Recent developments in Monte Carlo studies of superstring theory

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“Current Themes in High Energy Physics and Cosmology”

Niels Bohr Institute, Copenhagen

Large-N matrices

fundamental dynamical degrees of freedom
of string theory in its nonperturbative formulation

- double scaling limit in one-matrix model
  as a nonperturbative formulation of noncritical string theories

Brezin-Kazakov, Douglas-Shenker, Gross-Migdal (1990)

More recently,

- gauge-gravity duality
  Maldacena ('97), Gubser-Klebanov-Witten('98), …
- matrix model for superstring/M theory
  Banks-Fischler-Shenker-Susskind, ('96)
  Ishibashi-Kawai-Kitazawa-Tsuchiya ('96)

Emergence of space-time from matrices

Monte Carlo studies of supersymmetric $\left\{\begin{array}{l}\text{gauge theories}\\ \text{matrix models}\end{array}\right.$

Understanding of the Planck scale physics such as

- black holes (black branes, in general)

  the microscopic origin of their themodynamic properties
  effects of quantum gravity
  Hawking radiation and information loss paradox

- early universe

  birth of the universe (SSB from 9d to 3d)
  inflation, density fluctuation
  emergence of the Standard Model (and beyond)

The models we study are:

1) D0-brane system

$$S = \frac{1}{g^2} \int dt \, \mathrm{tr} \left\{ \frac{1}{2} (DX_i(t))^2 - \frac{1}{4} [X_i(t), X_j(t)]^2 \right\} + (\text{fermions})$$

dual to a black hole solution in type IIA SUGRA

2) plane wave matrix model

$$S = \frac{1}{g^2} \int dt \, \mathrm{tr} \left\{ \frac{1}{2} (DX_i(t))^2 - \frac{1}{4} [X_i(t), X_j(t)]^2 \right\}$$
$$+ \frac{1}{2} \mu^2 \sum_{i=1}^{3} (X_i)^2 + \frac{1}{8} \mu^2 \sum_{a=4}^{9} (X_a)^2 + i \mu \epsilon_{ijk} X_i X_j X_k + (\text{fermions})$$

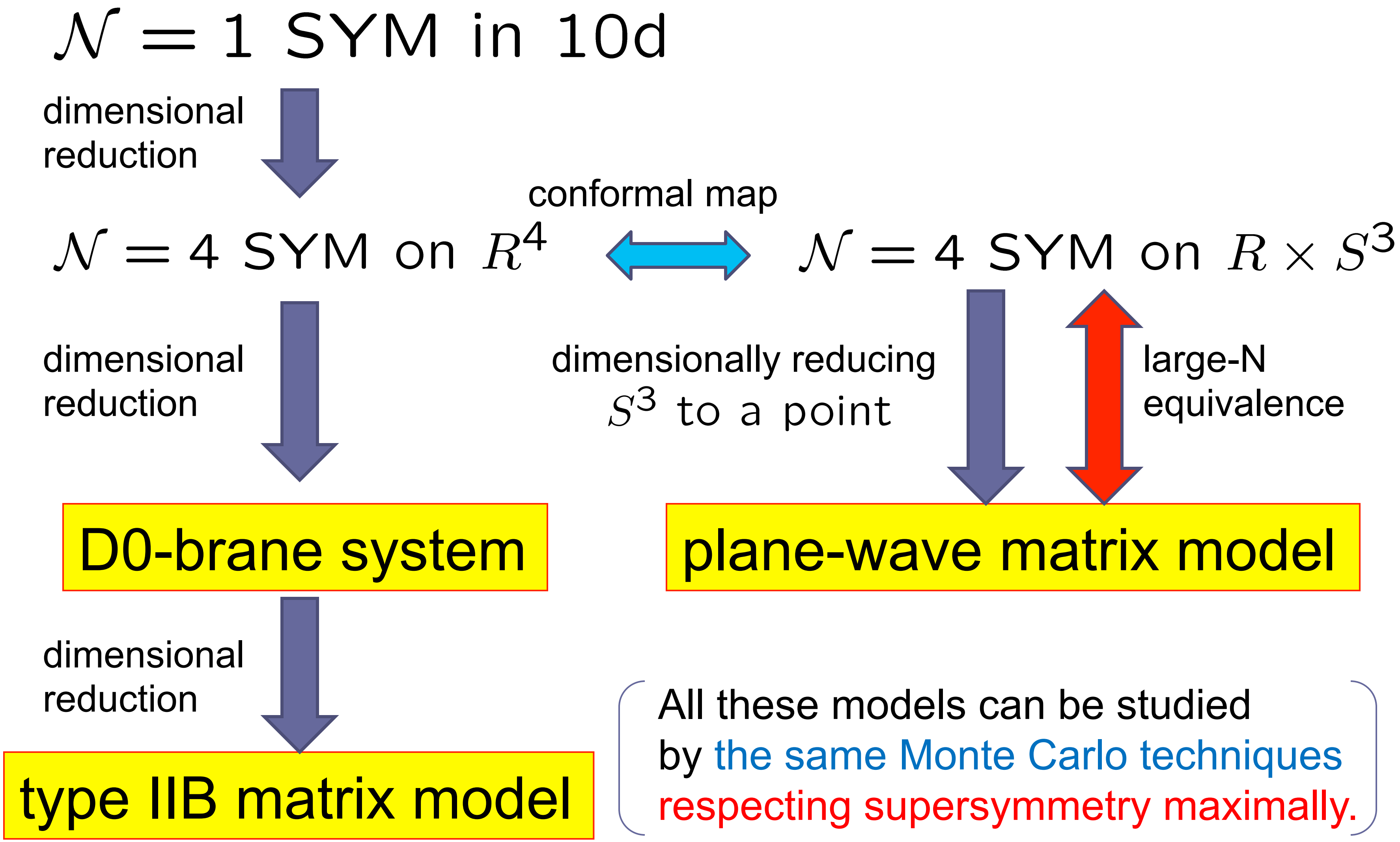
a nonperturbative formulation of $\mathcal{N} = 4$ SYM in 4d

3) type IIB matrix model

$$S = -\frac{1}{4g^2} \mathrm{tr}([A_\mu, A_\nu][A^\mu, A^\nu]) + (\text{fermions})$$

a nonperturbative formulation of superstring theory

Relationship among the models



$\mathcal{N}=1$ SYM in 10d

dimensional reduction

$\mathcal{N}=4$ SYM on R^4

conformal map

$\mathcal{N}=4$ SYM on $R\times S^3$

dimensional reduction

dimensionally reducing S^3 to a point

large-N equivalence

D0-brane system

plane-wave matrix model

dimensional reduction

type IIB matrix model

All these models can be studied by the same Monte Carlo techniques respecting supersymmetry maximally.

Plan of the talk

1. Introduction
2. Direct test of gauge/gravity duality in D0-brane system
3. Direct test of AdS/CFT duality using the large-N reduced model of $\mathcal{N}=4$ SYM
4. Studies of early Universe in type IIB matrix model
5. Summary and future prospects

2. Direct test of gauge/gravity duality in D0-brane system

Direc test of gauge/gravity duality

Difficult because the gauge theory side is strongly coupled.

D0 brane case Itzhaki-Maldacena-Sonnenschein-Yankielowicz ('98)

1d SYM with 16 supercharges

Monte Carlo simulation

- internal energy v.s. temperature

  microscopic origin of black hole thermodynamics

  α' corrections, string loop corrections

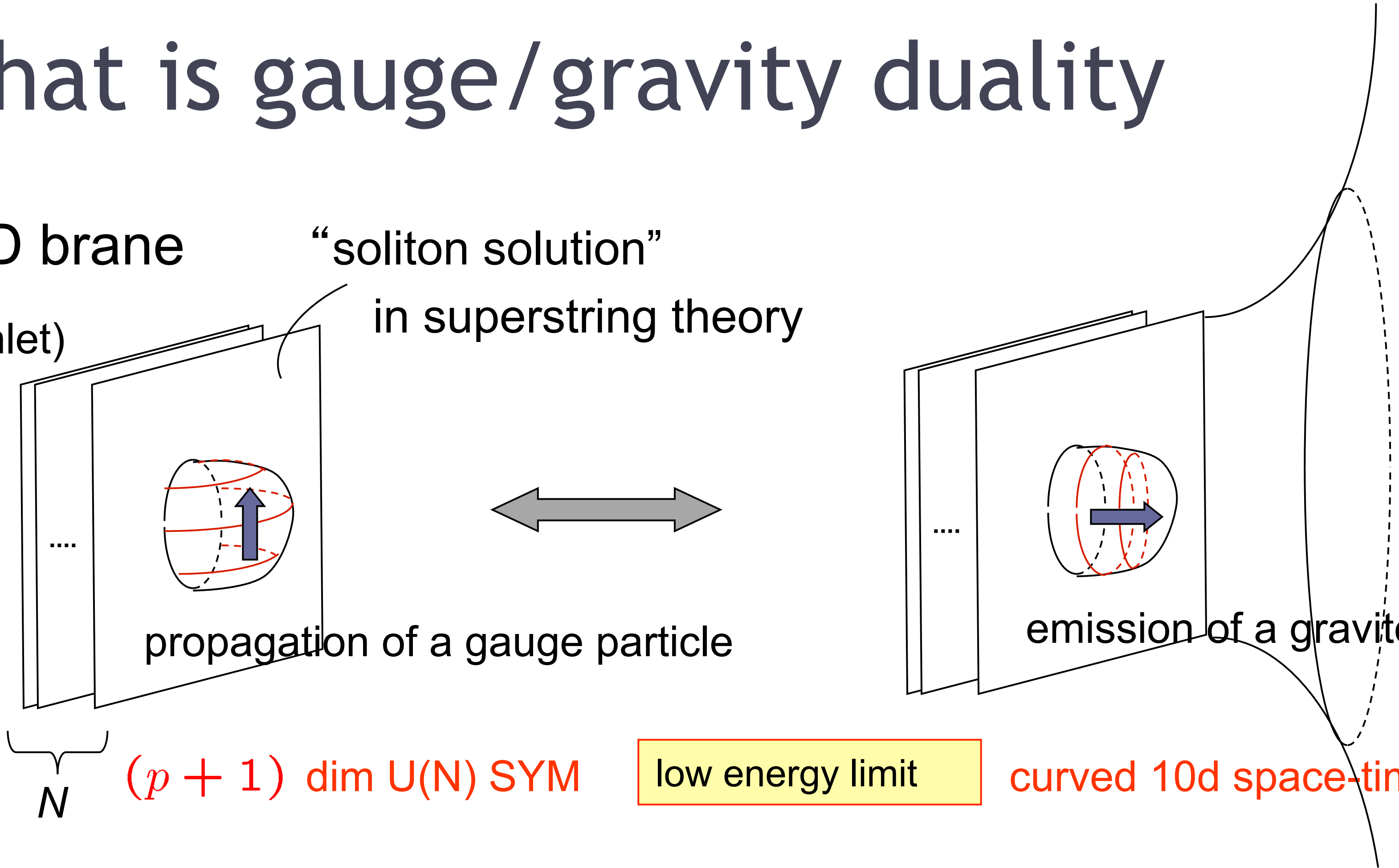
- Wilson loop

  One can see Schwarzschild radius from gauge theory!

- correlation functions

  confirmation of the GKP-Witten prescription

What is gauge/gravity duality



D brane

(Dirichlet)

"soliton solution"

in superstring theory

propagation of a gauge particle

emission of a graviton

$(p+1)$ dim U(N) SYM

low energy limit

curved 10d space-time

't Hooft $N \to \infty$ limit

($\lambda \equiv g^2_{\text{YM}} N$: fixed)

large λ

string loop corrections suppressed by $1/N$

α' corrections suppressed by $1/\lambda^q$

D0-brane system : 1d SYM with 16 SUSY

$$S_{\mathrm{b}} = \frac{1}{g^2}\int_0^\beta dt\ \mathrm{tr}\left\{\frac{1}{2}(DX_i(t))^2 - \frac{1}{4}[X_i(t), X_j(t)]^2\right\}$$

$$S_{\mathrm{f}} = \frac{1}{g^2}\int_0^\beta dt\ \mathrm{tr}\left\{\frac{1}{2}\Psi_\alpha D\Psi_\alpha - \frac{1}{2}\Psi_\alpha(\gamma_i)_{\alpha\beta}[X_i, \Psi_\beta]\right\}$$

1d gauge theory $D = \partial_t - i\,[A(t),\ \cdot\ ]$

$$\begin{cases} X_j(t) & (j = 1, \cdots, 9) \\ \Psi_\alpha(t) & (\alpha = 1, \cdots, 16) \end{cases}$$

p.b.c.

anti p.b.c.

