Universality results in quantum gravity

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based on work 1309.0804

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Phenomelogy of quantum gravity

Various quantum gravity effects could be of important physical implications:

- Infra-red physics
  - large extra dimensions
  - weakening of the 4d gravitational force at cosmological scales
  - Effective mass of a graviton: from geometry, fluxes, higher dimensional gravity models, string theory,...
- Ultraviolet completion dependent
  - early time quantum fluctuations
  - Non-decoupling of fundamental degrees of freedom from string theory
  - choice of vacuum ...

Quantum gravity as an effective field theory

- Some physical properties of quantum gravity are *universal* being independent of the UV completion

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[Donoghue] has explained that one can evaluate some long-range infra-red contributions in any quantum gravity theory and obtain reliable answers independent of the UV completion.

We are interested in quantum gravity contributions at loop order that depend only on the structure of the effective tree Lagrangian

At one-loop order these will be infra-red contributions involving only the structure of the tree amplitudes and independent of the UV completion

Black-Hole entropy and AdS/CFT

[Sen] has showed that the log correction to the entropy of non-extremal black holes can be computed in any quantum gravity theory

$$S = \frac{Area}{4\ell_p^2} + c\,\log\left(\frac{A}{\ell_p^2}\right) + \cdots$$

matches the prediction from string theory but fails to be reproduced by loop quantum gravity models.

The coefficient c is universal because it only depends on the low-energy spectrum determined by the massless fields and their coupling to the background

Corrections to Newton's potential

One-loop corrections to Newton's potential can be calculated reliably using effective field theory approach to gravity [Donoghue; Bjerrum-Bohr, Donoghue, Holstein]

$$V(r) = -\frac{G_N m_1 m_2}{r} \left(1 + C\,\frac{G_N(m_1+m_2)}{r} + Q\frac{G_N \hbar}{r^2}\right) + Q' G_N^2 m_1 m_2 \delta^3(\vec{x})$$

- C is the classical correction and Q and Q' are quantum corrections

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- If $\lambda = \hbar/(m_1+m_2)$ is the Compton wavelength

$$C\,\frac{G_N m_1 m_2(m_1+m_2)}{(r\pm\lambda)^2} \simeq C\,\frac{G_N m_1 m_2(m_1+m_2)}{r^2} \pm C\,\frac{G_N m_1 m_2\,\overbrace{(m_1+m_2)\lambda}^{\hbar}}{r^3}$$

- Q in the potential $V(r)$ is ambiguous but $V(r)$ is not observable

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- C is the classical correction and Q and Q' are quantum corrections
- The coefficients of $1/\sqrt{-q^2}$ and $\log(-q^2)$ in the amplitude are unambiguously defined

$$V(q^2) = \frac{G_N m_1 m_2}{q^2} + C\frac{G_N^2 m_1 m_2 (m_1+m_2)}{\sqrt{-q^2}} + Q G_N^2 m_1 m_2 \hbar \log(-q^2) + Q' G_N^2 m_1 m_2$$

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- C is the classical correction and Q and Q' are quantum corrections
- Q' is the short distance UV divergences of quantum gravity
- Need to add the R^2 term ['t Hooft-Veltman]

$$S = \int d^4x |-g|^{\frac{1}{2}} \left[\frac{2}{32\pi G_N} \mathcal{R} + c_1 \mathcal{R}^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \cdots \right]$$

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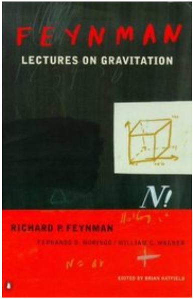
The coefficients C and Q are independent of the UV completion and any quantum gravity theory should give these computations

The aim of this talk is to discuss the computation of these constant and their physical interpretation

