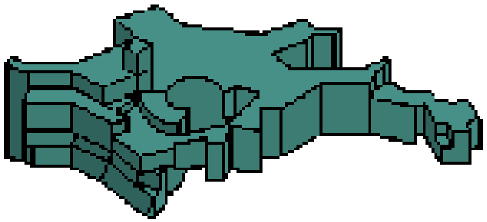
General relativistic hydrodynamics beyond bouncing polytropes



Pablo Cerdá-Durán
Max Planck Institut für Astrophysik
Garching bei München, Germany

MICRA2009

Copenhagen - August 24-28 2009

Punch line(s)

To model rotating compact objects we need MHD and relativity

To model MHD and relativity we need new physics and new methods

Outline

- Motivation
- GR (magneto)hydrodynamics
- New problems
- Conclusions

Motivation

Astrophysical scenarios

- Collapse to black hole
- Core collapse supernovae
- Proto-neutron stars
  - Pulsar/Magnetar formation
- Isolated neutron stars
  - Instabilities
  - Asterosismology
- Binary neutron stars

Motivation

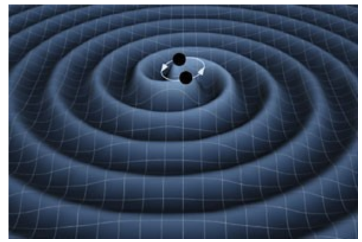
Why relativistic?

- Shock propagation ~ 0.1c
- Low density regions
  - Relativistic jets
  - Newtonian: Alfvén speed > c

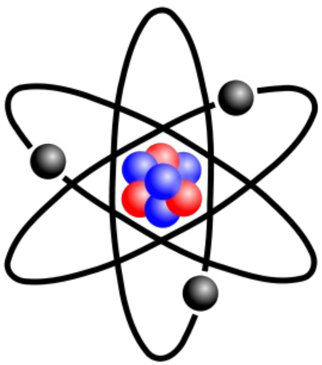
Why general relativistic?

- Compact objects (BH, NS) = strong gravity
- Black holes
- Gravitational waves

Status



Gravity →



Microphysics ↓

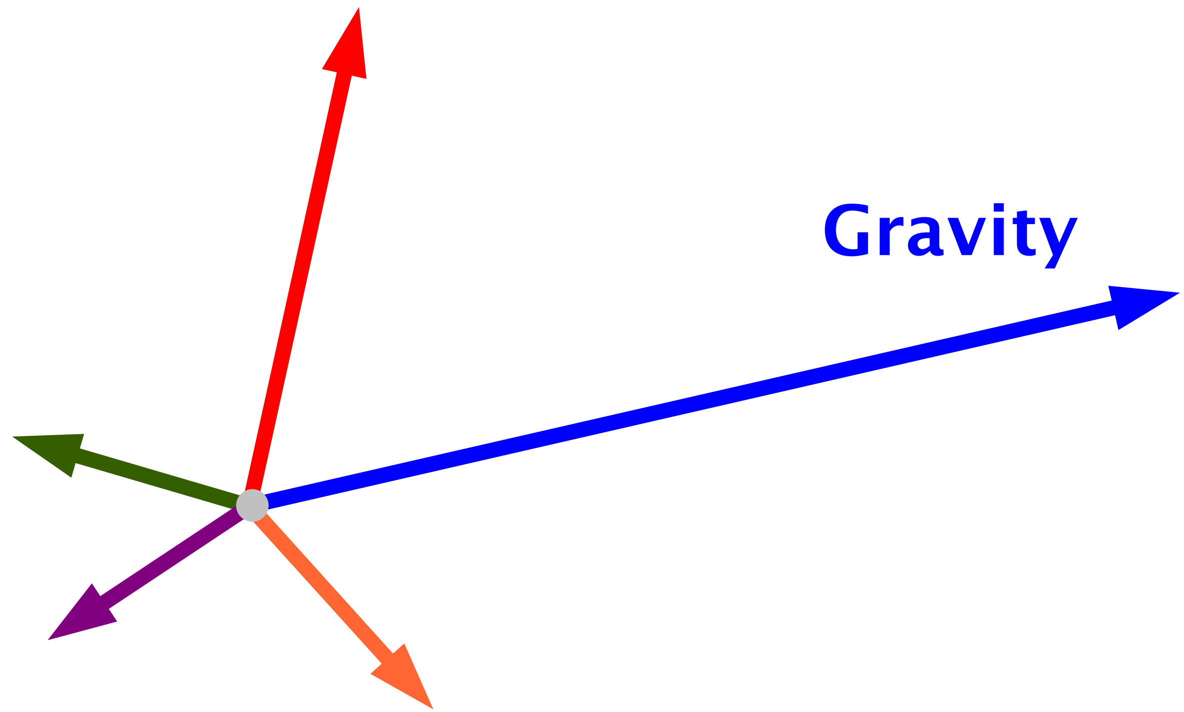
	Newtonian	Modified Newtonian (TOV-like potentials)	Approximately relativistic (PPN, CFC, CFC+)	Fully relativistic (ADM, BSSN, FCF)
Polytropes Ideal gas				
Tabulated EOS				
Simple neutrino treatment			**Ongoing work**	**Ongoing work**
Boltzmann neutrino transport	**Ongoing work**	**Ongoing work**	**Ongoing work**	

(adapted from the table by Harald Dimmelmeier)

Status

Microphysics

Gravity



Magnetic fields

Ideal MHD

Resistive MHD

Beyond MHD (Hall effect, ambipolar diffusion)

Small scales

Magneto-rotational instability (MRI)

Turbulence and dynamo

Multi-D performance

1D

2D

3D

Framework

3+1 decomposition

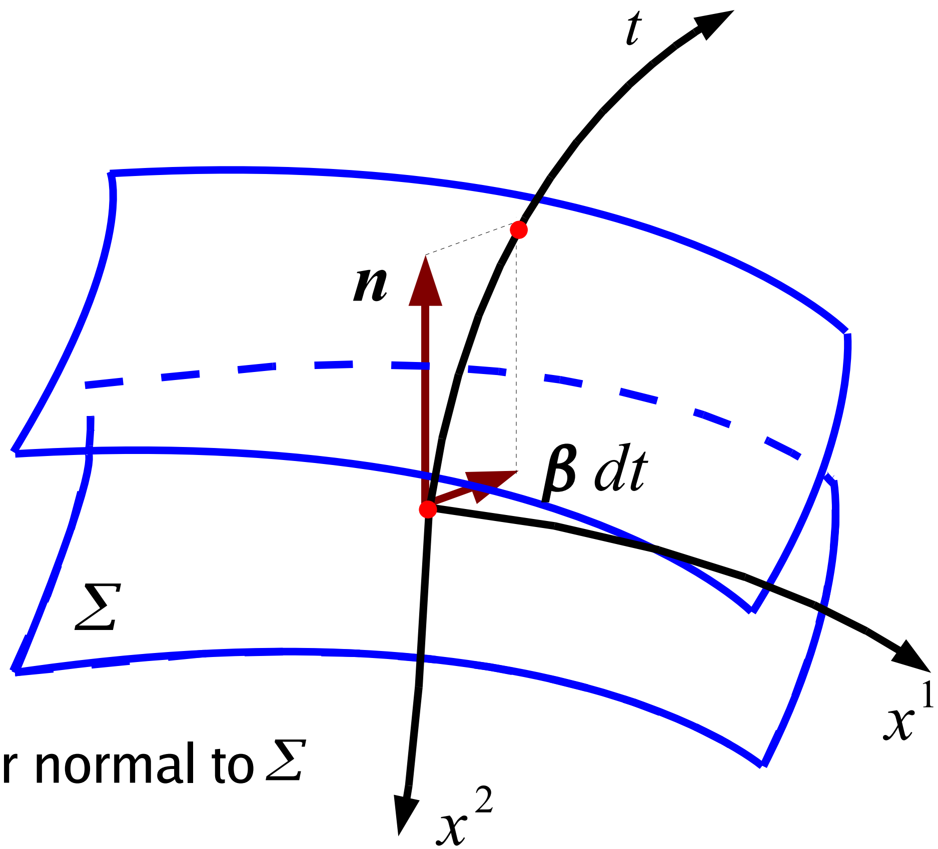
$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = (\beta^i \beta_i - \alpha^2) dt^2 + 2\beta_i dx^i dt + \gamma_{ij} dx^i dx^j$$

α : lapse

ϕ : conformal factor

β^i : shift vector

γ_{ij} : 3-metric



$n^\mu n_\mu = -1$: timelike 4-vector normal to Σ

$\perp^\mu_\nu \equiv g^\mu_\nu + n^\mu n_\nu$: proyector onto Σ

Energy-momentum tensor

Perfect fluid (one component)

$$J^{\mu}_{fluid} = \rho u^{\mu}$$

$$T^{\mu\nu}_{fluid} = \rho h u^{\mu} u^{\nu} + P g^{\mu\nu}$$

Electro-magnetic field (resistive MHD)

$$J^{\mu}_{q} = \rho_{q} u^{\mu} + \sigma F^{\mu\nu} u_{\nu}$$

$$T^{\mu\nu}_{EM} = F^{\mu\lambda} F^{\mu}_{\lambda} - \frac{1}{4} g^{\mu\nu} F^{\lambda\delta} F_{\lambda\delta}$$

Energy-momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_{fluid} + T^{\mu\nu}_{EM} + \ldots$$

