The Niels Bohr International Academy

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Non-Abelian strings in Yang-Mills theories and beyond: 2D-4D correspondence, spin-orbit interaction generating unexpected Goldstone modes, and all that

★ Hanany-Tong, 2003

★ ★ Auzzi et al., 2003

} SUSY

★ ★ ★ Shifman-Yung, 2003 - ...

❖ Gaiotto, 2012 & Gaiotto, Gukov, Seiberg, 2013 "surface defects"...

Outline: a) Non-Abelian strings in FT (N=2, N=1),
b) World sheet models,
c) 2D-4D correspondence,
d) Applications (e.g. in He-3B)

★ Abrikosov strings (1950s);

★ ★ Cosmic strings (Kibble 1970s, Witten 1985);

★ ★ ★ Non-Abelian strings in susy Yang-Mills:

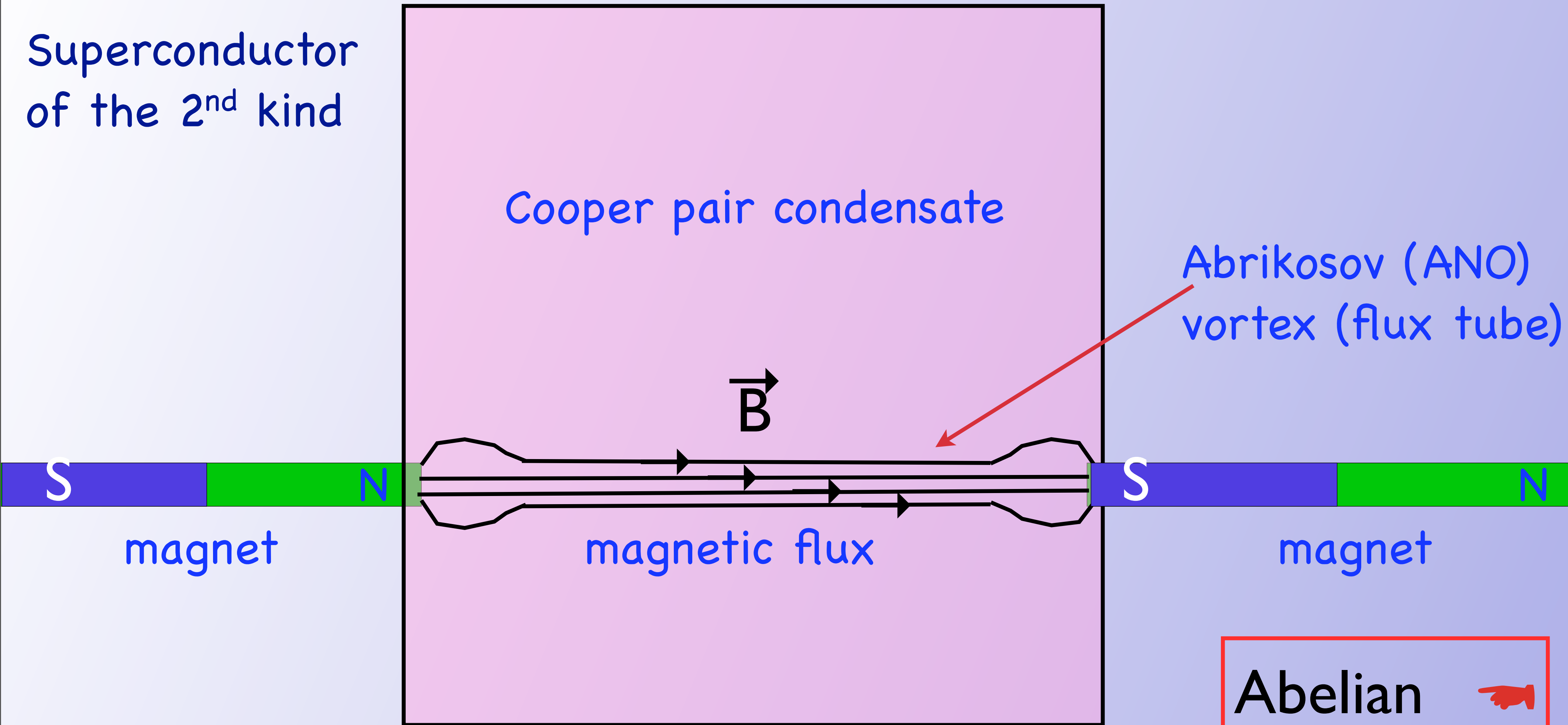
Bulk $G \times G \to$ CF locking $\to (G_{diag} \to H) \to G/H$ coset model on the world sheet $\to$ (susy in bulk $\to$ susy on ws)

