

Planet-Disk Interaction

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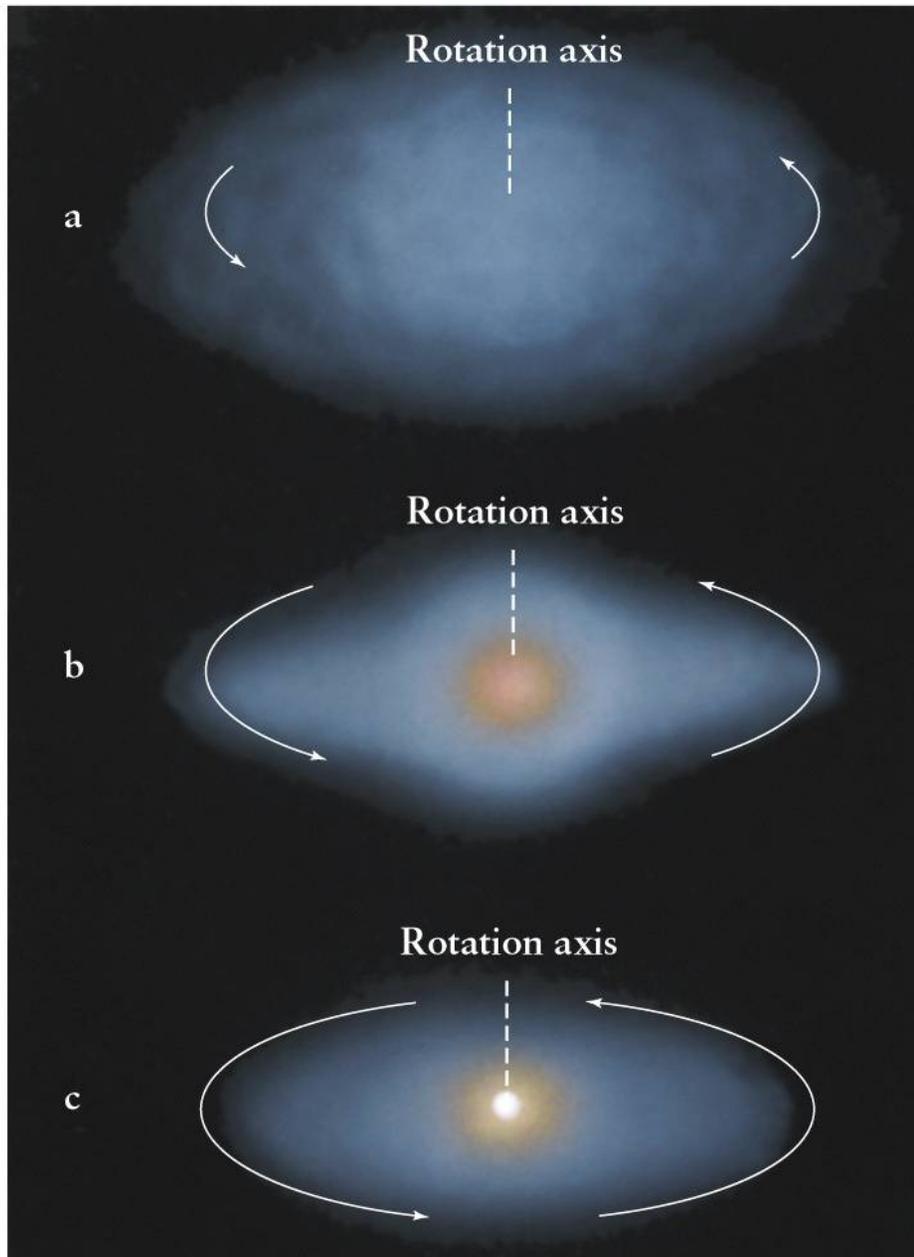


22. August, 2013



- Context
- Modelling/Numerics
- Torques
- Radiation
- Eccentricity
- Summary

(A. Crida)



Historic View:

(Leukippos, 480-420 BC)

“The worlds form in such a way, that the bodies sink into the empty space and connect to each other.”

Modern View:

Collaps of an interstellar
Molecular Cloud

Slight rotation \Rightarrow Flattening

Protosun in center / disk formation
(based on Kant & Laplace, 1750s)

Planets form in protoplanetary disks

\equiv Accretion Disks (99% Gas, 1% Dust)



Flat system, uniform rotation, circular
orbits

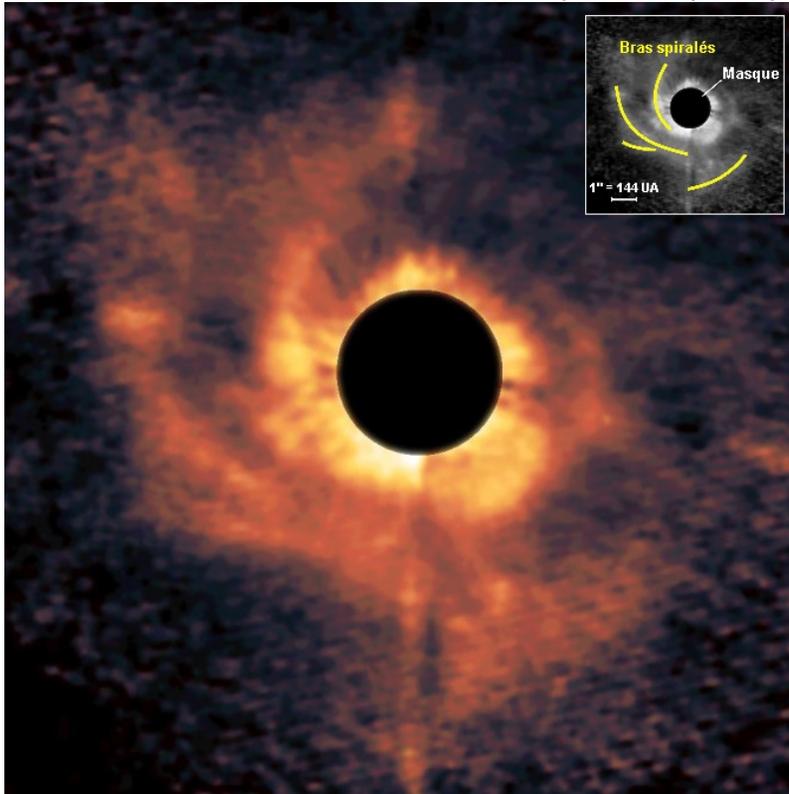
As in Solar System, Kepler Systems



Here: AB Aurigae (Herbig Ae star)

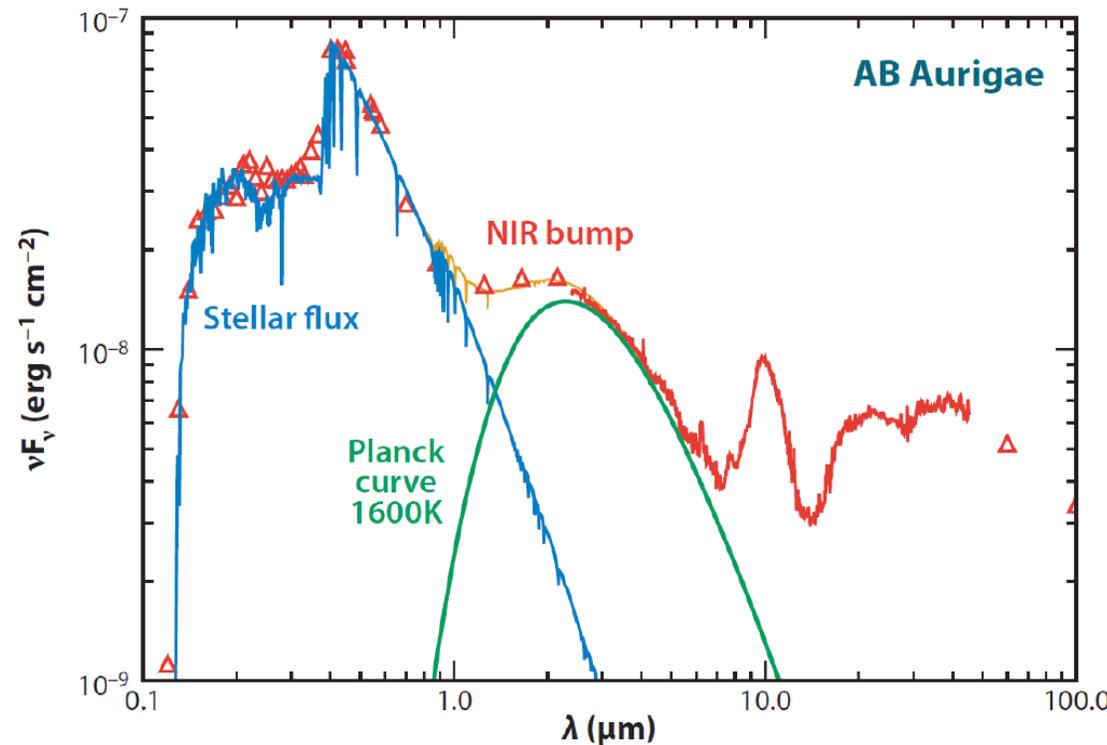
Direct Imaging :

Subaru AO, H-Band ($1.65 \mu\text{m}$)



(Fukagawa et al., 2004)

Spectral Energy Distribution (SED)
IR-Excess (over stellar contribution)



(Dullemond & Monnier, 2010)

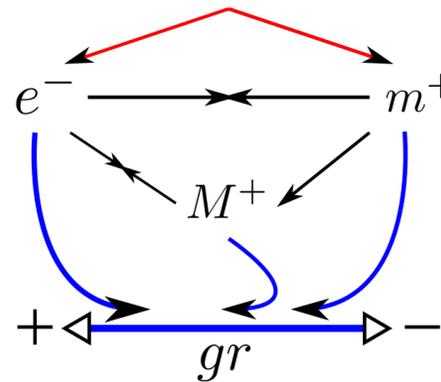
Idea: obtain disk structure and dynamics

- precondition for planet formation
- indicators for the presence of planets



3D MHD Simulations:

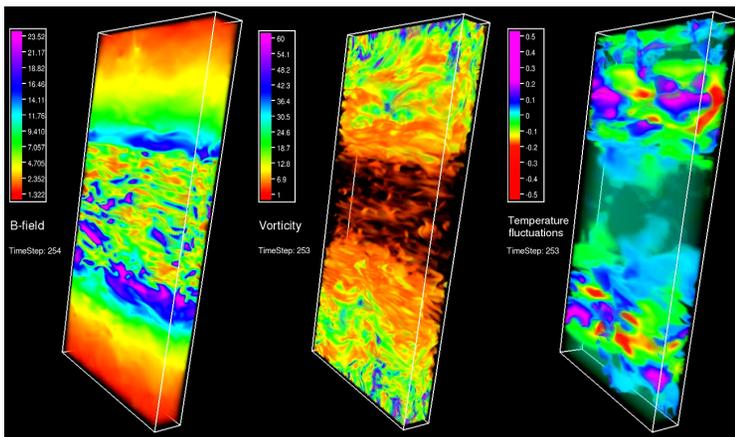
- Local stratified Box
- Non ideal MHD
- Radiation Transport
- Ionising sources
- Chemical network
- at different radii

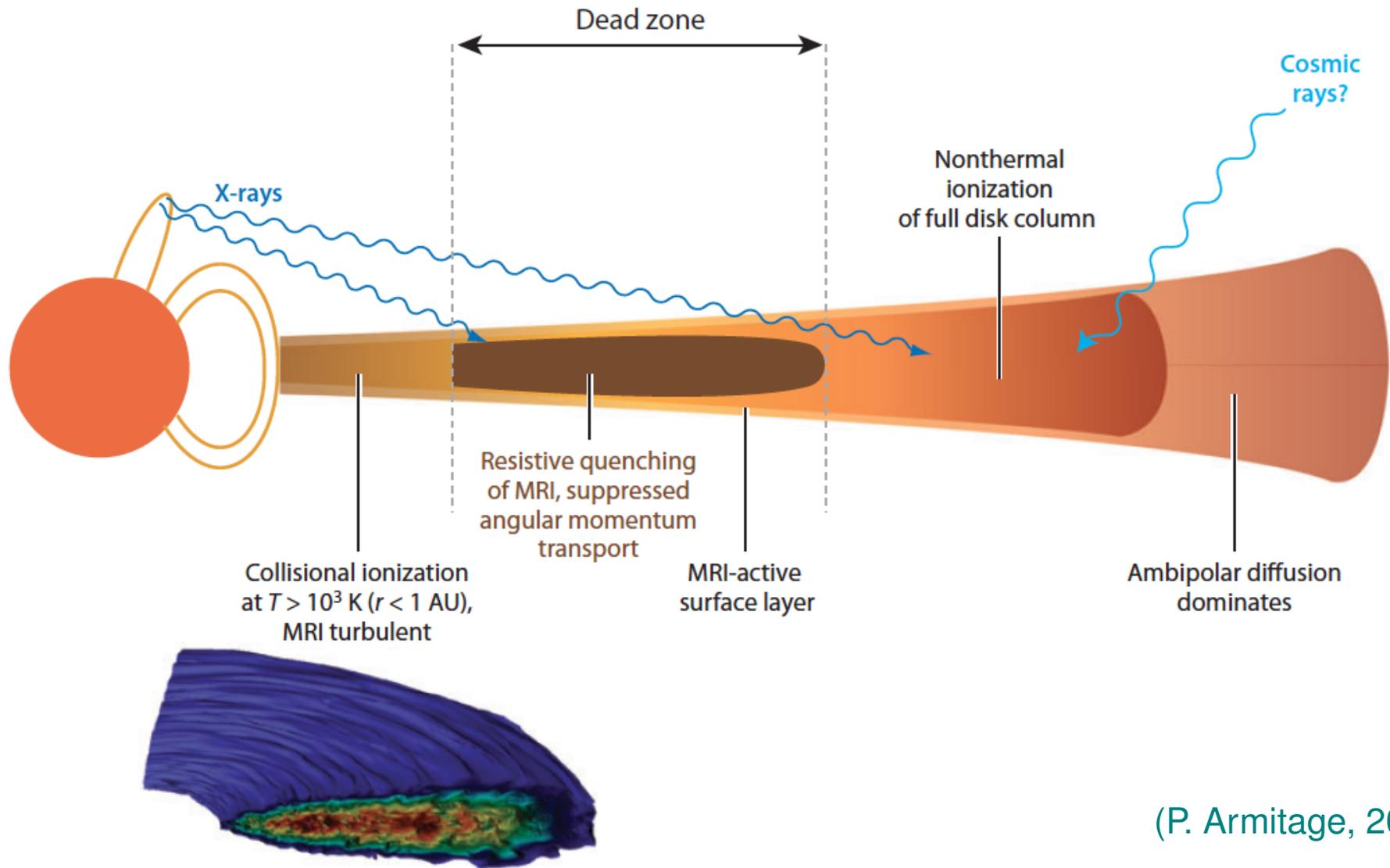
Chemical Network
(after Ilgner & Nelson, 2006)

Outcome:

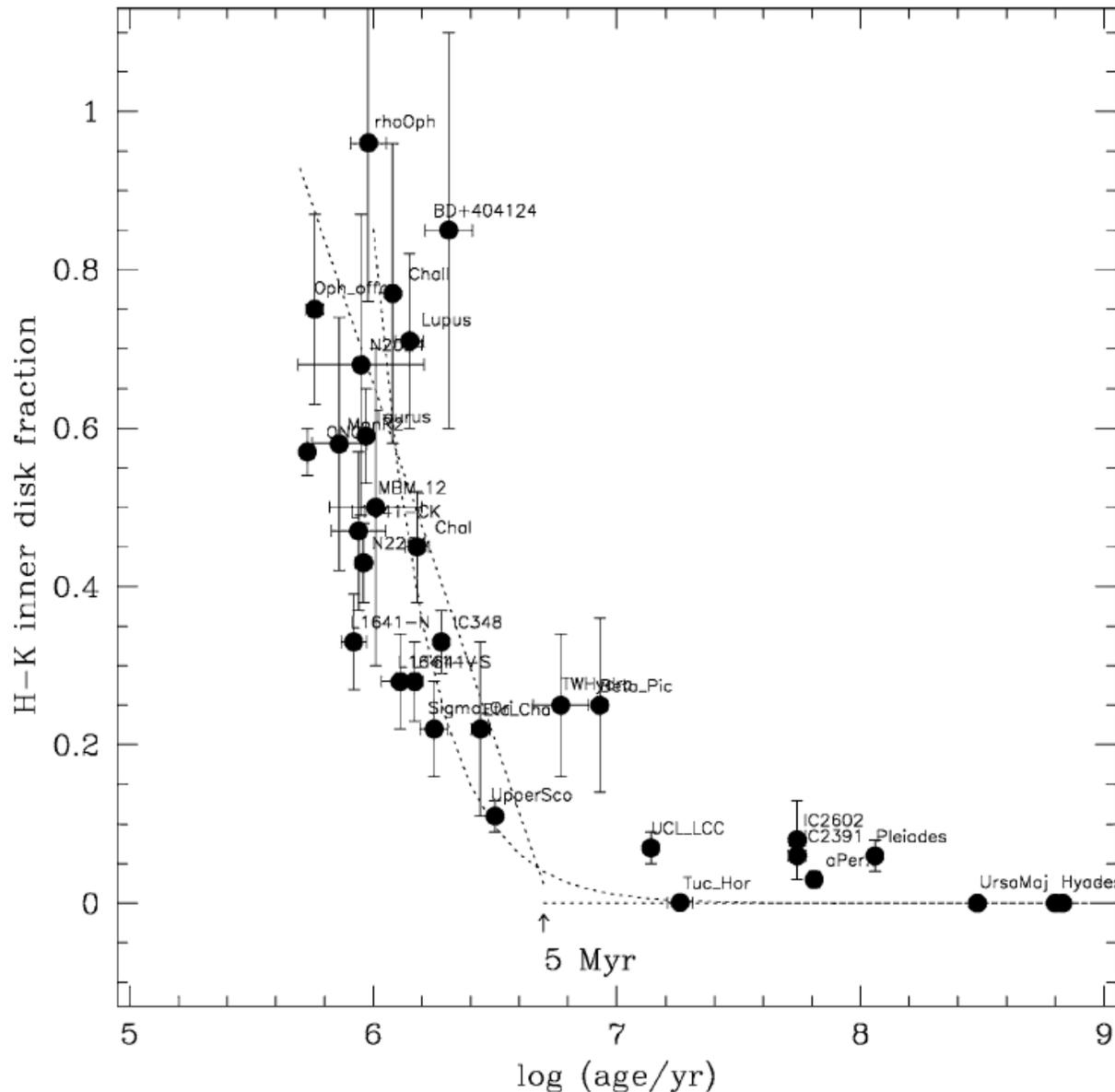
(Flaig ea. 2012)

- Saturation level
- Vertical structure
- Surface temperature
- Transport efficiency (α)
- Deadzones
- Velocity field for dust





The origin of the angular momentum transport in disks not clear (HD or MHD turbulence) Problem: low ionization and non-ideal MHD-effects



From IR-Excess

IR-Excess vs. Stellar Age

Lifetimes:

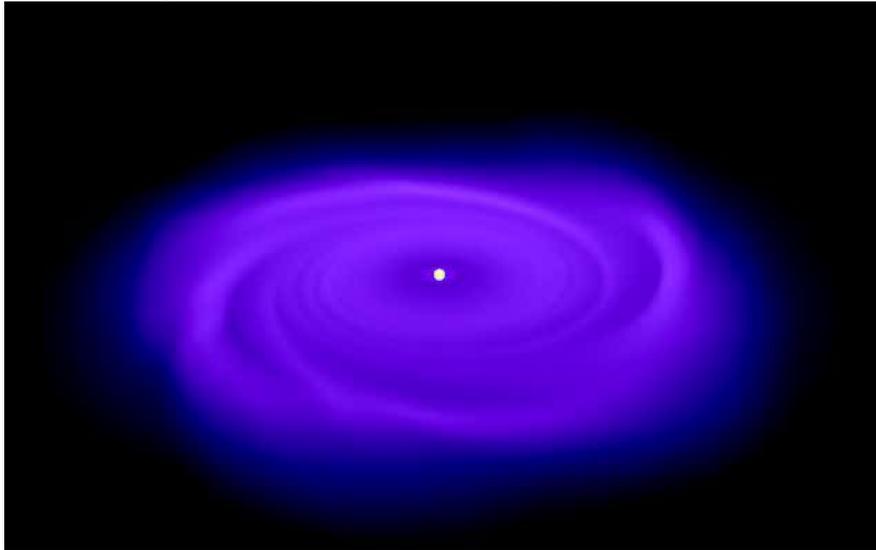
approx. 10^6 - 10^7 Years

(Montmerle et al. 2006)

- Need efficient ang.mom. transport
 - Anomalous Viscosity
 - **Turbulence**
- Planets form fast



Gravitational Instability in protoplanetary Disk



Growth of unstable modes in spiral arms

Advantage: Very fast formation

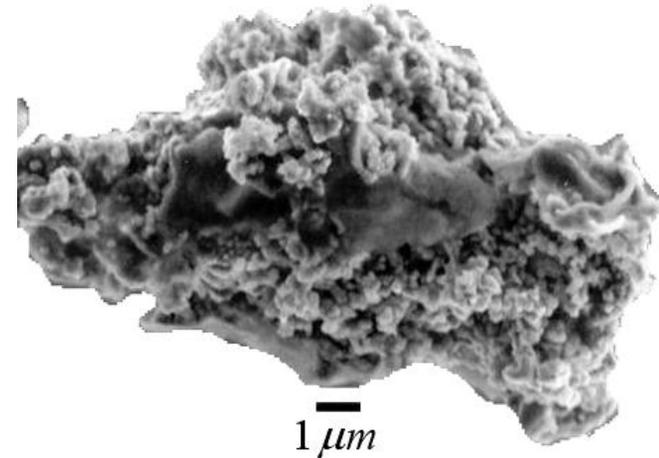
Disadvantage: Condition, cores

Planets are embedded in disk: interact gravitationally

Exert torques on each other: Orbital evolution ([Migration](#))

Evidence: **HOT** Planets (near the star), **resonant** configurations

Coagulation of Dust/Gas in Disk

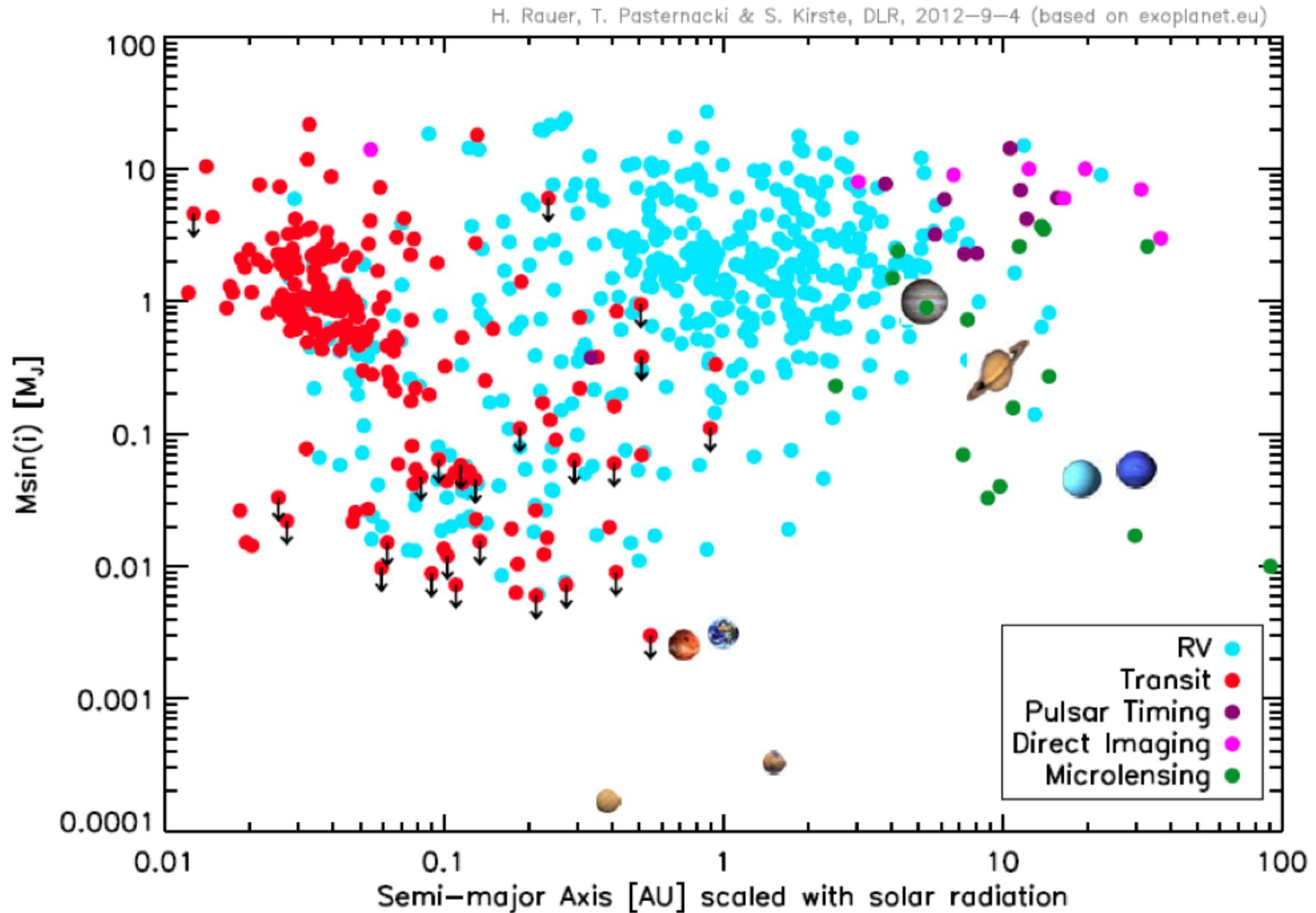


Growth through sequence of collisions & coagulations

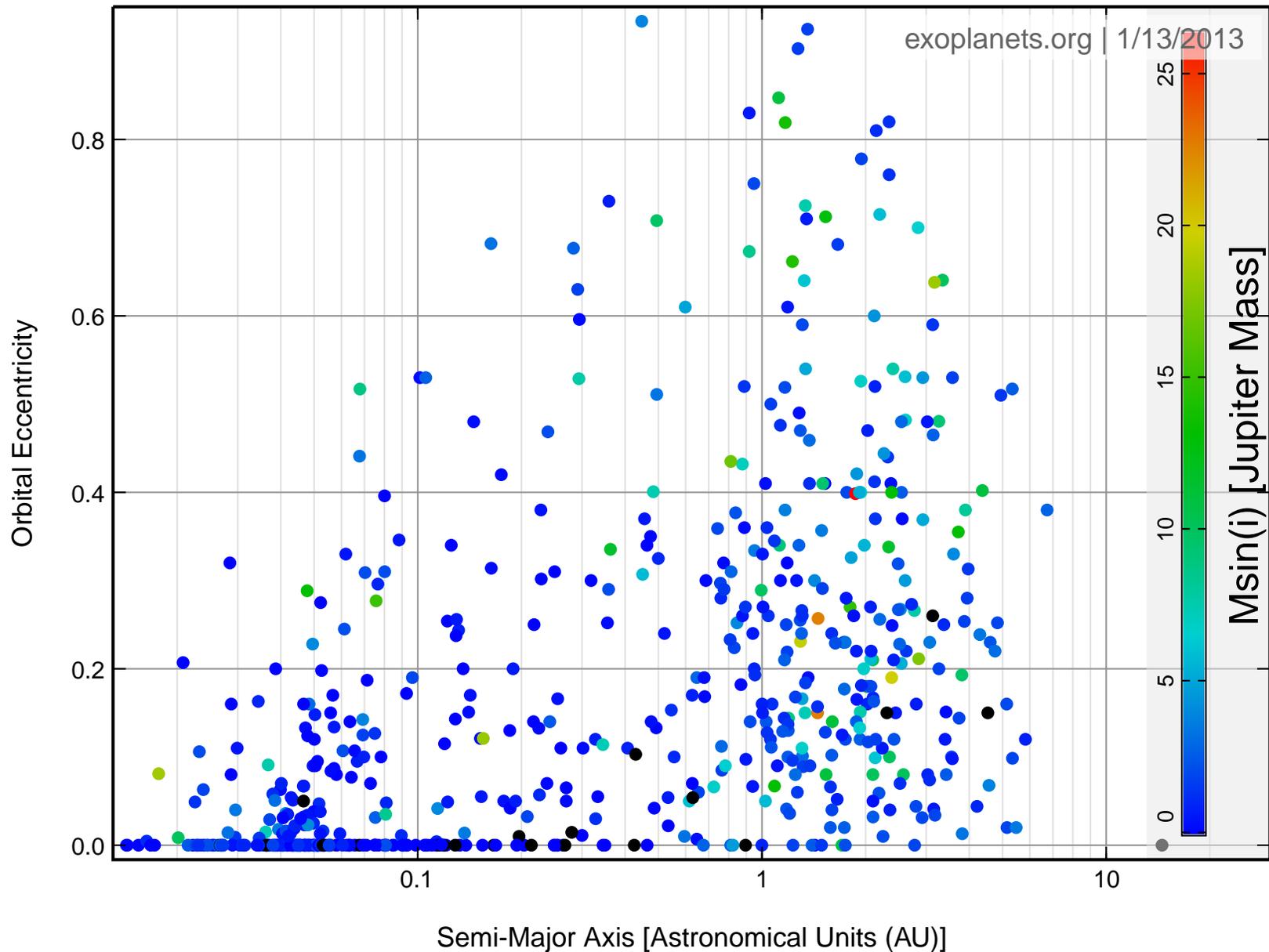
Late Phase: Gas accretion onto Core

Advantage: Cores of Planets

Disadvantage: Long timescales



High masses, very and close in, and very far away



High eccentricity and high inclinations (dynamically excited)



- Not possible to form hot Jupiters in situ
 - disk too hot for material to condense
 - not enough material
- Difficult to form massive planets
 - gap formation
- Eccentric and inclined orbits
 - planets form in flat disks (on circular orbits)

But planets grow **and** evolve in disks:

⇒ Have a closer look at planet-disk interaction

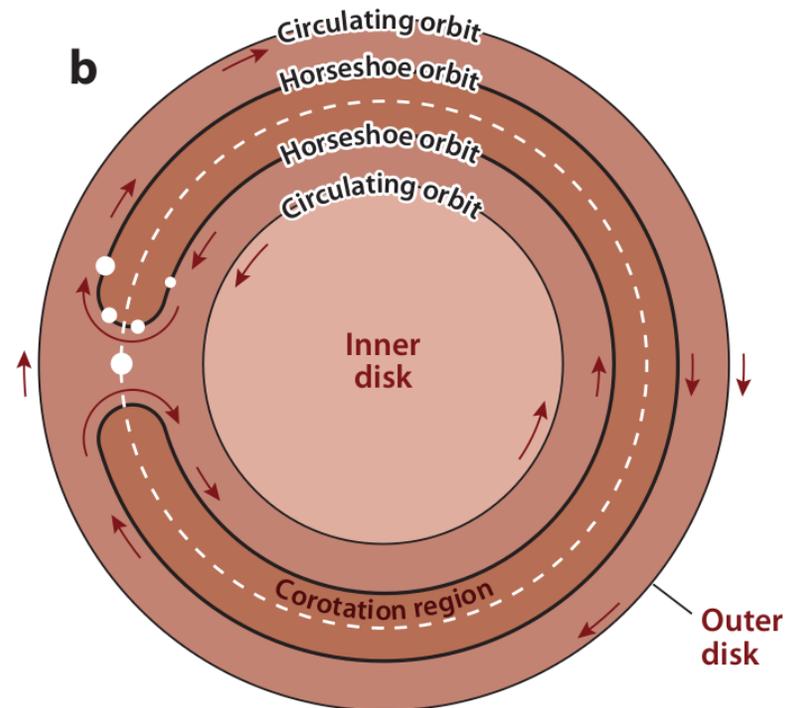
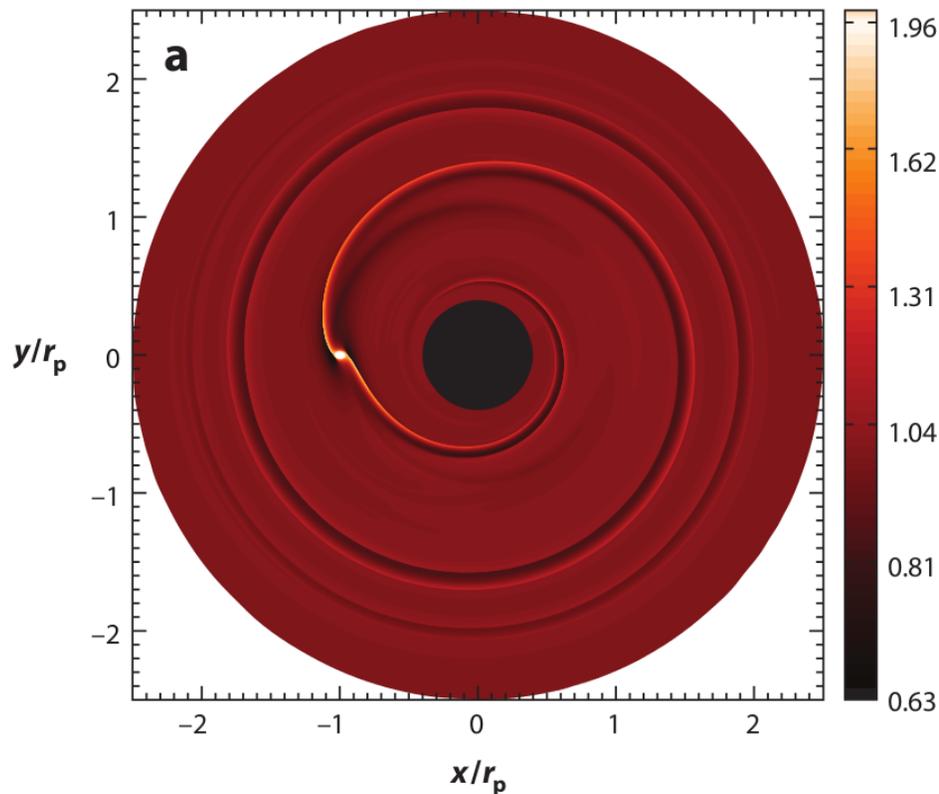
Have 2 contributions: Spiral arms & Corotation effects

(more details: [Annual Review article by Kley & Nelson, ARAA, 50, 2012](#))



1) Spiral arms
(Lindblad Torques)
(Typically inward)

2) Horseshoe region
(Corotation Torques)
(Typically outward)



Properties of Migration: in radiative disks, in turbulent disks, in self-gravitating disks, tidally driven migration (Kozai), Stellar Irradiation, runaway (type-III) migration (SolSys)

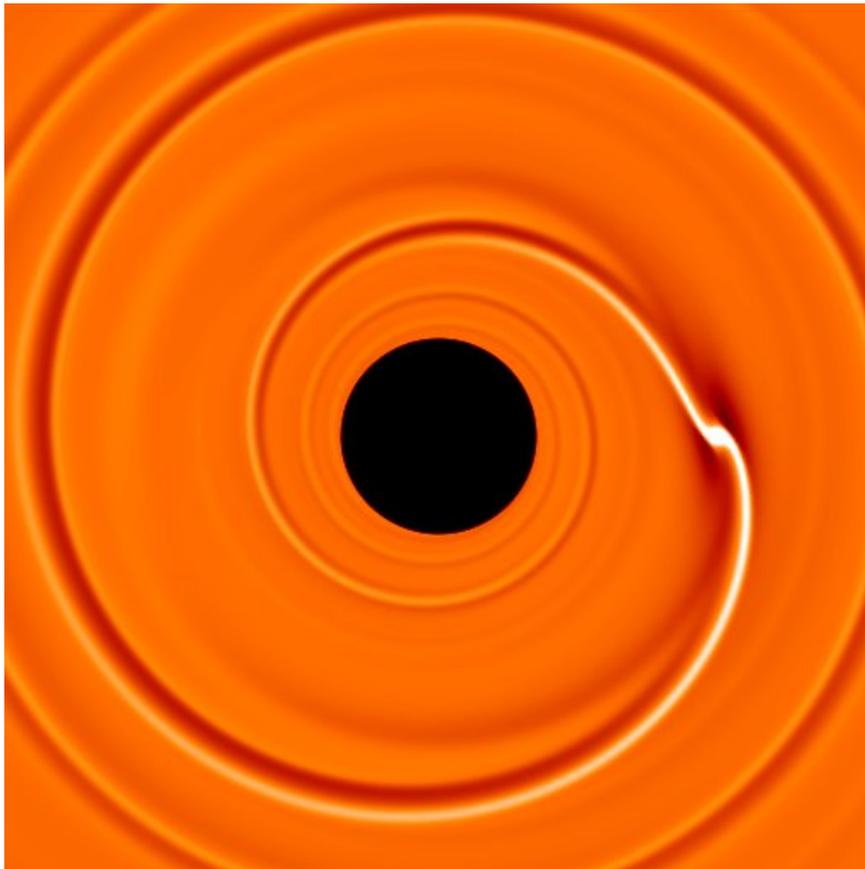
Eccentricity & Inclination: Check influence of the disk and Dynamical Processes



Young planets are embedded in gaseous disk

Creation of **spiral arms**:

- stationary in planet frame
- Linear analysis,
- 2D hydro-simulations



(Masset, 2001)

Inner Spiral

- pulls planet forward:
- positive torque

Outer Spiral

- pulls planet backward:
- negative torque

→ Net Torque

⇒ Migration

Most important:

Strength & Direction ?