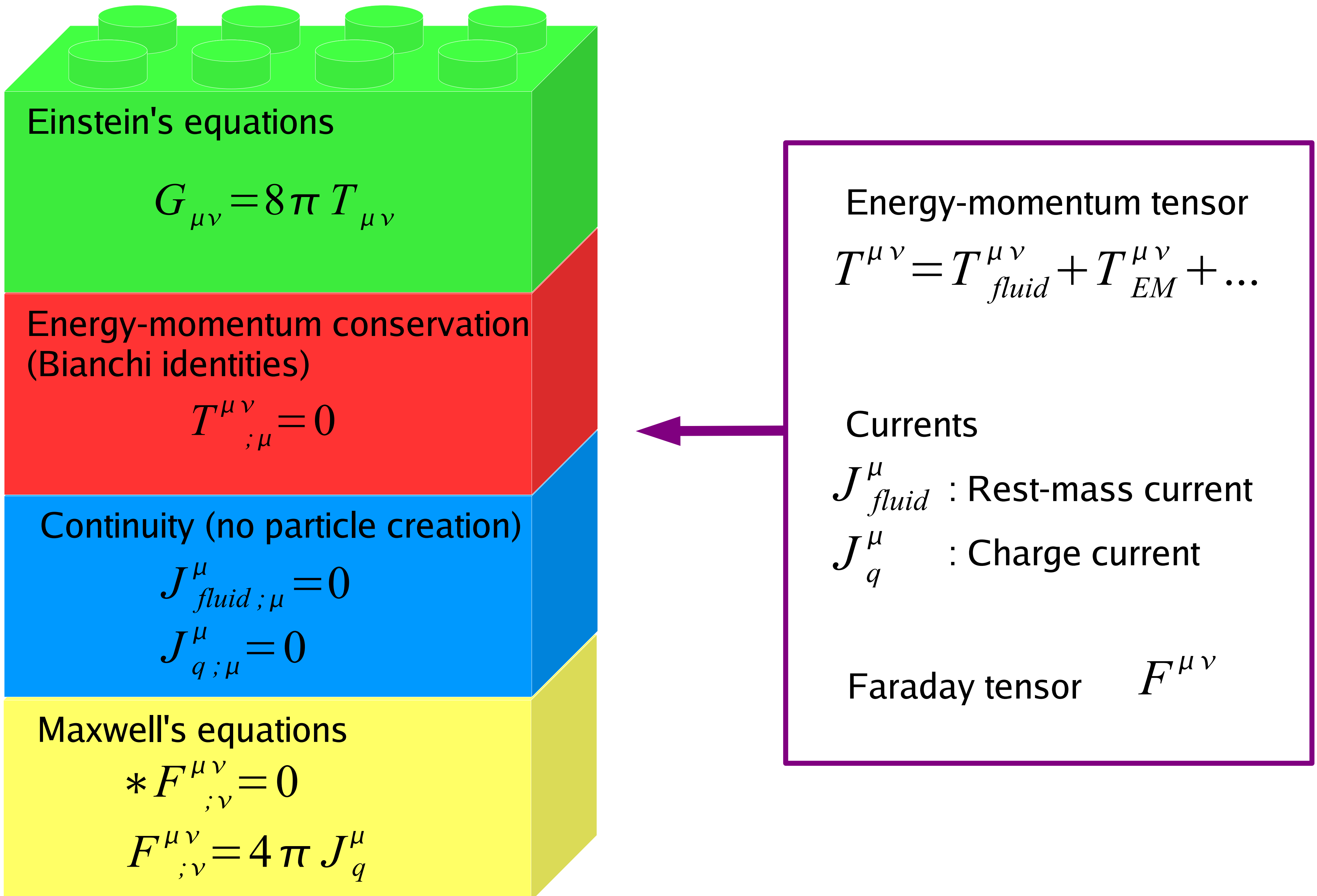
Currents

J^{μ}_{fluid} : Rest-mass current

J^{μ}_{q} : Charge current

Faraday tensor $F^{\mu\nu}$

Equations



Einstein's equations
$G_{\mu\nu} = 8\pi T_{\mu\nu}$
Energy-momentum conservation
(Bianchi identities)
$T^{\mu\nu}{}_{;\mu} = 0$
Continuity (no particle creation)
$J^{\mu}_{fluid\,;\mu} = 0$
$J^{\mu}_{q\,;\mu} = 0$
Maxwell's equations
$* F^{\mu\nu}{}_{;\nu} = 0$
$F^{\mu\nu}{}_{;\nu} = 4\pi J^{\mu}_{q}$
Energy-momentum tensor
$T^{\mu\nu} = T^{\mu\nu}_{fluid} + T^{\mu\nu}_{EM} + \ldots$
Currents
J^{μ}_{fluid} : Rest-mass current
J^{μ}_{q} : Charge current
Faraday tensor $F^{\mu\nu}$

Equations – Ideal MHD

Einstein's equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

Energy-momentum conservation (Bianchi identities)

$$T^{\mu\nu}{}_{;\mu} = 0$$

Continuity (no particle creation)

$$J^{\mu}{}_{;\mu} = 0$$

Maxwell's equations (ideal MHD)

$$*F^{\mu\nu}{}_{;\nu} = 0$$

Energy-momentum tensor

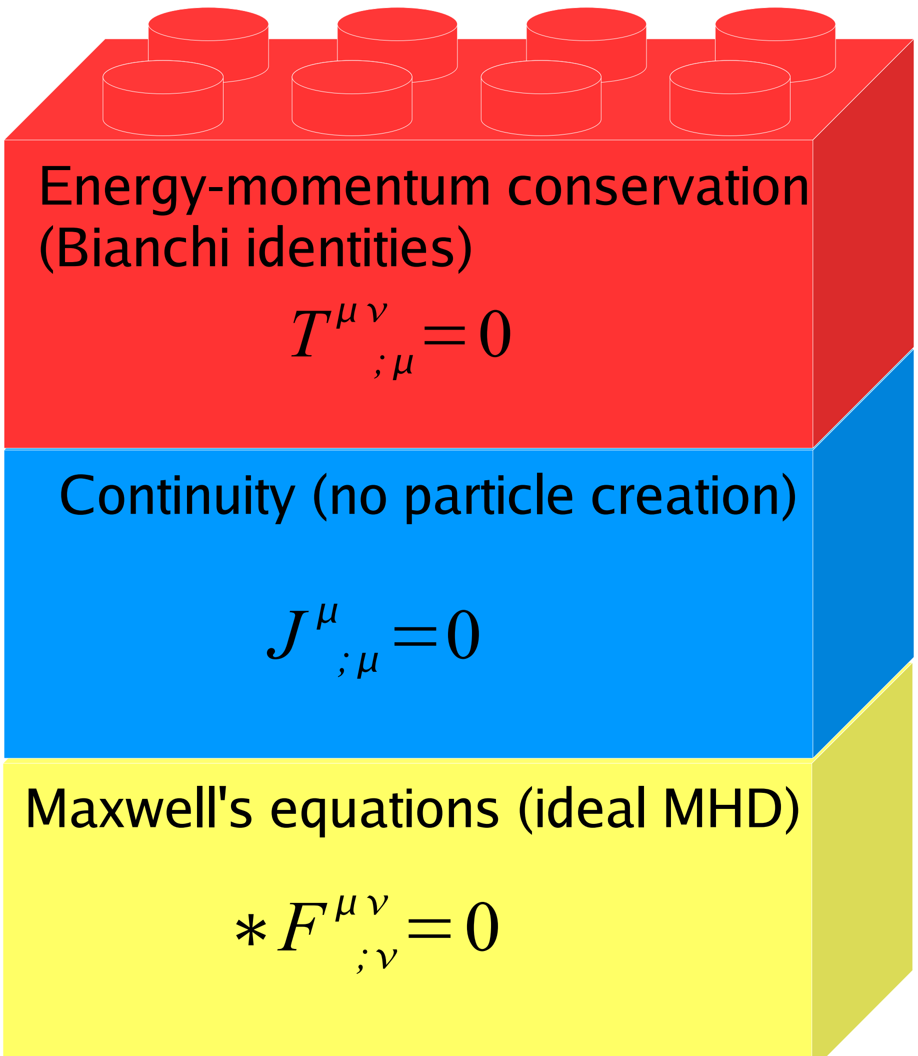
$$T^{\mu\nu} = T^{\mu\nu}_{fluid} + T^{\mu\nu}_{EM} + \ldots$$

Currents

J^{μ} : Rest-mass current

Faraday tensor $F^{\mu\nu}$

Equations – Ideal MHD



Hyperbolic system of conservation (balance) laws

$$\frac{\partial \hat{\boldsymbol{U}}}{\partial t}+\nabla_i \hat{\boldsymbol{F}}^i(\hat{\boldsymbol{U}})=\hat{\boldsymbol{S}}(\hat{\boldsymbol{U}})$$

$$\nabla_k \hat{B}^k=0$$

Conserved variables

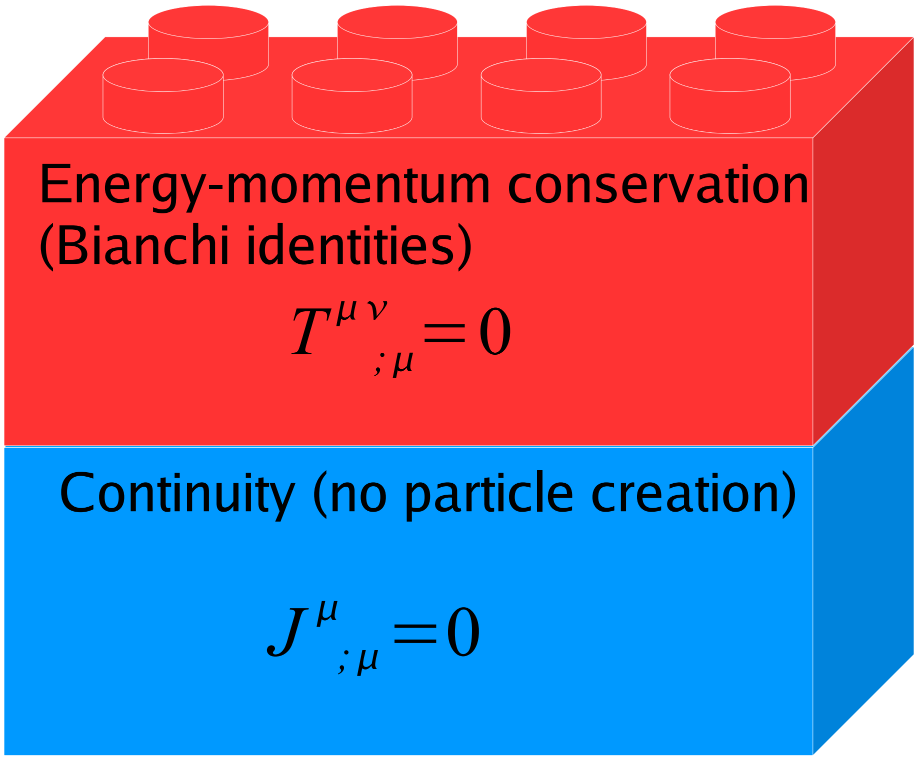
$$\hat{\boldsymbol{U}}\equiv\sqrt{\gamma}\,(D,S_i,\tau,B^k)$$
$$=(\hat{D},\hat{S}_i,\hat{\tau},\hat{B}^k)$$

$D\equiv -n_\mu J^\mu$: relativistic density

$S_i\equiv -\perp^{\mu}_{i} n^\nu T_{\mu\nu}$: momentum

$\tau\equiv n^\mu n^\nu T_{\mu\nu}-D$: energy (without density)

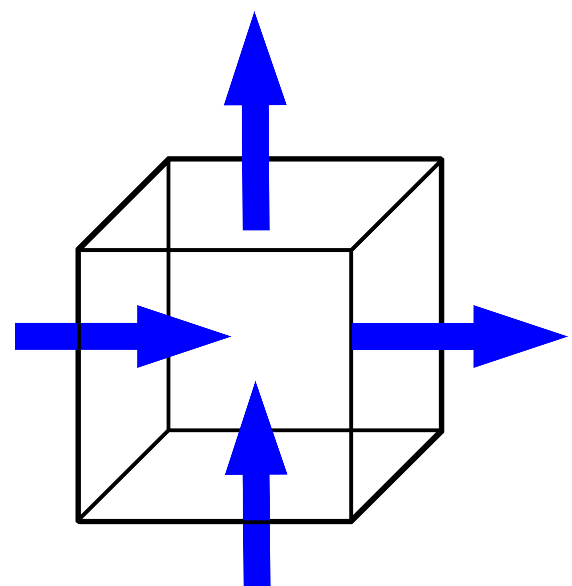
Equations – Ideal MHD



Conservation of
Number of particles
Momentum
Energy

$$\partial_t \int dV\, \hat{\boldsymbol{U}}_{fluid} + \int dA \cdot \hat{\boldsymbol{F}} = \int dV\, \hat{\boldsymbol{S}}$$