Effective theory of quantum gravity

One can try to treat quantum gravity as an ordinary quantum field theory

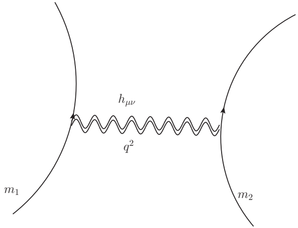


- In the 60's Feynman spelled the technics for perturbative gravity : Quanta = gravitons (massless, spin 2)
- Rules for Feynman diagrams given from the linearization of gauge fixed Einstein-Hilbert action $g_{mn} = \eta_{mn} + \kappa_{(4)} h_{mn}$
- Very similar to other gauge theories but *huge gauge symmetry* from diffeomorphism invariance

Perturbative technics

Classical Newton's potential is obtained in the non-relativistic limit

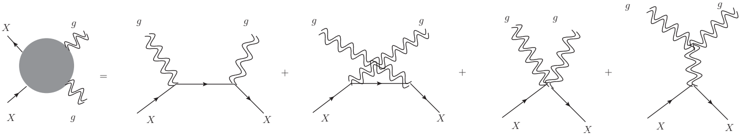
$$V(q^2) = \frac{G_N m_1 m_2}{q^2} \qquad V(r) = -\frac{G_N m_1 m_2}{r}$$



is derived by a tree-level graph exchanging a graviton

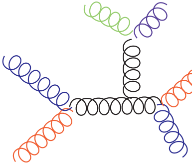
Gravitational compton scattering

Gravitational Compton scatting off a massive particle of spin $s = 0, \frac{1}{2}, 1$



using Feynman rules and DeWitt or Sannan's 3- and 4-point vertices this is a big mess

Tree-level amplitudes in Yang-Mills

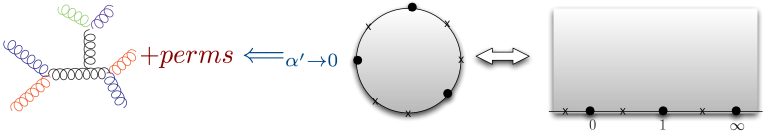


- We consider tree-level amplitudes in Yang-Mills with gauge group G
- Tree-level amplitudes are decomposed into color-ordered sub-amplitudes

$$\mathfrak{A}(1,\ldots,n) \sim \sum_{\sigma\in\mathfrak{S}_{n-1}/\mathbb{Z}_2} \mathrm{Tr}\left(\lambda^{a_1}\lambda^{a_{\sigma(2)}}\cdots\lambda^{a_{\sigma(n)}}\right) \mathcal{A}(1,\sigma(2,\ldots,n))$$

- λ^a are generators in the fundamental representation of the gauge group G
- $\mathcal{A}(1,\sigma(2,\ldots,n))$ are the $(n-1)!/2$ color-ordered amplitudes

Tree-level amplitudes in Yang-Mills



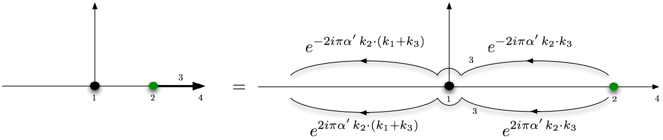
- Tree-level YM amplitudes from $\alpha' \to 0$ of disc open string amplitudes
- $PSL(2,\mathbb{R})$ invariance $z_1 = 0$, $z_{n-1} = 1$ and $z_n = +\infty$. (3 marked points)

$$\mathcal{A}(\sigma(1,\dots,n)) = \int_{x_{\sigma(1)}<\cdots<x_{\sigma(n)}} \prod_{1\leqslant i<j\leqslant n} f(x_i - x_j)\,(x_i - x_j)^{2\alpha' k_i\cdot k_j}\, d^{n-3}x$$

- The function $f(x)$ does not have branch cut but has poles.
- Depends on the polarisation and momenta of the external states.

