

# Microphysics, Methods, and Core-Collapse Supernovae

Adam Burrows, in collaboration with  
Todd Thompson and Ivan Hubeny

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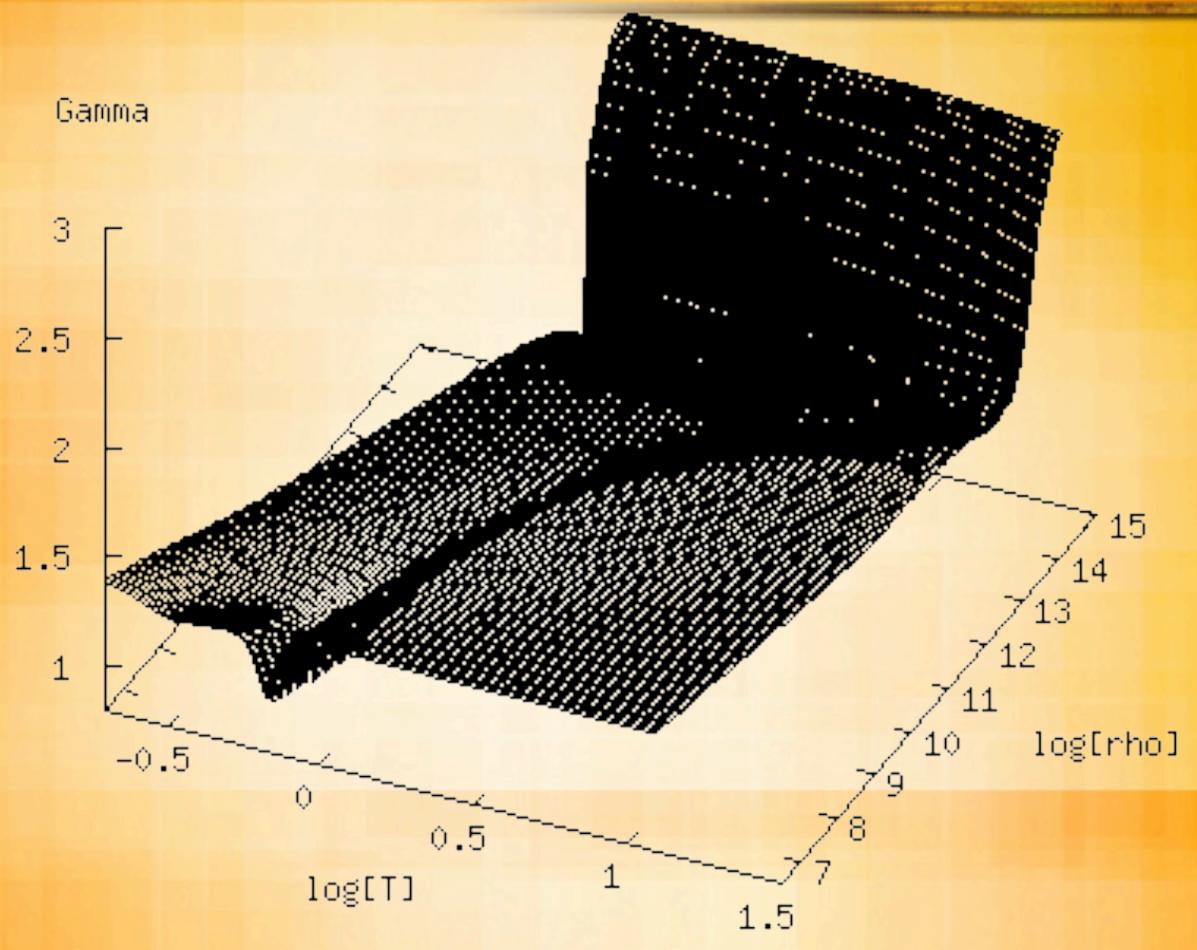
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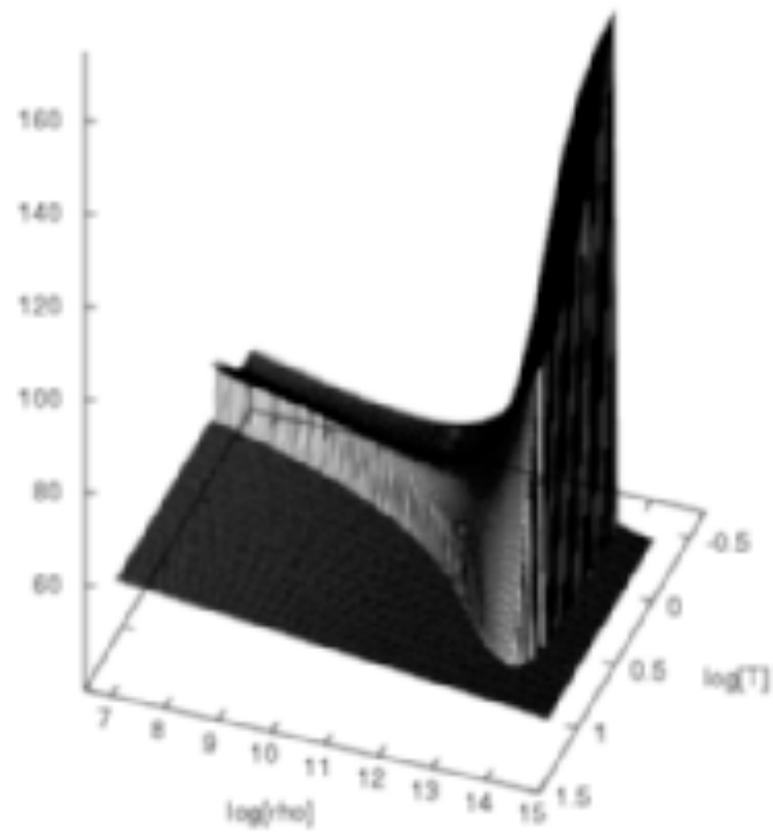
# **Nuclear Equation of State (EOS)**