$$\hat{\boldsymbol{U}}_{fluid} \equiv (\hat{D}, \hat{S}_i, \hat{\tau})$$

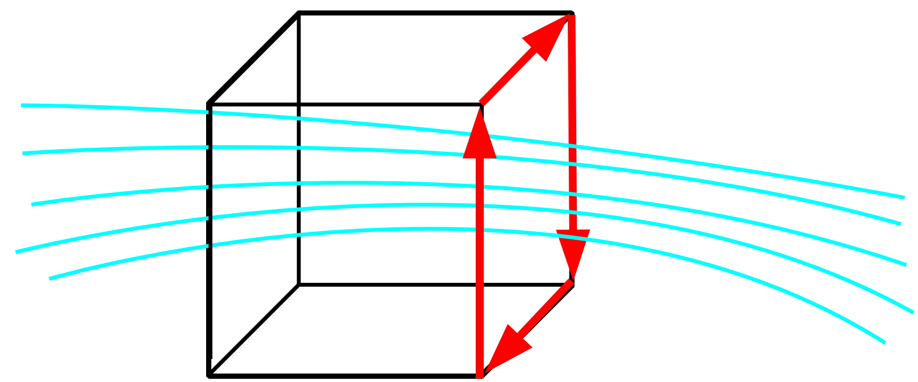


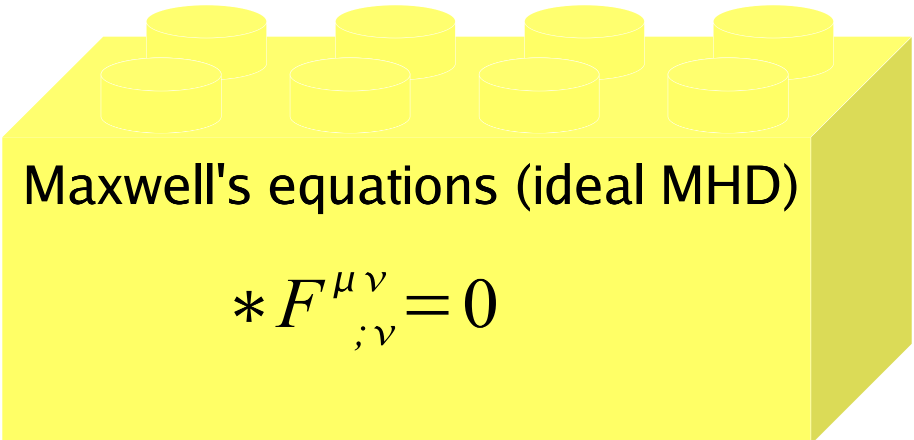
Equations – Ideal MHD

Conservation of magnetic flux

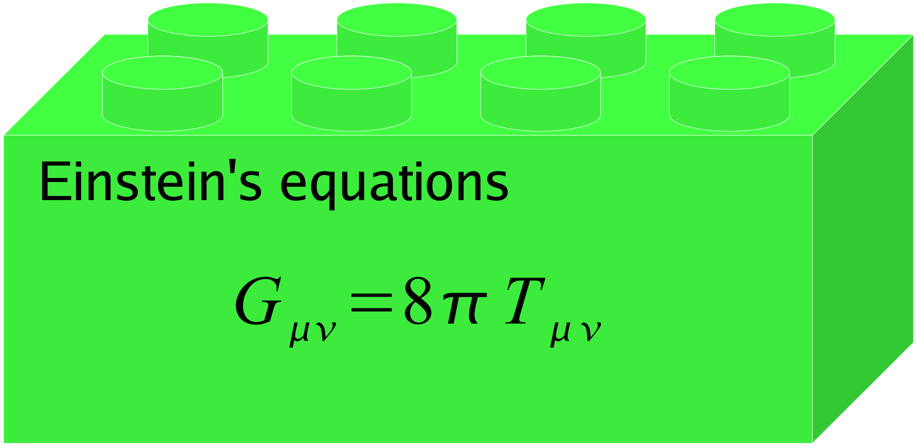
Induction equation → $$\partial_t \int dA_k \hat{B}^k + \oint dl \cdot \hat{E} = 0$$

Solenoidal condition → $$\oint dA_k \hat{B}^k = 0$$





Equations - space-time



$$\gamma_{ij} = \phi^4 f_{ij} + h_{ij}$$
$$\phi^4 \equiv Tr(\gamma_{ij})$$

10 variables (α , β^i , ϕ , h_{ij})
- 4 gauge conditions

6 degrees of freedom

Newtonian limit

$$\alpha^2 \simeq 1 - 2U$$
$$\phi^4 \simeq 1$$
$$\beta_i, \quad h_{ij} \simeq 0$$

1 degree of freedom

U : Newtonian potential

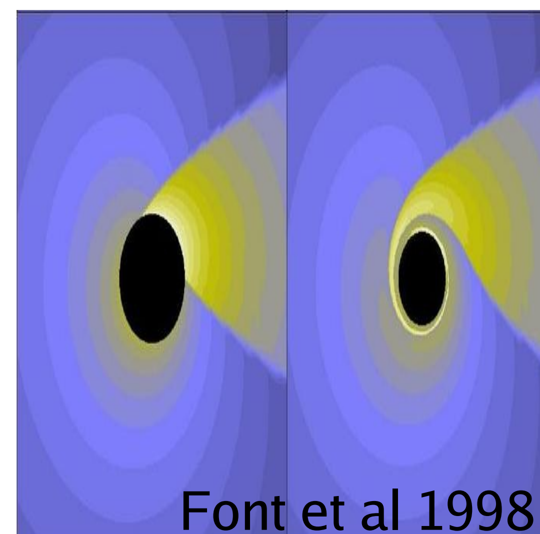
Vacuum-Minkowski limit

$$\Box h^{ij} = 0$$
$$\phi, \quad \alpha \simeq 1$$
$$\beta^i \simeq 0$$

2 degree of freedom

h^+ , $h^\times$: Gravitational waves

Frame-dragging



β_i

CoCoNuT code

General relativistic ideal MHD code

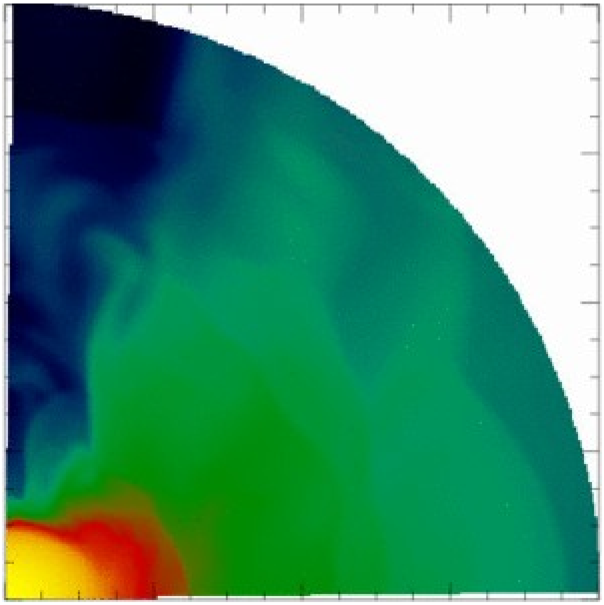
(Dimmelmeier et al. 2002, Cerda-Duran et al 2008)

Spherical coordinates: 2D (MHD) and 3D (only HD)

Godunov-type schemes + CT scheme

General relativity: dynamic space-time in the CFC approx. (isenberg 1979, Wilson 1989) with **spectral methods** (**LORENE:** www.lorene.obspm.fr) (Dimmelmeier et al. 2004)

Microphysics: finite temperature EOS (SHEN, LS) and deleptonization scheme (Liebendörfer 2005)
Applications: core collapse supernovae, isolated neutron star evolution and BH formation.



