Overview: The Equation of State For Numerical Simulations

James Lattimer

lattimer@astro.sunysb.edu

Department of Physics & Astronomy
Stony Brook University

We need more equation of state tables with decent gridding covering relevant ranges of density, temperature and composition that are thermodynamically consistent and for which nuclear parameters can be extensively varied.

Required Equation of State Conditions

- density ρ : 0 to 10^{15} g cm^{-3} ($\rho_s = 2.7 \times 10^{14}$ g cm^{-3}, $n_s = 0.16$ fm^{-3})
- temperature T: 0 to 60 MeV
- electron fraction Y_e : ~ 0.6 to ~ 0

For densities below 10^7 g cm^{-3}, large Y_e, low T: Known nuclear masses with Nuclear Statistical Equilibrium (NSE) with Coulomb corrections, or network

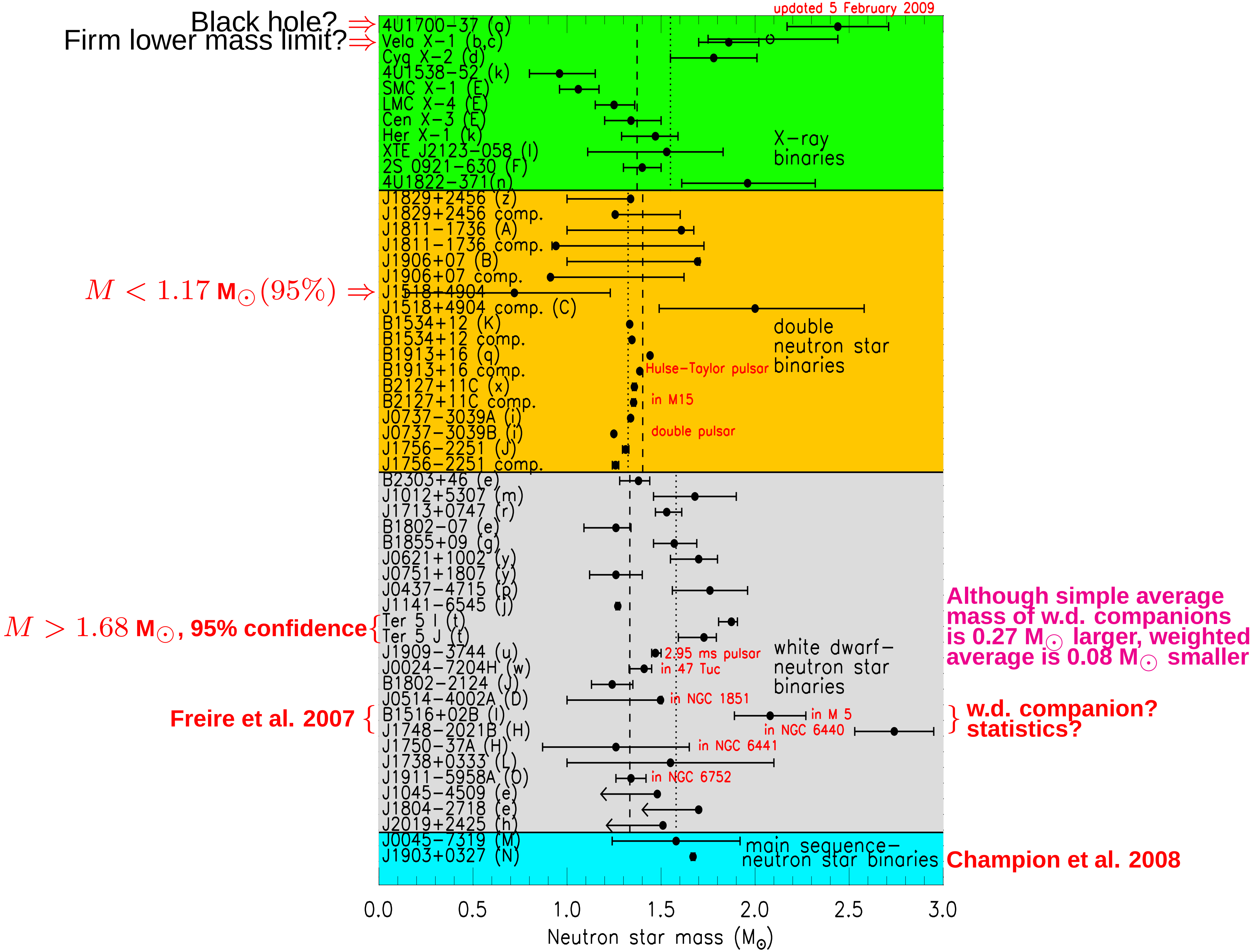
Focus here will be for densities $> 10^7$ g cm^{-3}:

- extrapolation of nuclear masses to lower Y_e, higher A
- inclusion of nuclei - external $n - p$ gas interactions
- inclusion of nuclear excited states
- phase transition to bulk nucleon matter around $\rho_s/3 - \rho_s/2$,
- vary uncertain bulk nuclear properties (K, K', S_v, $S_v(n)$, S_s)

Necessary table assumption: Single nucleus approximation

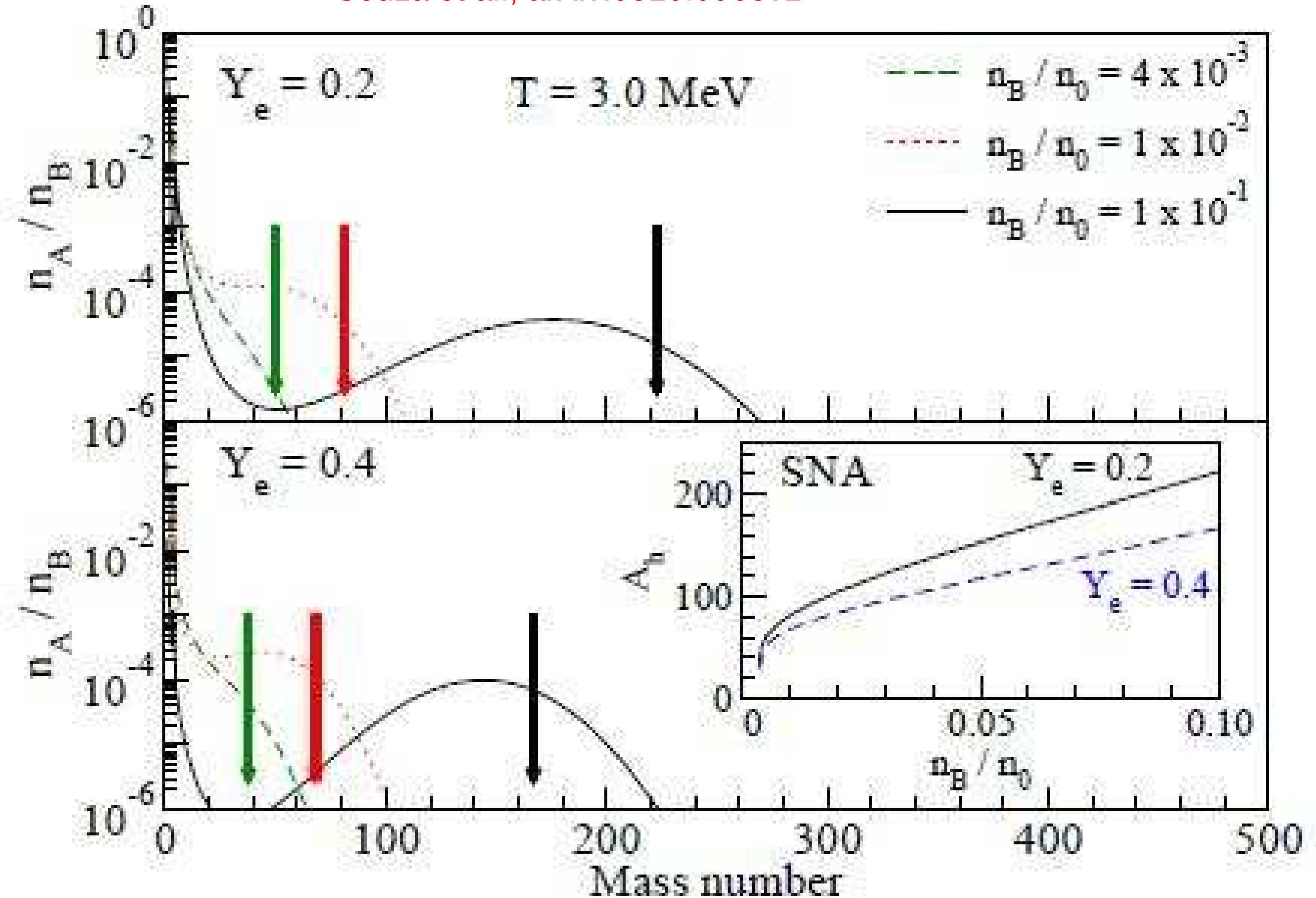
Distribution of nuclei replaced by most energetically favored nucleus. Although this introduces negligible errors in pressure $\simeq nT/A$ and chemical potentials $\simeq T/A$, not adequate for modelling neutrino opacities or electron capture/beta decay rates. Do regions of substantial light nucleus abundances exist?

