Holographic Conductivity

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Based on work with Gary Horowitz, Jorge Santos and Mike Blake

Basics of Conductivity

Ohm's Law

$$\vec{j}(\omega) = \sigma(\omega)\vec{E}(\omega)$$

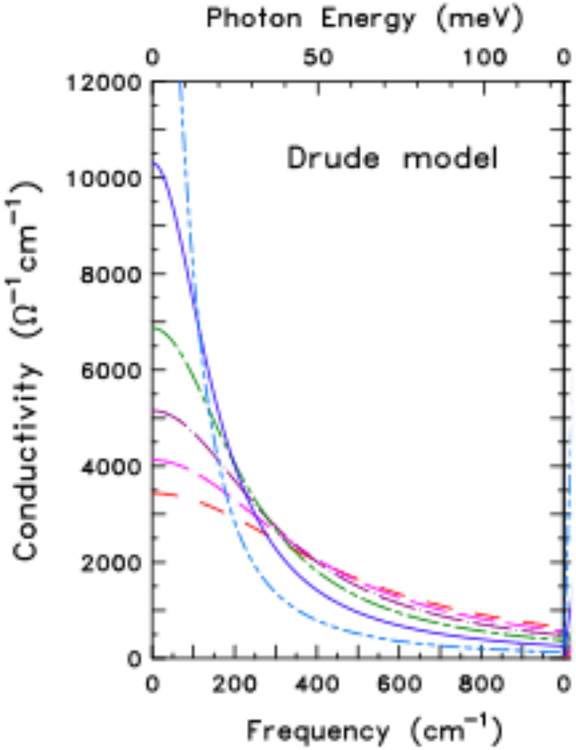
$$\vec{E} = \vec{E}(\omega)e^{-i\omega t}$$
$$\vec{j} = \vec{j}(\omega)e^{-i\omega t}$$

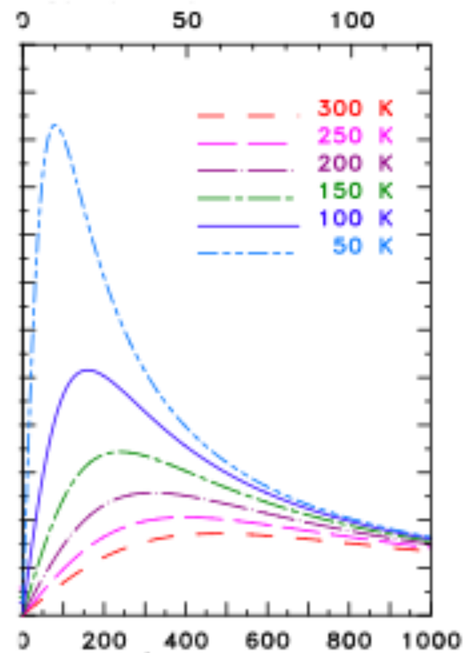
Drude Model

$$m\frac{d\vec{v}}{dt} + \frac{m}{\tau}\vec{v} = q\vec{E}$$

$$\vec{j} = nq\vec{v} \quad \Longrightarrow \quad \boxed{\sigma(\omega) = \left(\frac{nq^2\tau}{m}\right)\frac{1}{1 - i\omega\tau}}$$

Drude Model



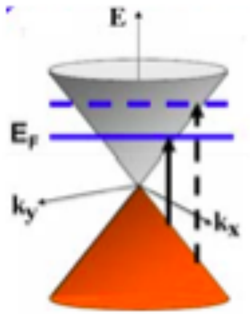


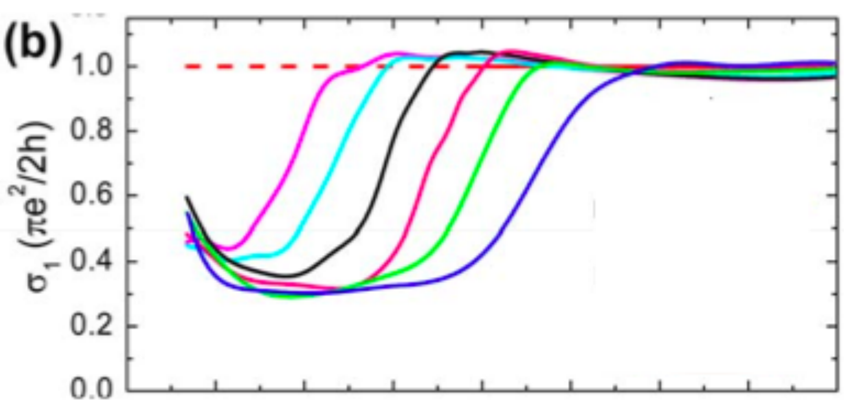
$$\mathrm{Re}(\sigma) = \frac{\sigma_0}{1 + \omega^2\tau^2}$$

