UNIVERSITY OF COPENHAGEN

NBIA Summer School on Computational Astrophysics

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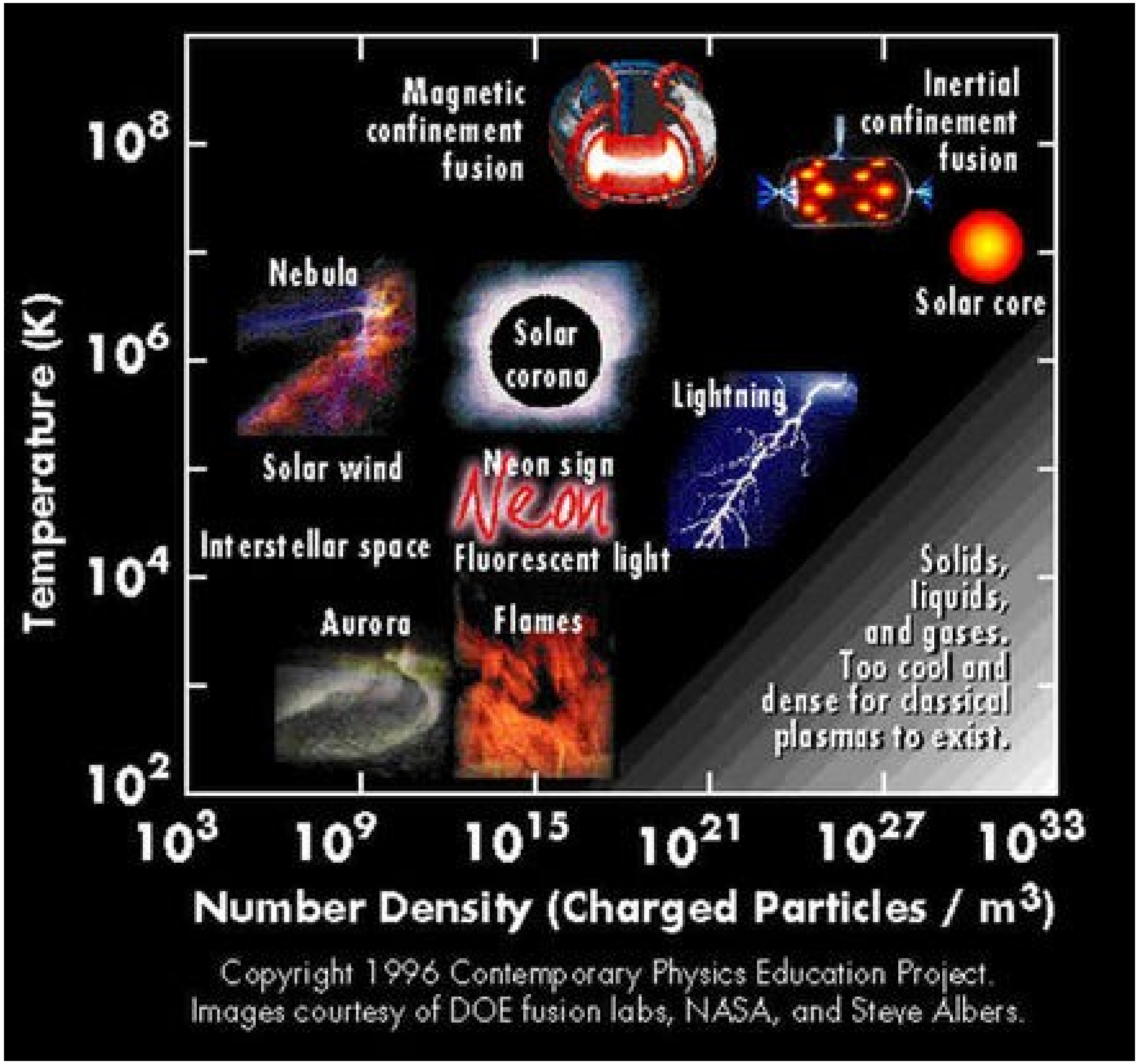
MagnetoHydroDynamics
Magnetic reconnection
Regime comparisons

What do we want to explorer?



How do we describe it?

- Ability to describe the time evolution of a magnetised plasma
- Many body problem
  - "random motions" of individual particles
  - Interaction via
    - Coulomb scattering
    - Magnetic fields

- Numerical expensive to solve for individual particle dynamics
  - Often done using Particle In Cell or Vlasov simulations
  - More on this later in Aakes lectures
- Often we can manage with a simpler approach

Which approach?

- Particle statistics can be described using distribution functions
  - $f(\vec{r}, \vec{v}, \rho, q, T)$
    - Maxwellian velocity distribution for a plasma in thermodynamic equilibrium
- Based on moments of the particle distribution functions
- Can derive various macroscopic quantities:
  - Temperature (Maxwellian velocity distribution..)
  - Density (number of particles per volume)
  - Velocity (mean velocity of particles)
  - Magnetic field (mean effect of moving charged particles)
- 8 physical quantities in 3D space

MHD equations

- 8 pde's + 3 conditional relations

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\vec{u}\,\rho)$$

$$\frac{\partial (\vec{u}\,\rho)}{\partial t} = -\nabla p - \nabla \cdot (\vec{u}\,\vec{u}\,\rho + \underline{\tau}) + \vec{j} \times \vec{B} + \vec{F}_{extrenal}$$

$$\frac{\partial e}{\partial t} = -\nabla \cdot (\vec{u}\,e) - p\,\nabla \cdot \vec{u} + Q_{visc} + Q_{joule} + Rad + Cond + \;.\;\;.\;\;.$$

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

$$\nabla \cdot \vec{B} = 0$$

$$P = (\gamma - 1)\,e$$

$$\vec{E} = -\vec{u} \times \vec{B} + \eta\,\vec{j}$$

Requirement for the validity of the MHD equations

- Particle properties determine how the plasma behaves
- Not easy to cope with large scale structures using such an approach
- Need to find a simplifying approach
  - Needs to take moments of the particle distribution functions

- Statistically mean values are physically well defined
- Collisional dominated plasma
  - Mean free path is short compared to minimum length scale in the problem
  - Gyro-radius is very short compared to minimum length scale in problem
- Velocities are much smaller than the speed of light
- Close to total ionisation

- → can introduce a single fluid approximation

Where/when does MHD apply?

- $L \gg \lambda_{coll} \gg L_D$
  - Coulomb collisions: $\lambda_{col} = 3 \times 10^{-12} \dfrac{v^4}{n \ln \Lambda}$
  - Charge neutrality: $L_D = 4.9 \left(\dfrac{T}{n} \right)^{0.5}$
- **Solar corona**
  - $n \approx 10^9\, cm^{-3}, \ln \Lambda \approx 20, v \approx 20\, km/s, T \approx 1\, mill.\, K \rightarrow \lambda_{col} \approx 30\, m, L_D \approx 10\, cm$
  - Obs. resolution is order 100-700 Km
  - Typical length scales are 50.000 km
- **Solar wind at earth**
  - $n \approx 6\, cm^{-3}, \ln \Lambda \approx 20, v \approx 500\, km/s, T \approx 50.000\, K \rightarrow \lambda_{col} \approx 1600\, m, L_D \approx 450\, m$
- **Interstellar medium**
  - $n \approx 10^6\, cm^{-3}, \ln \Lambda \approx 20, v \approx 10\, km/s, T \approx 100\, K \rightarrow \lambda_{col} \approx 1500\, m, L_D \approx 1\, m$
  - $n \approx 0.5\, cm^{-3}, \ln \Lambda \approx 20, v \approx 100\, km/s, T \approx 8000\, K \rightarrow \lambda_{col}\, LARGE, L_D \approx 620\, m$

Energy equation

- Total energy
  - Thermal, kinetic, potential, magnetic,...
- Different forms
  - Only internal energy (e) used here

$$\boxed{\frac{\partial e}{\partial t} = -\nabla\left(ue\right) - p\nabla\cdot u} + Q_{visc} + Q_{resistivity} + Radiation + Conduction + \ldots$$

Adiabatic

- Why this form?
  - Numerical reasons – accuracy when splitting total energy into its smaller contributions (pressure in low beta plasma)

Equation of state

- Need an additional equation to close the system
- Equation of state

$$P = P(n, T) \quad \left(P = \rho \frac{R}{\mu} T, \quad T = e / \rho \right)$$

- Number density
- Temperature
- Chemical composition
- Ionisation state

Maxwell's equations

- Induction eq $$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$
- Amperes law $$\cancel{\frac{\partial \vec{E}}{\partial t}} = \nabla \times \vec{B} - \vec{j}$$
- Solonoidal condition $\nabla \cdot \vec{B} = 0$
- Charge density $$\cancel{\nabla \cdot \vec{E} = \rho_q}$$
- Ohms law $$\vec{E} = -\vec{u} \times \vec{B} + \eta \vec{j}$$

Electromagnetic waves in vacuum → speed of light

$$\nabla \cdot (\nabla \times \vec{B}) = 0$$

Ignore these terms when investigating plasma moving with v<< c

Limitations

- Viscosity/Resistivity
  - Parameters replacing particle interaction
- Length scales
  - Much longer than particles mean free path
- Time scales
  - Much longer than particles gyro and collision times
- Requires close to a thermal plasma
  - Also works when in a magnetic dominated plasma

Induction equation

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} \qquad \vec{E} = -\vec{u} \times \vec{B} + \eta \vec{j}$$

Advection term and a diffusion term

Ratio defines the Magnetic Reynolds number

Often much larger than unity in astrophysics

Implies that diffusion is only important on very small length scales

- Why do we bother about it then?
- Seems like being unimportant...

Viscosity

Viscous diffusion "only" transfer kinetic energy in to thermal energy

Damp motions → small scale motions

- particle interaction process that is not correctly handled in numerical experiments
  - typically represented by a constant parameter
  - or numerical argumentation to handle special effects in the code implementation

  - important for the behaviour of turbulence...

