Coupling Multiple Scales
Magnetic Reconnection and Coupling Issues (for scalar hyperbolic PDEs)

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including work with O. Porth et al.

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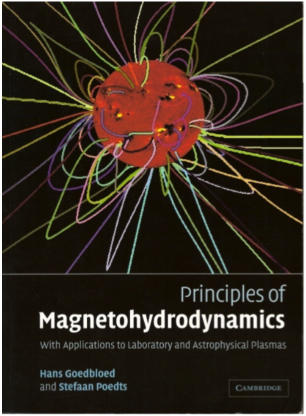
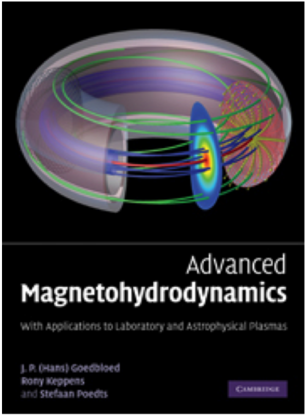
Outline

- Ideal to resistive MHD: magnetic reconnection basics
  - ⇒ double GEM challenge: long-term, chaotic dynamics
  - ⇒ coupling challenges for reconnection
- scalar hyperbolic PDE models for coupling strategies
- Outlook

- lecture material from modern (2004 & 2010) textbooks
  - ⇒ **Goedbloed** et al., Cambridge University Press
  - ⇒ chapter 14 on resistive MHD . . .





The induction equation:

- evolutionary equation for **B** in **ideal** MHD: Faraday's law

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times \underbrace{(\mathbf{v} \times \mathbf{B})}_{-\mathbf{E}} = 0$$

$\Rightarrow$ **field lines are frozen in plasma**

$\Rightarrow$ unimpeded flow along **B**, flow $\perp$ **B** displaces field line

$\Rightarrow$ analytically: if $\nabla \cdot \mathbf{B} = 0$ initially, then always

- electric field in co-moving frame for perfectly conducting fluid

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

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Ideal MHD and conservation laws:

- ideal MHD case: referred to as ‘frozen-in’ conditions
- equivalent formulation of ideal MHD induction equation

  $\Rightarrow$ conservation law

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\underbrace{\mathbf{vB} - \mathbf{Bv}}_{\text{second rank tensor}}) = \mathbf{0}$$

- term $\nabla \times (\mathbf{v} \times \mathbf{B})$ represents conversion of mechanical energy to electromagnetic induction

  $\Rightarrow$ when conductor moves with velocity $\mathbf{v}$ in magnetic field $\mathbf{B}$

  $\Rightarrow$ process creates an electromotive force $\mathbf{v} \times \mathbf{B}$ (emf)

Ideal versus resistive MHD

- consider medium with constant resistivity η, Ohm's law

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \, \mathbf{j} \, .$$

$\Rightarrow$ electric field in comoving frame proportional to current density, hence $\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta \frac{1}{\mu_0} \nabla \times \mathbf{B}$

$\Rightarrow$ induction equation then given by (for constant η)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{\eta}{\mu_0} \nabla^2 \mathbf{B}$$

- timescale for resistive diffusion

$$\tau_R \sim \frac{\mu_0 l_0^2}{\eta}$$

$\Rightarrow$ also: **Ohmic heating term** ηj^2 in energy equation

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Resistive MHD

- current $\mathbf{J} = \nabla \times \mathbf{B}$: dissipation through resistivity

  $\Rightarrow$ from ideal to resistive (non-ideal) MHD
- spatio-temporal resistivity profile $\eta(\mathbf{x}, t)$ introduces

  $\Rightarrow$ Ohmic heating term in energy equation

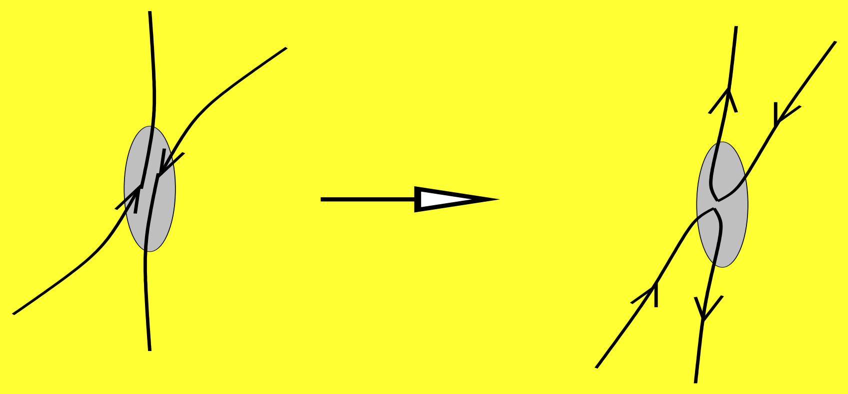
$$S_e = \nabla \cdot (\mathbf{B} \times \eta \mathbf{J})$$

$\Rightarrow$ diffusion term in induction equation

$$\mathbf{S}_B = -\nabla \times (\eta \mathbf{J})$$

$\Rightarrow$ uniform resistivity: $\eta \left(J^2 + \mathbf{B} \cdot \nabla^2 \mathbf{B}\right)$ and $\eta \nabla^2 \mathbf{B}$

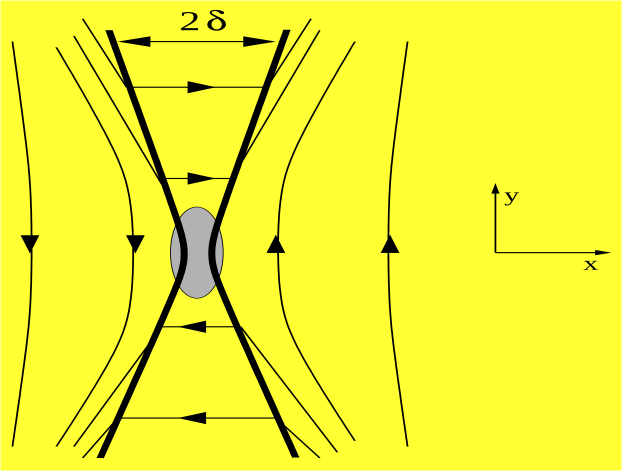
- ideal ($\eta = 0$) versus resistive MHD
  - $\Rightarrow$ topological constraint on **B** alleviated
  - $\Rightarrow$ field lines can reconnect in regions of strong currents



Petschek reconnection

- Petschek model (1964) for fast magnetic field annihilation
  - $\Rightarrow$ two regions containing oppositely directed field lines
  - $\Rightarrow$ realize steady-state with X-type magnetic neutral point
- steady state contains pair of stationary slow shocks
  - $\Rightarrow$ where **B** bends towards shock front normal
- at X-point: flow controlled by diffusion
- within region bounded by slow shocks: purely B_x, 'constant' ρ
  - $\Rightarrow$ shock front half-width $\delta(y) = \frac{\rho_e}{\rho_i} \frac{v_{x,e}}{V_{A,e}} \mid y \mid$ (external/internal)
  - $\Rightarrow$ fronts have fixed opening angle (away from neutral point)
  - $\Rightarrow$ fluid moves to boundary layer and is ejected along it

- stationary configuration



$\Rightarrow$ use symmetry to simulate corner region $[0, 1] \times [0, 4]$ only

- solve resistive MHD equations incorporating resistivity profile

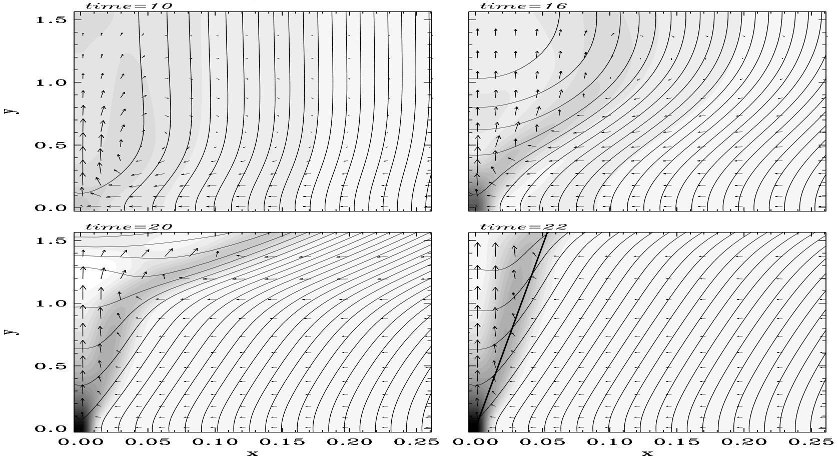
$$\eta(x, y) = \eta_0 \exp\left[-(x/l_x)^2 - (y/l_y)^2\right]$$

  - $\Rightarrow$ anomalous η centered on origin
  - $\Rightarrow$ parameters $\eta_0 = 0.0001$, $l_x = 0.05$, $l_y = 0.1$
- initial field configuration $\mathbf{B} = (0, \tanh(x/L))$
  - $\Rightarrow$ initial current sheet width $L = 0.1$
  - $\Rightarrow$ $\gamma = 5/3$, $p(x) = 1.25 - B_y^2(x)/2$ and $\rho(x) = 2p(x)/\beta_1$
  - $\Rightarrow$ isothermal initial condition with $\beta_1 = \beta(x = 1) = 1.5$
- fix Alfvén Mach number of inflow at $x = 1$: $v_x(x = 1) = -0.04$

consistently evolves to Petschek reconnection configuration

- VAC test for implicit scheme: Tóth et al, A&A, 332, 1159 (1998)

- field lines, velocity field, current density evolution



⇒ checks with theoretical opening angle in steady-state!