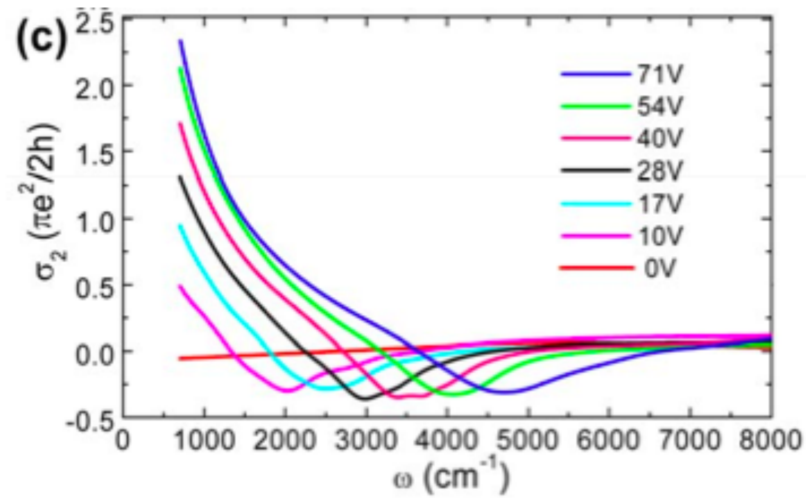
$$\mathrm{Im}(\sigma) = \frac{\sigma_0\omega\tau}{1 + \omega^2\tau^2}$$

Graphene

Li et al. (2008)

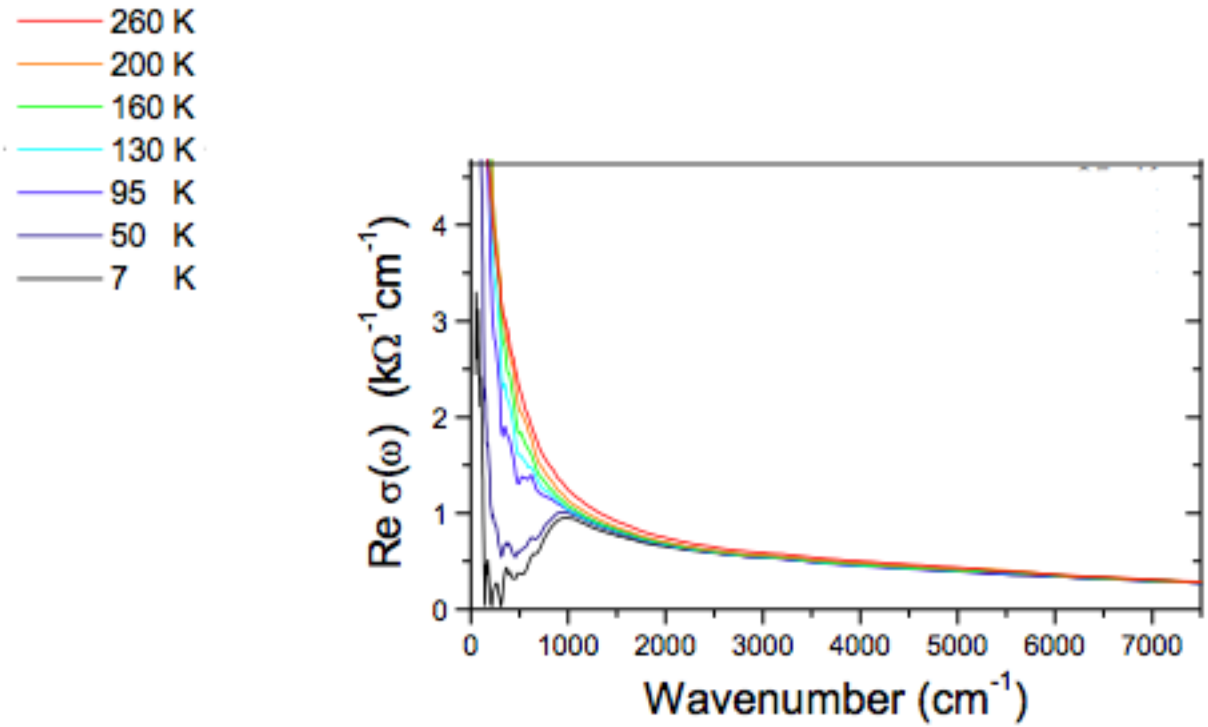






Cuprates

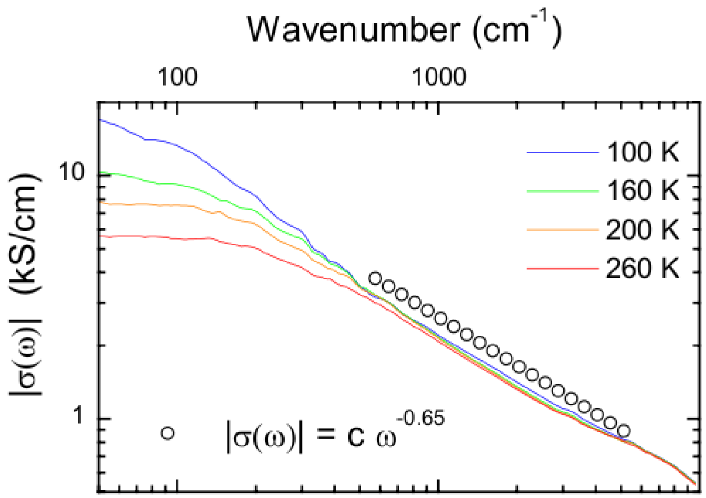
van der Marel
et al. (2004)

