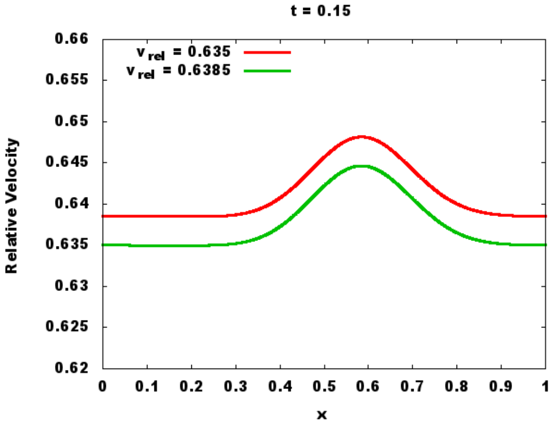

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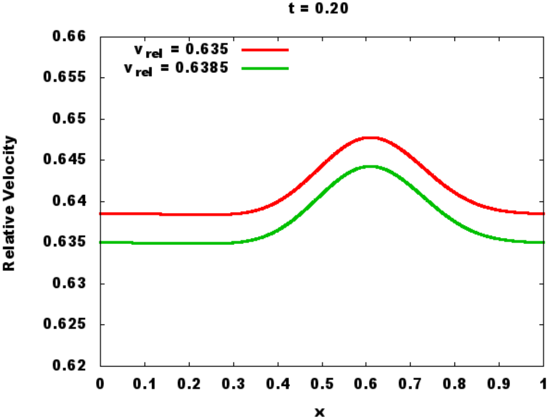

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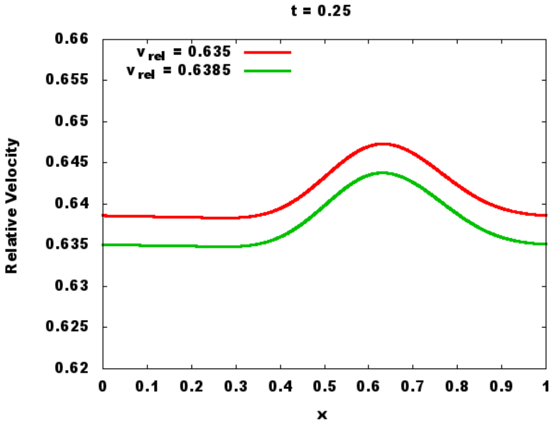

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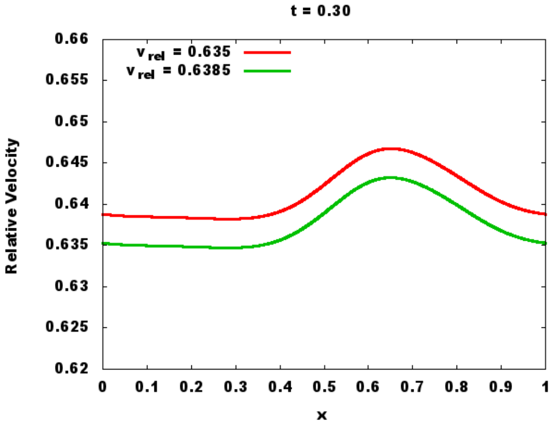

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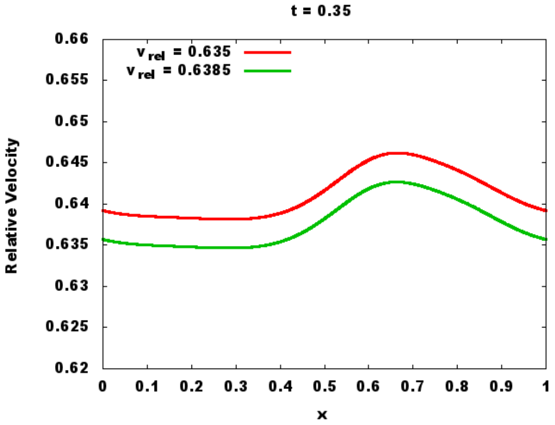