★ ★ ★ ★ $N=2 \to N=(2,2)$

$N=1 \to N=(2,0)$ nonminimal

$N=0 \to N=0$

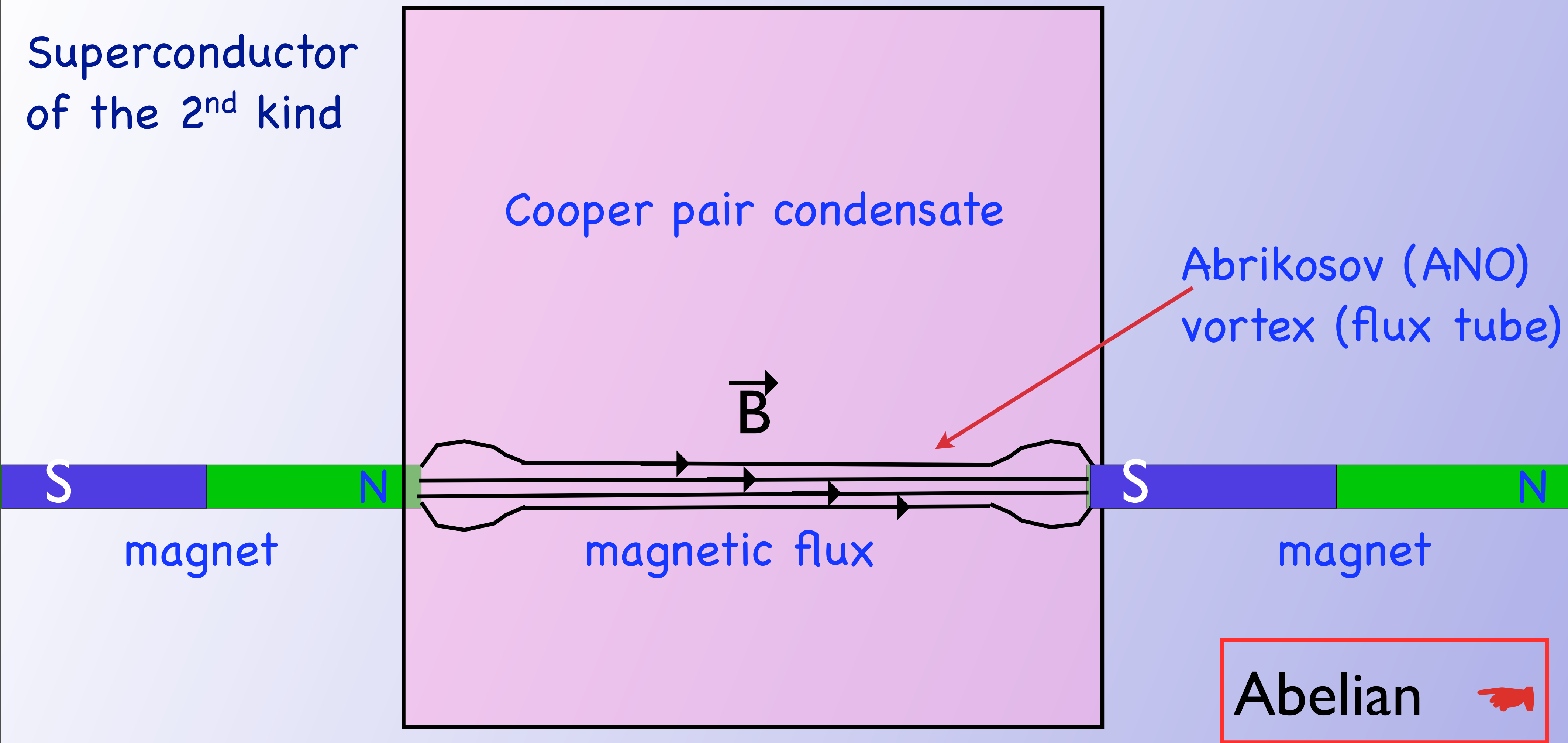
Superconductor of the 2nd kind



Abelian

DUAL MEISSNER EFFECT (Nambu-'t Hooft-Mandelstam, ~1975)

☞ The Meissner effect: 1930s, 1960s

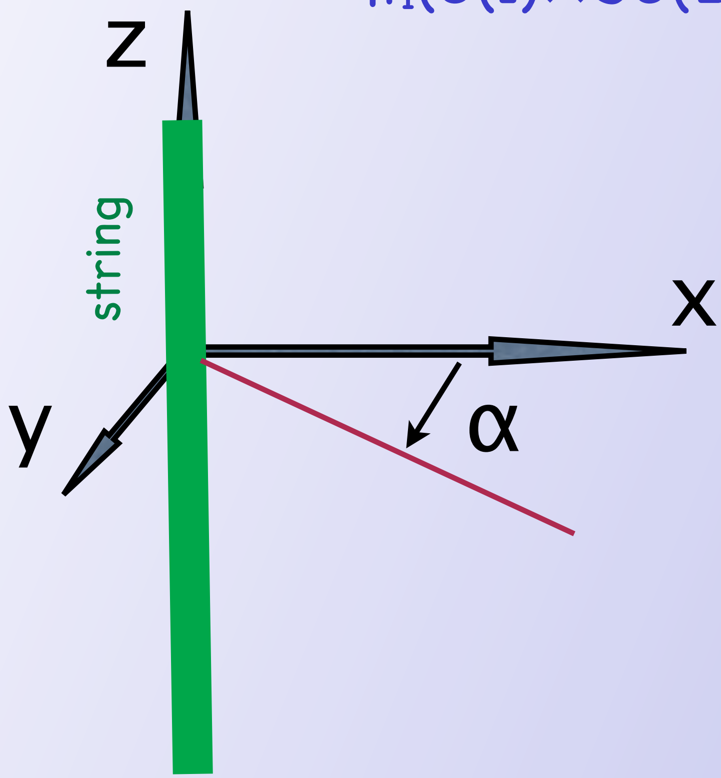


DUAL MEISSNER EFFECT (Nambu-'t Hooft-Mandelstam, ~1975)

★ ANO strings are there because of U(1)!

★ New strings:

$\pi_1(U(1)\times SU(2))$ nontrivial due to Z_2 center of SU(2)



ANO $\sqrt{\xi}\, e^{i\alpha} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$T=4\pi\xi$

Non-Abelian $\sqrt{\xi} \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & 1 \end{pmatrix}$

x_0 ← string center in perp. plane

$T_{U(1)} \pm T^3_{SU(2)}$

$T=2\pi\xi$

SU(2)/U(1) ←orientational moduli; O(3) σ model

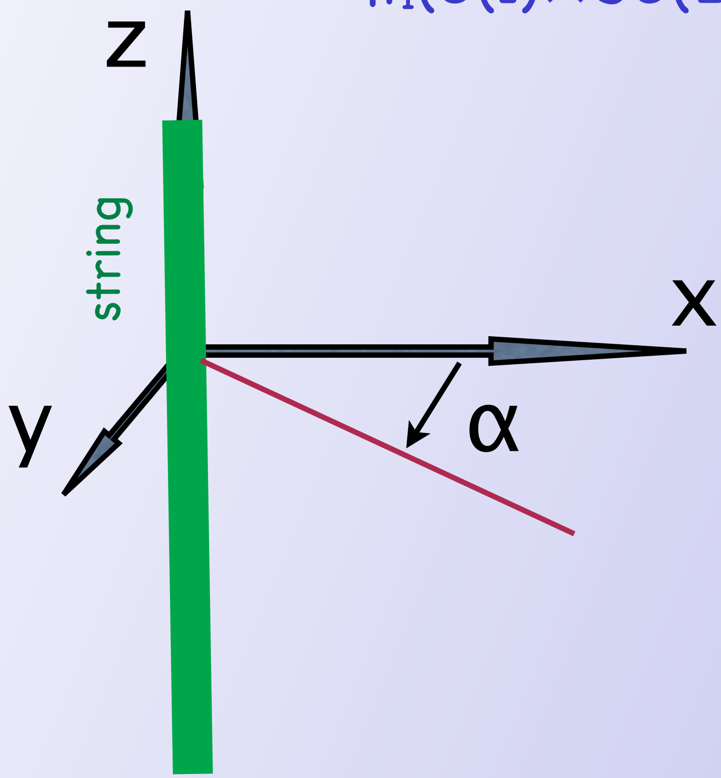
☆ ANO strings are there because of U(1)!

