# Isotropic Diffusion Source Approximation (IDSA, Liebendörfer et al. 2009)

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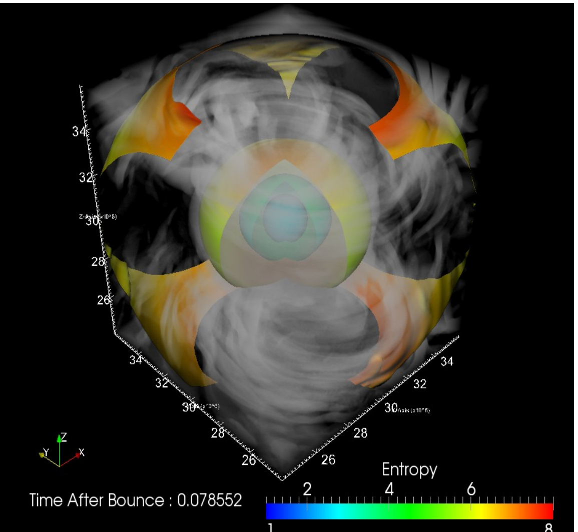


**Developed & implemented by**
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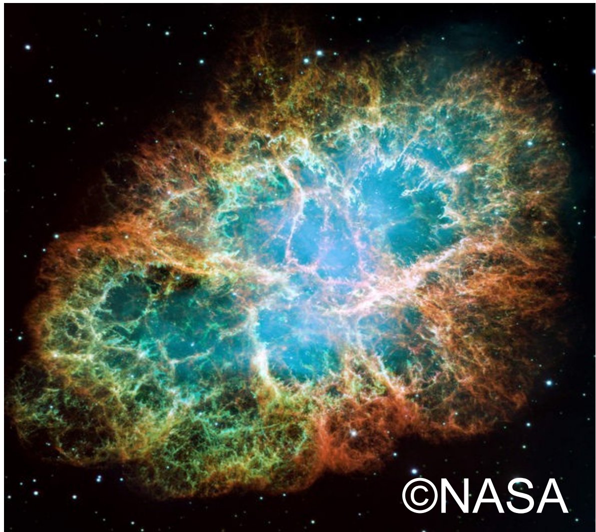


*"The IDSA implements the necessary---but only the necessary ---neutrino physics for productive 3D supernova models"*

*M. Liebendörfer, Aug. 09*

Or: *how to breathe life into the stalled shock wave in 3D.*

# Outline

-Multi-dimensional requirements for a core collapse supernova code.

-IDSA (For simplicity explained spherically symmetry).

-Verification for spherically symmetric SN models.

