Neutron star envelopes: (micro)physics and thermal radiation

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in collaboration with

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1. Motivation: Importance of the envelopes
2. Plasma EoS: fully ionized / partially ionized, nonmagnetic / magnetic
3. Radiative opacities: Magnetized atmospheres
4. Conduction: Thermal structure and luminosity
5. The effects of superstrong magnetic fields: Uncertainties
6. [An application example]
Motivation

Stellar mass–radius relation for different EOSs
Thermal evolution

“Basic cooling curve” of a neutron star (no superfluidity, no exotica)

Cooling of neutron stars with proton superfluidity in the cores

Neutron star cooling

• Relation between *internal* (core) temperature and *effective temperature* (surface luminosity)
  • requires studying *thermal conduction* and *temperature profiles* in heat-blanketing envelopes

• Knowledge of the shape and features of the *radiation spectrum* at given effective temperature
  • requires modeling neutron star *surface layers* and propagation of electromagnetic radiation in them

**Solution of both problems relies on**
modeling thermodynamic and kinetic properties of *outer neutron-star envelopes* –
dense, strongly magnetized plasmas

*Magnetic field affects* thermodynamics properties *and the heat conduction of the plasma,*
as well as *radiative opacities*
• **Strong magnetic field** $B$:
  \[ \tilde{\hbar} \omega_c = \tilde{\hbar} eB/m_e c > 1 \text{ a.u.} \]
  \[ B > m_e^2 c e^3/\tilde{\hbar}^3 = 2.35 \times 10^9 \text{ G} \]

• **Superstrong field**:
  \[ \tilde{\hbar} \omega_c > m_e c^2 \]
  \[ B > m_e^2 c^3 / e\tilde{\hbar} = 4.4 \times 10^{13} \text{ G} \]

• **Strongly quantizing**:
  \[ \rho < \rho_B = m_{\text{ion}} n_B \langle A \rangle / \langle Z \rangle \approx 7 \times 10^3 B_{12}^{3/2} (\langle A \rangle / \langle Z \rangle) \text{ g cm}^{-3} \]
  \[ T << T_B = \tilde{\hbar} \omega_c / k_B \approx 1.3 \times 10^8 B_{12} \text{ K} \]
Neutron star structure
Neutron star structure in greater detail

\[ M \approx 1 - 2 \, M_{\odot} \]
\[ B \approx 10^8 - 10^{15} \, G \]

Electrons, ions, atoms, molecules (gas/liquid)
electrons, nuclei (Coulomb liquid)
electrons, nuclei (Coulomb crystal)
e\text{, } n \text{ [superfluid], } n\text{-rich nuclei (Coulomb crystal)}
e\text{, } n \text{ (superfluid), exotic nuclei (liquid crystal)}
e\text{, } \mu^\text{e}, n \text{ (superfluid), } p \text{ (supercond.)}
nucleons, e^\text{e}, \mu^\text{e}, hyperons? kaon condensate? pion condensate? quarks? ...

Log \( \rho \) [g cm\(^{-3}\)]

Depth:
- \( \approx 11.6 \) \( \sim 0.3 \) km
- \( \approx 14 \) \( \sim 1 \) km
- \( \approx 14.7 \) \( \sim 10 \) km
- \( \approx 15 \) \( 10 - 13 \) km
Neutron star without atmosphere: possible result of a phase transition
Equation of state of electron-ion plasmas

Ideal Fermi gas

Fitting and asymptotic formulae:


An alternative – numerical calculation of tables and interpolation:


Exchange-correlation interaction of electrons

**Ion liquid**

Best Monte Carlo calculations of the internal energy at $1 < \Gamma < 190$:

Debye – Hückel formula + corrections up to $O(\Gamma^{9/2} \ln \Gamma)$:

Fit formula reproducing the Caillol’s results at $1 < \Gamma < 190$ with a fractional error about $1/10^6$, and also the Cohen – Murphy formula at $\Gamma < 0.3$

**Quantum corrections**


*Numerical results beyond perturbation theory are wanted for quantum liquid!*
Coulomb (Wigner) crystal

Harmonic approximation: analytic formulae


Classical anharmonic corrections


Quantum anharmonic corrections


EoS: fully ionized, nonmagnetic
Reliable and usable numerical results beyond perturbation theory and beyond the harmonic model are wanted for quantum crystal!


