Turbulence, Rotation and MHD: the von Neumann approach

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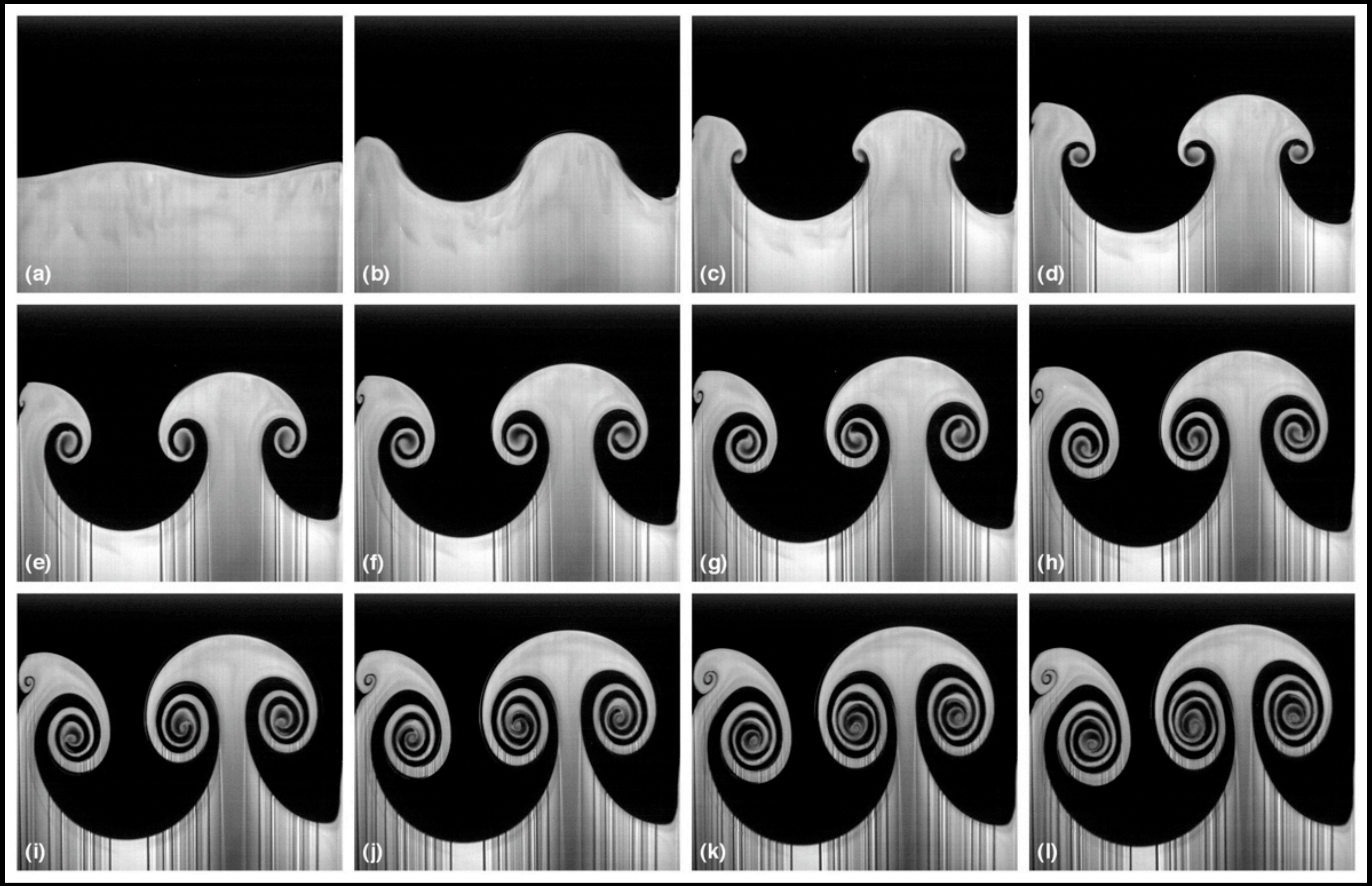
The purpose of computing is insight, not numbers.