Monodromies from contour deformation

Contour deformation



- The monodromy lead to a linear system of equations relating different ordering of the external states

- In string theory

$$0 = \sin(2\pi\alpha' k_2 \cdot k_3)\mathcal{A}(1324) + \sin(2\pi\alpha' k_2 \cdot (k_1 + k_3))\mathcal{A}(3124)$$

[Bern, Carrasco, Johansson; Bjerrum-bohr, Damgaard, Vanhove; Stieberger; Mafra, Schlotterer]

[Bjerrum-bohr, Damgaard, Feng, Søndergaard; Bjerrum-bohr, Damgaard, Søndergaard, Vanhove]

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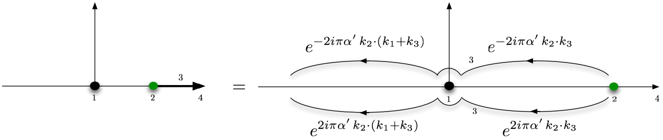

- The monodromy lead to a linear system of equations relating different ordering of the external states
- In field theory $\alpha' \to 0$

$$0 = (k_2 \cdot k_3)\mathcal{A}(1324) + k_2 \cdot (k_1 + k_3)\mathcal{A}(3124)$$

[Bern, Carrasco, Johansson; Bjerrum-bohr, Damgaard, Vanhove; Stieberger; Mafra, Schlotterer]

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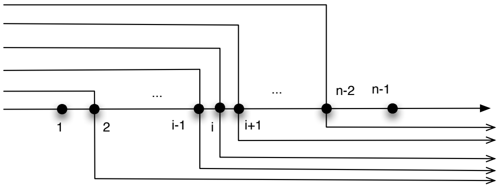

- The monodromy lead to a linear system of equations relating different ordering of the external states $\forall \beta\, \mathfrak{S}_{n-2}$

$$\sum_{\sigma\in\mathfrak{S}_{n-2}} \mathcal{S}[\sigma(2,\dots,n-1)|\beta(2,\dots,n-1)]_{k_1}\mathcal{A}_n(n,\sigma(2,\dots,n-1),1) = 0$$

[Bern, Carrasco, Johansson; Bjerrum-bohr, Damgaard, Vanhove; Stieberger; Mafra, Schlotterer]

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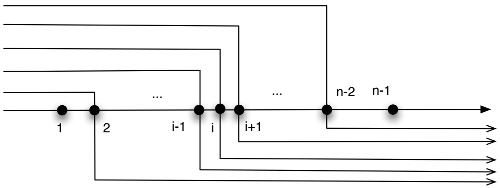
Momentum kernel

Contour deformation



- The $\alpha \to 0$ limit leads to an object named momentum kernel $\mathcal{S}$

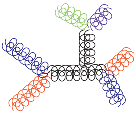
$$\mathcal{S}[i_1,\ldots,i_k|j_1,\ldots,j_k]_p := \prod_{t=1}^{k}\left(p\cdot k_{i_t} + \sum_{q>t}^{k}\theta(t,q)\,k_{i_t}\cdot k_{i_q}\right)$$

- $\theta(t,q) = 1$ if $(i_t - i_q)(j_t - j_q) < 0$ and 0 otherwise

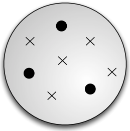
[Bern, Carrasco, Johansson; Bjerrum-Bohr, Damgaard, Vanhove; Stieberger; Mafra, Schlotterer]

[Bjerrum-Bohr, Damgaard, Feng, Søndergaard; Bjerrum-Bohr, Damgaard, Søndergaard, Vanhove]

Gravity as a square of Yang-Mills



$+perms \Longleftarrow_{\alpha'\to 0}$



- Holomorphic factorization. Relative ordering of the contours

$$\mathfrak{M}(1,\ldots,n) = \int_{C_x} d^{n-3}x \int_{C_y} d^{n-3}y \prod_{1\leqslant i<j\leqslant n} (x_i - x_j)^{\frac{\alpha' k_i\cdot k_j}{2}} (y_i - y_j)^{\frac{\alpha' k_i\cdot k_j}{2}} f(x_{ij}) g(y_{ij})$$

$$\mathfrak{M}_n = (-1)^{n-3} \sum_{\sigma,\gamma\in\mathfrak{S}_{n-3}} \mathcal{S}[\gamma(2,\ldots,n-2)|\sigma(2,\ldots,n-2)]_{k_1}$$
$$\times \mathcal{A}_n(1,\sigma(2,\ldots,n-2),n-1,n)\widetilde{\mathcal{A}}_n(n-1,n,\gamma(2,\ldots,n-2),1)$$

