Simulations of the magneto-rotational instability in core-collapse supernovae

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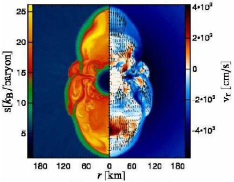
Exlosion mechanisms



Marek et al, 2008

How is the failed explosion revived?

Not a matter of energy ($e_{core} \gg e_{env}$), but of energy transfer.

- Spherical neutrino-driven explosion
- neutrino heating aided by hydrodynamic instabilities
- Energy transfer by (accoustic) waves
- rotation

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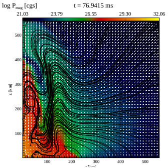

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Rotation

- tap into e_{rot} by magnetic fields (Thompson et al., 2004)
- successful?
- realistic?
  - rapid rotation: only certain stars
  - $|\vec{b}|$ sufficiently strong?

$\longrightarrow$ MRI? (Akiyama et al., 2003)

Field amplification in supernovae

Why magnetic fields?

- pulsar fields, magnetars
- asymmetric explosions: caused by large-scale fields?
- additional energy reservoir: rotation

But...

- strong (equipartition) fields needed for dznmical effects
- typical pre-collapse fields are too weak

$\Rightarrow$
- special class of progenitors
- strong amplification

field amplification mechanisms

- compression: gravitational infall $\Rightarrow$ magnetic energy
- winding: differential rotation $\Rightarrow$ magnetic energy
- hydromagnetic instabilities: differential rotation, entropy/composion gradients $\Rightarrow$ magnetic energy

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Magneto-rotational explosions

- effective viscosity due to small-scale MHD turbulence
- angular-momentum transport

$\longrightarrow$ loss of rotational equilibrium

- large-scale fields $\longrightarrow$ bipolar explosions, jets collimated by magnetic hoop stress
- potentially only important on long time scales

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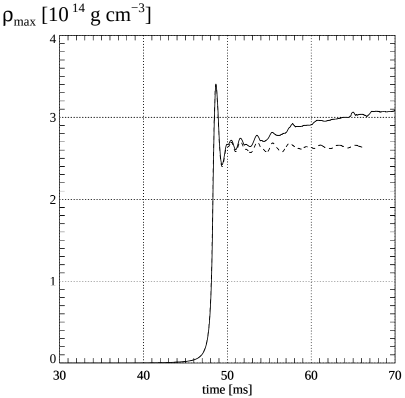

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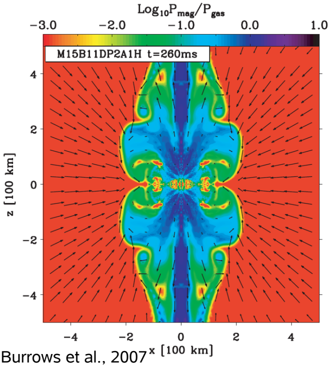

Burrows et al., 2007

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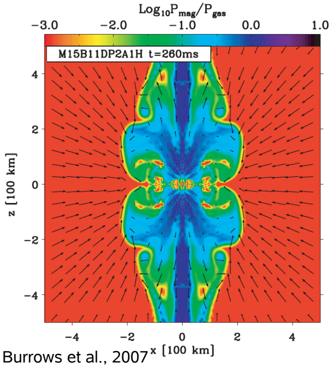

Burrows et al., 2007

General properties of the MRI

- local linear MHD instability of differentially rotating fluids
- weak initial magnetic field required
- run-away of angular-momentum transport along field lines
- instability criterion: negative Ω gradient
- growth time $\sim$ rotational period
- leads to MHD turbulence and efficient transport

Open Questions

Accretion discs

- Keplerian shear
- ⇒ Rayleigh-stable, MRI-unstable
- rapid growth
- MHD turbulence may provide viscosity required for accretion
- well-studied system, yet still many open questions

study the MRI in supernovae

- theoretical analysis of the instability criteria
- simulations of MRI-unstable systems