Typically: Outer spiral wins

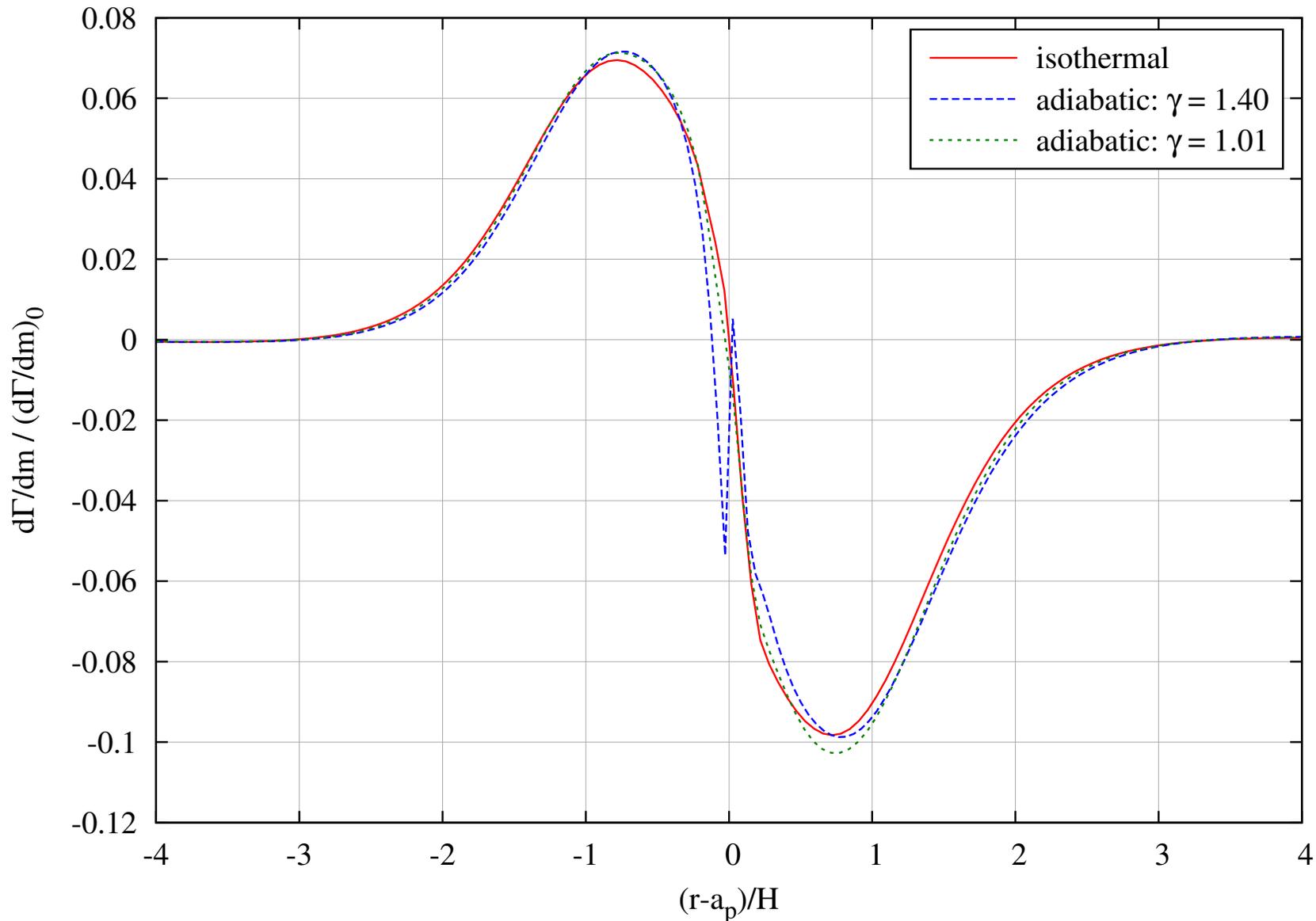
⇒ Inward Migration

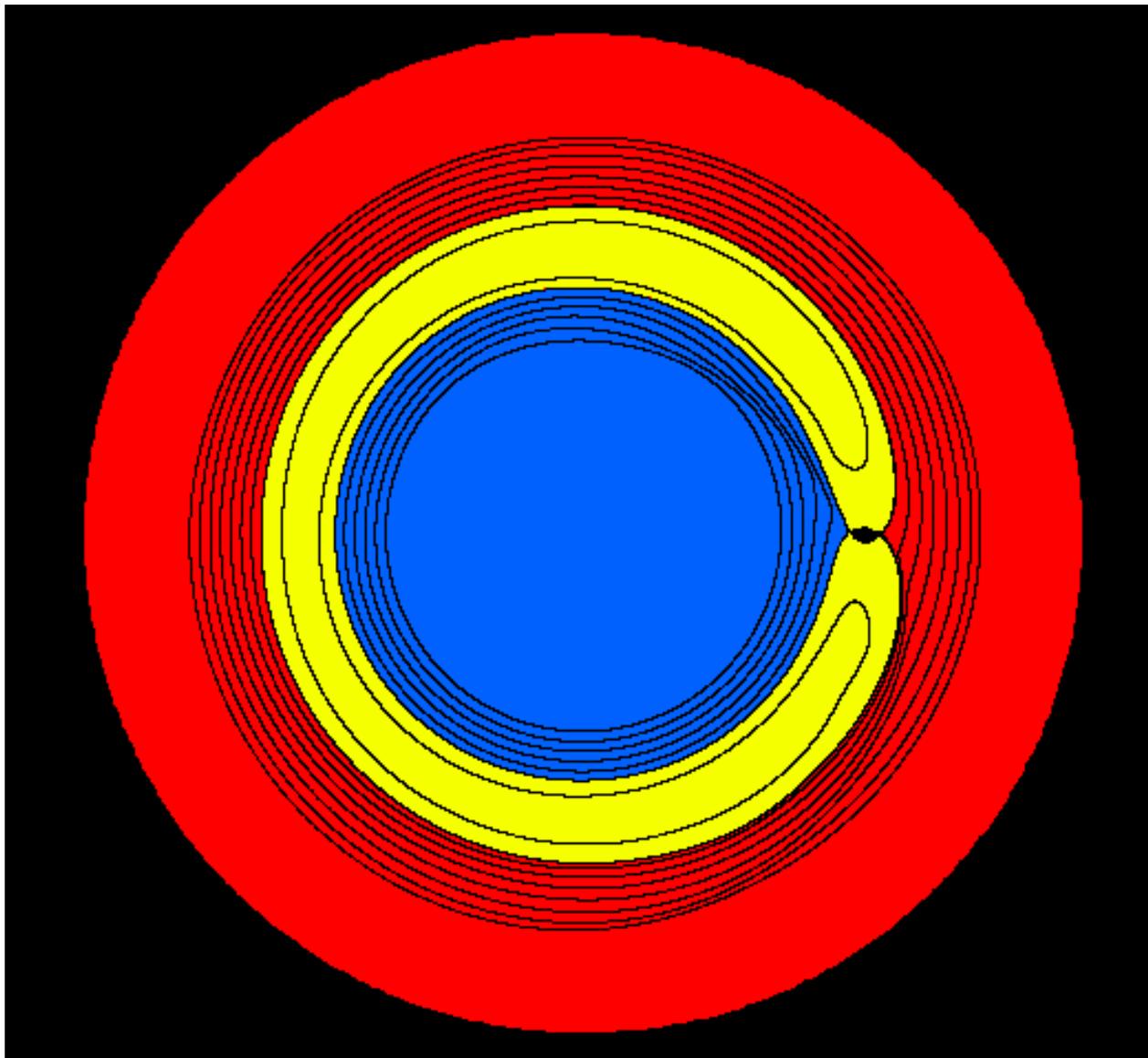
Torque scales with:

inv. Temp. $(H/r)^{-2}$, M_p^2 , M_d



$$\Gamma_{\text{tot}} = 2\pi \int \frac{d\Gamma}{dm}(r) \Sigma(r) r dr \quad \text{mit} \quad \left(\frac{d\Gamma}{dm}\right)_0 = \Omega_p^2(a_p) a_p^2 q^2 \left(\frac{H}{a_p}\right)^{-4}, \quad H_{\text{adi}} = \sqrt{\gamma} H_{\text{iso}}$$





3 Regions

Outer disk (spiral)

Inner disk (spiral)

⇒ Lindblad torques

Horseshoe (coorbital)

⇒ Corotation Torques
(Horseshoe drag)

Scaling with:

- Vortensity gradient
(Vorticity/density)

(F. Masset)



Evolution of planet in Disk

Require:

Torques

(disk pulls on planet)

 \implies Migration rate

Accretion

(planet attracts material)

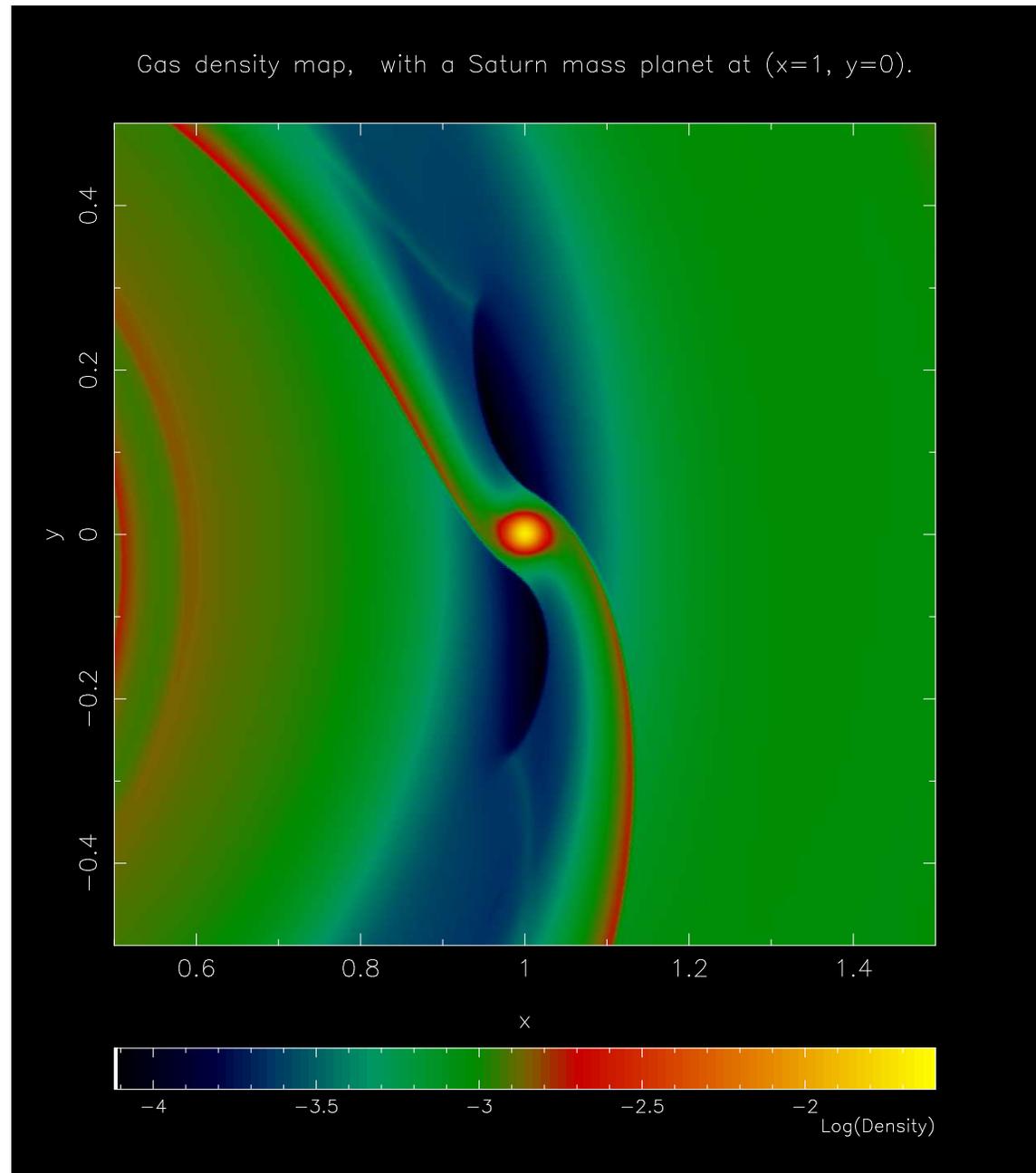
 \implies Mass growth rate

On the right:

Saturn mass planet in disk

- Spiral arms
- Envelope around planet
- Gap formation

(A. Crida)





The problem is full 3D: requires MHD, radiation transport, self-gravity

Often simplifications are used

- Dimensions
 - assume that the disk is flat \Rightarrow vertical averaging
 - perform simulations in 2D ($r - \varphi$ Coordinates)
 - recently simulations in 3D
- Physics
 - often isothermal, to avoid solving an energy equation
 - use pure hydrodynamics (no MHD)
 - model angular momentum transport through (eff.) viscosity
 - no self-gravity of disk
 - no radiative transport
- Numerics
 - in 2D: cylindrical coordinates
 - in 3D: spherical polar coordinates
 - velocities in ϕ are much larger \rightarrow special treatment
 - Planet: pointmass potential, special treatment in 2D required



Actors

Star ($1 M_{\odot}$), Disk ($0.01 M_{\odot}$), Planet ($M_p = M_{\text{Jup}}$)

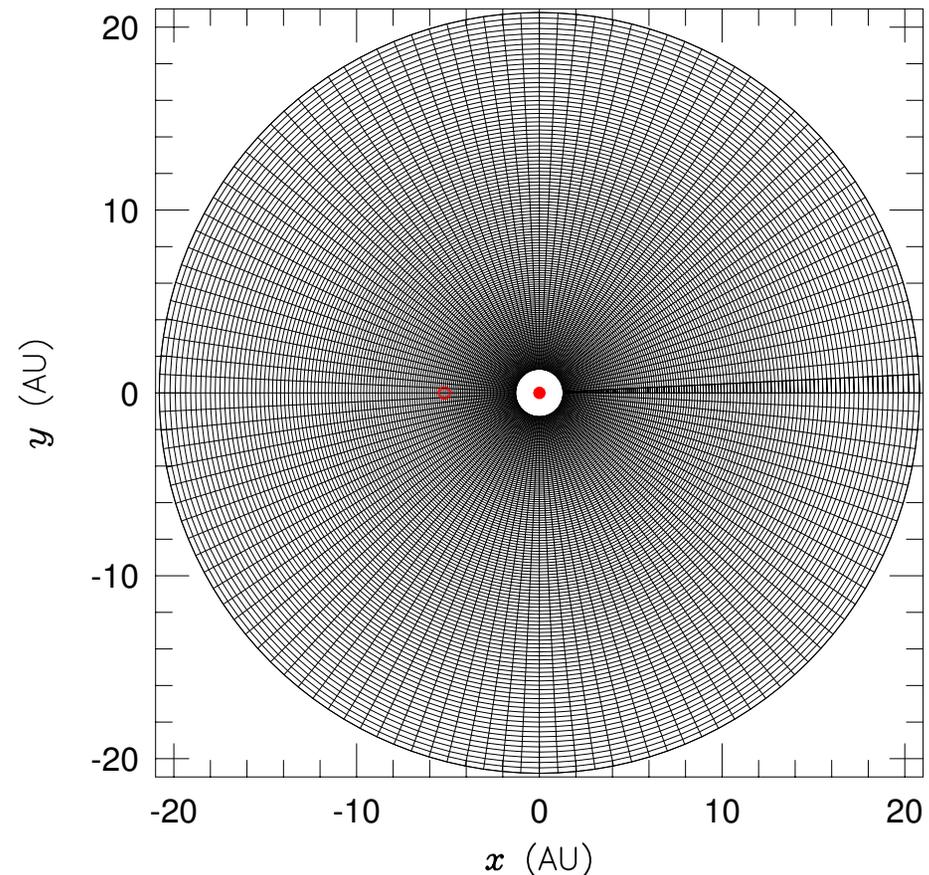
Disk: Hydrodynamical Evolution (Grid-Sample: 128x128)

- In potential of Star & Planet
- 2D (r, φ or $x - y$)
- $r_{\min} = 2.08$, $r_{\max} = 10.4$ AU
- locally isothermal: $T \propto r^{-1}$
- const. viscosity: $\nu = \text{const.}$

Planet: At 5.2 AU

- here on fixed orbit
- point mass (smoothed potential)

Compare **17 Codes** in
De Val-Borro & 22 Co-authors, MN
(2006)



EU comparison project: <http://www.astro.princeton.edu/~mdevalbo/comparison/>



Mass Conservation

$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma \mathbf{u}) = 0 \quad \mathbf{u} = (u_r, u_\varphi) = (v, r\omega)$$

Radial Momentum

$$\frac{\partial(\Sigma v)}{\partial t} + \nabla \cdot (\Sigma v \mathbf{u}) = \Sigma r(\omega + \Omega)^2 - \frac{\partial p}{\partial r} - \Sigma \frac{\partial \Phi}{\partial r} + f_r$$

Angular Momentum

$$\frac{\partial[\Sigma r^2(\omega + \Omega)]}{\partial t} + \nabla \cdot [\Sigma r^2(\omega + \Omega) \mathbf{u}] = -\frac{\partial p}{\partial \varphi} - \Sigma \frac{\partial \Phi}{\partial \varphi} + f_\varphi$$