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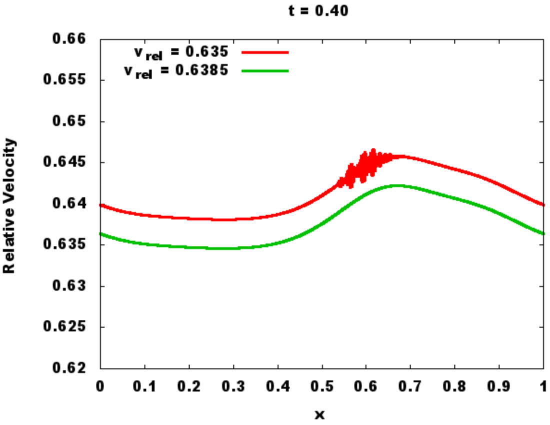

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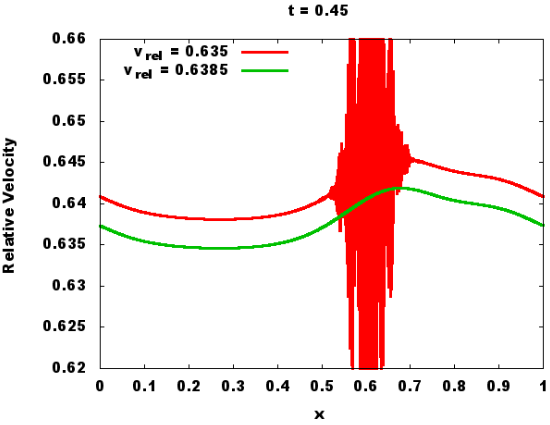

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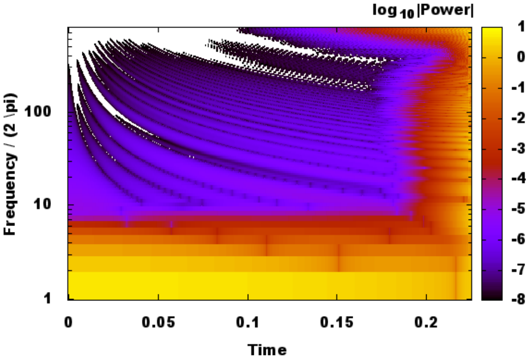

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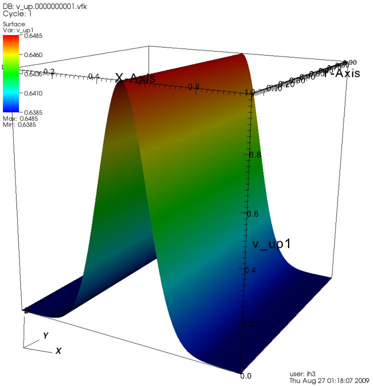
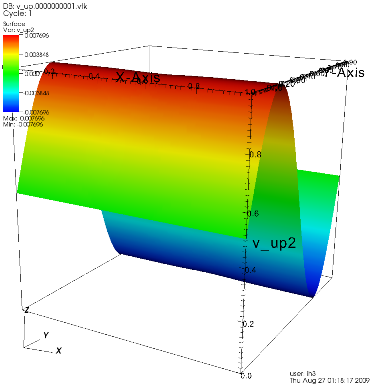
2d results





It is straightforward to extend to a 2d "shearing box"; perturbing the initial data in the other direction has no effect on the instability.

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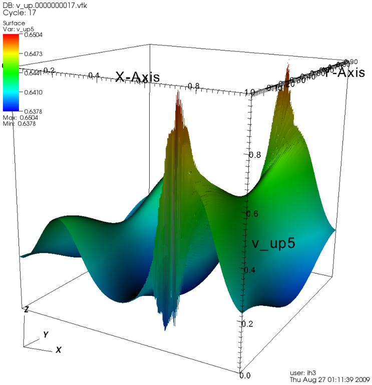

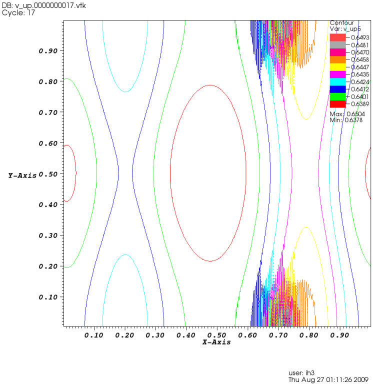

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Summary

- Multiple (ideal) fluids give a simple framework for modelling complex interactions.
- General conservation law forms are not obvious.
- Numerically expensive to convert between required types of variables.
- Linear instabilities are clear in nonlinear evolutions.