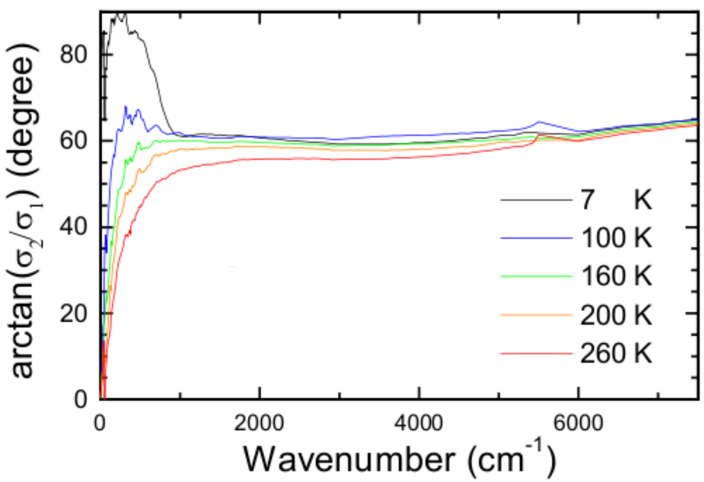


$Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$

Cuprates

van der Marel et al. (2004)

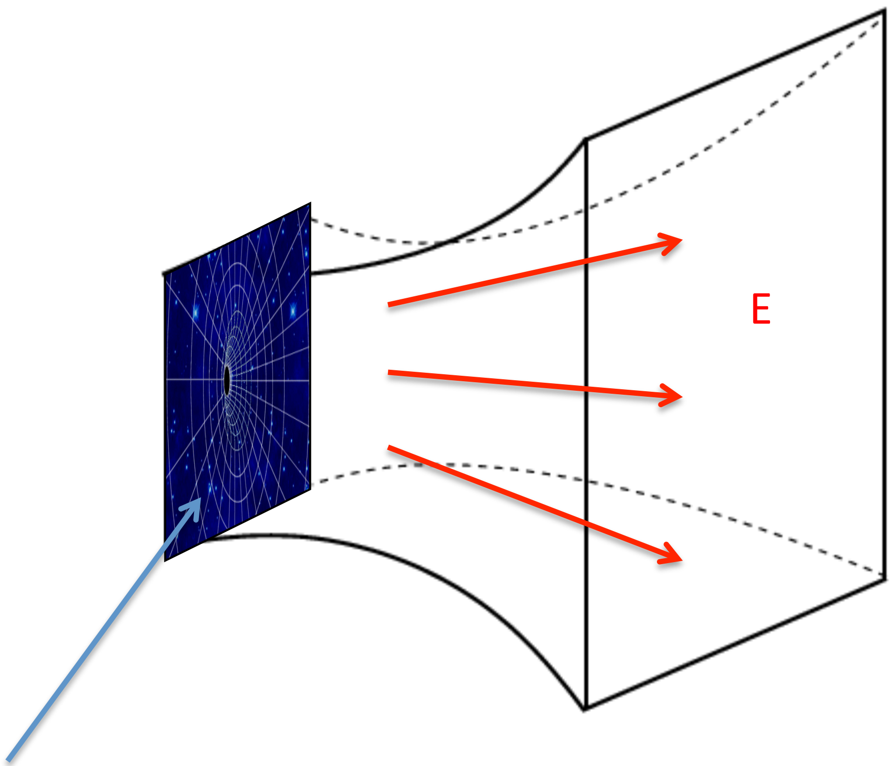




$$|\sigma(\omega)| \sim \frac{1}{\omega^{2/3}}$$

Holography

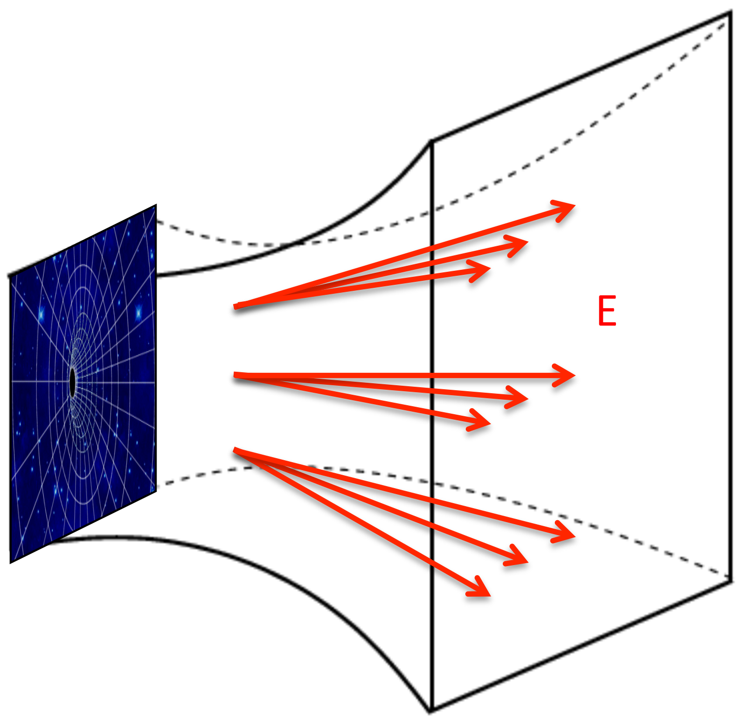
Finite Density Matter



Boundary field theory
d=2+1

- d=3+1 bulk
- Hawking radiation = finite temperature
- Electric field = chemical potential