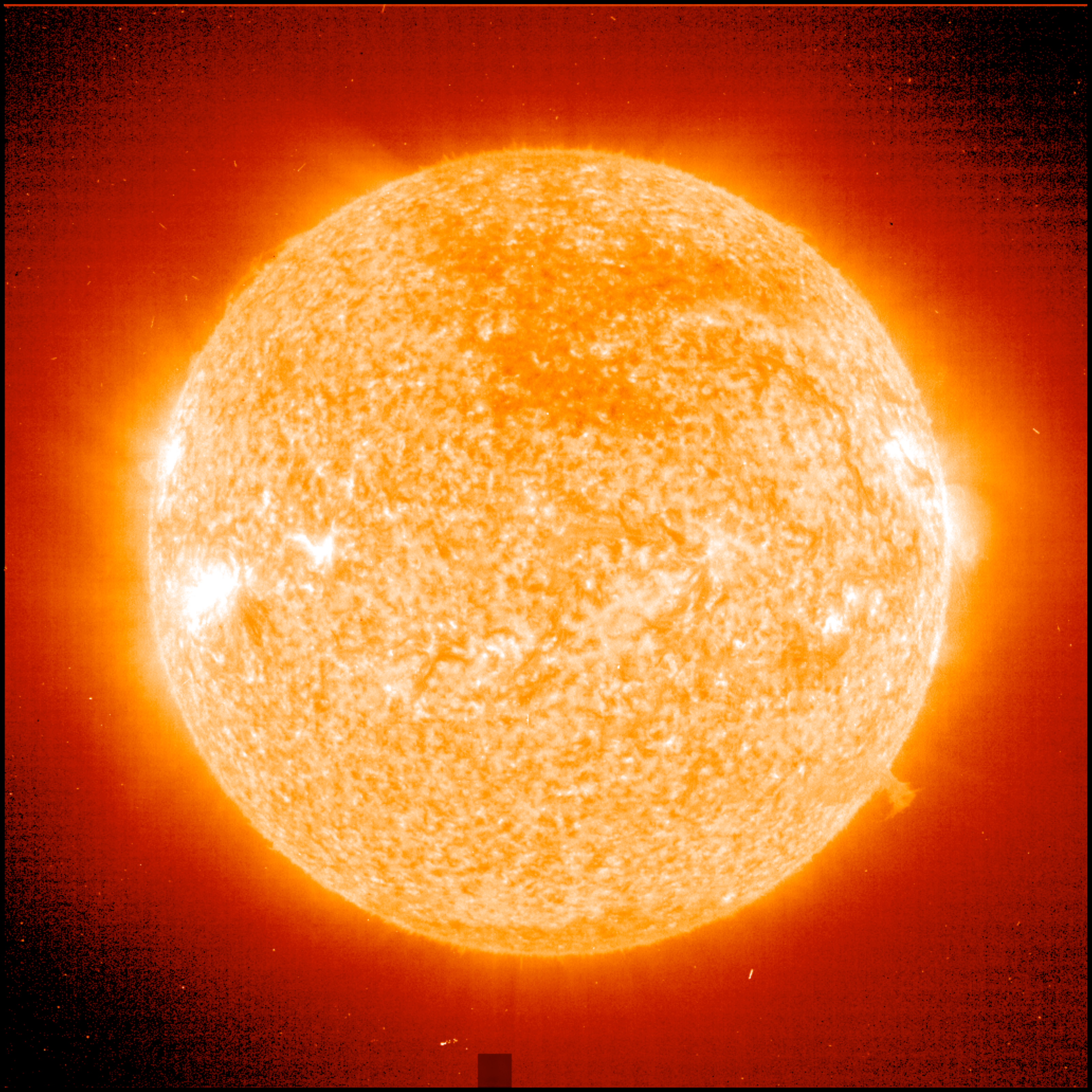
One can imagine obtaining insight into the nature of nonlinear solutions (of turbulence) by direct numerical computations, and using that insight to construct a theory (J. von Neumann)

Leave behind all hopes ye who enter here (Dante).

Jacobs' Experiments: Hydrocode validation



The Sun: a testbed



New Result from 2009 IAU General Assembly

- Sasha Brun (Saclay) has run the Boulder ASH code with higher resolution than ever before
- This gives a lower effective viscosity and higher Reynolds number
- Higher resolution calculations of turbulence "in a box" show that monotonicity preserving codes approximate Kolmogorov dissipation below the grid scale
- Brun finds that the Solar rotation inferred from helio-seismology is now reproduced!