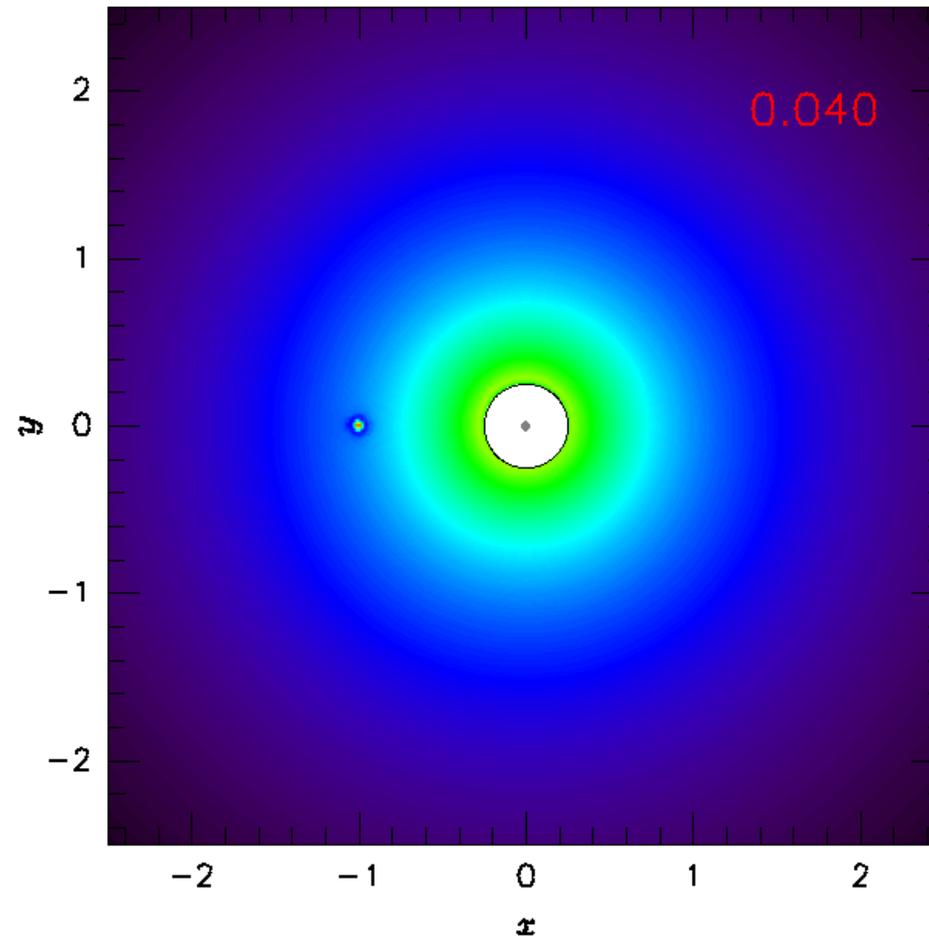
Energy

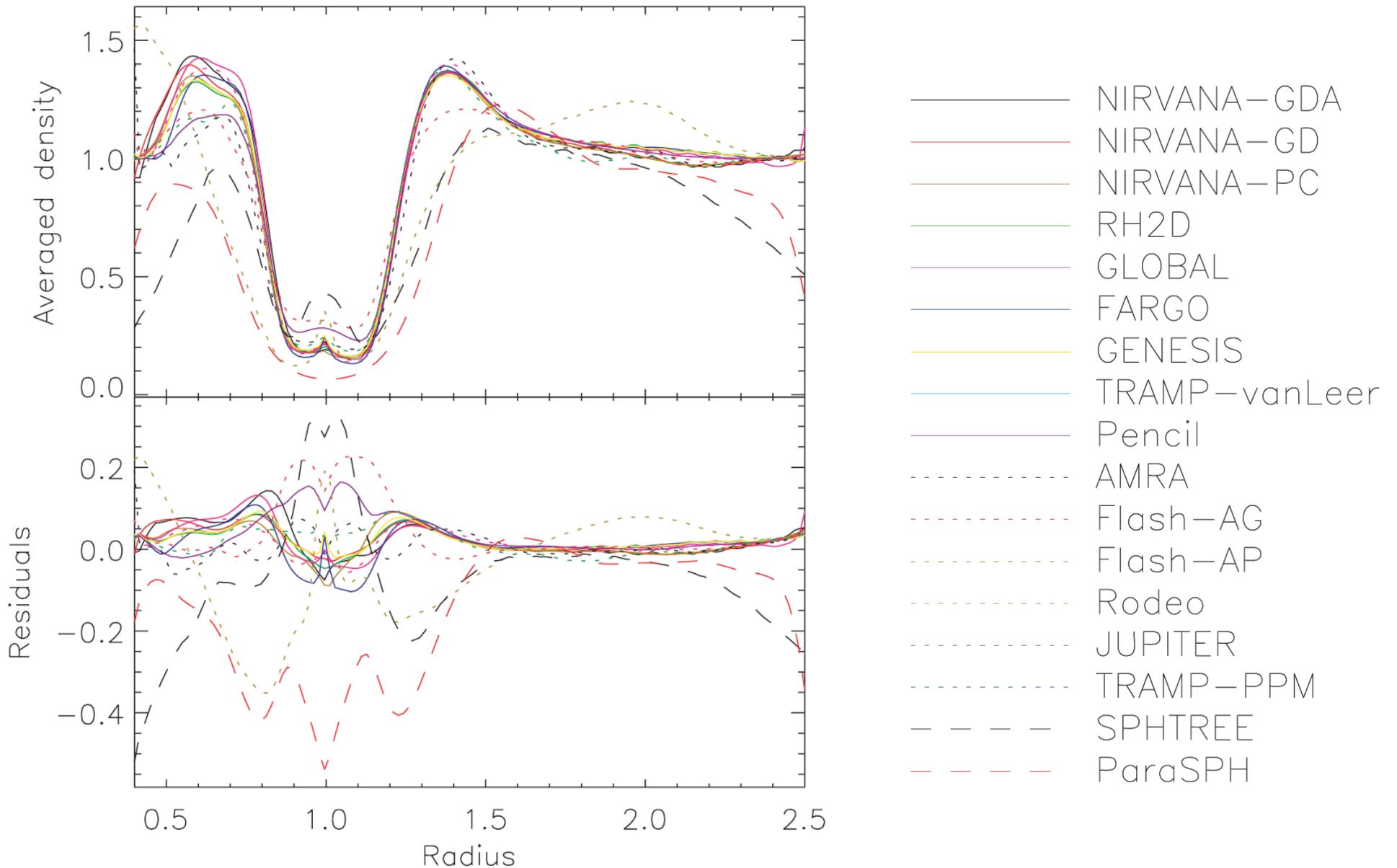
$$H/r = \text{const.} \implies T \propto 1/r \quad p = \Sigma T/\mu$$

Ω : rotation of coord. system = Ω_p Φ : planet potential, f_r, f_φ : viscosity



Viscous disc: $M_p = 1 M_{\text{Jup}}$, $a_p = 5.2 \text{ AU}$ ()





Types: solid - Upwind, **HighOrder**, short-dashed - Riemann, long-dashed SPH; (Cyl. vs. Cartesian)



Angular Momentum Equation $\omega =$ angular velocity in rotating frame

Conserved (as before)

$$\frac{\partial[\Sigma r^2(\omega + \Omega)]}{\partial t} + \nabla \cdot [\Sigma r^2(\omega + \Omega)\mathbf{u}] =$$

$$-\frac{\partial p}{\partial \varphi} - \Sigma \frac{\partial \Phi}{\partial \varphi} + f_\varphi$$

Not-Conserved (Explicit)

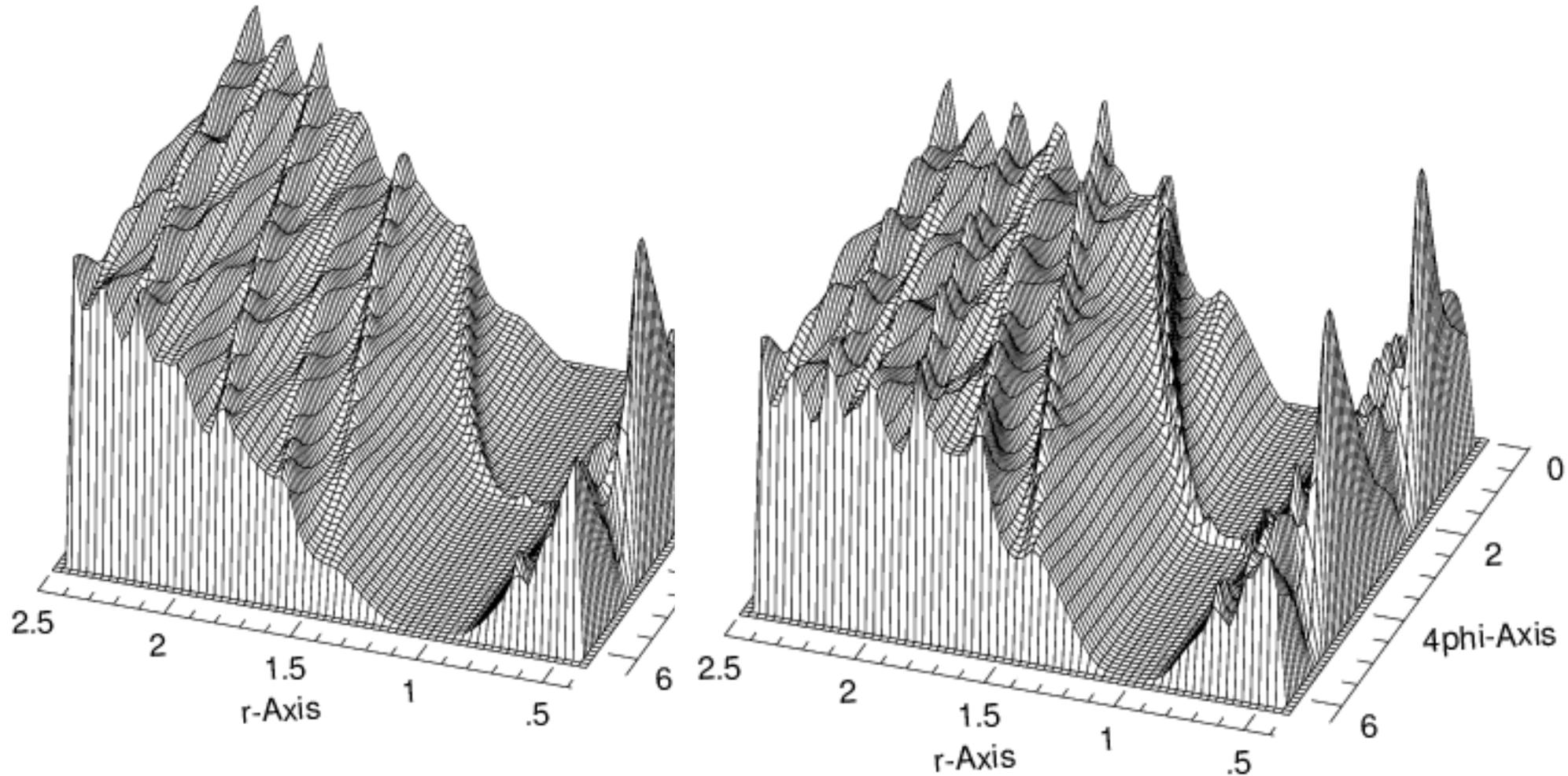
$$\frac{\partial[\Sigma r^2\omega]}{\partial t} + \nabla \cdot [\Sigma r^2\omega\mathbf{u}] = -2 \Sigma v\Omega r$$

$$-\frac{\partial p}{\partial \varphi} - \Sigma \frac{\partial \Phi}{\partial \varphi} + f_\varphi$$



Conservative

Non-Conservative



Conservative results: identical to inertial frame

⇒ If rotating frame: Use conservative formulation



A Fast eulerian transport Algorithm for differentially rotating disks

Description of the Fargo-Code: (Masset, A&A 2000) (<http://fargo.in2p3.fr/>)

The problem:

- Cylindrical coord. system
- explicit code
- Courant condition:

$$\Delta t < \frac{\Delta \varphi}{\omega}$$

with $\omega \propto r^{-3/2} \rightarrow \Delta t \propto r^{3/2}$

The solution:

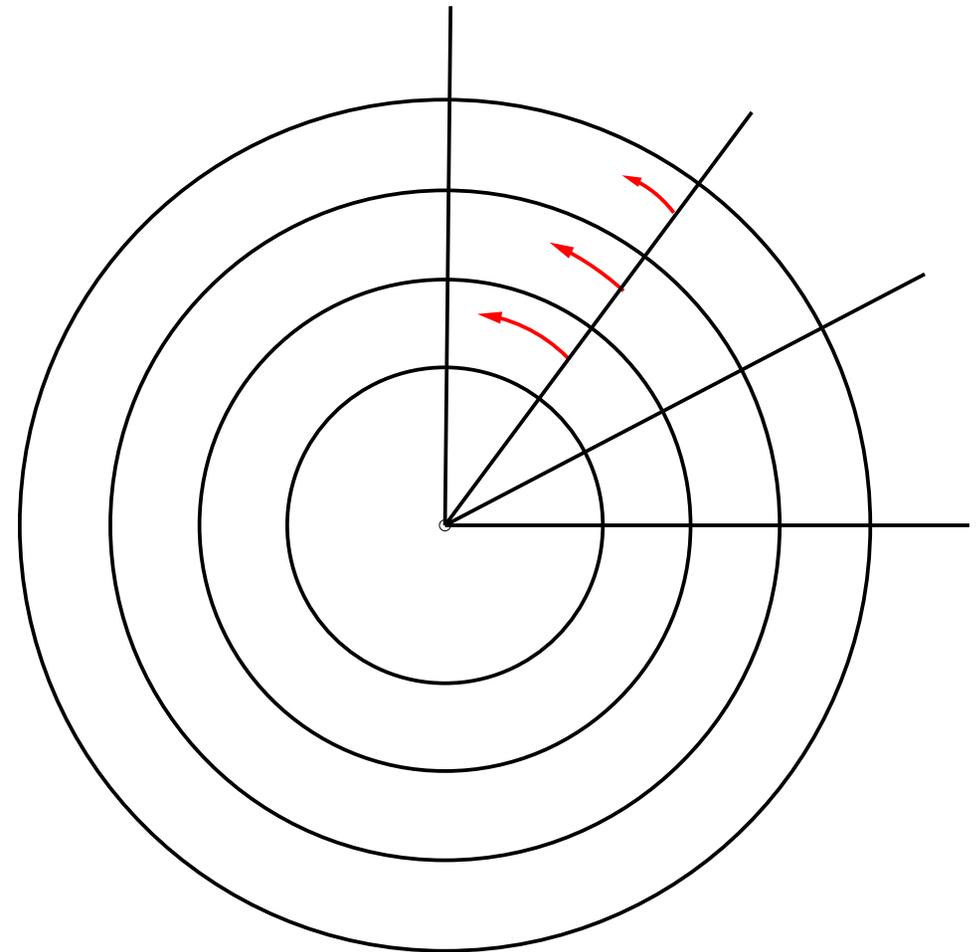
Calculate average velocity $\bar{\omega}_i$
of ring i , and perform **Shift** over
 $n_i = \text{Nint}(\bar{\omega}_i \Delta t / \Delta \varphi)$ gridcells

Transport on with remaining velocity

⇒ **much larger Δt**

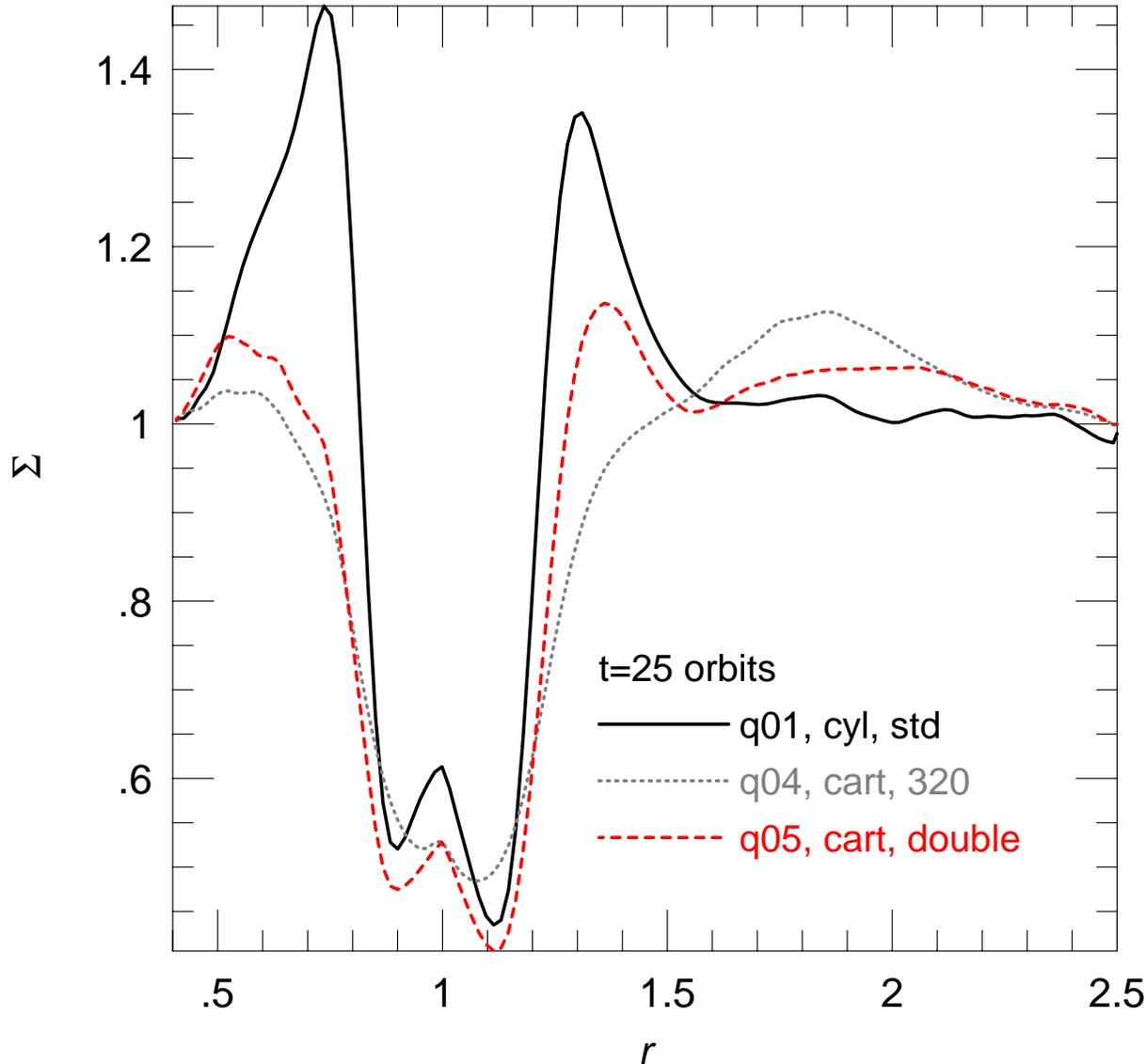
Take care: no ring separation

FARGO-method implemented in many codes: e.g. PLUTO





Azimuthally averaged surface density



Cartesian vs. Cylindrical

(inertial frame)

Jupiter mass planet
after 25 Orbits

Models

- Cyl.: 128×384
- Cart.: 320×320
- **Cart.: 640×640**

⇒ Cartesian problematic !