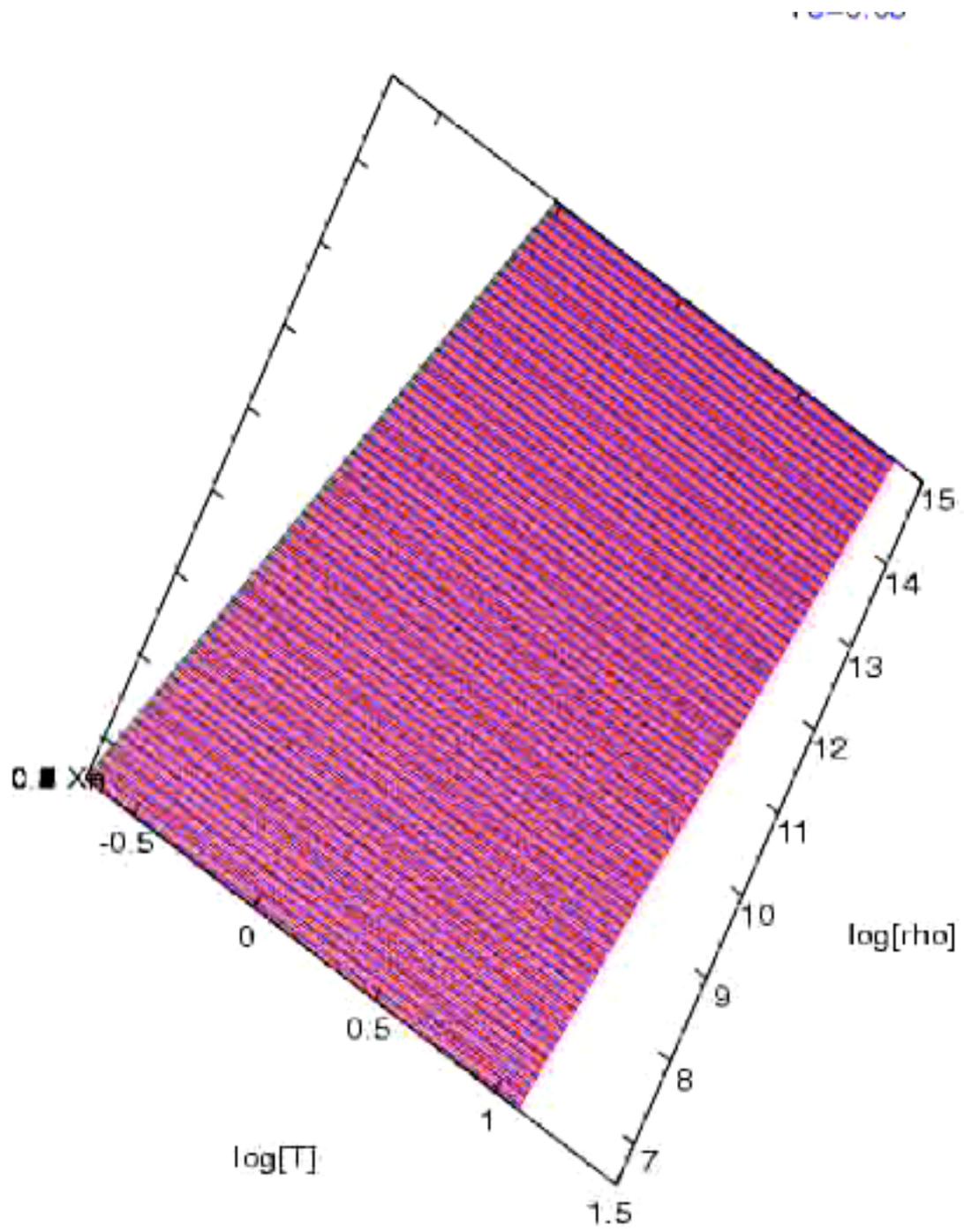
$Y_e = 0.50$  ———



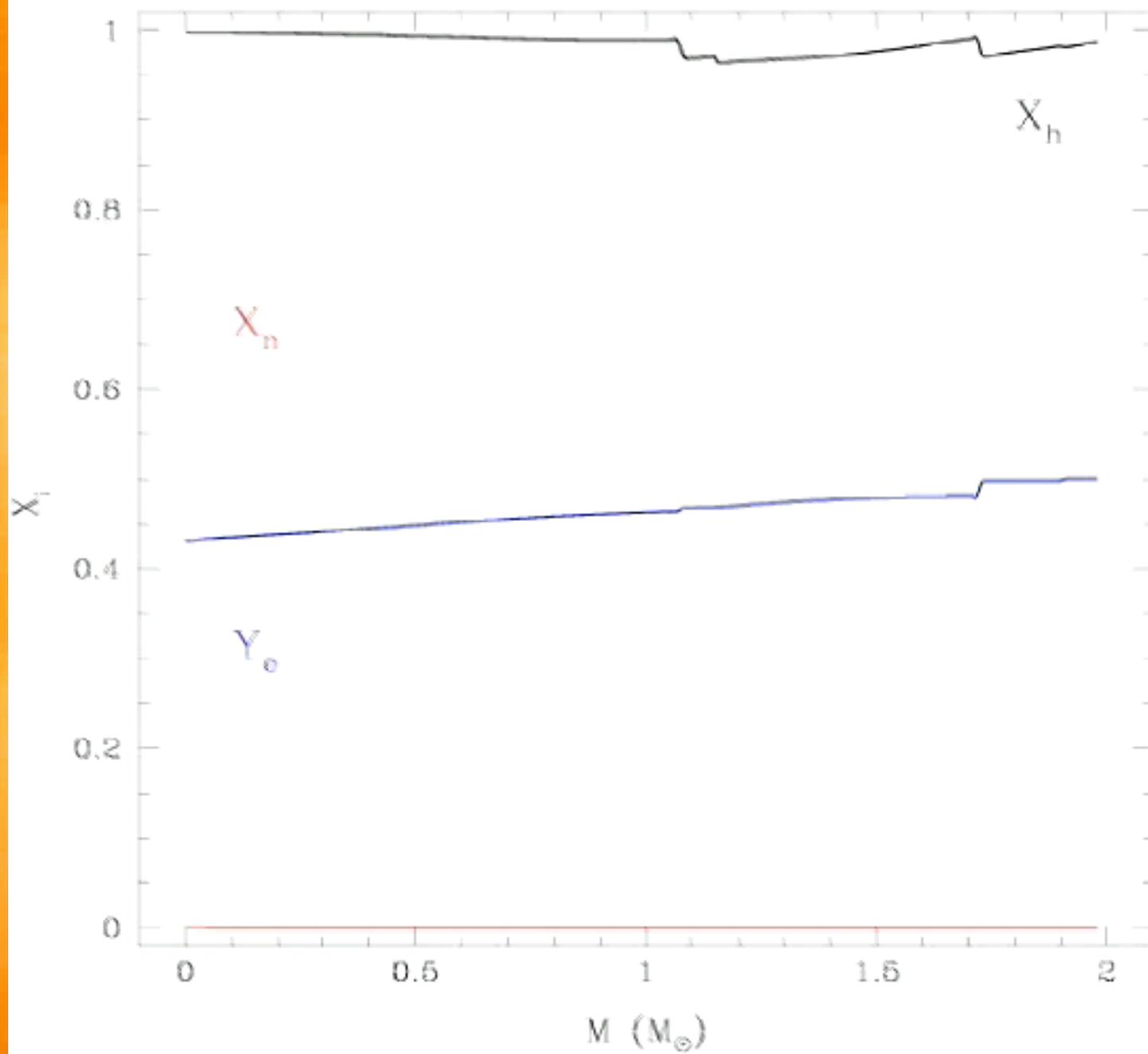
$Y_e = 0.49$  ———

A



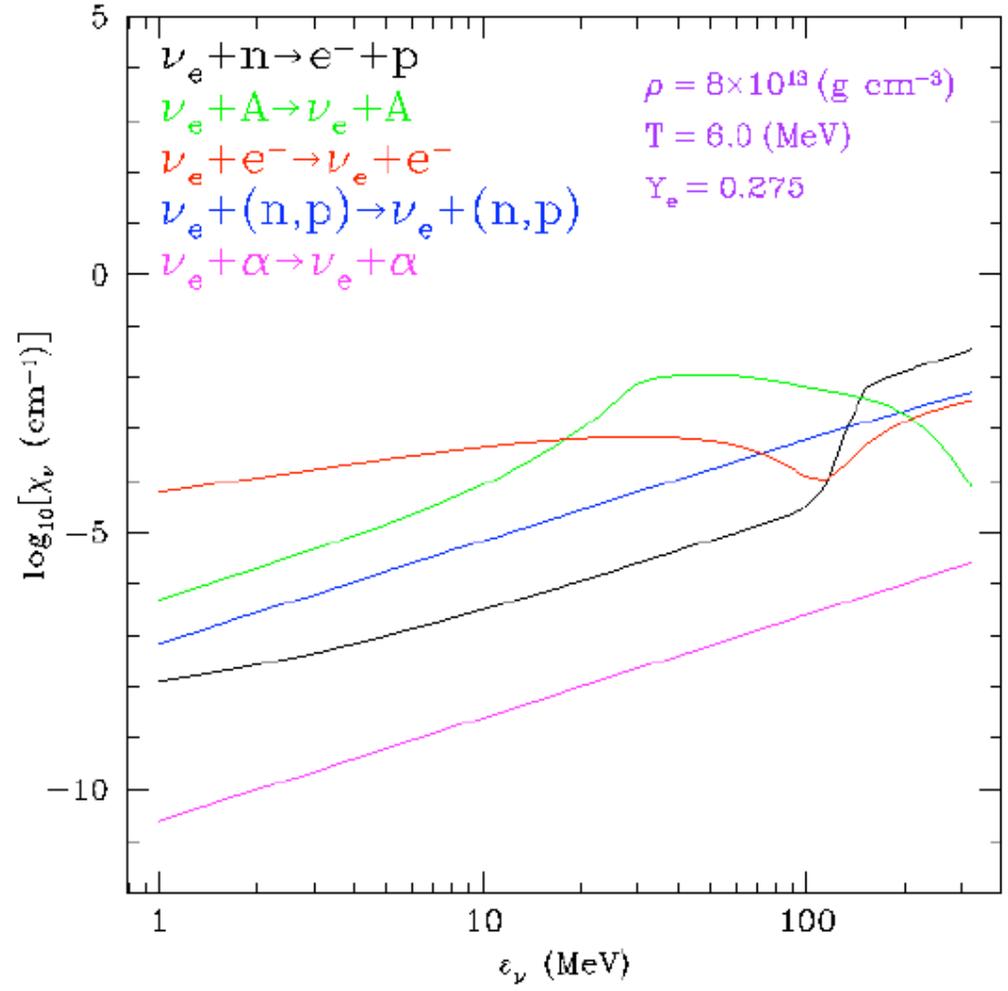


Model : S20

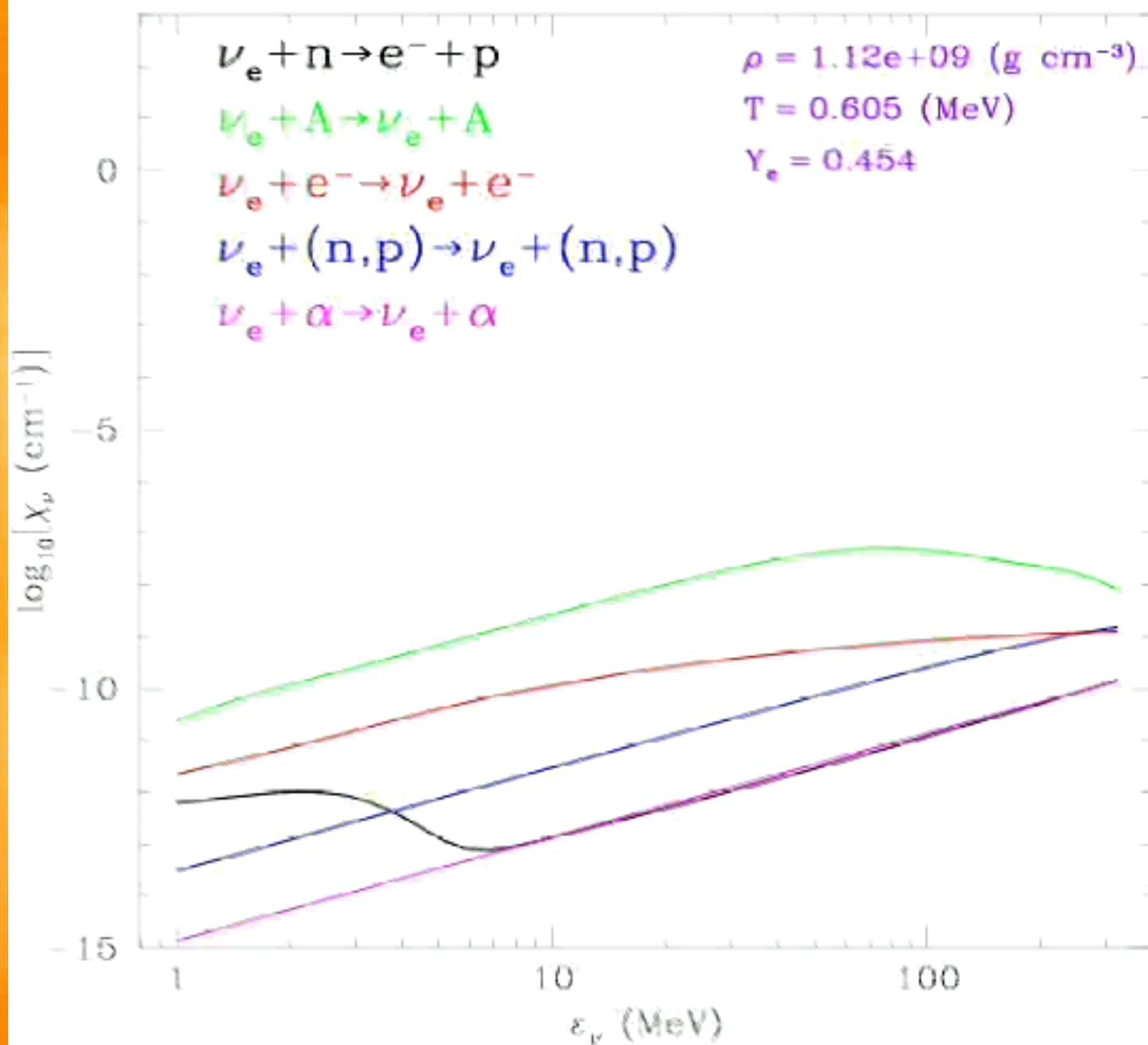


# Neutrino Opacities

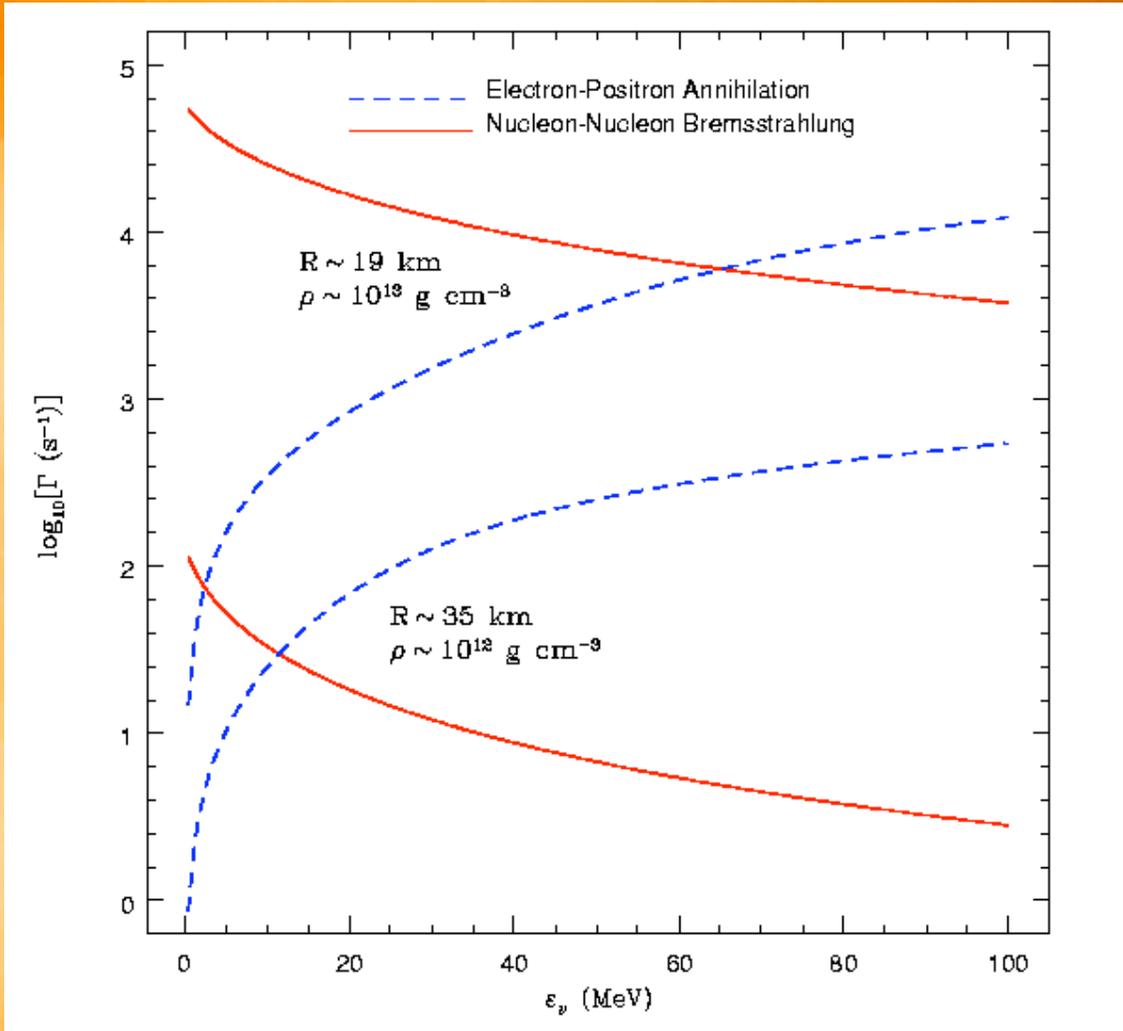
Model: 11  $M_{\odot}$ , Radius = 4.48 (km)



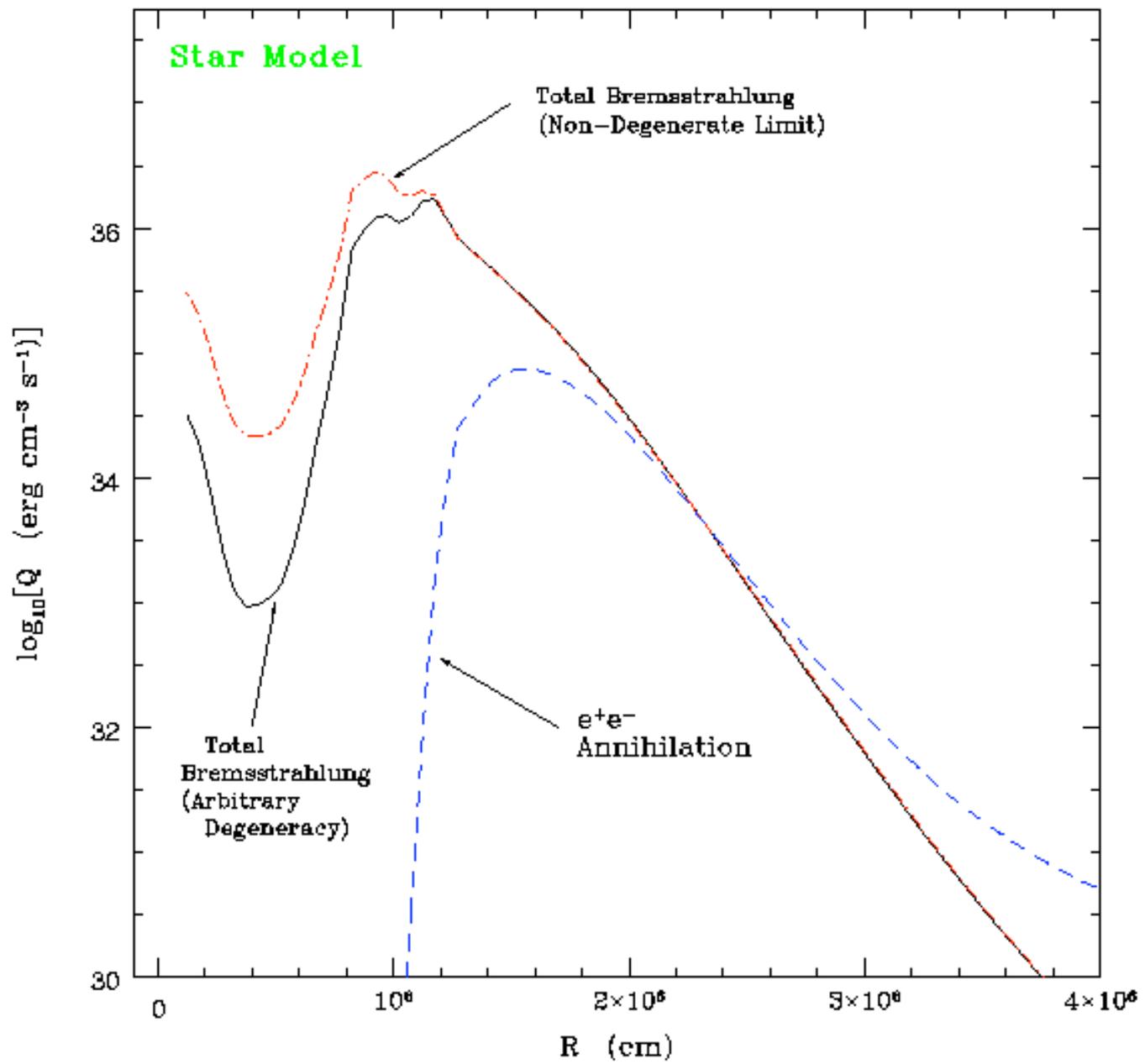
Model:  $11 M_{\odot}$ , Mass =  $0.9 M_{\odot}$ , Radius = 489 (km)