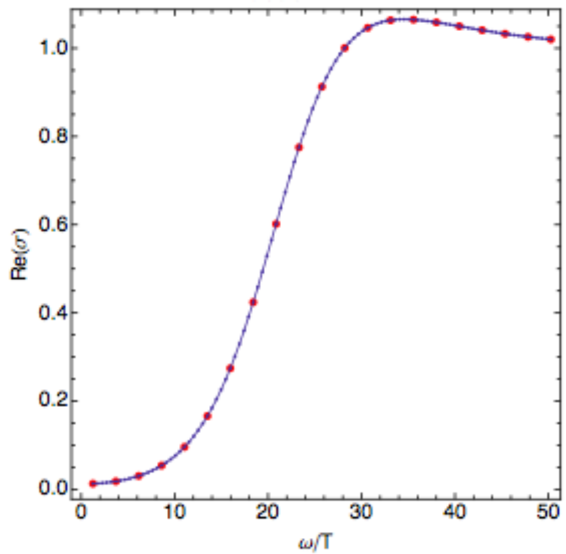
Optical Conductivity

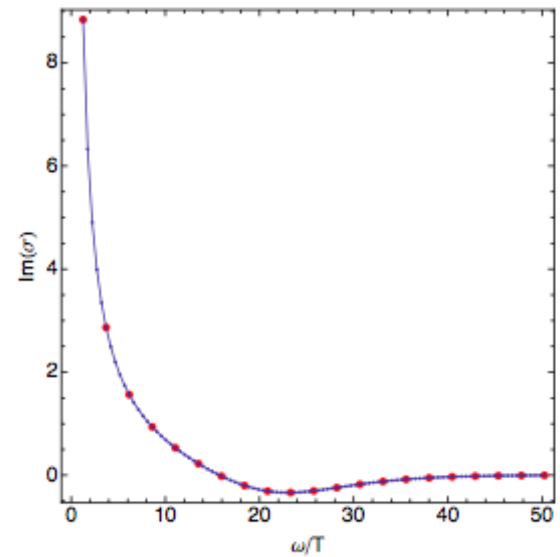


$$A_x = \frac{E}{i\omega} + \langle J_x \rangle r + \dots$$

Optical Conductivity

$$j(\omega) = \sigma(\omega) E(\omega)$$

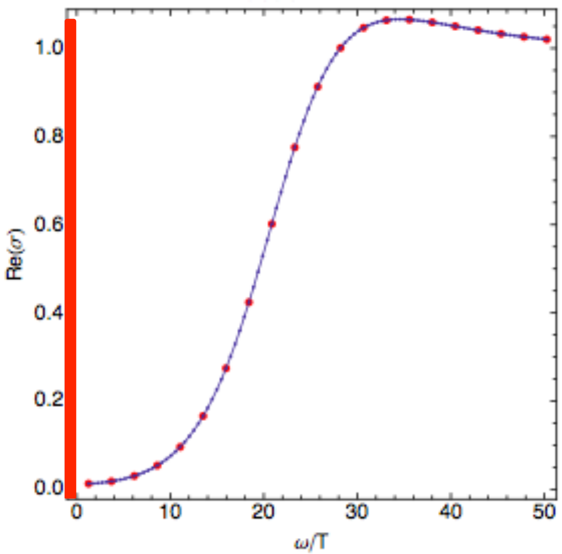


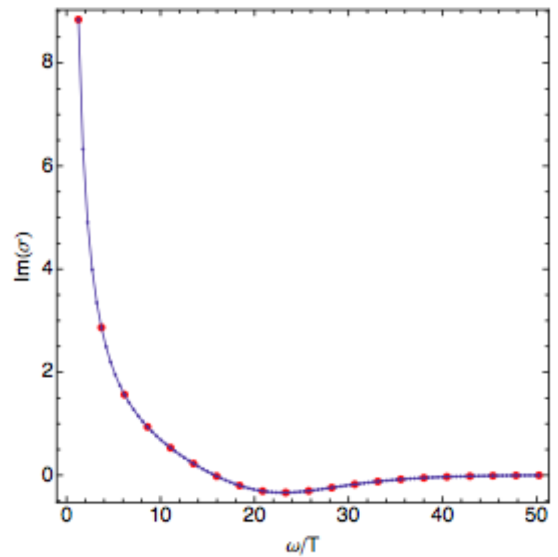


Herzog, Kovtun, Sachdev and Son; Hartnoll

Optical Conductivity

$$j(\omega) = \sigma(\omega) E(\omega)$$





$$\mathrm{Re}\,\sigma(\omega) \sim K\,\delta(\omega)$$

$$\mathrm{Im}(\sigma) \to \frac{K}{\pi\omega}$$

Herzog, Kovtun, Sachdev and Son; Hartnoll

The Delta-Function

$$\mathrm{Re}\,\sigma(\omega) \sim K\,\delta(\omega)$$

Due to:

- Finite density of charge carriers
- Translational invariance

Story 1:

Breaking Translational Invariance

How to Build a Lattice

Horowitz, Santos and Tong (2012)

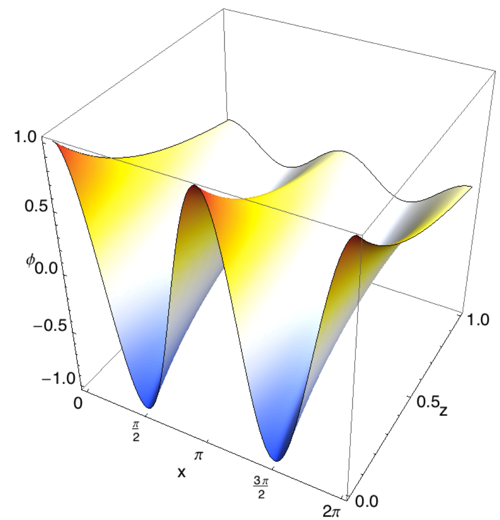
$$\mathcal{L} = \mathcal{L}_{\mathrm{QFT}} + \mu(x,y)\,\mathcal{Q}$$