[Bern, Carrasco, Johansson] [Kawai,Lewellen, Tye; Tye, Zhang;Bjerrum-Bohr, Damgaard, Feng, Søndergaard; Bjerrum-Bohr, Damgaard, Søndergaard, Vanhove]

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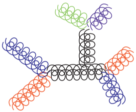

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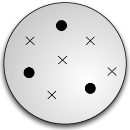

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$$\mathfrak{A}_n^{\mathrm{YM}}=\mathcal{A}^{\mathrm{vector}}\otimes\mathcal{S}\otimes\mathcal{A}^{\mathrm{scalar}}$$

$$\mathfrak{M}_n^{\mathrm{Grav}}=\mathcal{A}^{\mathrm{vector}}\otimes\mathcal{S}\otimes\mathcal{A}^{\mathrm{vector}}$$

[Bern, Carrasco, Johansson] [Kawai,Lewellen, Tye; Tye, Zhang;Bjerrum-Bohr, Damgaard, Feng, Søndergaard; Bjerrum-Bohr, Damgaard, Søndergaard, Vanhove]

Tree amplitudes with massive external legs

We are interested into *pure gravity* amplitudes of gravitons scattering off *massive particles*

$$\mathcal{A}^{\text{vector}}(\sigma(1,\ldots,n)) = \int_{x_{\sigma(1)}<\cdots<x_{\sigma(n)}} d^{n-3}x\, f(x_i - x_j) \prod_{1\leqslant i<j\leqslant n} (x_i - x_j)^{2\alpha' k_i\cdot k_j}$$

- Massive state vop are of the form $V =: (\partial X)^{n+1} e^{ik\cdot X}:$ with $\alpha' k^2 = n$
- The OPE between the plane-wave still gives $(x_i - x_j)^{2\alpha' k_i\cdot k_j}$

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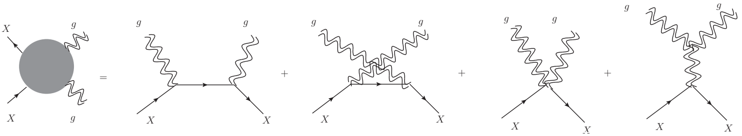
- The function $f(x_i - x_j)$ develops new poles $1/(x_i - x_j)^m$ with m integer
- This shifts $2\alpha'(k_i \cdot k_j) + m = 2\alpha'(k_i + k_j)^2$ to accommodate for the masses $\alpha'(k_i^2 + k_j^2) = m$

Since the momentum kernel and the amplitudes relations are expressed in terms of the scalars products $k_i \cdot k_j$ they are still valid in the same form for massive external states

Gravitational compton scattering

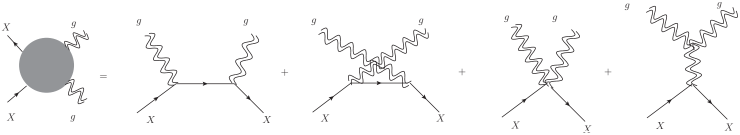


We express the gravity Compton scattering as a product of two Yang-Mills amplitudes

$$\mathfrak{M}(X^s g \to X^s g) = G_N \times (p_1 \cdot k_1)\, \mathcal{A}_s(1234) \tilde{\mathcal{A}}_0(1324)$$

$\mathcal{A}_s(1234)$ is the color ordered amplitudes scattering a gluon off a massive spin s state $X^s g \to X^s g$

Gravitational compton scattering



We express the gravity Compton scattering as a product of two Yang-Mills amplitudes