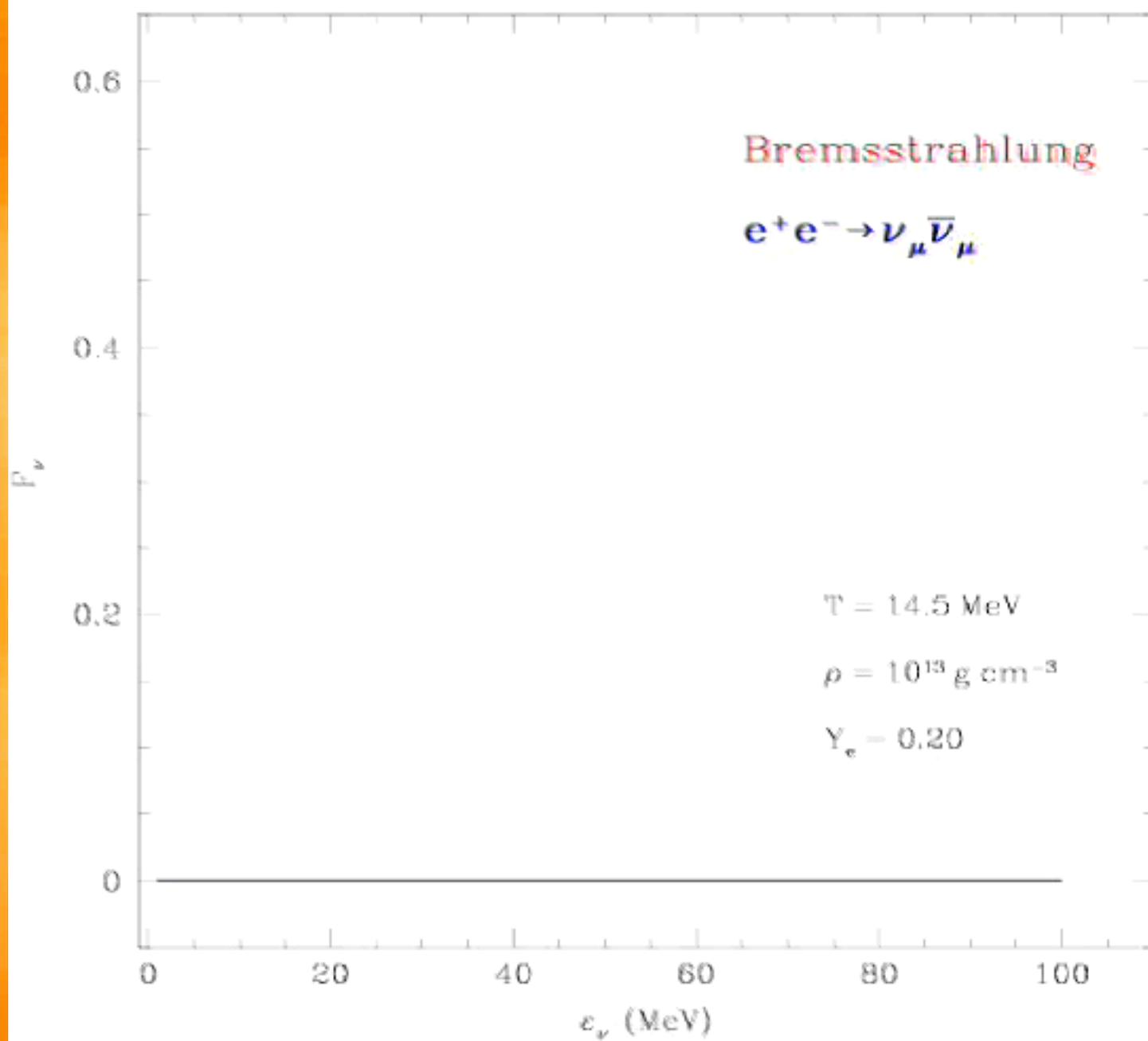


**Nucleon-nucleon  
Bremsstrahlung  
(and  $e^+e^-$  pairs)**

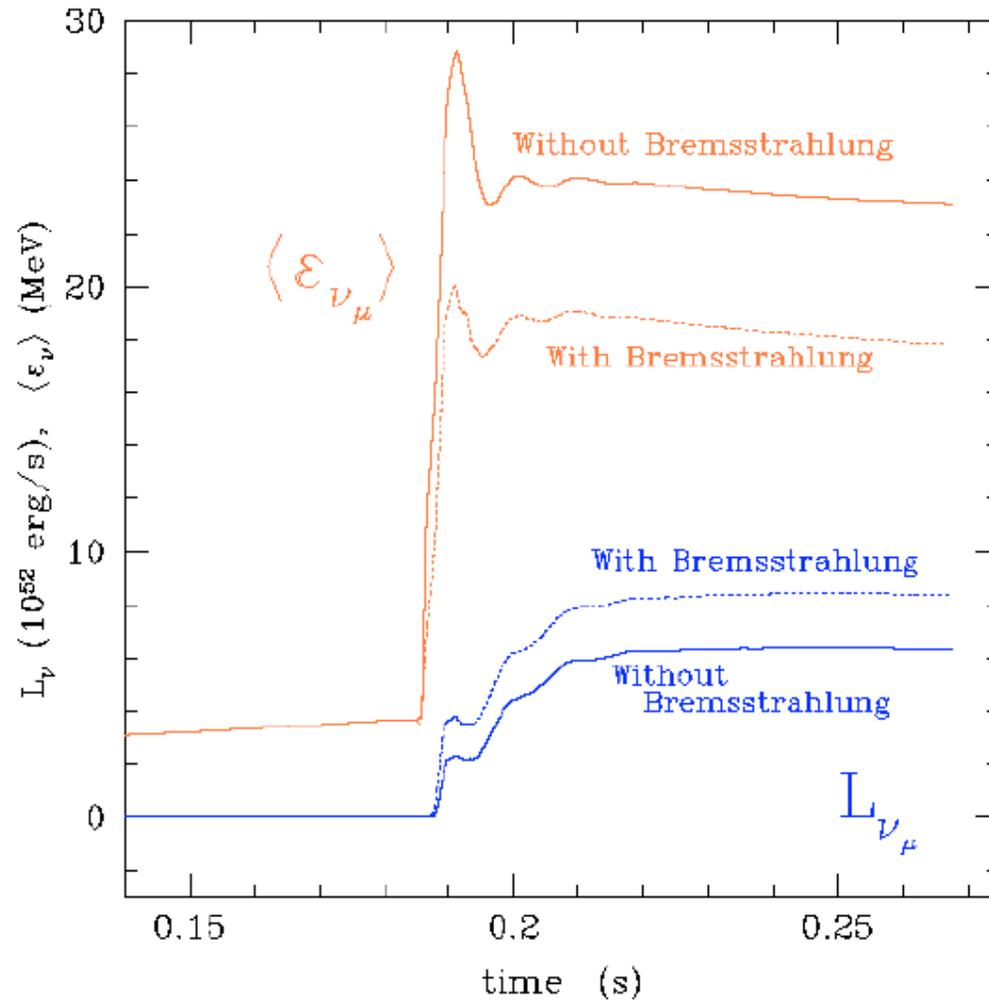


**Star Model**

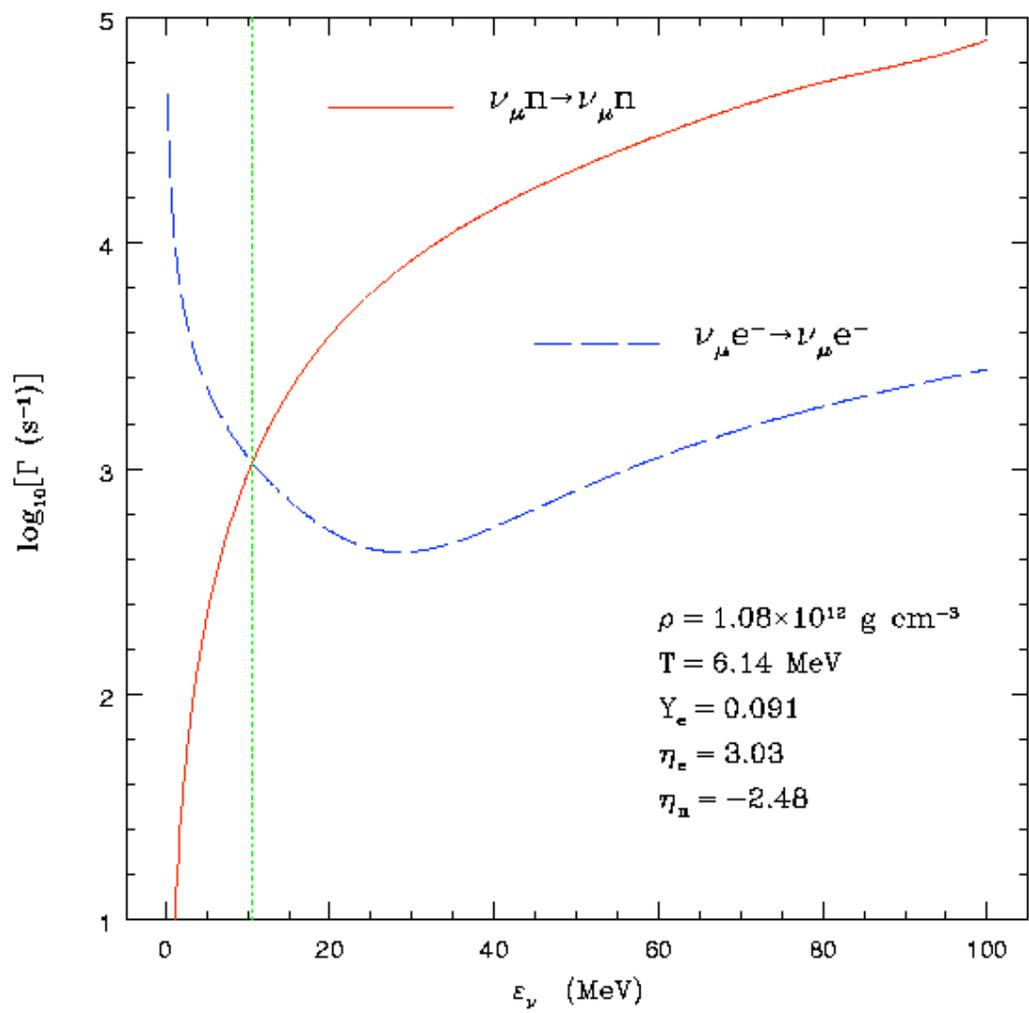




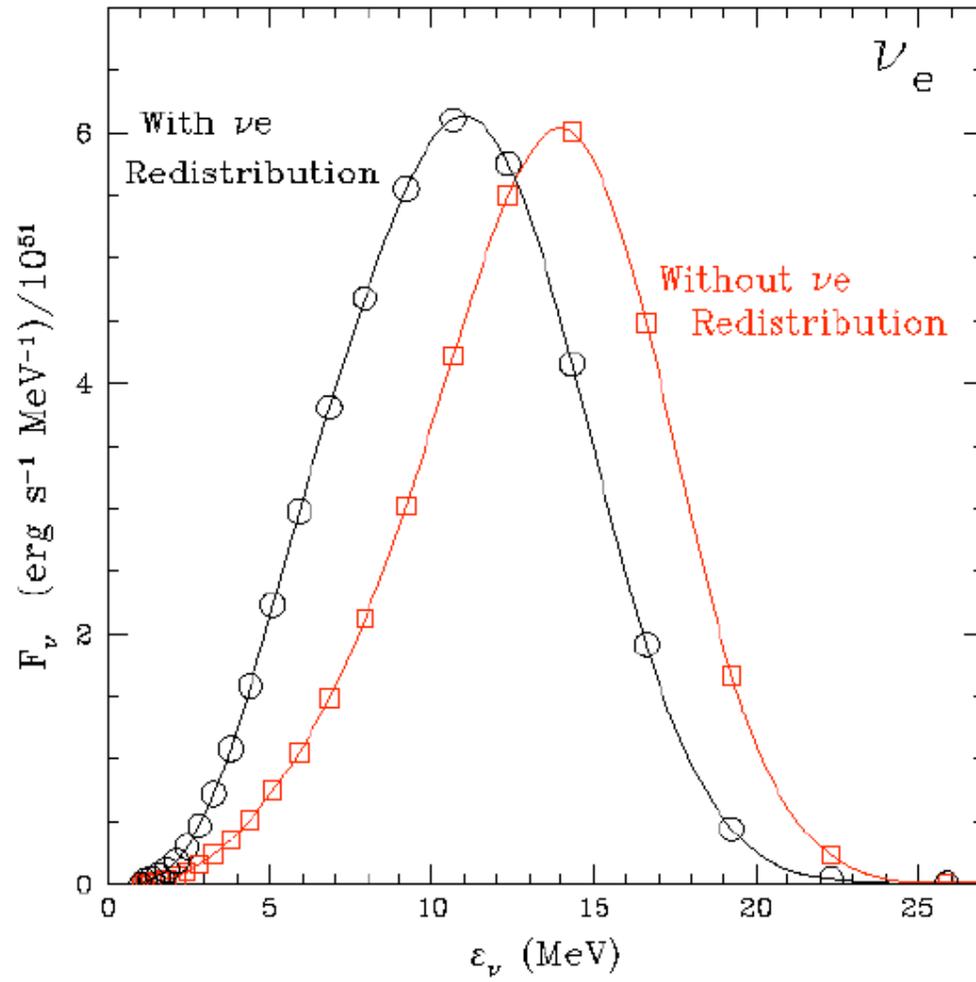
Progenitor Model:  $15 M_{\odot}$



# Inelastic Scattering



Progenitor Model: S15



# Microphysics, Methods, and Core-Collapse Supernovae: **Transport**

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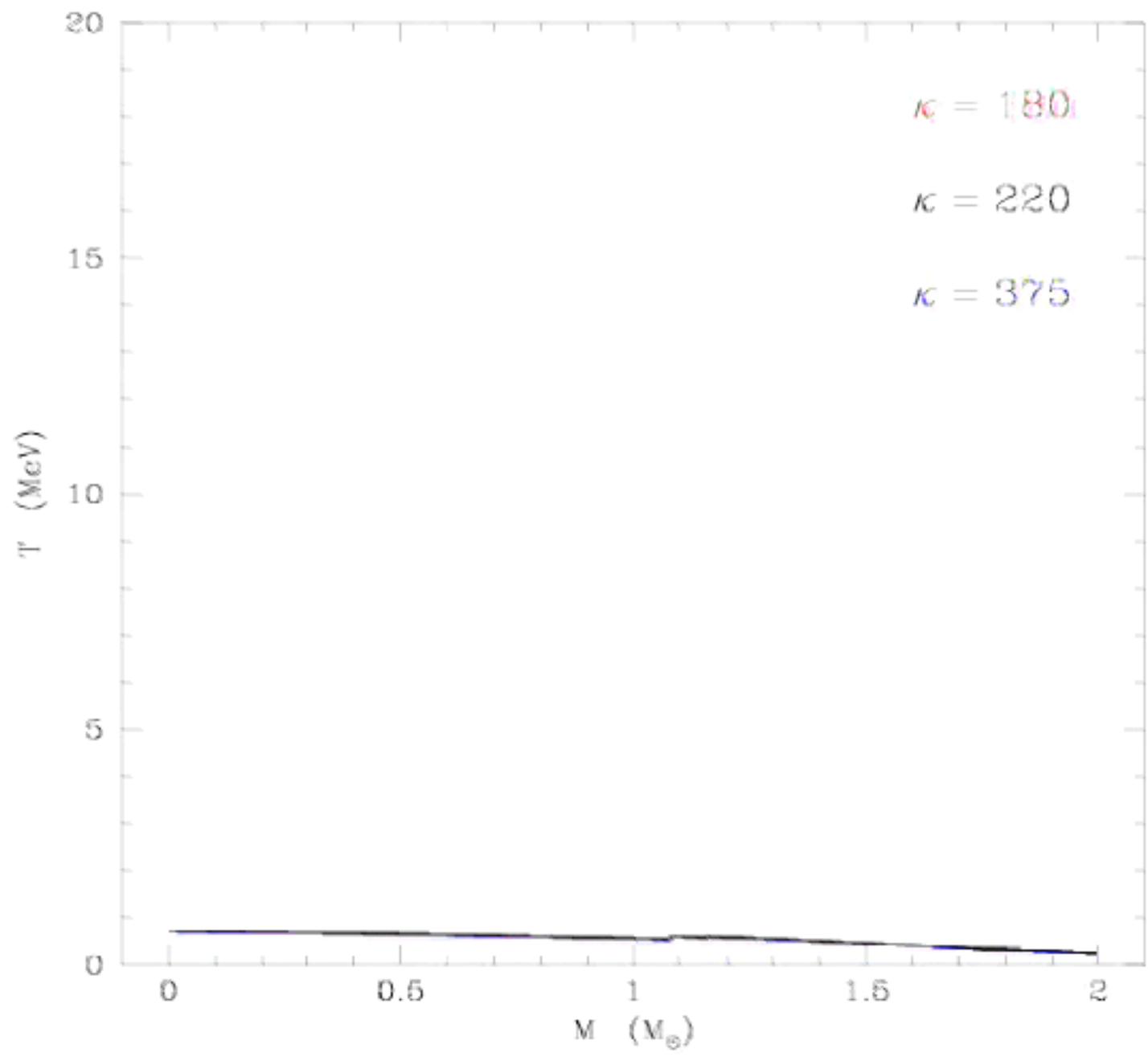
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## VULCAN/2D Multi-Group, Multi-Angle, Time-dependent Boltzmann/Hydro (6D)

- Only code with multi-D transport used in supernova theory
- Arbitrary Lagrangian-Eulerian (ALE); remapping
- 6 - dimensional (1(time) + 2(space) + 2(angles) + 1(energy-group))
- Moving Mesh, Arbitrary Grid; Core motion (kicks?)
- 2D multi-group, multi-angle,  $S_n$  (~150 angles), time-dependent, implicit transport (still slow)
- 2D MGFLD, rotating version (quite fast)
- Poisson gravity solver
- Axially-symmetric; Rotation
- MHD version ("2.5D") -  $\text{div } B = 0$  to machine accuracy; torques