$T = \beta^{-1}$ temperature

$\lambda = g^2 N$ 't Hooft coupling

$$\lambda_{\mathrm{eff}} = \frac{\lambda}{T^3}$$

$\lambda = 1$ (without loss of generality)

- low T ➡ strongly coupled dual gravity description
- high T ➡ weakly coupled high T exp.

Kawahara-J.N.-Takeuchi ('07)

D0 brane solution (gravity side)

After taking the decoupling limit : $\alpha' \to 0$

$$U \equiv \frac{r}{\alpha'} \quad , \quad \lambda \equiv g_s N {\alpha'}^{-3/2} \quad \text{(fixed)}$$

$$ds^2 = \alpha' \left\{ f(U) dt^2 + \frac{1}{f(U)} dU^2 + \sqrt{d_0 \lambda} U^{-3/2} d\Omega^2_{(8)} \right\}$$

$$f(U) \equiv \frac{U^{7/2}}{\sqrt{d_0 \lambda}} \left\{ 1 - \left(\frac{U_0}{U} \right)^7 \right\}$$

range of validity: $N^{-10/21} \ll \left(\frac{U}{\lambda^{1/3}} \right)^{5/2} \ll 1$

Black hole thermodynamics

- D0 brane solution

$$\begin{cases} \text{Hawking temperature :} & \dfrac{T}{\lambda^{1/3}} = \dfrac{7}{16\sqrt{15}\pi^{7/2}} \left(\dfrac{U_0}{\lambda^{1/3}}\right)^{5/2} \\ \text{Bekenstein-Hawking entropy :} & \mathcal{S} = \dfrac{1}{28\sqrt{15}\pi^{7/2}} N^2 \left(\dfrac{U_0}{\lambda^{1/3}}\right)^{9/2} \end{cases}$$

$$\Rightarrow \quad \frac{1}{N^2}\frac{E}{\lambda^{1/3}} = \underbrace{\frac{9}{14}\left\{4^{13}15^2\left(\frac{\pi}{7}\right)^{14}\right\}^{1/5}}_{7.41} \left(\frac{T}{\lambda^{1/3}}\right)^{14/5}$$

Klebanov-Tseytlin ('96)

range of validity for gauge/gravity duality:

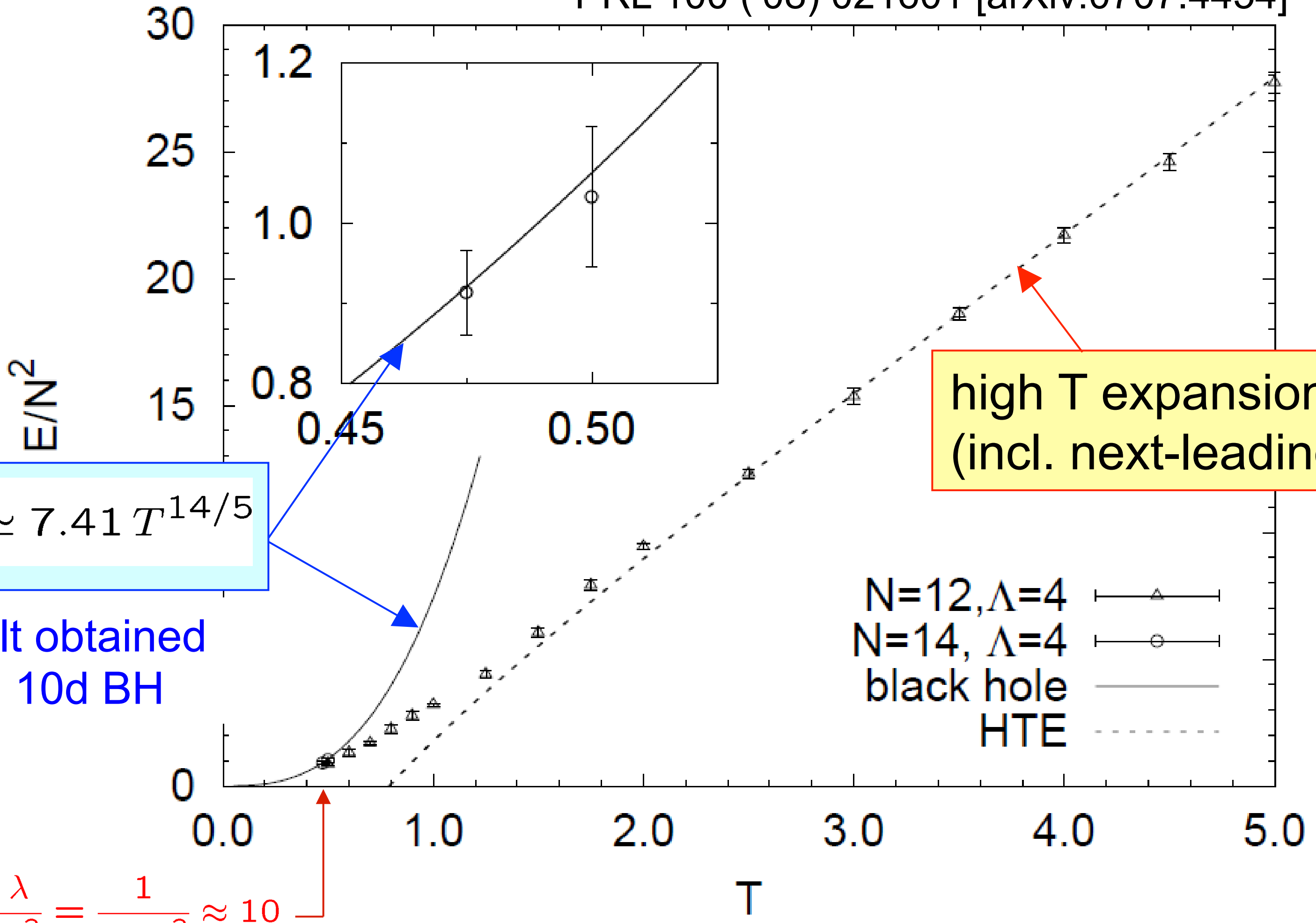
$$N^{-10/21} \ll \frac{T}{\lambda^{1/3}} \ll 1$$

Internal energy

$$E = \frac{\partial}{\partial \beta}(\beta \mathcal{F})$$

free energy

Anagnostopoulos-Hanada- J.N.-Takeuchi,
PRL 100 ('08) 021601 [arXiv:0707.4454]



high T expansion
(incl. next-leading order)

$$\frac{E}{N^2} \simeq 7.41\, T^{14/5}$$

result obtained
from 10d BH

$$\lambda_{\rm eff} = \frac{\lambda}{T^3} = \frac{1}{0.46^3} \approx 10$$

α' corrections to SUGRA action

low energy effective action of type IIA superstring theory

⇦ tree-level scattering amplitudes of the massless modes

leading term : type IIA SUGRA action

$$\mathcal{S}_{(0)} = \frac{1}{16\pi G_{\mathrm{N}}} \int d^{10}x\sqrt{-g}\,\left\{ \mathrm{e}^{-2\phi}(R + 4\partial_\mu\phi\partial^\mu\phi) - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} \right\}$$

$$G_{\mathrm{N}} \sim \alpha'^4 g_s^2$$

explicit calculations of 2-pt and 3-pt amplitudes

$$\Rightarrow \mathcal{S}_{(1)} = \mathcal{S}_{(2)} = 0$$

4-pt amplitudes ⇨ $\mathcal{S}_{(3)} = \frac{\alpha'^3}{16\pi G_{\mathrm{N}}} \int d^{10}x\sqrt{-g}\,\left\{ \mathrm{e}^{-2\phi}\mathcal{R}^4 + \cdots \right\}$

Complete form is yet to be determined,
but we can still make a dimensional analysis.

Black hole thermodynamics with α' corrections

curvature radius of the dual geometry

$$\rho^2 \sim \left(\frac{\lambda^{1/3}}{U_0}\right)^{3/2} \alpha'$$

α' corrections

$$\Rightarrow \quad \frac{\alpha'}{\rho^2} \sim \left(\frac{U_0}{\lambda^{1/3}}\right)^{3/2} \sim \left(\frac{T}{\lambda^{1/3}}\right)^{3/5} \qquad \frac{T}{\lambda^{1/3}} \sim \left(\frac{U_0}{\lambda^{1/3}}\right)^{5/2}$$

corrections at ${\alpha'}^3$ order gives

$$\frac{1}{N^2}\frac{E}{\lambda^{1/3}} = 7.41 \left(\frac{T}{\lambda^{1/3}}\right)^{14/5} \left\{1 + \boxed{c\left(\frac{T}{\lambda^{1/3}}\right)^{9/5}}\right\}$$

More careful treatment leads to the same conclusion.
(Hanada-Hyakutake-J.N.-Takeuchi, PRL 102 ('09) 191602

Setting $\lambda = 1$,

$$\frac{E}{N^2} = 7.41\, T^{14/5} - C\, T^{23/5}$$

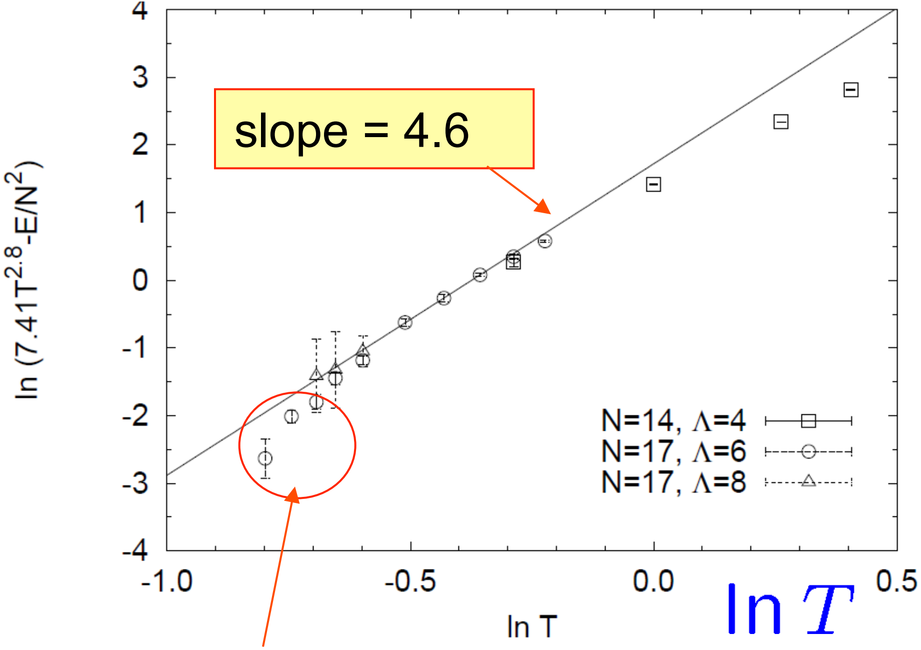
Comparison including α' corrections

Hanada-Hyakutake-J.N.-Takeuchi,
PRL 102 ('09) 191602 [arXiv:0811.3102]

$$\frac{E}{N^2} = 7.41\,T^{14/5} - C\,T^{23/5}$$

α' corrections

$\ln\left(7.41\,T^{14/5} - \frac{E}{N^2}\right)$

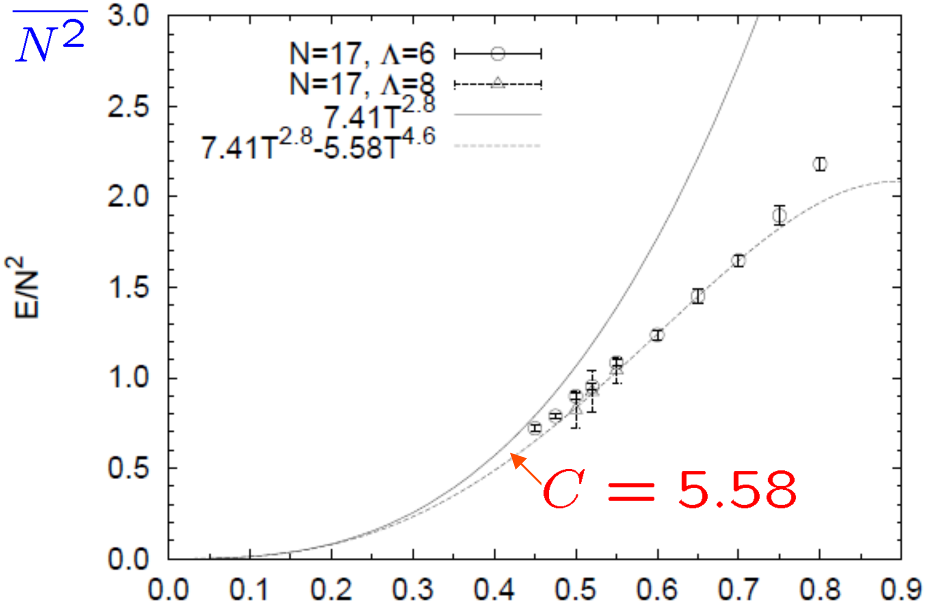


slope = 4.6

finite cutoff effects

$\ln T$

$\frac{E}{N^2}$



$C = 5.58$

T

MC data at $T \lesssim 0.7$ can be nicely fitted with $C = 5.58$

String loop corrections to SUGRA action

known from IIA string theory

1-loop $$g_{\mathrm{str}}{}^2\left(\frac{\alpha'}{\rho^2}\right)^{\textcircled{3}} \sim \frac{1}{N^2}\left(\frac{T}{\lambda^{1/3}}\right)^{-12/5}$$