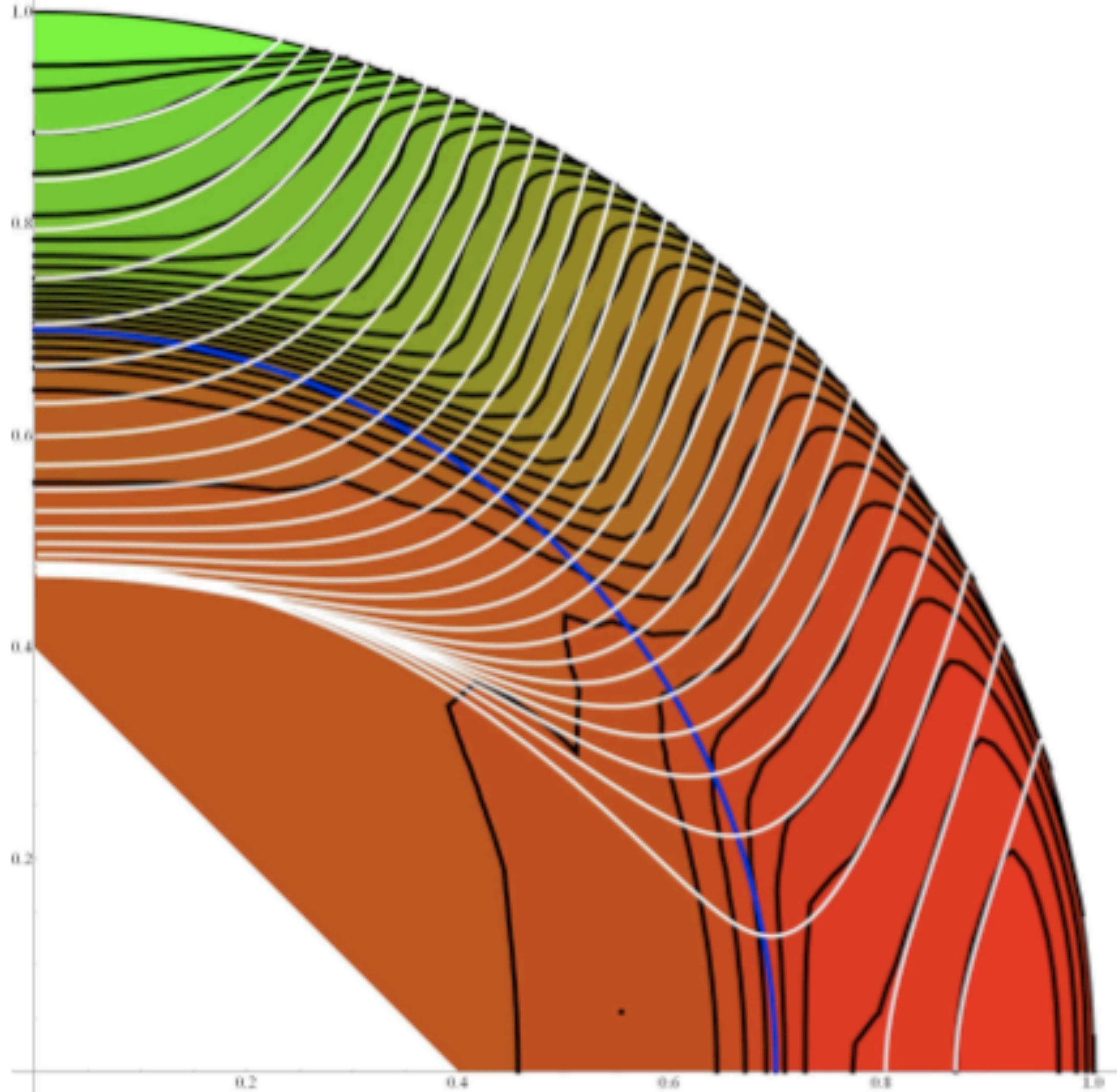


Fig. 1.— Constant Ω contours (7) of the thermal wind equation (2) (white curves) plotted on top of (black) isorotation contours from helioseismology data (GONG results courtesy of R. Howe). Blue contour is the bottom edge of the convective zone. Scale is in solar radii. Away from the tachocline and outer surface layers, the match is excellent. See text for further details.

Balbus, Bonart, Latter, & Weiss, arXiv:0907.5075v1

Thermal wind equation implies a deep connection between entropy and angular momentum