Leptons can be treated separately from baryons


updated 5 February 2009
Black hole? ⇒
Firm lower mass limit? ⇒
4U1700−37 (a)
Vela X−1 (b,c)
Cyg X−2 (d)
4U1538−52 (k)
SMC X−1 (E)
LMC X−4 (E)
Cen X−3 (E)
Her X−1 (k)
XTE J2123−058 (l)
2S 0921−630 (F)
4U1822−371 (n)
X−ray binaries
J1829+2456 (z)
J1829+2456 comp.
J1811−1736 (A)
J1811−1736 comp.
J1906+07 (B)
J1906+07 comp.
$M < 1.17$ M$_\odot$ (95%) ⇒
J1518+4904
J1518+4904 comp. (C)
B1534+12 (K)
B1534+12 comp.
B1913+16 (q)
B1913+16 comp.
Hulse−Taylor pulsar
B2127+11C (x)
B2127+11C comp.
in M15
J0737−3039A (i)
J0737−3039B (i)
double pulsar
J1756−2251 (J)
J1756−2251 comp.
double neutron star binaries
B2303+46 (e)
J1012+5307 (m)
J1713+0747 (r)
B1802−07 (e)
B1855+09 (g)
J0621+1002 (y)
J0751+1807 (y)
J0437−4715 (p)
J1141−6545 (j)
$M > 1.68$ M$_\odot$, 95% confidence {
Ter 5 I (t)
Ter 5 J (t)
J1909−3744 (u)
2.95 ms pulsar
J0024−7204H (w)
in 47 Tuc
B1802−2124 (J)
J0514−4002A (D)
in NGC 1851
Freire et al. 2007 {
B1516+02B (I)
in M 5
J1748−2021B (H)
in NGC 6440
J1750−37A (H)
in NGC 6441
J1738+0333 (L)
J1911−5958A (O)
in NGC 6752
J1045−4509 (e)
J1804−2718 (e)
J2019+2425 (h)
white dwarf−neutron star binaries
Although simple average mass of w.d. companions is 0.27 M$_\odot$ larger, weighted average is 0.08 M$_\odot$ smaller
} w.d. companion? statistics?
J0045−7319 (M)
J1903+0327 (N)
main sequence−neutron star binaries
Champion et al. 2008
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Neutron star mass (M$_\odot$)