Some new problems

Microphysics/recovery

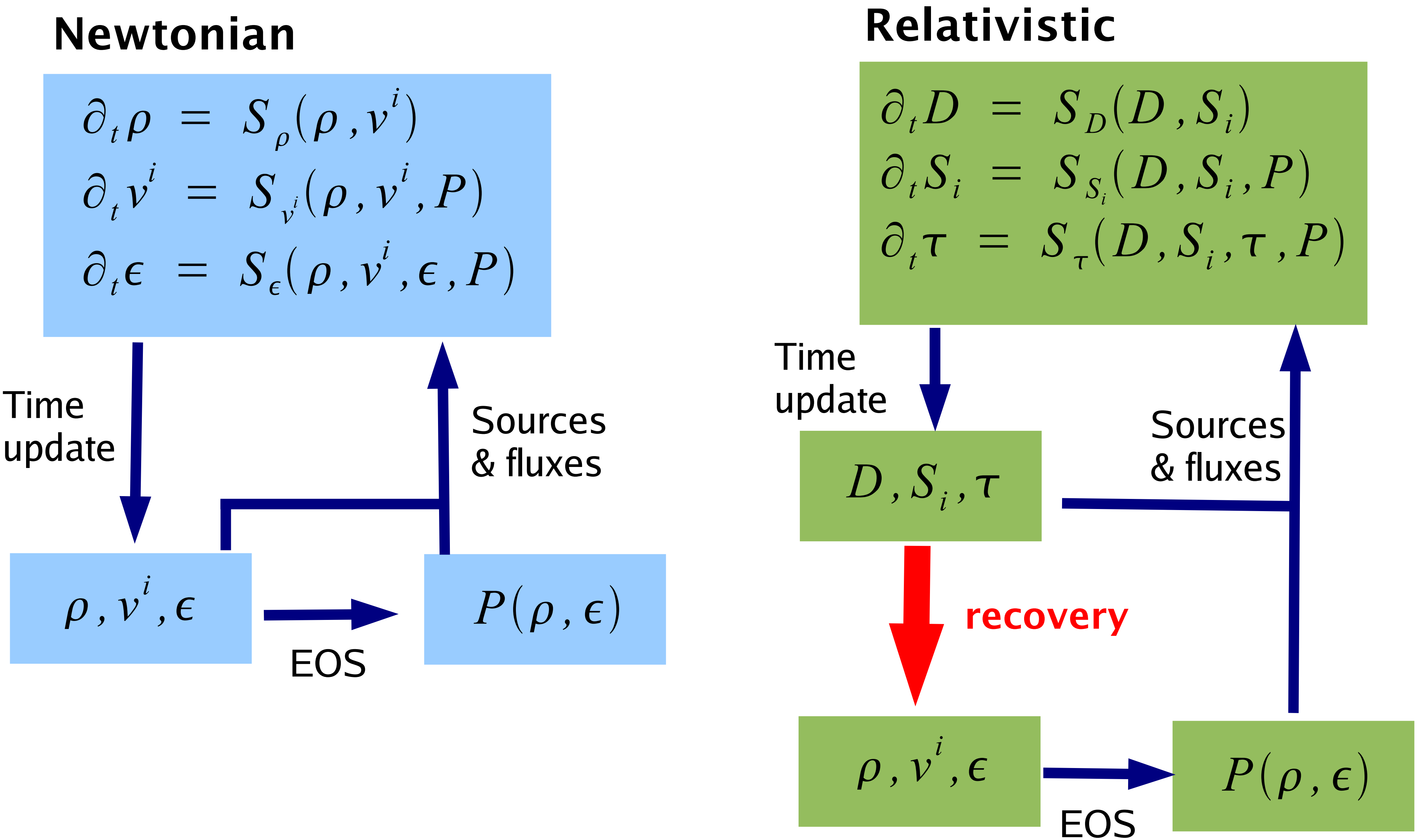
Multi-scale problem

3D-efficiency

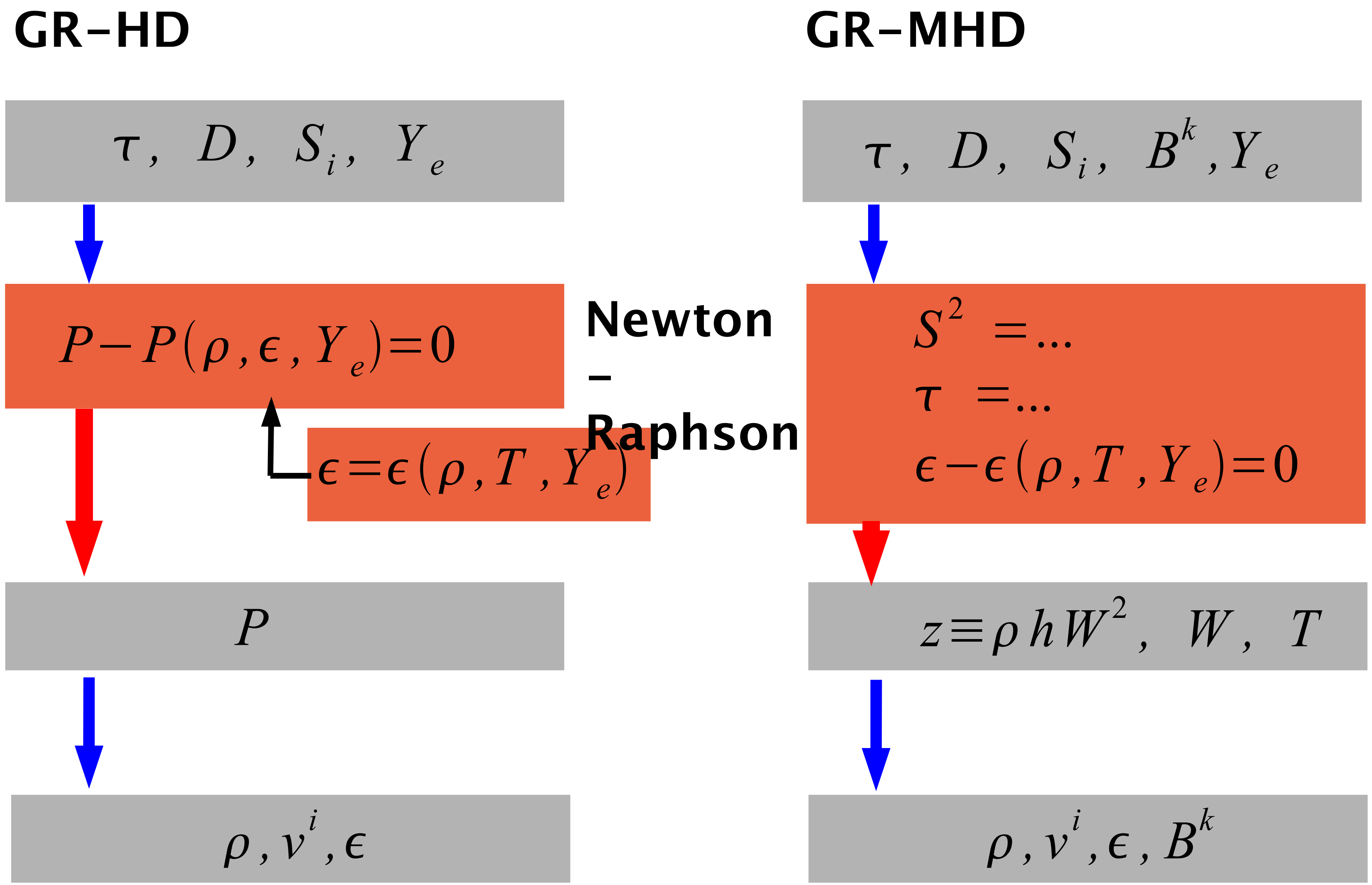
“realistic” MHD

Microphysics/recovery

Microphysics/recovery



Microphysics/recovery



Microphysics/recovery

Efficiency: number of iterations?

- Use the previous time-step value

Robustness: always converge?

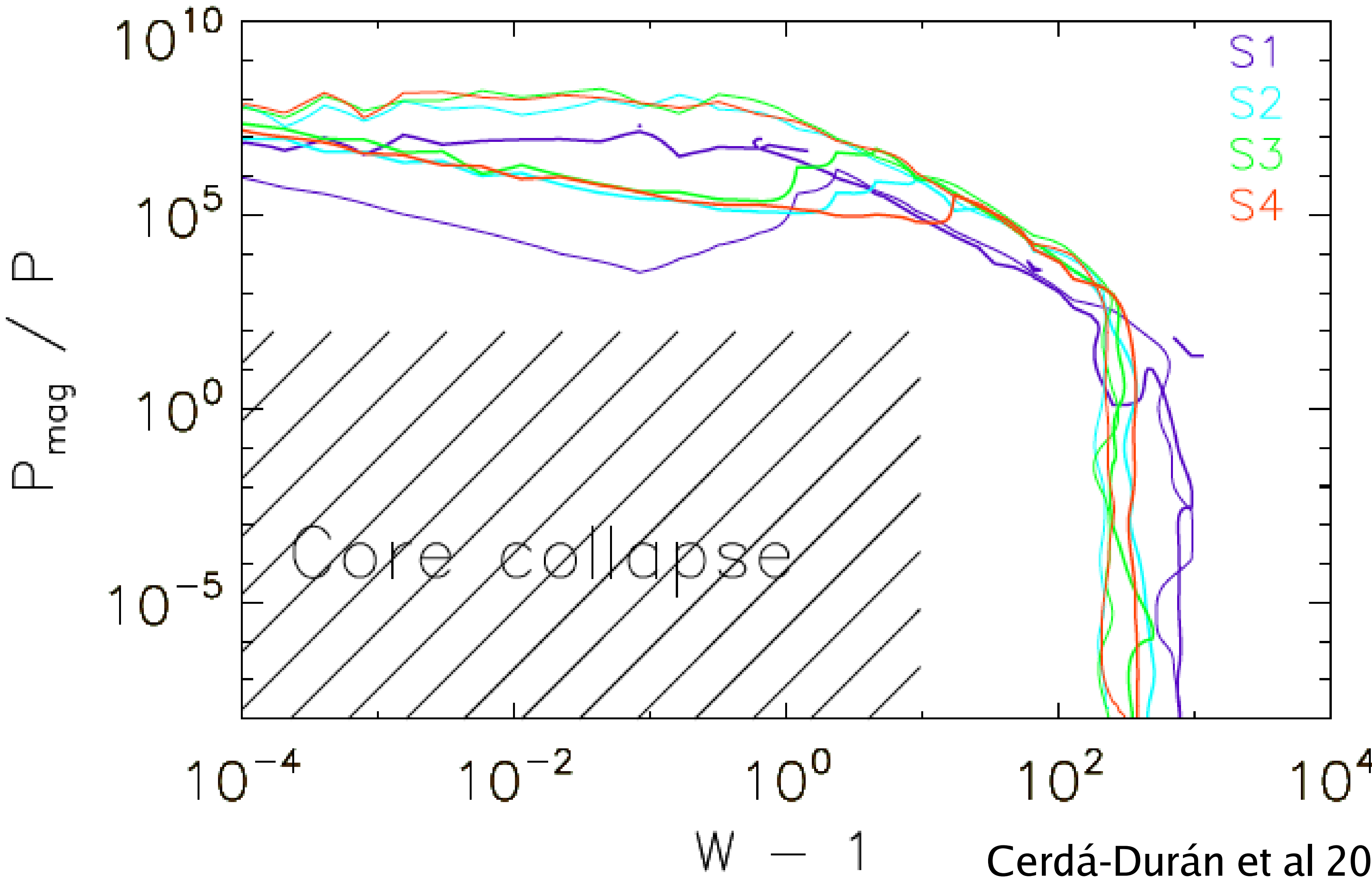
- “Safe” recovery values (next slide)

Parallel computation: can be a bottle neck!

- Recovery: ~10-20% CPU time
- Bad balance: Amdahl's law limits number of CPUs

Microphysics/recovery

"Safe" recovery values (SHEN EOS, collapse and bounce conditions)



Cerdá-Durán et al 2008

Multi-scale problem

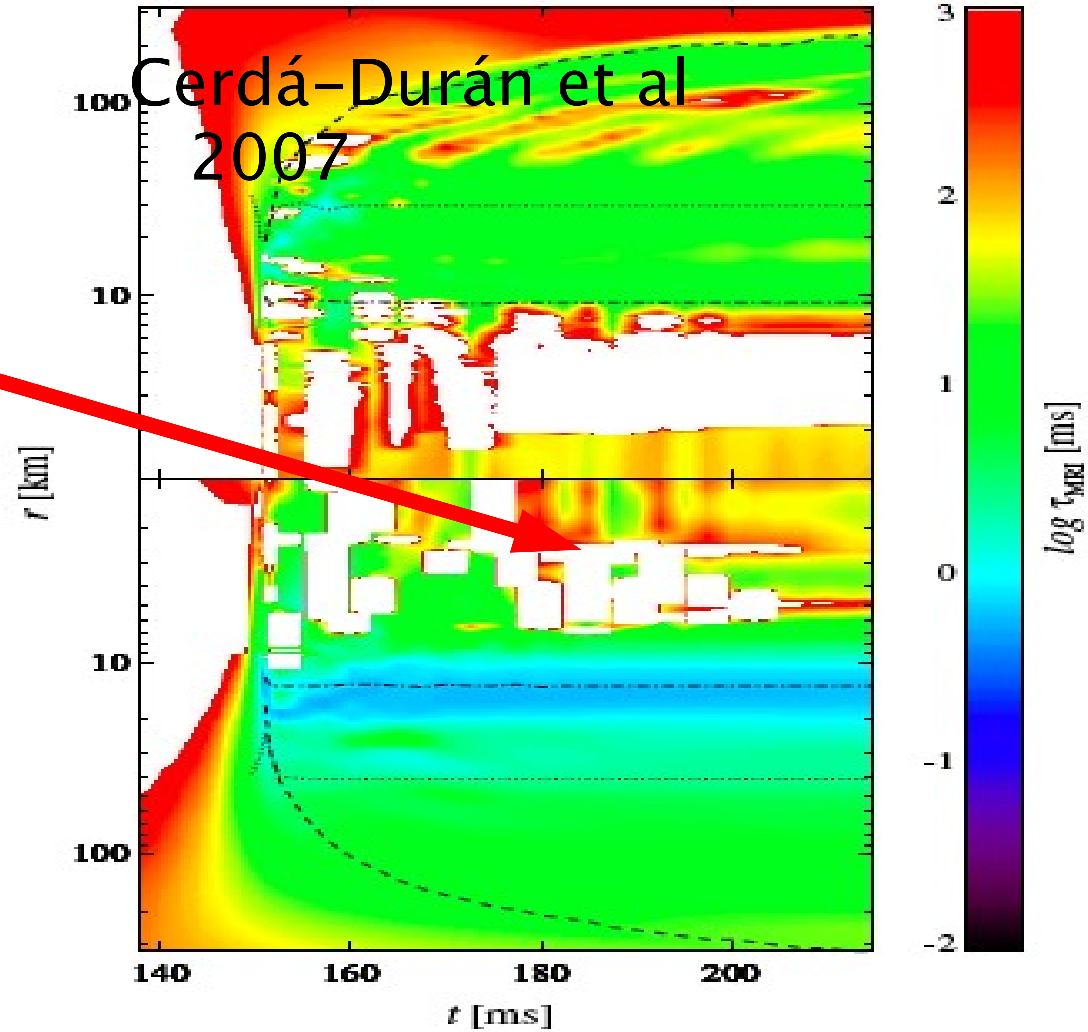
Multi-scale problem - MRI (small scales)

s20 model (Woosley et al 2002)+ rotation (T/|W|=0.05)

Unstable to hydromagnetic instabilities

(MRI, magneto-convection)

Timescale ~ 1 ms!