Isorotational contours and residual entropy

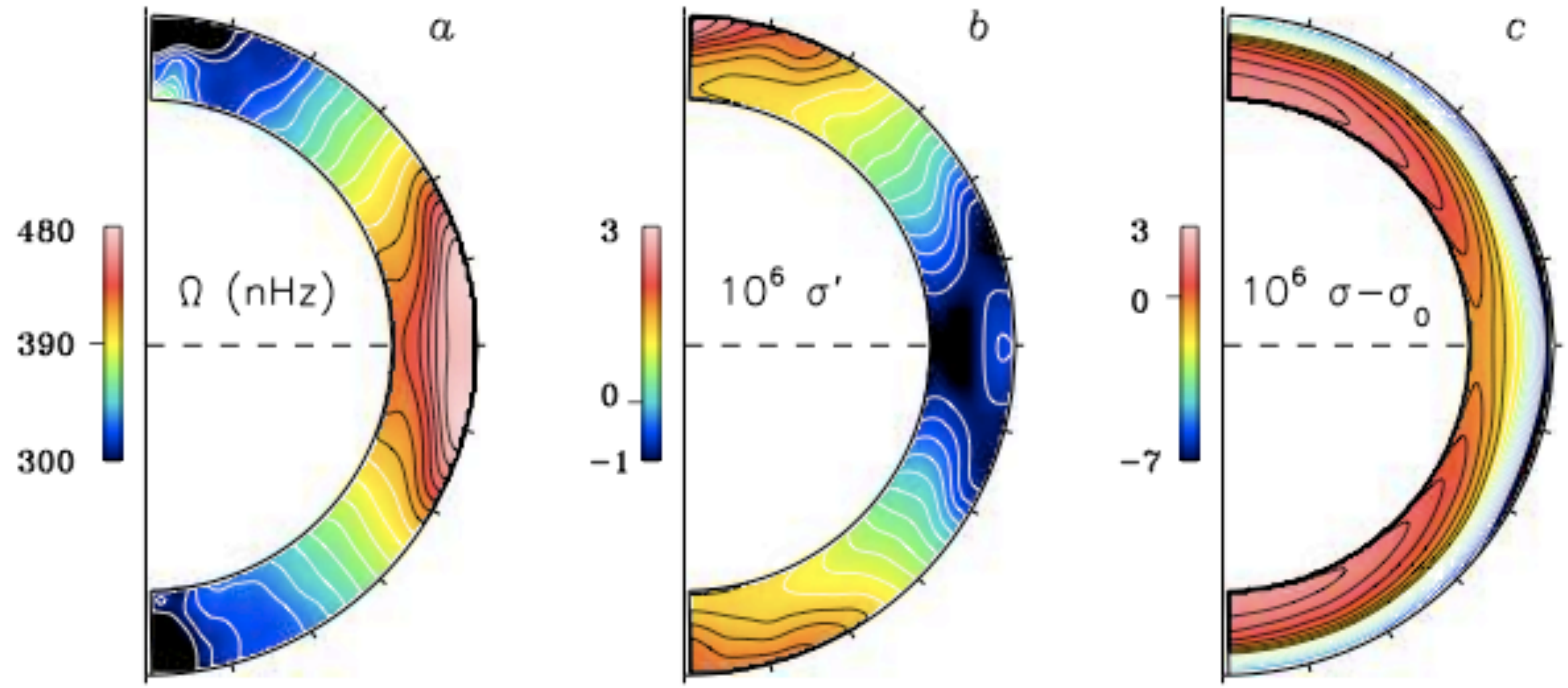
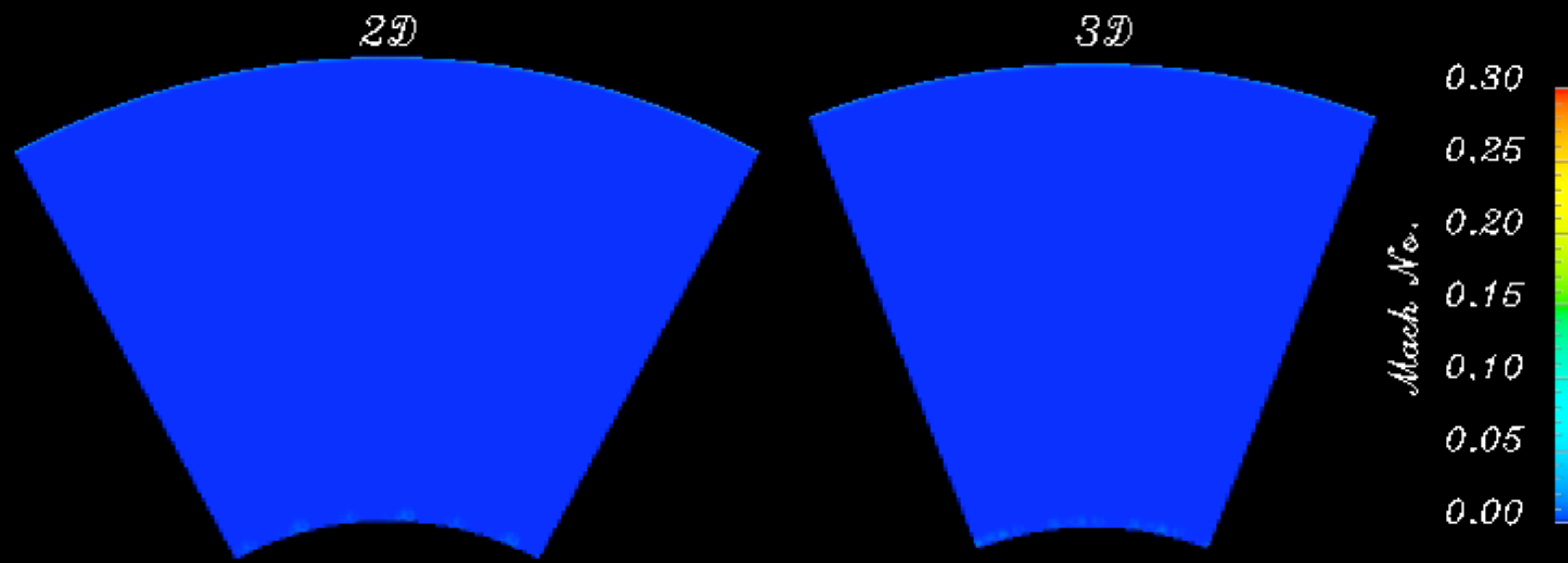


Fig. 2.— Isorotation contours (a), contours of constant residual entropy (b), and contours of total entropy (c), from the run AB3 of MBT06. (In [c], a constant background entropy has been subtracted out.) There is generally very good agreement between the angular velocity and residual entropy contours, but not between the angular velocity and total entropy contours. (Figures courtesy of M. Miesch.)