Resistivity

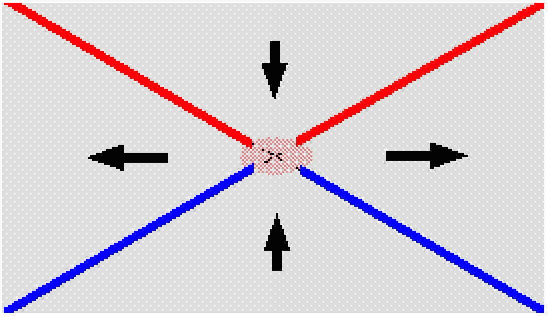
Handled through particle interaction

Conversion of magnetic energy in to:

- Thermal energy through Joule heating
- Electric field acceleration of sub population of particles
  - Non-Maxwellian velocity distributions
  - Not included in MHD, but very important for the energy budget in the diffusion region
    - In solar flares possible up to 50% of the energy conversion is using this channel!
- Indirect bulk flow acceleration
  - Lorentz force of reconnected field lines + pressure gradient

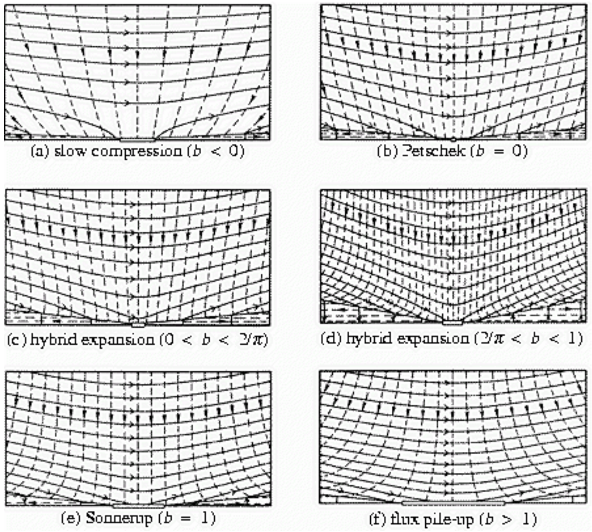
The process of magnetic reconnection

The 2D cartoon picture



Characteristics

- Change in magnetic field line connectivity
- Can only take place at a 2D magnetic null point
- Electric field is out of the plane
- There is a one-one mapping of field lines in the diffusion region
- Lots of analytical work over the years
    - Different regimes depending on the inflow conditions
        - Current sheet dimensions, standing slow shocks
    - Very different reconnection rates and scalings with magnetic Reynolds number



3D effects of reconnection

Where to find reconnection in 3D?

- 3D null points
- Separator lines connecting 3D nulls
- Regions with non-vanishing magnetic field

What is important for having reconnection in 3D

- Electric field parallel to the magnetic field
    - Can only arises from the non ideal term in Ohms law!

$$\vec{E} = -\vec{u} \times \vec{B} + \eta \vec{j}$$

Theoretical considerations

Assume the induction equation can be written in the following form

$$\frac{\partial \vec{B}}{dt} = \nabla \times (\vec{w} \times \vec{B})$$

Where *w* is the *field lines velocity* (not the flow velocity!)

When *w* is smooth and continues, file lines are frozen into the plasma and the topology of the magnetic file can't changed!

In 2D this conditions can be fulfilled everywhere (E . B =0) except at null points if E ne 0 there.

2D reconnection takes place in a singular point, where field lines are cut and rejoined → identical mapping of the involved field lines.

Schindler et al. (1988); Hornig and Schindler (1996); Hornig (2001, 2007)

Theoretical considerations

The results from 2D do not carry over to 3D!!

If there exist a diffusion region in 3D then the following applies:

- A flux transport velocity *w* does not exist (Priest et al. (2003))
- Follows magnetic field lines from footpoints comoving in the ideal flow, *they appear to split as soon as they enter the non-ideal region*, and *their connectivity changes continually and continuously* as they pass through the non-ideal region
- Magnetic field lines are not reconnected in a one-to-one fashion.

The effect of 3D reconnection

After Hornig and Priest (2003), with B = (y, k²x, 1)

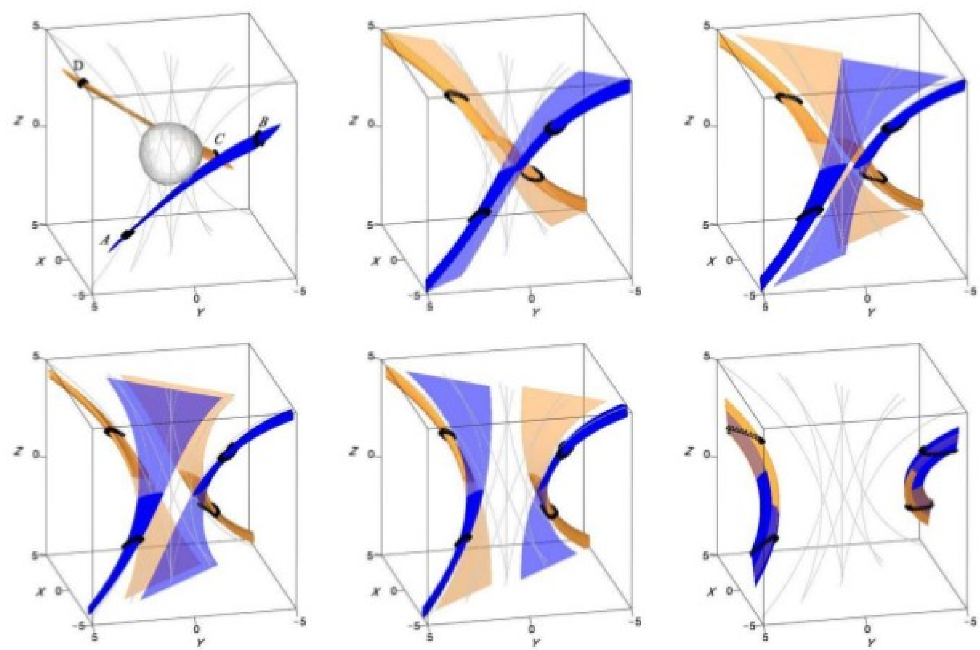


Figure 2. Reconnection of representative flux tubes in the magnetic field $\mathbf{B} = (y, k^2x, 1)$, with $k = 1.2$, traced from ideal comoving footpoints (marked black). The solid sections move at the local plasma velocity (outside the non-ideal region), while the transparent sections correspond to field lines that pass through the non-ideal region. The shaded surface in the first frame shows the localised non-ideal region. (After Pontin (2011).)

3D nulls

Location in 3D space where the magnetic field vanishes

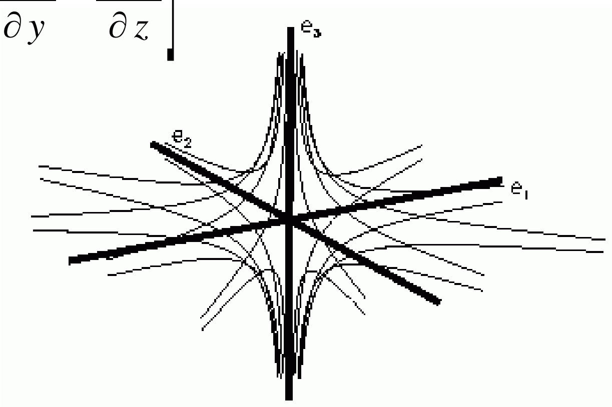
Linear expansion around the null point:

$$\vec{B}(\vec{r}) = \vec{B}_0(\vec{r}_0) + M(\vec{r} - \vec{r}_0)$$

$$M = \begin{bmatrix} \frac{\partial B_x}{\partial x} & \frac{\partial B_x}{\partial y} & \frac{\partial B_x}{\partial z} \\ \frac{\partial B_y}{\partial x} & \frac{\partial B_y}{\partial y} & \frac{\partial B_y}{\partial z} \\ \frac{\partial B_z}{\partial x} & \frac{\partial B_z}{\partial y} & \frac{\partial B_z}{\partial z} \end{bmatrix}$$

Concept of fan plane and spine axis

Divide space into two independent domains



Current accumulation at nulls

Two different main types of stress

Twist of the spine axis

- Current accumulation all over the fan plane/spine axis
- Diffusion of field lines

Tilt of spine – fan plane

- Current accumulation in fan plane -spine axis
- Structure depends on the shape of the null
- Flipping of field lines

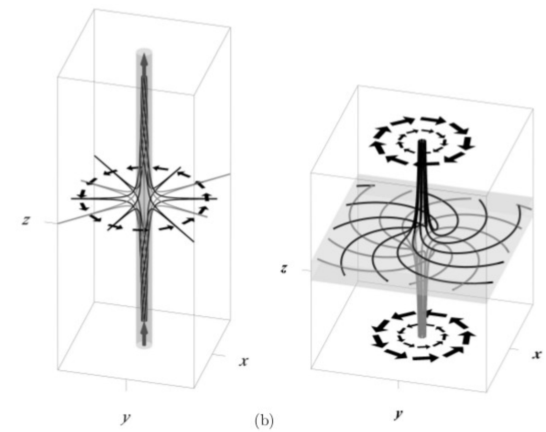


Figure 5. Schematic diagrams of (a) torsional spine and (b) torsional fan reconnection. Black and grey lines are magnetic field lines, the shaded surfaces are current density isosurfaces, grey arrows indicate the direction of the current flow, while black arrows indicate the driving plasma velocity. (After Pontin (2011).)