“**present**”: Potekhin & Chabrier – interpolation (unpublished)

**EoS**: fully ionized, nonmagnetic
Equation of state of multicomponent electron-ion plasmas (2009)

1. Strongly nonideal Coulomb plasma

For every component $j$ one can write

$$f_{ex} \equiv \frac{F_{ex}}{N_i k_B T} = f_{ii} + f_{ie} + Z_j f_{ee}$$

Linear Mixing Rule

$$f_{ex}^{LM}(\Gamma) \approx \sum_j x_j f_{ex}(\Gamma_j, x_j = 1), \quad \Gamma_j = \Gamma \frac{Z_j^{5/3}}{\langle Z^{5/3} \rangle}$$

2. Extremely weakly nonideal Coulomb plasma

Debye – Hückel approximation (nonlinear!)

$$f_{ee}^{DH} = -\frac{\Gamma_e^{3/2}}{\sqrt{3}} \quad f_{ii}^{DH} = f_{ee}^{DH} \zeta_{ii}^{DH}, \quad \zeta_{ii}^{DH} = \frac{\langle Z^2 \rangle^{3/2}}{\langle Z \rangle^{1/2}}$$

3. Moderate Coulomb coupling – ???

Examples for three-component plasmas

EoS: fully ionized, nonmagnetic
Electron-ion interaction

Electron polarization in Coulomb liquid


Electron polarization in Coulomb crystal


*Numerical results beyond perturbation theory and beyond Yukawa and harmonic models are wanted!*
Heat capacity of plasma in a white dwarf or a neutron star envelope

Various contributions to the heat capacity of carbon at density $10^5$ g cm$^{-3}$

EoS: fully ionized, nonmagnetic

http://www.ioffe.ru/astro/EIP/
Melting of a neutron star envelope

Top: Latent heat of carbon and iron as function of density.
Bottom: Coulomb coupling parameter $\Gamma$ value at the melting point.

EoS: fully ionized, nonmagnetic

http://www.ioffe.ru/astro/EIP/
Equation of state in *magnetic* neutron star envelopes

Normalized thermodynamic functions of fully ionized iron without magnetic field (dashed lines) and in a strong magnetic field (solid lines)

Normalized thermodynamic functions of fully ionized iron without magnetic field (dashed lines) and in a strong magnetic field (solid lines)
The effects of a strong magnetic field on the atoms and molecules.

- **a–c**: H atom in the ground state (a: $B < 10^9$ G, b: $B \approx 10^{10}$ G, c: $B \approx 10^{12}$ G).
- **d**: The field stabilizes the molecular chains (H$_3$ is shown).
- **e**: H atom moving across the field becomes decentered.
Main transition energies of the hydrogen atom in a magnetic field  

Binding energies of the hydrogen atom in the magnetic field \( B=2.35\times10^{12} \) G as functions of its state of motion across the field  
Partial ionization/recombination in hydrogen plasmas with strong magnetic fields

EoS: partially ionized, magnetic
Equation of state of hydrogen in strong magnetic fields: The effects of nonideality and partial ionization

EOS of ideal (dotted lines) and nonideal (solid lines) H plasmas at various field strengths.

Potekhin, Chabrier, & Shibanov, *Phys. Rev. E* 60, 2193 (1999);
Oscillator strengths for transitions between 2 levels of the hydrogen atom at $B=2.35 \times 10^{12}$ G, as functions of pseudomomentum

Photoionization cross sections for the ground-state H atom at $B=2.35 \times 10^{12}$ G

Photoionization cross sections for the ground-state H atom at $B=2.35 \times 10^{12}$ G

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Plasma absorption and polarizabilities in strong magnetic fields:
The effects of nonideality and partial ionization

Spectral opacities for 3 basic polarizations. 
Solid lines – taking into account bound states, 
dot-dashes – full ionization

To the right: top panel – basic components of the absorption coefficients; middle and bottom – components of the polarizability tensor
Opacities for two normal modes of electromagnetic radiation in models of an ideal fully ionized (dash-dot) and nonideal partially ionized (solid lines) plasma at the magnetic field strength $B=3\times10^{13}$ G, density 1 g/cc, and temperature $3.16\times10^5$ K. The 2 panels correspond to 2 different angles of propagation with respect to the magnetic field lines. An upper/lower curve of each type is for the extraordinary/ordinary polarization mode, respectively [Potekhin, Lai, Chabrier, & Ho (2004) ApJ 612, 1034]
Modeling results: temperature profiles and the atomic fractions

\[ B = 10^{12} \, \text{G} \]

\[ \log T_{\text{eff}} = 6.8 \]

\[ \log T_{\text{eff}} = 5.5 \]

\[ \rho \, (\text{g cm}^{-3}) \]
Result of modelling: spectra, dipole model
(Wynn Ho)
NS Magnetic Atmosphere Model