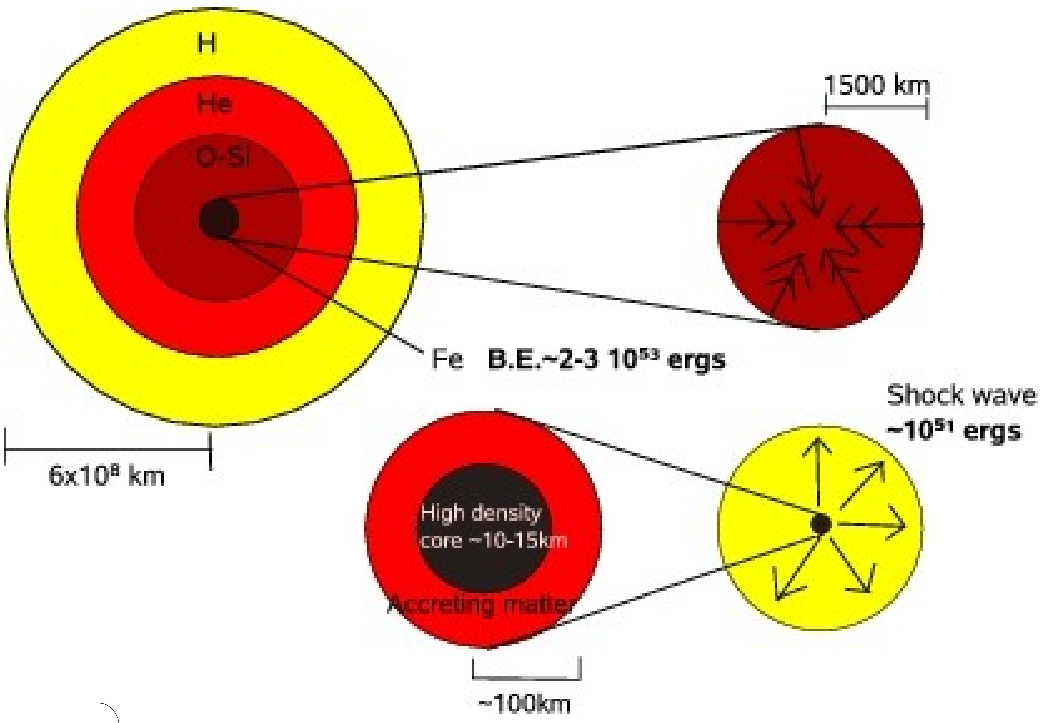
# The Core Collapse scenario

(SN-Mechanism see e.g. Bethe & Wilson (1985), Buras et al. (2006), Mezzacappa et al. (2006), Janka et al. (2007), Marek & Janka (2007), Foglizzo et al. (2007), Burrows et al. (2006),...)

- Explosion powered by gravitational binding energy of forming compact remnant.

$$E_b \approx 3 \times 10^{53} \left(\frac{M}{M_\odot}\right)^2 \left(\frac{R}{10\mathrm{km}}\right)^{-1} \mathrm{erg}$$

- O($10^{51}$) erg kinetic & internal energy of ejecta.
- 99% of energy radiated as neutrinos.
- Shock loses kinetic energy to neutrinos and dissociation of iron group nuclei ~8.8MeV/ nucleon.
- Shock wave stalls. “prompt mechanism” fails.
- What is SN-mechanism that revives the shock for an explosion?
  - -Neutrino mechanism (e.g. Bethe & Wilson (1985)
  - -Enhanced (SASI/convection-aided) Neutrino driven mechanism (e.g. Buras et al. (2006), Mezzacappa et al. (2006), Marek & Janka (2007),...)
  - -Acoustic Mechanism (Burrows et al. 2006)
  - -MHD-JET (e.g. Akiyama(2003), Wilson(2005), Kotake (2006), Burrows et al.(2007),..)
  - -QCD phase transition (Sagert, Fischer et al. (2008))




**<u>most scenarios rely on 3D effects!</u>**

# 3D Models with neutrinos

- Multi-Dimensional Hydrodynamics
no explosions in general in 1D, e.g. Thompson et al. (2003), Rampp & Janka (2002),Liebendörfer et al. (2002/2005))

- Plasmaphysics (Magnetic fields)

Magnetohydrodynamics (MHD):
~ 10 variable per fluid element; not a problem

-Full Boltzmann neutrino transport in 3D:

1 fluid element contains 4 types x 20 energies x 100 angles =8000 variables
For 3D domain with $1000^3$ cells →**64TB memory per time step required.**

-Not possible on today's supercomputers.

-Don't want to run a single simulation for several months (useless e.g. for GW parameter study; numerical errors can sum up...)

-Need to rely on simplifications!

-Basel code **ELEPHANT**: $Y_e$-parametrisation during collapse, **IDSA** postbounce
(Elegant Parallel Hydrodynamics with Approximate Neutrino Transport)

# Conceptual ansatz of the IDSA

- Goal: implement the **dominant features** of radiative/neutrino transfer efficiently.
- →Transport of electron neutrino and antineutrino.
- To do this we decompose problem into different subdomains and apply appropriate algorithms for each.
- Each subdomain can use a different method.
- Neutrino **opaque** & **transparent** region.