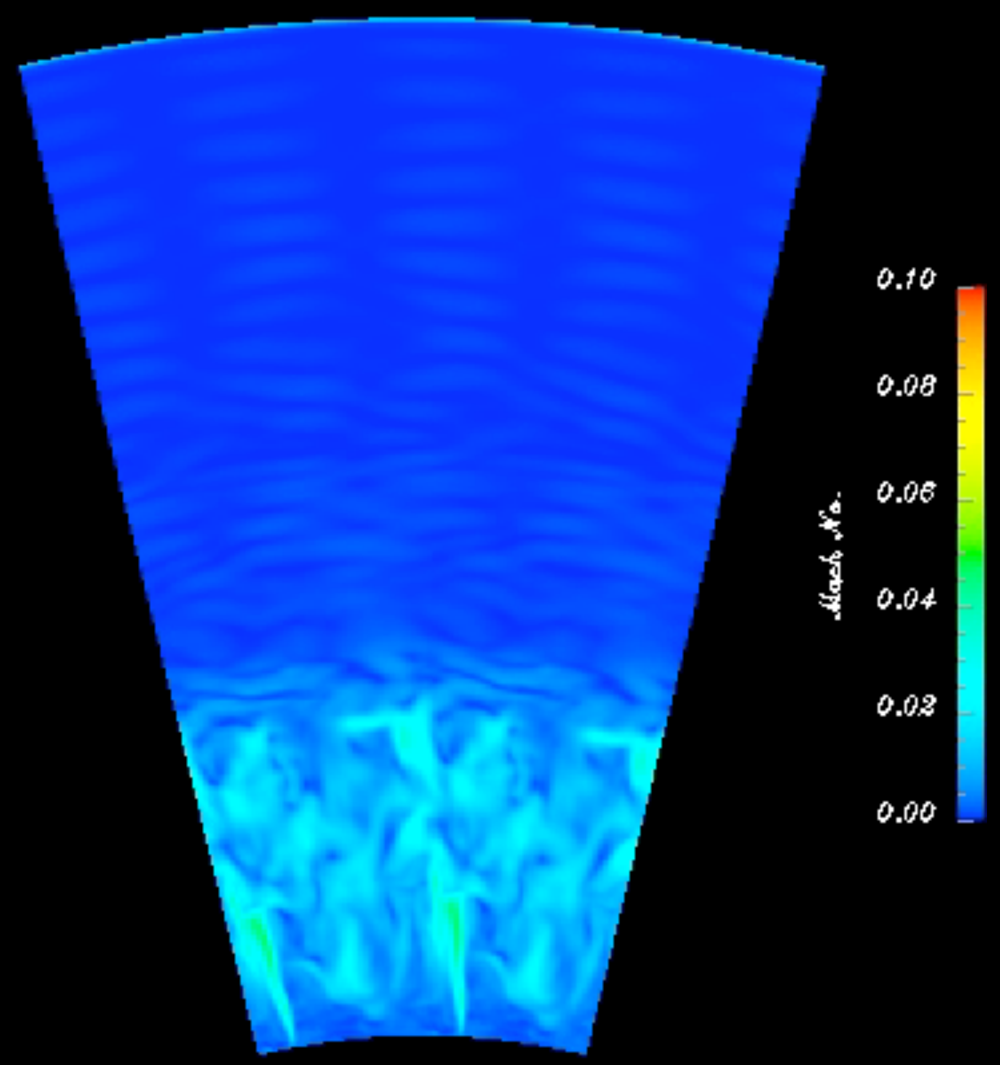
Miesch, M., 2005, Living Revs. Sol. Phys., 2, 1
(www.livingreviews,org/lrsp-2005-1)

- We should understand the solar rotation, at least qualitatively, before we extrapolate to core collapse and GRB's
- We should base our astrophysics on terrestrial experiment not astronomical calibration whenever possible
- We ignore related fields like geophysics, meteorology, and oceanography at our peril