Bern, Rozowsky, Yan

Bern, Dixon, Dunbar, Perelstein, Rozowsky

2-loop $$g_{\mathrm{str}}{}^4\left(\frac{\alpha'}{\rho^2}\right)^{\textcircled{5}} \sim \frac{1}{N^4}\left(\frac{T}{\lambda^{1/3}}\right)^{-27/5}$$

Using Kawai-Lewellen-Tye relation to SYM amp.

$$g_{\mathrm{str}} \sim \frac{1}{N}\left(\frac{T}{\lambda^{1/3}}\right)^{-21/10} \qquad \frac{\alpha'}{\rho^2} \sim \left(\frac{T}{\lambda^{1/3}}\right)^{3/5}$$

$$\frac{1}{N^2}\frac{E}{\lambda^{1/3}} = 7.41\left(\frac{T}{\lambda^{1/3}}\right)^{14/5}\left\{1+\frac{a}{N^2}\left(\frac{T}{\lambda^{1/3}}\right)^{-12/5}+\frac{b}{N^4}\left(\frac{T}{\lambda^{1/3}}\right)^{-27/5}\right\}$$

$$\sim 7.41\left(\frac{T}{\lambda^{1/3}}\right)^{14/5}+\frac{A}{N^2}\left(\frac{T}{\lambda^{1/3}}\right)^{2/5}+\boxed{\frac{B}{N^4}\left(\frac{T}{\lambda^{1/3}}\right)^{-13/5}}$$

negative specific heat !

Only meta-stable vacuum at finite N (flat directions)

small *N*, very low *T* regime

1/N corrections ⟺ string loop corrections

$$\frac{E}{N^2} = 7.41T^{2.8} - 5.58T^{4.6} + \frac{1}{N^2}(-\boxed{5.76}T^{0.4} + aT^{2.2} + \dots) + \frac{1}{N^4}(bT^{-2.6} + cT^{-0.8} + \dots) + \mathcal{O}\left(\frac{1}{N^6}\right)$$

fixed by explicit calculations
on the string theory side
(Hyakutake, to appear)

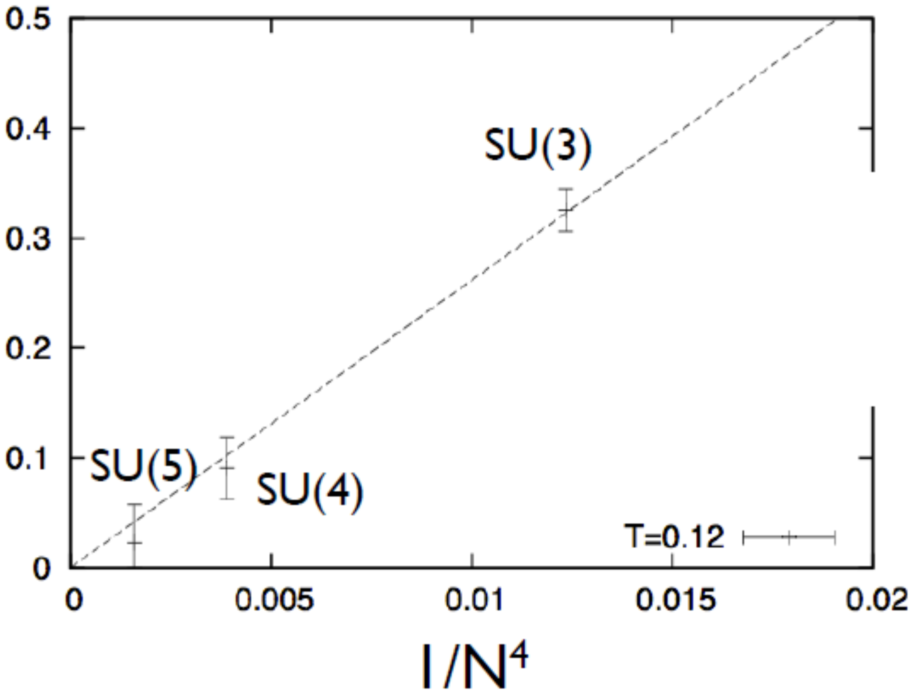
We can test it by looking at:

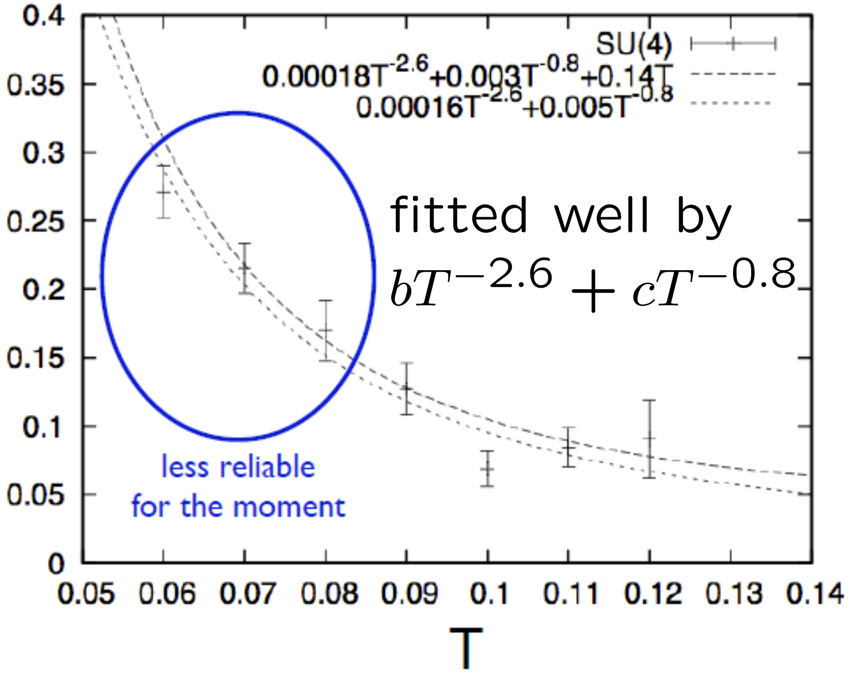
$$\frac{E}{N^2} - \left(7.41T^{2.8} - 5.58T^{4.6} - 5.76\frac{T^{0.4}}{N^2}\right) \text{ v.s. } \frac{1}{N^4}$$

Testing the string loop corrections

Ishiki-Hanada-Hyakutake-J.N., in preparation

$$\frac{E}{N^2} - \left(7.41T^{2.8} - 5.58T^{4.6} - 5.76\frac{T^{0.4}}{N^2}\right)$$



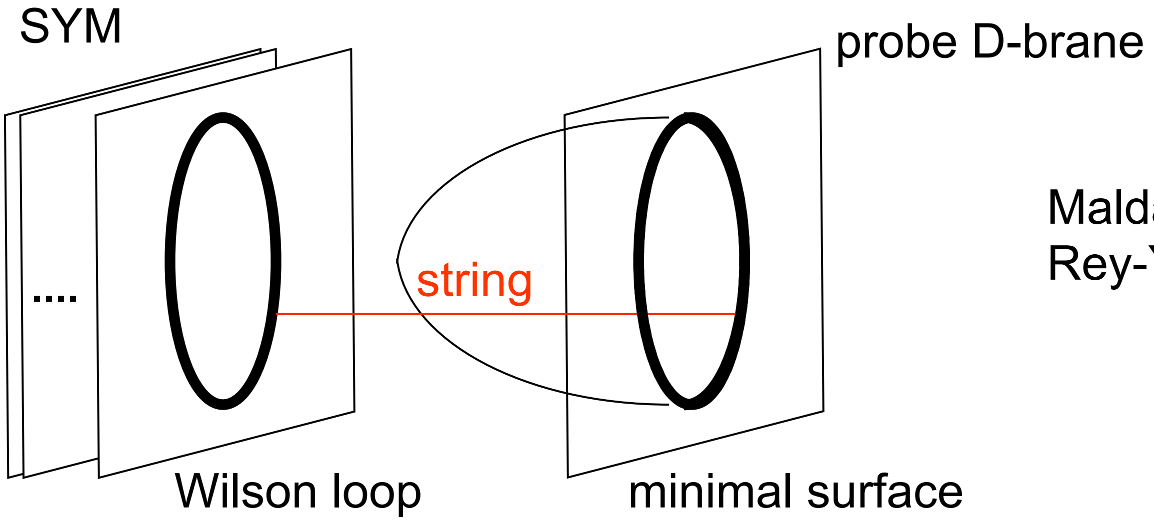


fitted well by $bT^{-2.6} + cT^{-0.8}$

less reliable
for the moment

MC data are indeed consistent with string loop corrections !

Calculating Wilson loop from gravity



Maldacena ('98)
Rey-Yee ('98)

- Schwarzschild radius from Wilson loop (in 1d SYM)

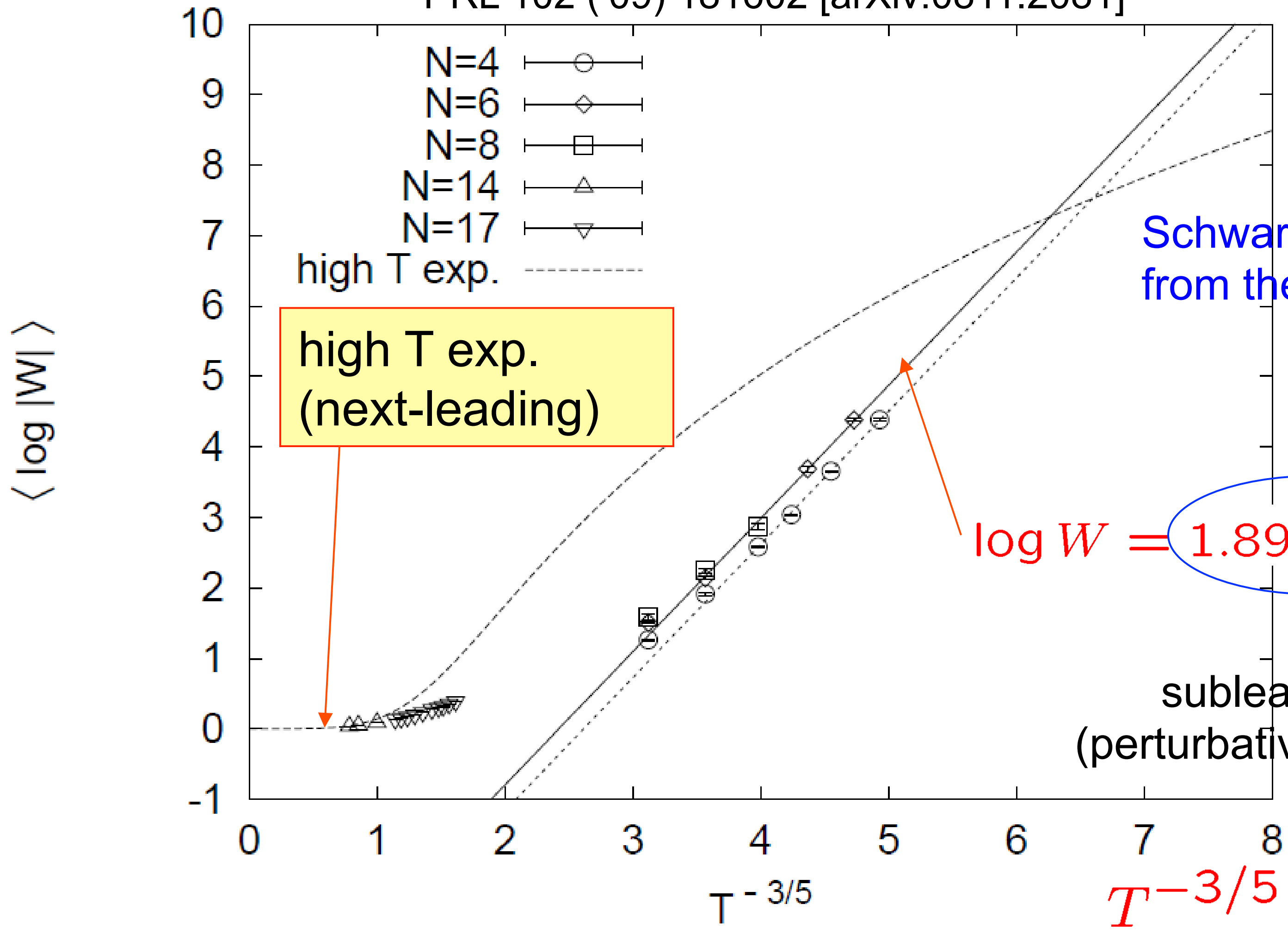
$$W \equiv \mathrm{tr}\,\mathcal{P}\exp\left[i\int_0^\beta dt\{A(t)+\boxed{iX_9(t)}\}\right] \sim \exp\left(\frac{\beta R_{\mathrm{Sch}}}{2\pi\alpha'}\right)$$