- no operator-splitting
- 2nd order RK-integrator

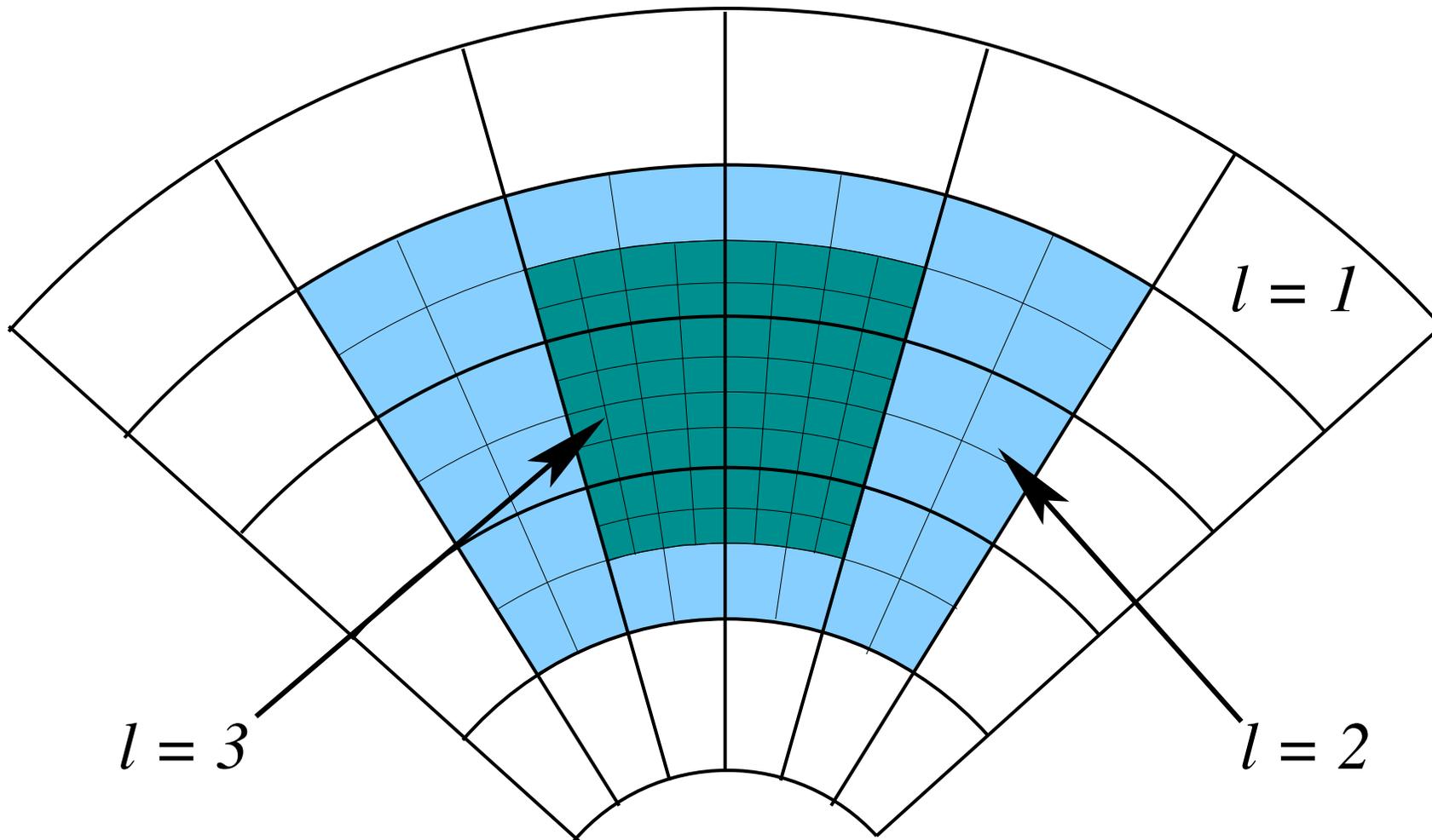
[only PENCIL code reasonable
(HiOrder)]

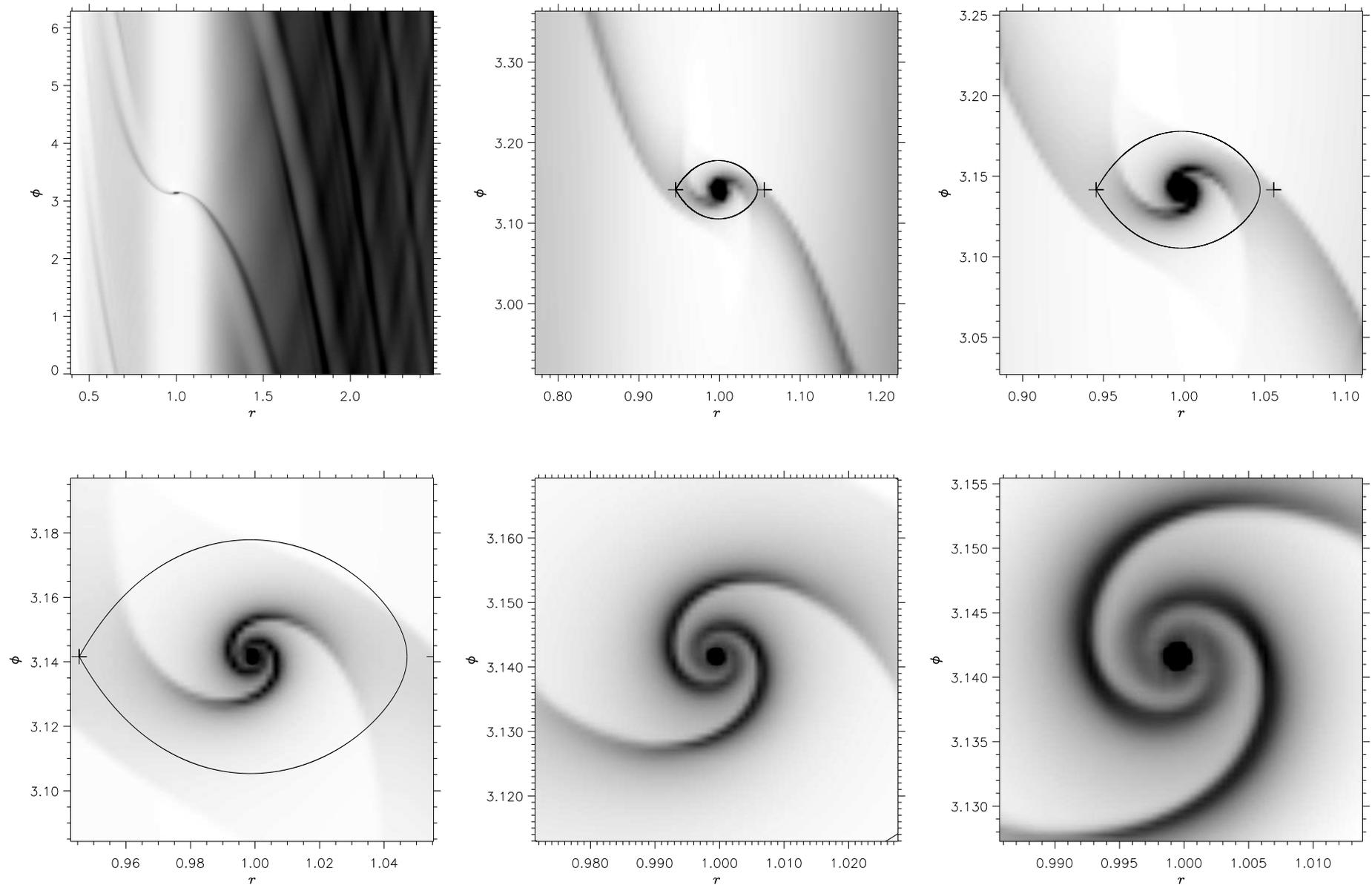


Nested-Grid System: Centered on Planet - corotating frame

Advantage: High resolution near planet, grid fixed in advance (no AMR)

(D'Angelo et al. 2002/03)



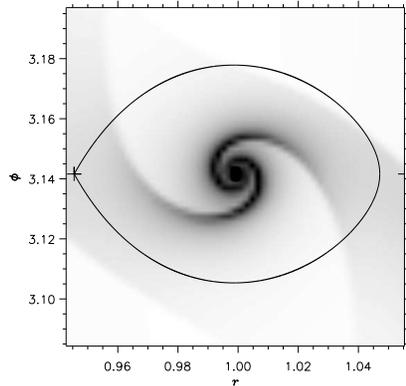


(D'Angelo et al., 2002, PhD Thesis Tübingen, 2004)



Sum over all grid-cells, $\Gamma_{tot} = \sum_{cells} (x_p F_y - y_p F_x)$

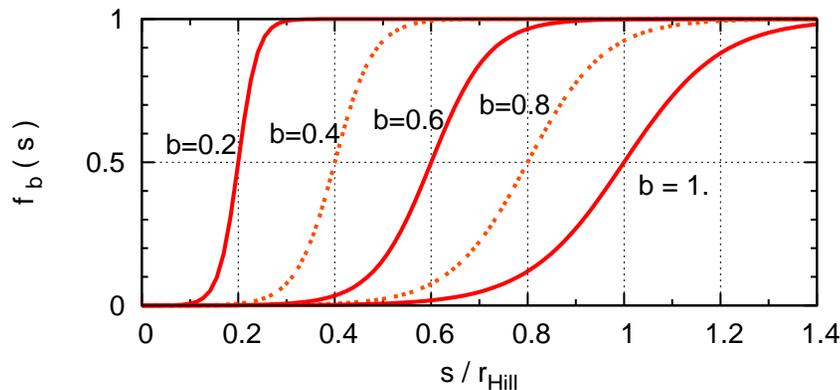
Which parts belong to planet ?



Use **Truncation Radius**

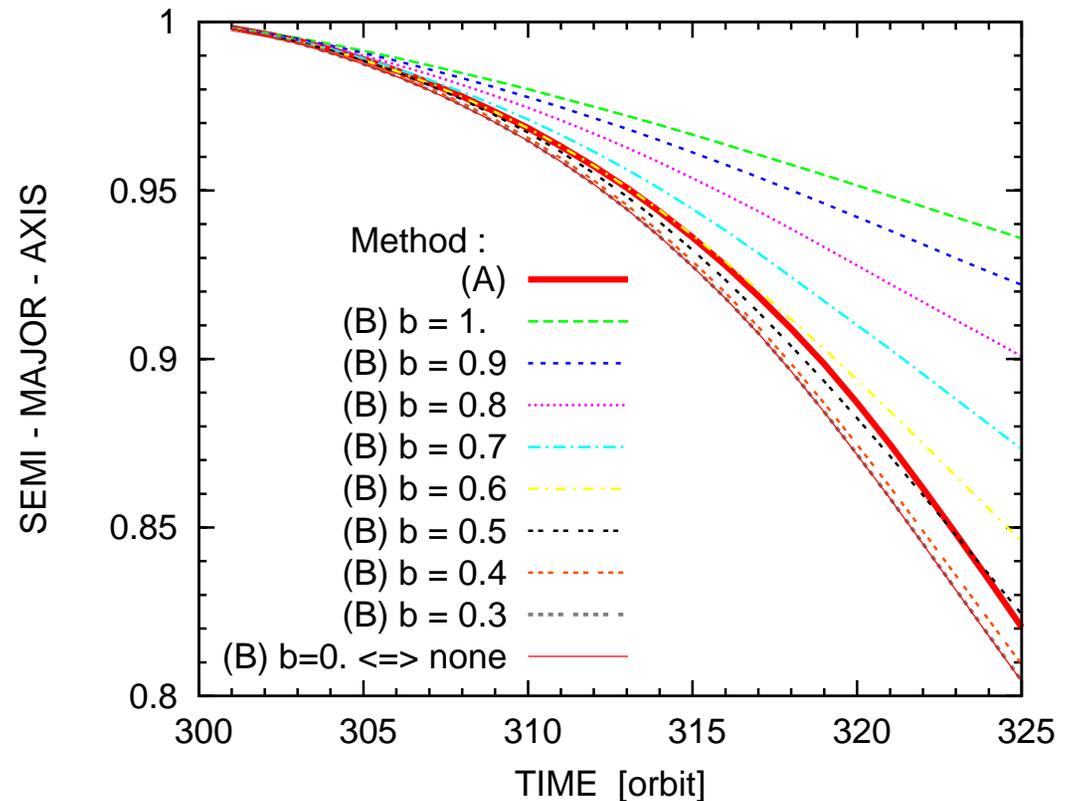
$$R_{trunc} \approx 0.5 - 1.0 R_{roche}$$

Smooth tapering-function f_b



Migration of Saturn mass planet

$M_p = 3.10^{-4} M_*$; $N_r = 460$; $N_s = 1572$



(A. Crida)

– (A) fully self-gravitating
 \implies need $b \approx 0.5 - 0.6$



Torque **on** the planet:

$$\Gamma_{\text{tot}} = - \int_{\text{disk}} \Sigma(\vec{r}_p \times \vec{F}) df = \int_{\text{disk}} \Sigma(\vec{r}_p \times \nabla \psi_p) df = \int_{\text{disk}} \Sigma \frac{\partial \psi_p}{\partial \varphi} df \quad (1)$$

3D analytical (Tanaka et al. 2002) and numerical (D'Angelo & Lubow, 2010)

$$\Gamma_{\text{tot}} = -(1.36 + 0.62\beta_{\Sigma} + 0.43\beta_T) \Gamma_0. \quad (2)$$

where

$$\Sigma(r) = \Sigma_0 r^{-\beta_{\Sigma}} \quad \text{and} \quad T(r) = T_0 r^{-\beta_T} \quad (3)$$

and normalisation

$$\Gamma_0 = \left(\frac{m_p}{M_*} \right)^2 \left(\frac{H}{r_p} \right)^{-2} (\Sigma_p r_p^2) r_p^2 \Omega_p^2, \quad (4)$$

$$\Rightarrow \text{Migration} \quad \dot{J}_p = \Gamma_{\text{tot}} \quad \Rightarrow \quad \frac{\dot{a}_p}{a_p} = 2 \frac{\Gamma_{\text{tot}}}{J_p} \quad (5)$$

Low Planet Mass (Type I): Typically Negative & fast



Parameter	Symbol	Value
mass ratio	$q = M_p/M_*$	6×10^{-6}
aspect ratio	$h = H/r$	0.05
nonlinearity parameter	$\mathcal{M} = q^{1/3}/h$	0.36
kinematic viscosity	ν	inviscid
potential smoothing	ϵ_p	$0.1H$
radial range	$r_{\min} - r_{\max}$	0.6 – 1.4
angular range	$\phi_{\min} - \phi_{\max}$	$0 - 2\pi$
number of grid-cells	$N_r \times N_\phi$	256×2004
spatial resolution	Δr	$H/16$
damping range at r_{\min}		0.6 – 0.7
damping range at r_{\max}		1.3 – 1.4

2D, isothermal, inviscid, high resolution, low mass planets. Parameter for a study to test the accuracy and stability of the FARGO-algorithm.

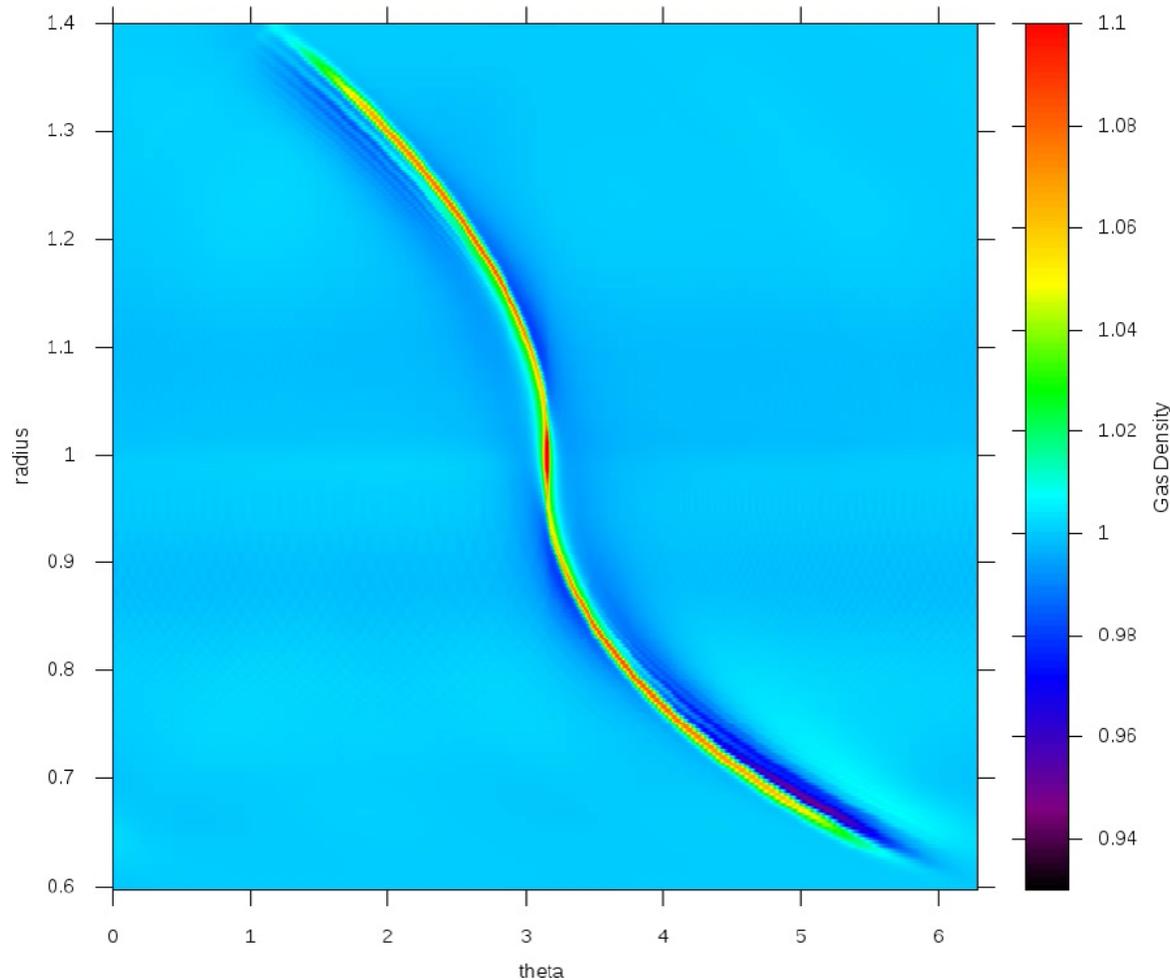
Analyze, torque distribution, wake profile, gravitational smoothing

(Kley ea. 2012)



2D Simulation, FARGO, $q = 6 \times 10^{-6}$, $h = 0.05$, $\Sigma_0 = \text{const.}$