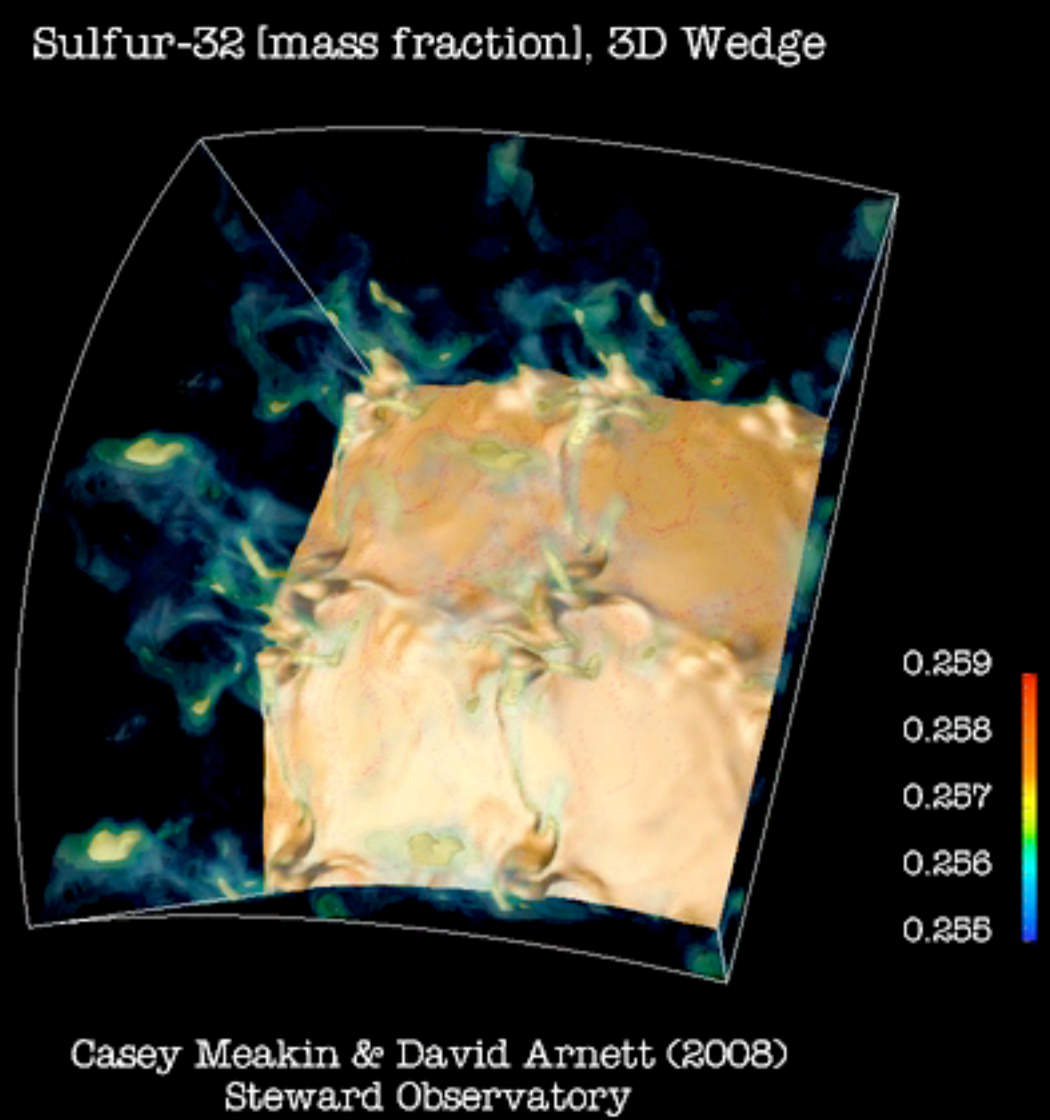
2D versus 3D



G-modes in radiative regions



O+O burning: Meakin and Arnett: entrainment



The Reynolds Decomposition

Taking the scalar product of the velocity with the equation of motion, we decompose the convective velocity $\mathbf{u}$, the density ρ, and the pressure p into mean and fluctuating components (e.g., $p = p_0 + p'$, so the time averages are $\overline{p} = p_0$ and $\overline{p'} = 0$. This choice of just $\mathbf{u}$, p, and ρ for this Reynolds decomposition into average and fluctuating parts gives the simplest equation for kinetic energy; the velocity $\mathbf{u}$ is derived from buoyancy and pressure forces (ρ and p fluctuations).

The Turbulent Kinetic Energy Equation

$$\partial_t \langle \overline{\rho E_K} \rangle + \nabla \cdot \langle \overline{\rho E_K \mathbf{u_0}} \rangle + \nabla \cdot \langle \overline{\mathbf{F_p} + \mathbf{F_K}} \rangle$$
$$- \langle \overline{p' \nabla \cdot u'} \rangle - \langle \overline{\rho' \mathbf{g} \cdot \mathbf{u}'} \rangle = \varepsilon_K .$$

where the acoustic flux is $\mathbf{F}_p = \overline{\langle p' \mathbf{u}' \rangle}$, the kinetic energy flux is $\mathbf{F_K} = \overline{\langle \rho (u')_z (u')^2/2 \rangle}$, and the enthalpy (convective "energy") flux is $\mathbf{F_e} = \overline{\langle \rho_0 \mathbf{u}' C_p T' \rangle}$.