$$\ln W = \frac{\beta R_{\mathrm{Sch}}}{2\pi\alpha'} = \underbrace{\frac{1}{2\pi}\left\{\frac{16\sqrt{15}\pi^{7/2}}{7}\right\}^{2/5}}_{1.89}\left(\frac{T}{\lambda^{1/3}}\right)^{-3/5}$$

Wilson loops

$$W = \mathrm{tr}\,\mathcal{P}\exp\left[i\int_0^\beta dt\{A(t)+iX_9(t)\}\right]$$

Hanada-Miwa-J.N.-Takeuchi,
PRL 102 ('09) 181602 [arXiv:0811.2081]



high T exp.
(next-leading)

Schwarzschild radius
from the Wilson loop

$$\beta\,\frac{R_{\mathrm{Sch}}}{2\pi\alpha'}$$

$$\log W = 1.89\,T^{-3/5} - 4.58$$

subleading term
(perturbative corrections)

$T^{-3/5}$

Correlation functions in gauge/gravity duality

correlation functions in gauge theory

$$\langle \mathcal{O}(0)\mathcal{O}(t)\rangle \Longleftarrow \quad Z[J]=\left\langle \exp\left\{\int dt\, J(t)\,\mathcal{O}(t)\right\}\right\rangle$$

generating functional

operator-field correspondence

gauge gravity

$$\mathcal{O}(t) \Longleftrightarrow \phi(t,z)$$

Gubser-Klebanov-Polyakov-Witten relation ('98)

$$Z[J]=\exp\{-I[\phi(t,0)=J(t)]\}$$

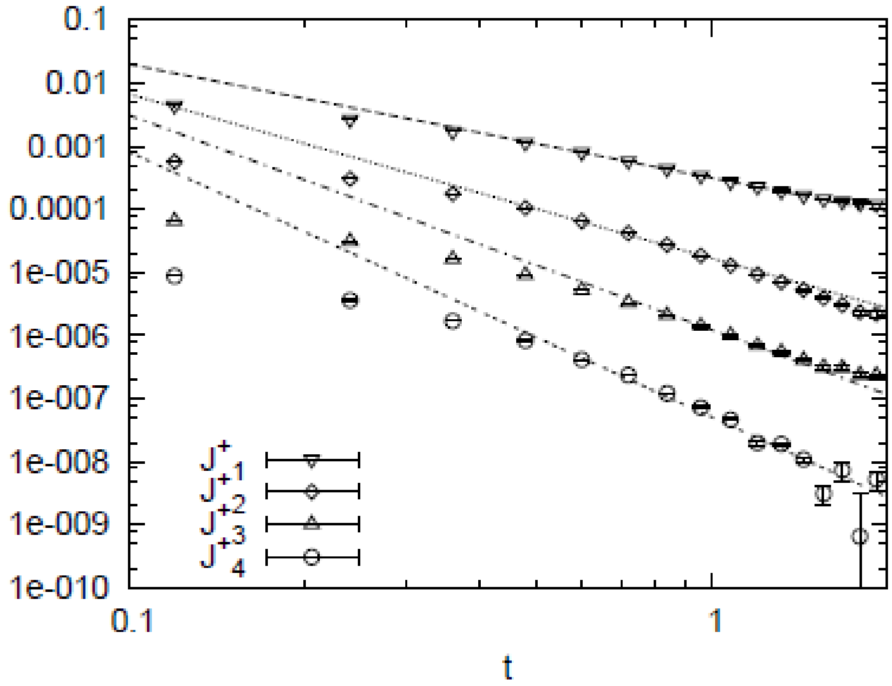
SUGRA action evaluated at the classical solution with the boundary condition $\phi(t,0)=J(t)$

Correlation functions

Hanada-J.N.-Sekino-Yoneya,
PRL 104 ('10) 151601,
JHEP 12(2011) 020

$$\langle \mathcal{O}(t)\mathcal{O}(t')\rangle \propto \frac{1}{|t-t'|^{2\nu+1}}$$

predicted from dual geometry by Sekino-Yoneya ('99)
using Gubser-Klebanov-Polyakov-Witten relation ('98)



$$J^{+ij}_{l,i_1,\cdots,i_l} \equiv \frac{1}{N}\mathrm{Str}\left(F_{ij}X_{i_1}\cdots X_{i_l}\right)$$

$$F_{ij} \equiv -i[X_i, X_j]$$

$$\nu = \frac{2l}{5}$$

$l = 1$

$l = 2$

$l = 3$

$l = 4$

Predicted power law confirmed clearly.

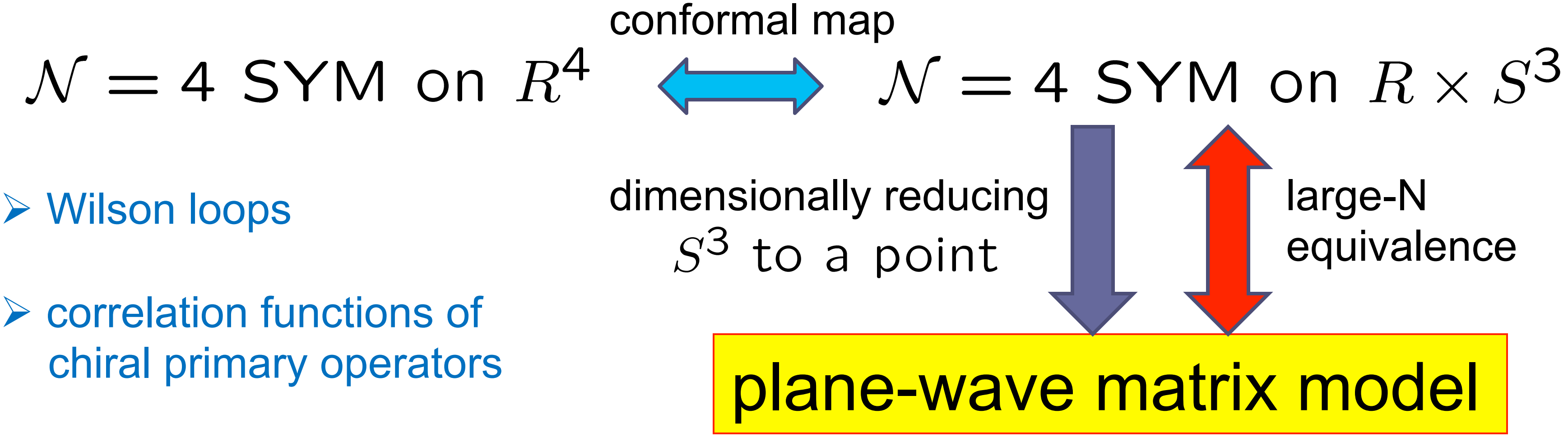
$$N = 3,\ \beta = 4,\ \Lambda = 16$$

3. Direct test of AdS/CFT duality using the large-N reduced model of SYM $\mathcal{N} = 4$

AdS/CFT correspondence

Maldacena ('97)

- gauge/gravity duality in the case of D3-brane
  4d $\mathcal{N}=4$ SYM superconformal (32 supercharges)
- confirmed by calculations of anomalous dimensions
  using spin-chains and assuming integrability
- We attempt a direct test of AdS/CFT from first principles
  by Monte Carlo studies of the large-N reduced model



conformal map

$\mathcal{N}=4$ SYM on R^4 $\Longleftrightarrow$ $\mathcal{N}=4$ SYM on $R\times S^3$

dimensionally reducing
S^3 to a point

large-N
equivalence

plane-wave matrix model

- ➢ Wilson loops
- ➢ correlation functions of
  chiral primary operators

Large N reduction

Eguchi-Kawai ('81)
Bhanot-Heller-Neuberger ('82)
Gonzalez-Arroyo & Okawa ('82)

U(N) gauge theory in D-dim. torus

$A_\mu(x)$

$$W[C] = \left\langle \mathcal{P}\mathrm{exp}\left(i \int A_\mu(x(\sigma))\, \dot{x}^\mu(\sigma)\, d\sigma \right) \right\rangle$$

$$C = \{x^\mu(\sigma)\}$$

reduce
to a point

large-N reduced model

A_μ

$$W[C]_{\mathrm{red}} = \left\langle \mathcal{P}\mathrm{exp}\left(i \int A_\mu\, k^\mu(\sigma)\, d\sigma \right) \right\rangle$$

$$k^\mu(\sigma) = \dot{x}^\mu(\sigma)$$

$$\lim_{N\to\infty} W[C] = \lim_{N\to\infty} W[C]_{\mathrm{red}}$$

A novel large N reduction for $\mathcal{N} = 4$ SYM on $R \times S^3$

Ishiki-Ishii-Shimasaki
-Tsuchiya ('08)

reduce
to a point

$$\boxed{R_{S^3} = \frac{2}{\mu}}$$

1d SYM + mass deformation
(preserving 16 SUSY)

$$\int dt\, \mathrm{tr} \left[\frac{1}{2}\mu^2 \sum_{i=1}^{3} (X_i)^2 + \frac{1}{8}\mu^2 \sum_{a=4}^{9} (X_a)^2 + i\mu\epsilon_{ijk}X_iX_jX_k + \frac{3}{8} i\mu\Psi\gamma_{123}\Psi \right]$$

many classical vacua preserving 16 SUSY

$$X_i = \mu L_i \qquad [L_i, L_j] = i\,\epsilon_{ijk}\, L_k$$

all degenerate

A novel large N reduction for $\mathcal{N}=4$ SYM on $R\times S^3$ (cont'd)

Ishiki-Ishii-Shimasaki-Tsuchiya ('08)

$$X_i = \mu \begin{pmatrix} L_i^{(1)} & & & \\ & L_i^{(2)} & & \\ & & \ddots & \\ & & & L_i^{(\nu)} \end{pmatrix} \otimes \mathbf{1}_k$$

