Full GR simulations with microphysics

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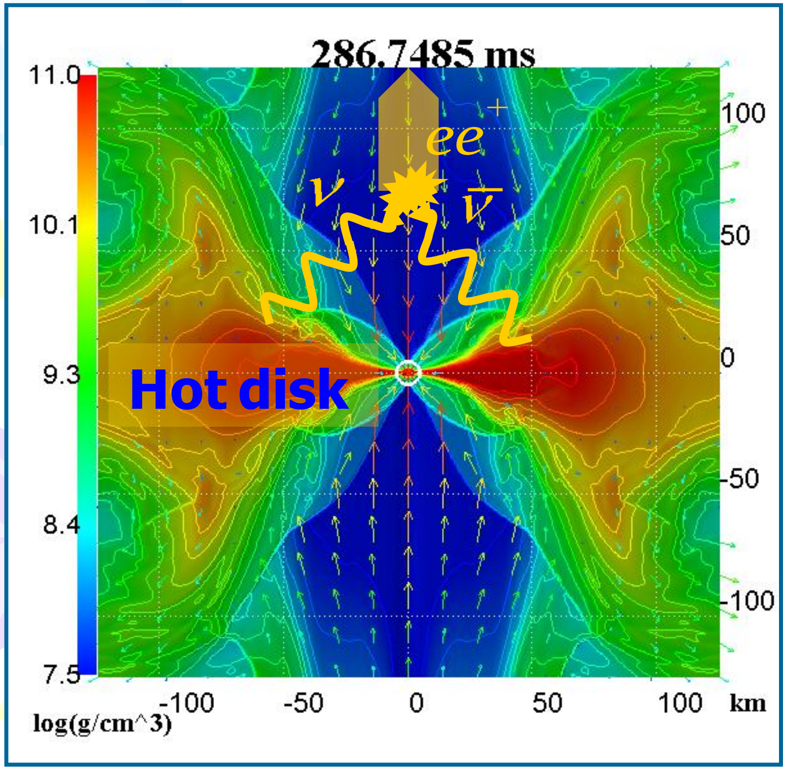
MICRA2009 @ Niels Bohr International Academy and NORDITA

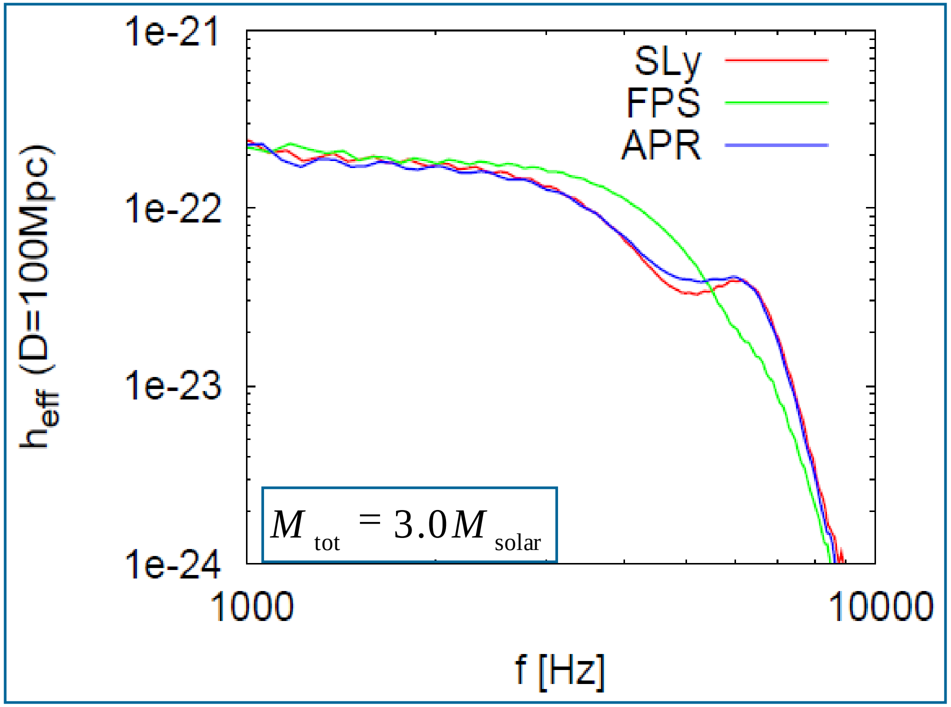
- In full GR simulation,
  - implicit full neutrino transfer is much harder
  - instead, **explicit approximate** solver may be good approach
    - targeting mainly GRB, BNS, and BH formation
  - (For example, our GR-leakage scheme gives good results)

Why GR with microphysics ?

- GR is essential for
  - BH formation
    - GRB, HNe
  - Accurate GWs
    - Compact-star merger



- Microphysics
  - SNe, GRB
    - EOS, weak rates, neutrinos
  - Realistic GWs
  - Time variability
    - e.g. convection

