General relativistic equilibrium configurations with purely toroidal magnetic fields and its stability

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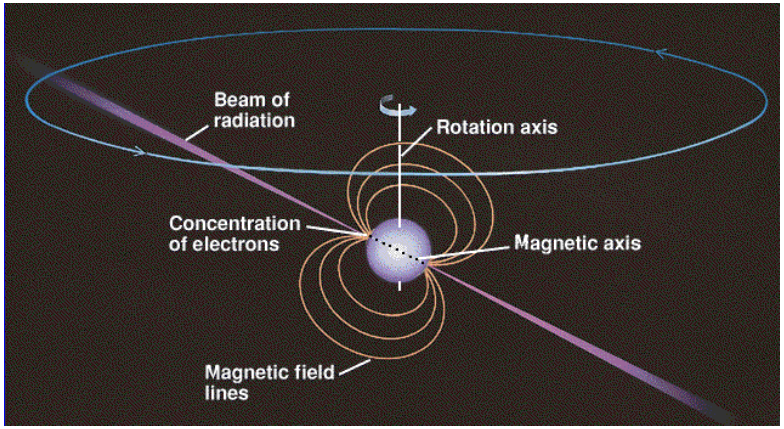
Talk Plan

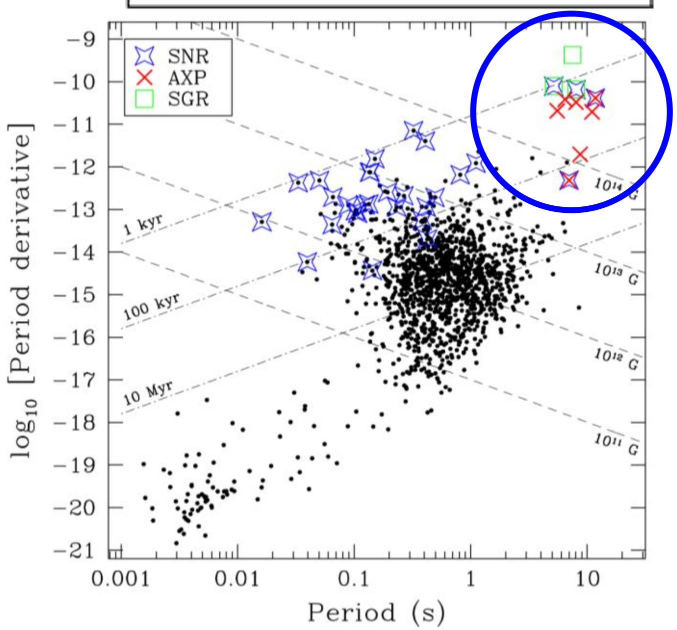
- Introduction
- (Part 1) Guide line to construct equilibrium configuration of magnetized star
- (Part 2) Stability of magnetic field in neutron star
- Summary

Introduction



Pulsars are believed to be neutron stars which have strong magnetic field ($B \sim 10^{12}$G)

$P - \dot{P}$ Diagram



There are some speical classes called magnetars which have a very strong magnetic field $B \sim 10^{14-15}$G.

What is an origin of such stong magnetic fields ?

Magnetically driven SN (many references) ? or dynamo process in proto NS ?(Thompson & Duncun 93 etc.).

Introduction

We don’t have information of interior magnetic fields because

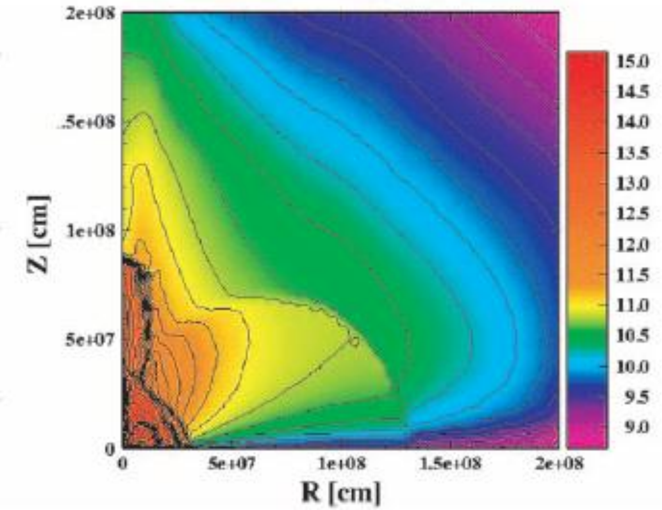
$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{1}{2}I\Omega^2\right) = -4\pi^2 I\frac{\dot{P}}{P^3}$$