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Issues in MRI theory

- Saturation mechanism
- Saturation level as a function of
  - physics, e.g., dissipation coefficients, thermodynamics of the disc
  - numerics (box size, boundaries, ...)
- formulate a simple model

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Supernovae

- differential rotation, thermal stratification

$\Rightarrow$ possibly: hydrodynamically unstable + MRI unstable

- growth: fast enough?
- saturation: strong enough?
- starting to receive interest

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Questions in SN MRI

- MRI with complex thermodynamics, in complex geometry
- regimes of the MRI: linear instability analysis
- physics of saturation

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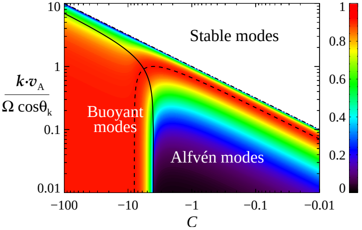
Instability analysis

local linear WKB analysis (Balbus, 1995; Urpin, 1996)

- hydrodynamic background model in equilibrium with
  - differential rotation, $\Omega \propto \varpi^{-|\alpha_\Omega|}$
  - entropy gradient, $S = S_0 + \partial_\varpi S \varpi$
- add a weak magnetic field and linearise (incompressible) MHD equations
- examine the dispersion relation of MHD waves

The dispersion relation of MRI modes

normalised growth rate



dashed line: fastest growing mode
solid line: boundary between modes branches

Stable modes short modes are stablised by magnetic tension

Alfvén modes fast growth only for finite wave number

Bouyant modes appear only for large entropy gradient; fast growth for long modes

Definition of symbols

$$\mathcal{C} = \frac{(N)^2 + \left(\varpi \times \partial_\varpi \Omega^2\right)^2}{\Omega^2}$$

N = bouyancy frequency

$\mathbf{v}_\mathrm{A}$ = Alfven velocity

$\mathbf{k}$ = wave number

θ_k = angle between $\mathbf{k}$ and the vertical

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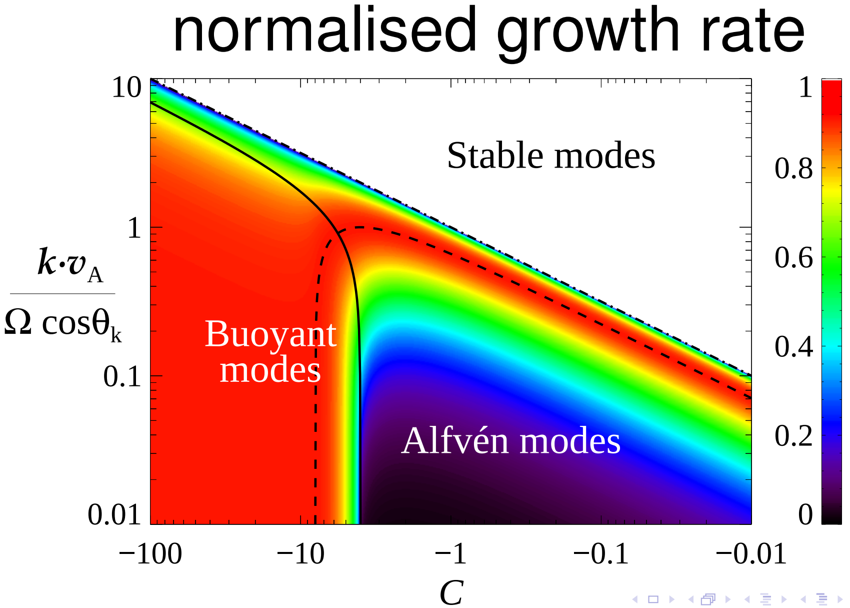

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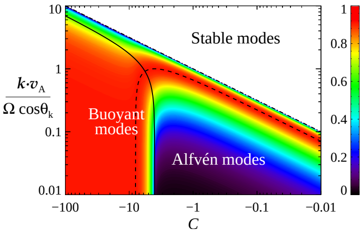