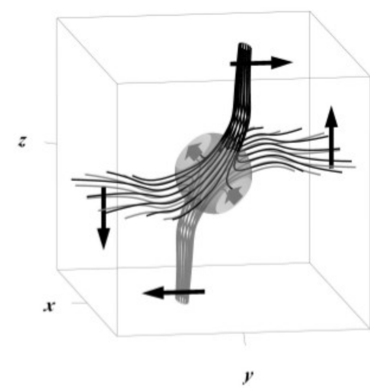
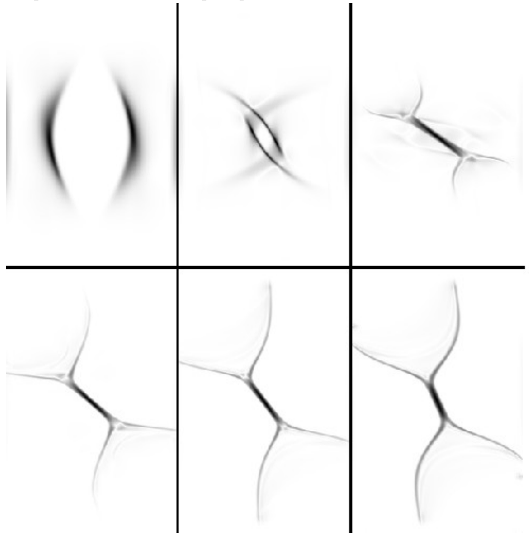


Figure 6. Schematic diagram of spine-fan reconnection at an isolated null. Black and grey lines are magnetic field lines, the shaded surface is a current density isosurface, the grey arrows indicate the direction of the current flow, while the black arrows indicate the driving plasma velocity. (After Pontin (2011).)

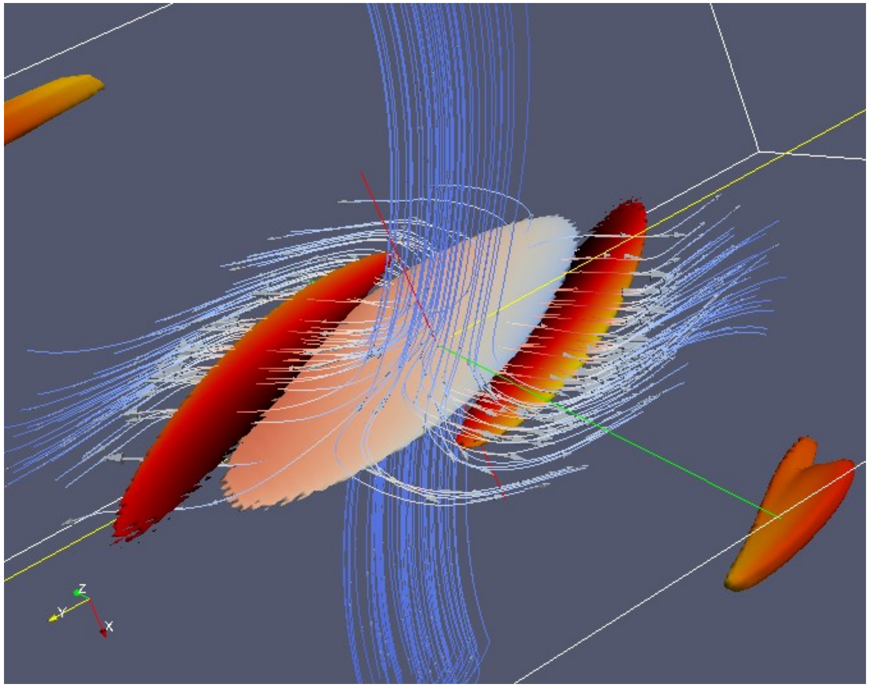
Examples of reconnection process

Boundary driven fan-spine reconnection – MHD experiment!

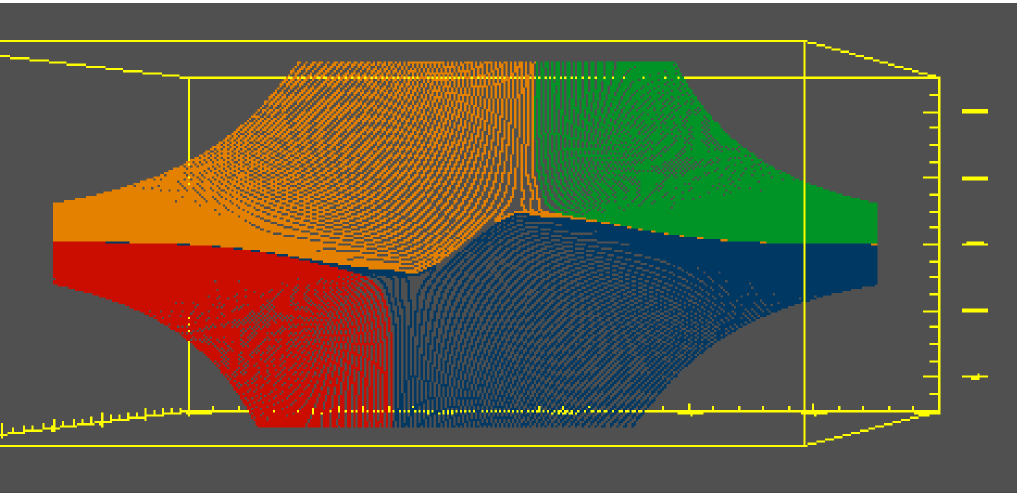
Current magnitude / time
Symmetry plane

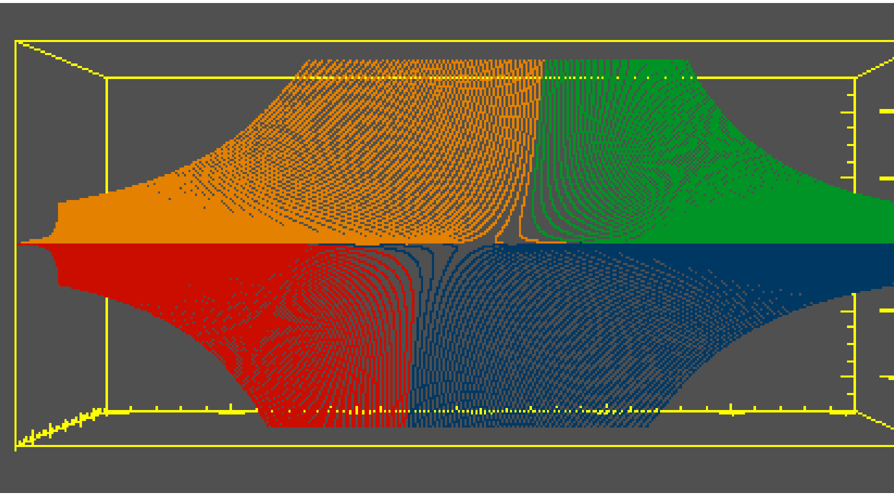


Asymmetric nulls
change in relative angles



Flipping of field lines   side view

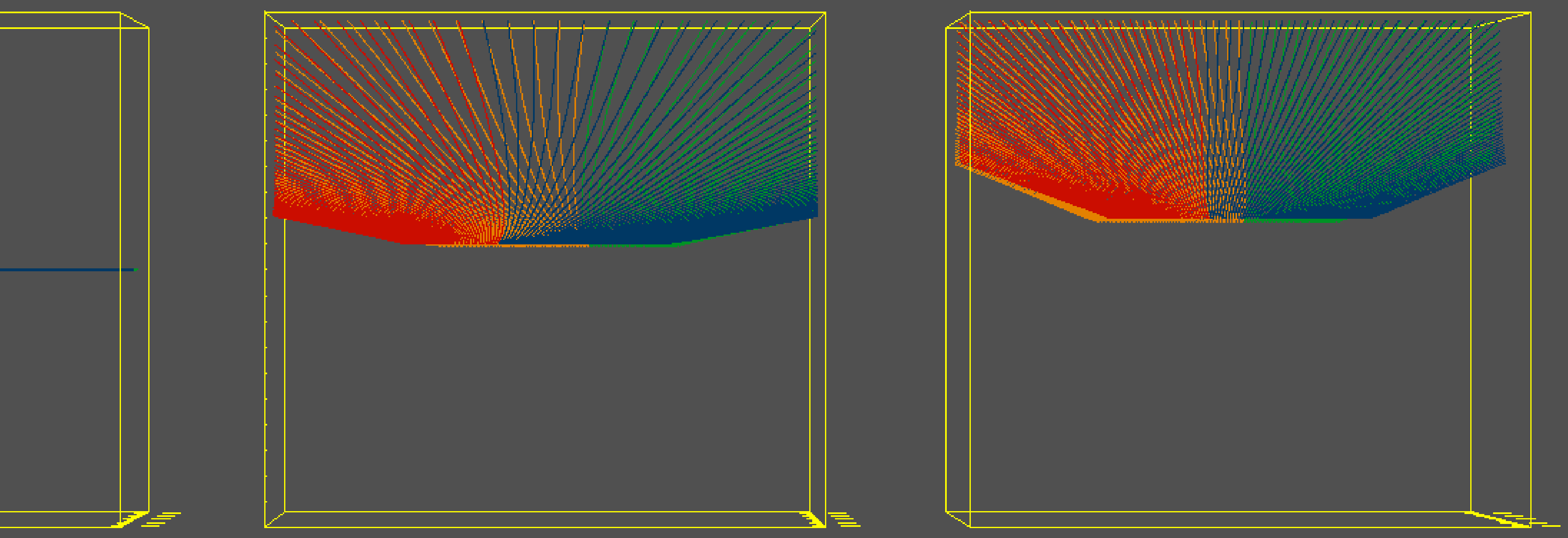




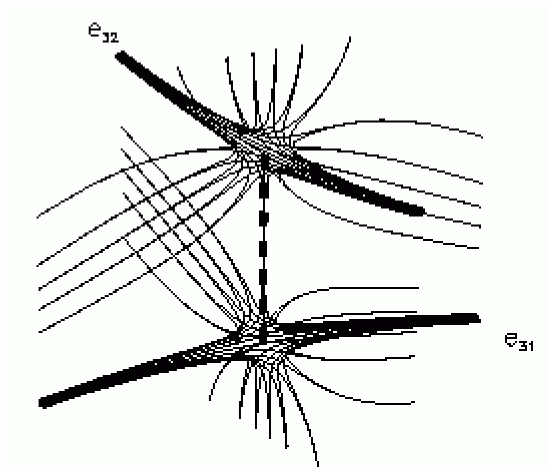
Symmetry plane

Off axis field lines

Flipping of field lines – top view



Field lines moving in opposite direction
on either side of the fan plane
Less effect as the distance increases from the symmetry plane