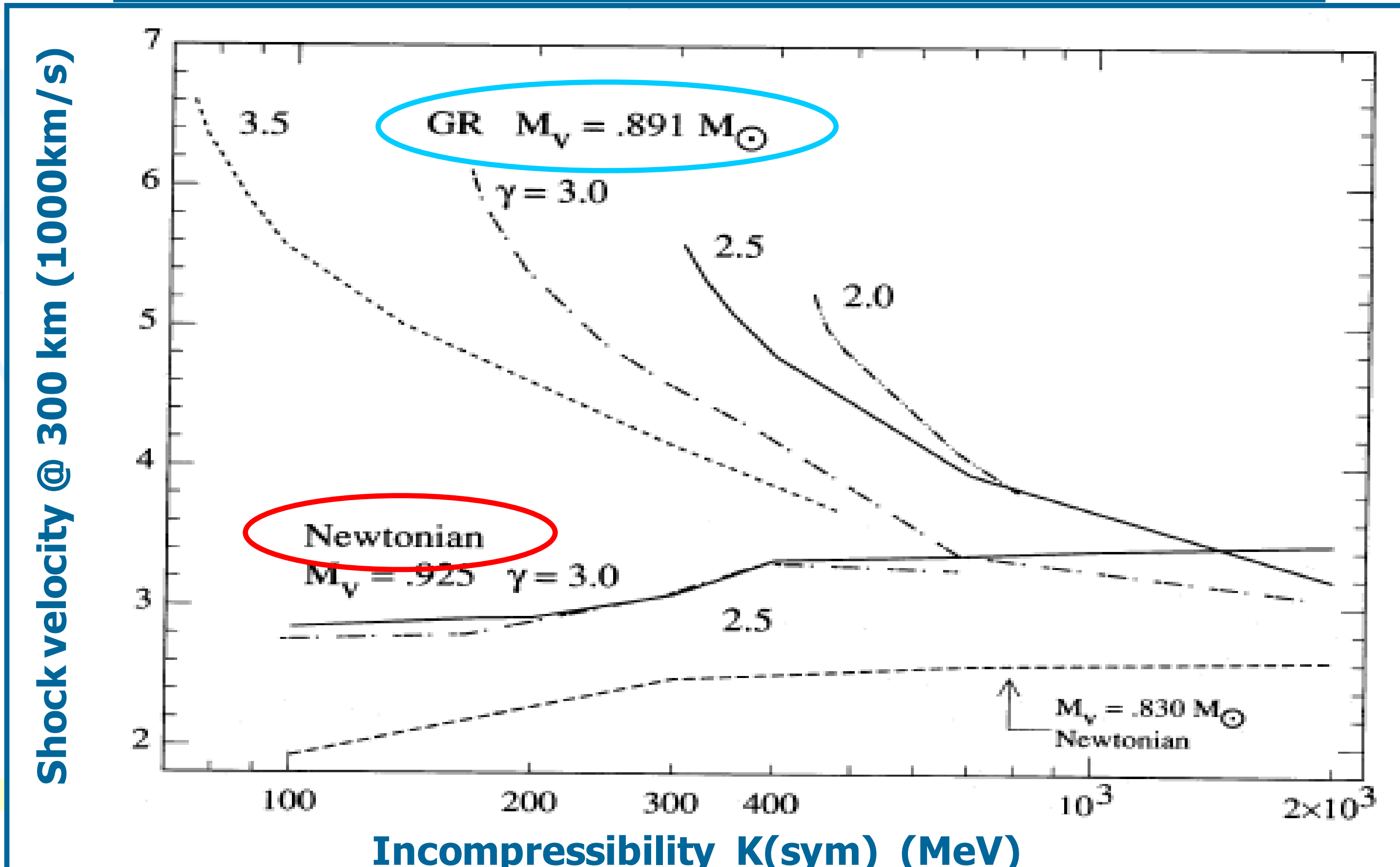


GR and EOS

Van Riper (1988) ApJ 326, 235

$$P_n = K\rho_0[(\rho/\rho_0)^\gamma - 1]/9\gamma \text{ MeV fm}^{-3}$$

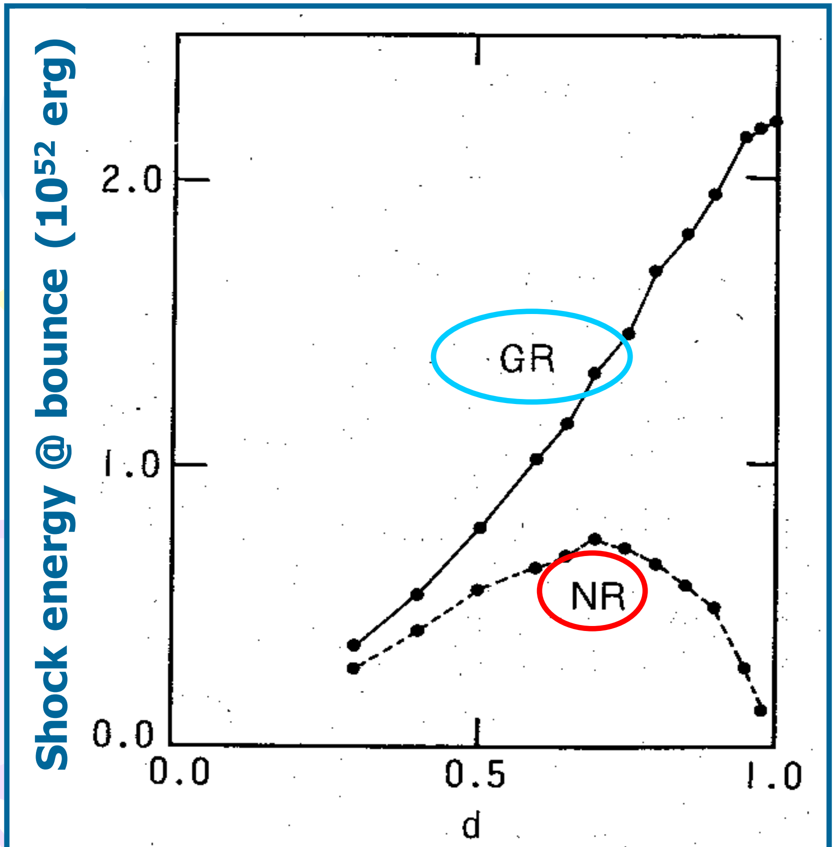
Kolehmainen, K., Prakash, M., Lattimer, J., and Treiner, J. 1985,

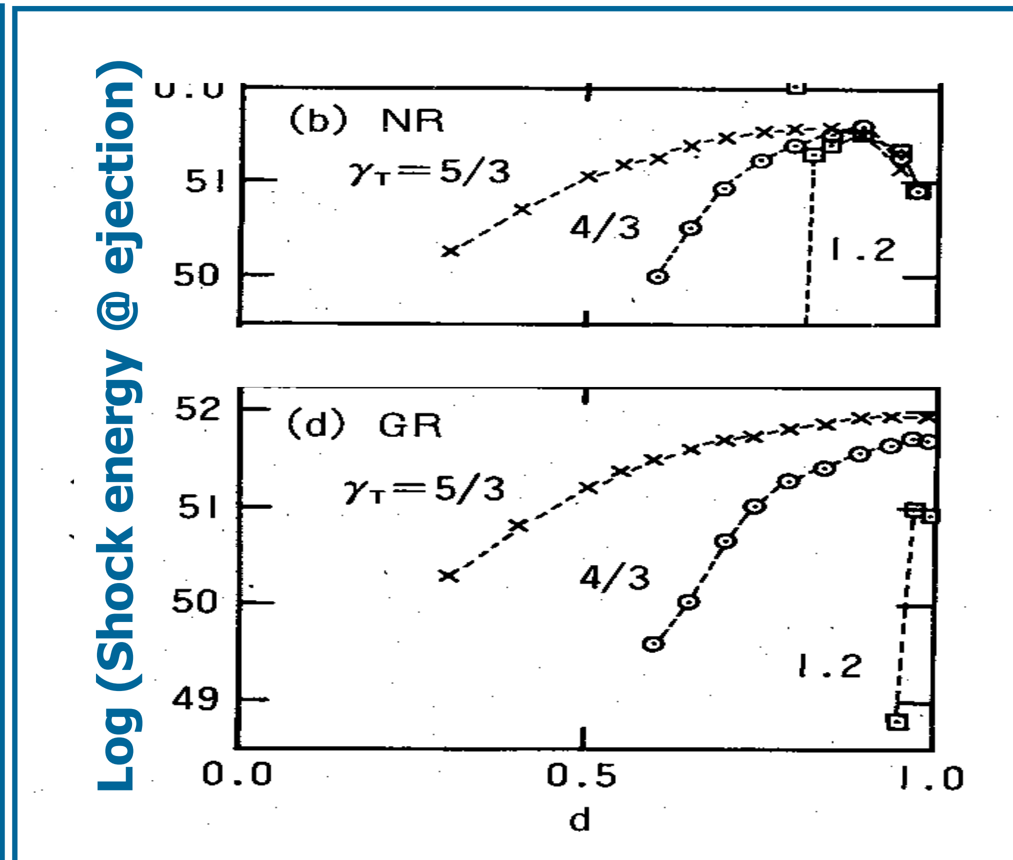


GR and weak rates

Takahara & Sato (1984) PTP 72, 978

$$d \sim \left(\frac{Y_{\text{lepton, bounce}}}{Y_{\text{lepton, init}}} \right)^{4/3} \quad \text{: depends on weak rates (e - capture, } \nu \text{ - trapping)}$$