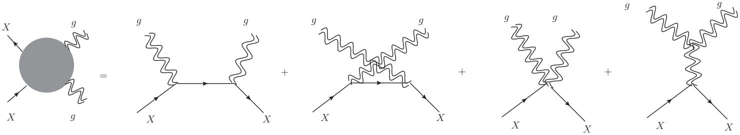
- using the monodromy relations

$$(k_1 \cdot k_2)\, \mathcal{A}_s(1234) = (p_1 \cdot k_2)\, \mathcal{A}_s(1324)$$

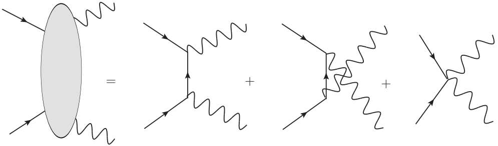
- then

$$\mathfrak{M}(X^s g \to X^s g) = G_N \frac{(p_1 \cdot k_1)(p_1 \cdot k_2)}{k_1 \cdot k_2}\, \mathcal{A}_s(1324)\tilde{A}_0(1324)$$

Gravitational compton scattering



The gravity Compton scattering is expressed as the square of QED (abelian) Compton amplitudes



$$\mathfrak{M}(X^s g \to X^s g) = G_N \frac{(p_1 \cdot k_1)(p_1 \cdot k_2)}{k_1 \cdot k_2} \mathcal{A}_s(1324) \tilde{A}_0(1324)$$

A natural value for the Gyromagnetic ratio I

A first physical consequence of the relation between the gravitational Compton amplitudes and the QED amplitudes is a natural derivation of the natural value $g = 2$ of the gyromagnetic ratio

The classical value of the g-factor for the electron is $g_0 = 2$ and Quantum mechanically $g = g_0 + \text{quantum corrections}$

$$g_{electron} = g_0 \left(1 + \frac{\alpha}{2\pi} + \cdots\right) = 2 \times 1.00115965$$

There was the question of the natural value of g_0 for spin S particle.

Belinfante conjectured that $g_0 = 1/S$ - but various arguments favored $g_0 = 2$ independently of the spin

A natural value for the Gyromagnetic ratio II

Massive spin 1 Compton amplitudes have a piece that diverges for $m^2 \to 0$ if $g_0 \neq 2$

$$\frac{(g_0 - 2)^2}{m^2} \left(\frac{n_s}{s} - \frac{n_t}{t}\right)$$

If $g \neq 2$

- Violation of unitarity for photon of energy $E \sim m$
- QED gets strongly coupled at energies $E \sim m/e$

A natural value for the Gyromagnetic ratio III

- from the expression of the gravity compton scattering as the product of the QED scattering on can extract expression of g factor

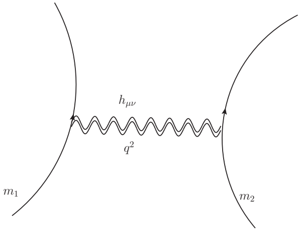
$$\mathfrak{M}(X^s g \to X^s g) = G_N \frac{p_1 \cdot k_1 \, p_1 \cdot k_2}{k_1 \cdot k_2} \mathcal{A}_s(1324) \tilde{\mathcal{A}}_0(1324)$$

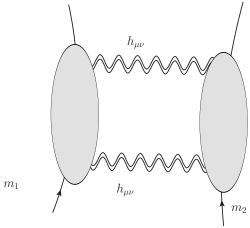
- [Holstein] showed that the amplitudes $\mathcal{A}_s(1324)$ are the QED Compton amplitude for spin s particle and a (bare) value of the g-factor $g_0 = 2$
- It is the two derivative nature of gravity that removes the $1/m^2$ for $m \to 0$

The relation $gravity \sim (gauge)^2$ leads to $g_0 = 2$ for *all* values of the spin S

Corrections to Newton's potential

The corrections to Newton's potential between two non-relativistic masses m_1 and m_2 extracted from a one-loop amplitude





Corrections to Newton's potential I

In the non-relativistic limit the second order potential reads

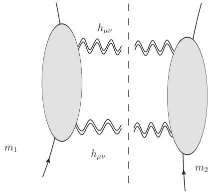
$$\mathfrak{M}^{(2)}(q^2) \simeq C\,\frac{G_N^2 m_1 m_2 (m_1+m_2)}{\sqrt{-q^2}} + Q G_N^2 m_1 m_2 \hbar \log(-q^2)$$

We consider cuts that will gives the $1/\sqrt{-q^2}$ and $\log(-q^2)$ coefficients the coefficients of $(q^2)^n/\sqrt{-q^2}$ and $(q^2)^n\,\log(-q^2)$ are negligible.