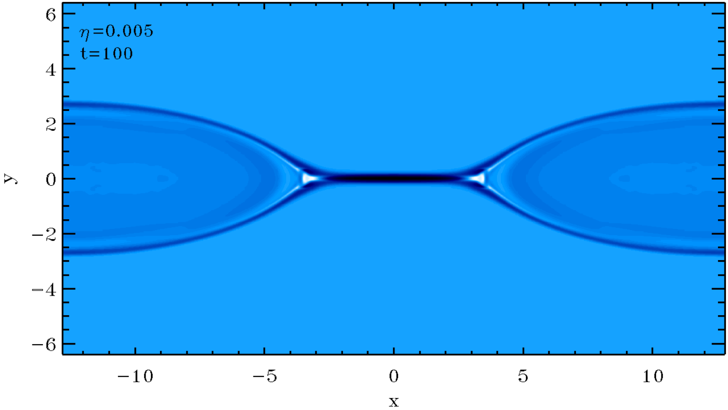
2D Harris sheet evolution: GEM

- 2D current-sheet setup: ‘Harris sheet’
  - $\Rightarrow$ horizontal field as $B_x(y) = B_0 \tanh(y/\lambda_B)$
  - $\Rightarrow$ constant T_0 and pressure-balancing density from

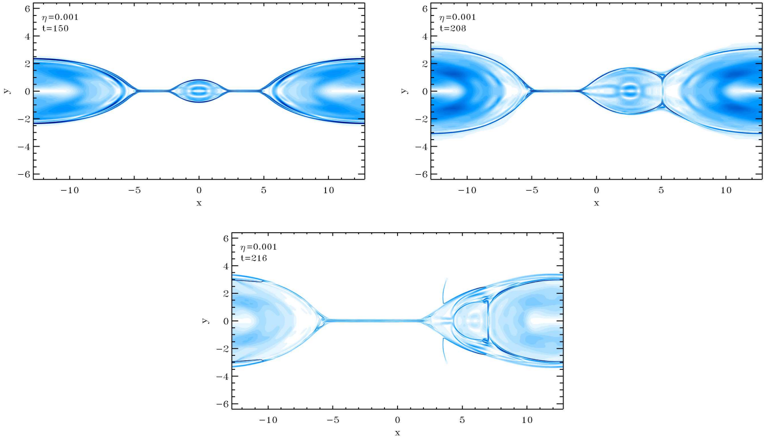
$$\rho(y) = \rho_0 \cosh^{-2}(y/\lambda_B) + \rho_\infty$$

  - $\Rightarrow$ add deterministic magnetic perturbation
  - $\Rightarrow$ solve compressible, resistive MHD with uniform η

- Harris sheet evolution, at fixed resistivity $\eta = 0.005$
  - $\Rightarrow$ 2D resistive MHD, GEM Challenge
  - $\Rightarrow$ reconnection at $\eta = 0.005$



- exactly same, at reduced resistivity $\eta = 0.001$



$\Rightarrow$ 2D resistive MHD, GEM Challenge, $\eta = 0.001$ case

- Harris Sheet evolution (`tanh` magnetic profile)
  - $\Rightarrow$ reconnection at $\eta = 0.005$, $\eta = 0.001$, $\eta = 0.0001$
  - $\Rightarrow$ **Rapid changes in complex flow!**
- run on Macbook pro with effective 1920×1920 resolution, several days . . .
  - $\Rightarrow$ **current evolution for** $\eta = 0.001$
  - $\Rightarrow$ **schlieren plot evolution for** $\eta = 0.001$

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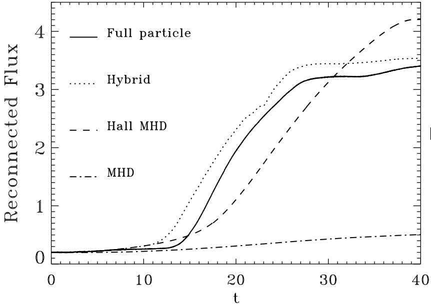
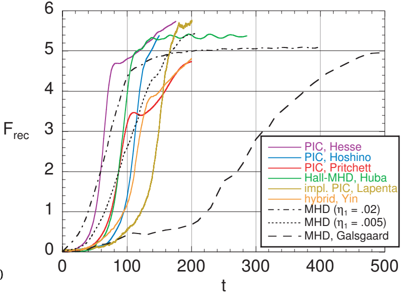
  - $\Rightarrow$ **schlieren plot evolution for $\eta = 0.001$**

- GEM/Newton (driven from boundary) challenges

  $\Rightarrow$ resistive MHD, Hall-MHD, hybrid and kinetic models





$\Rightarrow$ reconnection rate: smaller in resistive MHD [but η reached did not enter the chaotic, fast reconnection regime!]

$\Rightarrow$ at least having Hall term included speeds up reconnection

$\Rightarrow$ anomalously raised, local resistivity models can allow fast reconnection in resistive MHD

Double GEM setup

- recent (PoP, submitted) resistive MHD code comparison
  - $\Rightarrow$ double periodic setup on square $[-15, 15]^2$
  - $\Rightarrow$ lower/upper current layer

$$B_x(y) = B_0 \left[-1 + \tanh\left(y - y_{\rm low}\right) + \tanh\left(y_{\rm up} - y\right)\right]$$

$\Rightarrow$ again deterministic field perturbation, 10% amplitude (non-linear!)

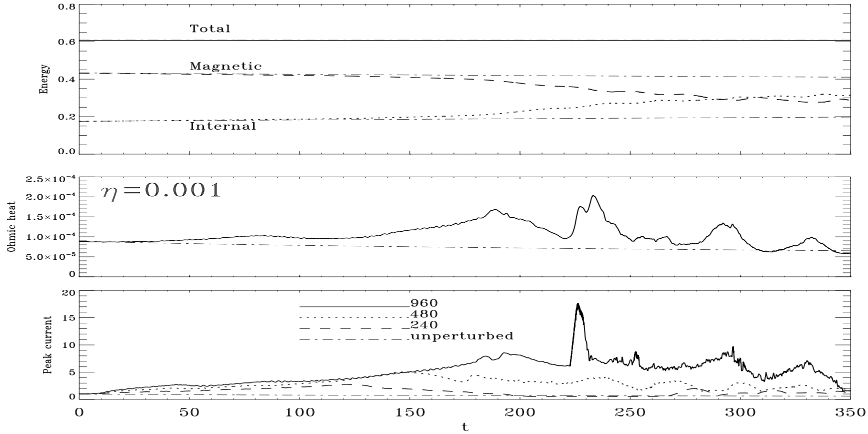
$\Rightarrow$ compared finite volume, difference and PIC-type (visco-)resistive MHD evolutions

- **resolving long-term, chaotic dynamics** for lower η

- Note: resistive MHD governed by conservation laws!
  - $\Rightarrow$ double periodic setup allows easy reality check
  - $\Rightarrow$ monitor total energy and its contributions on domain V

$$
\begin{aligned}
E_{\mathrm{Total}} &= \frac{1}{V}\int\int\left(\frac{p}{\gamma-1}+\frac{B^2}{2}+\frac{\rho v^2}{2}\right)\,dx\,dy \\
E_{\mathrm{Magnetic}} &= \frac{1}{V}\int\int\left(\frac{B^2}{2}\right)\,dx\,dy \\
E_{\mathrm{Internal}} &= \frac{1}{V}\int\int\left(\frac{p}{\gamma-1}\right)\,dx\,dy
\end{aligned}
$$