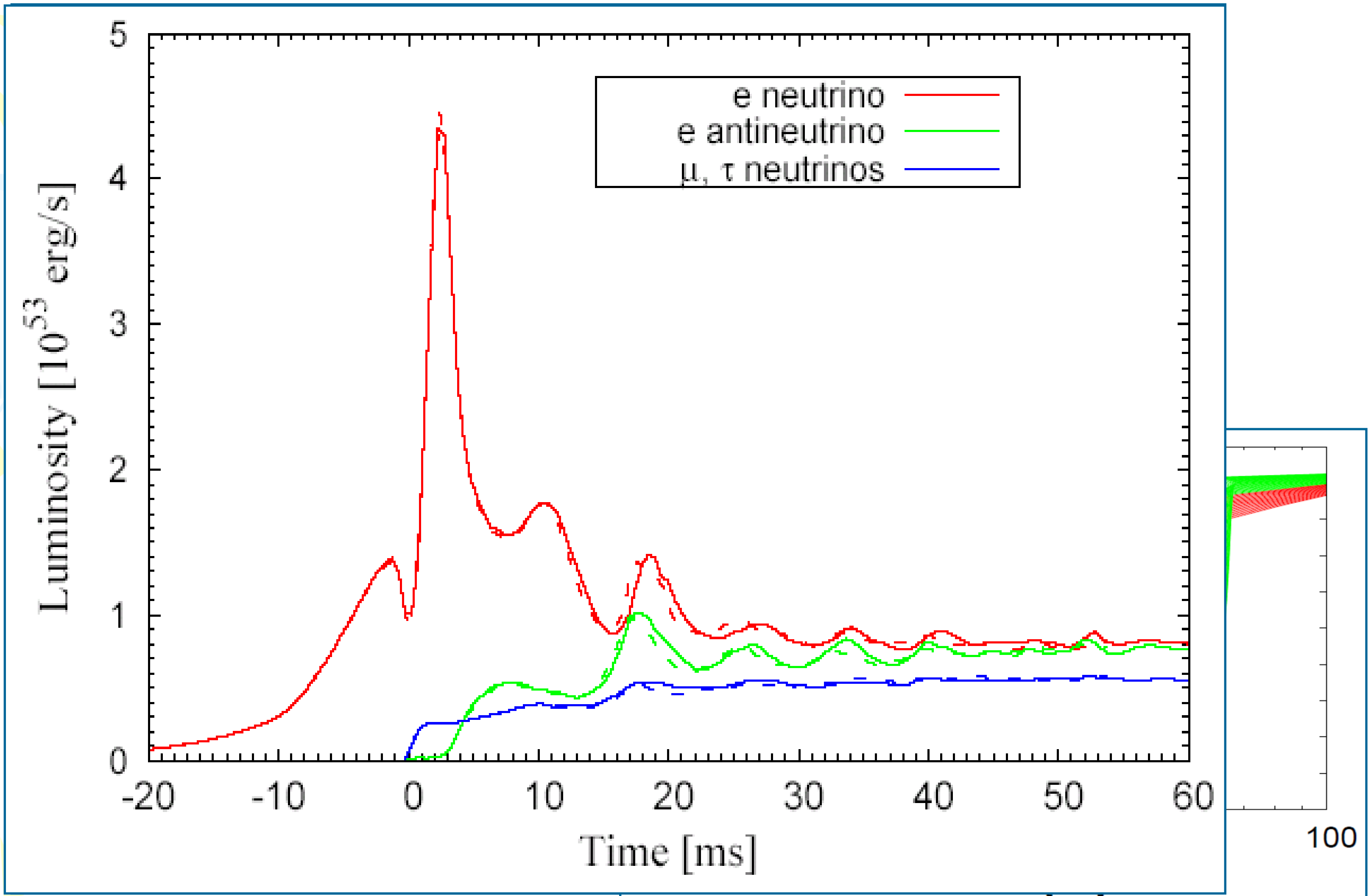


Summary of microphysics

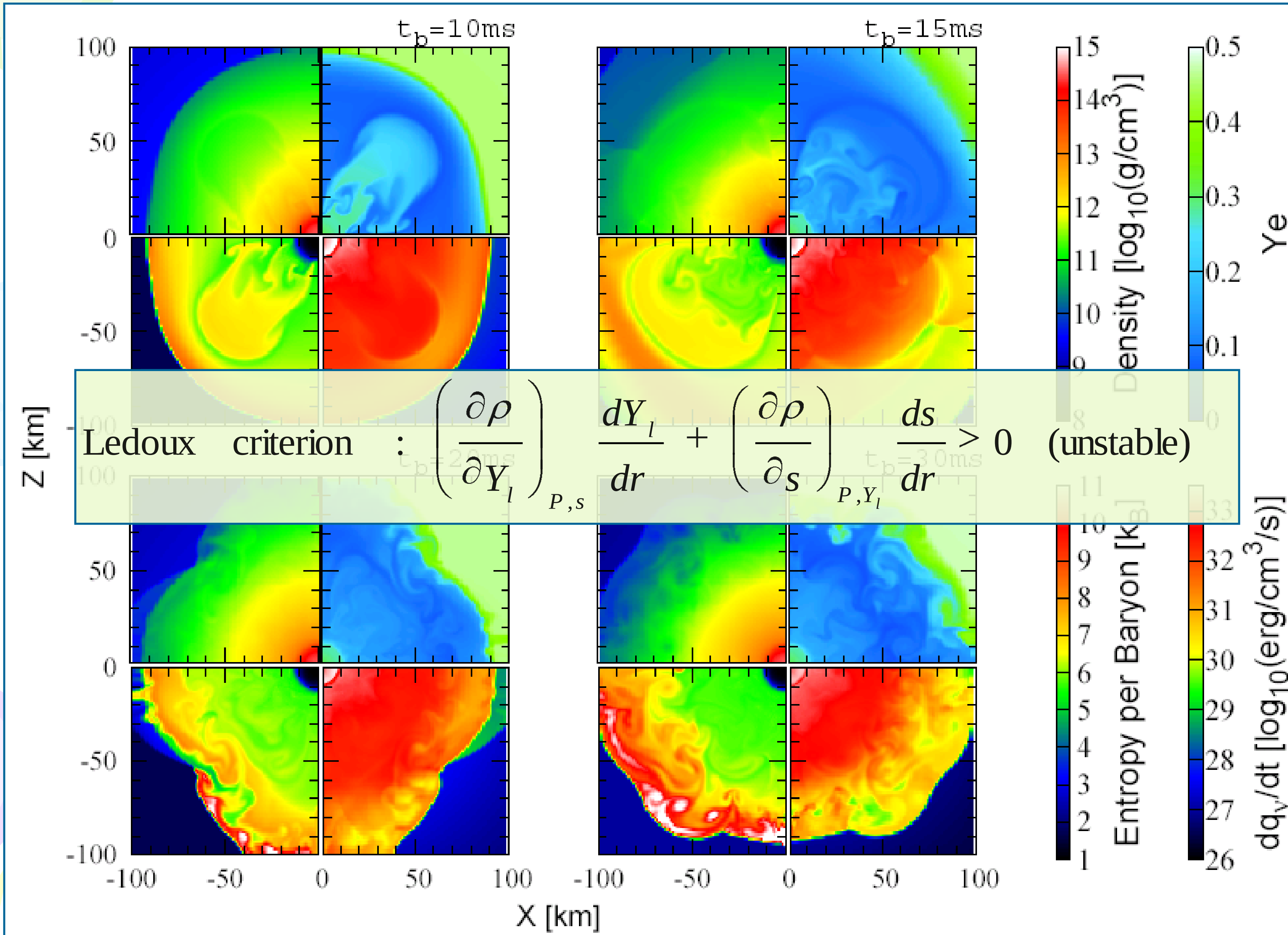
- EOS: Tabulated EOS can be used
  - Currently Shen EOS + electrons + radiation
- Weak rates
  - Electron capture: FFN1985, rate on NSE back ground
  - $e^{\pm}$annihilation: Cooperstein et al. 1985, Itoh et al. 1996
  - plasmon decay: Ruffert et al. 1996, Itoh et al. 1996