FARGO file: out1/gasdens286.dat



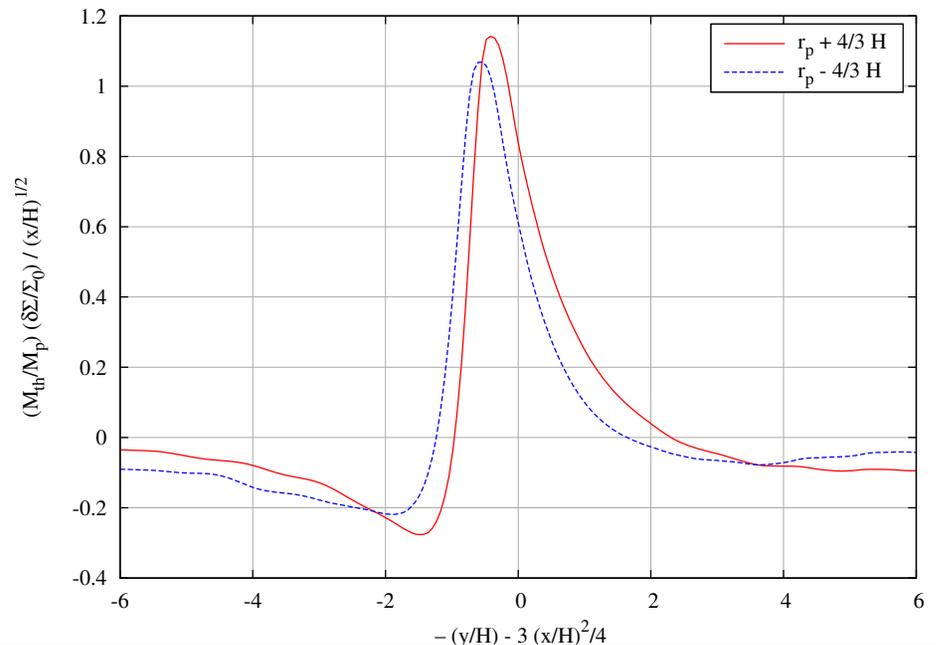
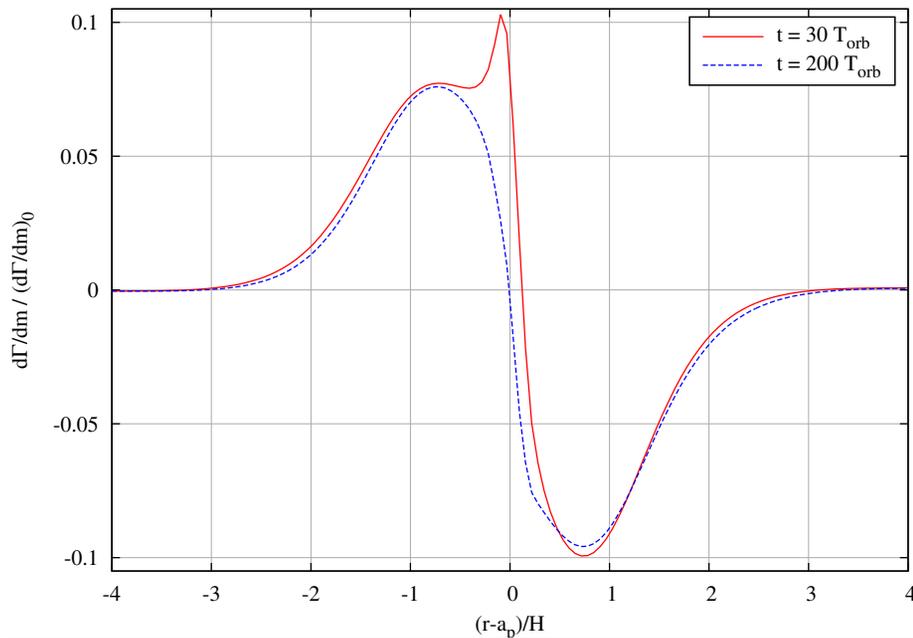
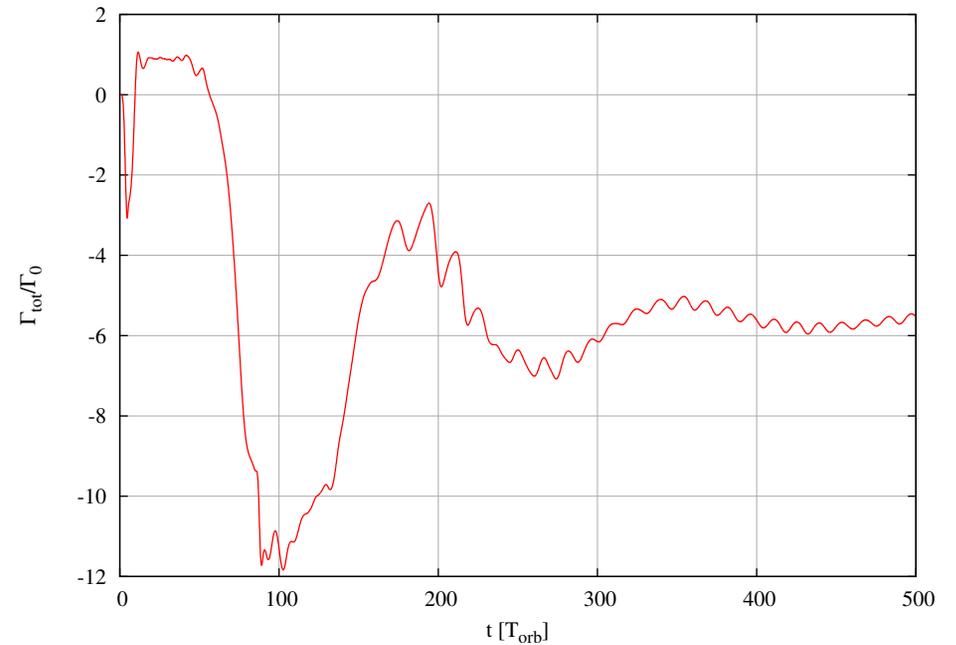
(Tobias Müller, Tübingen)

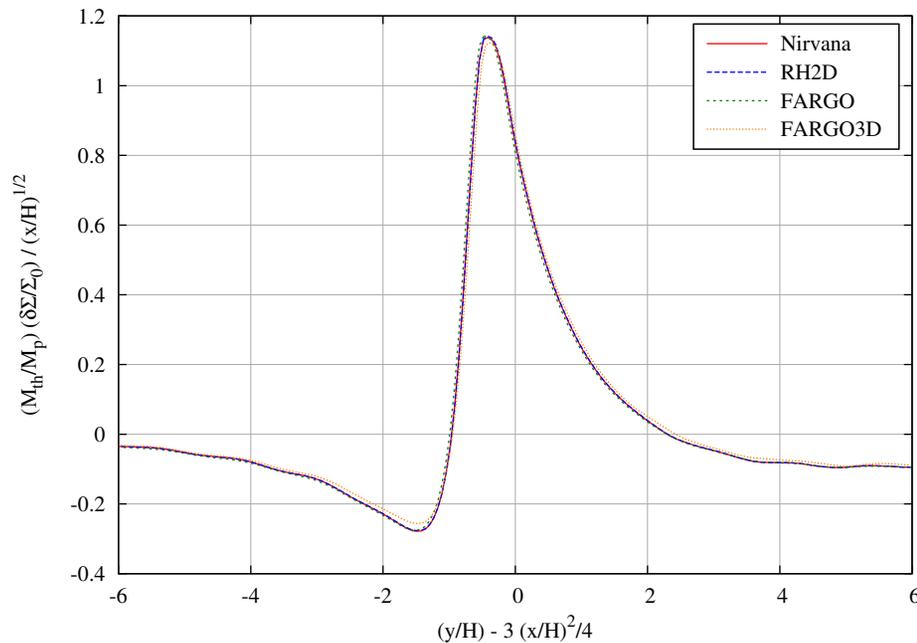
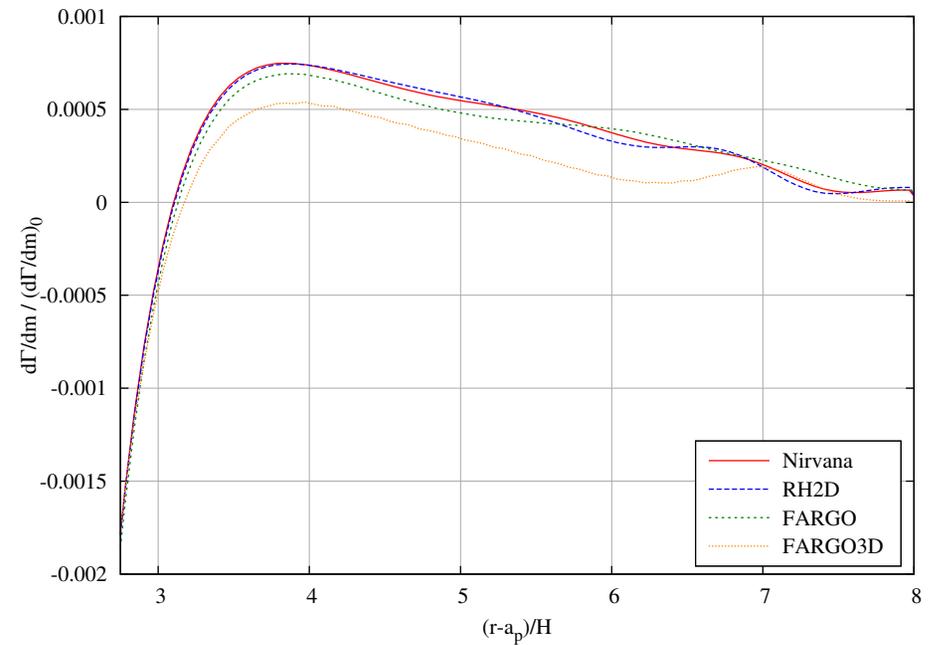
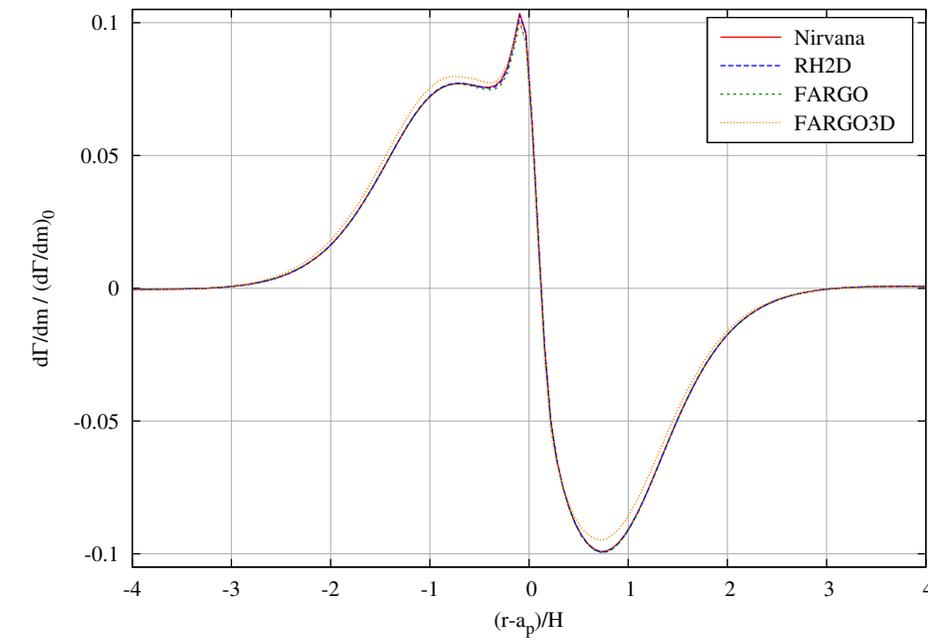


$$\Gamma_{\text{tot}} = 2\pi \int \frac{d\Gamma}{dm}(r) \Sigma(r) r dr$$

$$\left(\frac{d\Gamma}{dm}\right)_0 = \Omega_p^2(a_p) a_p^2 q^2 \left(\frac{H}{a_p}\right)^{-4}$$

$$\Gamma_0 = \Sigma_0 \Omega_p^2(a_p) a_p^4 q^2 \left(\frac{H}{a_p}\right)^{-2}$$





- all codes agree very well
 - on torque density and wake form
- all see the torque reversal at $r - r_p \approx 3.2H$

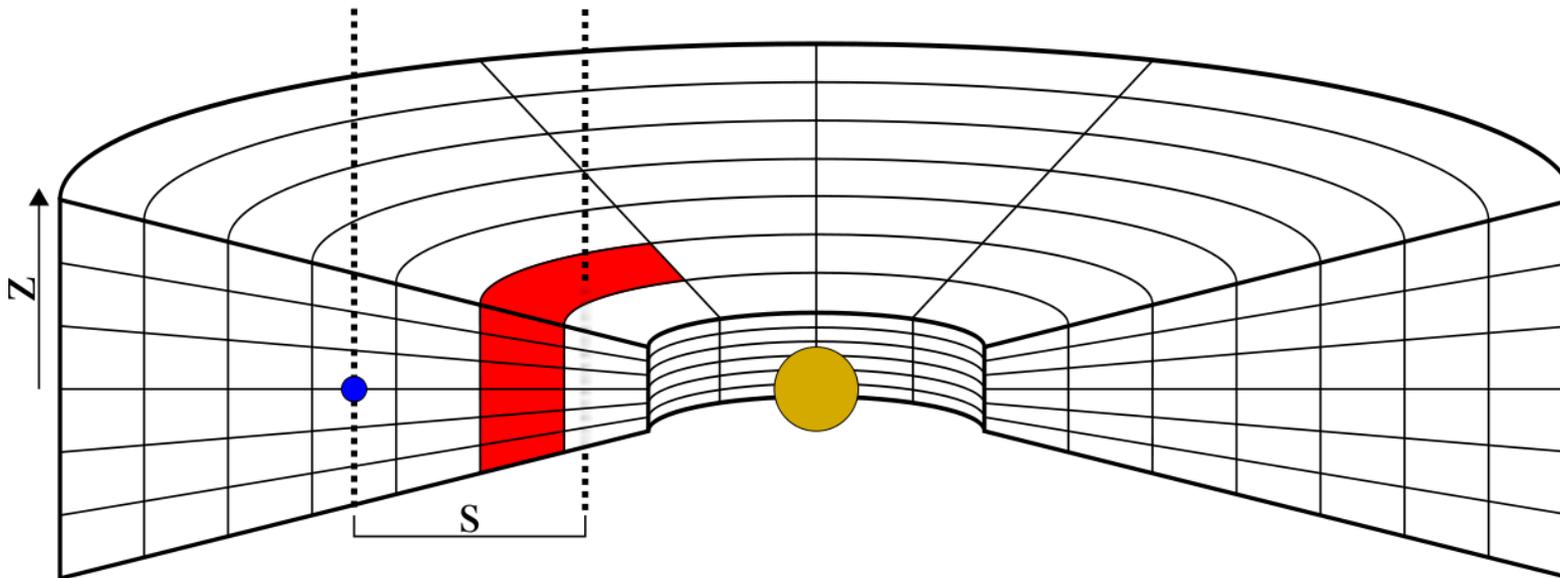


In 2D simulations: Epsilon-potential (s is distance from planet to gridpoint)

$$\Phi_p^{2D} = -GM_p \frac{1}{(s^2 + \epsilon^2)^{1/2}}$$

But: 2D eqns. are obtained from 3D by vertical averaging: \Rightarrow force density (force/area)

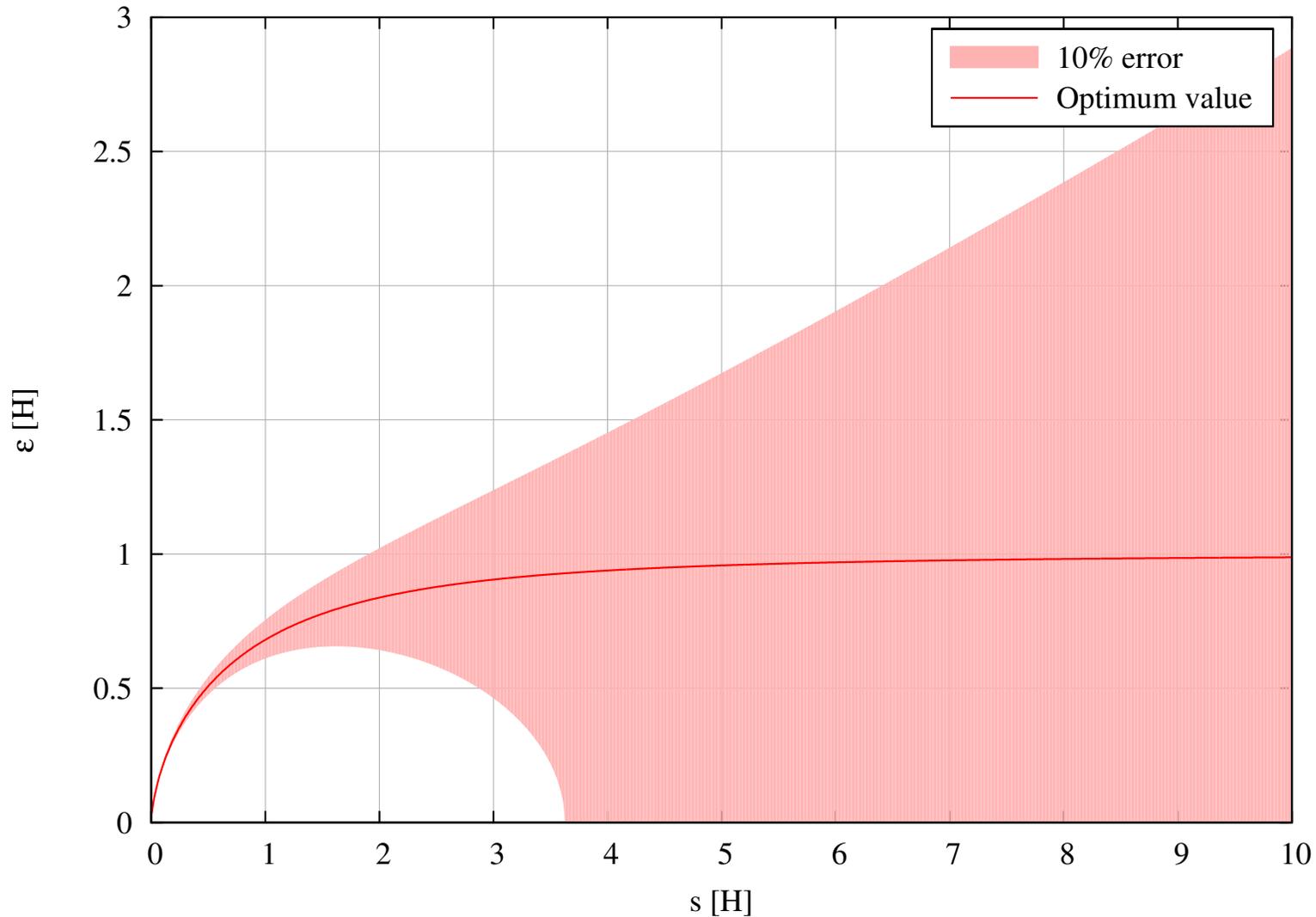
$$F_p(s) = - \int \rho \frac{\partial \Phi_p}{\partial s} dz = -GM_p s \int \frac{\rho}{(s^2 + z^2)^{3/2}} dz$$



NOTE: In general $\nabla \Phi_p^{2D} \neq \frac{\int \rho \nabla \Phi dz}{\Sigma}$ (specific force)