$L_i^{(n)}$: n-dim. irred. rep. of SU(2) algebra

c.f.) $\sum_{i=1}^{3} (L_i^{(n)})^2 = \frac{1}{4}(n^2-1)\,\mathbf{1}_n$

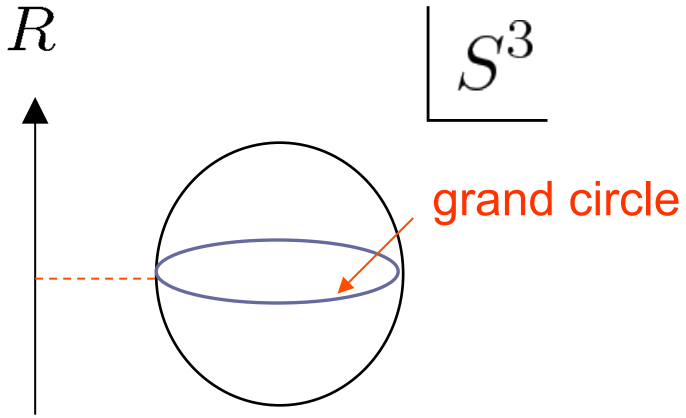
$k\to\infty\,,\ \nu\to\infty$ ⟹ $\mathcal{N}=4$ SYM on $R\times S^3$

$$R_{S^3} = \frac{2}{\mu}\,, \quad \lambda_{\mathrm{SYM}} = 2\pi^2 (R_{S^3})^3 \frac{g^2 k}{(\nu+1)/2}$$

circular Wilson loop in R^4

conformal mapping

$R \times S^3$



$$\langle W(C)\rangle = \sqrt{\frac{2}{\lambda_{\rm SYM}}}\, I_1\left(\sqrt{2\lambda_{\rm SYM}}\right)$$

Erickson-Semenoff-Zarembo ('00),
Drukker-Gross ('00),
Pestun ('07)

$$\simeq \frac{\boxed{e^{\sqrt{2\lambda_{\rm SYM}}}}}{(\frac{\pi}{2})^{1/2}(2\lambda_{\rm SYM})^{3/4}} \quad \text{at } \lambda_{\rm SYM} \gg 1$$

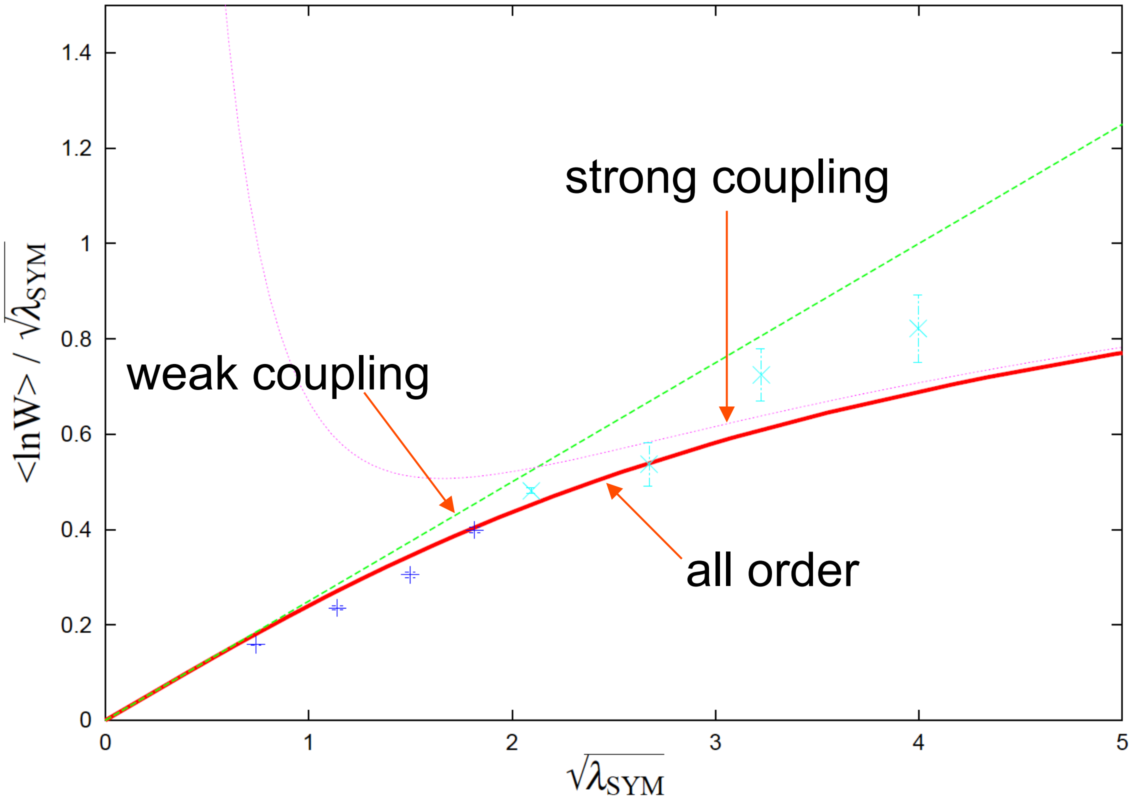
(agree with the result from dual geometry)

Result: circular Wilson loop

(preliminary)

$$\nu = 2 \qquad \begin{pmatrix} \boxed{1} & \\ & \boxed{\;2\;} \end{pmatrix} \otimes \mathbf{1}_k$$

work in progress
Honda-Ishiki-J.N.-Tsuchiya



Correlation functions of chiral primary operators in $\mathcal{N} = 4$ SYM on R^4

$$\mathcal{O}_{ab}(x) = \mathrm{tr}(\phi_a(x)\phi_b(x)) \quad \text{with} \quad a \neq b$$

Conformal symmetry implies:

$$\begin{aligned}
\langle \mathcal{O}_{45}(x_1)\mathcal{O}_{54}(x_2)\rangle &= \frac{\lambda^2}{(4\pi^2)^2}\frac{c^{(2)}(\lambda, N)}{(x_{12})^4} \\
\langle \mathcal{O}_{45}(x_1)\mathcal{O}_{56}(x_2)\mathcal{O}_{64}(x_3)\rangle &= \frac{\lambda^3}{(4\pi^2)^3 N}\frac{c^{(3)}(\lambda, N)}{(x_{12})^2(x_{23})^2(x_{31})^2} \\
\langle \mathcal{O}_{45}(x_1)\mathcal{O}_{56}(x_2)\mathcal{O}_{67}(x_3)\mathcal{O}_{74}(x_4)\rangle &= \frac{\lambda^4}{(4\pi^2)^4 N^2}\frac{c^{(4)}(\lambda, N; u, v)}{(x_{12})^2(x_{23})^2(x_{34})^2(x_{41})^2}
\end{aligned}$$

$$x_{ij} = |x_i - x_j| \ , \qquad u = \frac{(x_{12})^2(x_{34})^2}{(x_{13})^2(x_{24})^2} \ , \qquad v = \frac{(x_{12})^2(x_{34})^2}{(x_{14})^2(x_{23})^2}$$

For free theory, $c^{(2)} = c^{(3)} = c^{(4)} = 1$

Predictions from the AdS/CFT duality

Arutyunov-Frolov ('00)

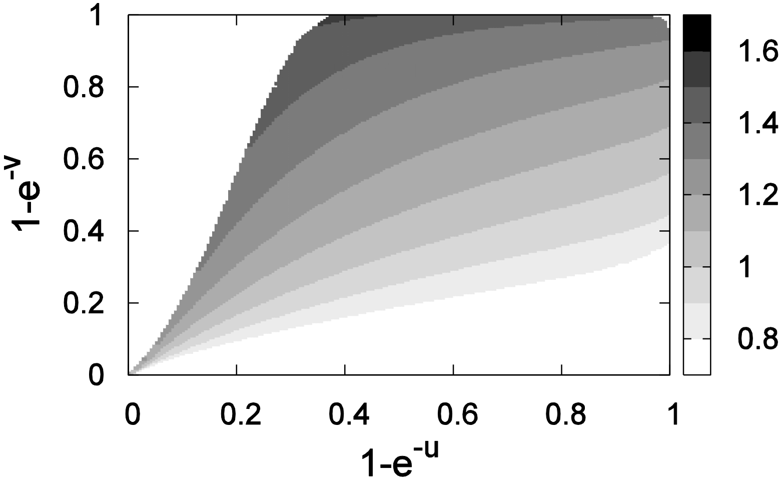
$$\lim_{N\to\infty,\lambda\to\infty} c^{(2)}(\lambda, N) = \zeta$$

$$\lim_{N\to\infty,\lambda\to\infty} c^{(3)}(\lambda, N) = \zeta^{3/2}$$

SUSY non-renormalization property implies: $\zeta = 1$

$$\lim_{N\to\infty,\lambda\to\infty} c^{(4)}(\lambda, N; u, v) = \zeta^2 \, c(u, v)$$

contour plot of $c(u, v)$



Note that $0.80 \lesssim c(u, v) \lesssim 1.66$

AdS/CFT predicts violation of SUSY non-renormalization property for 4-pt function

Conformal map from R^4 to $R \times S^3$

$$
\begin{aligned}
ds^2_{R^4} &= dr^2 + r^2 d\Omega_3{}^2 \\
&= e^{\mu t}\left(dt^2 + \left(\frac{2}{\mu}\right)^2 d\Omega_3{}^2\right) \\
&= e^{\mu t} ds^2_{R\times S^3}
\end{aligned}
$$

(polar coordinates)

$$\boxed{r = \frac{2}{\mu} e^{\frac{\mu}{2}t}}$$

fields on $R \times S^3$

$$\phi_a(t, \Omega_3) = e^{\frac{\mu}{2}t}\phi_a(x)$$

fields on R^4

Correspondence between correlation functions

large-*N* reduced model

$$\mathcal{O}_{ab}(t) = \mathrm{tr}(\phi_a(t)\phi_b(t)) \quad \text{with} \quad a \neq b$$

$$\langle \mathcal{O}_{I_1}(t_1) \cdots \mathcal{O}_{I_M}(t_M) \rangle$$

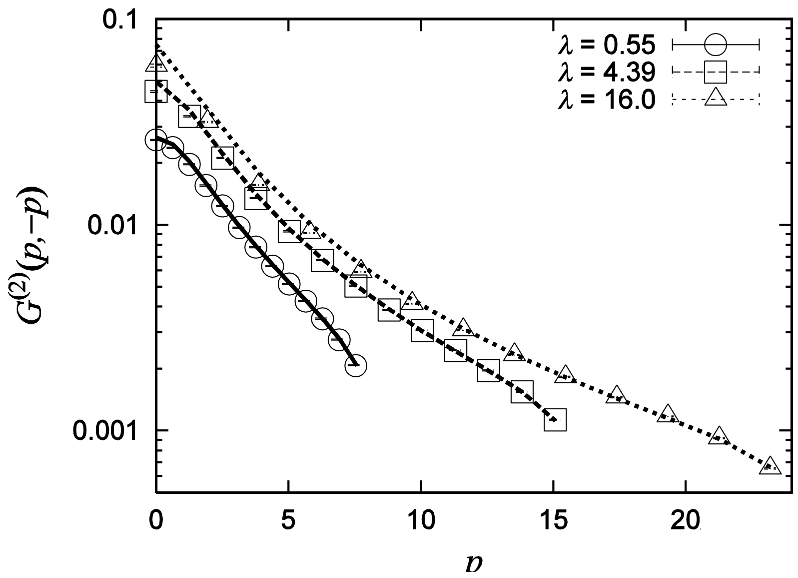
⇕

$\mathcal{N}=4$ SYM on $R \times S^3$

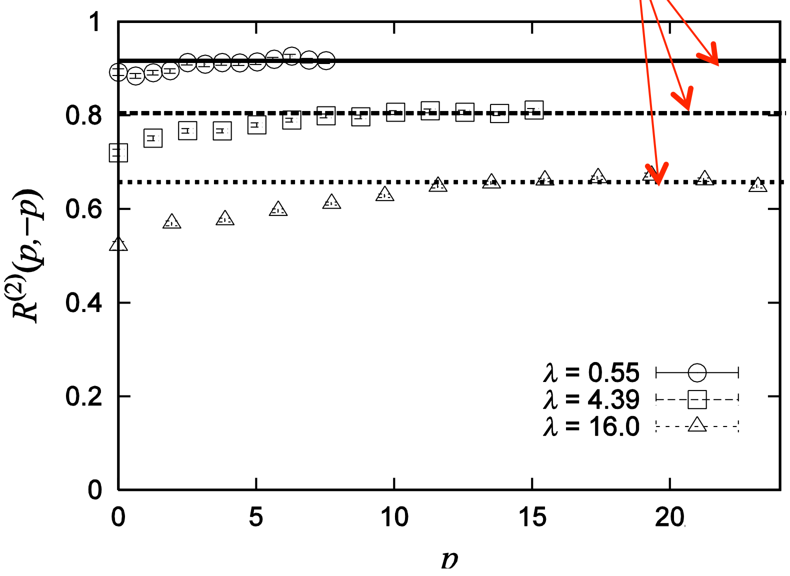
$$\bar{\mathcal{O}}_{ab}(t) \equiv \int \frac{d\Omega_3}{2\pi^2} \mathrm{tr}\left(\phi_a(t,\Omega_3)\phi_b(t,\Omega_3)\right)$$

$$\langle \bar{\mathcal{O}}_{I_1}(t_1) \cdots \bar{\mathcal{O}}_{I_M}(t_M) \rangle$$