- Flux-conservative; smooth matching to diffusion limit
- Parallelized in energy groups; almost perfect parallelism
- Livne, Burrows et al. (2004,2007a)
- Burrows et al. (2006,2007b), Ott et al. (2005,2008); Dessart et al. 2005ab,2006

## VULCAN/2D: Multi-Group, Multi-Angle Radiation/Hydrodynamics

- For the Simulation of Neutrino-Driven Supernovae
- Arbitrary Lagrangian-Eulerian (ALE)
- Moving Mesh
- Hydrodynamics has implicit mode
- 2D Multi-group, Multi-angle ( $S_N$ ) explicit transport
- Axially (Azimuthally) Symmetric
- Automatically Flux-Conservative Transport
- Smooth Matching of  $S_N$  method to Diffusion in interior (Diffusion-Synthetic Acceleration: DSA)

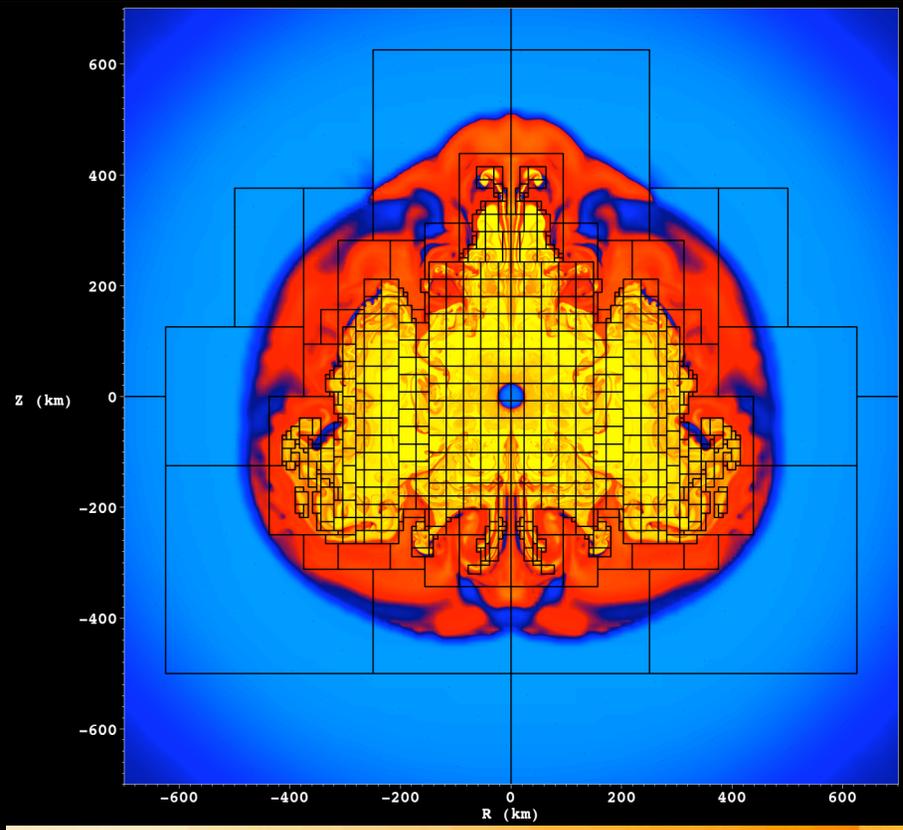
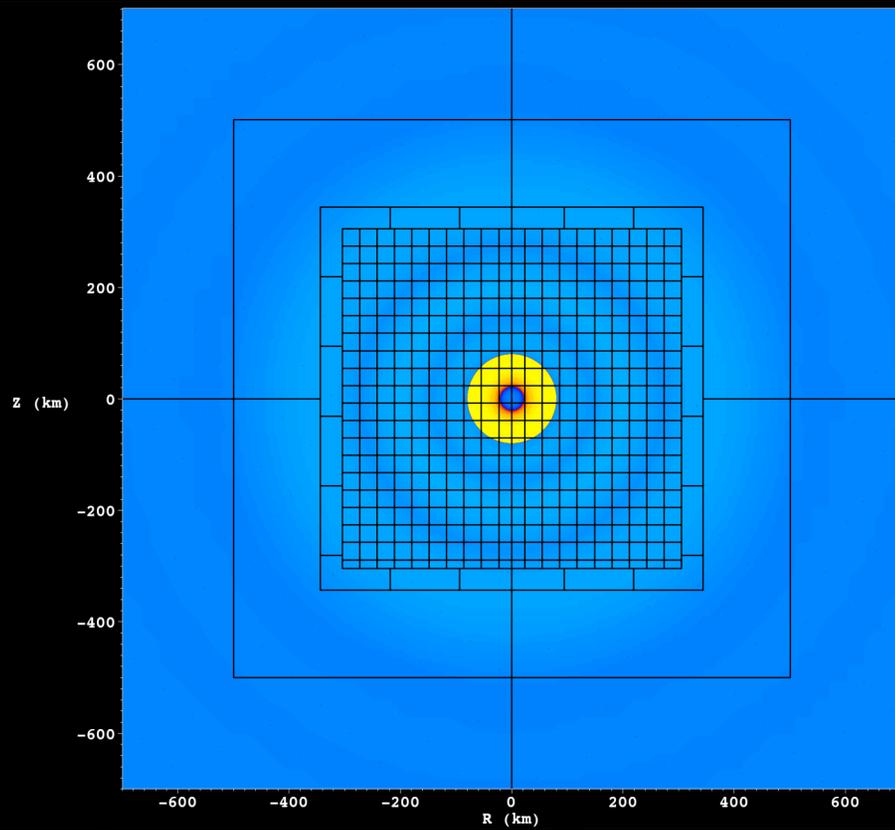
$$\frac{1}{c} \frac{\partial I}{\partial t} + \mu \frac{\partial I}{\partial r} + \frac{1 - \mu^2}{r} \frac{\partial I}{\partial \mu} + \sigma I = S \quad (1)$$

$$\frac{1}{c} \frac{\partial I}{\partial t} + \frac{1}{r^2} \frac{\partial (r^2 \mu I)}{\partial r} + \frac{1}{r} \frac{\partial [(1 - \mu^2) I]}{\partial \mu} + \sigma I = S \quad (2)$$