  - Bremsstrahlung: Burrows et al. 2006, Itoh et al. 1996
- Neutrino leakage
  - Opacity based on Burrows et al. 2006
    - (n, p, A) scattering
      - Including correction such as ion-ion correlation
    - (n, p, A) absorption

Spherical collapse to NS (S15)

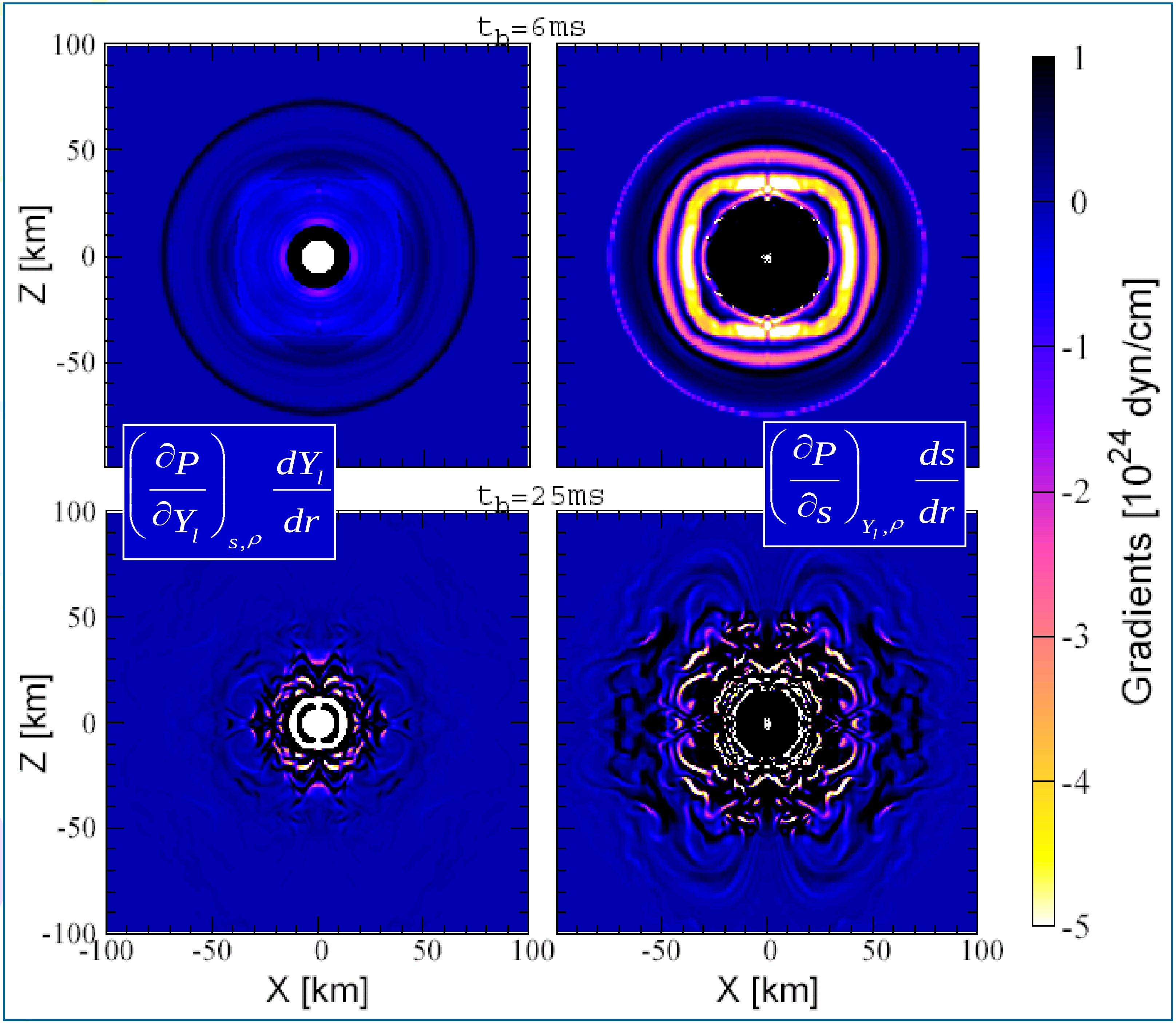
- Results consistent with Liebendorfer et al. 2004