☆ New strings:

$\pi_1(SU(2)\times U(1)) = Z_2$: **rotate by π around 3-d axis in SU(2)**

→ -1; another -1 rotate by π in U(1)

$\pi_1(U(1)\times SU(2))$ nontrivial due to Z_2 center of SU(2)



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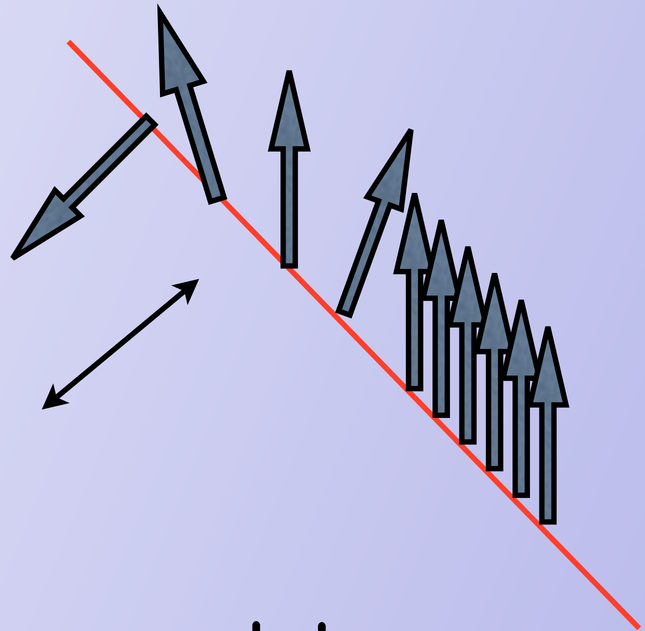
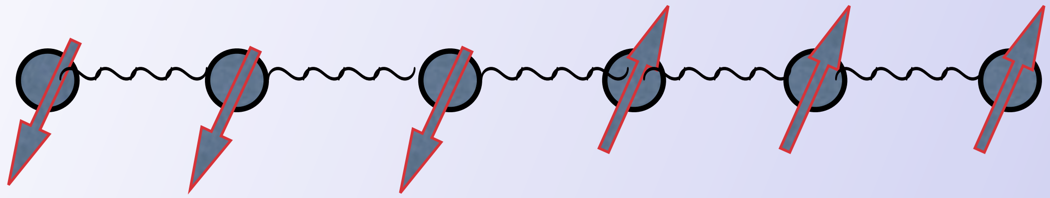
"Non-Abelian" string is formed if all non-Abelian degrees of freedom participate in dynamics at the scale of string formation