Cerdá-Durán et al 2007

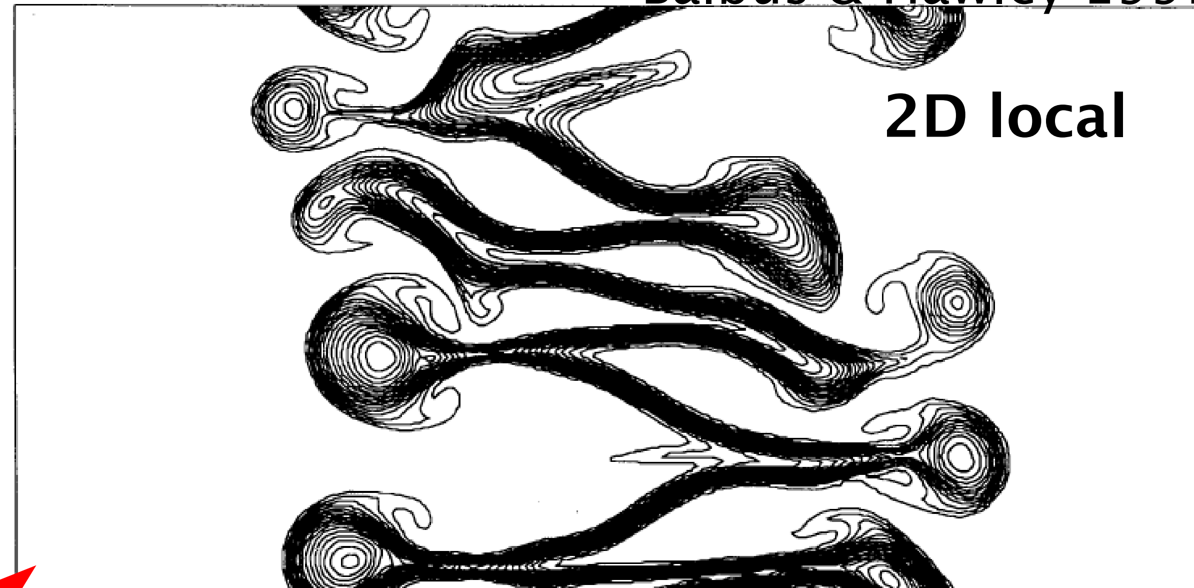
radius (km)

140 160 180 200

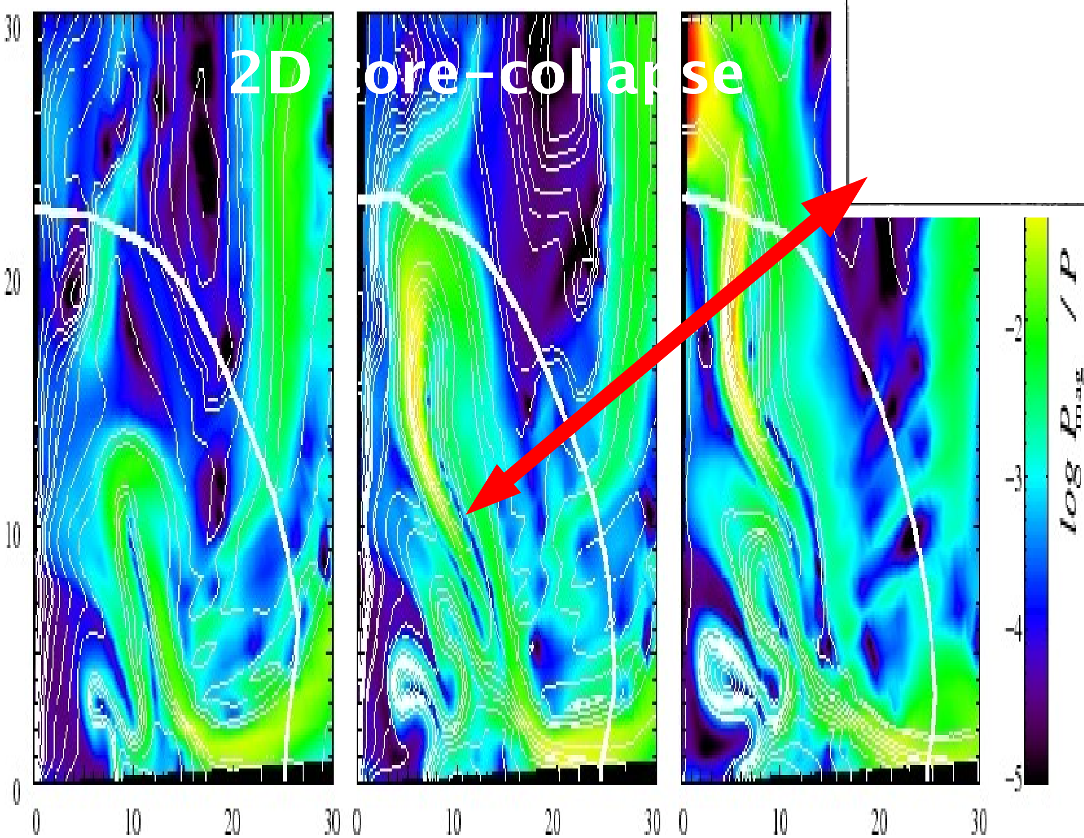
time (ms)

Multi-scale problem - MRI (small scales)

Balbus & Hawley 1991

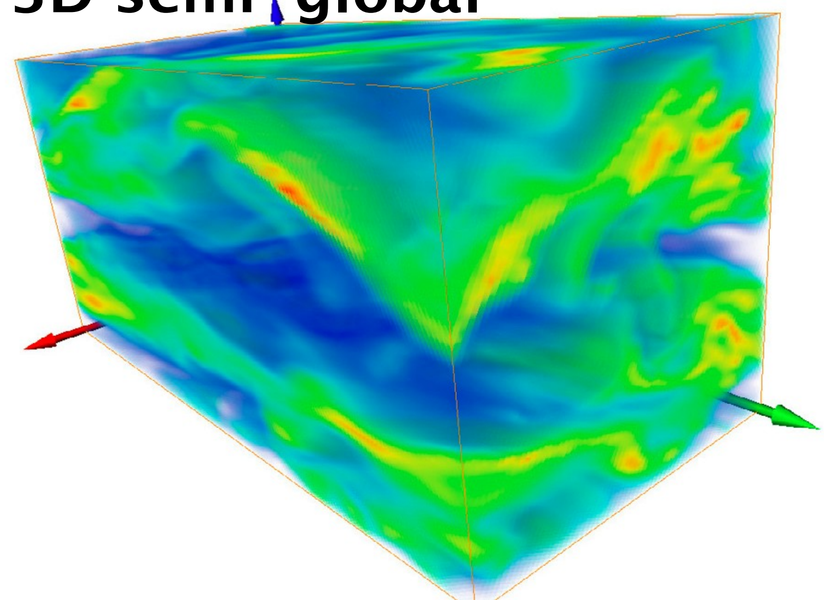


3D problem!