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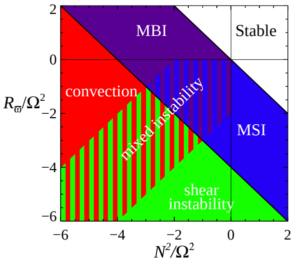
Regimes of the axisymmetric MRI

magneto-bouyant
convection stabilised by rotation, but destabilised by the magnetic field

stable stabilised by positive entropy or Ω gradients

convective
similar to hydrodynamic convection (Schwarzschild or Ledoux)



magneto-shear
classical MRI, e.g., accretion discs

mixed interplay of many effects

shear regime Rayleigh unstable

Max-Planck-Institut für Astrophysik

Physics and numerics

Simplified physics

- Full ideal MHD (rather than shearing box)
- simplified equation of state
- external gravity
- no neutrino transport

Code

- Eulerian, conservative
- high-order reconstruction (MP or WENO)
- MUSTA Riemann solver (Titarev & Toro, 2005)
- constraint transport

Models

- gas in hydrostatic equilibrium; uniform field or vanishing net flux
- axisymmetric and 3d simulations
- small (few kilometres) boxes resembling the equatorial region
- resolution between 0.625 and 40 metres
- shearing-disc boundary conditions (Klahr & Bodenheimer, 2003)

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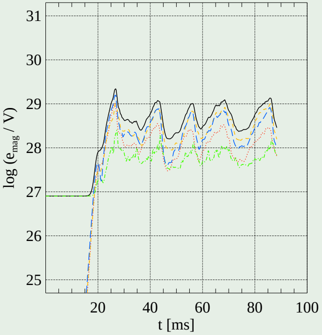
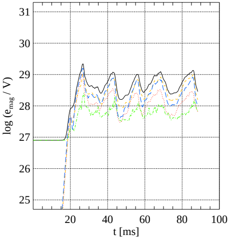
Dynamics

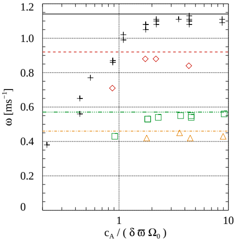
temporal evolution of the magnetic energy

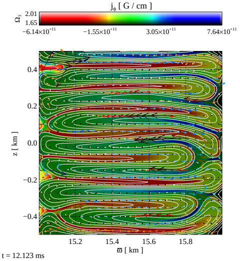


- confirm all (relevant) regimes of the linear analysis
- (de-)stabilisation by interplay of Ω and *S* gradients
- growth rates in agreement with linear analysis, i.e., a few milliseconds for rapidly rotating cores
- maximum field strength $\gtrsim 10^{15}$ G

Dynamics







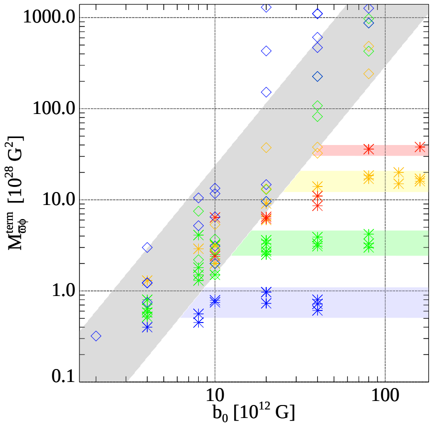
- early phase: exponential growth of channel flows
- termination of growth and breakup of channels

Scaling of the termination level

Termination ($\neq$ saturation)

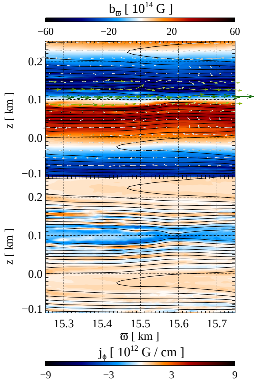
the Maxwell stress reached at the end of the growth of the MRI depends on (among other factors)

- the grid resolution:
  finer grid $\Rightarrow$ higher $M_{\varpi\phi}$
- the initial field:
  stronger b_0 $\Rightarrow$ higher $M_{\varpi\phi}$
- the rotational profile:
  slower $\Rightarrow$ higher $M_{\varpi\phi}$



Scaling of the termination level

- MRI growth terminates when channel flows are disrupted by resistive instabilities.
- Channels are generically unstable against secondary instabilities, here tearing modes (Goodman & Xu, 1994).
- MRI terminates approximately when the resistive instabilities grow faster than the MRI.