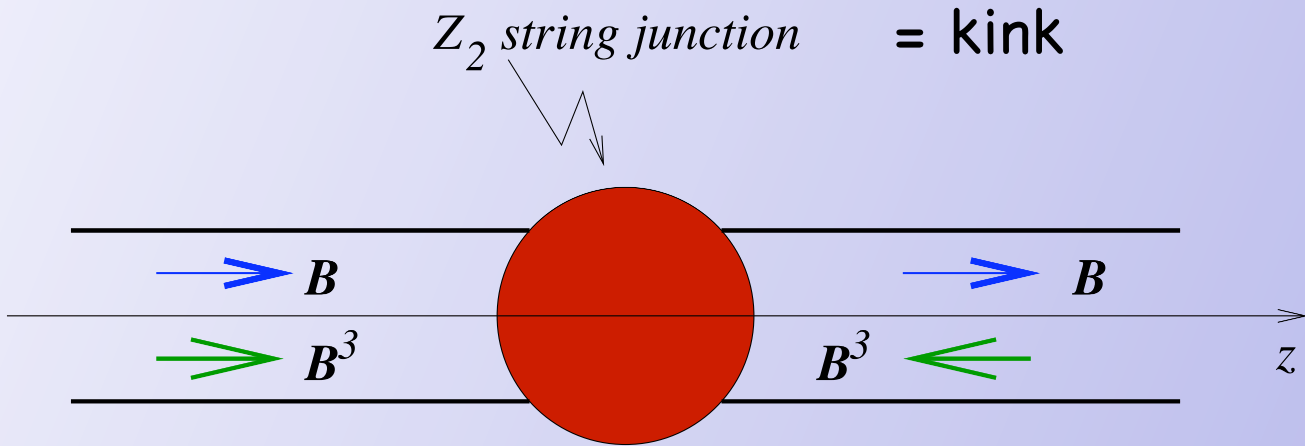


classically gapless excitation

SU(2)/U(1) = CP(1)~O(3) sigma model

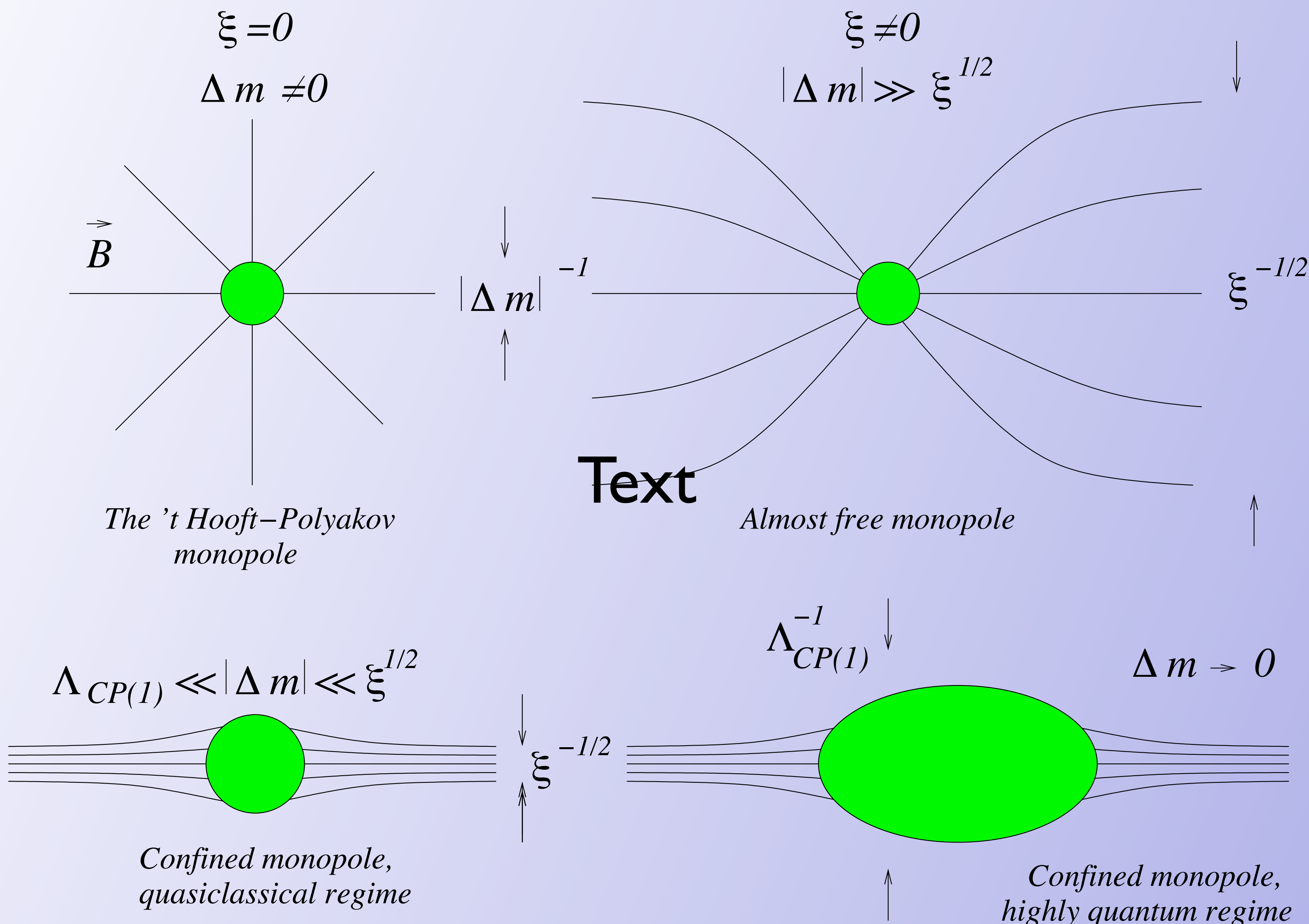
$$S=\int d^2x\left\{\frac{2}{g^2}\,\frac{\partial_\mu\bar\phi\,\partial^\mu\phi-(\Delta m)^2\bar\phi\phi}{(1+\bar\phi\phi)^2}+fermions\right\}$$





= kink

Evolution in dimensionless parameter m^2/ξ



$\xi = 0$
$\Delta m \neq 0$
$\vec{B}$
$|\Delta m|^{-1}$
$\xi \neq 0$
$|\Delta m| \gg \xi^{1/2}$
$\xi^{-1/2}$
Text
The 't Hooft–Polyakov monopole
Almost free monopole
$\Lambda_{CP(1)} \ll |\Delta m| \ll \xi^{1/2}$
$\xi^{-1/2}$
$\Lambda_{CP(1)}^{-1}$
$\Delta m \to 0$
Confined monopole, quasiclassical regime
Confined monopole, highly quantum regime

✳ Kinks are confined in 4D (attached to strings).

✳ ✳ Kinks are confined in 2D:

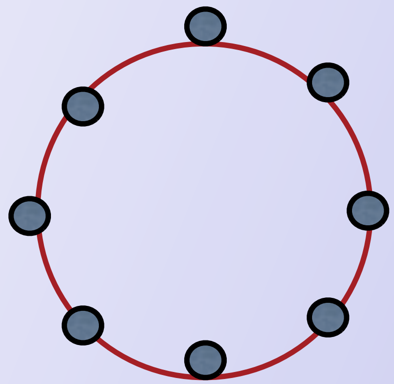
Kink = Confined Monopole

4D ↔ 2D Correspondence

☛ World-sheet theory ↔ strongly coupled bulk theory inside

Dewar flask

I. **N** = 2 SUSY bulk $\longrightarrow$ **N** = (2,2) CP(N-1) model



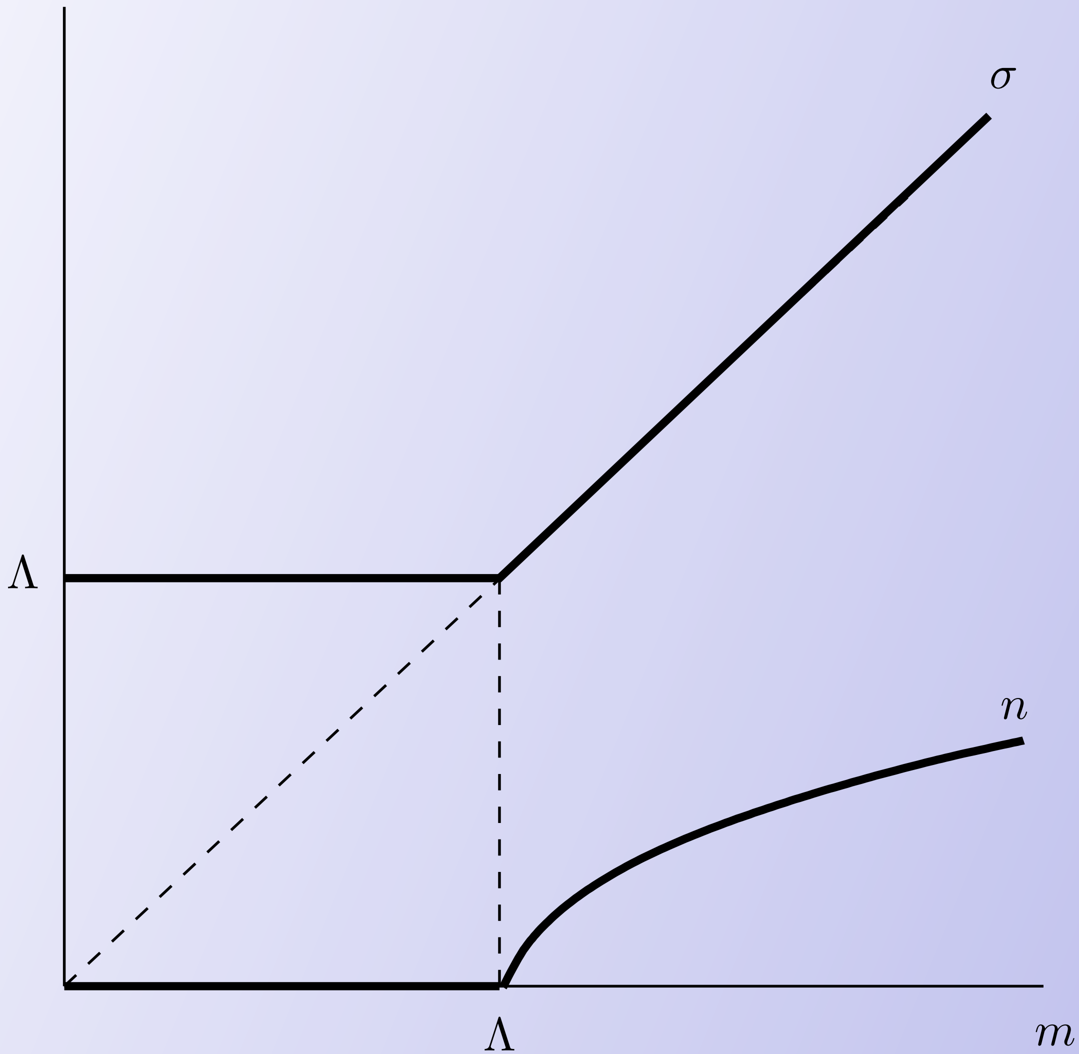
$m_i \sim$

$$e^{2\pi i/N},\ e^{4\pi i/N},\ \ldots,\ e^{2(N-1)\pi i/N},\ 1$$

$$\mathcal{L} \;=\; \frac{1}{e_0^2}\left(\frac{1}{4}F_{\mu\nu}^2 + |\partial_\mu \sigma|^2 + \frac{1}{2}D^2\right) \;+ i\,D\,(\bar{n}_i n^i - 2\beta)$$

$$+\;\left|\nabla_\mu n^i\right|^2 \;+2\sum_i \left|\sigma - \frac{m_i}{\sqrt{2}}\right|^2 |n^i|^2$$

\+ fermions



E_{vac}=0 always, SUSY unbroken,

Z_{2N} always broken, (N degenerate vacua)

Crossover instead of phase transition

Strong-coupling ↔ Higgs regime

II. $\mathcal{N} = 1$ SUSY bulk

$\mathcal{N} = (0,2)$ CP(N-1) model

Supersymmetry is broken, generally speaking !!!

Phase transitions possible

All phase transitions are of the second kind!

Break **N** = 2 down to **N** = 1 in the bulk

Deformation of the bulk: $W = \mu (A^a)^2 + \mu' A^2$

Heterotic deformation the of the World-sheet theory:

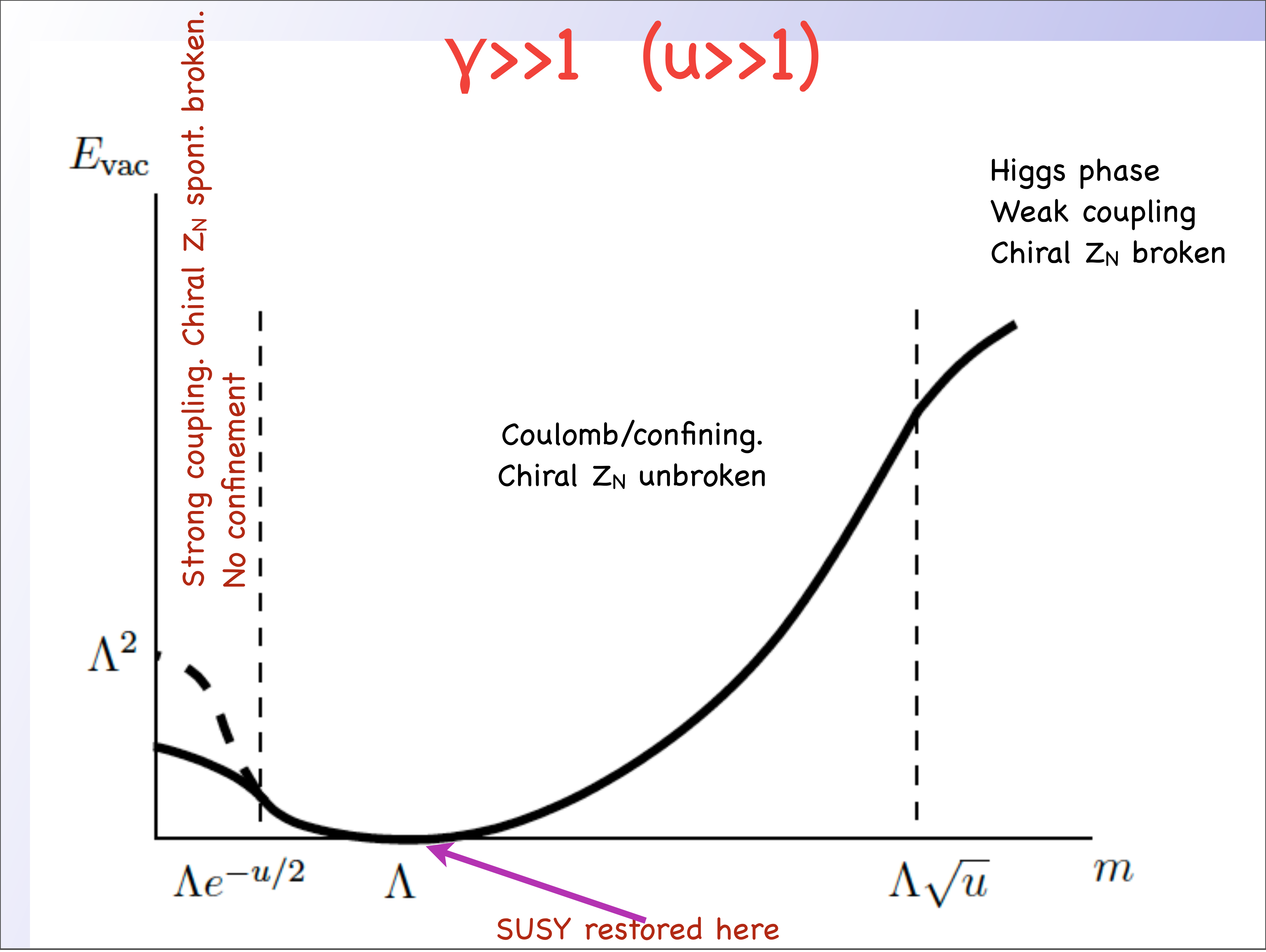
(2,2) supersymmetry is broken down to (0,2)

$$L_{heterotic} = \zeta_R^\dagger i\partial_L \zeta_R + \left[\gamma \zeta_R R \left(i\partial_L \phi^\dagger\right) \psi_R + H.c.\right] - g_0^2 |\gamma|^2 \left(\zeta_R^\dagger \zeta_R\right) \left(R \psi_L^\dagger \psi_L\right)$$

at small γ
ζ_R is Goldstino

$$\mathcal{E}_{vac} = |\gamma|^2 \left| \langle R \psi_R^\dagger \psi_L \rangle \right|^2$$

(0,2) supersymmetry is spontaneously broken!