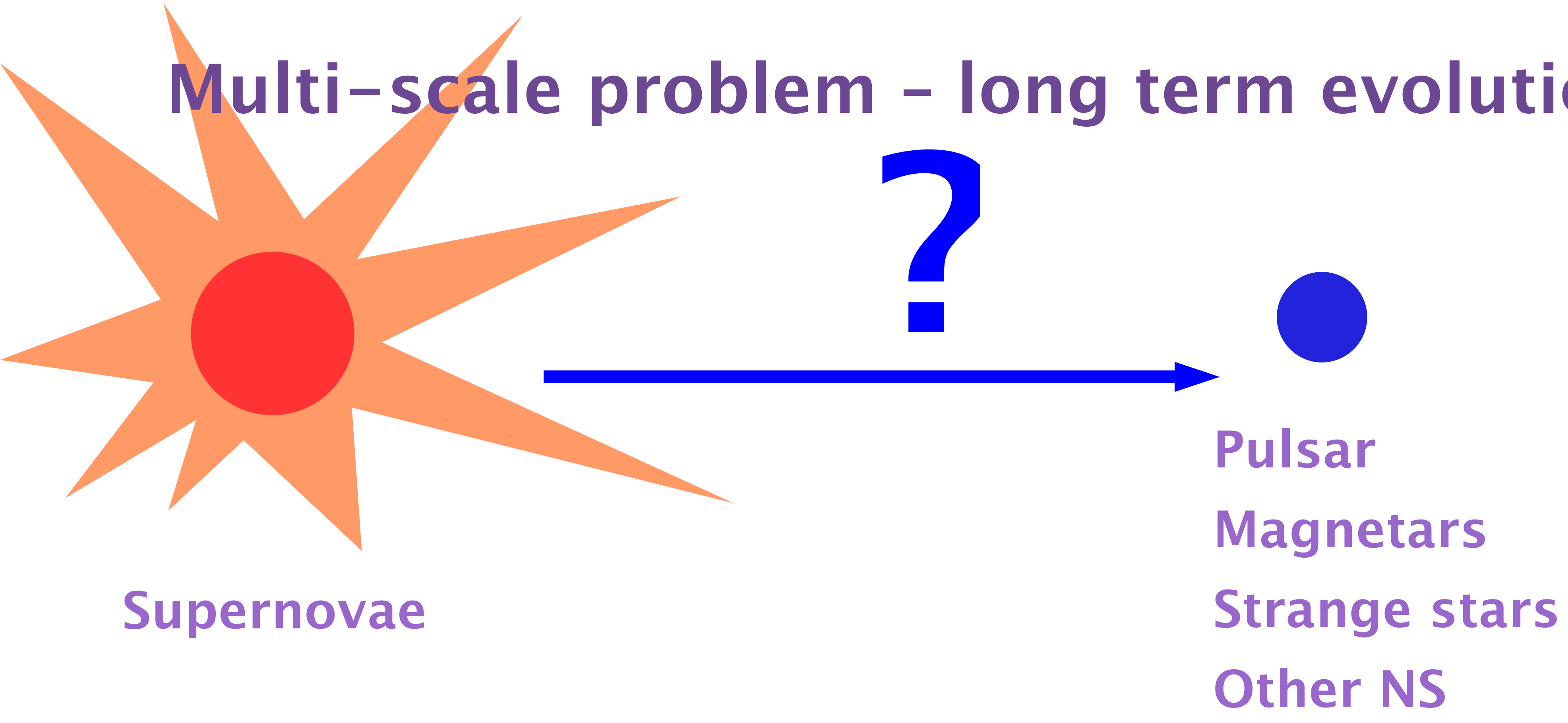
Cerdá-Durán et al 2008

3D semi-global



Obergaulinger et al 2008

Multi-scale problem – long term evolution

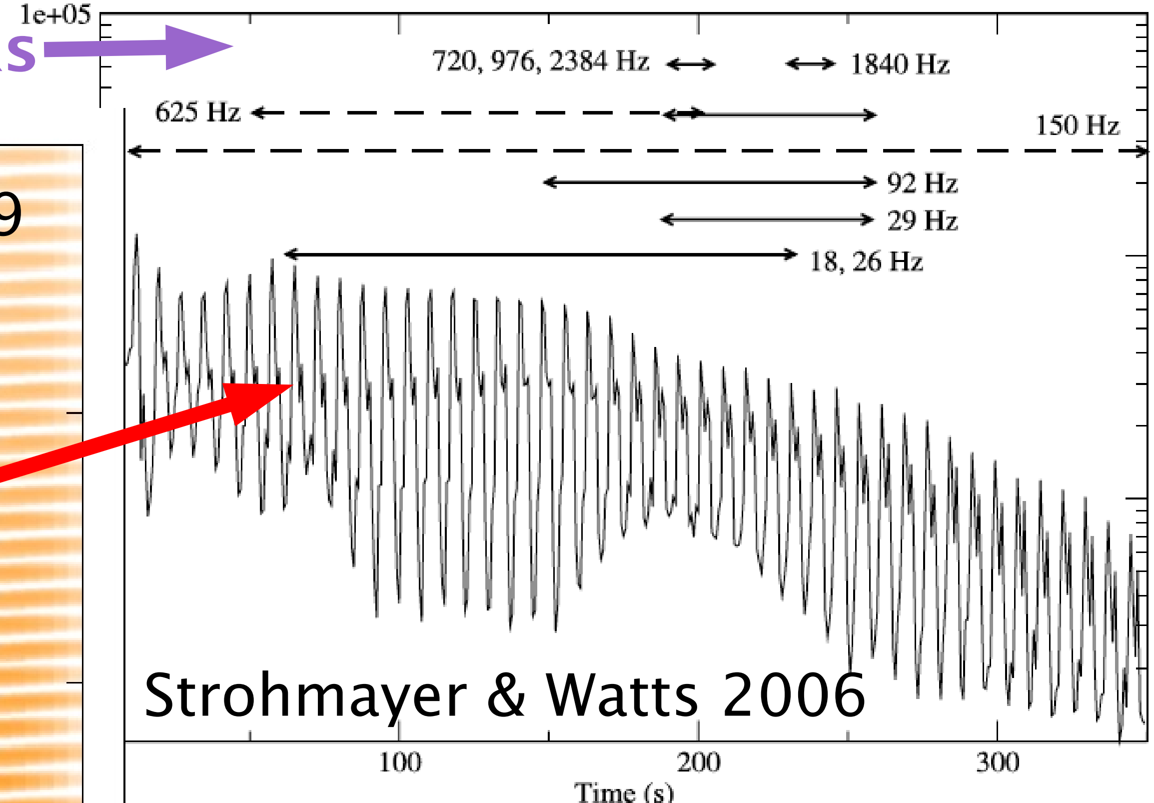


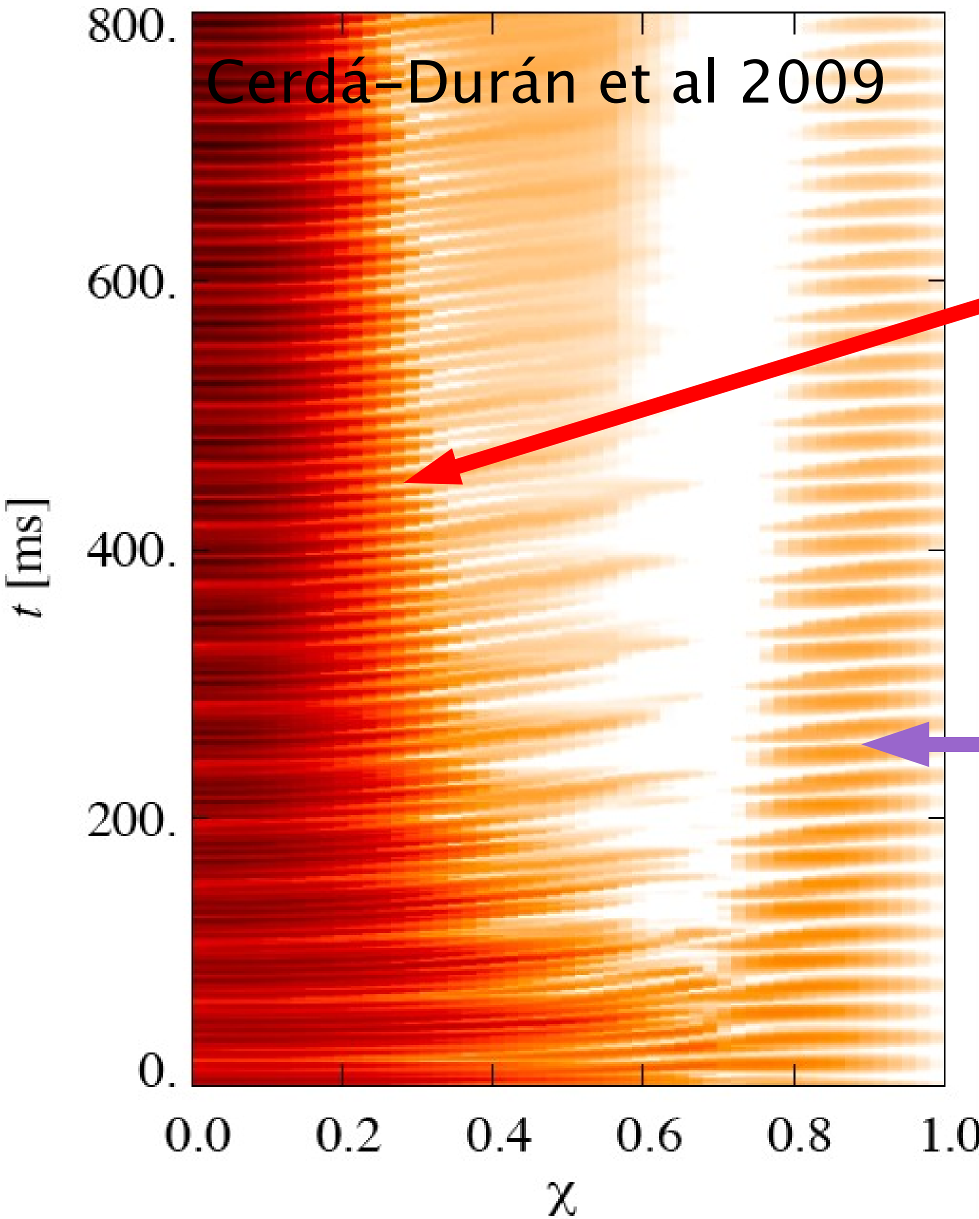
Problems: Dynamo, MRI, convection, cooling …

Methods: Implicit?, anelastic? …

Multi-scale problem - long term evolution

QPOs in giant flares of SGRs





Alfvén torsional oscillations:
Relativistic anelastic approx.

(Bonazzolla et al 2007)

~20 Alfvén crossing-time

~800 sound crossing-time

3D efficiency

Efficient 3D

Cartesian coordinates

+ Easy to implement

+ Best for scenarios without any symmetry (e.g. binary)

– Not possible to recover spherical symmetry and difficult for axisymmetry (Cartoon method)

– Difficult to treat large domains (AMR, FMR)

Spherical polar coordinates

– Geometric terms everywhere!

+ Well adapted for quasi-spherical objects (isolated tars)

+ Spherical and axisymmetry easily imposed

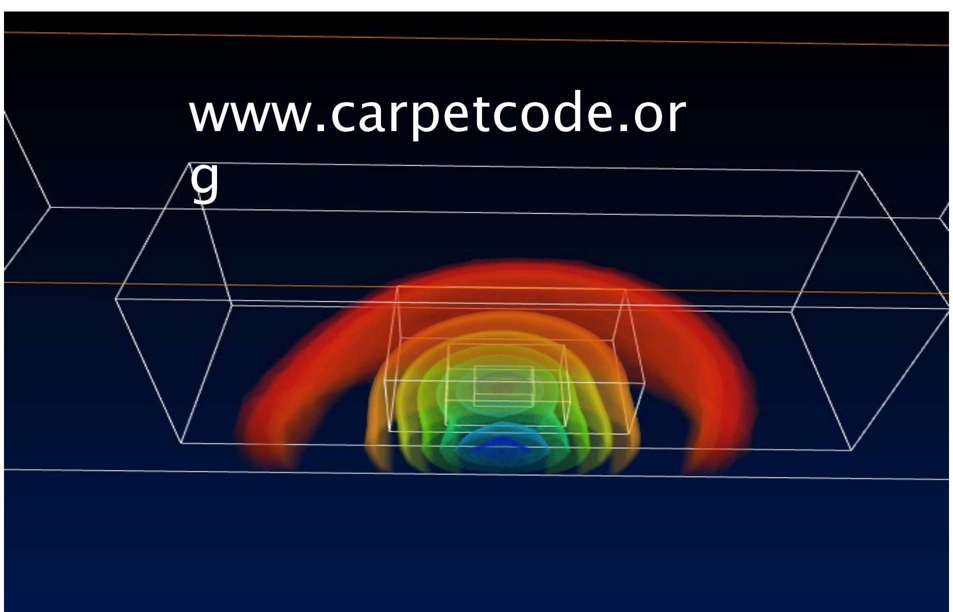
– Severe time-step restrictions for explicit methods

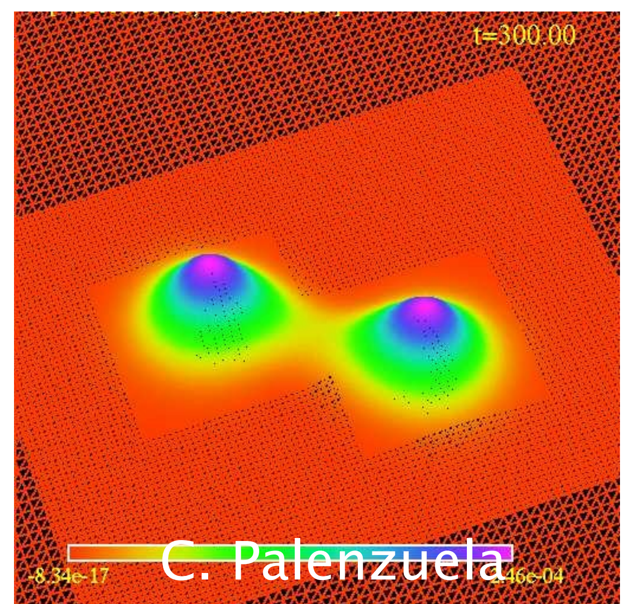
+ Large radial domains and compactification

Adaptive Mesh refinement (AMR)

- Extremely complicated
+ Long expertise / available infrastructure
+ No time-step problem
- Non-adapted to the problem (Cartesian)
- MHD+AMR difficult

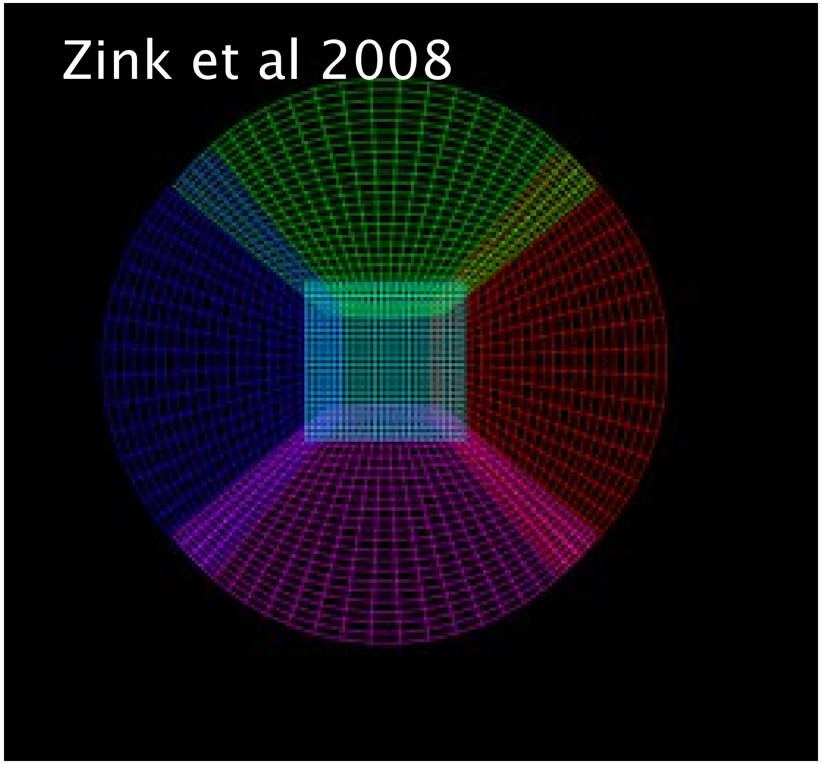
Examples:
- Carpet (Schnetter et al 2006) used in Cactus/Whisky
- HAD used for GRMHD simulations(Anderson et al 2006)