- case $\eta = 0.001$: long-term evolution

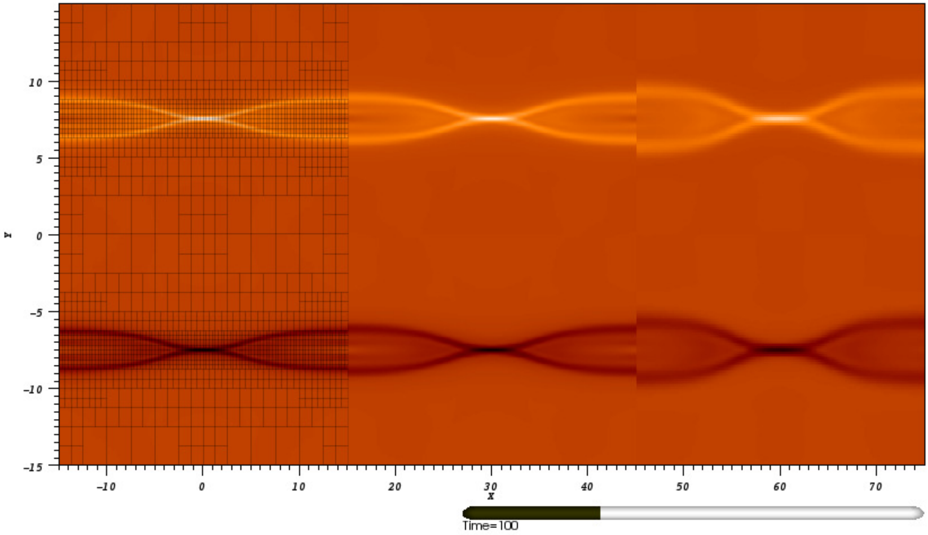


$\Rightarrow$ energy evolution for perturbed-unperturbed case: deviations beyond $t \approx 150$

$\Rightarrow$ Ohmic heating remains small (integral under curve)

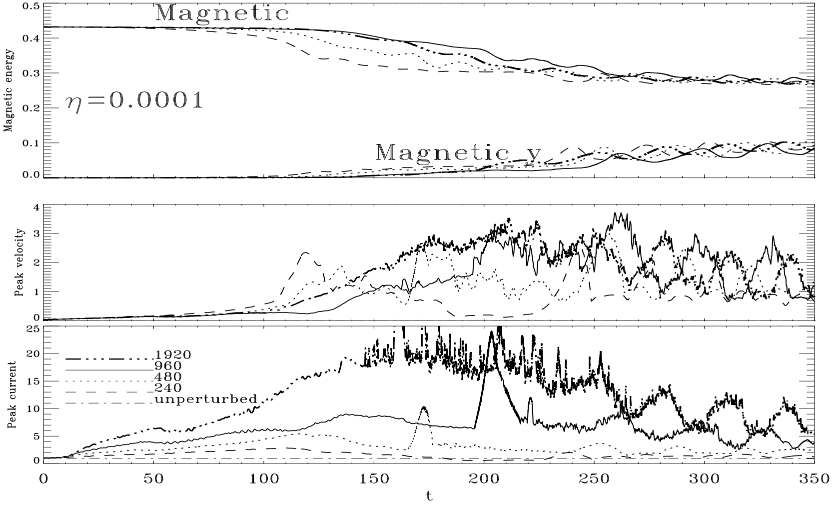
$\Rightarrow$ peak current enhancements at sufficient resolution!

- resolution study 960^2 to 240^2
  $\Rightarrow$ **current evolution for $\eta = 0.001$**

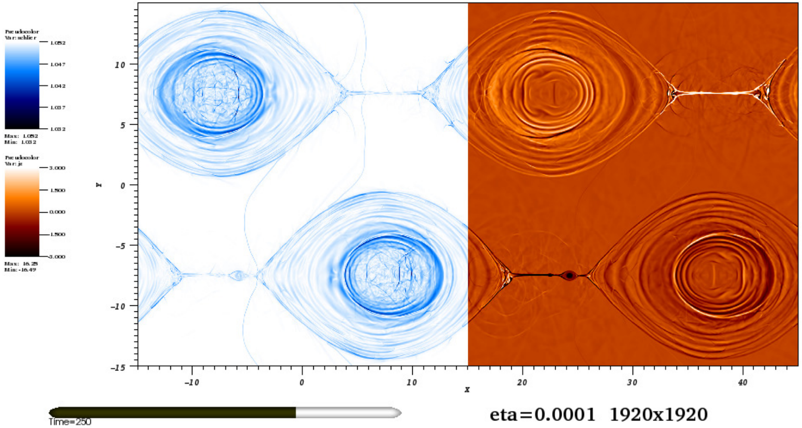


- secondary islands appear (induced tearing), merge with larger island structure, when resolution suffices!
  - $\Rightarrow$ initial phase and final endstates rather insensitive
  - $\Rightarrow$ no ‘strong convergence’ (perturbations grow from noise)
  - $\Rightarrow$ similar for FLIP-MHD (PIC) or FD Stagger (hyperdiffusion!)

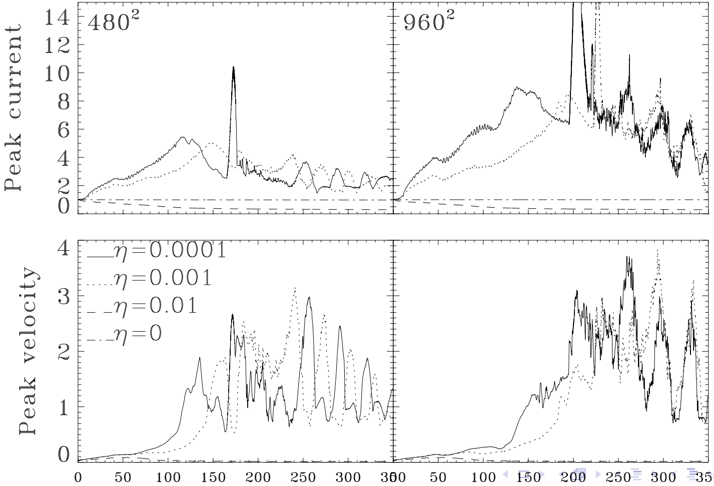
- lowering $\eta = 0.0001$: modern computational challenge!
  - $\Rightarrow$ energetic views at increasing resolution 240^2 to 1920^2



- global trend in energetics
  - ⇒ magnetic↔internal through compressive interactions
  - ⇒ peak current/velocity: chaotic phase agrees qualitatively
  - ⇒ **evolution for $\eta = 0.0001$**



- stringent local peak current/velocity trends
  - $\Rightarrow$ variations with η: chaotic phase beyond $\eta = 0.001$
  - $\Rightarrow$ shock-mediated island-coalescence, complex wave interferences, Petschek-like realizations at islands



Summary on resistive MHD

- high magnetic reynolds number regime: challenging

  $\Rightarrow$ anomalous resistivity or hyperdiffusion treatments exploited: difficult to quantify precise Reynolds number; discretization versus physics effects

  $\Rightarrow$ smaller scales: may necessitate beyond resistive MHD approach!

  $\Rightarrow$ resistive to Hall-MHD, 2-fluid, multi-species, kinetic . . . ?