assuming dipole radiation

$$\Rightarrow \quad B_p \propto (IP\dot{P})^{1/2}R^{-3}$$

Note that 10^{14-15}G is poloidal component.

Toroidal magnetic field (Takiwaki 04’)



Also, toroidal field is easily generated by magnetic winding.

So, there is a possibility that toroidal field dominates the poloidal component.

How to construct magnetized star in equilibrium

Basic assumptions

0. Stationary and axisymmetry
1. Perfect fluid
2. No meridional flow
3. Ideal MHD
4. Pure toroidal magnetic field
5. Barotropic EOS

How to construct magnetized star in equilibrium

Basic equations

Einstein equation (Gravitational field)

Maxwell equation (Magnetic field)

Hydrostatic equilibrium equation (Matter field)

For pure toroidal magnetic field case,

~~Grad-Shafranov eq. in terms of A_φ~~

Integrability condition

4 elliptic equations for metric potentials

Bernoulli eq. (1$^{\text{st}}$ integral of EOM)

Simple analogy for intergrability conditions

Newtonian rotating equilibrium configurations without magnetic field

hydrostatic eq. $$\frac{\nabla P}{\rho} + \nabla \phi_g + \varpi \Omega^2 \nabla \varpi = 0$$

if $\Omega = \Omega(\varpi)$

Bernoulli eq. $$\int \frac{dP}{\rho} + \phi_g + \int \Omega^2 \varpi d\varpi = C$$

Purely toroidal magnetic field in general relativistic equilibrium is given by

Faraday tensor $F_{r\theta} = (\text{metric}) \times K(u)$

where $u = \rho_0 h (r \sin\theta)^2 \times (\text{metric})$.

We simpliy choose $K(u)$ as

$$K(u) = bu^k \text{ with } k \geq 1$$

How to construct magnetized star in equilibrium

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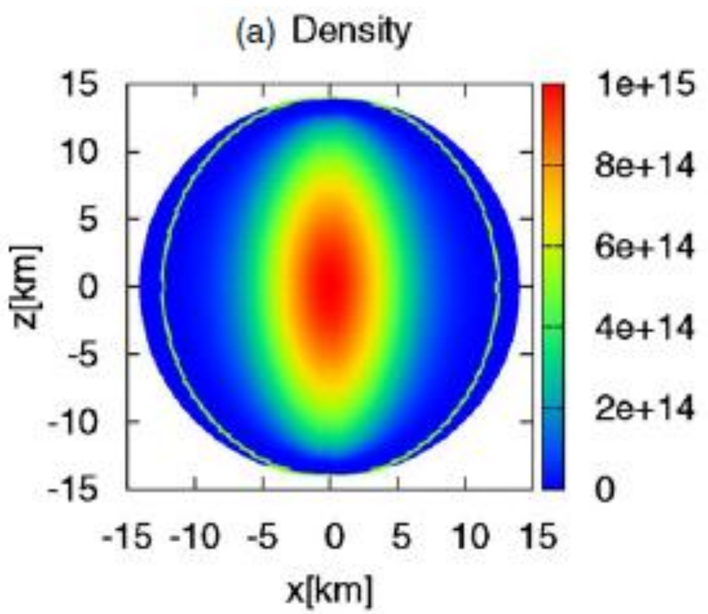
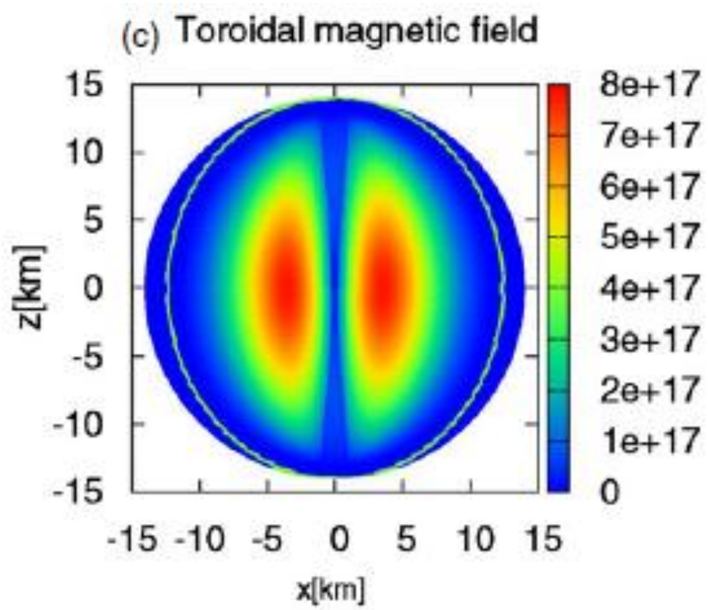
Numerical scheme : KEH (Komatsu et al.1987)