The NSMAX model interpolates from a grid of neutron star (NS) atmosphere spectra to produce a final spectrum that depends on the parameters listed below. The atmosphere spectra are obtained using the latest equation of state and opacity results for a partially ionized, strongly magnetized hydrogen plasma. The models are constructed by solving the coupled radiative transfer equations for the two photon polarization modes in a magnetized medium, and the atmosphere is in radiative and hydrostatic equilibrium. The atmosphere models mainly depend on the surface effective temperature $T_{\text{eff}}$ and magnetic field strength $B$ and inclination $\theta_B$; there is also a dependence on the surface gravity $g=(1+z_g)GM/R^2$, where $1+z_g=(1-2GM/R)^{1/2}$ is the gravitational redshift and $M$ and $R$ are the NS mass and radius, respectively.

Two sets of models are given: one set with a single surface $B$ and $T_{\text{eff}}$ and a set which is constructed with $B$ and $T_{\text{eff}}$ varying across the surface according to the magnetic dipole model (for the latter, $\theta_m$ is the angle between the direction to the observer and the magnetic axis). The effective temperatures span the range $\log T_{\text{eff}}=5.5-6.8$. The models with single ($B,T_{\text{eff}}$) cover the energy range 0.05-10 keV, while the models with ($B,T_{\text{eff}}$)-distributions cover the range 0.09-5 keV.

The model parameters are:

- $par_1 = \log T_{\text{eff}}$ surface (unredshifted) effective temperature
- $par_2 = 1+z_g$ gravitational redshift
- $par_3 = \text{switch indicating model to use (see nsmax.dat or list)}$
- $A = (R_{\text{em}}/d)^2 (1+z_g)^{-1}$, normalization, where $R_{\text{em}}$ is the size (in km) of the emission region and $d$ is the distance (in kpc) to the object.

Note: A is added automatically by XSPEC.

The source code, Inmodel.dat entries, input model list, and model data files (in one tar file) are available. A list of the models currently available can be found here. The model data files should either be placed in the $XANADU$/spectral/modellonData (v12), $XANADU$/spectral/xspec/manager (v11) directory, or the XSPEC command xset NSMAX(DIR directory-path) should be used to define the directory containing the model data files.
An alternative (or supplement):
**Radiation from condensed surface**
(Matt van Adelsberg)


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Monochromatic flux from the condensed surface in various cases

Stellar heat conductivities

Basic data sources

<table>
<thead>
<tr>
<th>Source</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.G.Yakovlev et al. (1980 – 2001)</td>
<td><strong>ei</strong>: (i) <strong>liquid</strong>: classical ions (strongly and weakly coupled) with a good structure factor; non-Born correction; (ii) <strong>solid</strong>: quantum treatment, account of multi-phonon processes. Allowance for strong magnetic fields. <strong>ee</strong>: strongly degenerate electrons; inaccurate treatment at relativistic densities.</td>
</tr>
<tr>
<td>S.Cassisi et al. (2007)</td>
<td><strong>ee</strong>: extension to arbitrary degeneracy.</td>
</tr>
<tr>
<td>A. Chugunov &amp; P. Haensel (2007)</td>
<td><strong>le, ii</strong>: ion thermal conduction.</td>
</tr>
</tbody>
</table>
Improvement of conduction opacities in RGB stellar cores relative to some previous models.

Thermal conductivities in a strongly magnetized envelope

Solid – exact, dots – without \( T \)-integration, dashes – magnetically non-quantized


Heat flux: \[ F = -\kappa_{\parallel} \nabla_{\parallel} T - \kappa_{\perp} \nabla_{\perp} T - \kappa_{\wedge} \mathbf{b} \times \nabla T, \quad \mathbf{b} = \frac{B}{B} \]
Thermal structure with a magnetic field

$B = 10^{15}$ G, different $T$

$T_b = 10^7$ K, different $B$
Thermal structure with a magnetic field and different chemical compositions of the envelope.