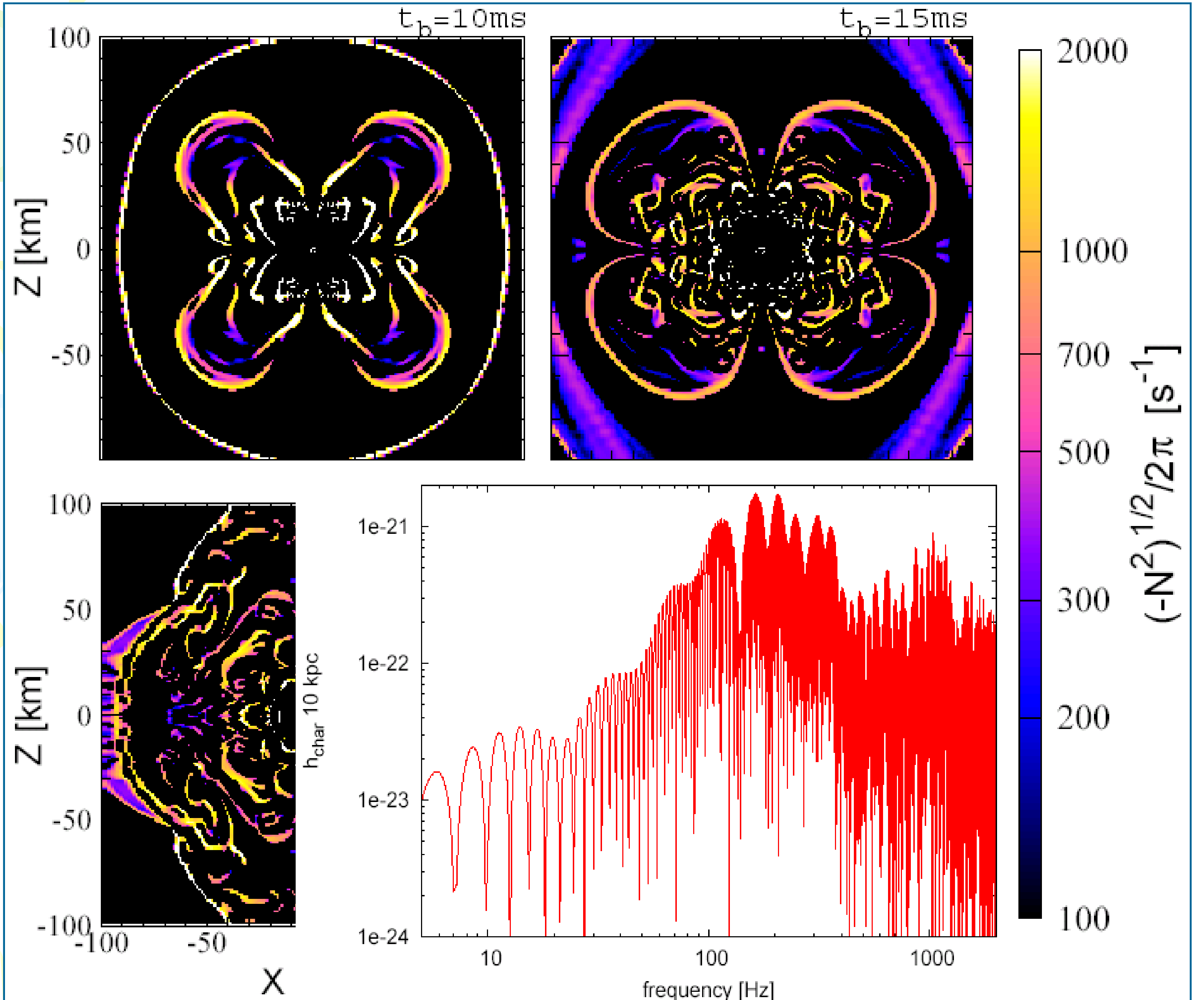
Convective activities



$$\text{Ledoux criterion}: \left(\frac{\partial \rho}{\partial Y_l}\right)_{P,s} \frac{dY_l}{dr} + \left(\frac{\partial \rho}{\partial s}\right)_{P,Y_l} \frac{ds}{dr} > 0 \quad \text{(unstable)}$$



t_b=6ms
t_b=25ms
$\left(\frac{\partial P}{\partial Y_l}\right)_{s,\rho} \frac{dY_l}{dr}$
$\left(\frac{\partial P}{\partial s}\right)_{Y_l,\rho} \frac{ds}{dr}$
Z [km]
X [km]
Gradients [10^{24} dyn/cm]
1
0
-1
-2
-3
-4
-5
100
50
0
-50
-100

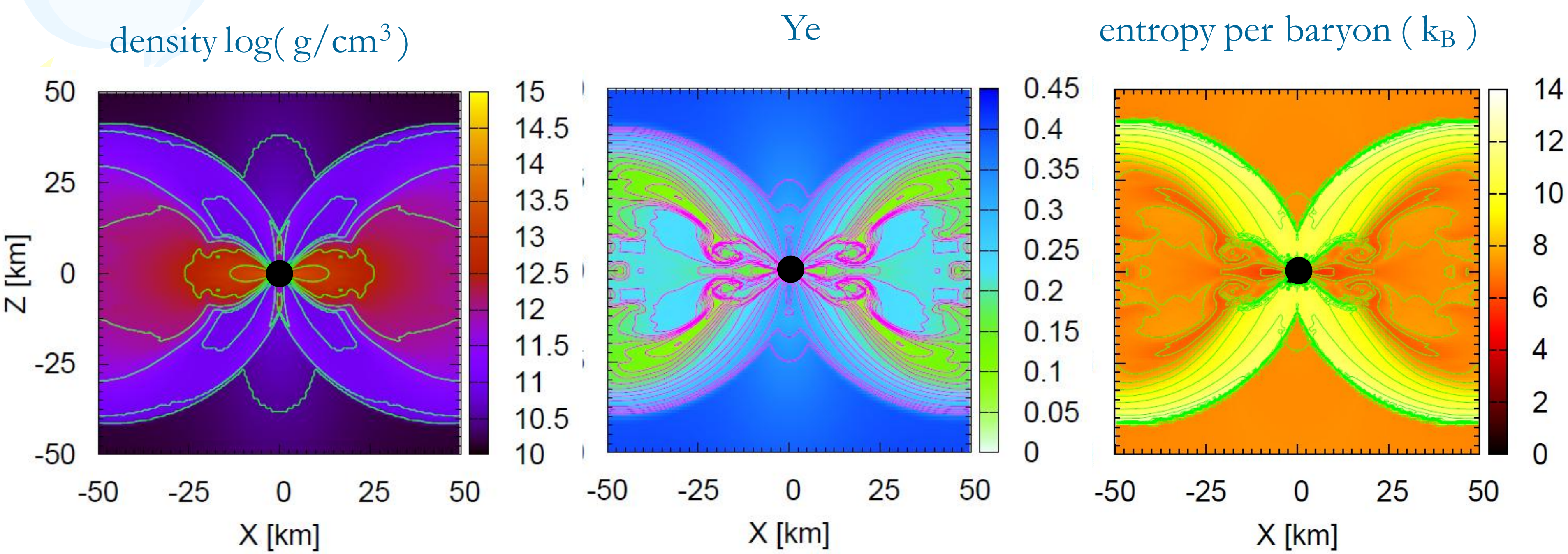


t_b=10ms
t_b=15ms
Z [km]
100
50
0
-50
-100
X
h_char 10 kpc
1e-21
1e-22
1e-23
1e-24
10
100
1000
frequency [Hz]
2000
1000
700
500
300
200
100
(-N^2)^1/2/2π [s^-1]