Multi-patched grids

+ Very well adapted to the problem
+ No time-step problem
- Still complicated
- Patch boundaries

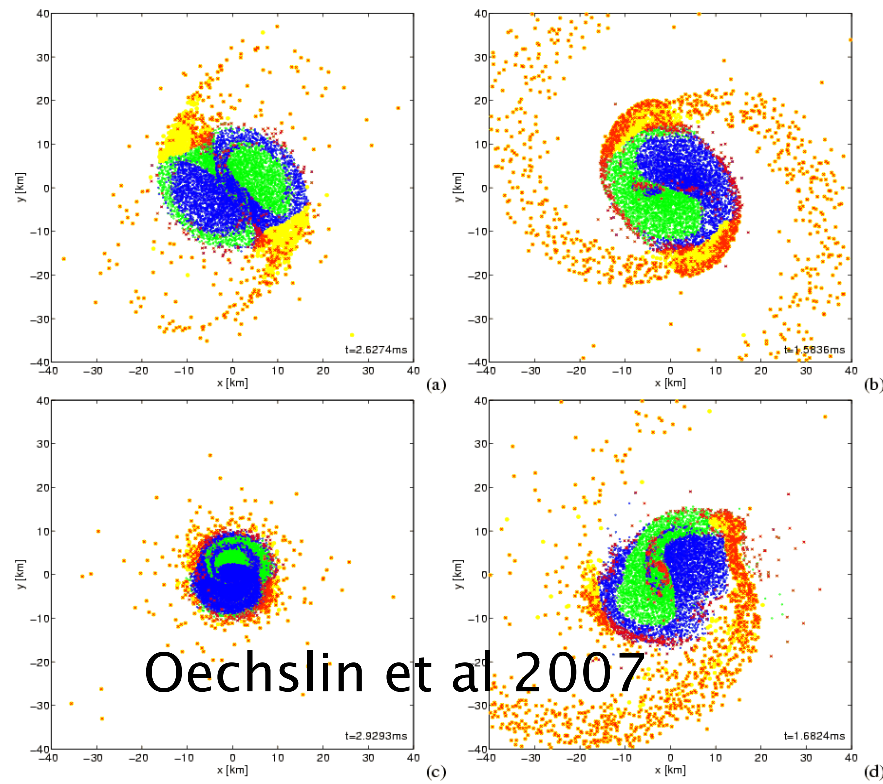


Examples:
- Zink et al 2008 (cube-sphere)
- Scheidegger et al 2008 (3D Cartesian embeded in 1D spherical)

Smoothed particle hydrodymanics (SPH)

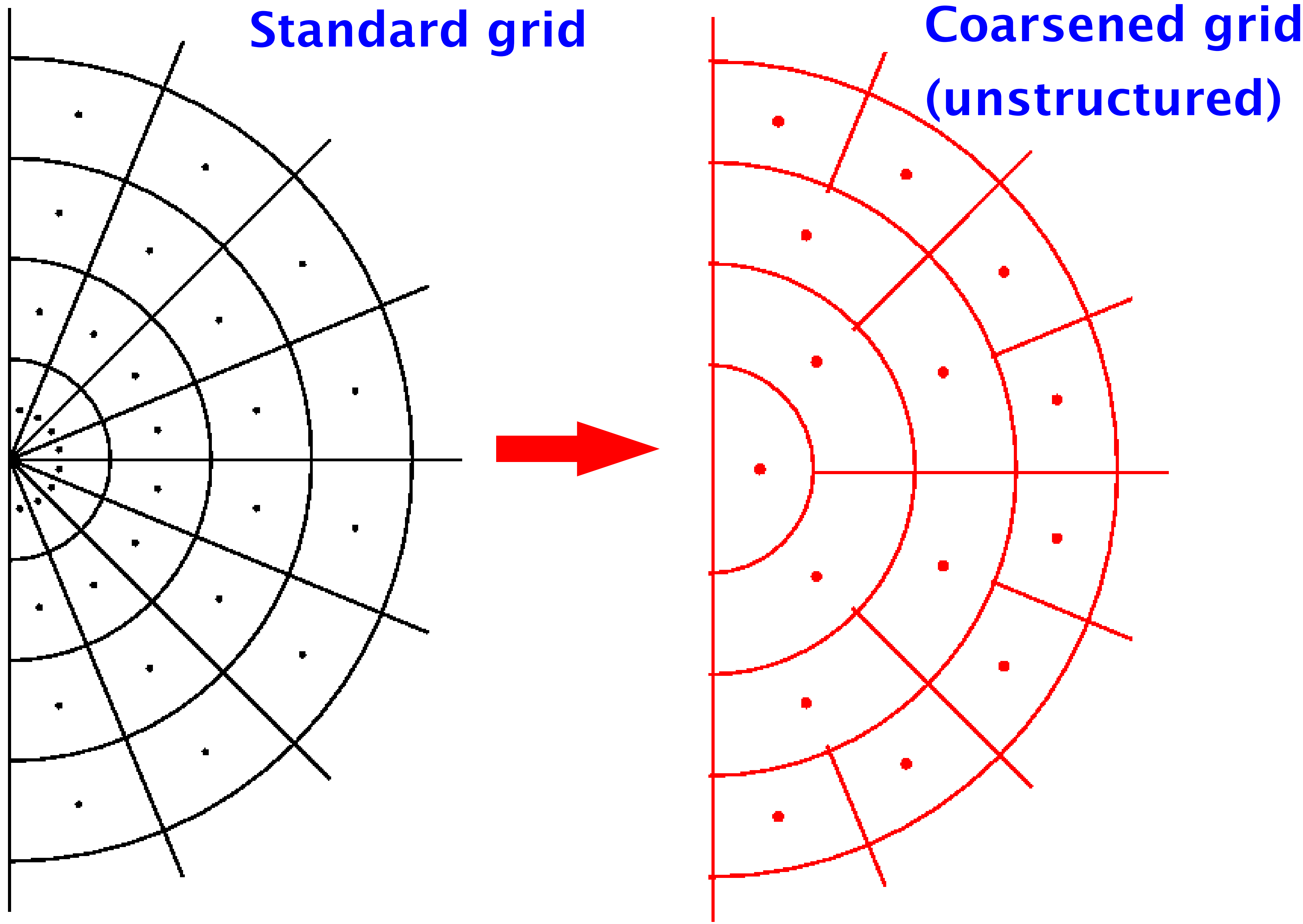
+ Well adapted to the problem
+ No time-step problem
- GR gravity is not a force
(Auxiliary grid for the metric)
- Difficult to handle discontinuities

Examples:
- Oechslin et al 2001



Oechslin et al 2007

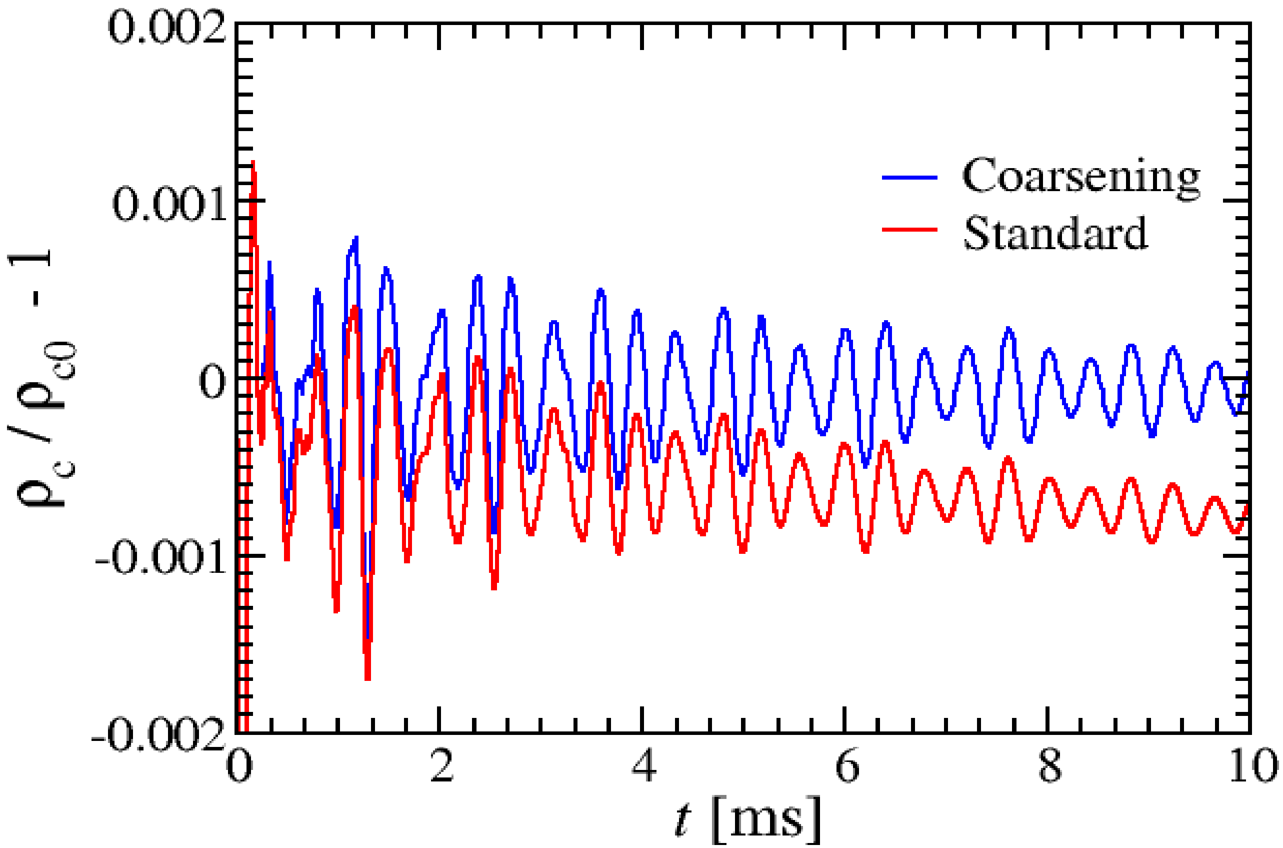
Mesh coarsening (preliminary)



Mesh coarsening (preliminary)

Central density evolution: coarsened vs standard

Rotating neutron star: $M=1.63M_{\odot}$, $R_e=17.3$, $T/|W|=0.074$, rigid rot., 80x32