Spatially dependent chemical potential

Aside: We have also built other lattices which have smooth charge density. The following results remain unchanged

The lattice black hole

Stripes: $\mu = \mu[1 + Acos(k_L x)]$



Parameters

$T,\, \mu,\, k_L,\, A$

$$ds^2 = \frac{L^2}{z^2}\Big[-g_{tt}(z,x)dt^2 + g_{zz}(z,x)dz^2 + g_{xx}(z,x)(dx + a(z,x)dz)^2 + g_{yy}(z,x)dy^2 \Big]$$

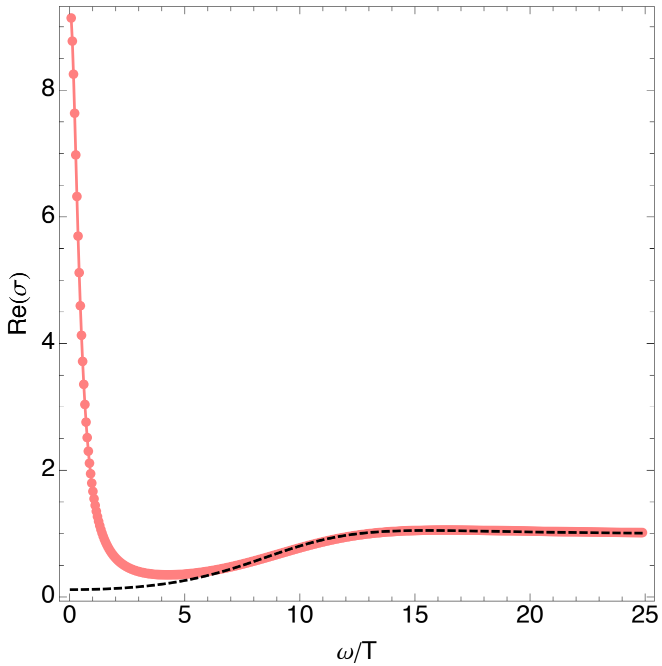
Perturbing the Lattice

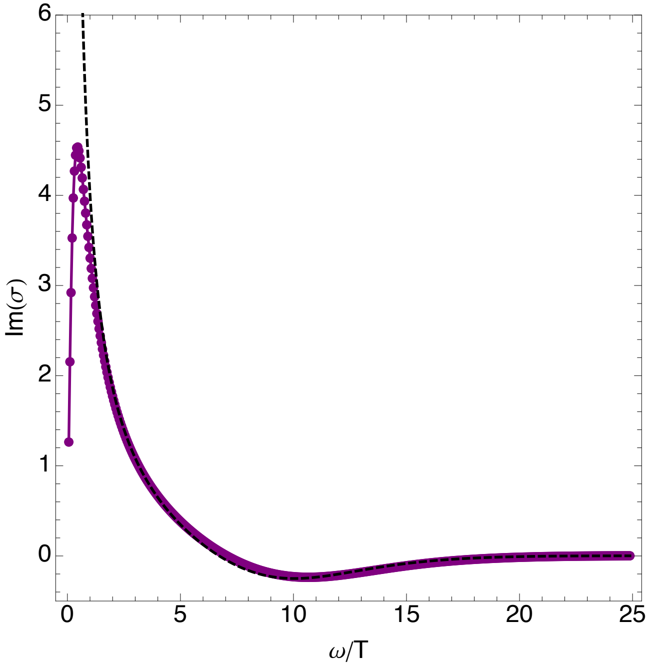
$$\delta g_{tt},\ \delta g_{tz},\ \delta g_{tx},\ \delta g_{zz},\ \delta g_{zx},\ \delta g_{xx},\ \delta g_{yy}$$