Single Nucleus Approximation

Souza et al., arXiv:0810.0963v1



Main Classes of Nucleon Force Models

- Non-relativistic potential models
  - Momentum- and density-dependent contact or finite-range potentials
  - Momentum dependence greatly restricted for calculational ease
  - Density-dependent effective nucleon masses
  - Relatively slowly varying $S_v(n)$, smaller neutron star radii (but adjustable)
  - Can become acausal
  - Can be constrained to fit low-density neutron matter properties
- Relativistic field-theoretical models
  - Interactions mediated by bosons (ω, σ, ρ)
  - Implicitly causal
  - Generally have linearly increasing $S_v(n)$, larger radii (but adjustable)
  - Not as easily constrained to fit low-density neutron matter properties
- High-density 'exotica'
  - Strangeness in form of hyperons, kaon/pion condensates, deconfined quark matter; many fewer laboratory constraints
  - Possibility of stable strange quark matter at zero pressure (Witten)
  - Are these important in supernova models (or suppressed by large μ_e)?

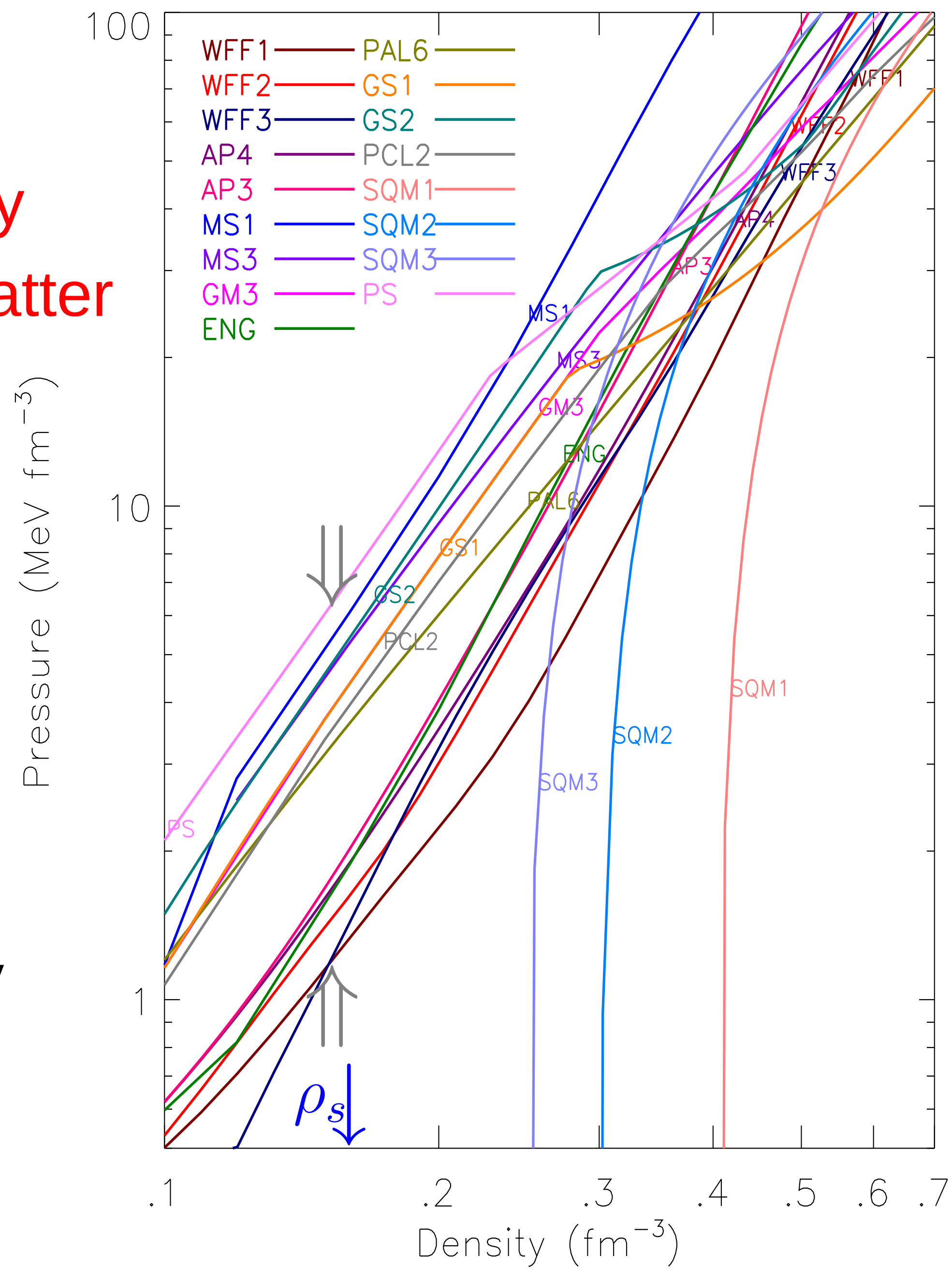
Neutron Star Matter Pressure

Extrapolation in symmetry
to nearly pure neutron matter

Wide variation:

$$1.2 < \frac{P(\rho_s)}{\text{MeV fm}^{-3}} < 7$$

Short-term goal:
Reduce this uncertainty



Major existing approaches for EOS tables

- Liquid droplet models
  - Myers & Swiatecki 1969, Baym, Bethe & Pethick 1971, Lattimer et al. 1985; Lattimer & Swesty 1990

    Non-relativistic contact potential, surface energy from semi-infinite calculation,

    Coulomb energy from droplet model, many nuclear parameter combinations
- Hybrid Droplet/Thomas-Fermi unit cell calculations
  - Oyamatsu 1993, Shen et al. 1998

    Relativistic mean field theory, suface and Coulomb energies from

    parametrized density profile optimizations, only 1 set of nuclear parameters
- Finite-temperature Hartree-Fock unit cell calculations
  - Bonche & Vautherin 1980, Wolff & Hilledbrandt (ca. 1985)

    Not well documented, only 1 set of nuclear parameters

Lattimer & Swesty 1991, *NPA* **535**, *331*

- Based on Lattimer, Pethick, Ravenhall & Lamb [NPA **432**, 646, (1985)] liquid droplet model merged with bulk equilibrium
- Free energy density is minimized