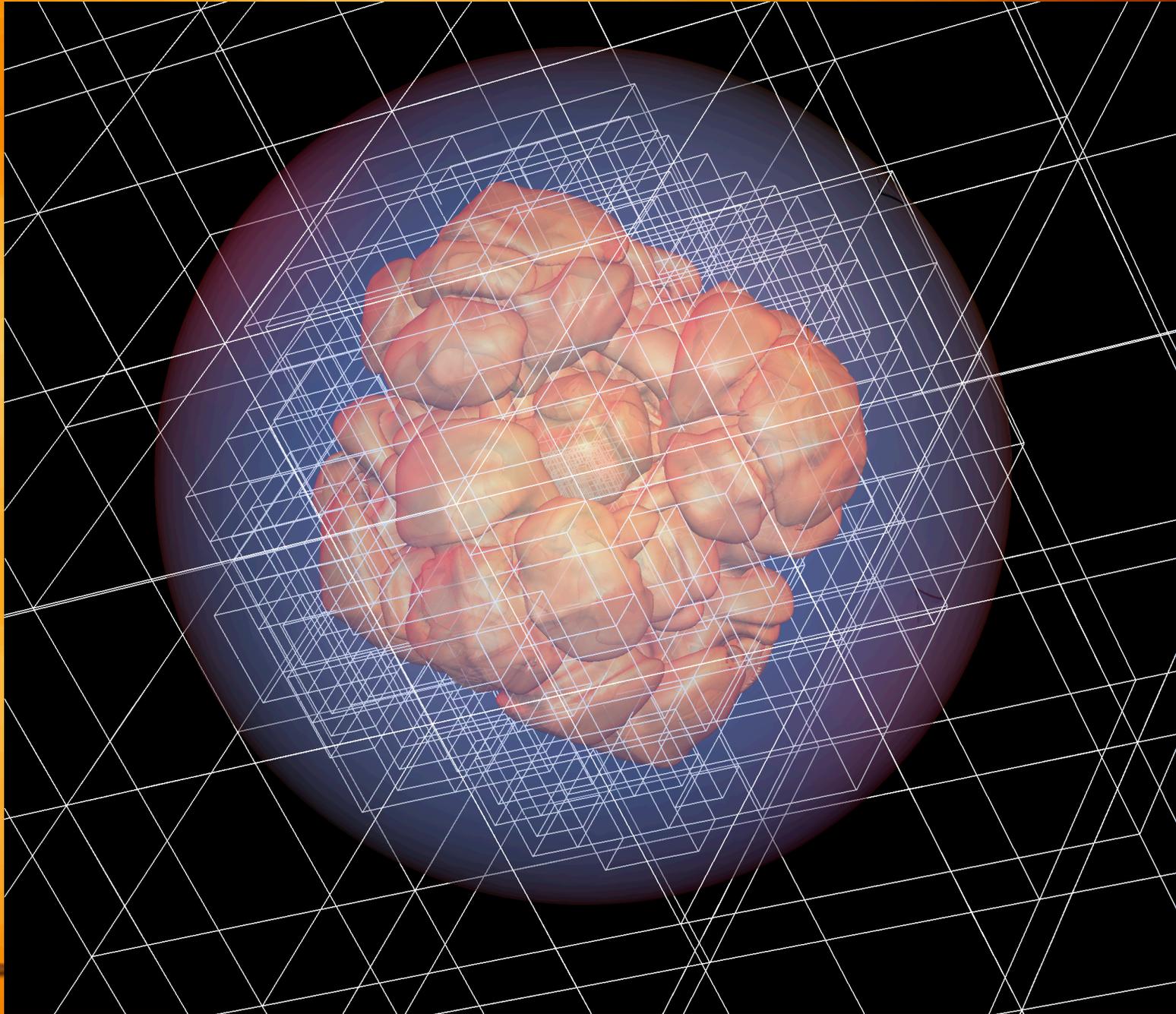
# CASTRO - 3D AMR, Multi-Group Radiation-Hydrodynamic Supernova Code

- 2nd-order, Eulerian, unsplit, compressible hydro
- PPM and piecewise-linear methodologies
- Multi-grid Poisson solver for gravity
- Multi-component advection scheme with reactions
- Adaptive Mesh Refinement (AMR) - flow control, memory management, grid generation
- Block-structured hierarchical grids
- Subcycles in time (multiple timestepping - coarse, fine)
- Sophisticated synchronization algorithm
- BoxLib software infrastructure, with functionality for serial distributed and shared memory architectures
- 1D (cartesian, cylindrical, spherical); 2D (Cartesian, cylindrical); 3D (Cartesian)
- Transport is a conservative implementation of mixed-frame method of Hubeny & Burrows (2007), with  $v/c$  terms and inelastic scattering
- Uses scalable linear solvers (e.g., hypre) with high-performance preconditioners that feature parallel multi-grid and Krylov-based iterative methods
- Developers: John Bell, Ann Almgren, Louis Howell, Mike Singer, Jason Nordhaus, Adam Burrows - LBNL, LLNL, Princeton

## Sample Block Grid Structures of CASTRO: Pre-collapse, Post-bounce

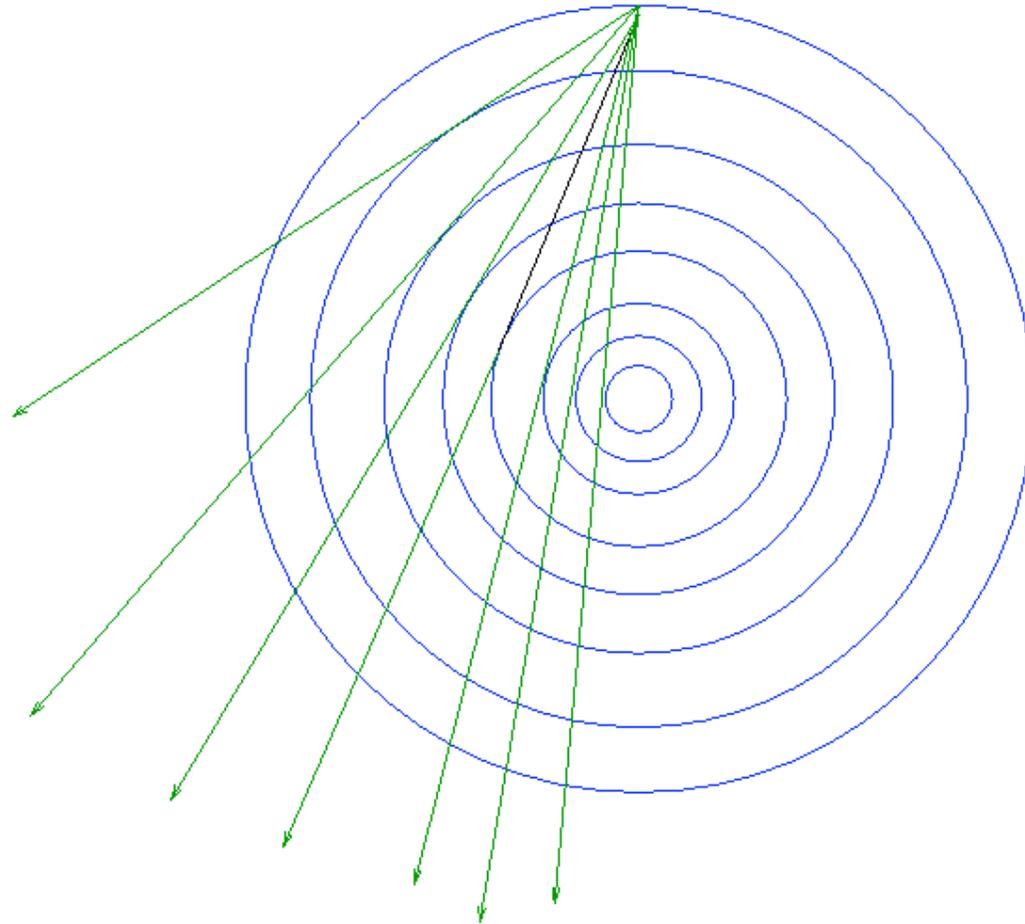


## CASTRO 3D AMR Mesh: Explosion Model

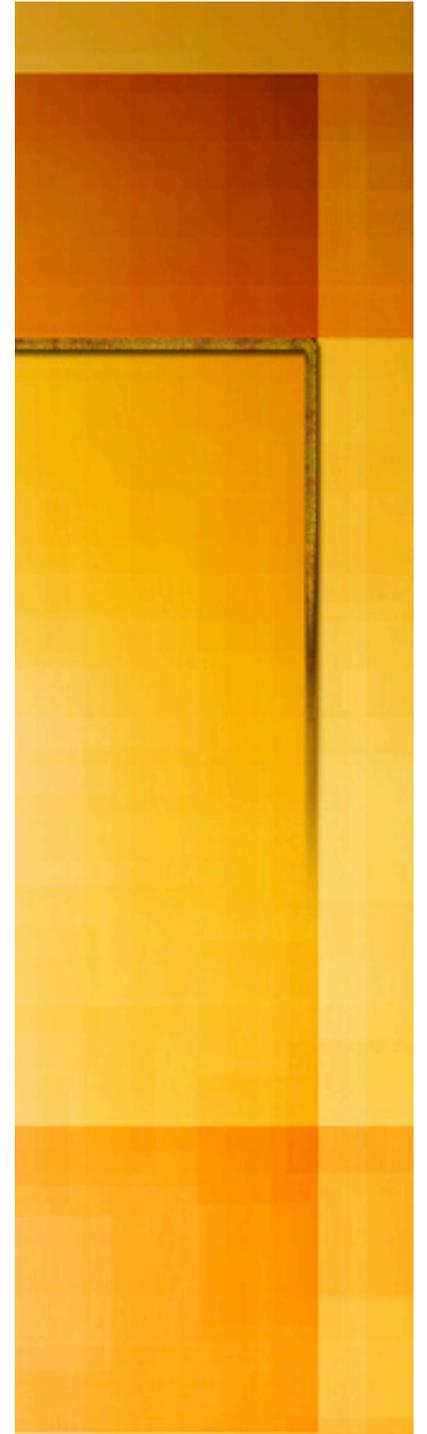
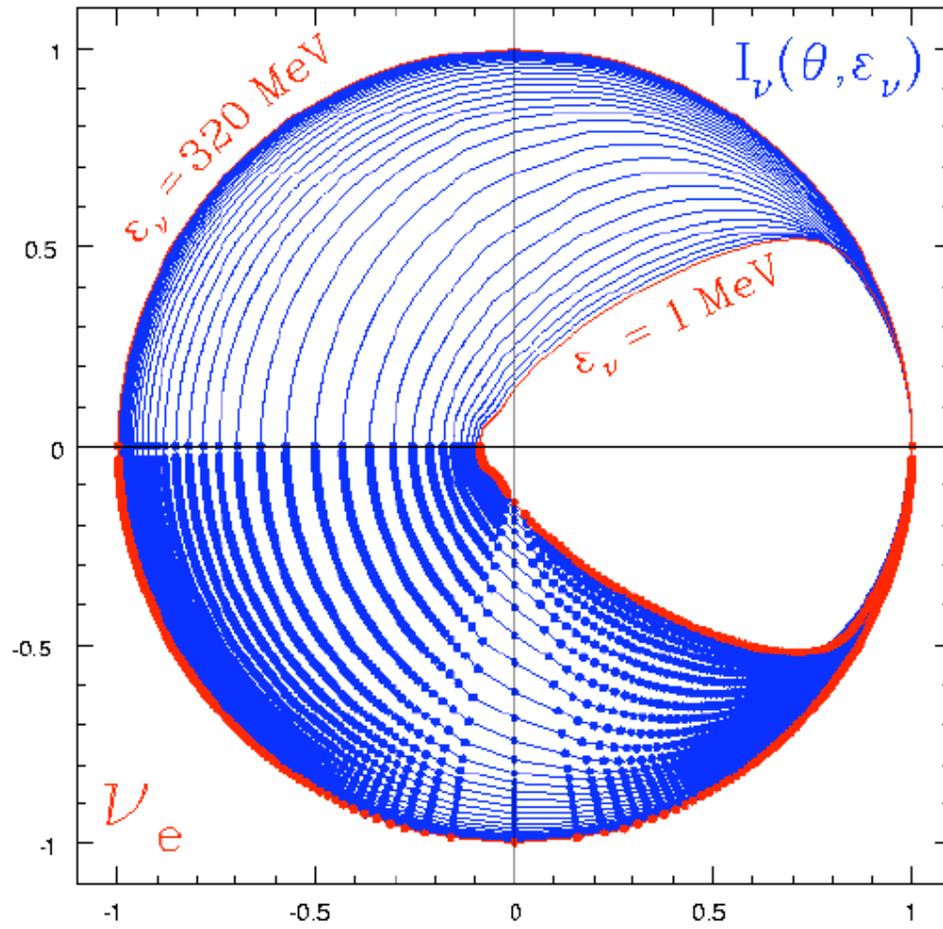


# **Precision Angular Distributions (SESAME)**

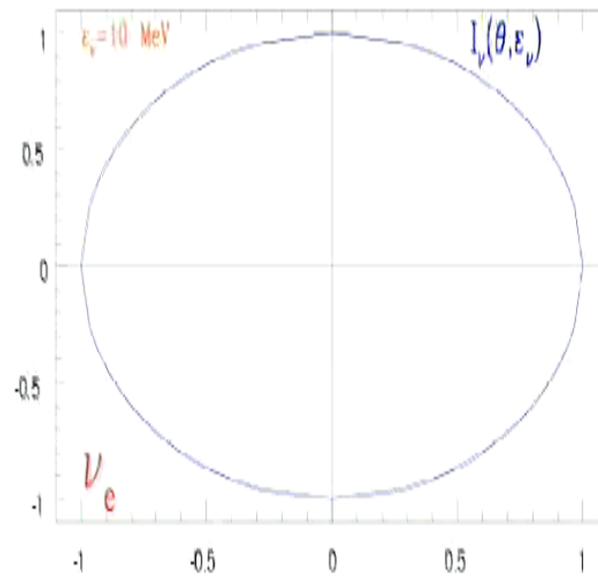
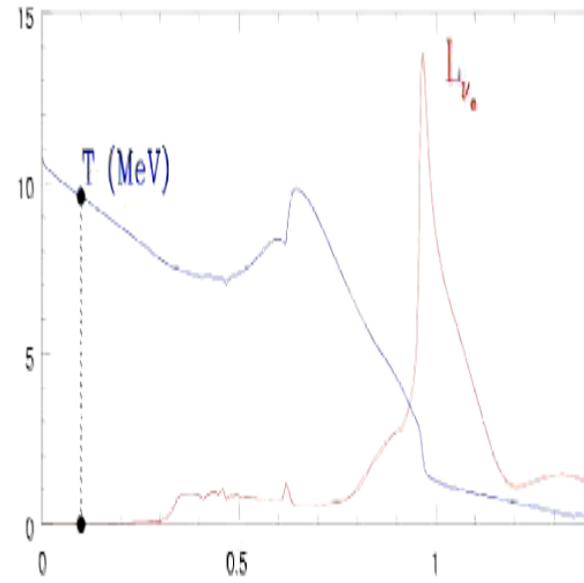
## Tangent Ray Construction