Hall-MHD

- extend to generalized Ohm's law with electric field

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \frac{1}{e n_e} \mathbf{J} \times \mathbf{B} + \eta \mathbf{J}$$

$\Rightarrow$ rewrite with Hall parameter $\eta_h \propto m_i / eZ$ to

$$\mathbf{E} = -\left(\mathbf{v} - \frac{\eta_h}{\rho} \mathbf{J} \right) \times \mathbf{B} + \eta \mathbf{J}$$

$\Rightarrow$ minimal ion-electron decoupling, as $\mathbf{v} = \mathbf{u}_i$ while electron bulk speed is $\mathbf{u}_e = \mathbf{v} - \mathbf{J} / e\, n_e$

- Hall-MHD: simple one-fluid extension to ideal-resistive MHD, extra term in induction equation

  $\Rightarrow$ for $\eta = 0$ (ideal case): modifies wave speeds

  $\Rightarrow$ linearize about cold state $p_0 = 0$ with uniform $\mathbf{B}_0$

  $\Rightarrow$ modified dispersion relation for plane waves $e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$

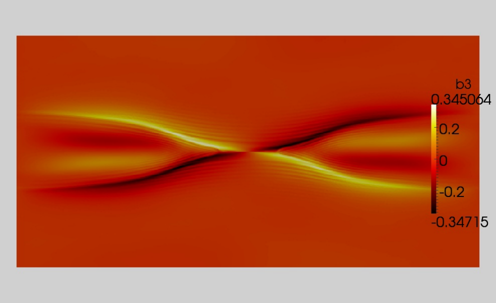
$$\left(\omega^2 - \omega_A^2\right)^2 = (\omega_A^4/\Omega_i^2)\,\omega^2$$

  $\Rightarrow$ fast/Alfvén waves (LHS) dispersive due to finite ion gyrofrequency $\Omega_i = ZeB_0/m_i$, where $\omega_A = k_{||} v_A$

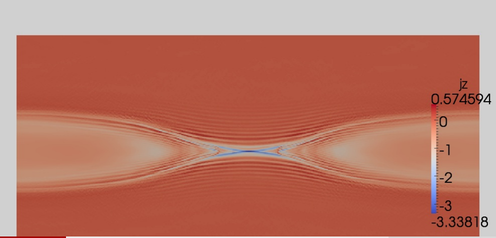
  $\Rightarrow$ shortest wavelengths travel fastest, highest ω arrive first

  $\Rightarrow$ 'whistler' waves, trouble for (explicit) numerical schemes

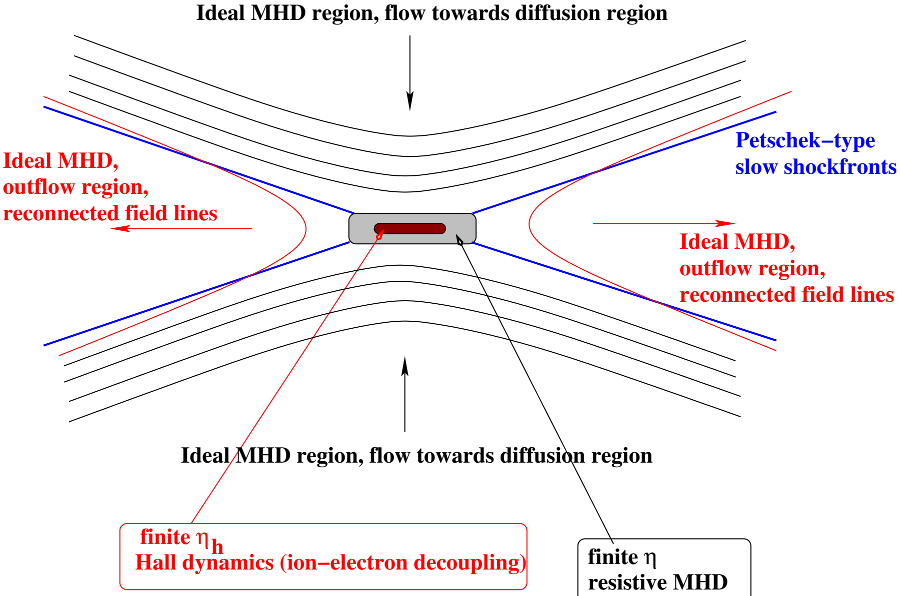
- redo GEM reconnection in Hall-MHD with $\eta_h = 1$, $\eta = 0.005$
  $\Rightarrow$ Hall-MHD implies out-of-plane (2.5D) field components



$\Rightarrow$ wave interference patterns due to whistler wave dynamics



- reconnection can show multi-scale character, schematic view



- schematic suggests: use $\eta(\mathbf{x})$ and $\eta_h(\mathbf{x})$ prescriptions where the spatial dependence incorporates that all models (ideal, resistive, Hall-MHD) are one-fluid representations, 'coupled' through (known overall dimensions of the) diffusion region

  $\Rightarrow$ **any effect at boundaries/overlap regions**?

- reality for collisionless reconnection much worse: need to descend in model hierarchy

  $\Rightarrow$ one-fluid MHD, Hall-MHD, two-fluid, hybrid, kinetic (particle based) prescriptions

  $\Rightarrow$ latter require **coupling of different sets of PDEs**, different number of variables, characteristic speeds: how to address this?

Coupling strategies

- address coupling strategies in analytically tractable case

  $\Rightarrow$ instead of full plasma-physical (reconnection) setup, idealize to scalar hyperbolic PDEs

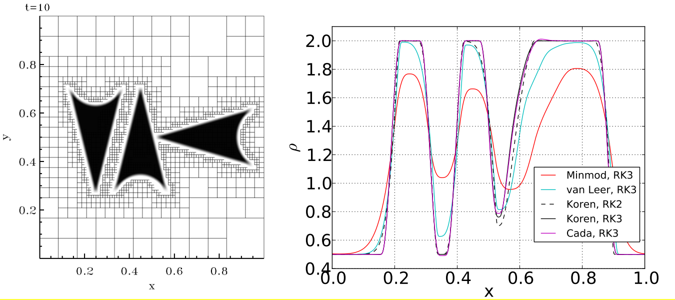
  $\Rightarrow$ multi-dimensional solutions of generic conservation law

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{F}(\rho, \mathbf{x}, t) = 0$$

  $\Rightarrow$ linear advection for $\mathbf{F}(\rho, \mathbf{x}, t) = \rho \mathbf{v}_0$

  $\Rightarrow$ nonlinear generalizations for $\mathbf{F}(\rho, \mathbf{x}, t) = \mathrm{F}\left(\rho(\mathbf{x}, t)\right) \mathbf{v}(\mathbf{x}, t)$

- **Test module: pure advection**
  - ⇒ with $\mathbf{U} = \rho$, $\mathbf{F} = \rho \mathbf{v}_0$ with $\mathbf{v}_0$ uniform velocity
  - ⇒ testing novel functionality in discretization or adaptivity
  - ⇒ demonstrating convergence, order of accuracy, ...
- Discontinuity dominated 2D profile: `VAC` logo
  - ⇒ **advected diagonally on unit square**
  - ⇒ after 10 passages, with horizontal cut (different limiters)



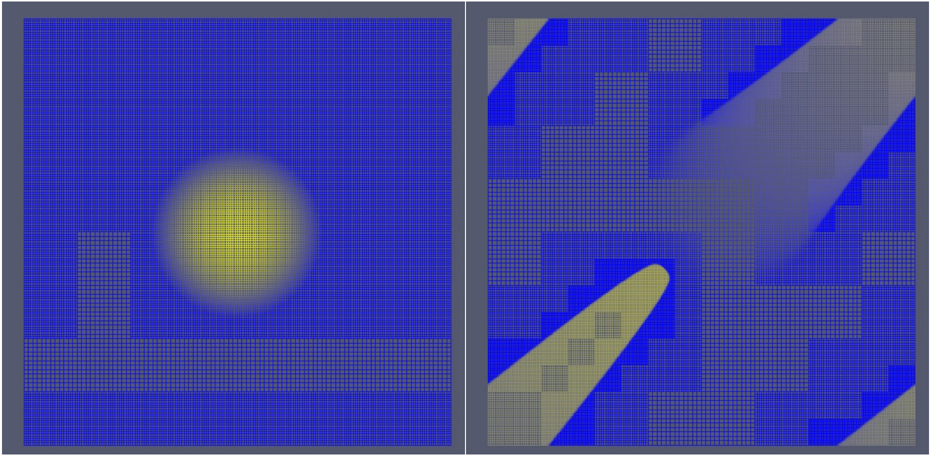
- **Nonlinear Scalar equation**: `amrvacphys.t.nonlinear`
  - ⇒ `eqpar(fluxtype_)` switch for different flux expressions
  - ⇒ inviscid Burgers (case 1), nonconvex equation (case 2)

$$\rho_t + \nabla \cdot \left(\frac{1}{2} \rho^2 \mathbf{e} \right) = 0$$

$$\rho_t + \nabla \cdot \left(\rho^3 \mathbf{e} \right) = 0$$

  - ⇒ in any dimensionality as $\mathbf{e} \equiv \sum_{i=1}^{D} \hat{\mathbf{e}}_i$

- Burgers for 2D: ‘advection’ of Gaussian bell profile

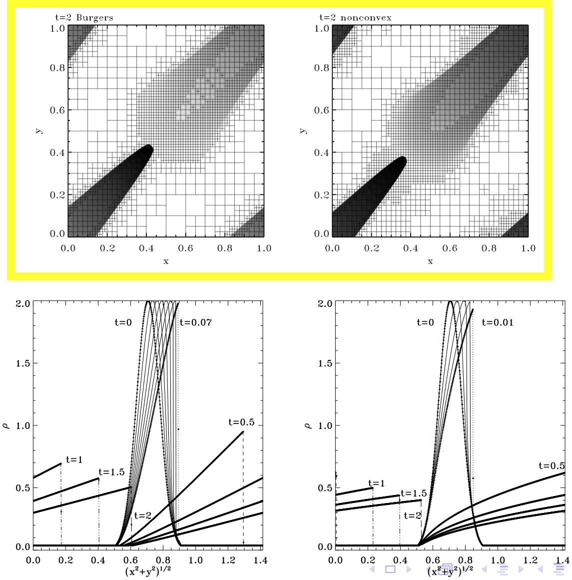


⇒ smooth initial condition steepens, shock formation

- **compare Burgers to nonconvex case**

⇒ **Rankine-Hugoniot relations** explain the different propagation speeds

- Burgers and nonvonvex evolution of Gaussian profile, analytically verified



- what for nested situation: advection+Burgers region?