# Decomposition

Decompose the distribution function f of transported particles

$$f = f^t + f^s$$

$f^T$ = Trapped particles – opaque regime
$f^S$ = Streaming particles – transparent regime

Boltzmann equation: $D(f = f^t + f^S) = C = C^t + C^S$

The two components are evolved separatly:

$$D(f^t) = C^t - \Sigma$$
$$D(f^s) = C^s + \Sigma$$

Σ: isotropic **diffusion source** term, converts trapped into streaming particles and vice versa.

# Evolution of trapped component

Assumptions:

-Distribution function $f^T$ is isotropic: $f^t = f^t(t,r,E)$
-Source function Σ is isotropic

Angular integration reduces Boltzmann equation to (co-moving frame):

$$\frac{df^t}{cdt} + \frac{1}{3}\frac{d\ln\rho}{cdt}E\frac{\partial f^t}{\partial E} = j - (j+\chi)f^t - \Sigma$$

Evolution of trapped particles must reproduce correct diffusion limit.
-particles slowly drain or replenish in a fluid element.

Comparison with diffusion gives diffusion source Σ.

Collision integral in the diffusion limit:

$$s = \frac{df_0}{cdt} + \frac{1}{3}\frac{\partial\ln\rho}{c\partial t}E\frac{\partial f_0}{\partial E} - \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{3(j+\chi+\phi)}\frac{\partial f_0}{\partial r}\right)$$

# The diffusion source Σ

Comparision with diffusion limit suggests the Σ:

$$\Sigma = \min\left\{\max\left[\alpha + (j+\chi)\frac{1}{2}\int f^{s} d\mu, 0\right], j\right\}$$

$$\alpha = \frac{1}{r^2}\left(\frac{-\lambda}{3}\frac{\partial f^t}{\partial r}\right) \qquad \lambda = \frac{1}{j+\chi+\phi}$$

-α is diffusive.
-λ mean free path.
-second term of RHS is absorption of streaming particles.

Min, Max conditions prevent unphysical particle fluxes!

-Upper limit: emissivity of matter (Σ ≤ j ).
-Lower limit: matter equilibrium (0 ≤ Σ).

# Streaming component

Streaming component weakly coupled to matter.

Laboratory frame more convenient (cumbersome energy-terms drop).

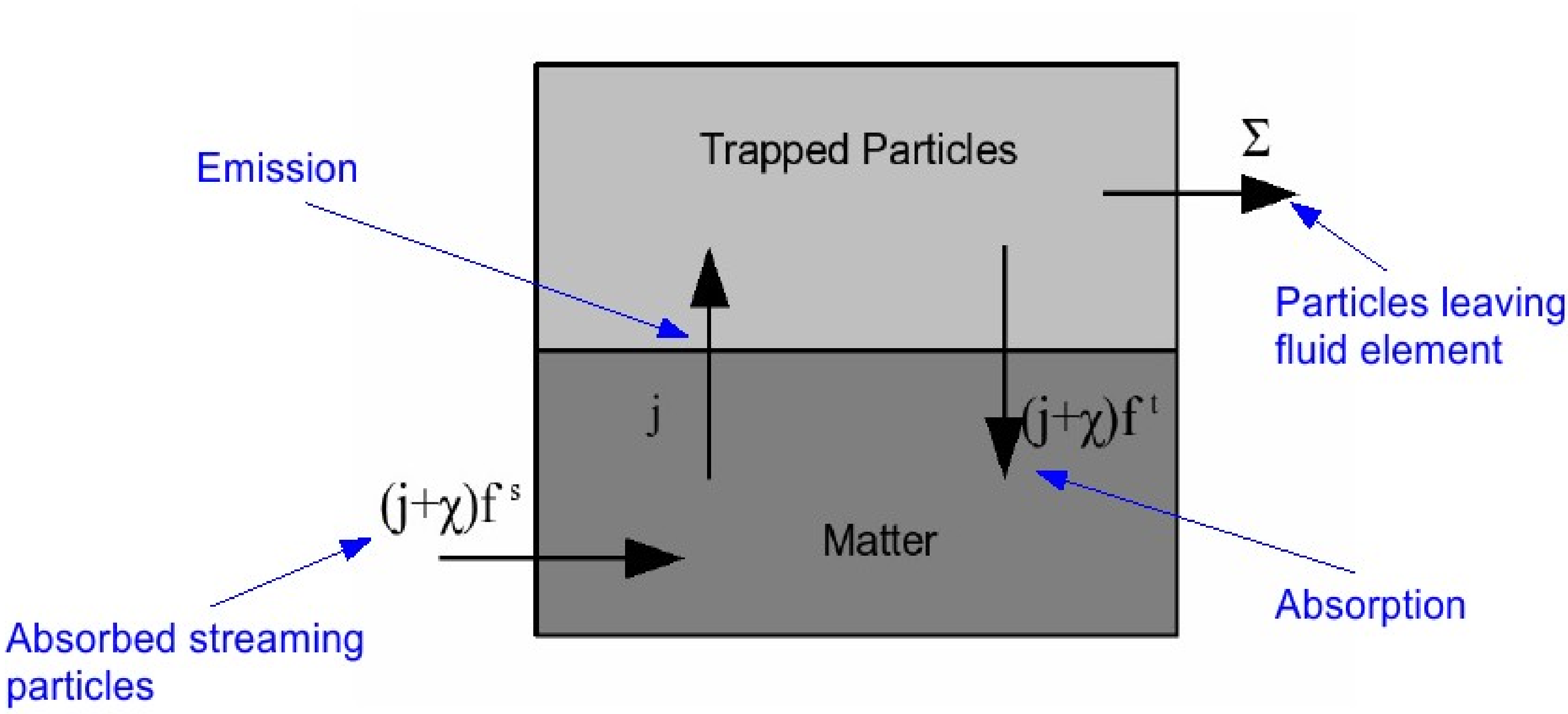
Transport equation becomes:

$$\frac{\partial \hat{f}^s}{c\partial \hat{t}} + \hat{\mu}\frac{\partial \hat{f}^s}{\partial r} + \frac{1}{r}\left(1-\hat{\mu}^2\right)\frac{\partial \hat{f}^s}{\partial \hat{\mu}} = -\left(\hat{j}+\hat{\chi}\right)\hat{f}^s + \hat{\Sigma}$$

Stationary state approximation (can drop *d/dt*-term).

All sources are assumed to be **isotropic**.
→Integration over angles: poisson equation.

# A fluid element



# Coupling to Hydrodynamics

Use thermal spectrum for $f^{t}$: $f_l^{\mathrm{t}}(E) = \{\exp[\beta_l(E - \mu_l)] + 1\}^{-1}$

This assumption is only made for trapped particles within a fluid element.

Streaming particles, which communicate between fluid elements, keep their detailed spectral information.

Characterise the thermal equilibrium by a

- neutrino fraction $Y^T$: $$Y^{\mathrm{t}} = \frac{m_{\mathrm{b}}}{\rho}\frac{4\pi}{(hc)^3}\int f^{\mathrm{t}}E^2 dE\, d\mu$$

- particle mean specific energy $Z^T$: $$Z^{\mathrm{t}} = \frac{m_{\mathrm{b}}}{\rho}\frac{4\pi}{(hc)^3}\int f^{\mathrm{t}}E^3 dE\, d\mu$$

# Coupling to Hydrodynamics II

Conservation law of Hydrodynamics:

$$\frac{\partial}{\partial t}U + \frac{\partial}{r^2 \partial r}\left(r^2 F\right) = 0$$

U: vector of primitive variables
F: flux vector

$$U = \begin{pmatrix} \rho \\ \rho v \\ \rho\left(e + \frac{1}{2}v^2\right) \\ \rho Y_{\mathrm{e}} \\ \rho Y_l^{\mathrm{t}} \\ \left(\rho Z_l^{\mathrm{t}}\right)^{\frac{3}{4}} \end{pmatrix}, \quad F = \begin{pmatrix} v\rho \\ v\rho v + p \\ v\rho\left(e + \frac{1}{2}v^2 + \frac{p}{\rho}\right) \\ v\rho Y_{\mathrm{e}} \\ v\rho Y_l^{\mathrm{t}} \\ v\left(\rho Z_l^{\mathrm{t}}\right)^{\frac{3}{4}} \end{pmatrix}$$