Magnetized Neutron Stars

Stellar structure on the meridional plane





1. Concentration of toroidal field deep inside the star
2. Prolate stellar configuration due to the strong magnetic fields
3. It is possible to construct hyper strong magnetized star, e.g., H/|W|=O(0.1)

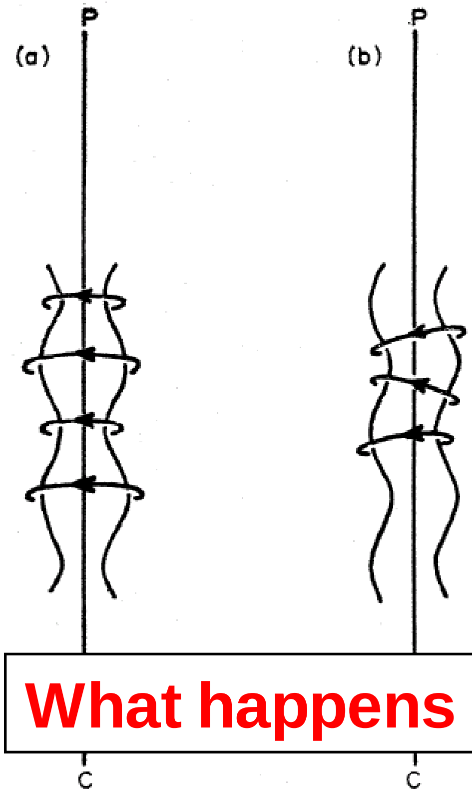
Stability of toroidal magnetic field

It is known that pure toroidal magnetic fields are unstable (Tayler instability).

Tayler instability = interchange type instability

m=0 perturbation

m=1 perturbation



Instability condition (Recall $B_\varphi \propto \varpi^{2k-1}$)

For m=0 mode $k > 1$

For m$\neq$ 0 mode $k > m^2/4$

Note that $k \geq 1$ for the regularity condition near the axis.

What happens after the onset of the instability ?

(Tayler 1973)