We have dotted the velocity and the momentum equation, and averaged over solid angle (brackets) and two turn-over times (over-bar).

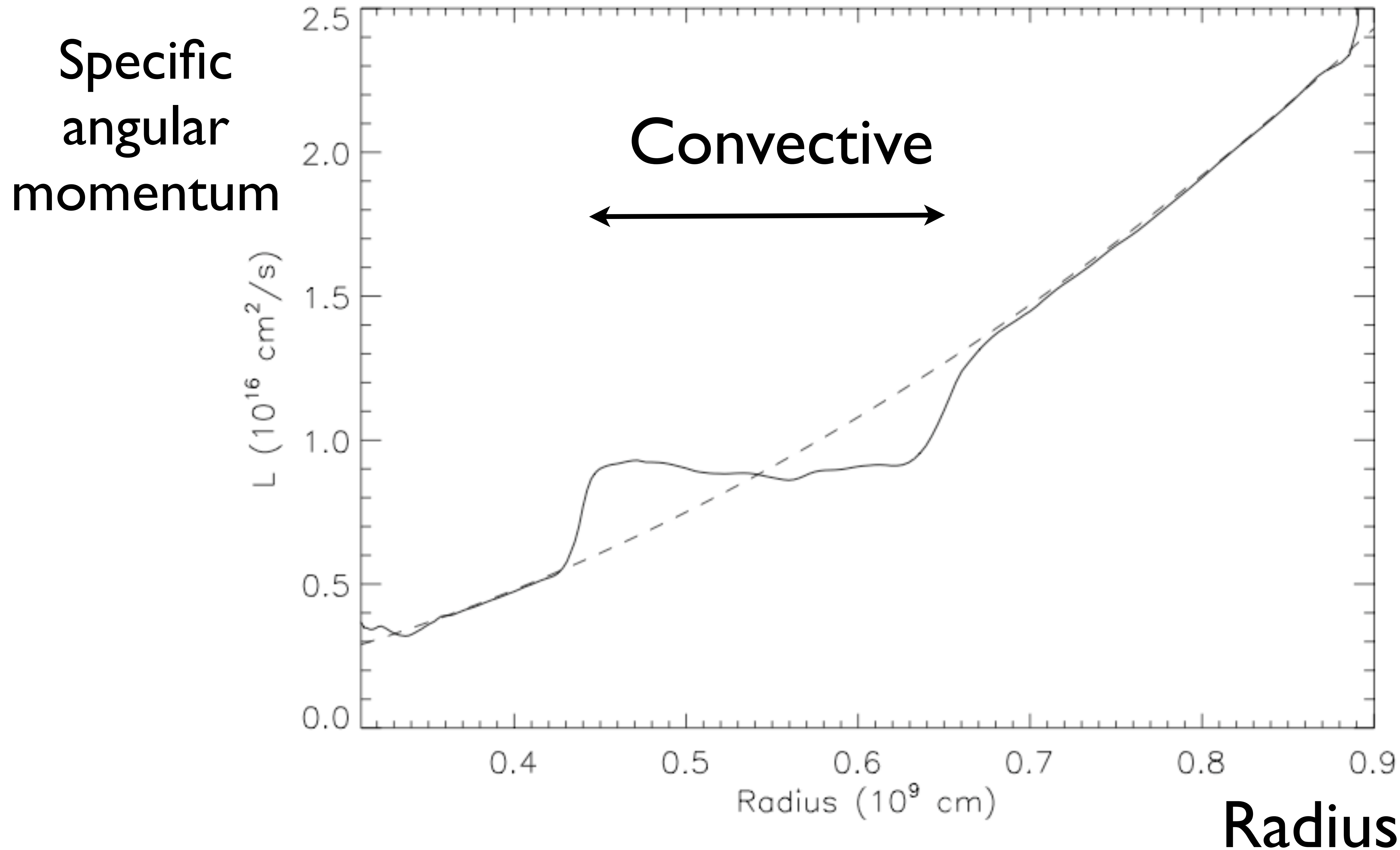
The viscosity term (Navier-Stokes) becomes the same as the Kolmogorov dissipation term for a turbulent cascade! A second-order spatial derivative becomes a scalar, a great simplification. This magic is demanded by the numerical solutions, and solves the closure problem that plagues the Reynolds theory (1922) of turbulence.

$$\begin{aligned} \varepsilon_K &= \overline{\langle \nu \rho \nabla^2 (\mathbf{u}^2/2) \rangle} \\ &\rightarrow \rho_0 \overline{\langle (u')^3 \rangle} / \ell_D \end{aligned}$$