  ⇒ interface treatments needed, two options

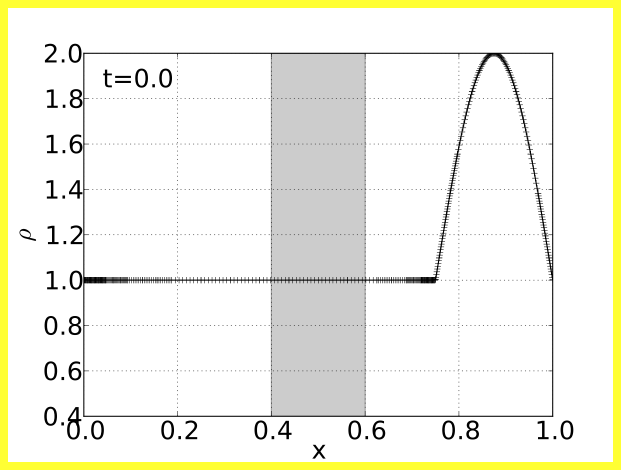
  ⇒ (1) **conservative coupling**: unique flux at interface, conservative

  ⇒ (2) **boundary coupling**: communication through scalar values in boundary

- Naive expectation: what happens with a Gaussian pulse when it is advected into a region where Burgers equation holds?

Conservative coupling

- greyzone: Burgers and rest of periodic domain is linear advection



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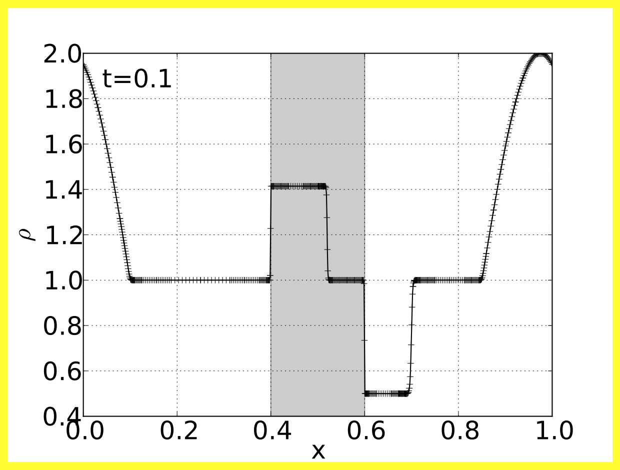

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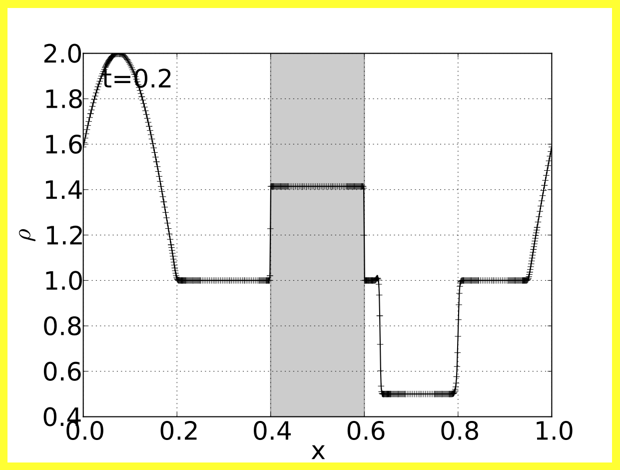

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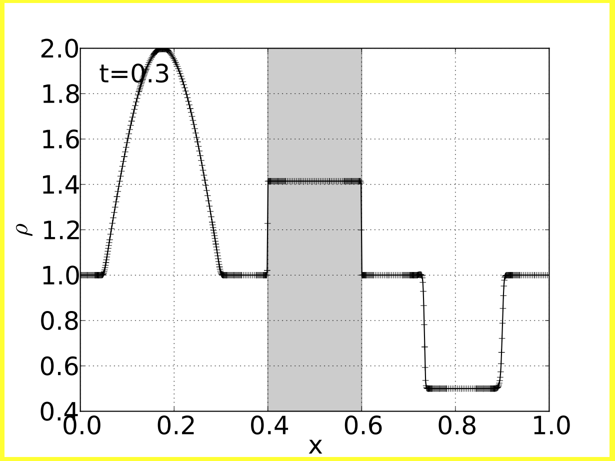

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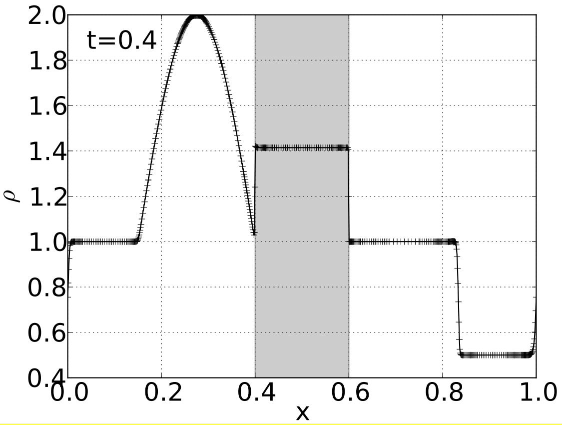

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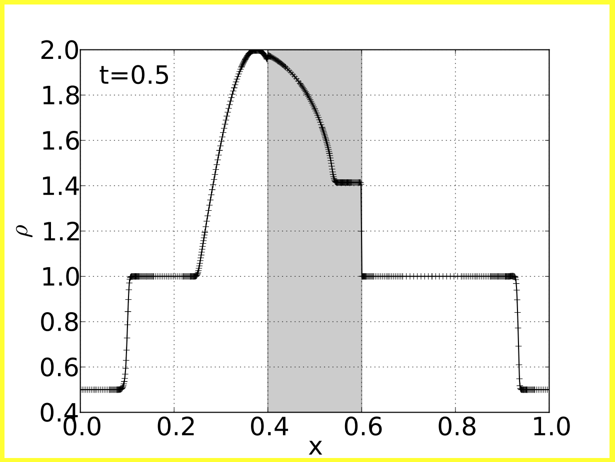

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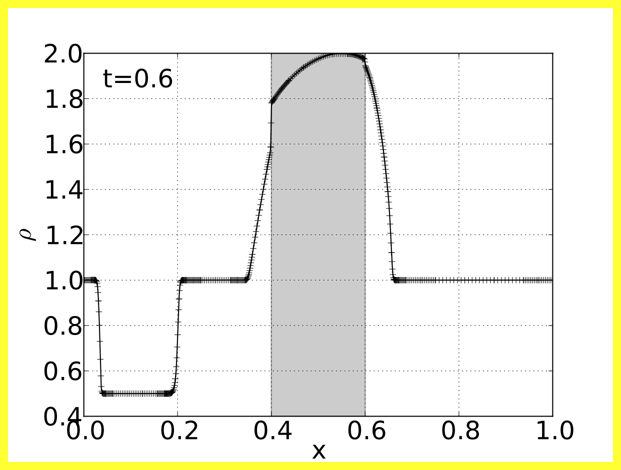

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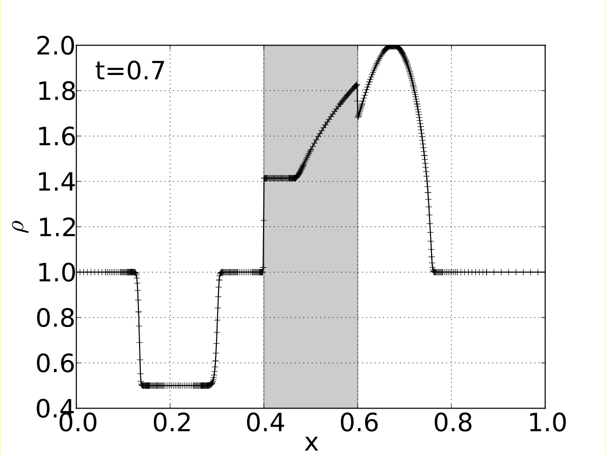