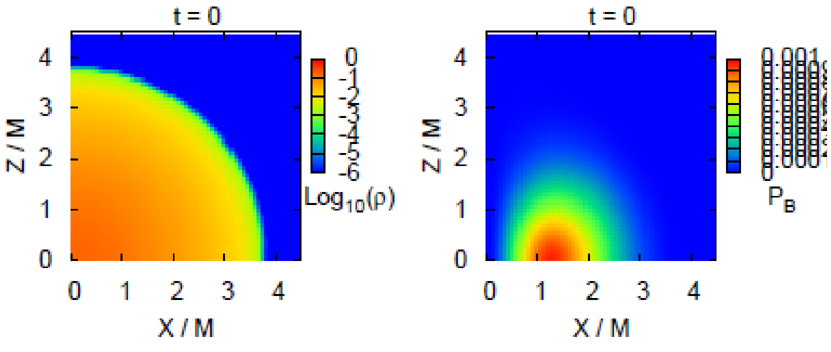
Stability of toroidal magnetic field

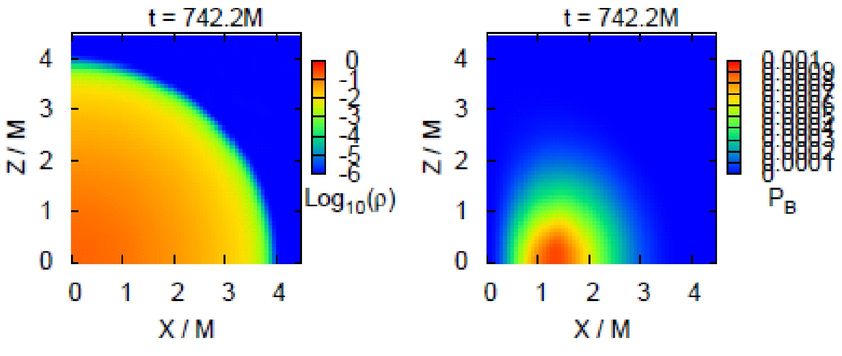
Set up

1. Prepare the equilibrium configurations as initial conditions (with varying the value of k)
2. Start the axisymmetric GRMHD simulation

k=1 configuration (Stable)

Density and toroidal magnetic field (t=0)	Density and toroidal magnetic field (t=742.2M)