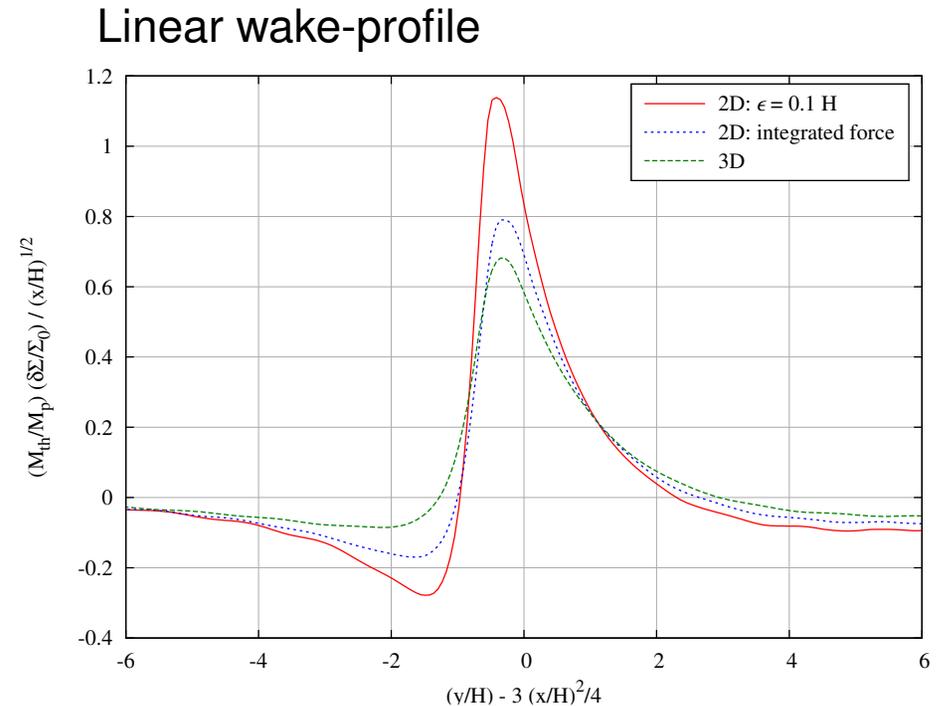
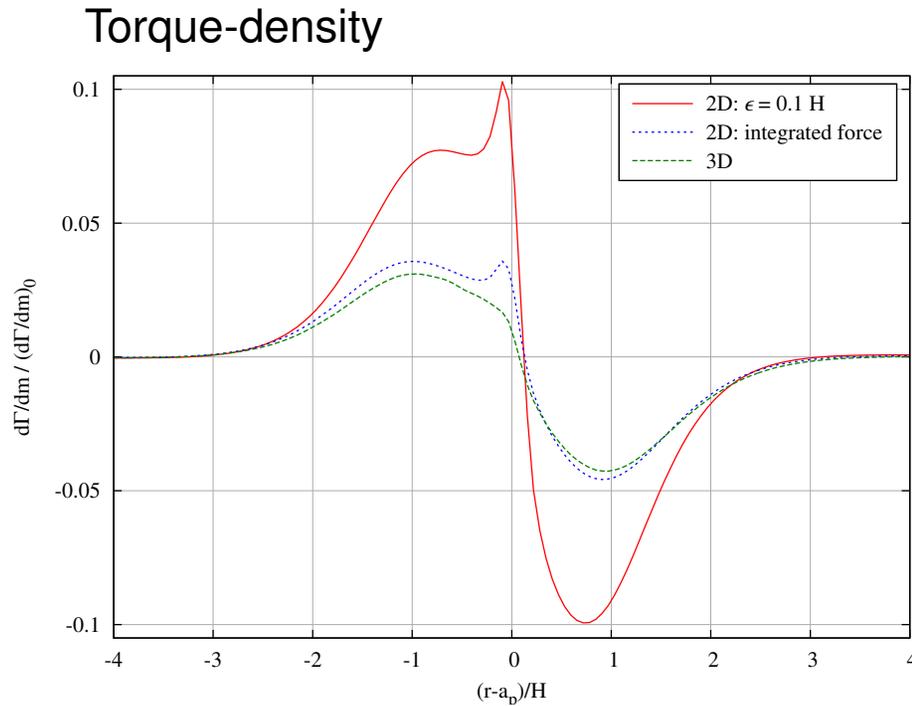


What ϵ makes $\nabla \Phi_p^{2D} = \frac{\int \rho \nabla \Phi dz}{\Sigma}$

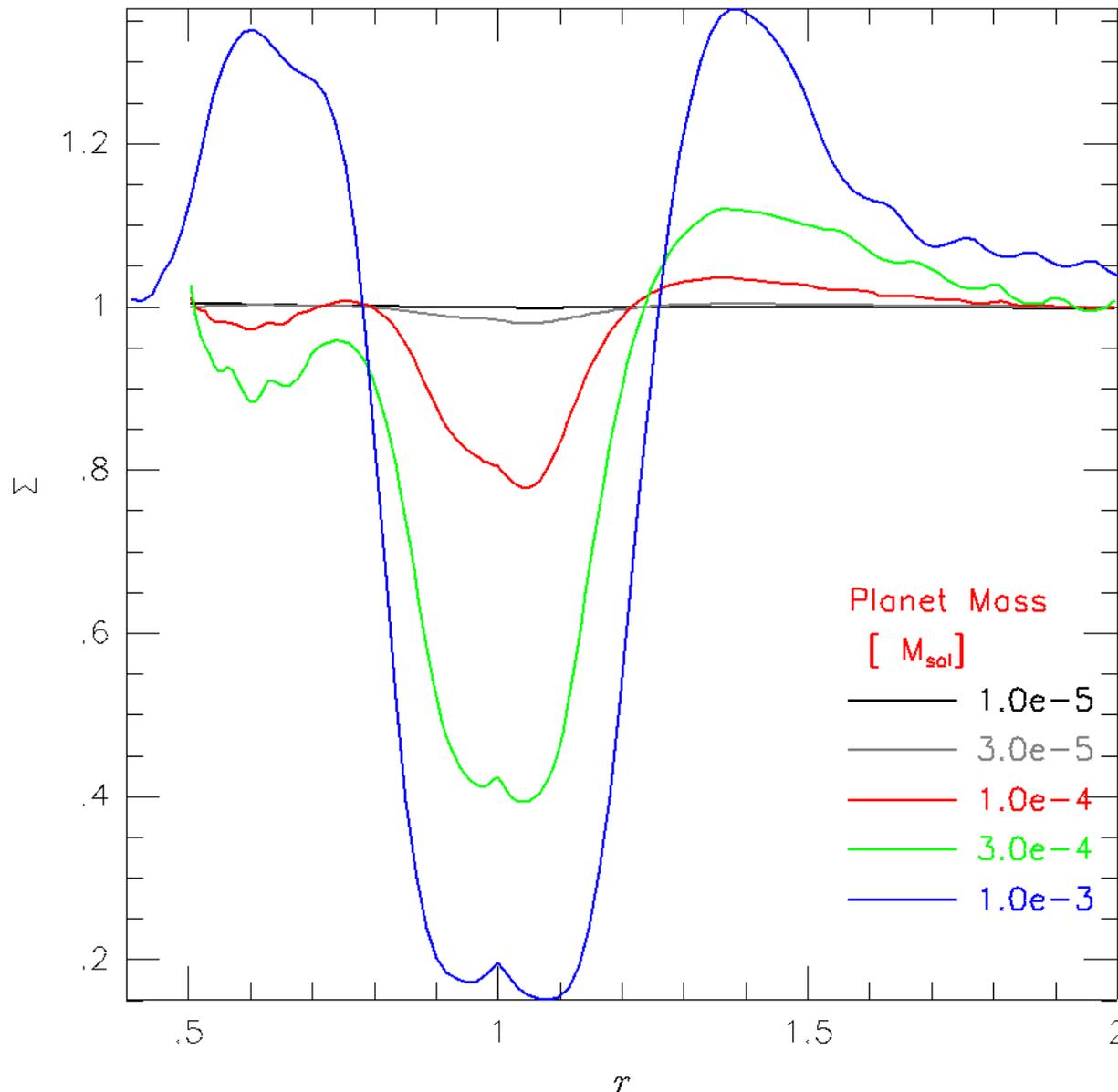


Müller ea 2012

- optimum ϵ distance dependend,
- $\epsilon \Rightarrow 1$ for $s \Rightarrow \infty$



- $\epsilon = 0.1$ leads to large overestimate of torques
- $\epsilon = 0.6 - 0.7$ agrees with 3D results
- good agreement vertical integrated force with 3D results



$$M_p = 0.01 M_{\text{Jup}}$$

$$M_p = 0.03 M_{\text{Jup}}$$

$$M_p = 0.1 M_{\text{Jup}}$$

$$M_p = 0.3 M_{\text{Jup}}$$

$$M_p = 1.0 M_{\text{Jup}}$$

Depth depends on:

- M_p
- Viscosity
- Temperature

Torques reduced:

Migration slows

Type I \Rightarrow Type II

linear \Rightarrow non-linear

Planet moves with
(viscous) disk



2D hydro-simulations:

- small mass
- inviscid

Note:

for barotropic and inviscid flows:

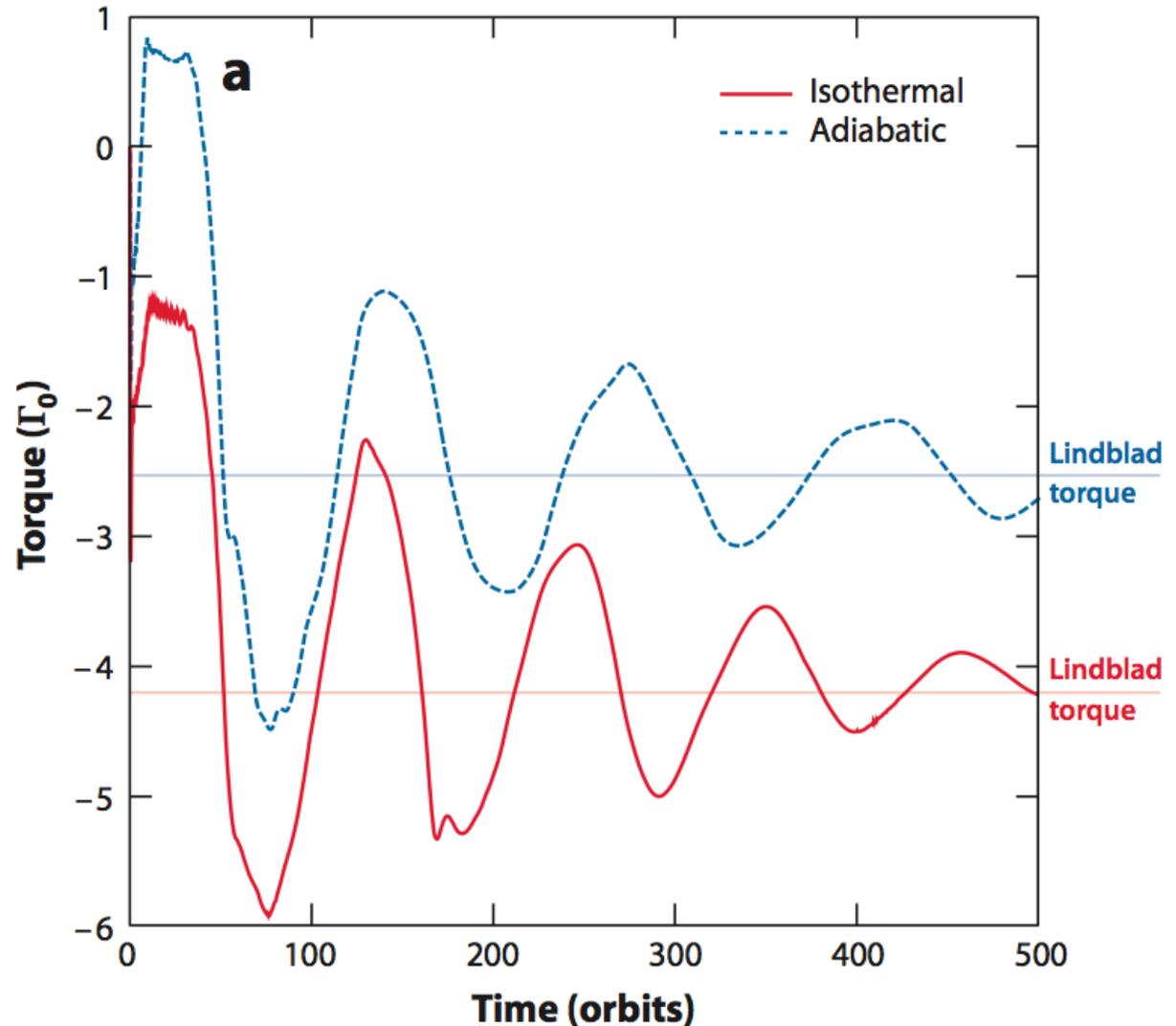
$$\frac{d}{dt} \left(\frac{\omega_z}{\Sigma} \right) = 0$$

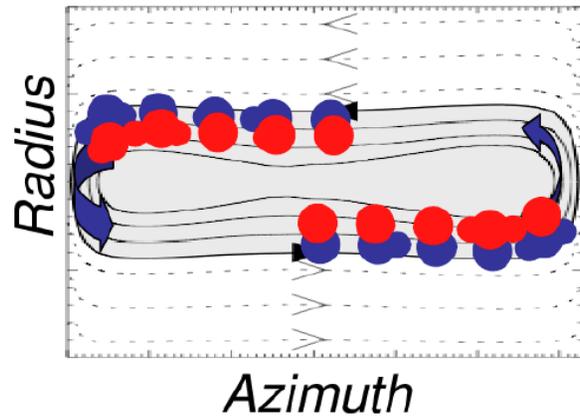
$$\frac{dS}{dt} = 0$$

Torque depends on:
 Gradients of ω_z/Σ , S
 ω_z Vorticity
 S Entropy

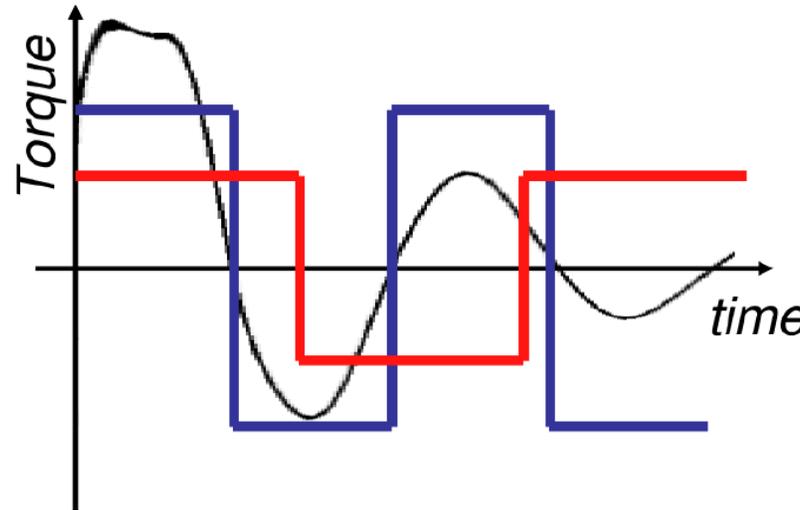
Gradients are wiped out in the corotation region

Total torque vs. time - isothermal - adiabatic



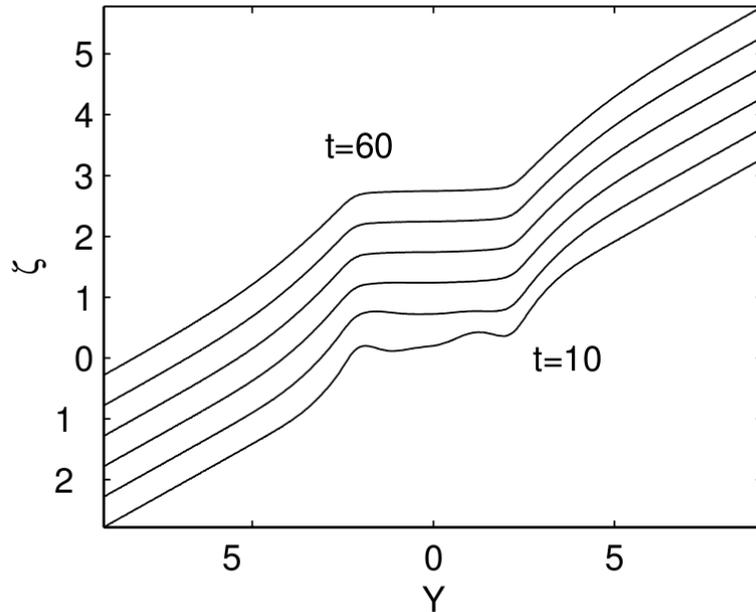


(F. Masset)

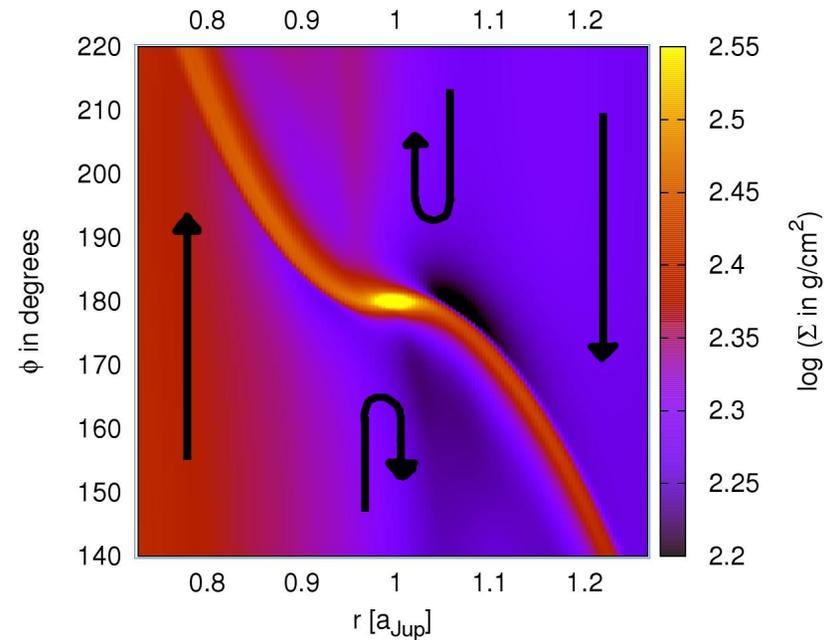


Red and Blue
Orbits have
different periods
⇒ Phase mixing

(a) Mean vorticity



(Balmforth & Korycanski)

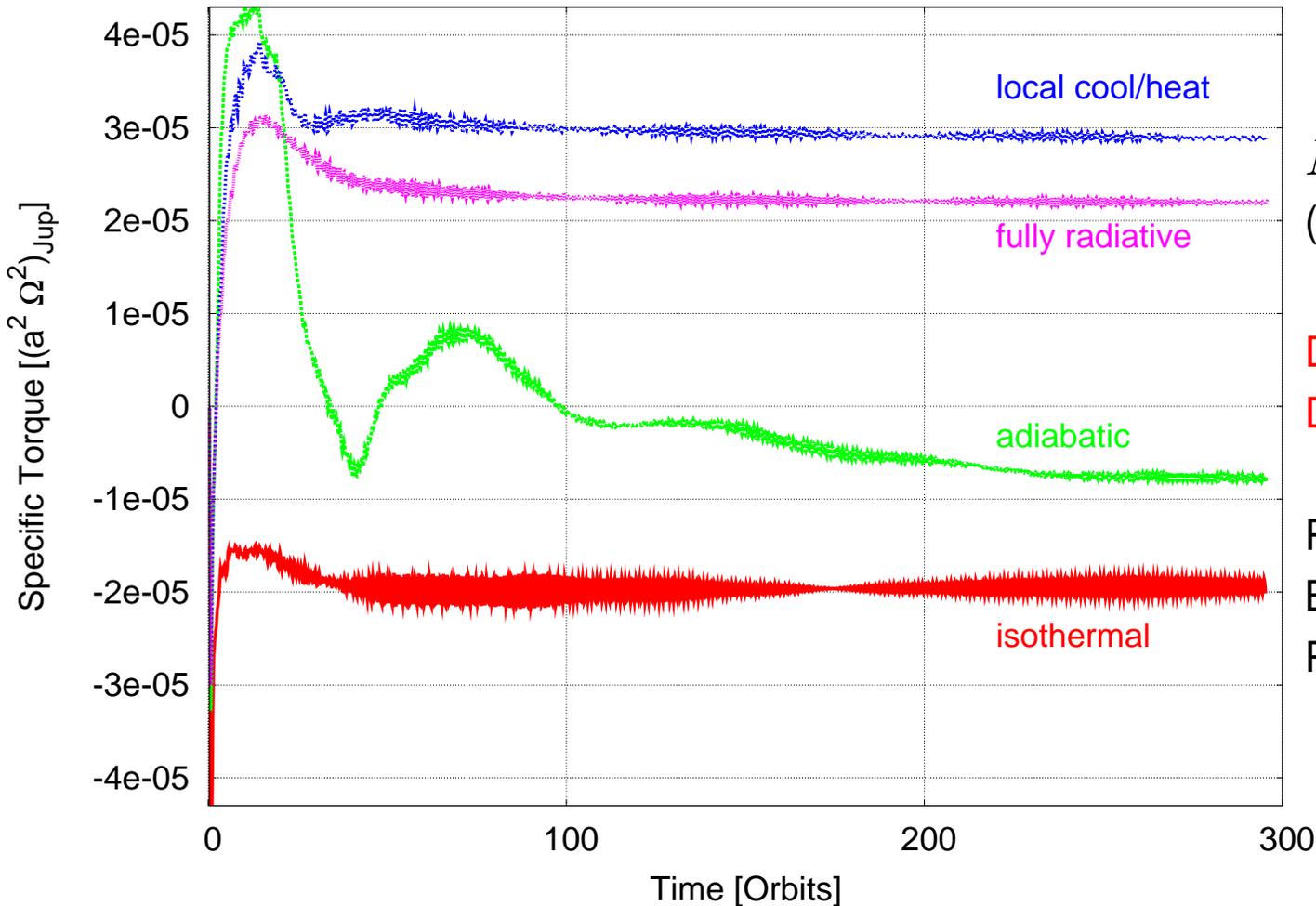


3D radiative disk (B. Bitsch)



$$\frac{\partial \Sigma c_v T}{\partial t} + \nabla \cdot (\Sigma c_v T \mathbf{u}) = -p \nabla \cdot \mathbf{u} + D - Q - 2H \nabla \cdot \vec{F}$$