Monte Carlo data for 2pt functions



fitted well by free theory results
with some normalization factor

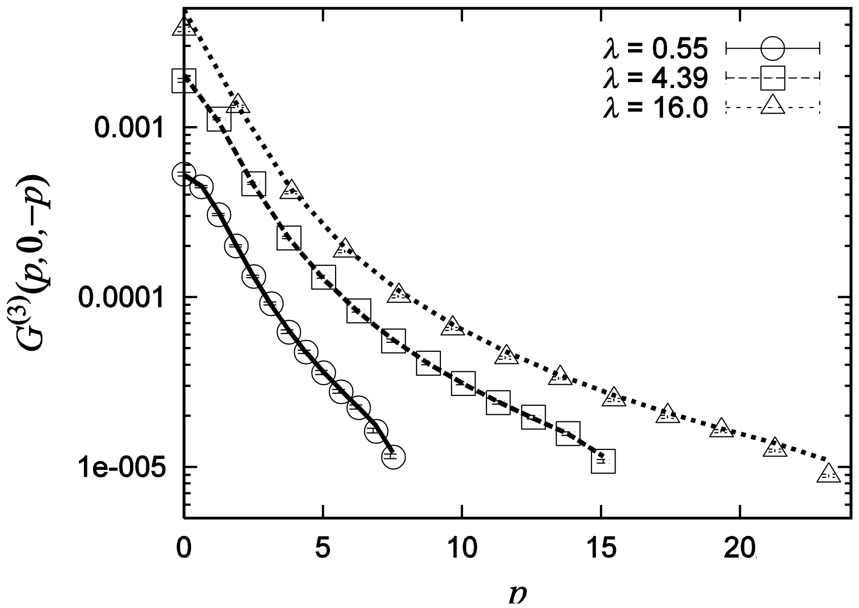


estimates for $c^{(2)}$

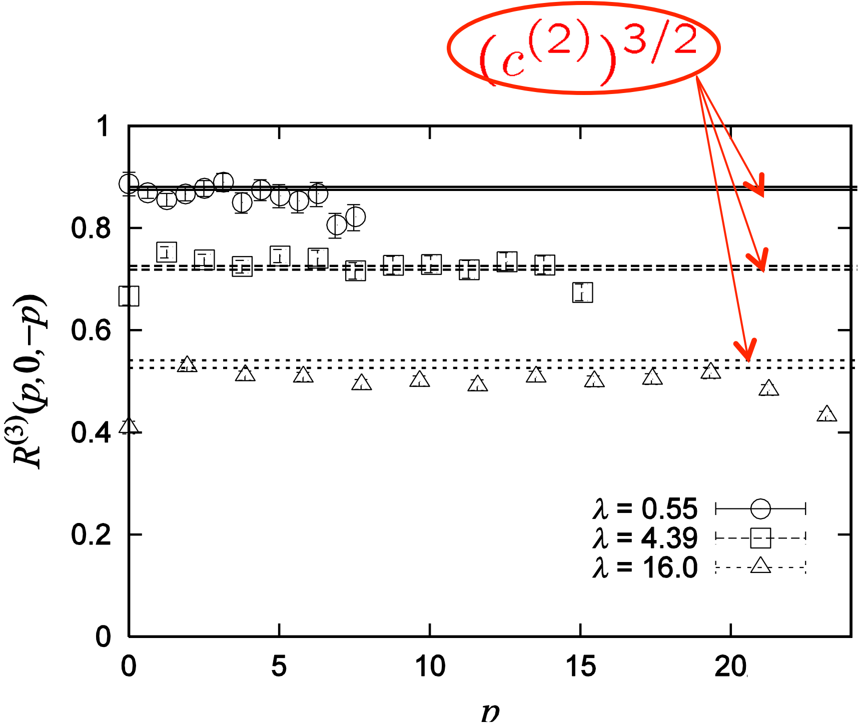
ratio to free theory results

We expect $c^{(2)} \to 1$ (SUSY non-renormalization property)
in the large-*N* limit. (c.f., *N*=6 in the present study.)

Monte Carlo data for 3pt functions



fitted well by free theory results
with some normalization factor



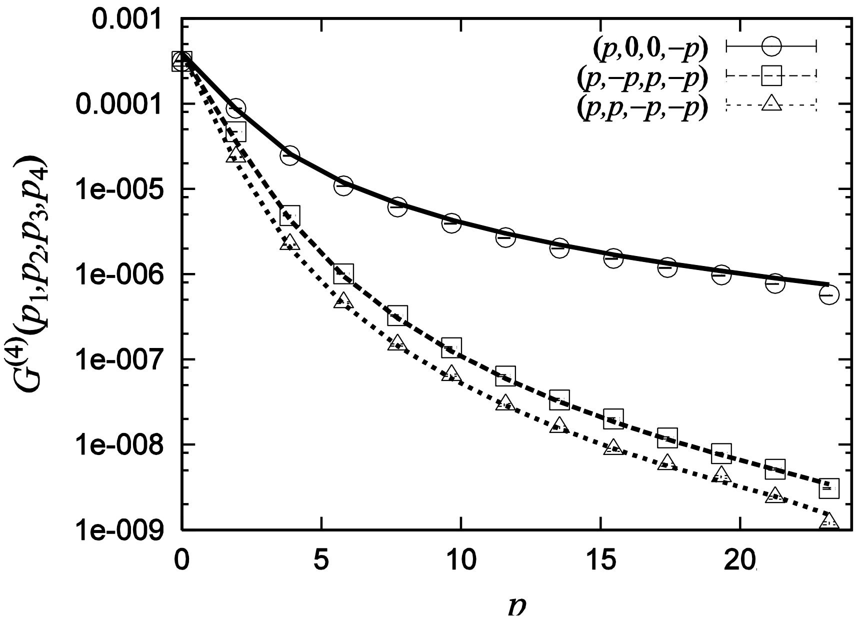
ratio to free theory results

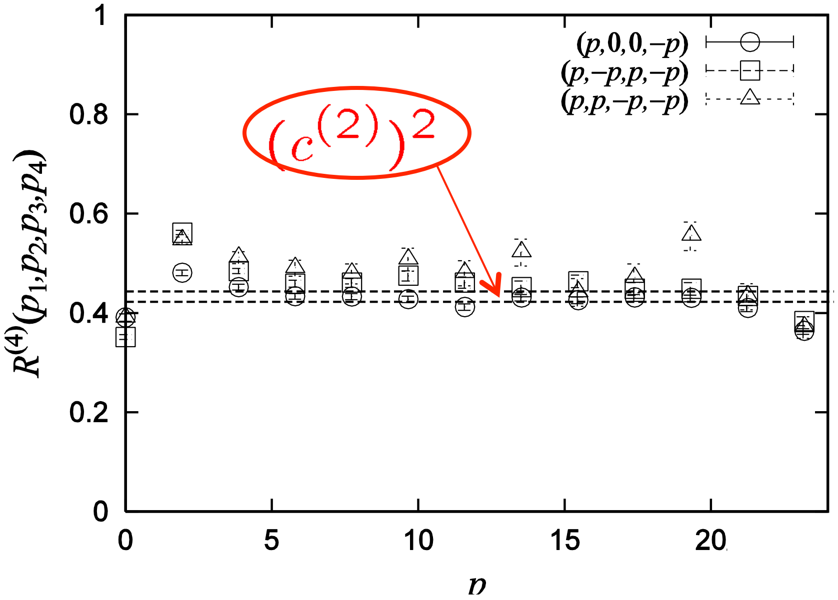
MC data exhibit SUSY non-renormalization property in the weak form at the regularized level.

Monte Carlo data for 4pt functions

Here we focus on $\lambda = 16.0$ case

with 3 types of momentum configuration
$$\begin{cases} (p, 0, 0, -p) \\ (p, -p, p, -p) \\ (p, p, -p, -p) \end{cases}$$



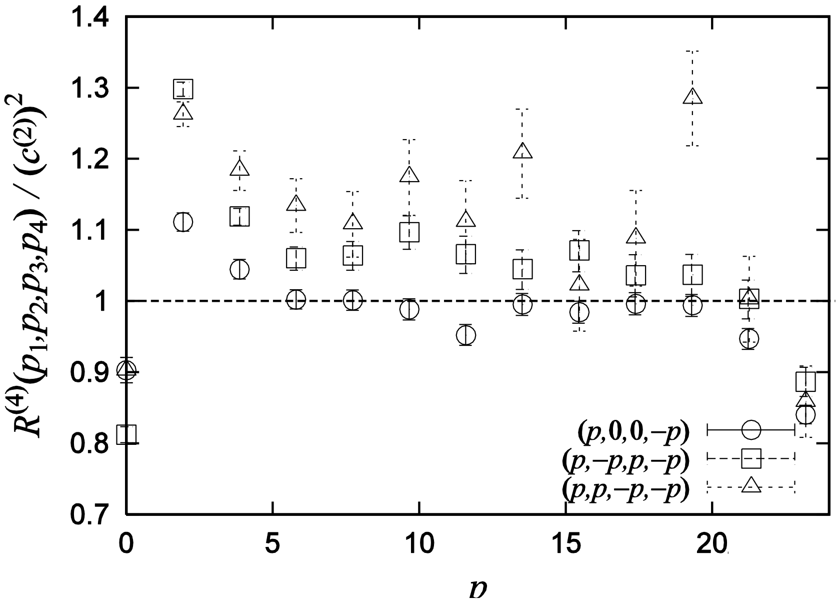


ratio to free theory results

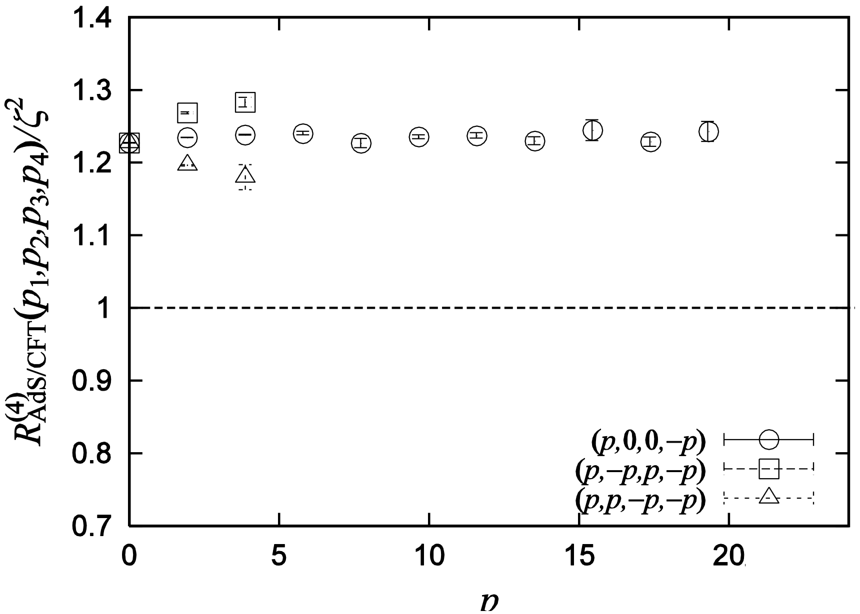
Slight violation of the SUSY non-renormalization property

Comparison with the prediction from the AdS/CFT duality

$R^{(4)}(p_1,p_2,p_3,p_4)/(c^{(2)})^2$



$R^{(4)}_{\mathrm{AdS/CFT}}(p_1,p_2,p_3,p_4)/\zeta^2$



The violation of the SUSY non-renormalization property agrees with the prediction in orders of magnitude.