Double null structure

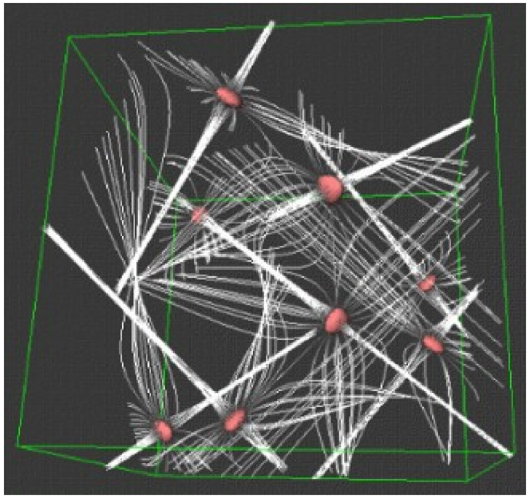
Nulls are formed in pairs

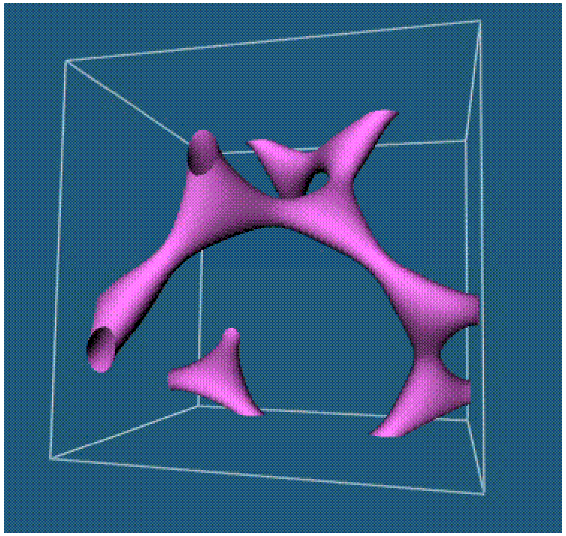
- Bifurcation process of the magnetic topology

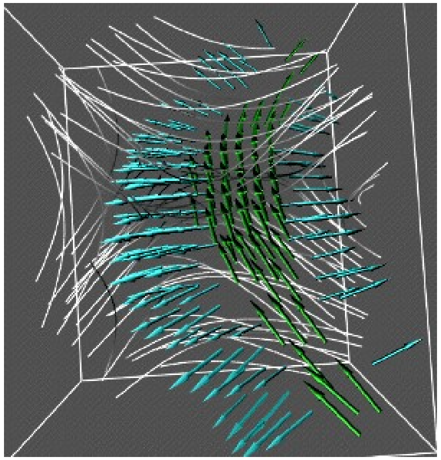
2D x-line topology along the separator line

- Not obvious how reconnection takes place along the separator.
- Initially one expected it to take place all along it
  - Kinematic investigation
  - Numerical experiments show different behaviour
    - Multiple separators between two nulls
    - Reversal of diffusive electric field along it

Driven ABC field – Separator reconnection

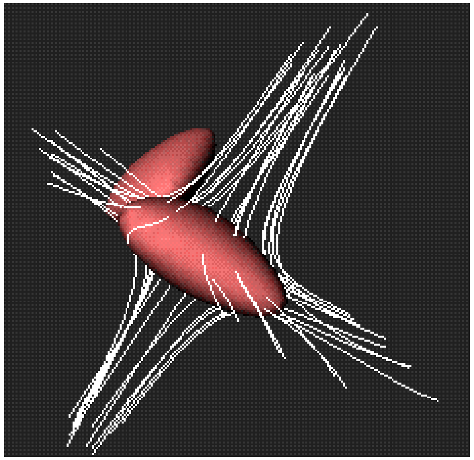


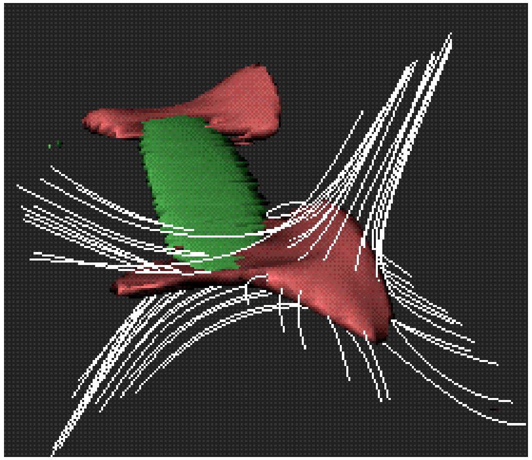


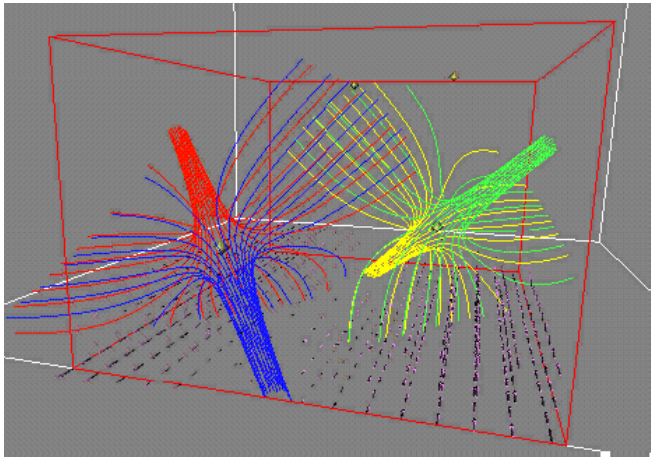


Galsgaard & Nordlund 97
Boundary stress forces the week field channels to collapse and form current sheets

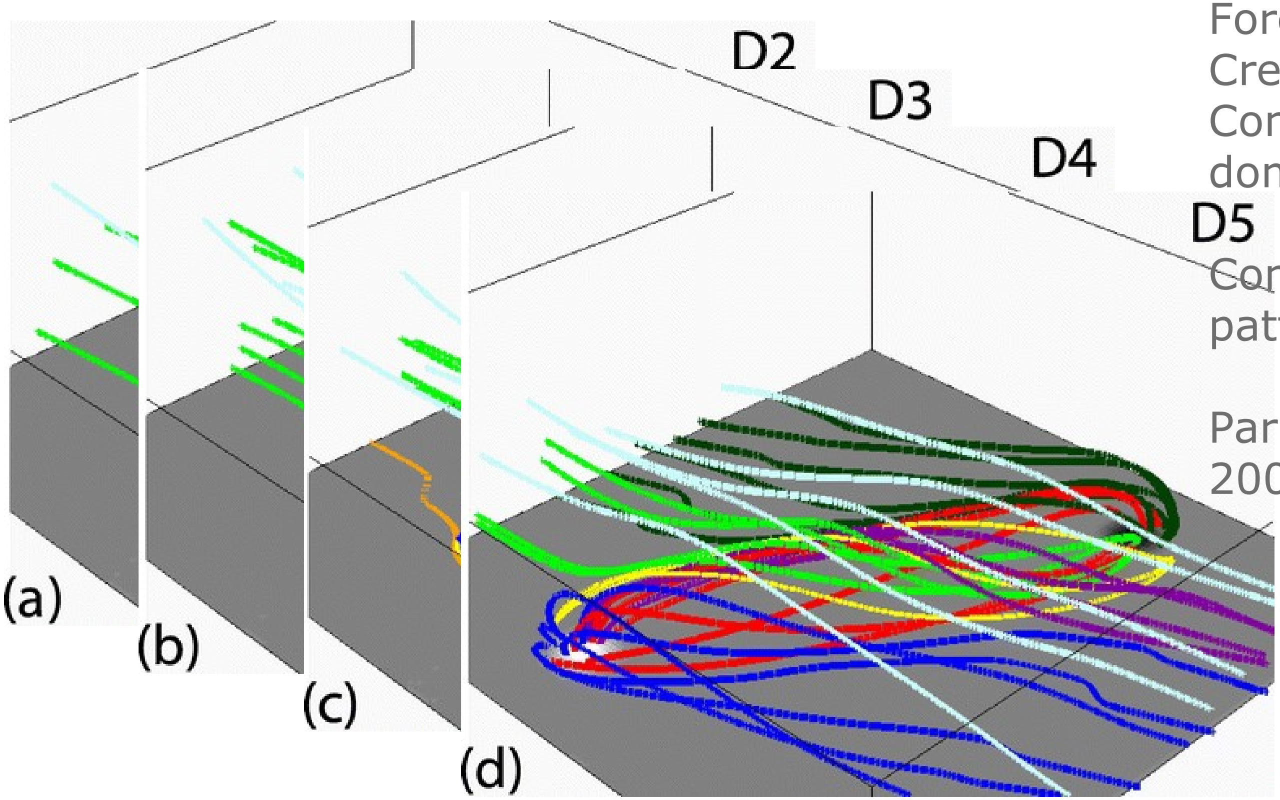
Separator reconnection







Example of separator reconnection



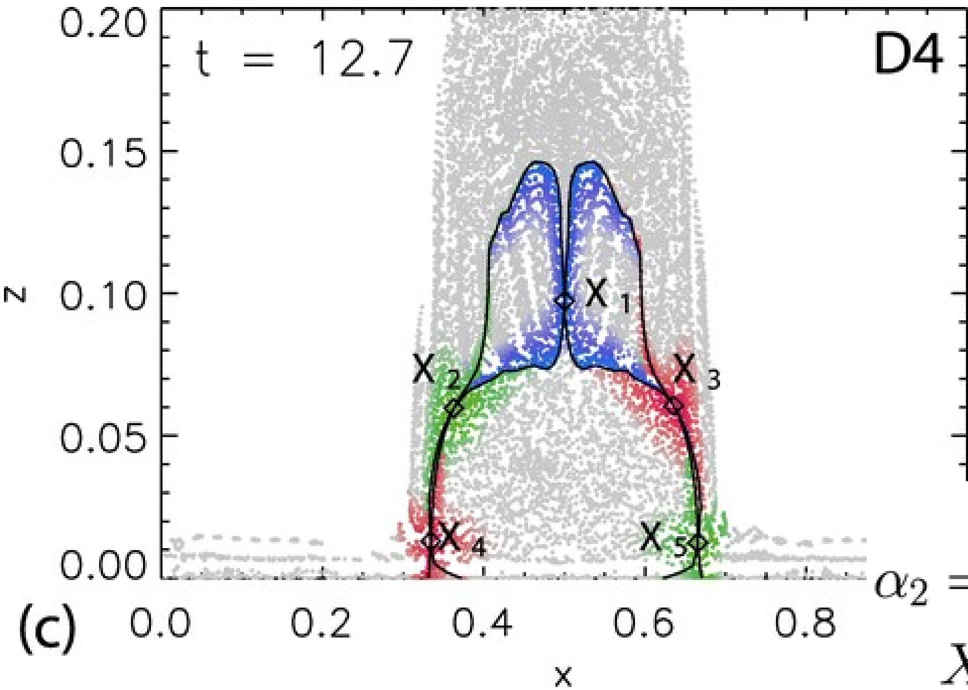
Two flux sources
Overlying B-field
2 nulls + fan surfaces
3 independent flux regions