γ>>1 (u>>1)
E_{vac}
Strong coupling. Chiral Z_N spont. broken.
No confinement
Higgs phase
Weak coupling
Chiral Z_N broken
Coulomb/confining.
Chiral Z_N unbroken
Λ^2
$\Lambda e^{-u/2}$
Λ
$\Lambda\sqrt{u}$
m
SUSY restored here

No-supersymmetry applications

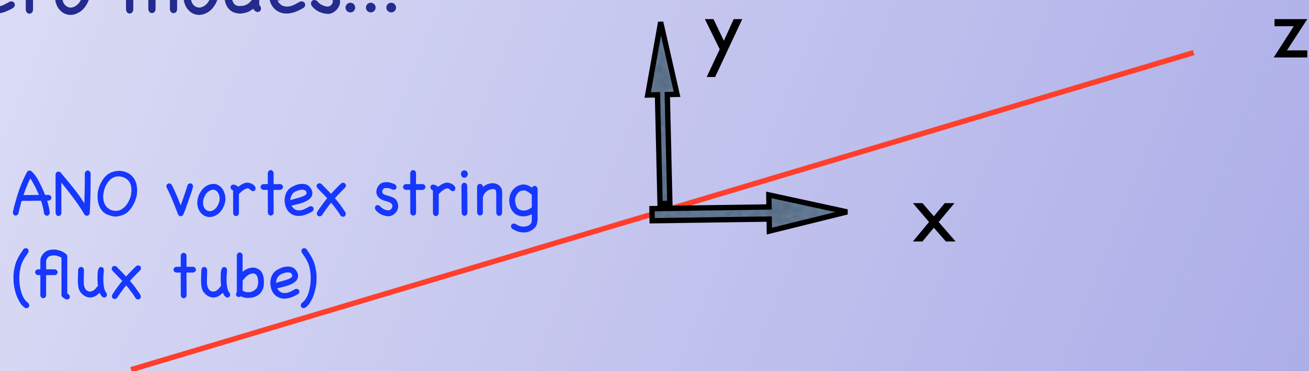
★ Internal symmetries $G \to H$,

$$V_{\text{"rel"}} = V_G - V_H\,, \quad V_{\text{"non-rel"}} = (V_G - V_H)/2.$$

★ ★ This is not the case for geometric symmetries! (Ivanov-Ogievetsky, 1975; Low-Manohar, 2002)

★ ★ ★ E.g. (structureless) string breaks two translations and two rotations, but one should consider only translational zero modes!!!

M_{zx} & M_{zy} broken,
T_x & T_y broken .