40 ms Post-Bounce, R = 42 km, 40 Energy Groups, 149 Angles

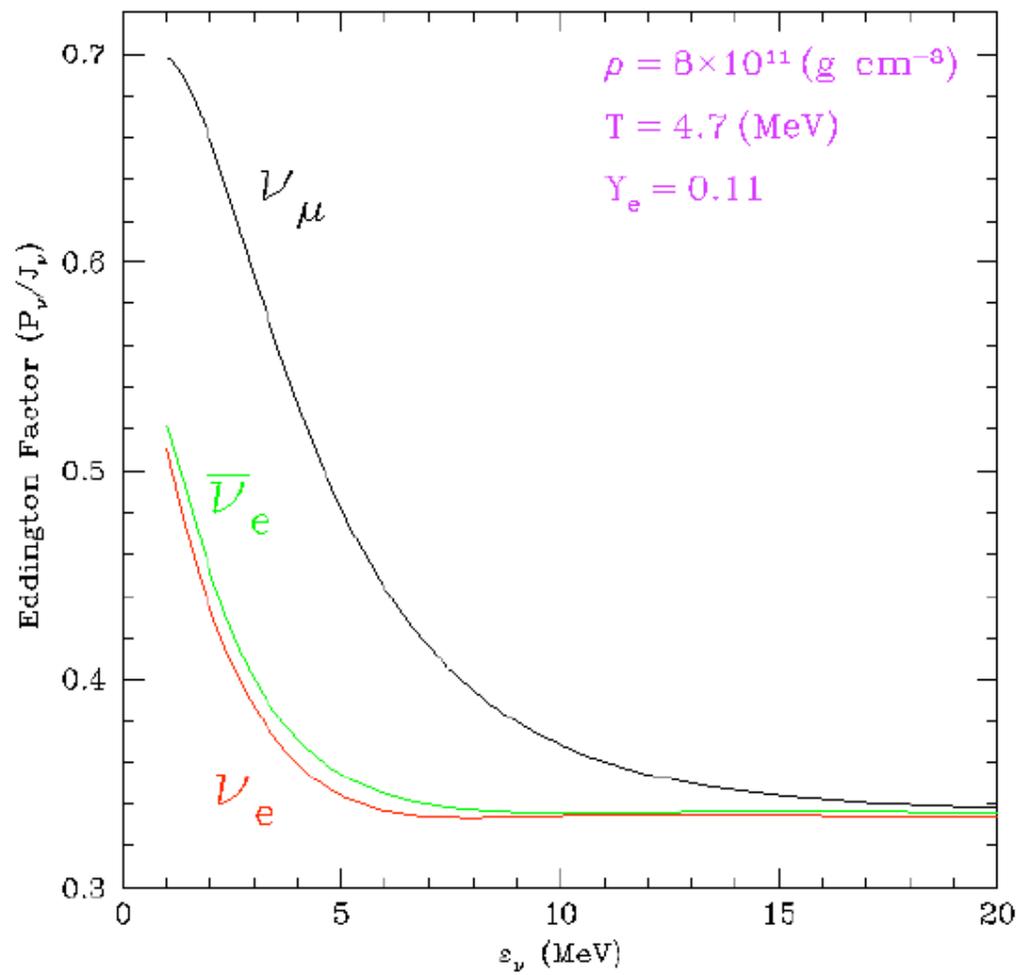


Progenitor Model:  $11 M_{\odot}$ ,  $R = 5.67$  km, 21 Angles

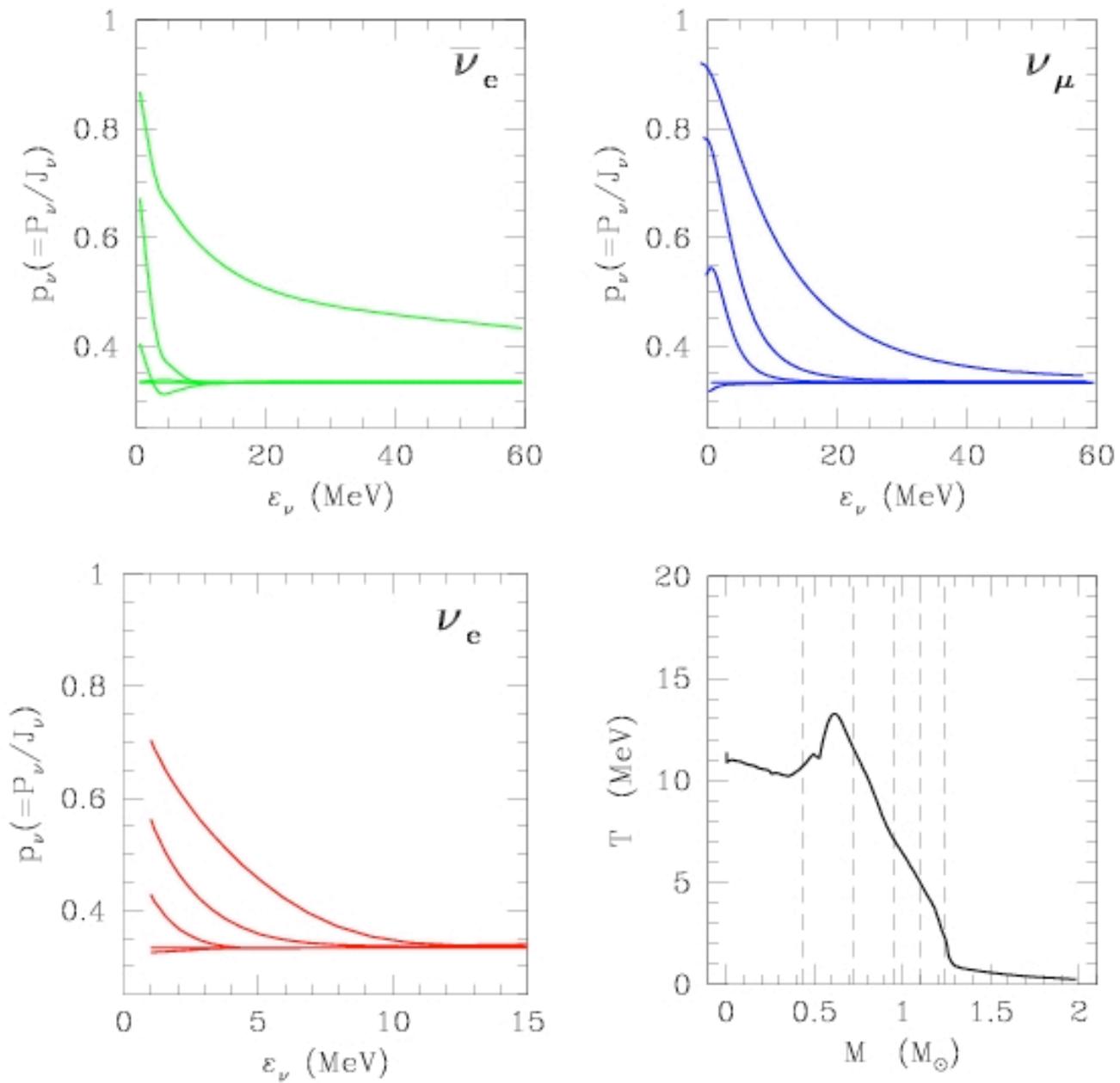


# **Eddington Factors**

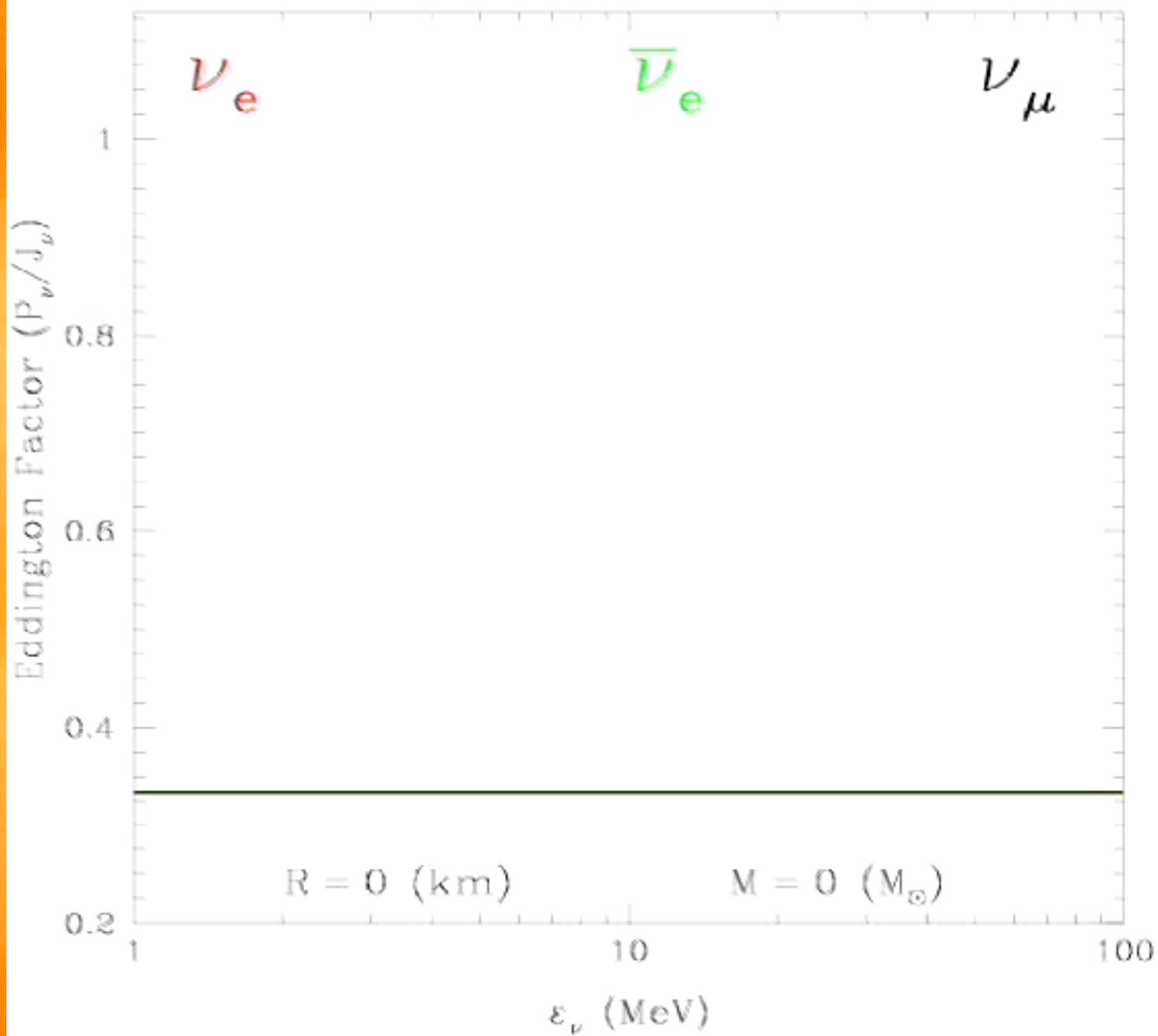
15 ms Post-Bounce, Model: 20  $M_{\odot}$