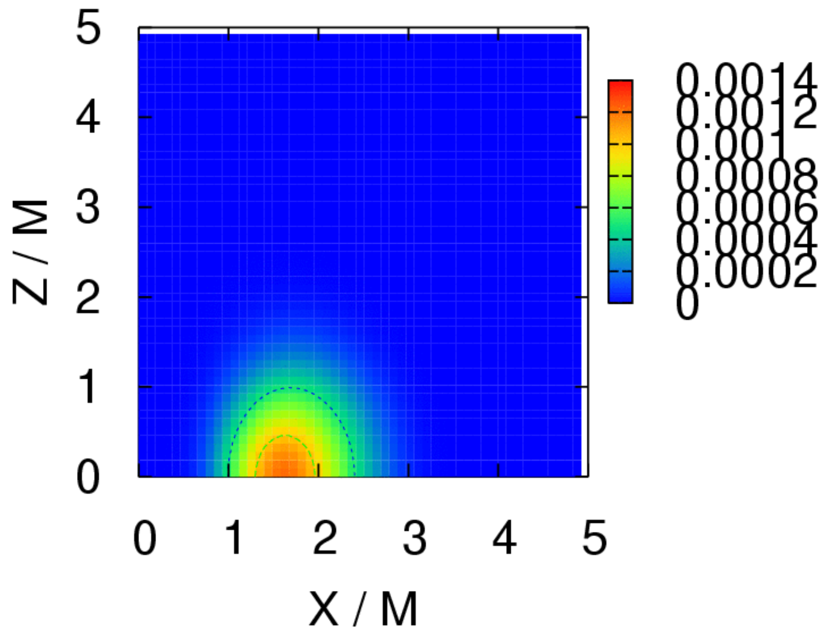




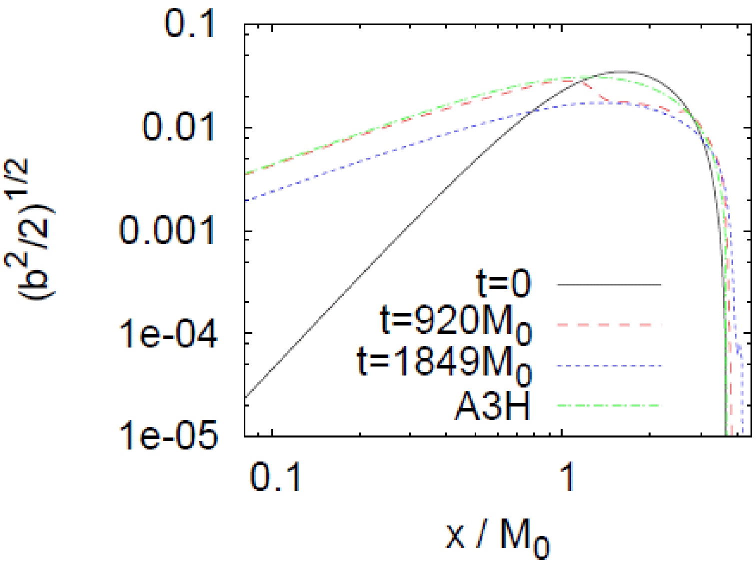
Stability of toroidal magnetic field

k=2 configuration (Unstable)

Toroidal magnetic field



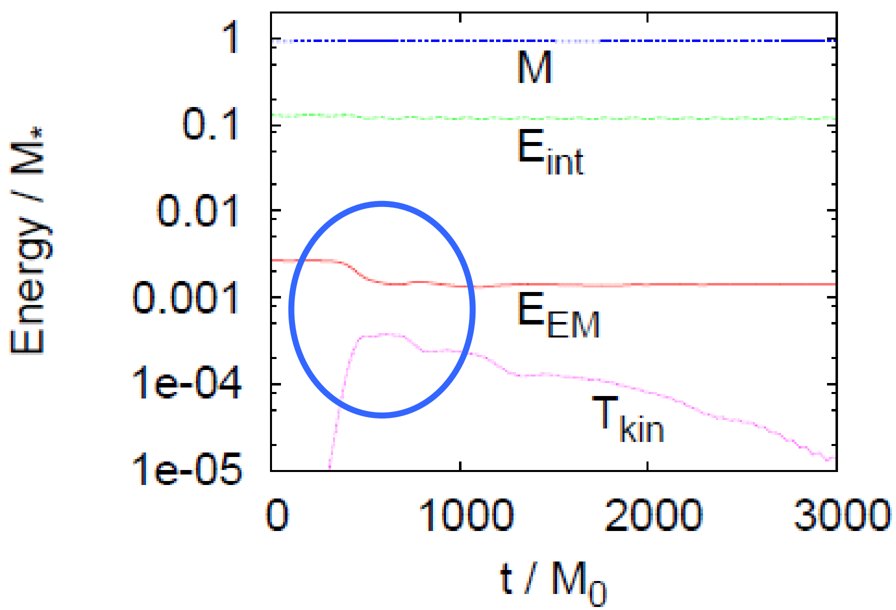
Snapshot

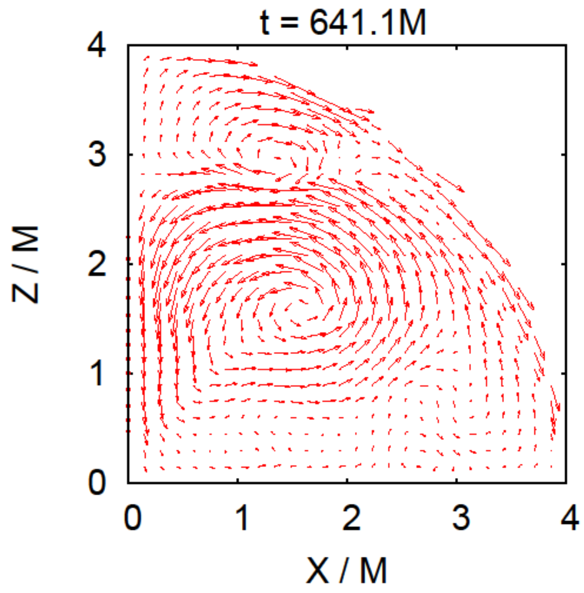


Unstable configuration (k=2) comes to the stable one (k=1)