$$\delta A_t,\ \delta A_z,\ \delta A_x,\ \delta\Phi$$

Optical Conductivity

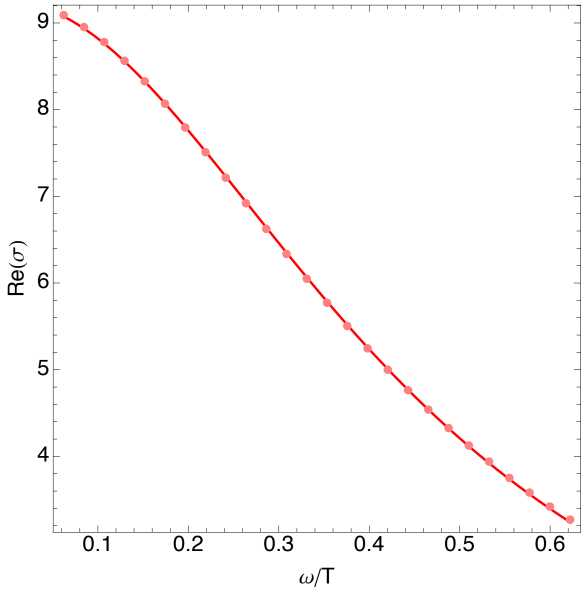
$$j(\omega) = \sigma(\omega)E(\omega)$$

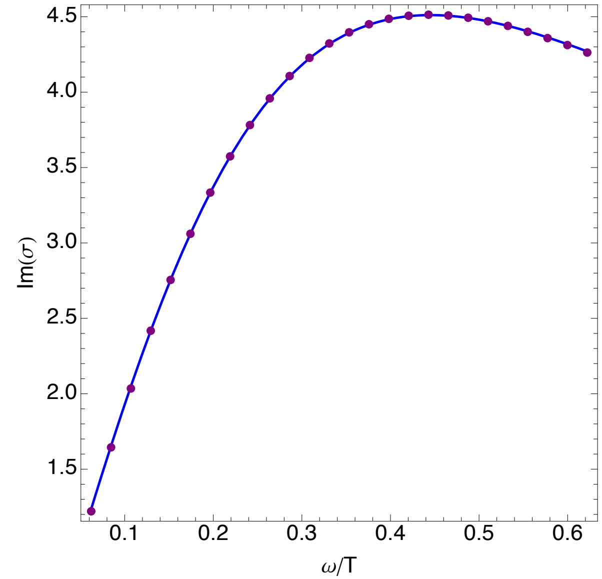




$\mu = 1.4 \qquad T = 0.115\mu \qquad k_L = 2 \qquad A = 1.5$

Low Frequency Drude Behaviour $\omega \lesssim T$





$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$

Note: No quasiparticles

DC Resistivity

$$\rho \sim T^{2\nu-1}$$

- Exponent depends on lattice spacing: $\nu = \frac{1}{2}\sqrt{5 + 2(k/\mu)^2 - 4\sqrt{1+(k/\mu)^2}}$

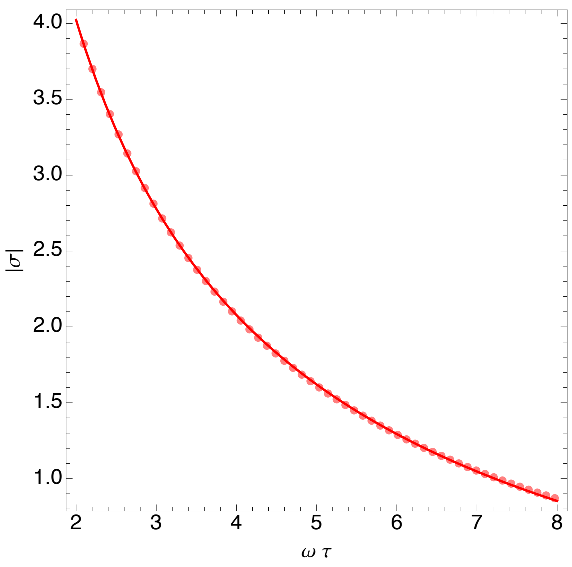
- This is characteristic of a *locally critical* theory
  i.e. geometry: $AdS_2 \times \mathbf{R}^2$
  field theory $z \to \infty$

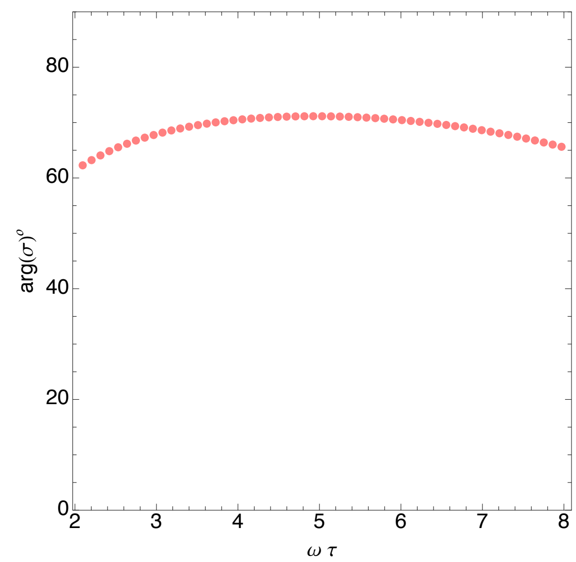
  Hartnoll, Hofman (2012)

- Model building: a mechanism suggested to drive this to linear resistivity.

  Donos, Hartnoll (2012)