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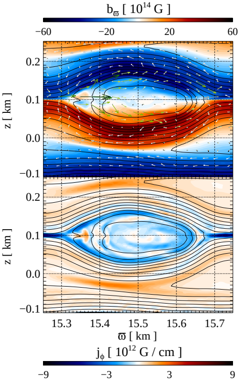

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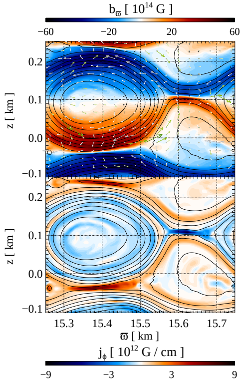

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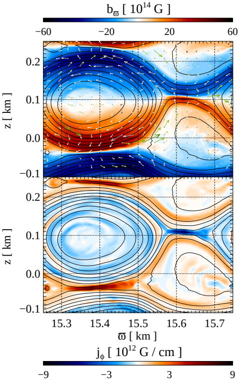

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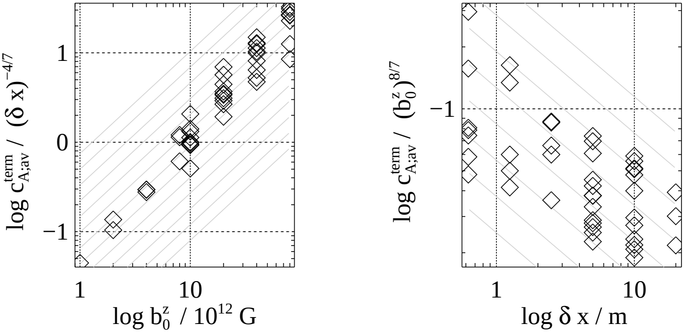
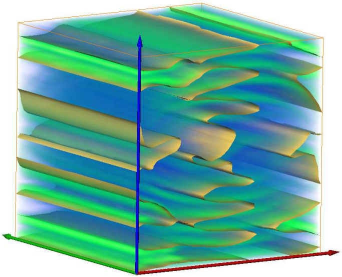


scaling laws for MRI termination

competing growth of MRI modes and parasites allows for an explanation

Saturation: turbulence and coherent flows

- Saturation: turbulent state
- efficient transport of angular momentum
- coherent flow and field patterns can be identified
- stable over several rotational periods
- example: average value of the toroidal field on slices $z = \text{const.}$ as a function of time (cf. Lesur & Ogilvie, 2008)



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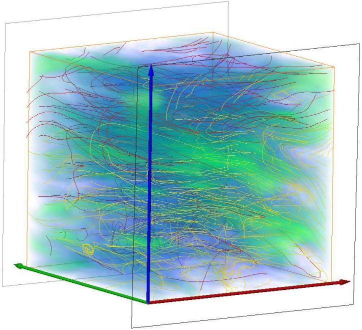

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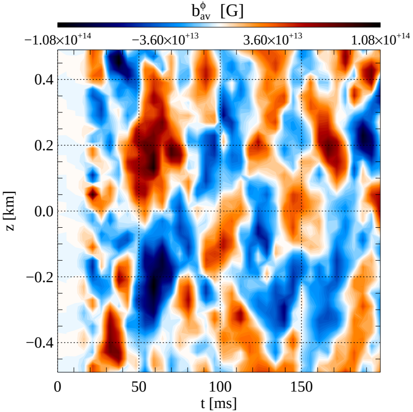

Summary

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- analysis of the dispersion relation: MRI can be relevant
- high-resolution simulations agree with linear regime
- turbulence and enhanced transport in saturation

Open issues

- influence of global geometry and progenitor structure
- interplay with additional physics
- physics of saturation
- formulation of a model for use in lower-resolution simulations

Conclusion

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