$$F = un_i[f_i + f_{surf} + f_{Coul} + f_{trans}] + (1 - u - n_N n_\alpha v_\alpha) f_o + n_\alpha f_\alpha (1 - u)$$
$$\mu_{no} = \mu_{ni} + \Delta_n, \quad \mu_{po} = \mu_{pi} + \Delta_p, \quad P_o = P_i + \Delta_P, \quad f_{surf} = 2 f_{Coul}$$

- f_i, f_o from non-relativistic potential (Skyrme-like) model
- f_{surf} from semi-infinite plane-parallel calculations using f_i, f_o and gradient contributions, but ignoring Coulomb effects
- f_{Coul}, f_{trans} from liquid-drop model including lattice effects
- f_α for Maxwell-Boltzmann particles to represent "light" nuclei
- uniform densities inside and outside nucleus
- L-S ignores "neutron skin" and nucleon effective masses to simplify minimization although LPRL includes these
- Phase transition to uniform matter treated with Maxwell construction
- L-S includes minimization wrt nuclear shape (i.e., nuclear pasta)
- LPRL assumed SI′ model; L-S contains arbitrary parameters to match input incompressibility and bulk and surface symmetry energies
- New tables including neutron skin and various nuclear parameter sets available `www.astro.sunysb.edu/lattimer`

Shen, Toki, Oyamatsu & Sumiyoshi 1998,

PTP **100**, *1013*

- Based on Oyamatsu [NPA **561**, 431 (1993)]
- Thomas-Fermi spherical cell
- Relativistic field-theoretical (RFT) model with σ, ω, ρ mesons
- Self-consistent gradient contributions to RFT energy density replaced with a single-parameter, ad-hoc density-gradient term with no symmetry dependence
- Full Coulomb energies included
- Alpha particles represented as Maxwell-Boltzmann particles
- Incomplete energy minimization using parametrized Fermi-like nucleon density radial profiles
- Phase transition to uniform matter ignored, but low density of table points makes this largely irrelevant
- Variations of nuclear shapes ignored
- Table exists for just one set of RFT parameters

Current problems

L-S

- α-particle binding energy error
- Unity effective masses underestimate nuclear specific heats
- Doesn't work well for extremely small temperatures and proton fractions and some points near critical temperature/density (convergence issues)
- Ignores neutron skin and Coulomb corrections to surface energy
- Extension to other nuclear models restricted by physical labor involved in computation of phase boundaries

Shen et al.