Mid-Frequency Behaviour $\omega \gtrsim T$



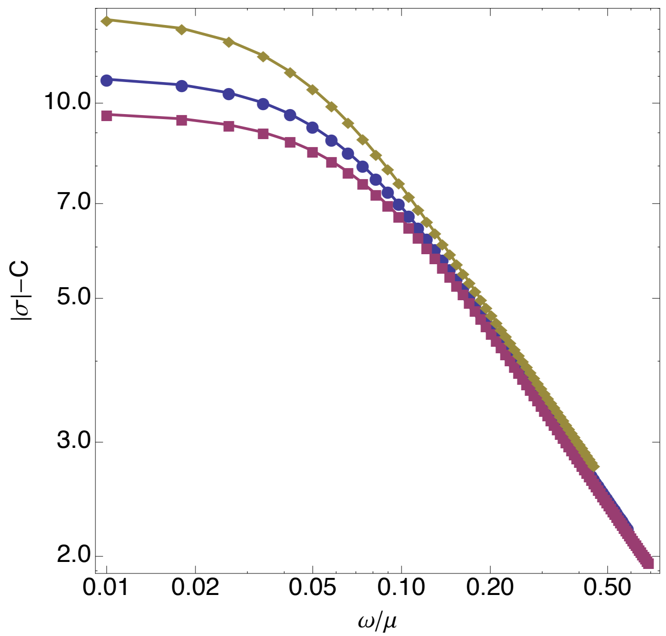


$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

Robust Power-Law

$T = 0.98\mu,\ T = 0.115\mu,\ T = 0.13\mu$

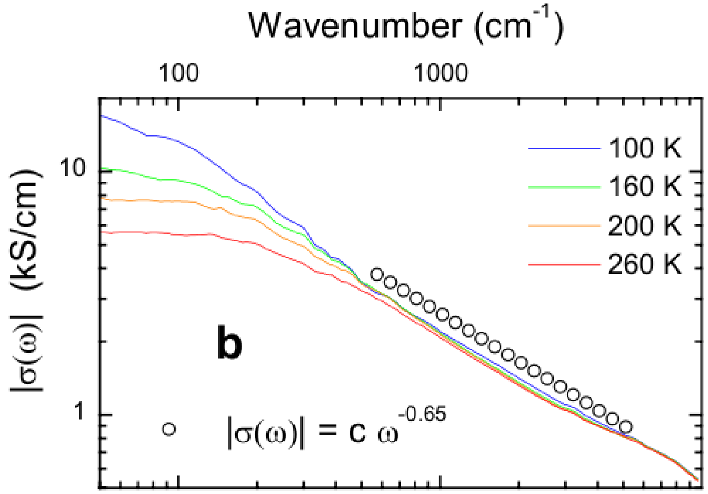
Log-log plots



$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

B temperature independent

Comparison to Cuprates

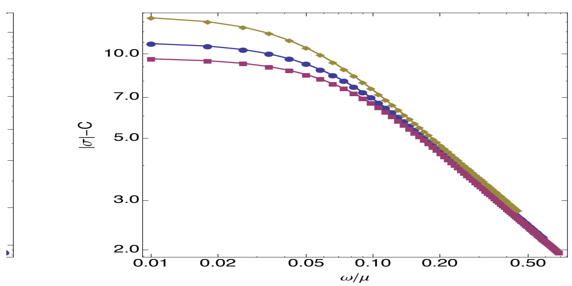


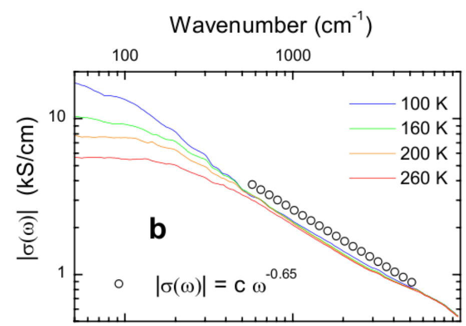
$$|\sigma(\omega)| \sim \frac{1}{\omega^{2/3}}$$

More Scaling

- Also seen in thermoelectric conductivity
- Also seen in AdS_5

Why?





Story 2:

A Holographic Model for Momentum Dissipation

Momentum Dissipation in Holography

Vegh (2013)

Bulk diffeomorphisms $\Longrightarrow$ $\partial_\mu T^{\mu\nu} = 0$

No diffeomorphisms $\Longrightarrow$ $\partial_\mu T^{\mu\nu} \neq 0$

This means massive gravity. (Gulp!)

Is massive gravity an effective holographic theory
for models with disorder or a lattice?

Massive Gravity

Expanding around flat space: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$S_{FP} = \int d^4x\sqrt{-g}\,\frac{m^2}{4}\left(h^{\mu\nu}h_{\mu\nu} - h^{\mu}{}_{\mu}h^{\nu}{}_{\nu}\right)$$

But expanding around a general background

Add further non-linear terms to try to remedy this…

Massive Gravity…ugly, but it works

de Rahm, Gabadadze, Tolley (2010)
Hassan, Rosen (2011)

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(R + \Lambda + \sum_{i=1}^{4}\alpha_i\, e_i[\mathcal{K}]\right)$$

$$\mathcal{K}^2 = g^{-1} f$$

Fixed background metric

$$e_1 = \mathrm{Tr}\,\mathcal{K}$$

$$e_2 = (\mathrm{Tr}\,\mathcal{K})^2 - \mathrm{Tr}\,(\mathcal{K}^2)$$

$$e_3 = (\mathrm{Tr}\,\mathcal{K})^3 - 3(\mathrm{Tr}\,\mathcal{K})\,\mathrm{Tr}\,(\mathcal{K}^2) + 2\mathrm{Tr}\,(\mathcal{K}^3)$$

$$e_4 = (\mathrm{Tr}\,\mathcal{K})^4 - 6(\mathrm{Tr}\,\mathcal{K})^2\,\mathrm{Tr}\,(\mathcal{K}^2) + 3\mathrm{Tr}\,(\mathcal{K}^2)^2 + 8(\mathrm{Tr}\,\mathcal{K})\,\mathrm{Tr}\,(\mathcal{K}^3) - 6\mathrm{Tr}\,(\mathcal{K}^4)$$

Holographic Massive Gravity

Does it give sensible answers?