The coefficient C and Q have a spin-independent and a spin-orbit contribution

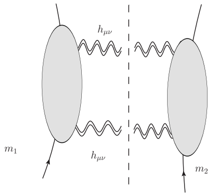
$$C, Q = C, Q^{S-I}\,\langle S_1|S_1\rangle\,\langle S_2|S_2\rangle + C, Q_{1,2}^{S-O}\,\langle S_1|S_1\rangle\,\vec{S}_2\cdot\frac{p_3 \vec{\times} p_4}{m_2} + (1\leftrightarrow 2)$$

The one-loop amplitude



We are not interested in the full amplitude so only the massless graviton cut is enough.

The one-loop amplitude



We are not interested in the full amplitude so only the massless graviton cut is enough.

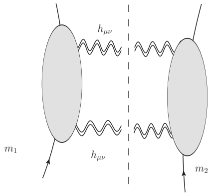
- The singlet cut gives a scalar box

$$\mathfrak{M}|_{non-singlet\ cut} = \int \frac{d^{4-2\epsilon}\ell}{\ell_1^2 \ell_2^2 \prod_{i=1}^4 \ell_1 \cdot p_i}$$

- The non-singlet cut gives

$$\mathfrak{M}|_{non-singlet\ cut} = \int d^{4-2\epsilon}\ell \frac{\Re\mathrm{e}\left(\mathrm{tr}_-(\not\ell_1 \not p_1 \not\ell_2 \not p_2)\right)^4}{\ell_1^2 \ell_2^2 \prod_{i=1}^4 \ell_1 \cdot p_i}$$

The one-loop amplitude



We are not interested in the full amplitude so only the massless graviton cut is enough.

- The result is given by

$$\mathfrak{M}^{(2)} = G_N^2 m_1 m_2 \left(\underbrace{6\pi}_{C} \frac{m_1 + m_2}{\sqrt{-q^2}} \underbrace{-\frac{41}{5}}_{Q} \log(-q^2) \right)$$

Universality of the result

Remarkably the coefficients are universal independent of the spin of the external states, a property noticed by [Holstein, Ross].

This is a consequence of

- The reduction to the product of QED amplitudes
- the low-energy theorems of [Low, Gell-Mann, Goldberger] and [Weinberg]

In the non-relativistic limit the QED Compton amplitudes take a simplified form given by

$$\mathcal{A}(X^s\gamma \to X^s\gamma) \simeq \langle S|S\rangle\, \mathcal{A}(X^0\gamma \to X^0\gamma) + \hat{\mathcal{A}}\, \vec{S}\cdot \frac{p_1 \times p_2}{m}$$

The KLT formula gives that the tree gravity amplitude take the same generic form

$$\mathfrak{M}(X^s g \to X^s g) \simeq \langle S|S\rangle\, \mathfrak{M}(X^0 g \to X^0 g) + \hat{\mathfrak{M}}\, \vec{S}\cdot \frac{p_1 \times p_2}{m}$$

- In the cut this leads to universality of the result [Bjerrum-Bohr, Donoghue, Vanhove]

Outlook

Recent progresses from string theory technics, on-shell unitarity, double-copy formalism simplifies a lot perturbative gravity amplitudes computations

- The amplitudes relations discovered in the context of massless supergravity theories extend to the pure gravity case with massive matter
- The use of quantum gravity as an effective field theory allows to compute universal contributions from the long-range corrections