# Movie

**Movie:**

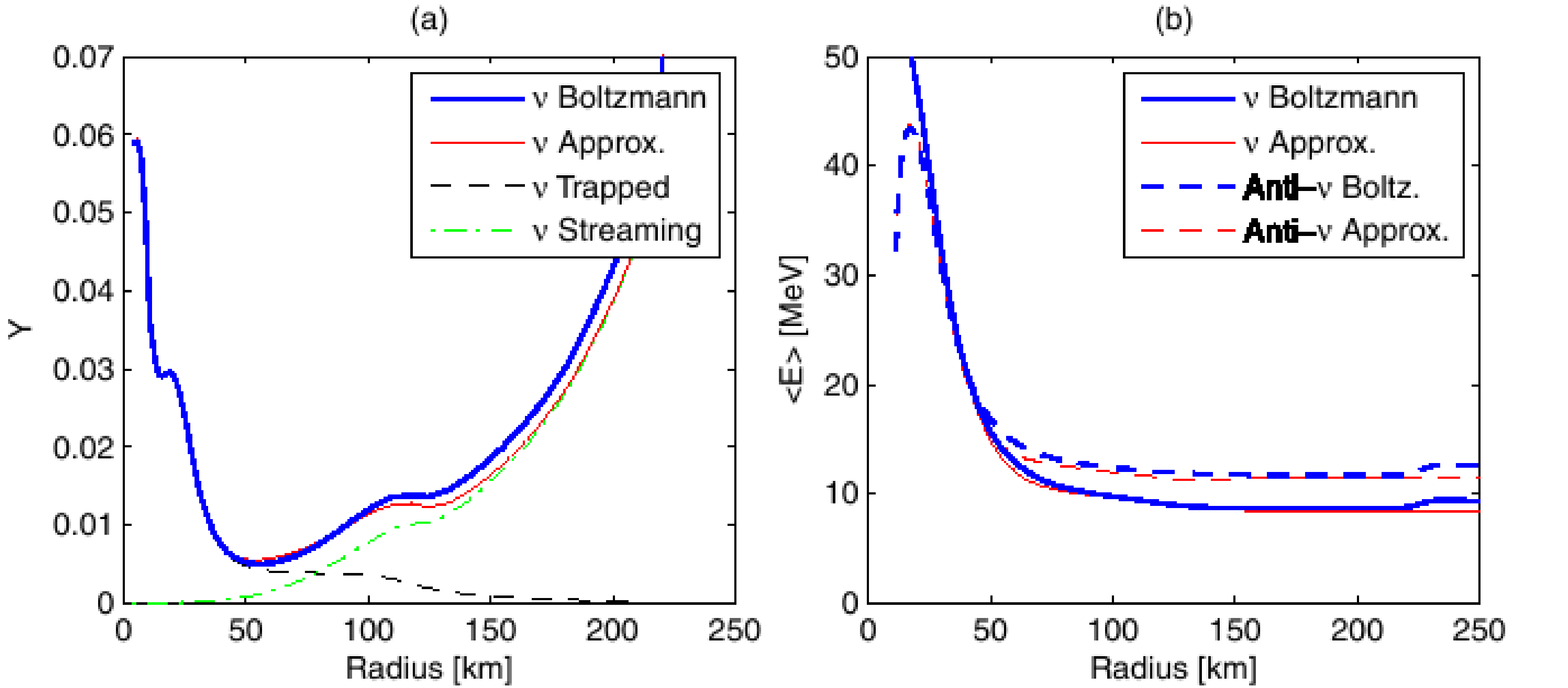
Initial conditions:

- $\Omega_{i,central}$=0[rad/sec]

- EoS: LS(K=180)

-15Msol (WW95)

# Neutrino abundance & mean energy 150 ms post-bounce




note IDSA: ~several 10 x faster than solving 1D GR Boltzmann transport equation.