Model: S20

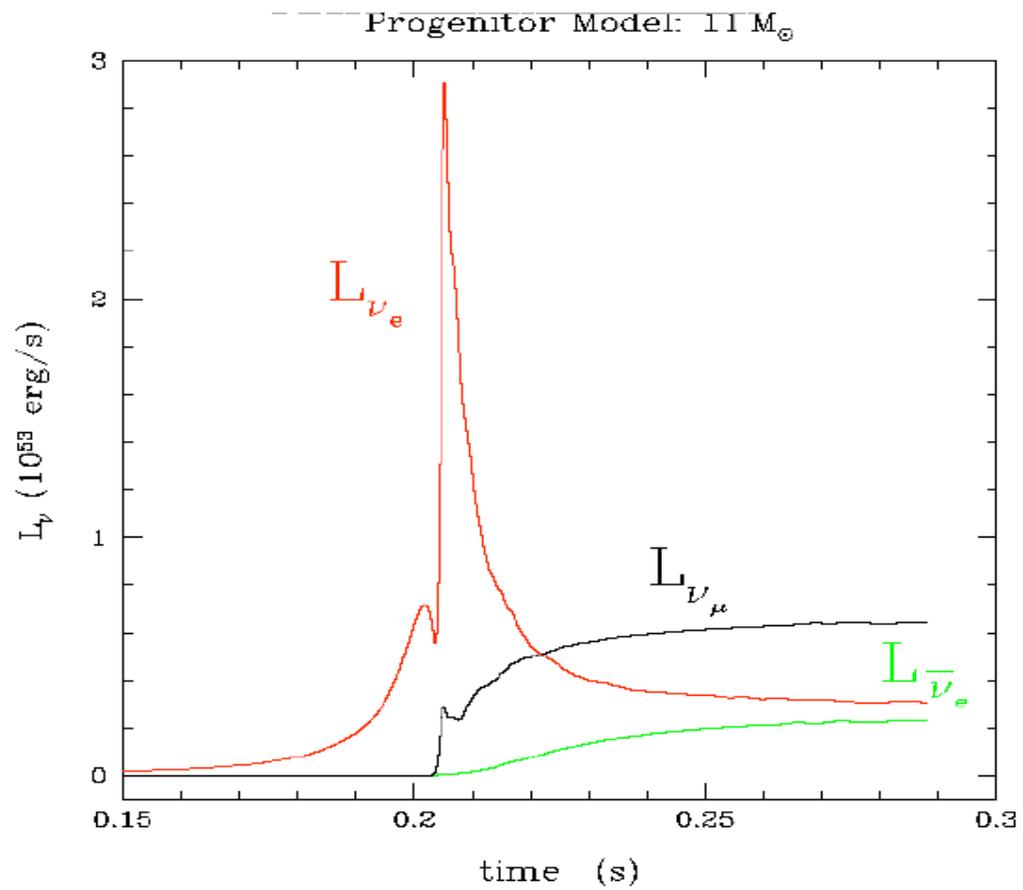


15 ms Post-Bounce, Model:  $20 M_{\odot}$

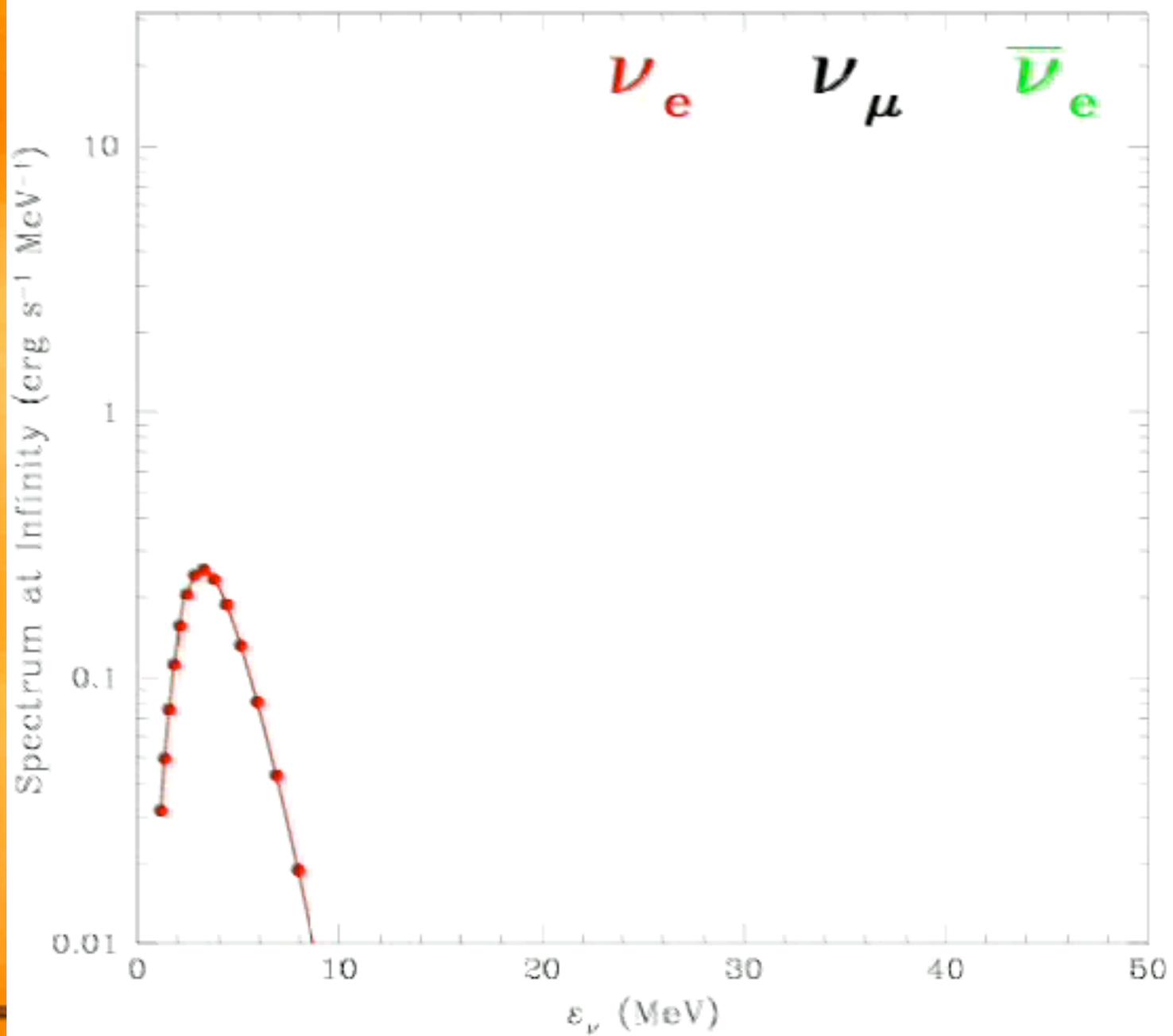


# Neutrino Fluxes

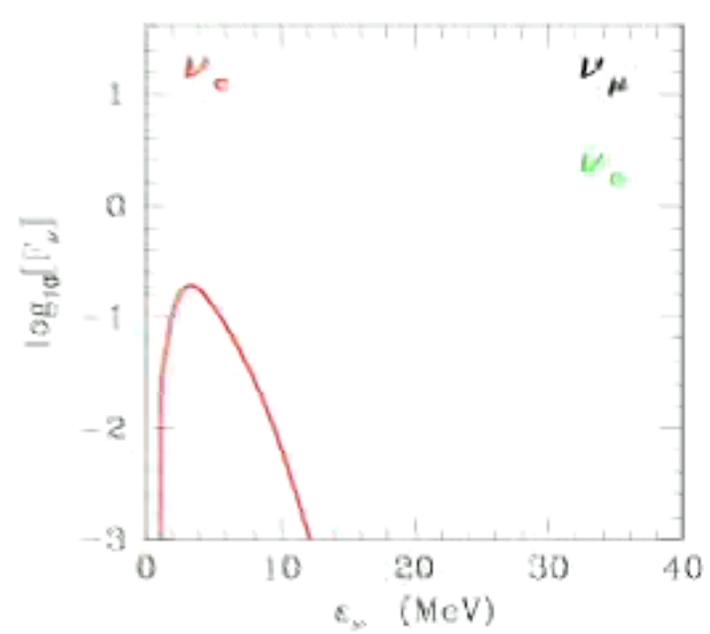
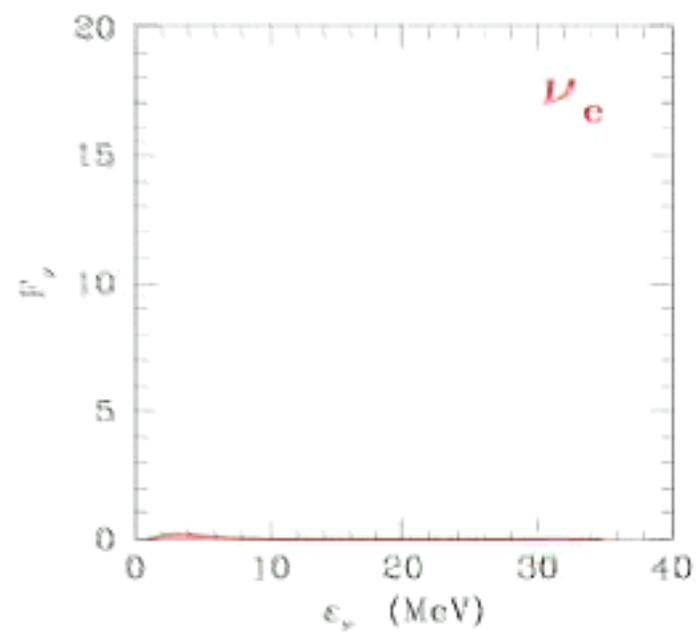
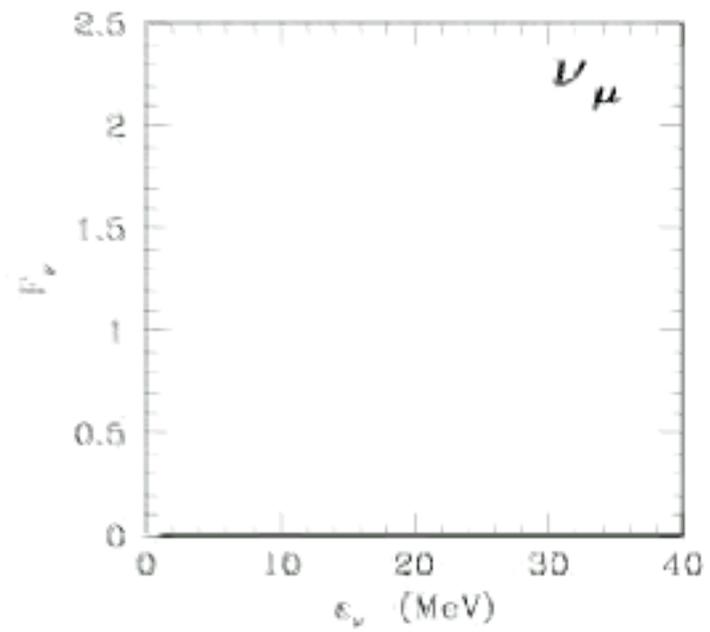
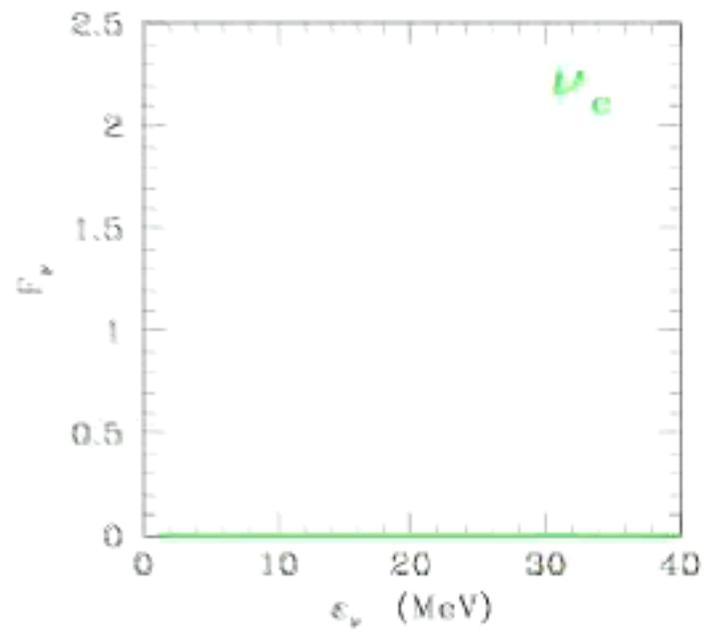
# Breakout Burst of Neutrinos: Precision Boltzmann Transfer



Model: 15  $M_{\odot}$ , time = 0.08823 s



$F_\nu$  (erg s<sup>-1</sup> MeV<sup>-1</sup>)/10<sup>51</sup>, Model: S11, time = 0.10798E+00 (s)



**BETHE-Transport: A NEW MIXED-FRAME  
ALGORITHM FOR NEUTRINO  
TRANSPORT FOR CORE-COLLAPSE  
SIMULATIONS**

**I. Hubeny & A. Burrows (2007)**

# MOTIVATION

- Need for fast and efficient multi-D transport solver for supernova simulations:
- Time-dependent, implicit
- At least 2-D + rotation --> 2 1/2-D
- All terms  $O(v/c)$
- Multi-group, multi-angle
- Anisotropic scattering
- Real transport; no Flux-Limited Diffusion

# GENERAL TRANSPORT EQUATION

$$\left( \frac{1}{c} \frac{\partial}{\partial t} + \mathbf{n} \cdot \nabla \right) I(\nu, \mathbf{n}) = \eta(\nu, \mathbf{n}) - \kappa(\nu, \mathbf{n}) I(\nu, \mathbf{n})$$

## FUNDAMENTAL COMPLICATION

l.h.s - lives in the **Laboratory** (Eulerian) Frame

r.h.s - lives in the **Co-moving** (Lagrangian) Frame

-- interaction of radiation and matter is most naturally expressed in this frame

-- thermodynamic quantities (e.g.  $T$ ) **DEFINED** relative to this frame

# VARIOUS FORMALISMS

- Fully **Laboratory (Eulerian) Frame:**
  - l.h.s. - simple and natural
  - r.h.s. - complicated, awkward
- Fully **Comoving (Lagrangian) Frame:**
  - r.h.s. - simple and natural
  - l.h.s. - complicated
  - difficult in multi-D, difficult to implement to hydro
  - **BUT: very successful in 1-D with spectral line transfer (CMFGEN, PHOENIX)**
- **Mixed Frame (Mihalas & Klein 1982):**
  - **combines advantages of both**
  - l.h.s. - simple
  - r.h.s. - uses **linear expansions of co-moving-frame cross-sections** => also simple (at least relatively)
  - **BUT: cross-sections have to be smooth functions of energy and angle**
  - **not appropriate for photon transport (with spectral lines), but perfect for Neutrinos**

## Mixed-frame: formulation

A generalization of [Mihalas & Klein \(1982\)](#), taking into account anisotropic scattering and the energy dependence of the scattering rate:

Hubeny & Burrows 2007

$$\kappa(\nu, \mathbf{n}) = \kappa_0(\nu) - \frac{\mathbf{n} \cdot \mathbf{v}}{c} \left[ \kappa_0(\nu) + \nu \frac{\partial \kappa_0}{\partial \nu} \right], \quad (11)$$

$$\sigma(\nu, \mathbf{n}) = \sigma_0(\nu) - \frac{\mathbf{n} \cdot \mathbf{v}}{c} \left[ \sigma_0(\nu) + \nu \frac{\partial \sigma_0}{\partial \nu} \right]. \quad (12)$$

$$\eta^{\text{th}}(\nu, \mathbf{n}) = \eta_0^{\text{th}}(\nu) + \frac{\mathbf{n} \cdot \mathbf{v}}{c} \left[ 2\eta_0^{\text{th}}(\nu) - \nu \frac{\partial \eta_0^{\text{th}}}{\partial \nu} \right], \quad (13)$$

$$\eta_0^{\text{sc}}(\nu_0, \mathbf{n}_0) = \frac{\sigma_0(\nu_0)}{4\pi} \oint d\omega'_0 I_0(\nu_0, \mathbf{n}'_0) g_0(\mathbf{n}'_0, \mathbf{n}_0), \quad (14)$$

$$g_0(\mathbf{n}'_0, \mathbf{n}_0) = 1 + \delta \mathbf{n}'_0 \cdot \mathbf{n}_0. \quad (15)$$