Forced into each other
Creates several separators
Complicated time dependent domain

Complicated reconnection pattern, multiple separators

Parnell, Galsgaard & Haynes 2002-2006

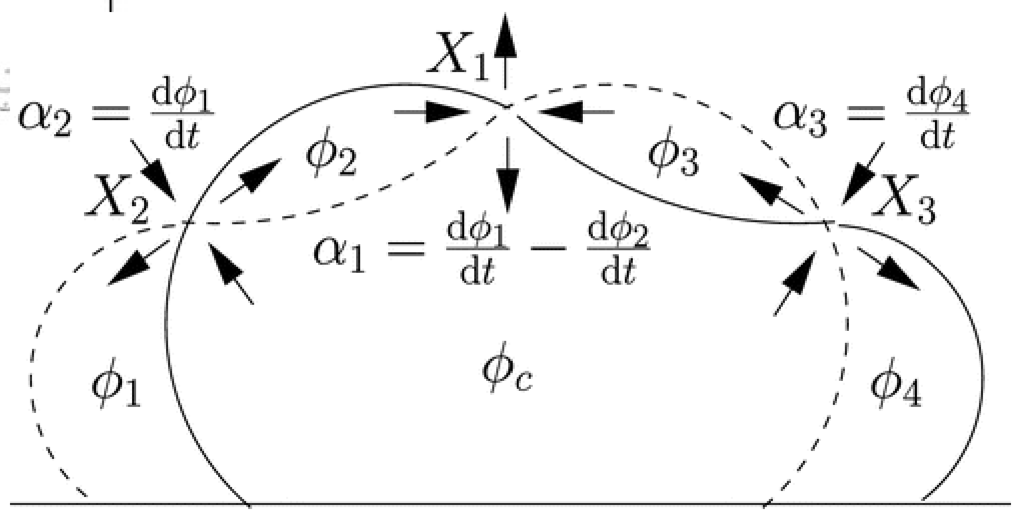
How does it works?



2D slice of the central plane
Field liens traces, colour coding of tracer particles that has changed connectivity.

X-points indicates locations of separators.

Recycling of flux → much more reconnection than a potential passage would indicate.



The total reconnection rate is the sum of all contributions.

Reconnection in non-null regions

Where do we find such locations?

Typically looking for regions were there is a continues but rapid change in the field line mapping.

A generally used method is to look for *Quasi Separatrix layers*

These are defined as location where the mapping of field lines changes rapidly in space (Priest & Demoulin 96,... Titov 07)

$$N=\sqrt{\left(\frac{\partial X(x,y)}{\partial x}\right)^2+\left(\frac{\partial X(x,y)}{\partial y}\right)^2+\left(\frac{\partial Y(x,y)}{\partial x}\right)^2+\left(\frac{\partial Y(x,y)}{\partial y}\right)^2}$$

Measure the change in the mapping between the two end points

$$Q=N^2\left|\frac{\hat{B}_n}{B_n}\right|$$

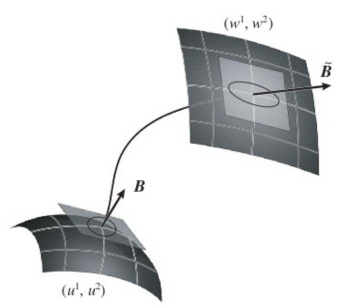
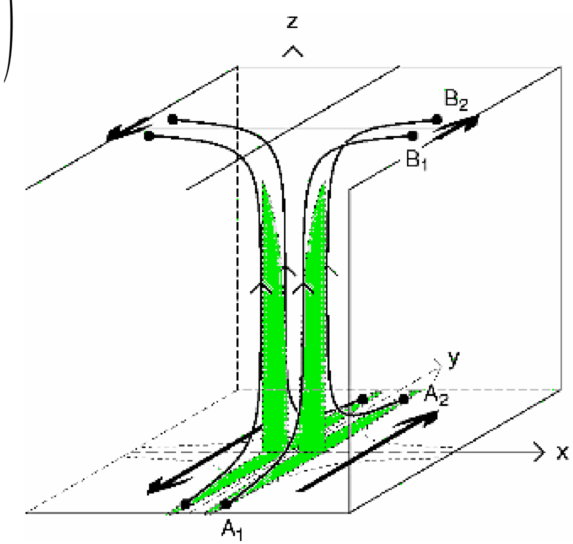
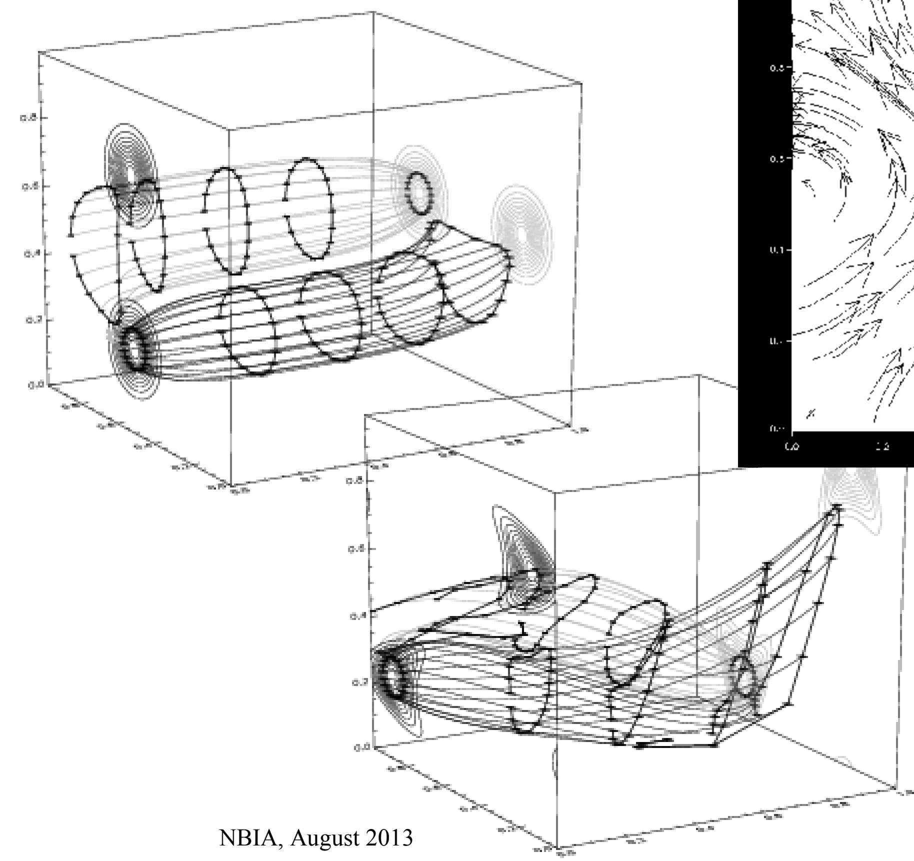


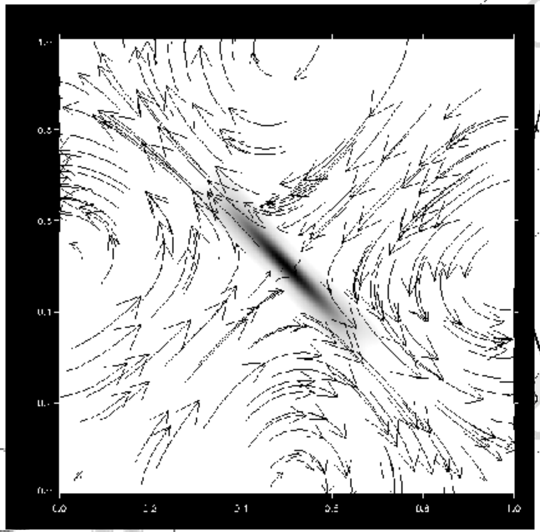
Figure 1. Circle is mapped into an ellipse by a linearized field-line mapping acting between the tangent planes of the launch and target boundaries, where two different curvilinear coordinates (u^1, u^2) and (w^1, w^2), respectively, exist. The aspect ratio of the ellipse, when it is large, coincides with a high value of the squashing factor Q.