Stability of toroidal magnetic field

Magnetic energy -> Kinetic energy



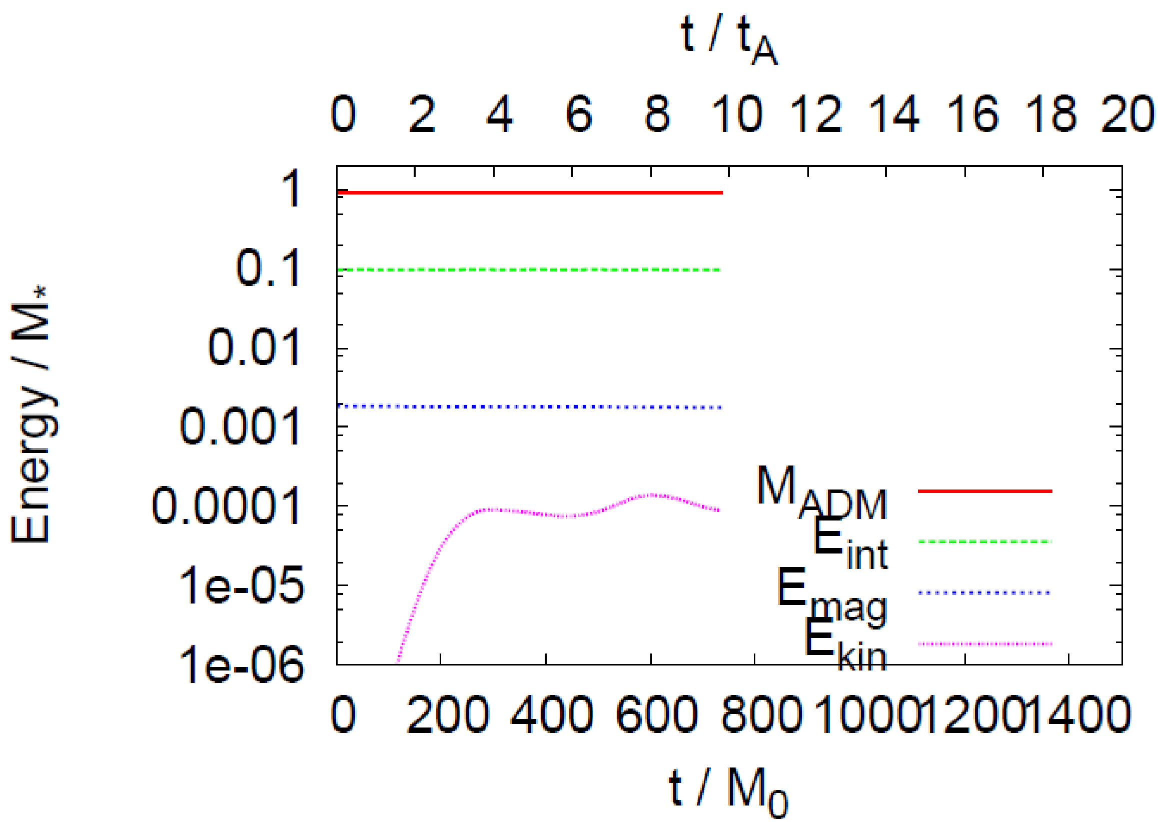


Tayler instability induces the circular motion on the meridional plane and settles down to a quasi equilibrium state.

Tayler instability in 3D GRMHD simulation (preliminary)

Recall that $m = 1$ perturbation inevitabley induces the instability because the instability condition is $k > m^2/4$.

k=1 configuration



Summary

1. Part 1: We formulate GR equilibrium configuration with purely toroidal magnetic field.
2. Part 2: With 2D GRMHD simulation, we explore the Tayler instability and confirm the linear analysis prediction.
3. Part 2: Toroidal magnetic fields settle down to “new” quasi equilibrium state with the circular motion.

Future issues

1. Effect of magnetic field on EOS for hyper strong magnetic field case
2. Tayler instability for non-axisymmetric perturbation (on going)
3. GR equilibrium configuration with poloidal-toroidal magnetic field.