# Mixed-frame: angle-dependent transfer equation

Hubeny & Burrows 2007

$$\left(\frac{1}{c} \frac{\partial}{\partial t} + \mathbf{n} \cdot \nabla\right) I(\nu, \mathbf{n}) = r_{00}(\nu, \mathbf{n}) + r_{01}(\nu, \mathbf{n}) \\ + r_{10}(\nu, \mathbf{n}) + r_{11}(\nu, \mathbf{n}), \quad (22)$$

$$r_{00} = \eta_0^{\text{th}} - (\kappa_0 + \sigma_0)I(\nu, \mathbf{n}) + \sigma_0 J, \quad (23)$$

$$r_{01} = \sigma_0 \delta H^j n_j, \quad (24)$$

$$r_{10} = n_j w^j \left[ \eta_0^{\text{th}} \left( 2 - \frac{\partial \ln \eta_0^{\text{th}}}{\partial \ln \nu} \right) + \kappa_0 \left( 1 + \frac{\partial \ln \kappa_0}{\partial \ln \nu} \right) I(\nu, \mathbf{n}) \right. \\ \left. + \sigma_0 \left( 1 + \frac{\partial \ln \sigma_0}{\partial \ln \nu} \right) I(\nu, \mathbf{n}) + \sigma_0 J \left( 2 - \frac{\partial \ln \sigma_0}{\partial \ln \nu} - \frac{\partial \ln J}{\partial \ln \nu} \right) \right] \\ - \sigma_0 w_j H^j \left( 1 - \frac{\partial \ln H^j}{\partial \ln \nu} \right), \quad (25)$$

and

$$r_{11} = \sigma_0 \delta \left[ -n_j w^j J + n_j w_k \frac{\partial \ln K^{jk}}{\partial \ln \nu} \right] \\ + \sigma_0 \delta H^j \left[ -w_j + n_j n_k w^k \left( 3 - \frac{\partial \ln \sigma_0}{\partial \ln \nu} - \frac{\partial \ln H^j}{\partial \ln \nu} \right) \right],$$

## Mixed-frame: Moment eqs. (cylindrical geometry)

Hubeny & Burrows 2007

$$\frac{1}{c} \frac{\partial J}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r H_r) + \frac{\partial H_z}{\partial z} = \eta_0^{\text{th}} - \kappa_0 J + \Xi_r H_r + \Xi_z H_z + \Xi_\phi H_\phi, \quad (85)$$

$$\begin{aligned} \frac{1}{c} \frac{\partial H_r}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r f_{rr} J) + \frac{\partial}{\partial z} (f_{rz} J) - \frac{1 - f_{rr} - f_{zz}}{r} J \\ = -(\kappa_0 + \sigma_{\text{tr}}) H_r + w_r \tilde{\eta}_0 + \xi_r J, \end{aligned} \quad (86)$$

$$\begin{aligned} \frac{1}{c} \frac{\partial H_z}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r f_{rz} J) + \frac{\partial}{\partial z} (f_{zz} J) \\ = -(\kappa_0 + \sigma_{\text{tr}}) H_z + w_z \tilde{\eta}_0 + \xi_z J, \end{aligned} \quad (87)$$

$$\begin{aligned} \frac{1}{c} \frac{\partial H_\phi}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r f_{r\phi} J) + \frac{\partial}{\partial z} (f_{z\phi} J) \\ = -(\kappa_0 + \sigma_{\text{tr}}) H_\phi + w_\phi \tilde{\eta}_0 + \xi_\phi J, \end{aligned} \quad (88)$$

# Computational Strategy

- Solve a full transport problem (perhaps not in all time steps)  $\Rightarrow$  Eddington tensor components
- Solve the moment equations (much simpler) at every time step
- True heart of the problem is a determination of the Eddington tensor - full transfer solution
  - **It has to be made as efficient and fast as possible**

# Computational strategy

- Discretize in space
- Discretize in angles
  - $S_N$  scheme
  - tangent rays/planes
- Discretize in energy ==> energy groups
  - Instead of treating frequency derivatives of moments explicitly (which may be insufficient) or implicitly (would prohibit parallelization in energy groups), we use a **hybrid scheme**:
  - $d(\text{moment})/d(\text{energy}) \sim \text{moment times logarithmic derivative}$ , and
    - “moment” part is treated **implicitly**
    - “logarithmic derivative” part treated **explicitly**
- Backward time differencing ==> **IMPLICIT SCHEME**

# Iterative solution: basics

- Transfer equation:  $D [I] = R[I] + r$ 
  - D - spatial and angular transport operator
  - R - source operator (scattering term)
  - r - “true” source term (thermal emission + previous-time-step terms)
- Source function:  $S = R [I] + r$ 
  - generally function of all specific intensities
  - however, in the present case, function of only moments  $x = \{J, H, K\}$
- Formal solution of the transfer equation:  $I = \Lambda [S]$
- Analogous set of equations for moments:  $x = M' [S]$
- Resulting system for moments:  $x = M [x] + b$
- Problem is reduced to the system of  $9 N^{\text{spatial}} \sim O(10^6)$  unknowns per time step
- **Preconditioning** (Accelerated Lambda Iteration; ALI):
  - $x^{\text{new}} = M^* x^{\text{new}} + (M - M^*) x^{\text{old}} + b$
  - Operator  $M^*$  (preconditioner) should be **cheap and easy to invert!**

# Iterative Solution: Choice of Preconditioner

- Operator  $M$  - appropriate angular integrals of the basic operator  $\Delta$
- Easiest choice for preconditioner: diagonal (local) part of  $M$  -- Jacobi method
- Inversion of  $M^*$  is thus reduced to inverting one  $9 \times 9$  matrix per spatial cell (3 x 3 matrix in the case of 1-D; 6 x 6 matrix in the case of true 2-D without rotation)
- Standard Jacobi method:
  - $\mathbf{x}^{\text{new}} - \mathbf{x}^{\text{old}} = (M^*)^{-1} \text{res}$
  - where the residue is given by  $\text{res} = \mathbf{x}^{\text{FS}} - \mathbf{x}^{\text{old}}$
  - and  $\mathbf{x}^{\text{FS}} = \mathbf{R} [\mathbf{x}^{\text{old}}] + \mathbf{b}$  (a result of Formal Solution with old source function)
- **New iterate** = action of **approximate operator** (preconditioner) on **residue**
- **BUT**, this may not be sufficiently fast (convergence speed determined by the spectral radius - magnitude of largest absolute eigenvalue) of the preconditioner

# Iterative Solution and Acceleration

- **Krylov subspace methods** - using pseudo-residual chosen to be orthogonal to current Krylov subspace
- Many variants of Krylov
- **GMRES** (Generalized Minimum Residual) method, and/or Ng method
- ~Equivalent scheme: **ORTHOMIN(k)** (Orthogonal minimization)
  - Truncate the orthogonalization process to  $k$  most recent vectors

# Formal solution of the transfer equation

- Short characteristics
- Feautrier scheme - 2nd order form
- **Discontinuous Finite Element**

# Convergence behavior: 1D

Krylov subspace  
methods

GMRES

ORTHOMIN(k)

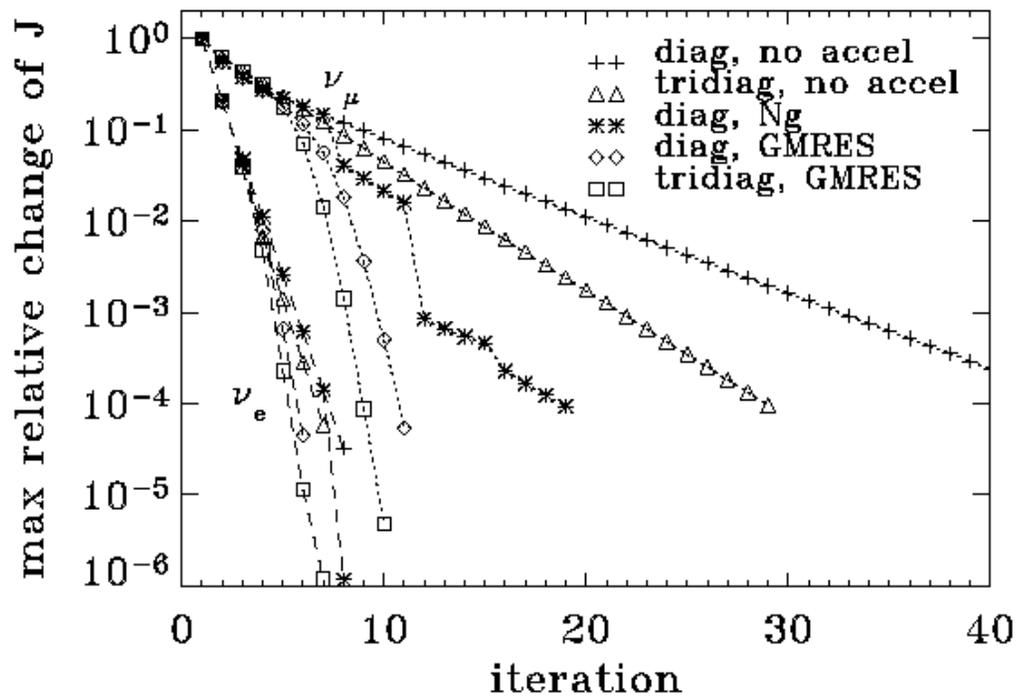


FIG. 2.—Convergence pattern for  $\nu_e$  and  $\nu_\mu$  neutrinos for  $E = 8.6$  MeV and for five different setups of the iterative transport solver. See text for a discussion.

Hubeny & Burrows 2007

# Future Work on Mixed-Frame Technique

- We are developing an efficient numerical scheme for treating neutrino transport in 2-D (2 1/2-D) with:
  - Mixed frame formulation of the transfer equation
  - All terms to  $O(v/c)$ ; rotation
  - Multi group; multi-angle; anisotropic scattering
  - Formal solver using the Discontinuous Finite Element (DFE) scheme
  - Global solution using Jacobi preconditioner (ALI) + GMRES or Ng acceleration
- Initial tests of performance on 1-D spherically symmetric model show great potential
- Will finish debugging code for 1- and 2-D cylindrical and spherical geometry
- Future improvements:
  - Inelastic scattering (energy redistribution); technique already devised
  - Neutrino flavor oscillations
  - Spatial 3-D
  - AMR, etc.

# **A Boltzmann Formalism for Oscillating Neutrinos**

**P. Strack and A. Burrows**

Phys.Rev.D 71:093004,2005 (hep-ph/0504035) & hep-ph/0505056

# Quasi-classical Boltzmann equations

- Wigner density matrix, ensemble-averaging →

$$\mathcal{F} = \langle n_i | \rho | n_j \rangle = \begin{pmatrix} f_{\nu e} & f_{e\mu} \\ f_{e\mu}^* & f_{\nu\mu} \end{pmatrix}$$



Diagonal elements: real numbers:  
Phase-Space densities



Off-diagonal elements: complex numbers: Macroscopic Overlap densities

$$f_r = \frac{1}{2} (f_{e\mu} + f_{e\mu}^*) \quad (\text{Real part})$$

$$f_i = \frac{1}{2i} (f_{e\mu} - f_{e\mu}^*) \quad (\text{Imaginary part})$$

$$L = \frac{4\pi\hbar c \varepsilon}{\Delta m^2 c^4}$$

$$A = \left(\frac{L}{\pi c}\right) \frac{2\sqrt{2}G_F}{\hbar} n_e(\mathbf{r})$$

$$\begin{aligned} \frac{\partial f_{\nu_e}}{\partial t} + \mathbf{v} \cdot \frac{\partial f_{\nu_e}}{\partial \mathbf{r}} + \dot{\mathbf{p}} \cdot \frac{\partial f_{\nu_e}}{\partial \mathbf{p}} &= -\frac{2\pi c}{L} f_i \sin 2\theta + C_{\nu_e} \\ \frac{\partial f_{\nu_\mu}}{\partial t} + \mathbf{v} \cdot \frac{\partial f_{\nu_\mu}}{\partial \mathbf{r}} + \dot{\mathbf{p}} \cdot \frac{\partial f_{\nu_\mu}}{\partial \mathbf{p}} &= \frac{2\pi c}{L} f_i \sin 2\theta + C_{\nu_\mu} \\ \frac{\partial f_r}{\partial t} + \mathbf{v} \cdot \frac{\partial f_r}{\partial \mathbf{r}} + \dot{\mathbf{p}} \cdot \frac{\partial f_r}{\partial \mathbf{p}} &= -\frac{2\pi c}{L} f_i (\cos 2\theta - A) \\ \frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \frac{\partial f_i}{\partial \mathbf{r}} + \dot{\mathbf{p}} \cdot \frac{\partial f_i}{\partial \mathbf{p}} &= \frac{2\pi c}{L} \left( \frac{f_{\nu_e} - f_{\nu_\mu}}{2} \sin 2\theta + \right. \\ &\quad \left. f_r (\cos 2\theta - A) \right) \end{aligned}$$