Black Hole Thermodynamics

Blake and Tong (to appear)

Yes

$$S = S_{BH} \sim r_h^2$$

But horizon radius is $r_h = r_h(T, \mu, m^2)$

graviton mass parameters
(typically depend on position in bulk)

Hydrodynamics

Davison (2013)

$$\partial_\mu T^{\mu i} = -\tau^{-1} T^{0i}$$

with relaxation time $\tau^{-1} \sim r_h m^2(r_h)$

DC Conductivity

Blake and Tong (to appear)

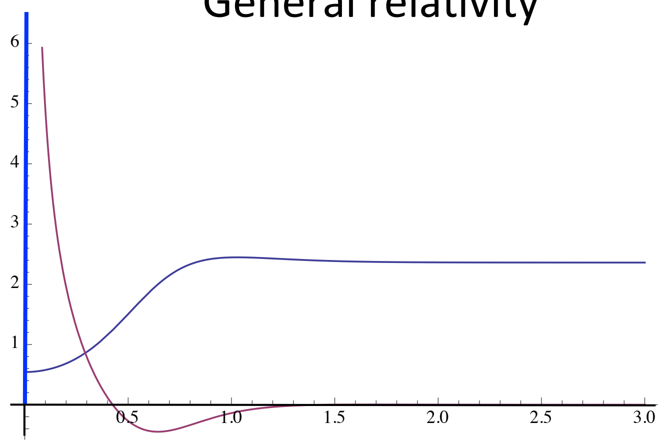
$$\sigma_{DC} = \frac{\mu^2}{m^2(r_h)} + 1$$

Universal behaviour, governed only by horizon

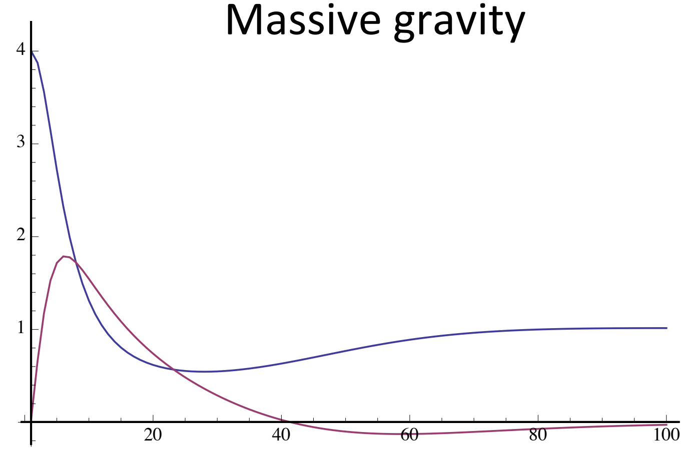
Optical Conductivity

Delta function

General relativity



Massive gravity

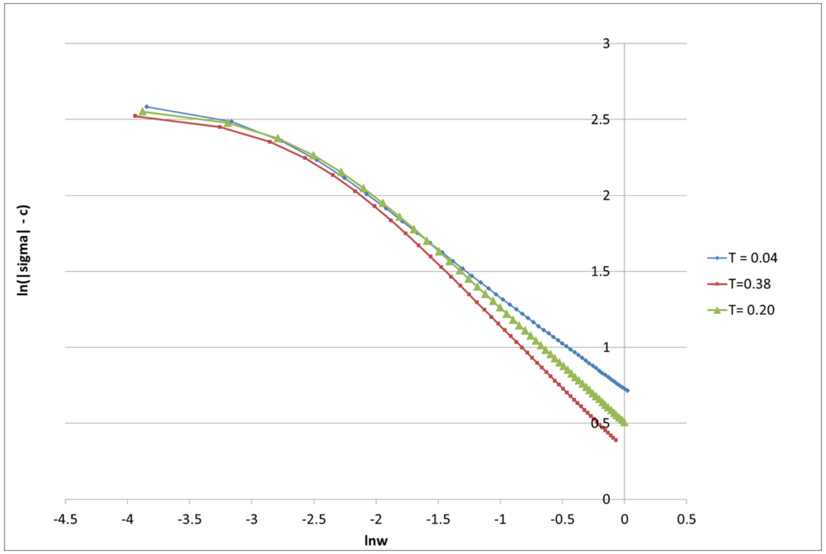


For small frequencies: $$\sigma(\omega) \sim \frac{\sigma_{\rm DC}}{1 - i\omega\tau}$$

Vegh (2013)
Davison (2013)

Mid-Frequency Behaviour

Vegh (2013)
Blake and Tong



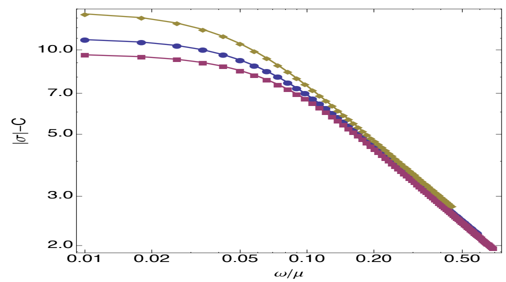
$$|\sigma(\omega)| = \frac{B}{\omega^{\gamma}} + C$$

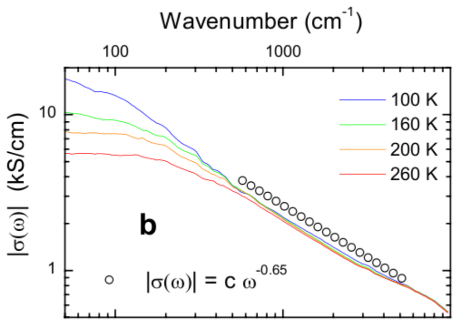
Some evidence for scaling, but all parameters now temperature dependent.

Summary

Story 1: General Relativity with a Lattice

Why Scaling?





Story 2: Massive Gravity

- Limited scaling, but sensible results.
- What is this a theory of? Disorder?