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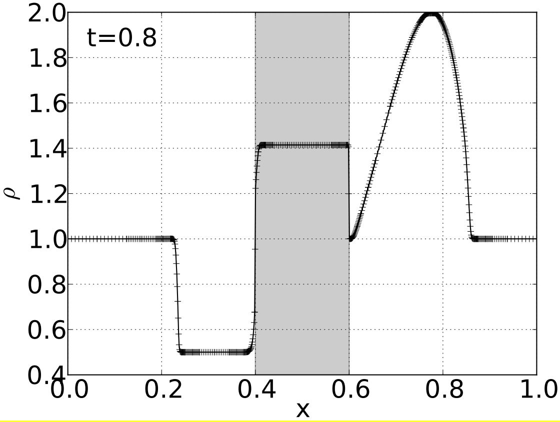


- perfectly conservative (area under curve kept)

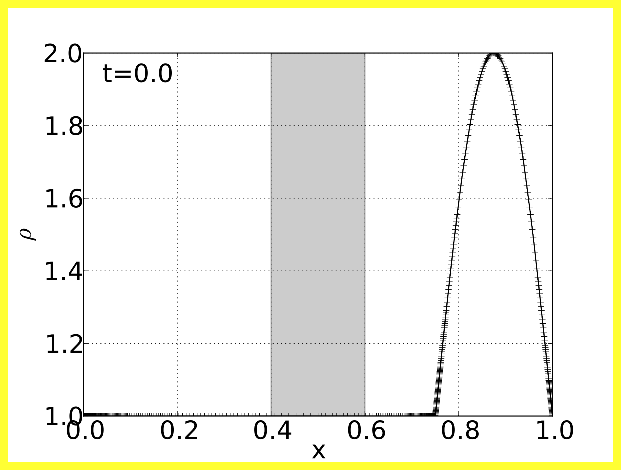
  $\Rightarrow$ instantly develops discontinuities at interfaces

  $\Rightarrow$ can be understood from Rankine-Hugoniot for stationary case at interface

  $\Rightarrow$ fully ok with AMR, but ‘undesired’ evolution

Boundary coupling

- greyzone: Burgers and rest of periodic domain is linear advection



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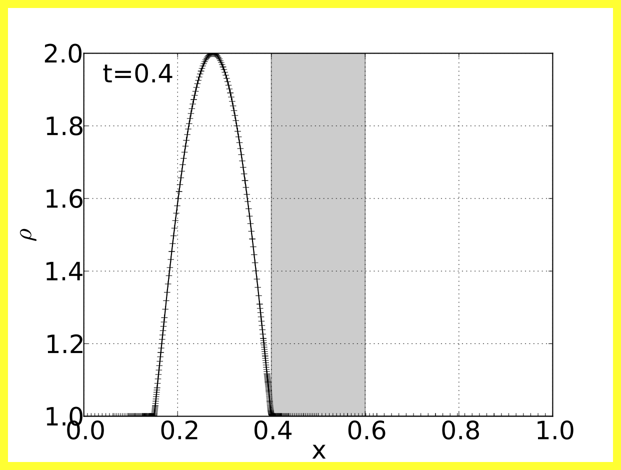

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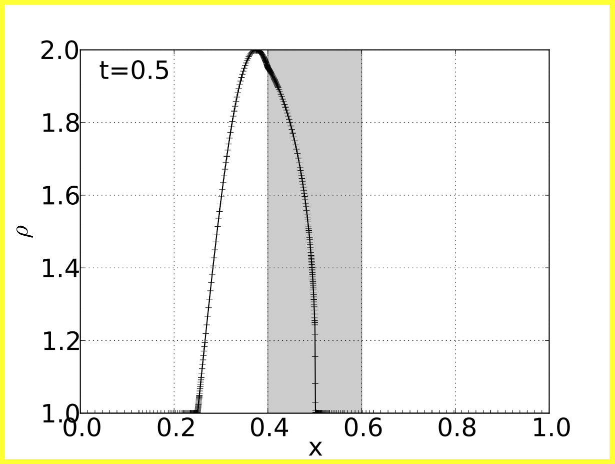

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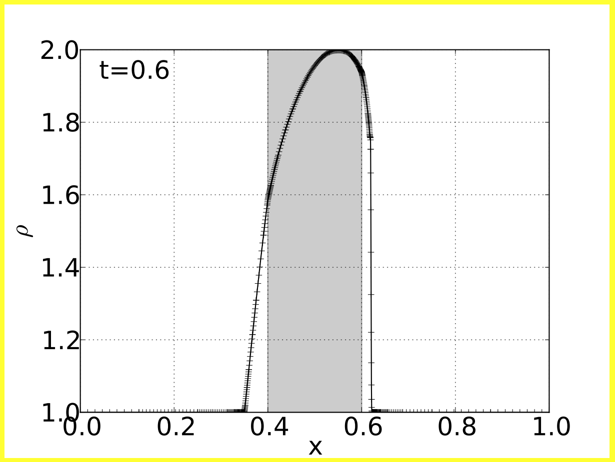

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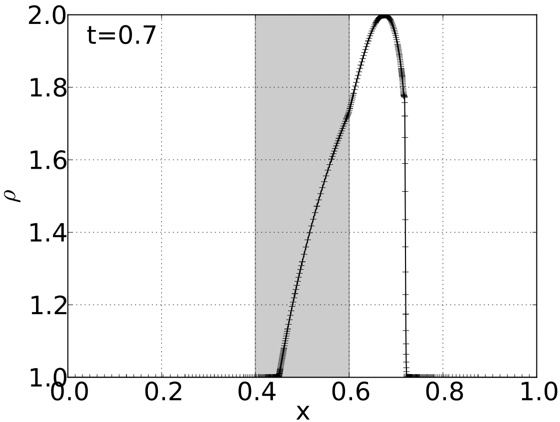

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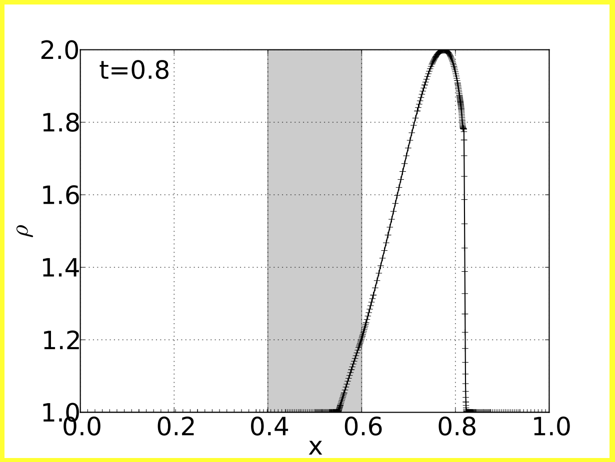

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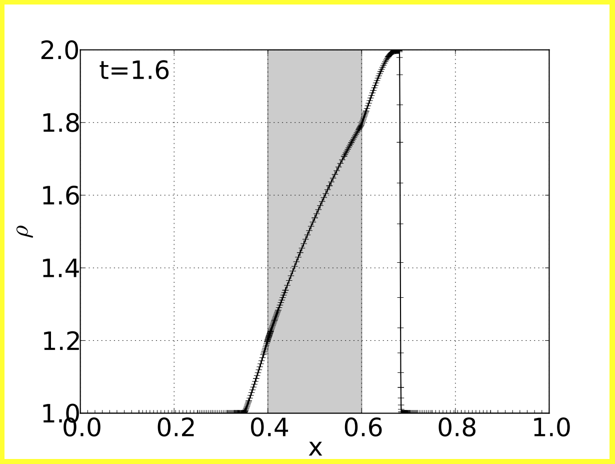

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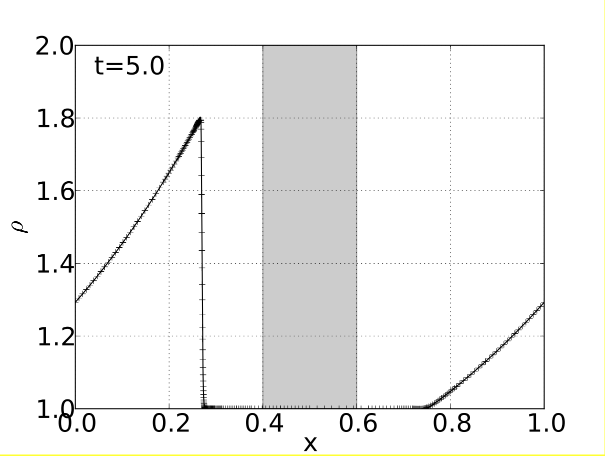