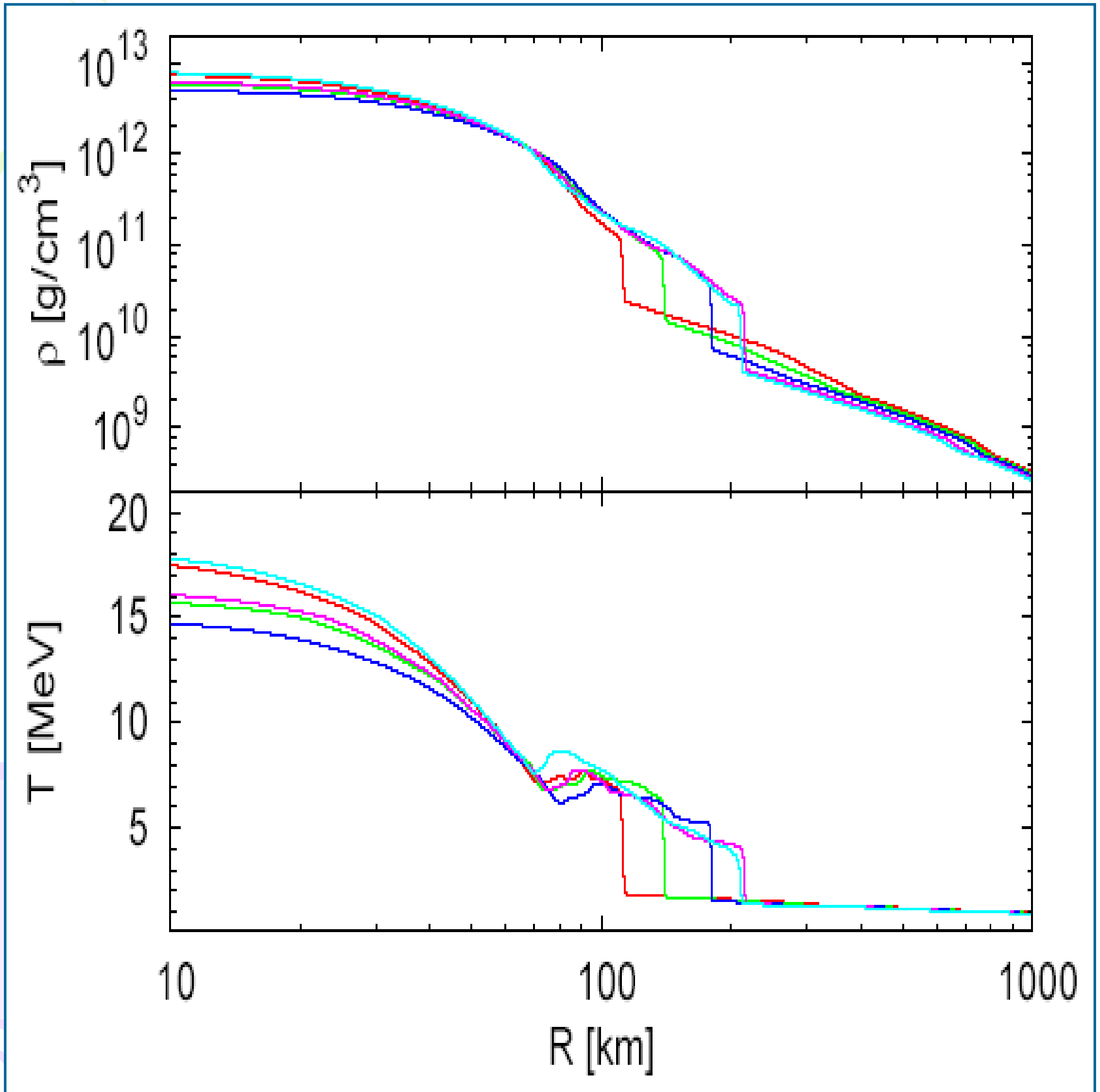
PopIII core collapse

Preliminary results

- BH formation with microphysics
  - black hole excision technique for hydrodynamics & microphysics
  - puncture evolution for geometry
- Initial condition
  - Simplified model (S = Ye = const core)
  - S=7kB, 8kB; Ye=0.5