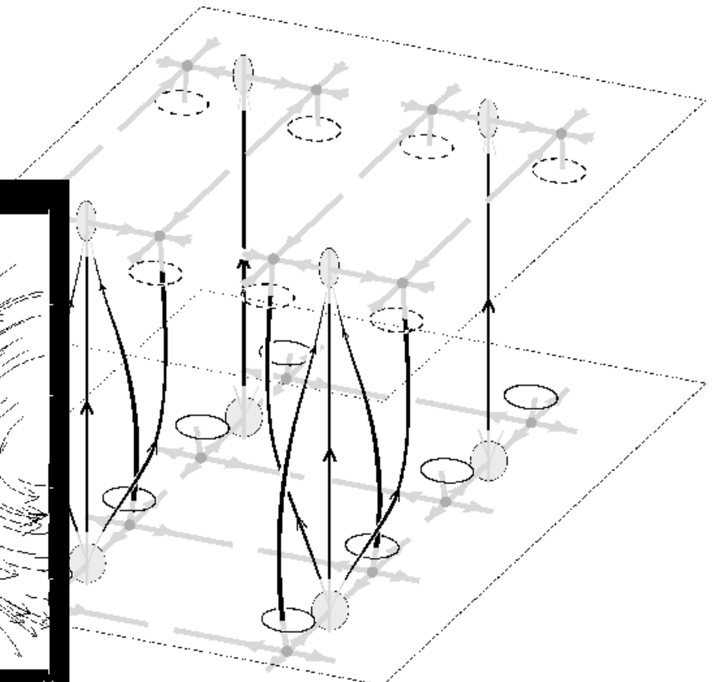


QSL regions in experiments I

Galsgaard et al. 2002





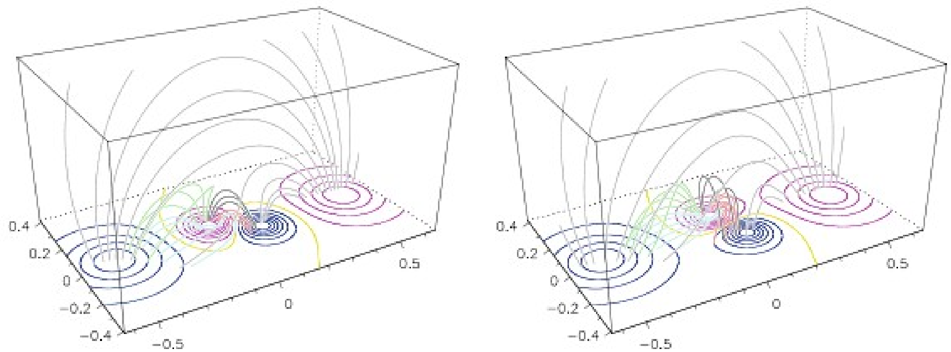


Hyperbolic flux tube

Boundary driving initiates a stagnation flow, but only when the driving is correct

QSL regions in experiments II

Aulanier et al. 2009



Hyperbolic flux tube

Looks like the previous experiment
In a more complicated structure!?

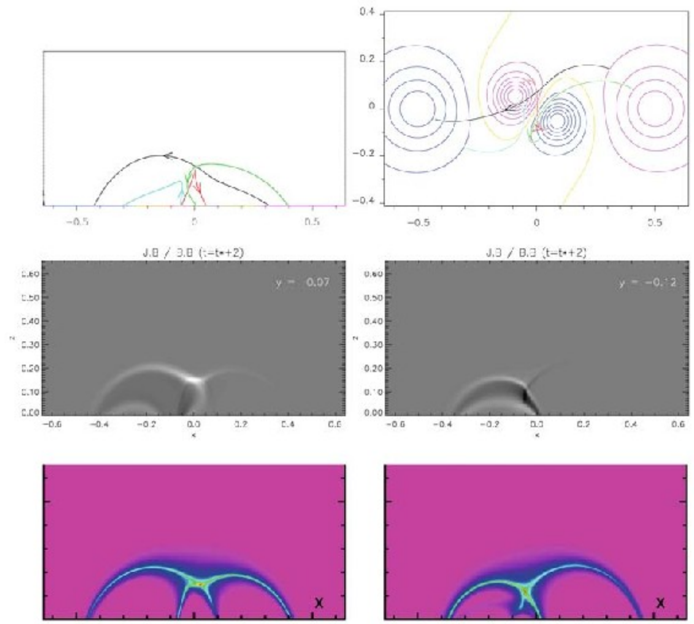


Figure 2. Magnetic field lines, electric currents and QSLs during the resistive relaxation of the configuration $\Phi = 150°$, at $t = t^{\star} + 2$, using $\eta = 15 \times 10^{-6}$. *Top row*: 3D projection views of four magnetic field lines, passing in the vicinity of the two narrowest current layers that are present in the QSLs at $y = 0.07$ and $y = -0.12$. Two projections are shown, along the y axis (*left panel*) and along the z axis, i.e. as viewed from above (*right panel*). The contours of $b_z(z = 0)$ are the same as in Figure 1. *Middle row*: 2D maps of $\alpha = \jmath \cdot \mathbf{b}/b^2$, drawn at $y = 0.07$ (*left panel*) and at $y = -0.12$ (*right panel*). The color coding is saturated, so that (*black; white*) stand for $\alpha = (-36; 36)$. *Bottom row*: 2D maps of the squashing degree Q in the QSLs in the same $y =$ constant planes. The (*red ; yellow; green; cyan; blue; pink*) colors stand for $Q = (10^8; 3\,10^6; 10^5; 5\,10^3; 50; 2)$. The central part of the HFT is thus drawn in *red*.

Braided magnetic field example

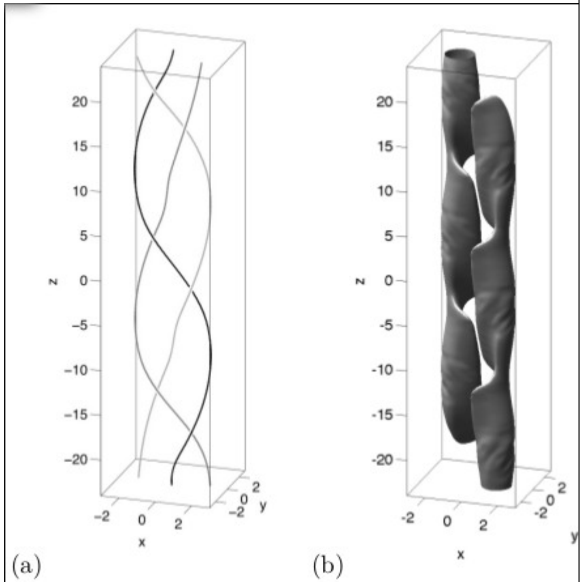


Figure 1: (a) Three representative magnetic field lines from the initial magnetic field ($t = 0$) which demonstrate the braiding present in the field. (b) Isosurface of current density $|\mathbf{J}|$ at $t = 0$, at 25% of its maximum value.

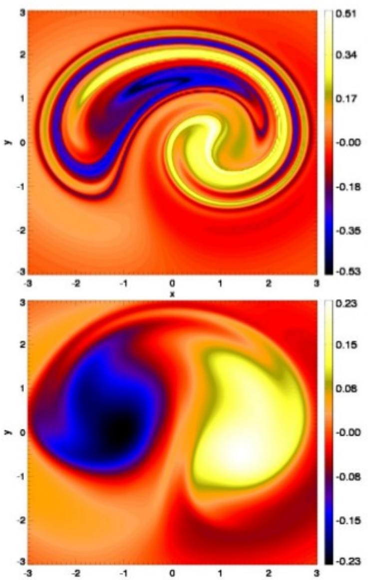


Figure 6: The mean value of α^* along field lines shown on a section of the lower boundary ($z = -24$) of the domain in the initial (*top*, $t = 0$) and final (*bottom*, $t = 290$) states showing a smoothing of α^* during the resistive relaxation with $\eta = 10^{-3}$.

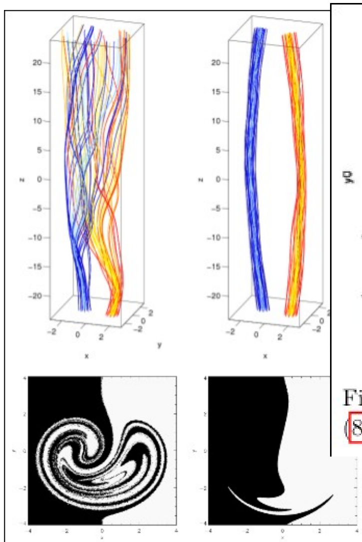


Figure 7: Above: field lines traced from fixed locations on the lower boundary (and coloured according to location on the lower boundary). Below: locations of intersection with the plane $z = +24$ of field lines traced from a regular grid in the plane $z = -24$. Field lines traced from locations $x \geq 0$ on the lower boundary are coloured white, from $x < 0$ black. Plots are made for $t = 0$ (*left*) and $t = 290$ (*right*) for the run with $\eta = 10^{-3}$.

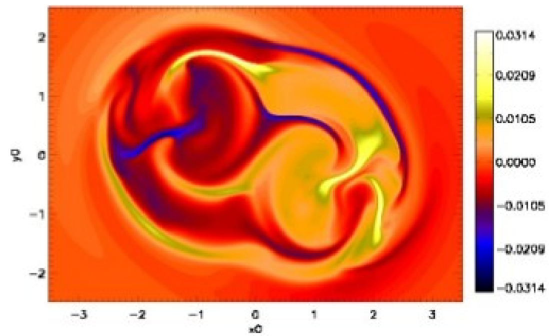


Figure 8: Plot of $\Phi(x_0, y_0)$ (as defined in Equation (8)) for $t = 50$ for the run with $\eta = 10^{-3}$.

Pontin et al 2010

How to find potential locations of reconnection?