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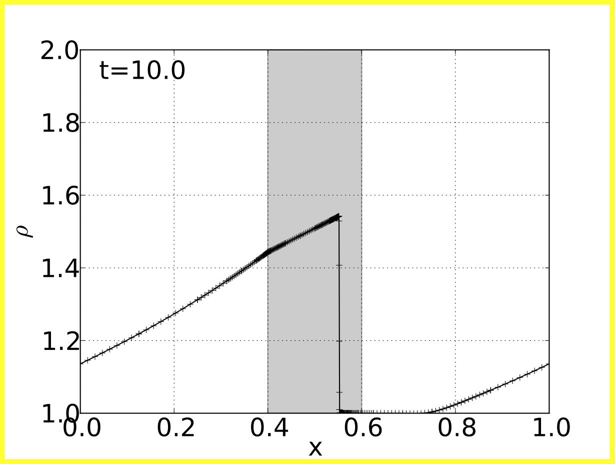
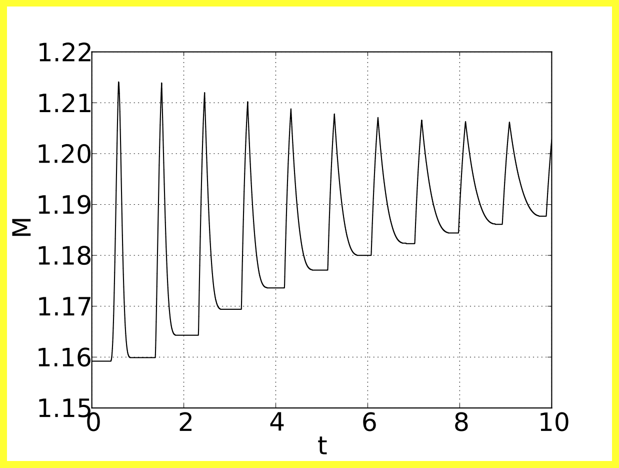


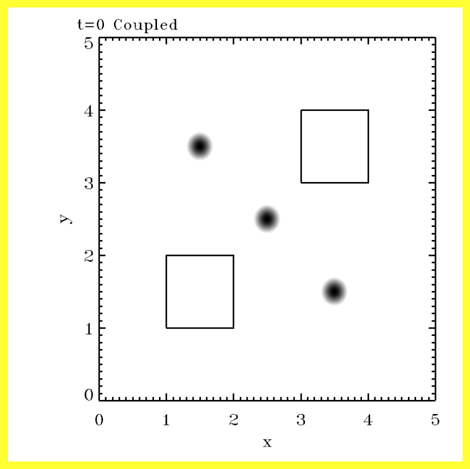
- naively expected evolution, but non-conservative (area under curve varies!)



$\Rightarrow$ full AMR, extension to 2D and multiple regions feasible

Boundary coupling:multi-D

- 2D advection on square, with embedded Burgers (UR) and nonconvex region (LL)



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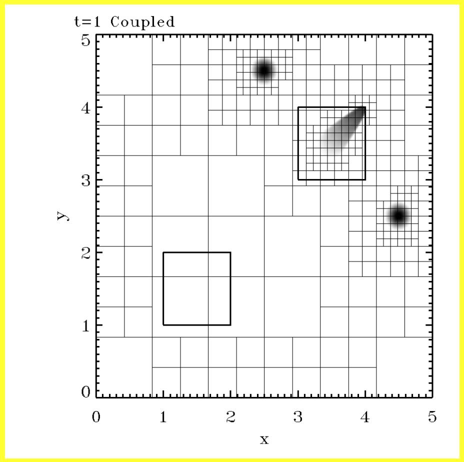

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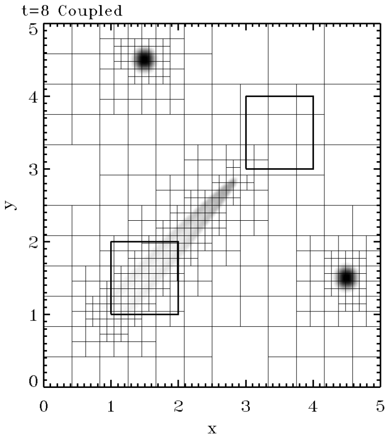

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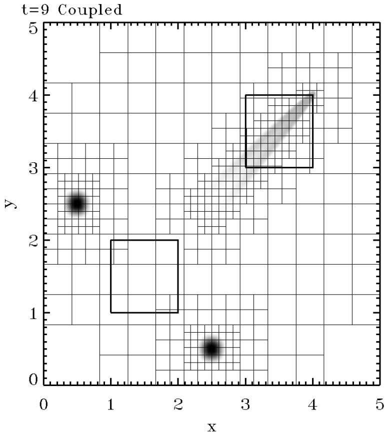


- boundary value exchange: relaxes conservation, allows multi-physics coupled evolutions

  $\Rightarrow$ so far AMR is adaptive, region where model changes is known/fixed geometrically

  $\Rightarrow$ model for ideal/resistive/Hall-MHD schematic
- what for varying $\eta(\mathbf{x})$? $\rightarrow$ mimic by setting $\mathbf{v}(\mathbf{x})$ in flux

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{F}(\rho, \mathbf{x}, t) = 0$$

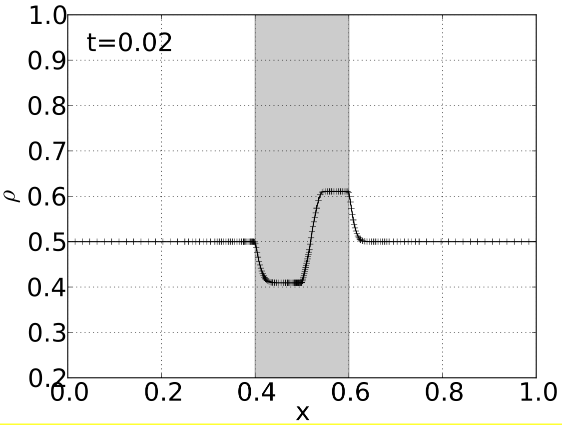
  $\Rightarrow$ nonlinear generalizations for $\mathbf{F}(\rho, \mathbf{x}, t) = \mathrm{F}\left(\rho(\mathbf{x}, t)\right) \mathbf{v}(\mathbf{x}, t)$

- solve $\partial_t \rho + \partial_x(\rho v(x)) = 0$ with spatially varying

$$v(x) = \begin{cases} 1 & \text{if } x < 0.4 \text{ or } x > 0.6, \\ 1 + h + \frac{h}{0.1}(x - 0.5) & \text{if } x \in [0.4, 0.5], \\ 1 + h - \frac{h}{0.1}(x - 0.5) & \text{if } x \in [0.5, 0.6]\,. \end{cases}$$

Space-varying advection speed

- 1D advection with linear-hat function for speed in grey region



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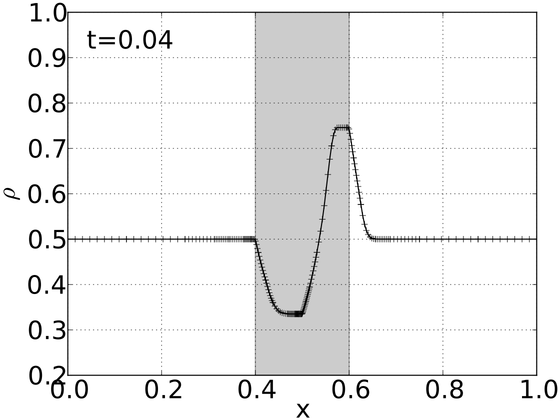

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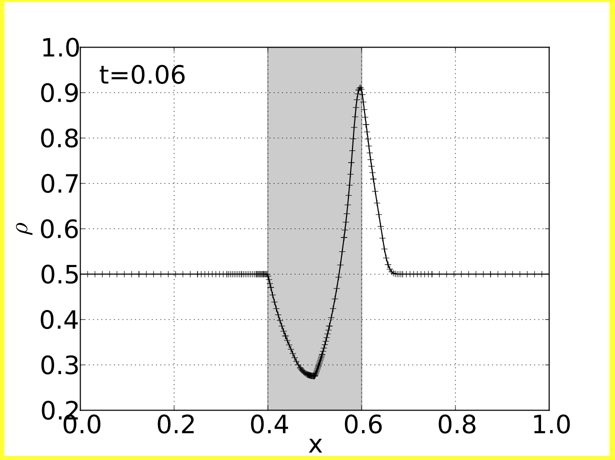