4. Studies of early Universe in type IIB matrix model

type IIB matrix model

Ishibashi-Kawai-Kitazawa-Tsuchiya ('96)

$$S_{\mathrm{b}} = -\frac{1}{4g^2}\mathrm{tr}([A_\mu, A_\nu][A^\mu, A^\nu])$$

$$S_{\mathrm{f}} = -\frac{1}{2g^2}\mathrm{tr}(\Psi_\alpha(\mathcal{C}\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta])$$

a nonperturbative formulation of superstring theory

$N \times N$ Hermitian matrices

$A_\mu \quad (\mu = 0, \cdots, 9)$ Lorentz vector

$\Psi_\alpha \quad (\alpha = 1, \cdots, 16)$ Majorana-Weyl spinor

raised and lowered by the metric

$$\eta = \mathrm{diag}(-1, 1, \cdots, 1)$$

Wick rotation $(A_0 = -iA_{10}\ , \quad \Gamma^0 = i\Gamma_{10})$

$\Rightarrow$ Euclidean model with SO(10) symmetry

Connection to the worldsheet formulation

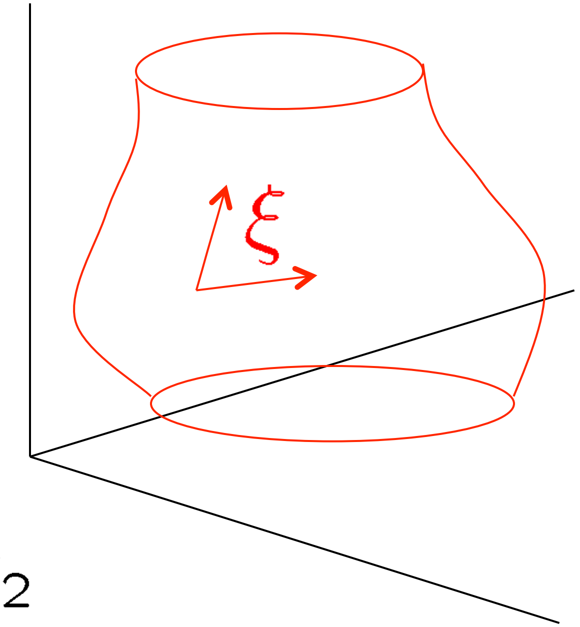
- worldsheet action

$$S = \int d^2\xi\sqrt{g}\left(\frac{1}{4}\{X^\mu, X^\nu\}^2 + \frac{1}{2}\bar{\Psi}\gamma^\mu\{X^\mu, \Psi\}\right)$$

$$\{X, Y\} \equiv \frac{1}{\sqrt{g}}\epsilon^{ab}\frac{\partial X}{\partial\xi^a}\frac{\partial Y}{\partial\xi^b}$$

Poisson bracket (regarding ξ_1 and ξ_2 as p and q in Hamilton dynamics)

$X^\mu(\xi)$, $\Psi(\xi)$

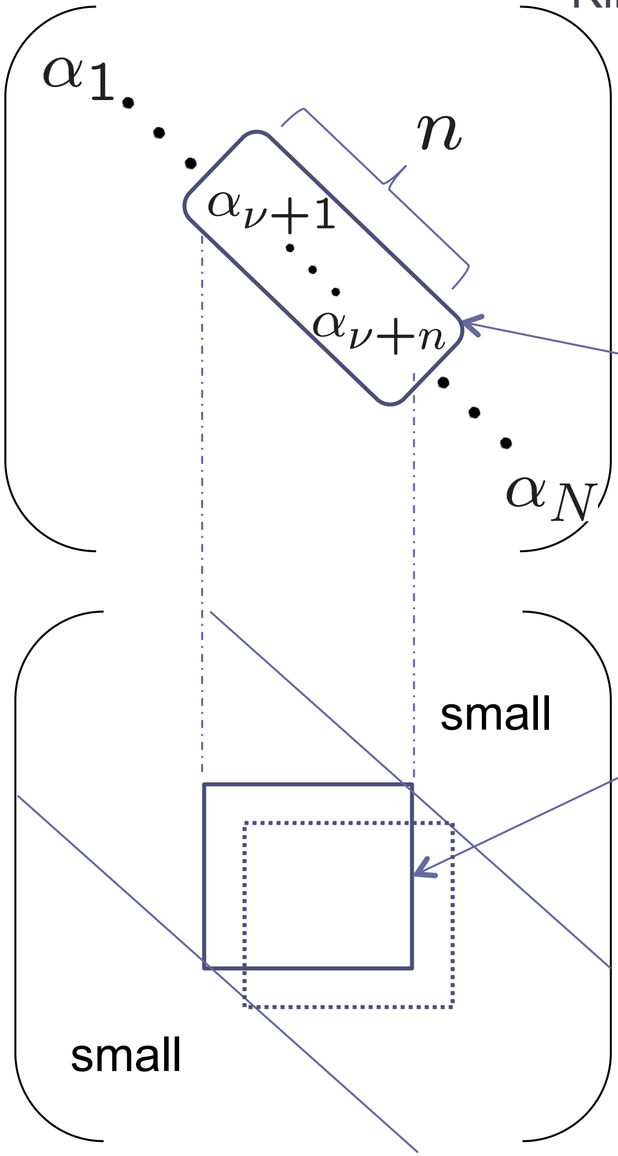


quantization $\Longrightarrow$ type IIB matrix model $\quad (\hbar \sim \frac{1}{N})$

$$\{X^\mu(\xi), X^\nu(\xi)\} \mapsto -i[A^\mu, A^\nu]$$

How to extract time-evolution

Kim-J.N.-Tsuchiya, PRL 108 (2012) 011601



diagonalize A_0

$\alpha_1 < \cdots < \alpha_N$

SU(N) transformation

definition of time "t"

$$t = \frac{1}{n} \sum_{i=1}^{n} \alpha_{\nu+i}$$

The state of the universe $\bar{A}_i(t)$ at time t

A_i has a band diagonal structure

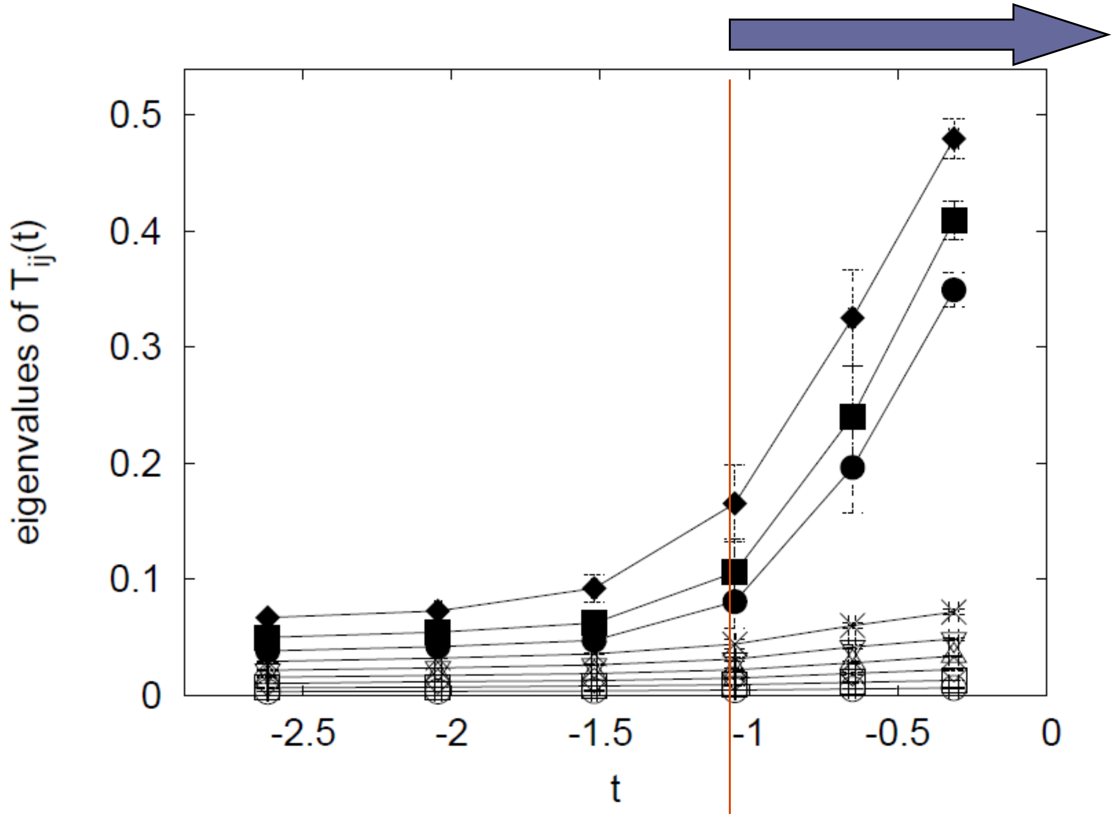
non-trivial dynamical property

Spontaneous breaking of SO(9)

Kim-J.N.-Tsuchiya, PRL 108 (2012) 011601

$$T_{ij}(t) = \frac{1}{n} \mathrm{tr}\, \{ \bar{A}_i(t) \bar{A}_j(t) \}$$

$$\mathrm{SO}(9) \overset{\mathrm{SSB}}{\rightarrow} \mathrm{SO}(3)$$



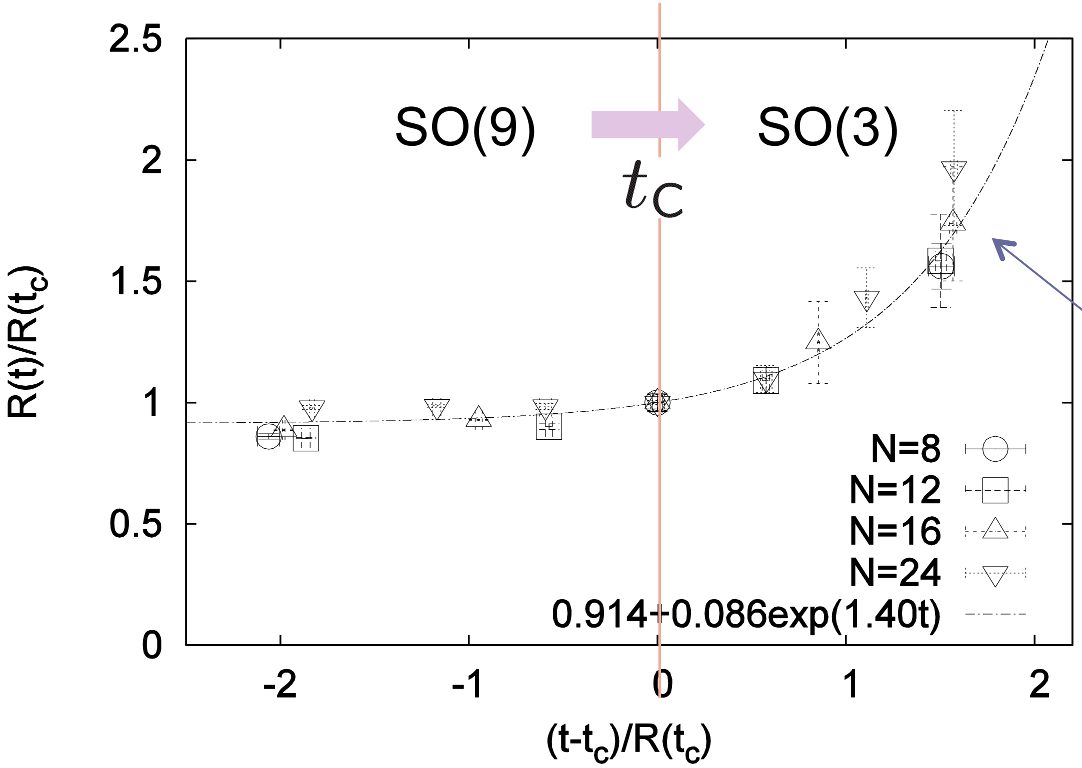
$N = 16$, $\kappa = 4.0$

"critical time"