- table relatively sparse; not possible to implement "thermodynamically consistent" table generation scheme (Swesty & Timmes)
- Incomplete energy minimization may make the table inherently thermodynamically inconsistent
- Tables for alternate incompressibility and symmetry parameters not available
- Inconsistent surface energies with no symmetry dependence (possibly reflected in anomalously small neutron skin thicknesses)
- Does not consider aspherical geometries

Current improvements to L-S

- α-particle binding energy error corrected (comparison to NSE calculations at low density satisfactory [Hix])
- Nuclear force generalized for arbitrary effective masses, both NRP and RFT models utilized
- Re-introduction of neutron skin
- Energy minimized without algebraic substitutions results in relatively automatic table generation – fewer convergence issues
- Works to very low temperatures and electron fractions
- Technical problems exist for $Y_e > 0.5$
- Two finely gridded tables are generated to identify table points within the phase transition region and replace appropriate values

Nuclear Structure Considerations

Information about E_{sym} can be extracted from nuclear binding energies and models for nuclei. For example, consider the schematic liquid droplet model (Myers & Swiatecki):

$$E(A,Z) \simeq -a_v A + a_s A^{2/3} + \frac{S_v}{1+(S_s/S_v)A^{-1/3}} A + a_c Z^2 A^{-1/3}$$

Fitting binding energies results in a strong correlation between S_v and S_s, but not definite values.

Blue: $\Delta E < 0.01$ MeV/b

Green: $\Delta E < 0.02$ MeV/b

Gray: $\Delta E < 0.03$ MeV/b

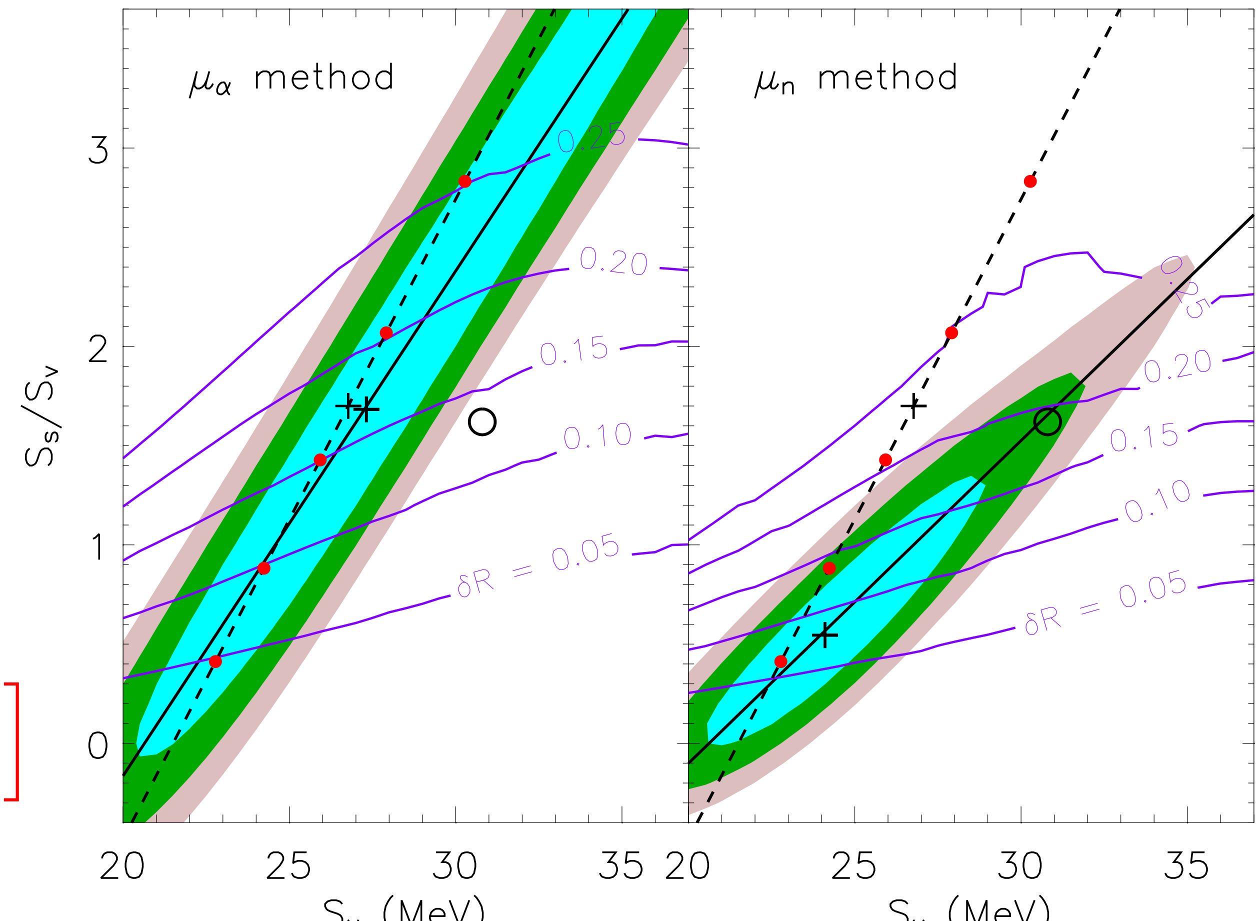
Circle: Moeller et al. (1995)