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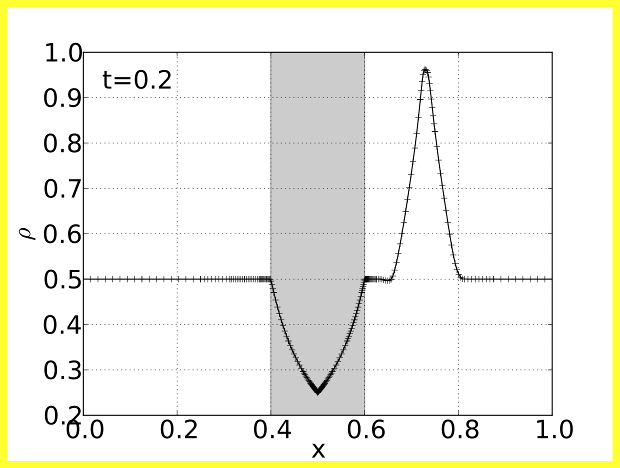

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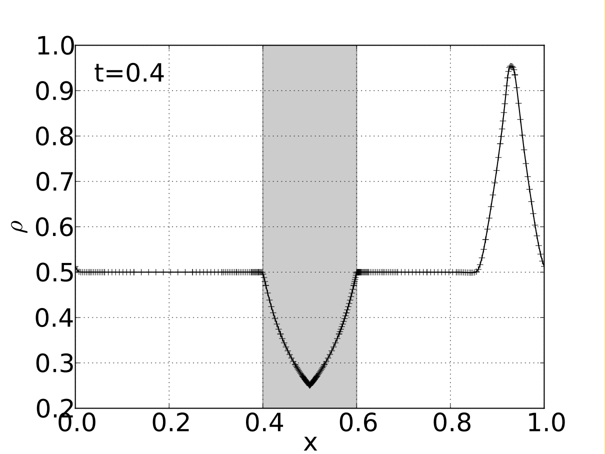

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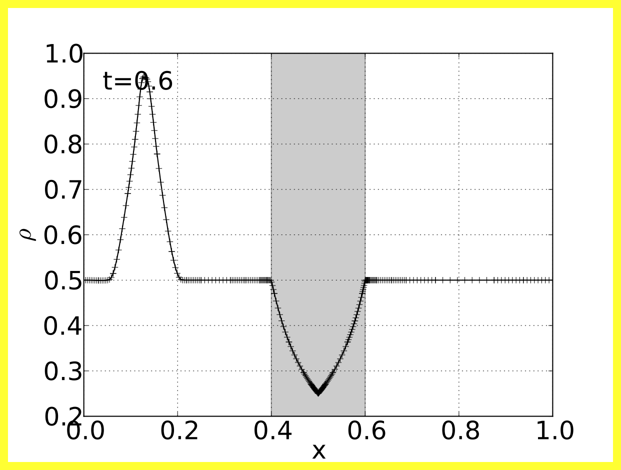

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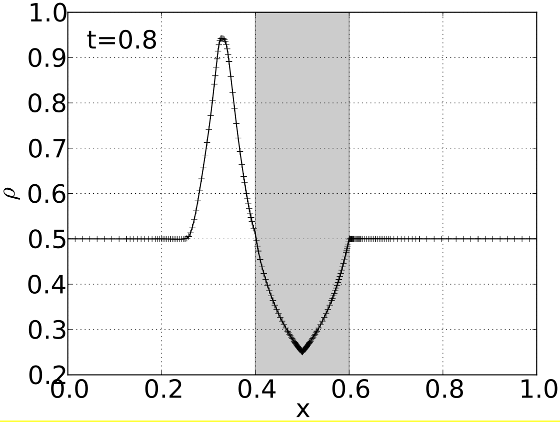

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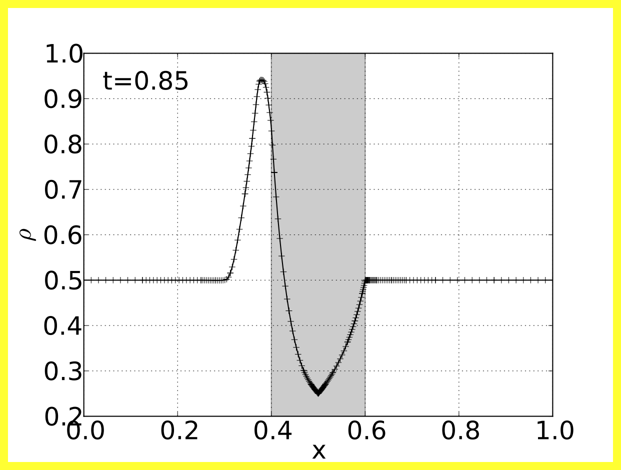

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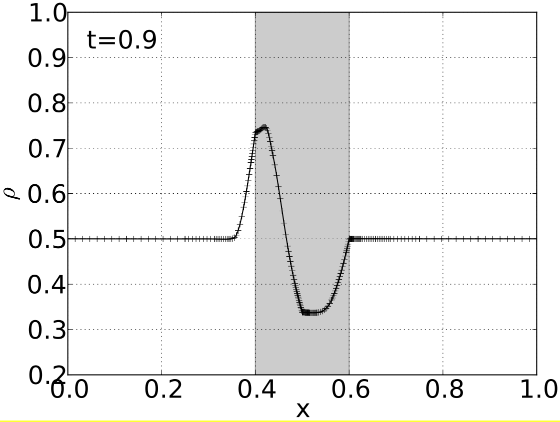
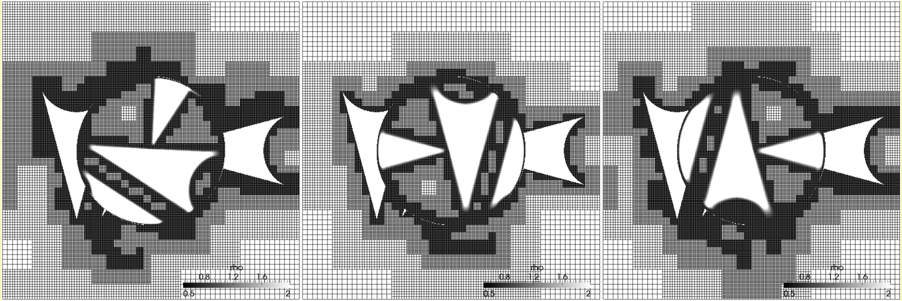


- density adjusts to create constant mass flux ρv

  $\Rightarrow$ (de)compressions, no discontinuities as velocity profile is continuous

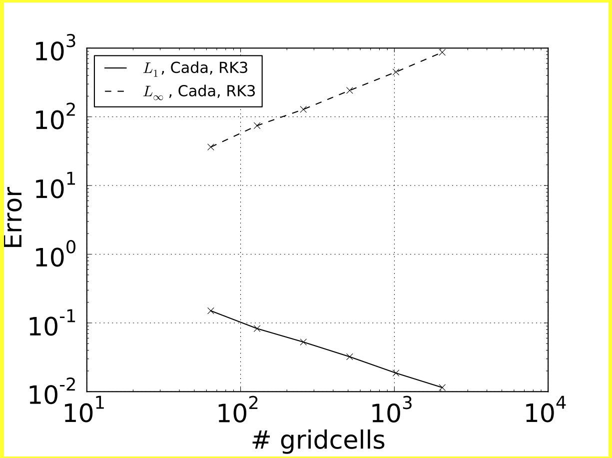
Space-varying advection speed:2D

- 2D advection with rotating disk



Space-varying advection speed:2D

- 2D advection with rotating disk: discontinuity at disk interface
  $\Rightarrow$ first order convergence in L_1, error grows in L_∞



Outlook

- Coupling strategies idealized to scalar nonlinear conservation
  $\Rightarrow$ potential issues identified for future plasma-physical coupled setups
  $\Rightarrow$ BC versus conservation; profiles with/without discontinuities in multiplying parameters
- reconnection also studied widely in full kinetic (PIC) setup, bottom-up approach feasible
  $\Rightarrow$ multi-level, multi-domain strategy [see Innocenti et al., JCP 238, 115 (2013)]