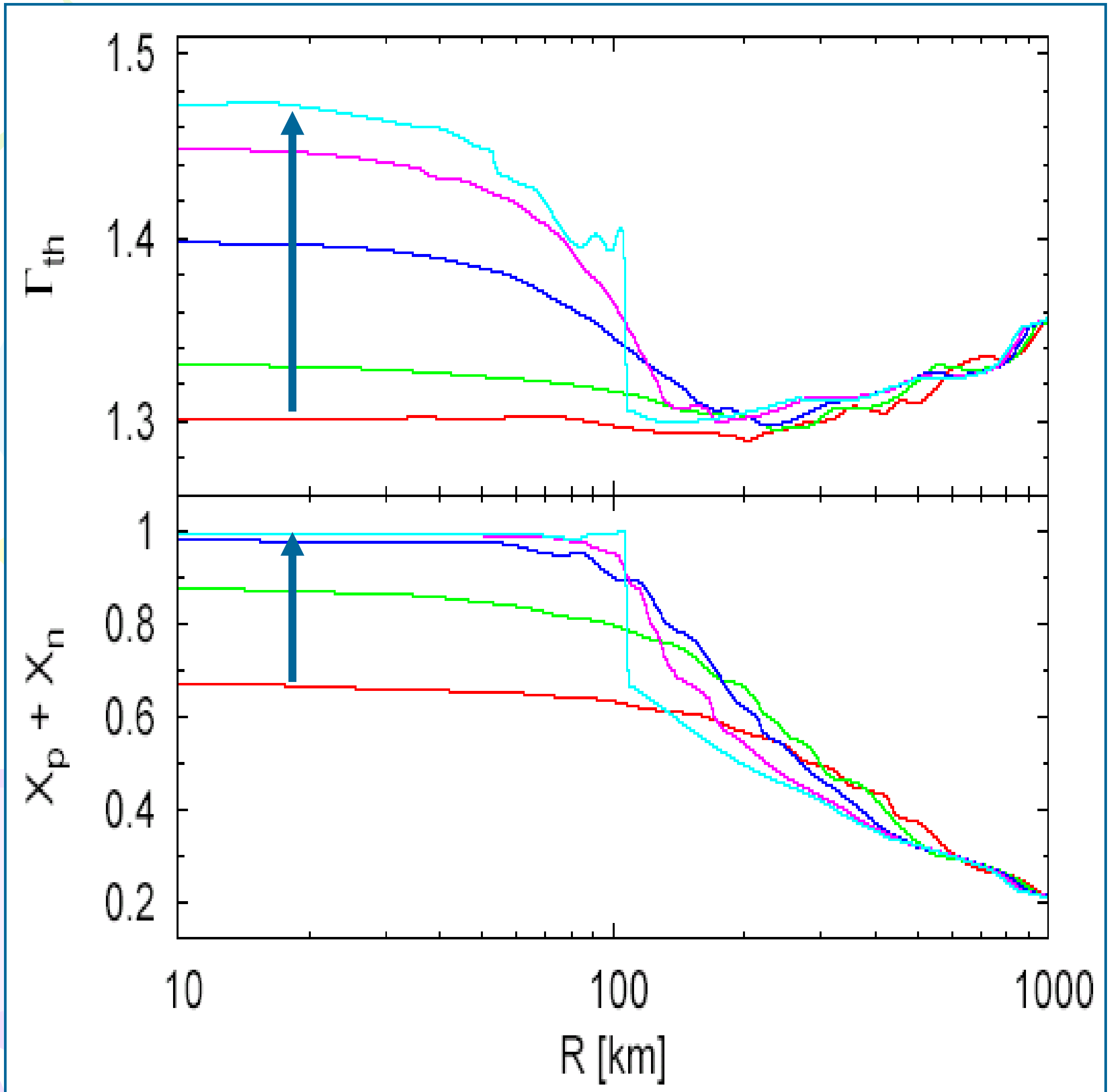


Weak bounce



- Do not directly collapse to BH
  - Weak bounce
- At bounce
  - $\rho \sim 10^{13}$ g/cm^3
    - subnuclear !
  - T ~ 18 MeV
  - Ye ~ 0.2

Bounce due to gas pressure



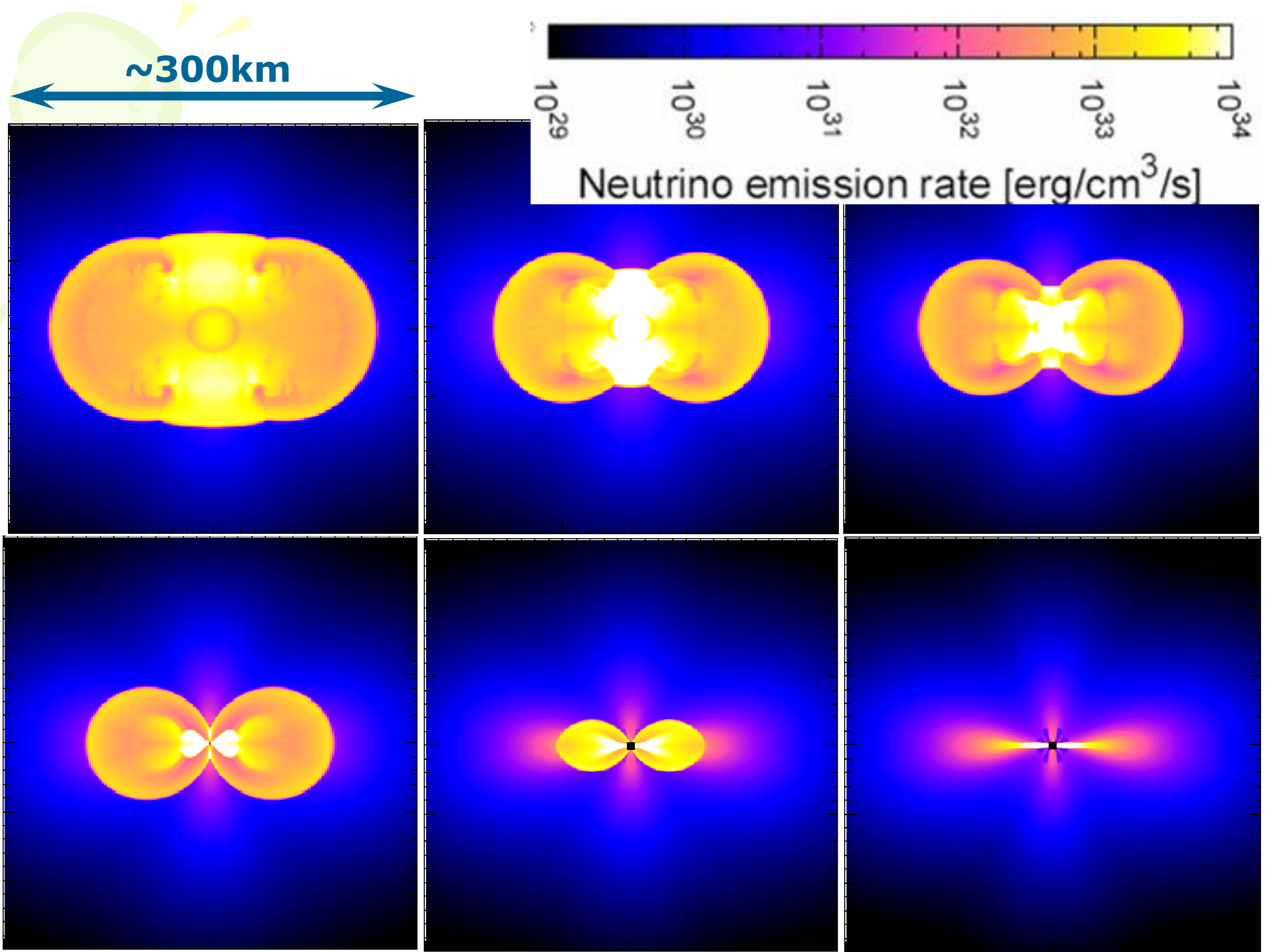
- He → 2p + 2n
  - Gas pressure (Γ=5/3) increase
- Indeed $\Gamma_{th} > 4/3$

$$P_{deg} \sim 1 \times 10^{32} \rho_{13}^{4/3}$$

$$P_{rad} \sim 1 \times 10^{31} T_{18\,\mathrm{MeV}}^{4}$$

$$P_{gas} \sim 2 \times 10^{32} \rho_{13} T_{18\,\mathrm{MeV}}$$

- Gas pressure dominates at $\rho \sim 10^{13}$ g/cm^3, T~18 MeV
- EOS becomes stiffer ⇒ weak bounce


~300km
Neutrino emission rate [erg/cm^3/s]
10^29
10^30
10^31
10^32
10^33
10^34

Why leakage scheme?