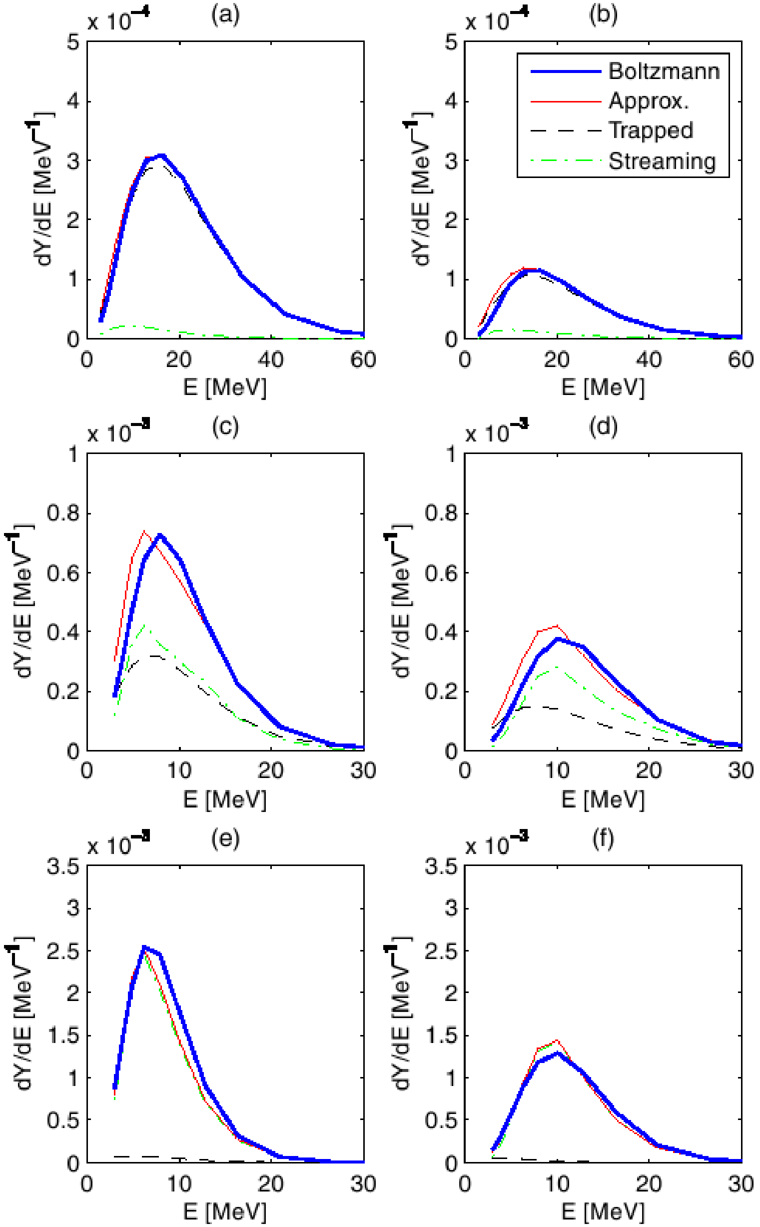
# Comparison of IDSA spectra

**Figure**: Particle spectra 150ms after bounce.

at 40 km radius
(trapped regime)

at 80 km radius
(semi-transparent regime)

at 160 km radius
(free streaming)



# Conclusions

- IDSA is an efficient way of including the neutrino transport in 3D core collapse supernovae.
- Agrees well with spherically symmetric Boltzmann transport with same input physics.
- Contains ONLY the necessary neutrino physics for productive 3D models of core collapse supernova postbounce phase. (The IDSA does not contain the neutrino-electron scattering that is essential in the collapse phase, we will treat the collapse phase with the parametrisation scheme.)
- Soon: $\mu/\tau$ - cooling by a leakage scheme. (→Albino Perego)
- Algorithm could be applied to other areas of astrophysics (e.g. accretion discs, star formation).