Crosses: Best fits

Dashed: Danielewicz (2004)

Solid: Steiner et al. (2005)

$$\alpha_c \rightarrow \alpha_c \left[1 + \frac{2 S_s Z I A^{1/3}}{3 S_v}\right] \times \left[1 + \frac{S_s}{S_v A^{1/3}}\right]^{-1}$$



Finite-Range Thomas-Fermi Model

Based on Seylar-Blanchard and Myers & Swiatecki [AP, **204**, 401 (1990)], but extended to finite temperature and Wigner-Seitz approximation

$$
\begin{aligned}
W &= -\frac{1}{h^3}\int d^3r_1 \int d^3r_2 f\left(\frac{r_{12}}{a}\right) \times \\
&\quad \sum_{t1,t2,t'2=n,p}\left[\int\int C_L f_{t1} f_{t2} d^3p_{t1} d^3p_{t2} + \int\int C_U f_{t1} f_{t'2} d^3p_{t1} d^3p_{t'2}\right] \\
C_{(L,U)} &\propto \alpha_{(L,U)} - \beta_{(L,U)}\left(\frac{p_{12}}{P_o}\right)^2 - \sigma_{(L,U)}\left(\frac{2\bar\rho}{\rho_o}\right)^{2/3} \\
f\left(\frac{r_{12}}{a}\right) &= \frac{1}{4\pi r_{12} a^2} e^{-r_{12}/a}, \quad r_{12} = |\vec{r_1} - \vec{r_2}|, \quad \int d^3r_2 f\left(\frac{r_{12}}{a}\right) = 1
\end{aligned}
$$

In contrast, Skyrme force Hamiltonian:

$\mathcal{H}_{Skyrme} = \mathcal{H}_{uniform}(n,\tau) + \sum_{i,j=n,p} Q_{ij} \nabla n_i \cdot \nabla n_j$

Energy minimization – Euler equations

FRTF

Integral equations:

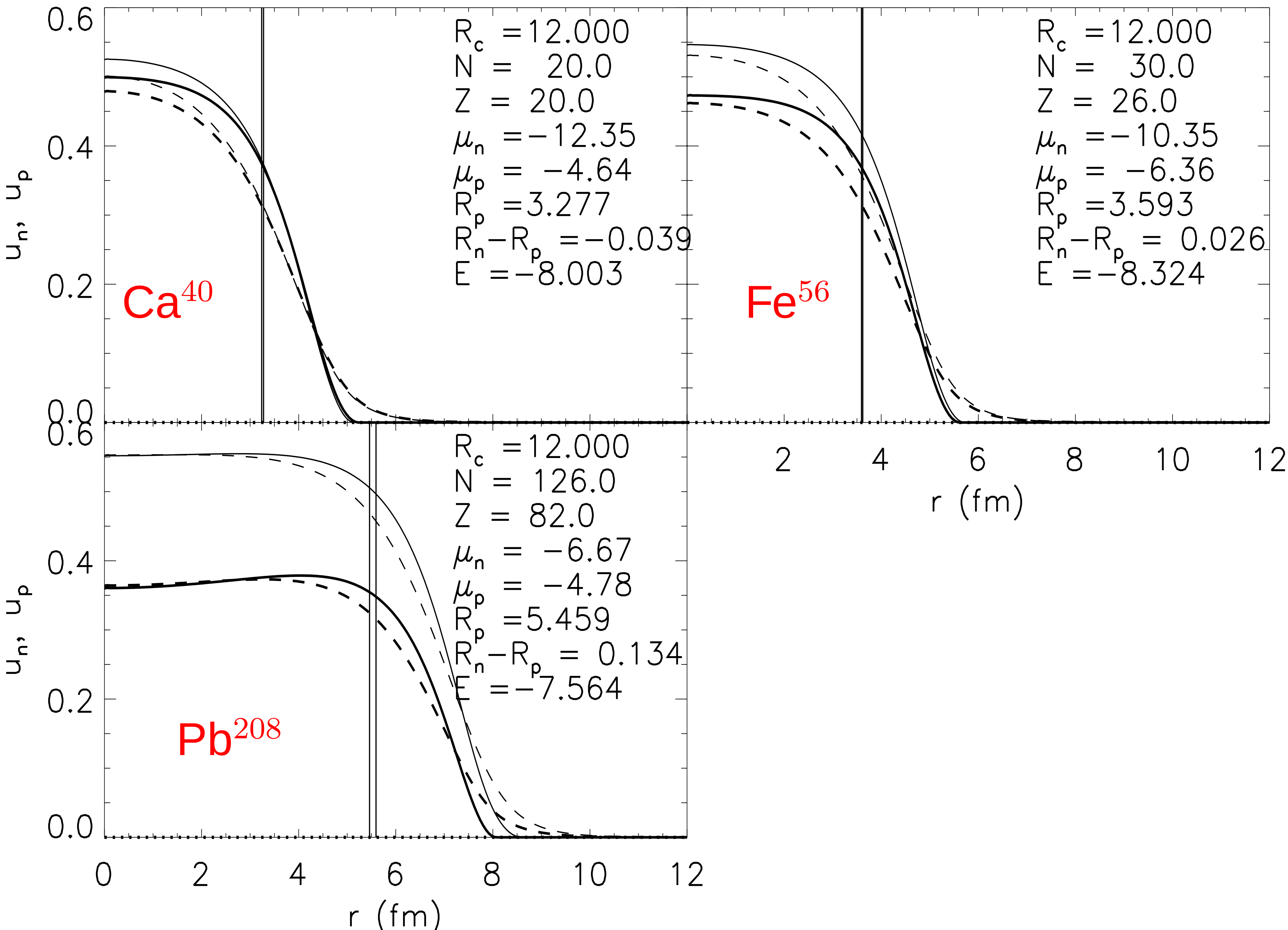
$$\mu_n(r) = \text{ constant}, \qquad \mu_p(r) = \text{ constant}$$

NRP

Differential equations:

$$\sum_{j=n,p} Q_{ij} \nabla^2 n_j = \frac{\partial \mathcal{H}_{uniform}}{\partial n_i} - \mu_{i0}$$

Finite-Range Thomas-Fermi Nuclear Model



Nuclei in Dense Matter