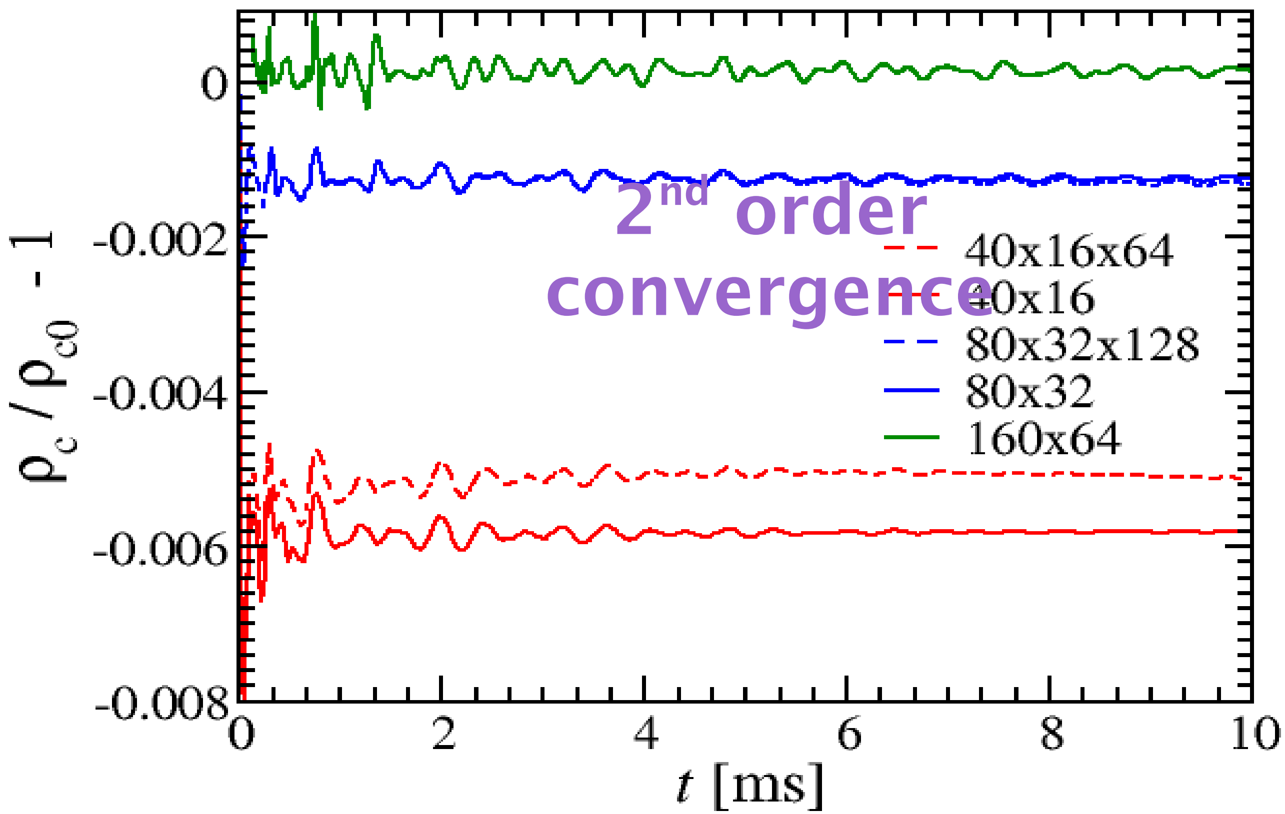


$T_{CPU}<4'$

$T_{CPU}=1h\ 44'$

Mesh coarsening (preliminary)

Central density evolution: convergence (PPM+MC)



33' (2D)

12h 57' (3D)

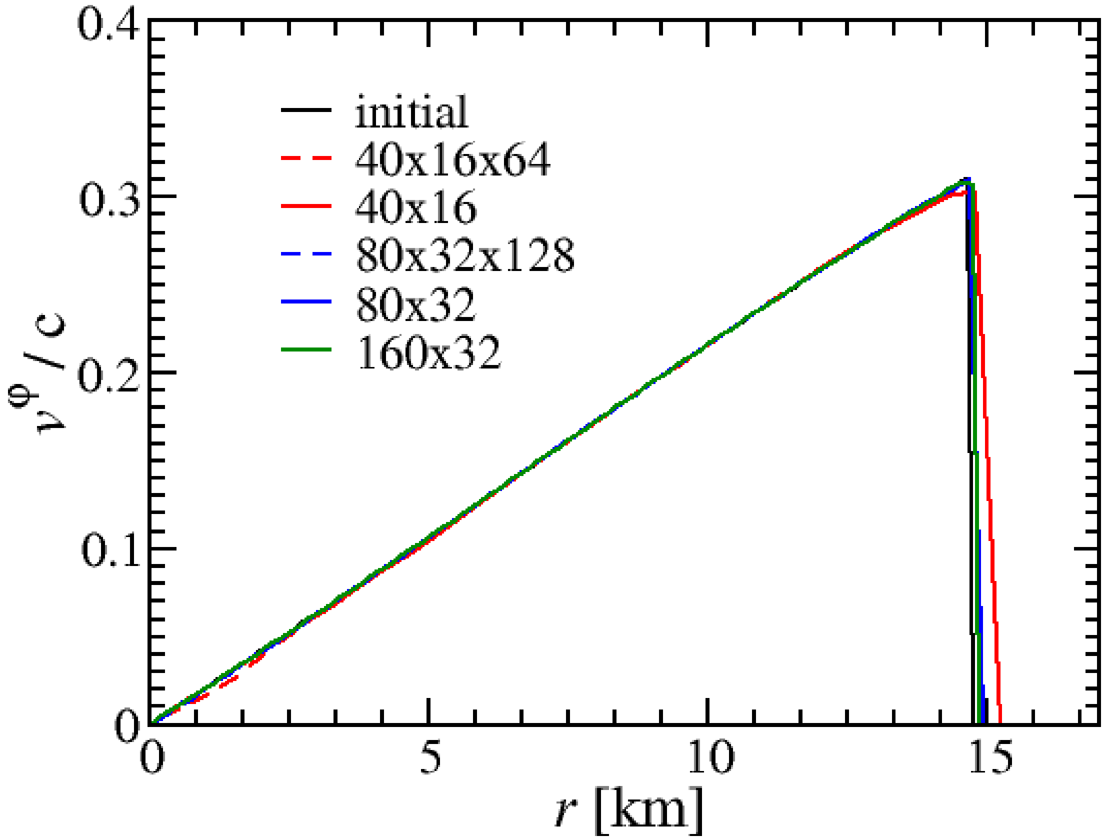
4' (2D)

32' (3D)

<1' (2D)

Mesh coarsening (preliminary)

Rotation profile **(after 10 ms)**



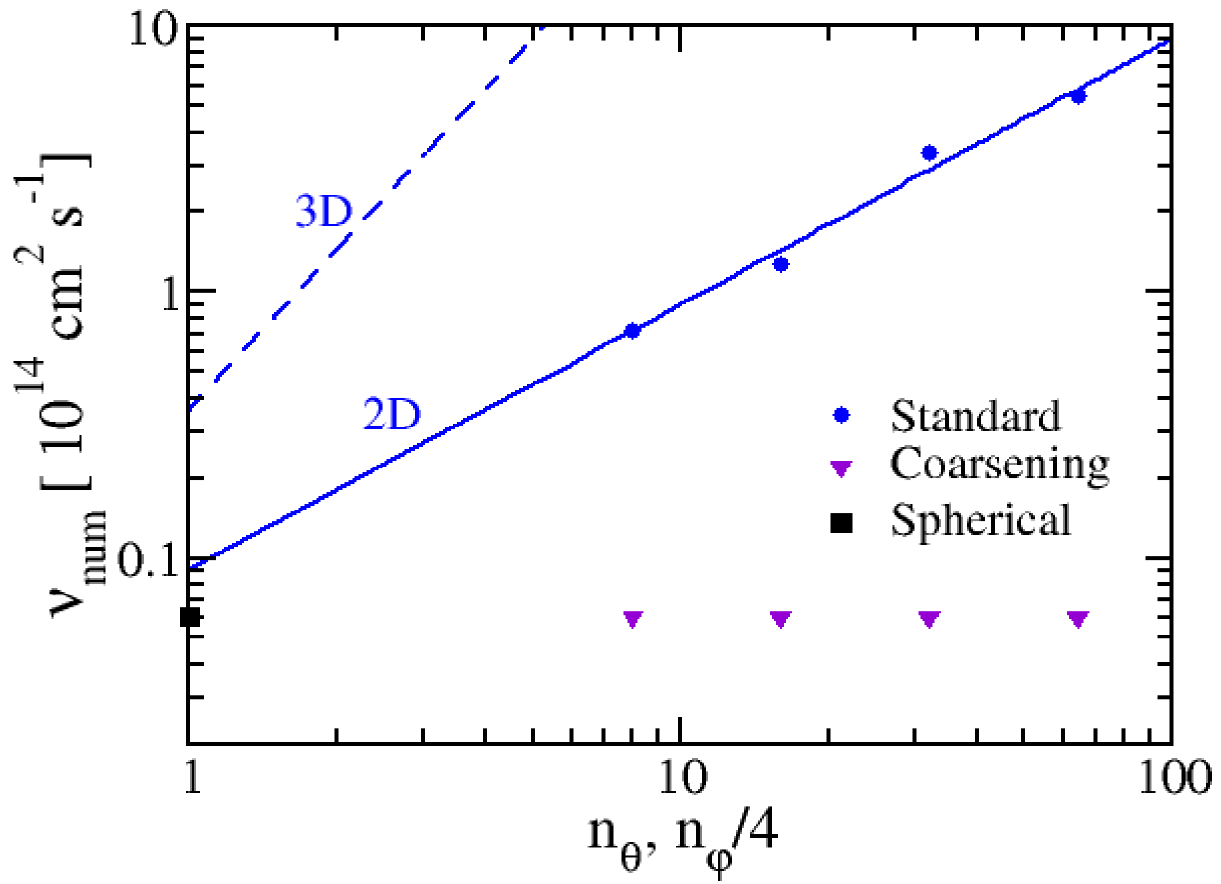
Mesh coarsening (preliminary)

Numerical viscosity (damping of radial modes) $\nu \sim \frac{R^2}{\tau_{damping}}$

Non-rotating NS (2D)

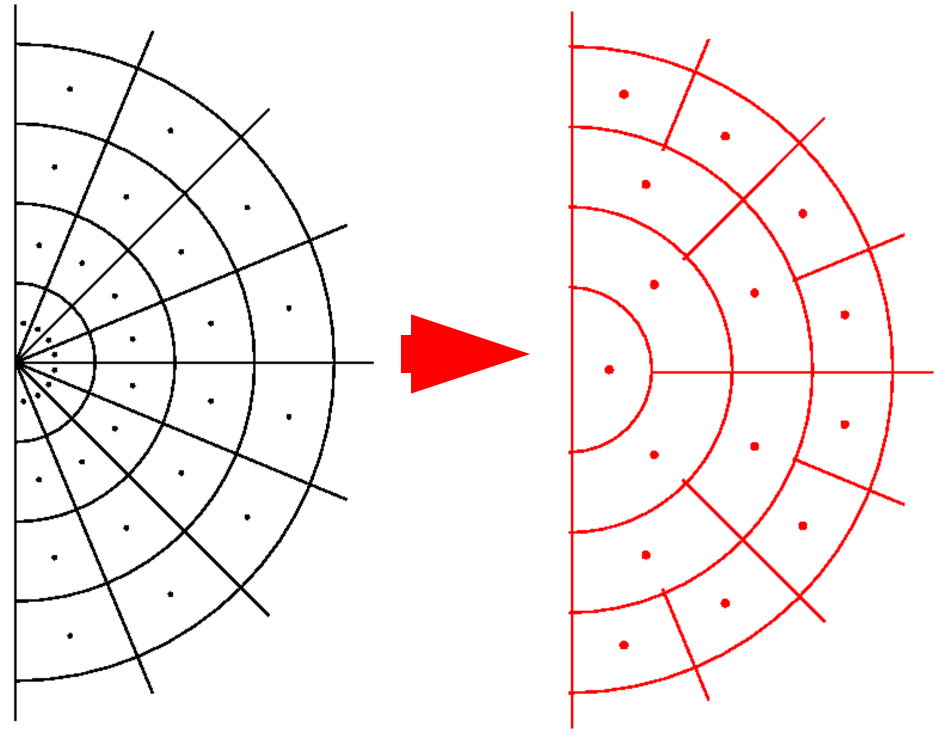
M=1.4 M

R=14.2 km



Mesh coarsening (preliminary)

+ Well adapted to the problem (spherical coordinates)
+ No time-step problem
+ Small modification in spherical coordinates code
– Still experimental



Conclusions

Rotating compact objects (CC, NS) can be much more difficult than standard Supernovae

- General relativity
- MHD and small scale instabilities
- Recovery (maybe no)

Mesh coarsening: poor man's approach to 3D