- Implicit solver is required in general (very hard in GR)

$$\dot{\rho}_* = 0 \quad [\rho_* = \alpha e^{6\phi} u^t \rho, \quad u^t = u^t(u_i, \rho, Y_e, T)]$$

$$\dot{e} = S_e(\rho, Y_e, T, f_\nu)$$

$$\dot{u}_i = S_u(\rho, Y_e, T, f_\nu)$$

$$\dot{Y}_e = S_{Ye}(\rho, Y_e, T, f_\nu)$$

$$\dot{f}_\nu = S_\nu(\rho, Y_e, T, f_\nu)$$

Evolved quantities

Argument quantities

$$u^a u_a = -1 \Rightarrow u^t$$

Nonlinear eq. with EOS table search

e depends on u^t

Nonlinear eq. with EOS table search

- Problem of phase space
  - Fluid rest ⇔ Tetrad ⇔ computing.
- Space time curvature

GR leakage scheme

- Basic equation : $\nabla_a (T^{\text{Total}})^a_b = 0$ ⇒ $\nabla_a T_b^{a\,(\text{fluid})} = -Q_b$, $\nabla_a T_b^{a\,(\nu)} = Q_b$
- Energy momentum tensor of neutrinos : $\nabla_a (T_b^{a\,(\nu,\text{trap})} + T_b^{a\,(\nu,\text{stream})}) = Q_b$
  - "Trapped neutrino" and "streaming neutrino" parts

$$T_{ab}^{(\nu)} = T_{ab}^{(\nu,\text{trap})} + T_{ab}^{(\nu,\text{stream})}$$

⇒

$$\nabla_a T_b^{a\,(\nu,\text{trap})} = Q_b - Q_b^{(\text{leak})}$$

$$\nabla_a T_b^{a\,(\nu,\text{stream})} = Q_b^{(\text{leak})}$$

Cooling-term like inclusion of neutrino emission violates constraint equations in full GR

neutrino emission in terms of energy momentum tensor is required

- Trapped neutrino part is included into Fluid part

$$T_{ab} = T_{ab}^{(\text{fluid})} + T_{ab}^{(\nu,\text{trap})}$$

$$T_{ab} = \rho h u_a u_b + P g_{ab}$$

$$T_{ab}^{(\nu,\text{stream})} = E n_a n_b + F_a n_b + F_b n_a + P_{ab}$$

$$P^{ab} = \frac{1}{3} E \gamma^{ab}$$

- The equation to be solved

$$\nabla_a T_b^a = -Q_b^{(\text{leak})}$$

$$\nabla_a T_b^{a\,(\nu,\text{stream})} = Q_b^{(\text{leak})}$$

Lepton conservation

- Trapped neutrinos $\Rightarrow Y_\nu \Rightarrow \mu_\nu \Rightarrow$ blocking

$$\frac{dY_e}{dt} = -\gamma_{\text{e-cap}} + \gamma_{\text{ep-cap}}$$

$$\frac{d(Y_{\nu_e})}{dt} = \gamma_{\text{e-cap}} + \gamma_{\text{pair}} + \gamma_{\text{plasmon}} - \gamma_{\nu_e\text{leak}}$$

$$\frac{d(Y_{\bar{\nu}_e})}{dt} = \gamma_{\text{ep-cap}} + \gamma_{\text{pair}} + \gamma_{\text{plasmon}} - \gamma_{\bar{\nu}_e\text{leak}}$$

$$\frac{d(Y_{\nu_x})}{dt} = \gamma_{\text{pair}} + \gamma_{\text{plasmon}} - \gamma_{\nu_x\text{leak}}$$

Leakage rate

Based upon Rosswog & Liebendoerfer (2004)

- **Neutrino Leakage rate**
  - "Cross sections" : $\sigma_i(E_\nu) = \tilde{\sigma}_i E_\nu^2$
  - "Opacities" : $\kappa(E_\nu) = \sum \kappa_i(E_\nu) = \tilde{\kappa} E_\nu^2$
  - "Optical depth" : $\tau(E_\nu) = \int \kappa \, ds = \tilde{\tau} E_\nu^2$
  - Diffusion time : $T_\nu^{\mathrm{diff}}(E_\nu) \equiv \dfrac{\Delta x(E_\nu)}{c} \tau(E_\nu) = \dfrac{\tilde{\tau}^2}{c\tilde{\kappa}} E_\nu^2$
  - **Neutrino energy and number leakage rate :**

$$\langle Q_\nu^{\mathrm{diff}} \rangle \equiv \int \frac{E_\nu \hat{n}(E_\nu)}{T_\nu^{\mathrm{diff}}(E_\nu)} dE_\nu \propto T^2 F_1(\eta_\nu)$$

$$\langle R_\nu^{\mathrm{diff}} \rangle \equiv \int \frac{\hat{n}(E_\nu)}{T_\nu^{\mathrm{diff}}(E_\nu)} dE_\nu \propto T \; F_0(\eta_\nu)$$

$$n_\nu = \int \hat{n}(E_\nu) dE_\nu$$

$$Q_b = \dot{Q}^{\mathrm{diff}} u_b$$

$$R^{\mathrm{diff}} \Rightarrow \gamma_{\mathrm{leak}}$$

Treatment of βequilibrium

- We solve all equations **explicitly**
  - Special treatment near β-equilibrium required
- Check if β-equilibrium is achieved or not
- In β-equilibrium, evolve the total lepton fraction
  - $$\frac{d(Y_l)}{dt} = -\gamma_{l\mathrm{leak}}$$
  - **EOS table with argument variables (ρ,Y_l, T) is used**
  - ⇒ One dimensional table search
    - Otherwise two dimensional search (Y_l, e) ⇒ (Y_e, T) required

Summary, Future work

- In full GR, approximate explicit treatment of neutrinos will be a good approach
- Our GR-leakage scheme provides reasonable results for SNe, and first results for BH formation in PopIII core collapse
- On going work
  - Binary neutron star merger (stable evolution of single NS is OK)
  - Approximate treatment of neutrino heating
  - Beyond-leakage, more sophisticated approximation

$$P = P_{\deg} + P_{\text{rad}} + P_{\text{gas}}$$

$$P_{\deg} \sim 1 \times 10^{32} \rho_{13}^{4/3}$$

$$\frac{\mu_e}{k_B T} \sim 51.6 \frac{(Y_e \rho)^{1/3}}{k_B T} \sim 5$$

$$P_{\text{rad}} \sim 1 \times 10^{31} T_{18\,\text{MeV}}^{4}$$

$$P_{\text{gas}} \sim 5 \times 10^{31} \rho_{13} T_{18\,\text{MeV}} \quad (\text{He case})$$

$$\sim 2 \times 10^{32} \rho_{13} T_{18\,\text{MeV}} \quad (p, n \text{ case})$$

- Gas pressure dominates at $\rho \sim 10^{13}$ g/cm^3, T~18 MeV
- EOS becomes stiffer ⇒ weak bounce