☞ Goldstone modes (gapless excitations) on strings

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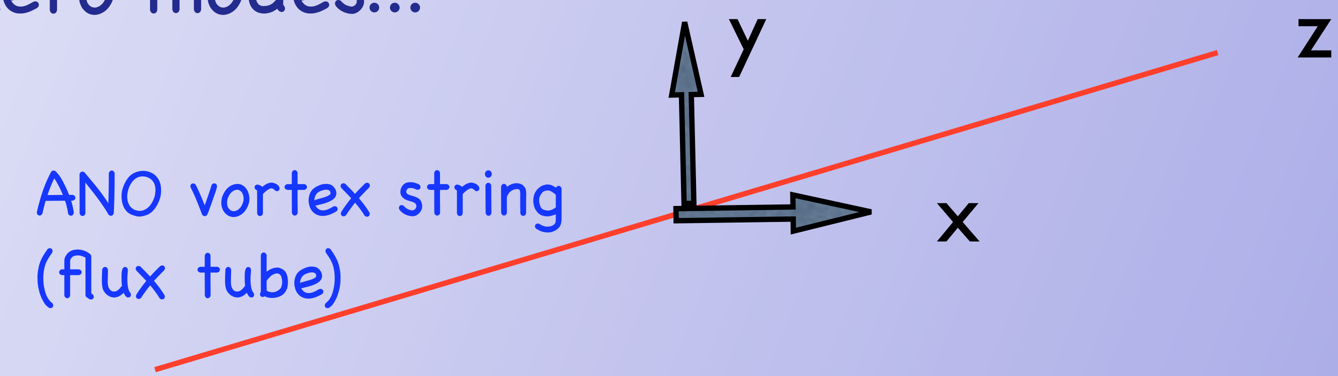
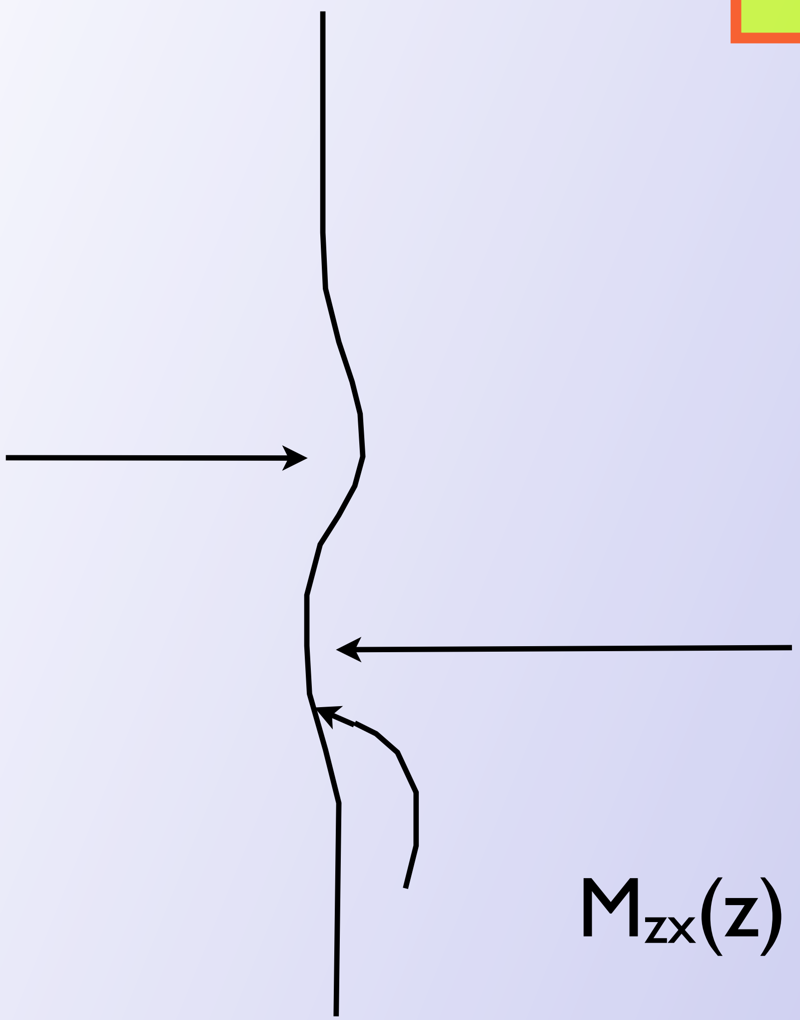
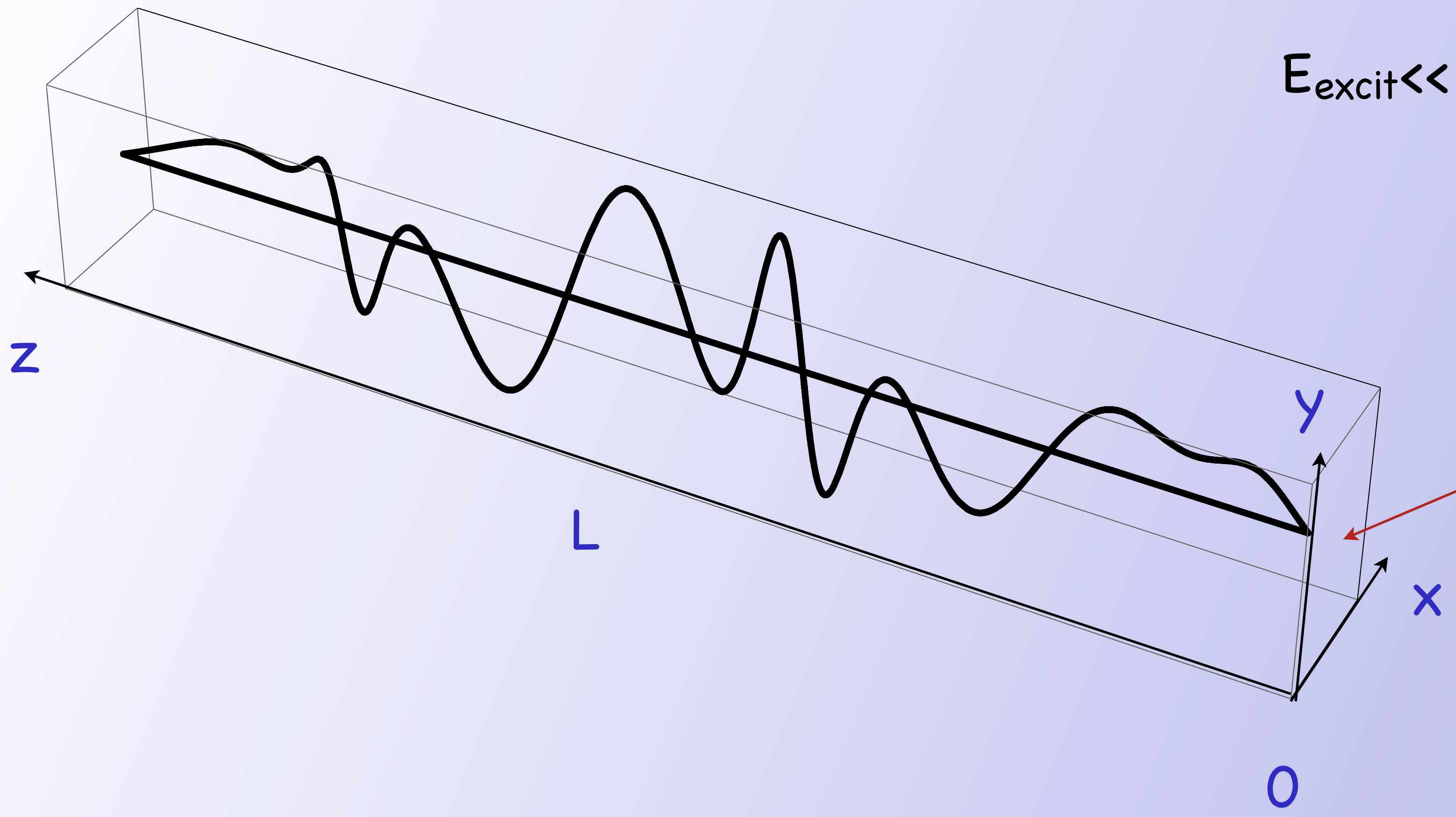


What if order parameter carries Lorentz indices?



$M_{zx}(z) \sim T_x$

Low-energy excitations (gapless modes) on vortices



$E_{excit} \ll m_\gamma \sim ev$

◇◇ $\Delta H_{GL} = (T/2)(\partial_z \vec{x}_{perp}\, \partial_z \vec{x}_{perp}) + h.d.$ ➠ time derivatives can be linear or quadratic.

Nambu-Goto → String Theory

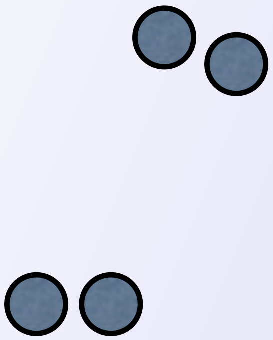
Kelvin modes or Kelvons
2 NG gapless modes in relat.
1 NG gapless mode in non-rel.

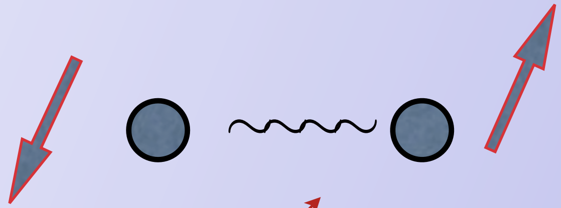
$E_{str} = TL + C/L$

Counts # of gapless modes !