Graphs showing the relationship between temperature ($T$) and density ($\rho$) for different magnetic field strengths ($B$), with lines representing iron envelopes and light elements.
Temperature drops in magnetized envelopes of neutron stars

Cooling of neutron stars with **accreted envelopes**

![Graph showing thermal evolution of neutron stars with accreted envelopes.](image)

Cooling of neutron stars with **magnetized envelopes**

![Graph showing thermal evolution of neutron stars with magnetized envelopes.](image)

Challenges from the **superstrong** fields
(B > $10^{14}$ G)

1. **Mechanical structure**: field affects EoS also in the inner crust
2. **Thermal structure**: field affects luminosity
3. [Possibly] **non-isotropic heat transport in the inner crust**
4. **Surface layers**: molecules, chains, and magnetic condensation
5. **Radiative transfer**: vacuum polarization and mode conversion
6. **Energy transport below the plasma frequency**
7. **Non-LTE distribution of ions over Landau levels**
Superstrong field affects EOS

Dependence of pressure on density for ground-state matter with zero and superstrong magnetic fields
Superstrong field affects total luminosity

Dependence of the mean effective temperature on the magnetic field strength for the light-element (dashed lines) and iron (solid lines) envelopes.
Thick or thin atmosphere?

Superstrong fields

Solid lines – extended atmosphere, dot-dashed lines – condensed surface
Importance of energy transport below plasma frequency

 Photon-decoupling densities for X- and O-modes for a partially ionized H atmosphere, for magnetic field strengths typical of pulsars (blue lines) and magnetars (red lines).

 Dot-dashed lines correspond to the radiative surface, the shadowed region corresponds to $E < E_{pl}$. 

$$E_{pe} = \left( \frac{4\pi\hbar^2 e^2 n_e}{m_e} \right)^{1/2} \approx 28.7 \rho^{1/2} \text{ eV}$$
Temperature profiles in the accreted envelope of a neutron star with “ordinary” (left panel) and superstrong (right) magnetic field, for the local effective temperature $10^{5.5}$ K, with (solid lines) and without (dashed lines) plasma-frequency cut-off [Potekhin et al. (2003) *ApJ* 594, 404]
Heating and cooling of magnetars

\[ M = 1.1 \, M_\odot \]
\[ B = 5 \times 10^{14} \, G \]
Heating and cooling of magnetars

![Graph showing cooling curves of magnetars with data points and labels.](image)

- **Viewing Points:**
  - 1=SGR 1900+14
  - 2=SGR 0526-66
  - 3=1E 1841-045
  - 4=CXOU J010043.1-721134
  - 5=1RXS J170849-400910
  - 6=4U 0142+61
  - 7=1E 2259+586

- **Other Data Points:**
  - 1=Crab
  - 2=PSR J0206+64
  - 3=RX J0822-43
  - 4=1E 1207-52
  - 5=CTA 1
  - 6=Vela
  - 7=PSR 1706-44
  - 8=PSR J0538+28
  - 9=Geminga
  - 10=RX J1856-37
  - 11=PSR 1055-52
  - 12=RX J0720-31

- **Equations:**
  - $M = 1.1 \ M_\odot$
  - $B = 5 \times 10^{14} \ G$

- **Legend:**
  - black: DH
  - blue: APR III
Heating and cooling of magnetars with accreted envelopes


\[
M = 1.1 \, M_\odot \\
B_p = 5 \times 10^{14} \, G
\]
Heating and cooling of magnetars with accreted envelopes

Heating and cooling of magnetars with accreted envelopes

Heating and cooling of magnetars with accreted envelopes

Conclusions

- **Equation of state** in neutron-star envelopes is basically known, but there remain uncertain ingredients.
- **Opacities** with strong magnetic fields are known for hydrogen at relatively high temperatures. [Note: For middle-Z elements, there are atmosphere models by Kaya Mori and Wynn Ho, with a restricted account of the atomic motion.]
- Practical models of the conductivities, applicable to neutron stars, are developed in recent years.
- A superstrong magnetic field
  - on the average, makes the envelope more heat-transparent,
  - accelerates cooling at late epochs,
  - leads to theoretical uncertainties, which require further study.

THANK YOU FOR YOUR ATTENTION!