Have to know the skeleton of the magnetic field

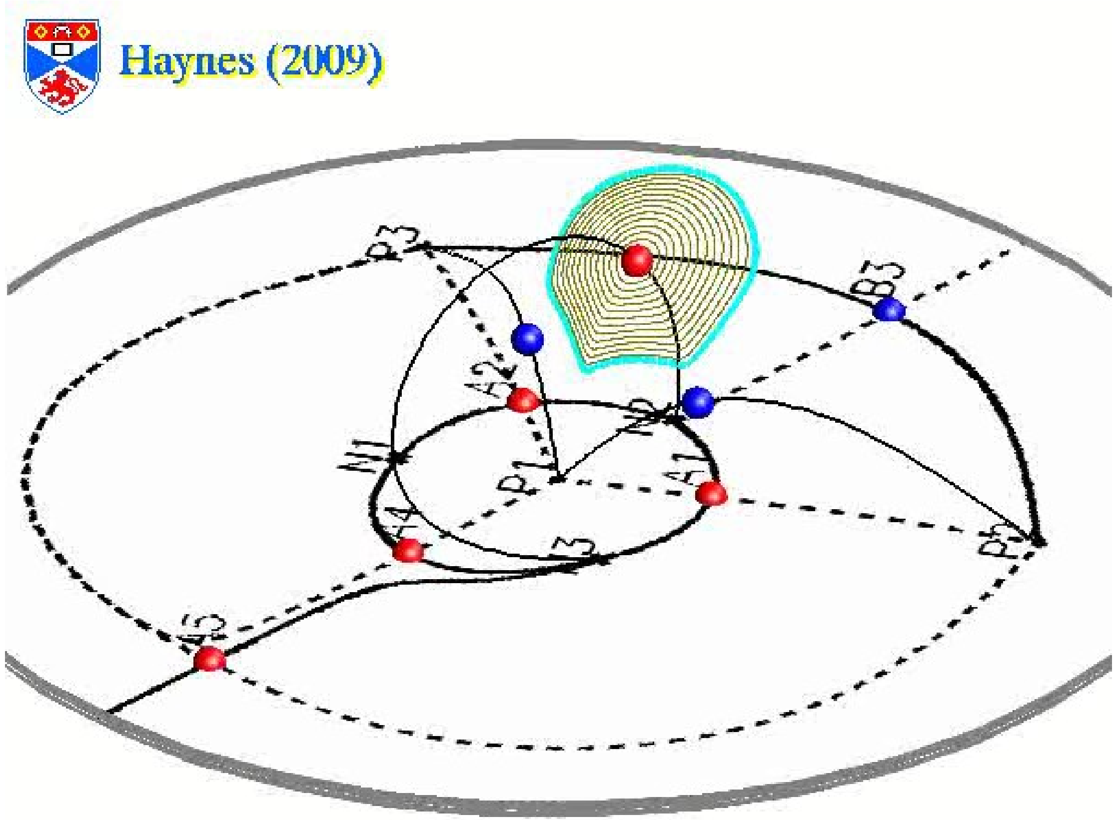
Need to locate nulls and their separators

- Nulls are simple to find under some simplifying assumptions
- Separators are much more challenging to identify!
  - Need null position, eigenvectors, intensive field line tracing and checking

A general method for doing this is developed by Haynes & Parnell 2006--

- Need further development to become more robust in more complex fields to locate separators

Haynes separator finder



Blue/red nulls, Ring of distances from the null, separator lines connecting nulls

Does this find all locations in space?

The skeleton approach MAY find the basic regions of topology for the magnetic field

Identify the key locations in 3D where topological domains are clearly defined

Approach works well in relative simple cases

Problems in finding separators when the field is complicated

QSL approach find potential locations for current accumulations

In both cases the field needs to be stressed appropriately to drive current

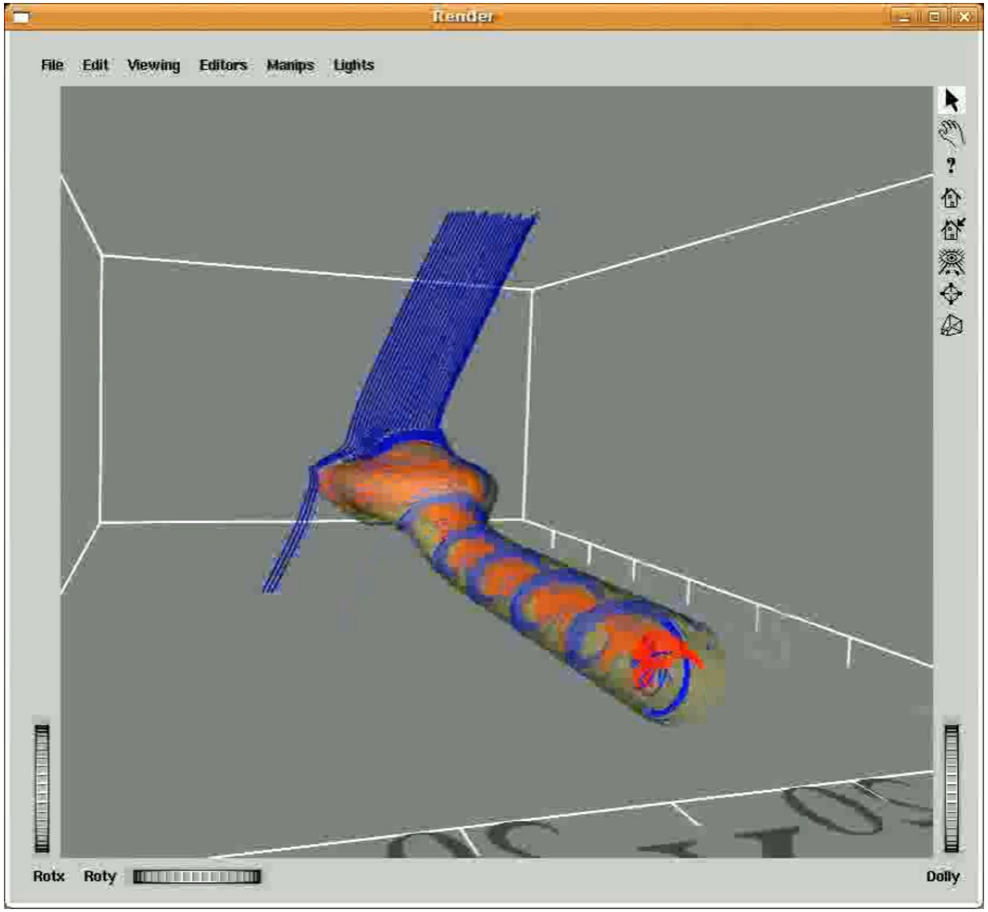
A case study – magnetic flux emergence and jet formation – solar model

Highly stratified model

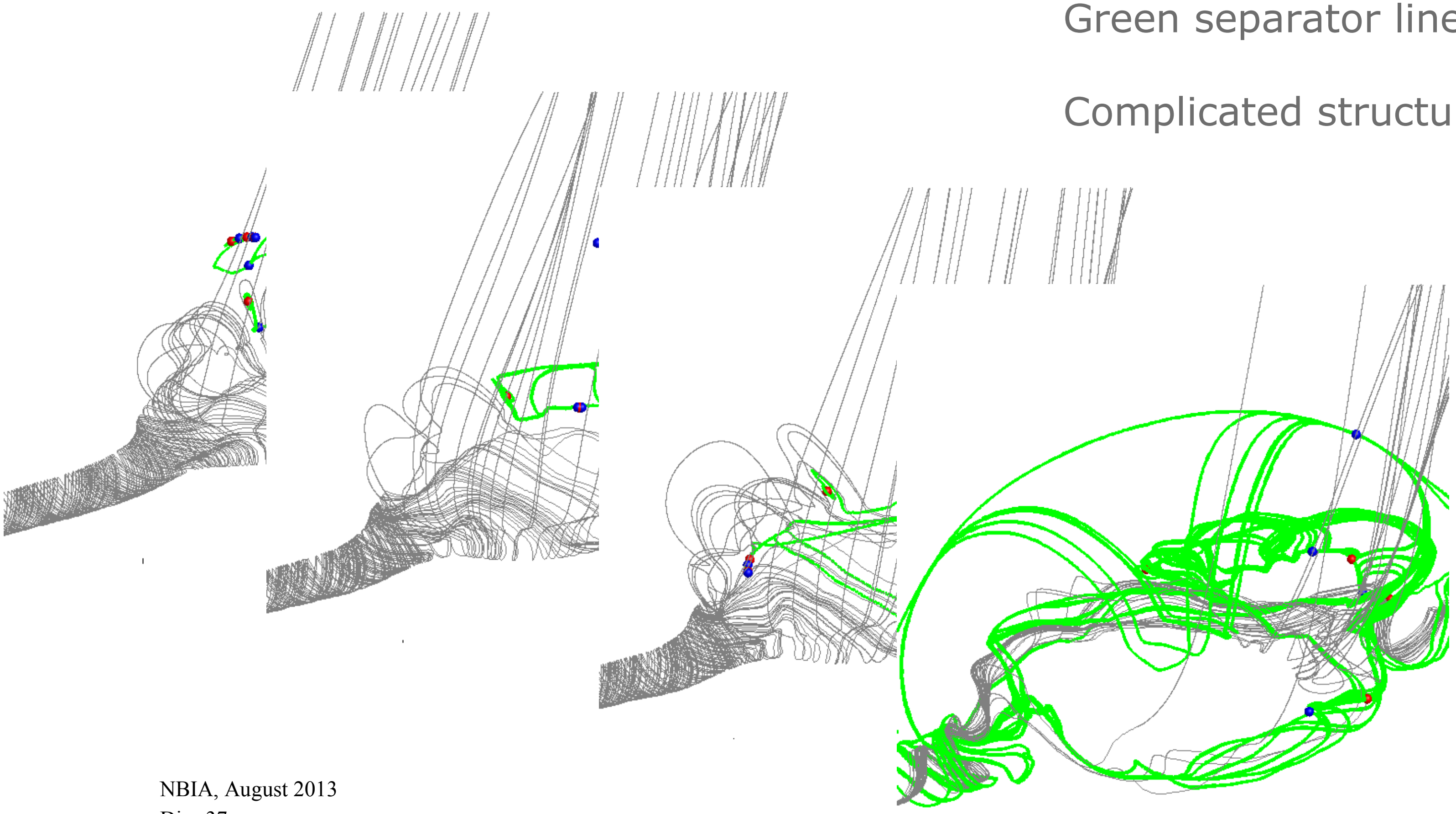
- Convection, transition region, corona
- Twist magnetic flux robe, open background field
- Local buoyancy

Eventually strong interaction between emerging and original B-field

What is the skeleton of this dynamical evolution?