^{3}He-B example

^{3}He atoms





Spin 1/2

P-wave paring

L=1, S=1 ➟ Cooper pair order parameter $e_{\mu i}$ $\leftarrow$ 3×3 matrix

$\boxed{\text{Spin-orbit small, symmetry of H is} \quad G = U(1)_p \times SO_S(3) \times SO_L(3)}$

In the ground state $U_p(1) \times SO_S(3) \times SO_L(3) \to H_{\rm B} = SO(3)_{S+L}$

Hence, contrived NG modes in the bulk!

$$S_{\text{world sheet}} = (\mu^2/2\beta)\int d^2x\, (\partial_\mu S^i)(\partial_\mu S^i)$$

$$\boxed{S^i S^i = 1}$$

Clasically two "rotational" zero modes.

QMechanically may be lifted

◇ Assume χ^i is spin field!

◇◇ Add $\Delta L = \varepsilon \, (\partial_i \chi^i)(\partial_k \chi^k)$

$$\Downarrow$$

✱ If $\varepsilon \to 0$, geometric symmetry is enhanced

Poincaré $\times$ O(3)

$$\Downarrow$$

✱✱ Two extra zero modes

What if ε≠0 but small?

$$\Delta_{CP(1)} S_{\text{world sheet}} = \varepsilon \int d^2x \, \{(\partial_z S^3)^2 - M^2[1-(S^3)^2]\}$$

$$\mathcal{L}_{x_\perp} = \frac{T}{2}\left(\partial_a \vec{x}_\perp\right)^2 - \tilde{M}^2 \left(S^3\right)^2 \left(\partial_z \vec{x}_\perp\right)^2 ,$$

★ EXTRA (quasi)gapless modes ★

★ ★ Translational (Kelvon) and orientational (spin) modes mix with each other ★ ★

◇ S. Dubovsky, R. Flauger, V Gorbenko, 2013

$\partial\partial x_{perp}\ \partial\partial x_{perp}$

$$S_{int} = \frac{\alpha}{8\pi}\int d^2\sigma\,\phi K^i_{\alpha\gamma}K^{j\gamma}_{\beta}\epsilon^{\alpha\beta}\epsilon_{ij}\,, \qquad (11)$$

where $K^i_{\alpha\gamma}$ is the extrinsic curvature of the worldsheet.

SQED with **two** flavors

$$S_0 = \int d^4x \left\{ -\frac{1}{4g^2} F_{\mu\nu}^2 + \left| \mathcal{D}_\mu \varphi^A \right|^2 - \lambda \left(|\varphi^A|^2 - \xi \right)^2 \right\}.$$

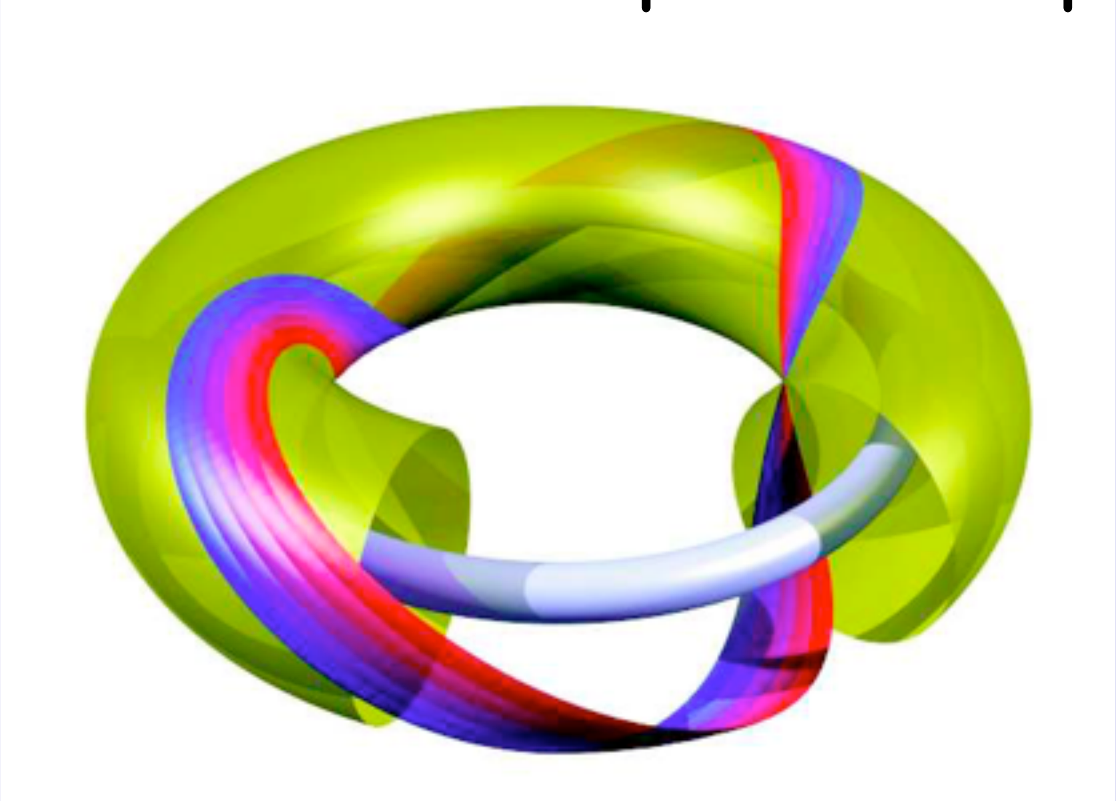
$$\beta = \frac{2\lambda}{g^2}$$ =1 → SUSY → β>>1 ~~SUSY~~

ξ → Fayet-Iliopoulos parameter

$$m_\gamma = \sqrt{2} g \sqrt{\xi}\,, \qquad m_H = m_\gamma \sqrt{\beta}\,.$$

$$\boxed{m_H \gg m_\gamma}$$ ← Abrikosov/London limit

Hopf solitons in two-component superconductors



$$\mathcal{L} = \frac{F^2}{2}\,\partial_\mu \vec{S}\,\partial^\mu \vec{S} - \frac{\lambda}{4}\left(\partial_\mu \vec{S} \times \partial_\nu \vec{S}\right) \cdot \left(\partial^\mu \vec{S} \times \partial^\nu \vec{S}\right)$$

4D Skyrme-Faddeev model: nontriv. $S_3 \to S_2$

Conclusions

★ Non-Abelian strings in SUSY bulk →

CP(N-1) models (heterotic & nonheterotic) on string; a wealth of phase transitions.

★ 2D ↔ 4D Correspondence brings fruits

and a treasure trove of novel 2D models with intriguing dynamics!

★ Unexpected applications in condensed

matter (poorly explored).