Exponential expansion

Ito-Kim-J.N.-Tsuchiya, work in progress

$$R(t)^2 \equiv \frac{1}{n} \mathrm{tr}\, \bar{A}_i(t)^2$$



fitted well to

$$f(x) = a + (1-a)e^{bx}$$

Exponential expansion

Inflation

Effects of fermionic action

$$\begin{aligned} S_{\mathsf{f}} &= \mathrm{tr}(\bar{\Psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu,\Psi_\beta]) \\ &= \boxed{\mathrm{tr}(\bar{\Psi}_\alpha(\Gamma^0)_{\alpha\beta}[A_0,\Psi_\beta]}) + \boxed{\mathrm{tr}(\bar{\Psi}_\alpha(\Gamma^i)_{\alpha\beta}[A_i,\Psi_\beta])} \end{aligned}$$

dominant term
at early times

keep only the first term

simplified model at early times

$$\boxed{\mathrm{Pf}\mathcal{M}(A) \simeq \Delta^{d-1} = \prod_{i<j}(\alpha_i - \alpha_j)^{2(d-1)}}$$

repulsive force between eigenvalues of A_0

dominant term
at late times

simplified model at late time

$$\boxed{\mathrm{Pf}\mathcal{M}(A) \simeq 1}$$

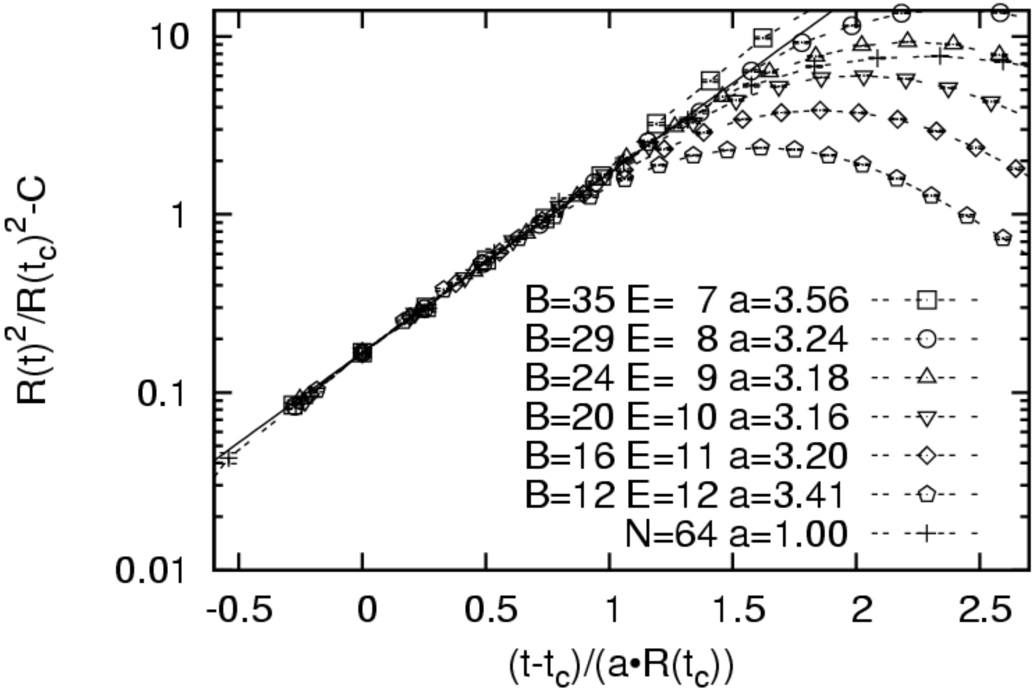
quench fermions

Exponential expansion at early times

Ito-Kim-Koizuka-J.N.-Tsuchiya, in prep.

- simplified model at early times

$$\mathrm{Pf}\mathcal{M}(A) \simeq \Delta^{d-1} = \prod_{i<j} (\alpha_i - \alpha_j)^{2(d-1)}$$



exponential expansion

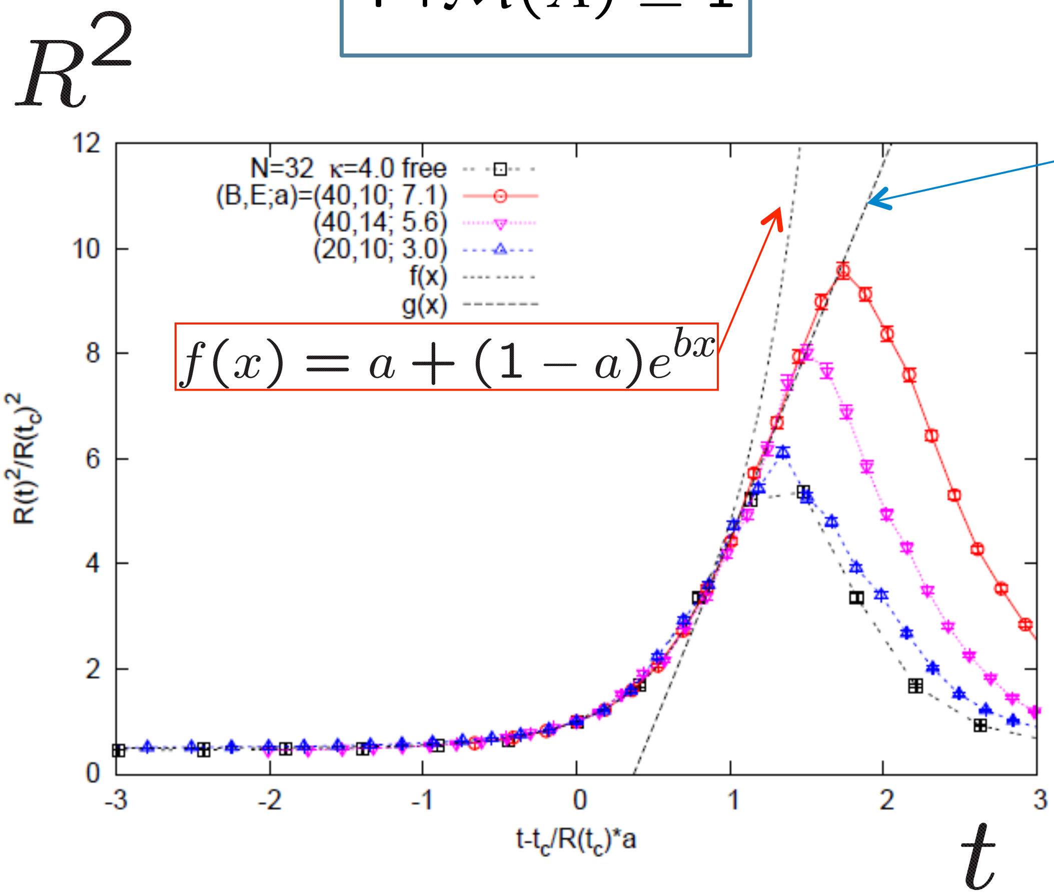
The first term is important for exponential expansion.

Power-law expansion at late times

Ito-Kim-J.N.-Tsuchiya, work in progress

- simplified model at late times

$$\boxed{\mathrm{Pf}\mathcal{M}(A) \simeq 1}$$



$$f(x) = a + (1-a)e^{bx}$$

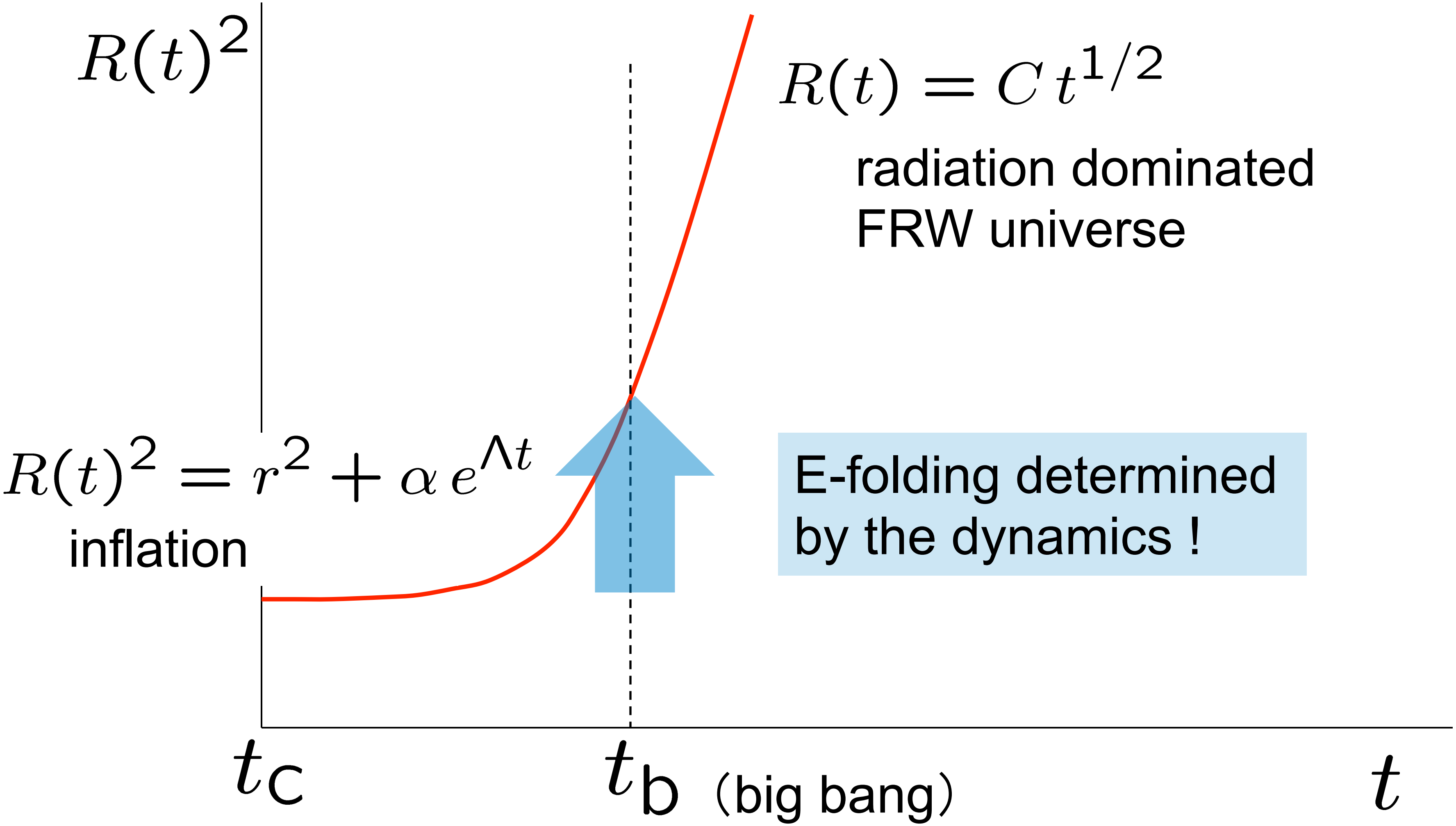
$$g(x) = ax + b$$

$$R^2 \sim t \Rightarrow R \sim t^{1/2}$$

$t^{1/2}$ behavior

Radiation dominated FRW universe

Expected scenario for the full Lorentzian IIB matrix model



$R(t)^2$

$R(t) = C\, t^{1/2}$

radiation dominated
FRW universe

$R(t)^2 = r^2 + \alpha\, e^{\Lambda t}$

inflation

E-folding determined
by the dynamics !

t_{c}

t_{b} (big bang)

t

5. Summary and future prospects

Large-*N* as a key to the Planck scale physics

- 1d SYM with 16 supercharges
  - Black hole thermodynamics gauge/gravity duality holds including α' corrections and string loop corrections
- 4d $\mathcal{N}=4$ SYM with 32 supercharges
  - The novel large-*N* reduction enables nonperturbative studies respecting 16 SUSYs
- Lorentzian type IIB matrix model
  - Expanding behavior of the early Universe (SSB from 9d to 3d, inflation + graceful exit) from nonperturbative dynamics of superstring theory
  - **No** initial value problem nor the multiverse problem.

Future prospects

- 1d SYM as a nonperturbative definition of M theory
  (BFSS conjecture)   Banks-Fischler-Shenker-Susskind ('96)

- studies of non-BPS operators in $\mathcal{N}=4$ SYM
  further tests of AdS/CFT correspondence

- the end of Inflation in the Lorentzian IIB matrix model
  calculation of the density fluctuation

- classical solution valid at late times suggests
  a natural solution of the cosmological constant problem
  Kim-J.N.-Tsuchiya ('12)

- a realization of chiral fermions and the Standard Model
  using fuzzy spheres in the extra dimensions

  Chatzistavrakidis-Steinacker-Zoupanos ('11), J.N.-Tsuchiya ('13)

A review article on all the topics I discussed:

J. Nishimura

“The Origin of space-time as seen from matrix model simulations”

PTEP 2012 (2012) 01A101 [arXiv:1205.6870].