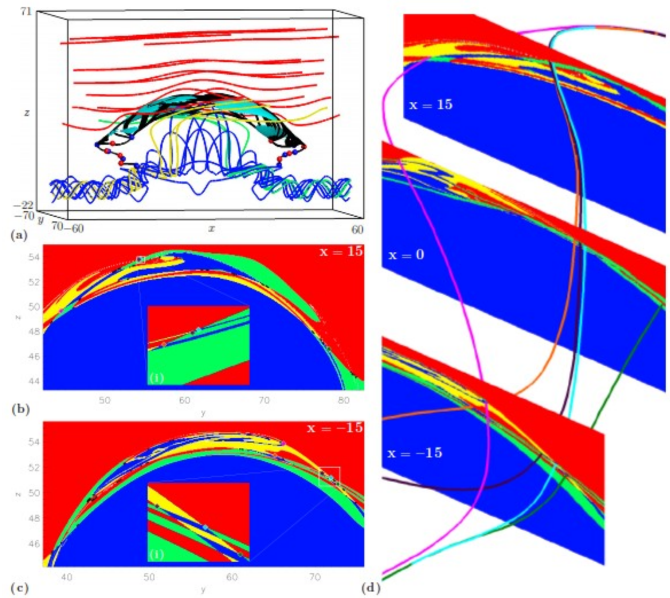
Skeleton of flux interaction



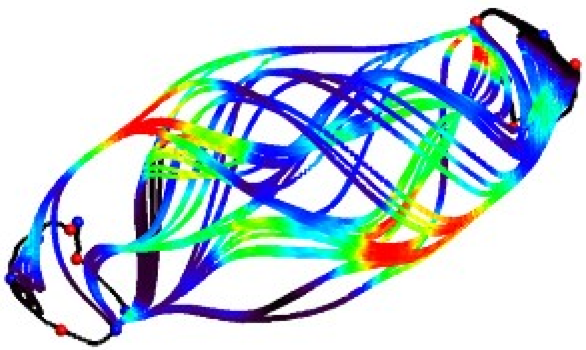
Red/blue 3D nulls
Green separator lines

Complicated structure

Electric field and separators



Complexity of the field line mapping
Different colours indicate different connectivity domains
X-lines/points in 2D cuts are due to separators



The mapping of the parallel electric field along separators

Red is large, blue green is small

Localised regions with important reconnection

QSL maps

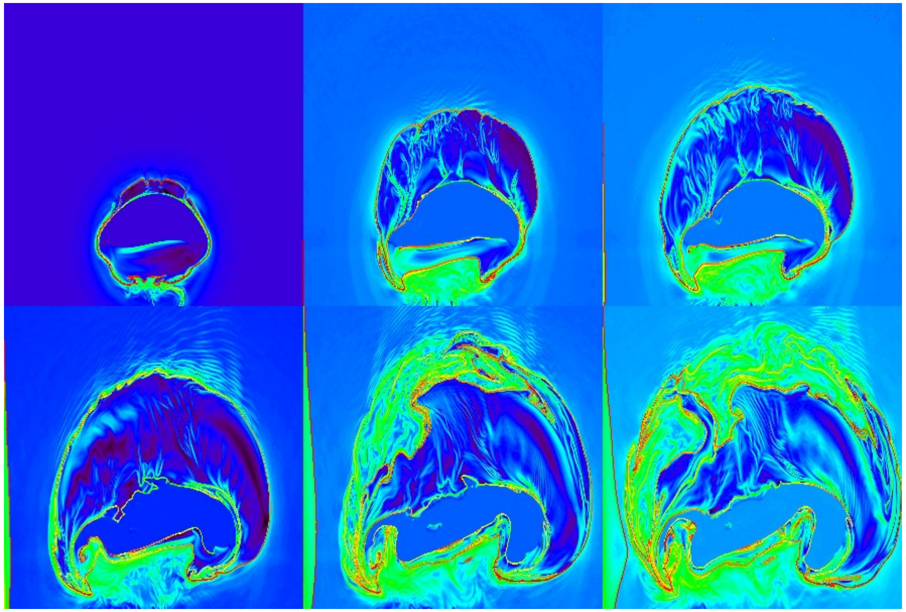
QSL map for the jet experiment
Time series during the last complex phase

Red is high values blue is neutral (the invariant measure of N)

Need to compare this with the skeleton to see if this provides additional information

Has often been used to argue for the reason for solar flares

This also shows the locations of skeleton boundaries!!!



Can't reach locations which are not directly connected to the local boundary region!

Depends on the chosen surface

The Newton Challenge Birn et al 2005

Driven Harris sheet 2D reconnection process

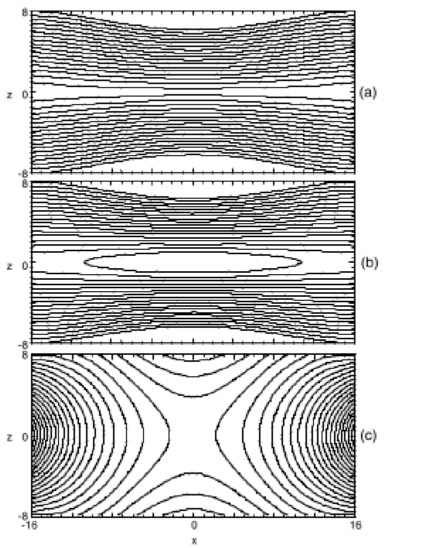


Figure 2. Equilibrium configurations for the same pressure function $p(A)$ as the initial Harris sheet but perturbed flux at $|z| = z_{max}$, which is the same for all cases. (a) This configuration has the same topology as the initial Harris sheet but contains a surface current at $z = 0$. (b) This configuration is characterized by continuous current distribution but changed topology. (c) This is the lowest energy configuration under the given constraints.

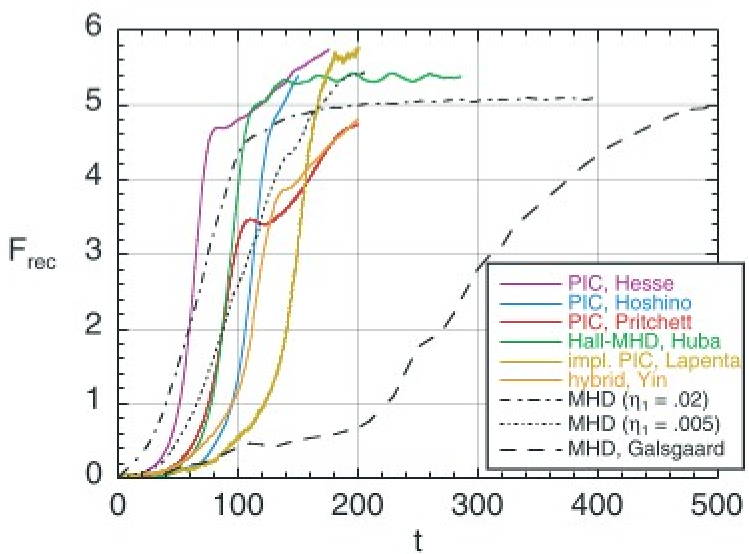


Figure 3. Time variation of the reconnected flux for various simulations of forced reconnection, as indicated.

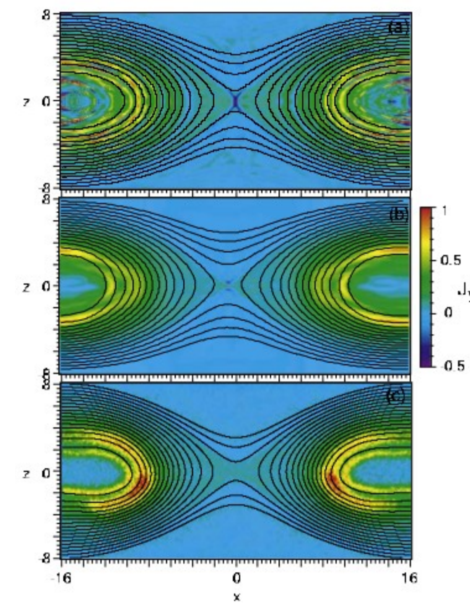


Figure 4. Late stages of the magnetic field (contour lines) and current distribution (color coded) for various simulations: (a) MHD simulations using spatially localized resistivity (J. Birn), (b) Hall-MHD simulation without explicit dissipation term (J. Huba), (c) PIC simulation (M. Hesse).

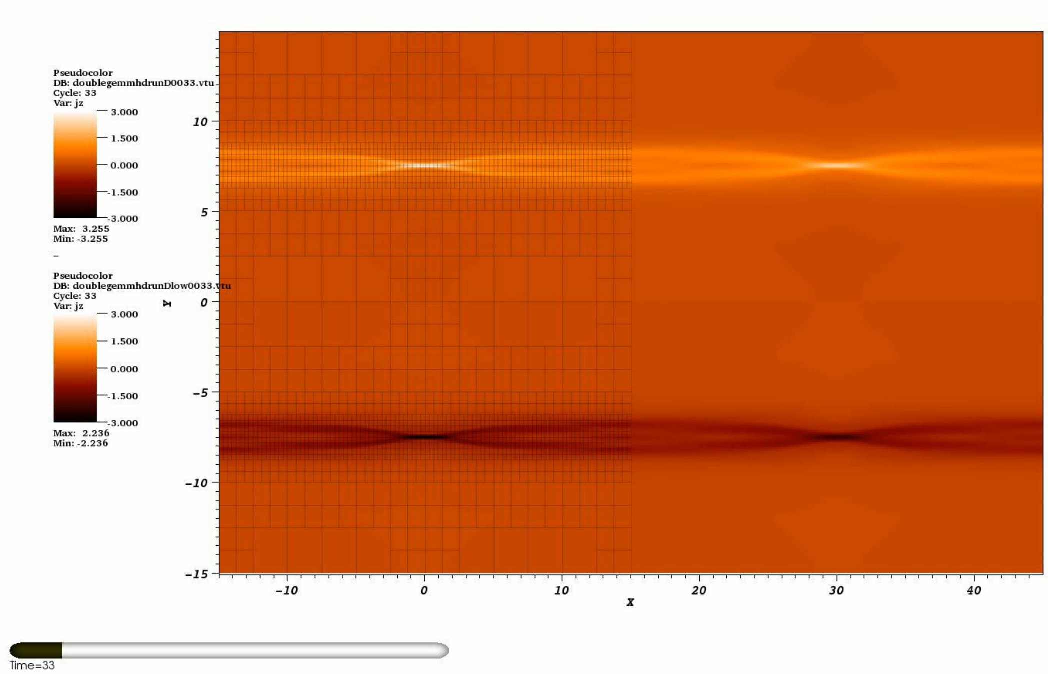
Details are different, but the general evolution is comparable

Double GEM code comparison

Double periodic Harris sheet, initial perturbation

CPH stagger, AMRVAC, FLIP3D

Shows that linear phase is comparable, while the non-linear becomes different in details, while typical evolution is comparable



Resolution comparison