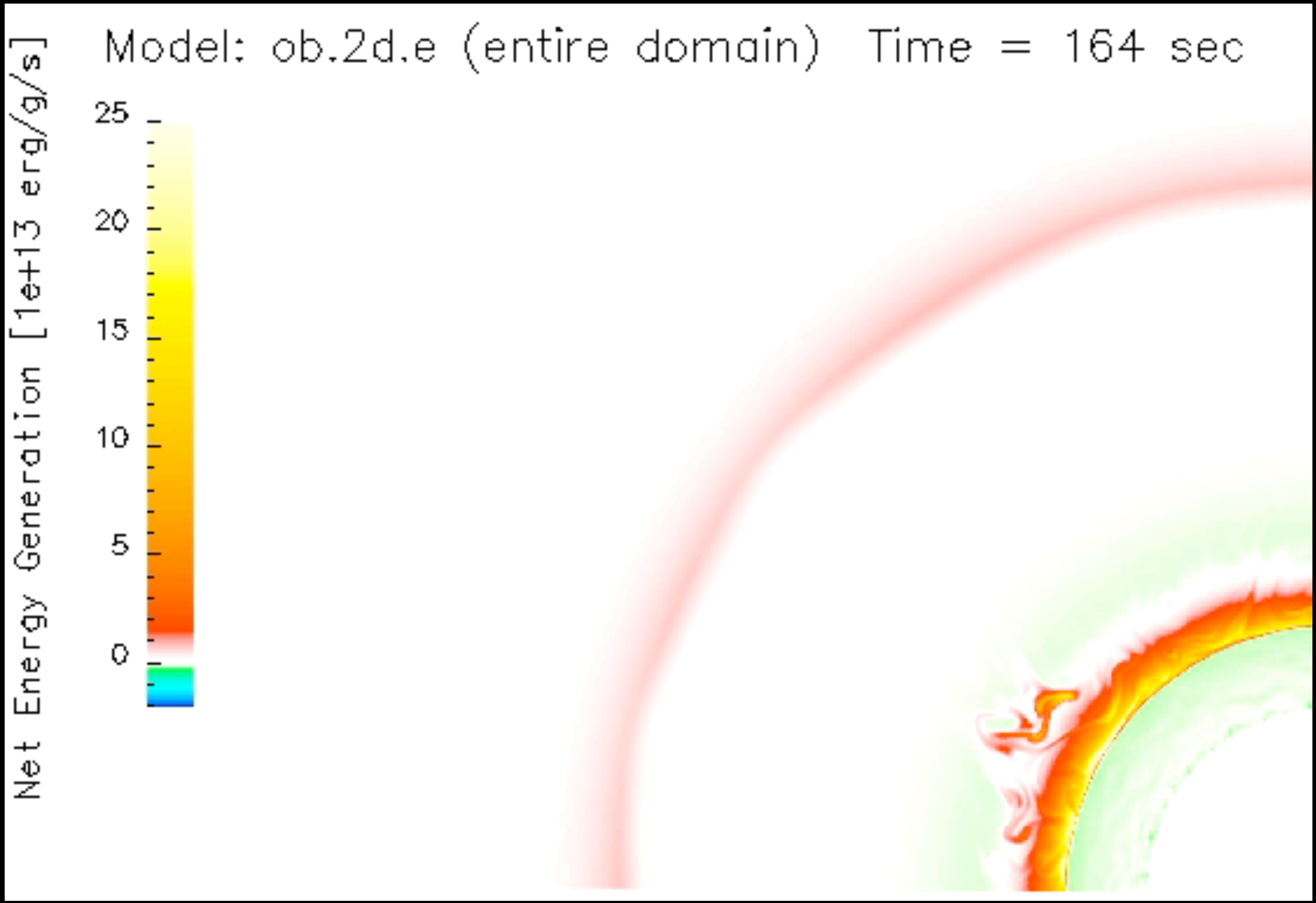
- Many astrophysicists model turbulent convection as diffusion
- Using this particle kinetic analogue, convective dissipation would average out rotational gradients over a mixing length
- Radiative zones, with smaller dissipation, would have differential rotation
- THIS IS WRONG!

Rotation and Turbulent Convection (shearing box)

Meakin & Arnett, in preparation



C-Ne-O-Si shell interaction in 2D: Meakin and Arnett



- 2D may be deceptive, so beware, but burning shells probably interact

- O+O burning is not static, but has vigorous bursts of burning

- Convection drives g-modes at all boundaries; these couple to p-modes as they run down density gradients

- O+O and Si burning get steadily more violent prior to core collapse

Open questions we must answer (and my guesses):

- Are rotating stars shellular? (maybe not)
- Is the anelastic approximation adequate for the Sun? (probably)
- Is the anelastic approximation adequate for oxygen and later burning stages? (no)
- Do shell burning stages interact strongly prior to collapse? (yes)