Adiabatic: Pressure Work, Viscous heating (D), radiative cooling (Q) ($\propto T_{eff}^4$)
 radiative Diffusion: in disk plane (realistic opacities)



$M_p = 20 M_{earth}$
 (Kley&Crida '08)

Disk Physics determines
 Direction of motion
 see also:

- Paardekooper & Mellema '06
- Baruteau & Masset '08
- Paardekooper & Papaloizou '08



Total torque
vs. viscosity:

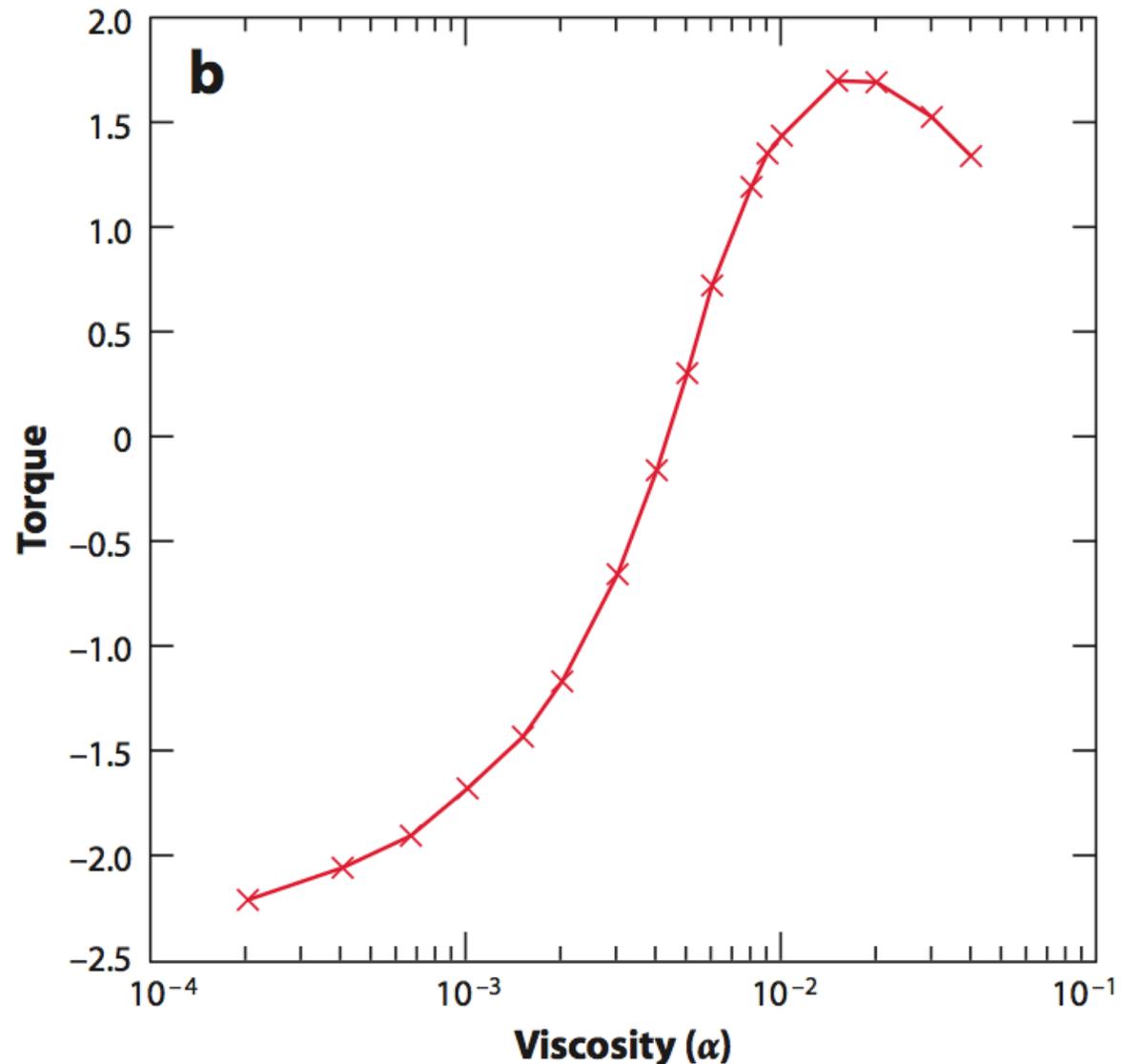
in **viscous** α -disk
2D **radiative** model
- in equilibrium

Efficiency depends
of ratio of timescales

$$\frac{\tau_{\text{visc}}}{\tau_{\text{librat}}}$$

$$\frac{\tau_{\text{rad}}}{\tau_{\text{librat}}}$$

Local disk properties
matter

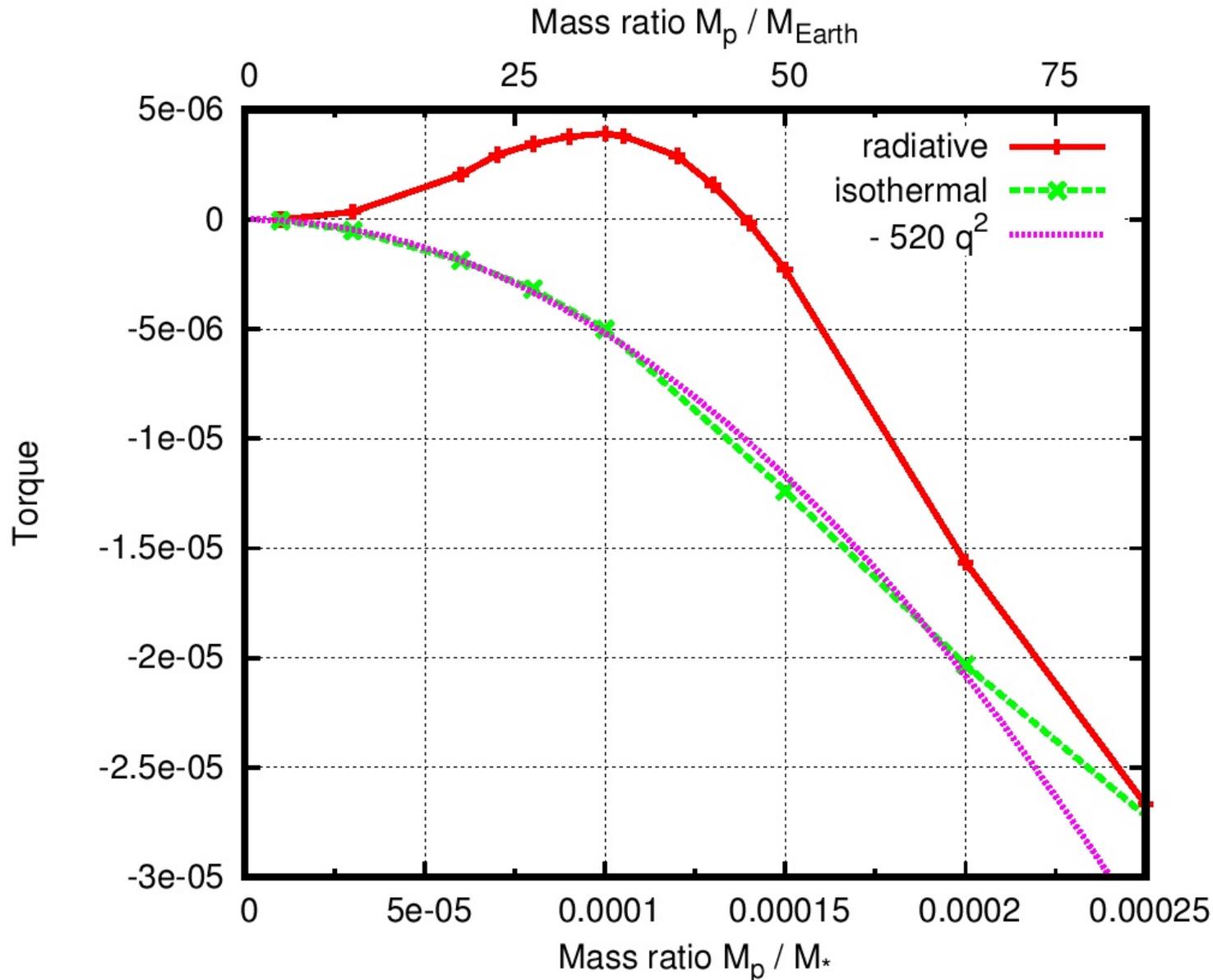


(Kley & Nelson 2012)

\Rightarrow Need viscosity to prevent saturation !



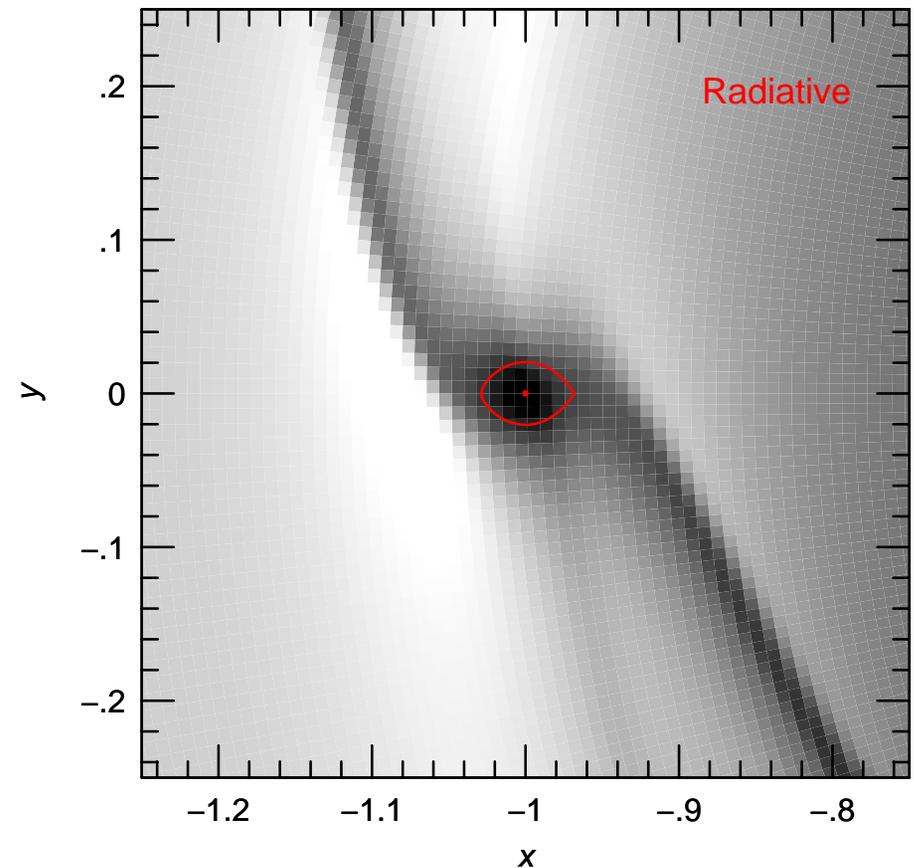
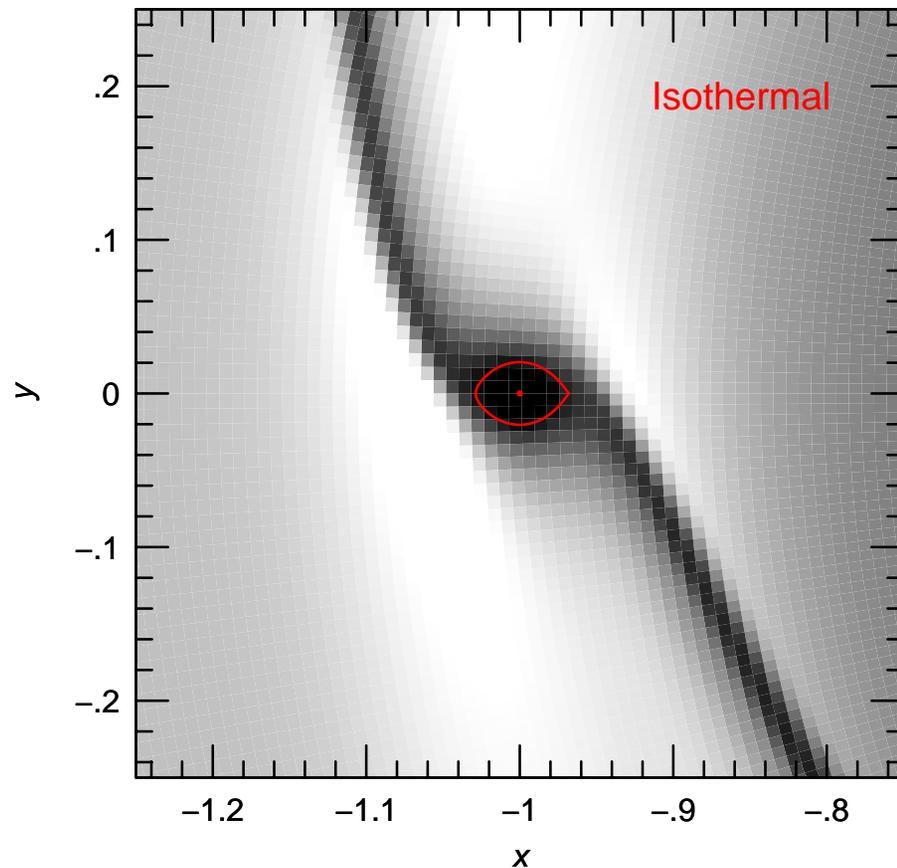
Isothermal and radiative models. Outward migration for $M_p \leq 40 M_{\text{Earth}}$



Vary M_p
(Kley & Crida 2008)

In full 3D
(Kley et al. 2009)

With irradiation:
Bitsch et al. (2012)



- wider spirals in radiative case (reduce torque)
- density maximum 'ahead' of planet (pulls planet forward)

⇒ possibly positive torques



Total torque
vs. viscosity:

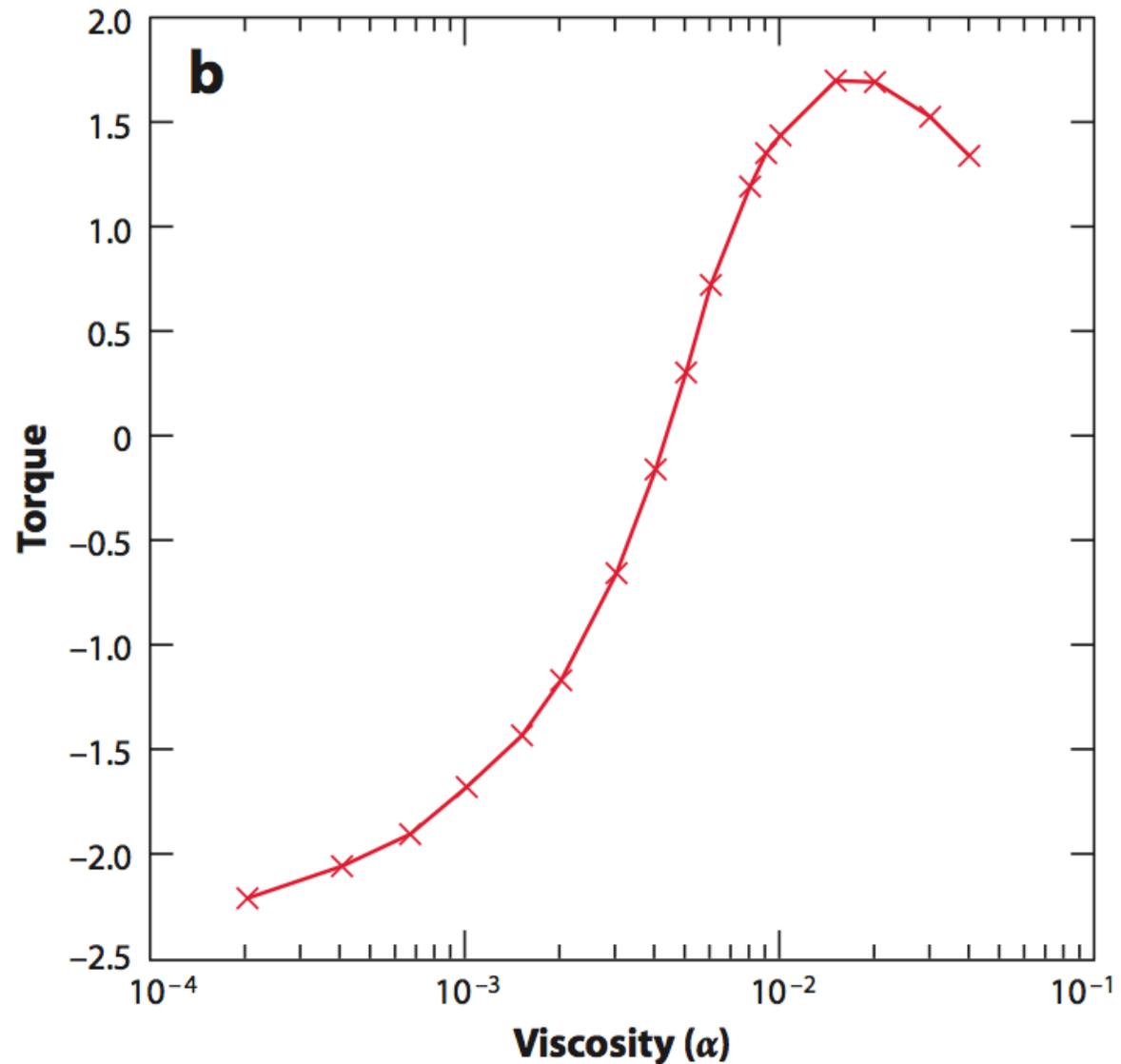
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Local disk properties
matter



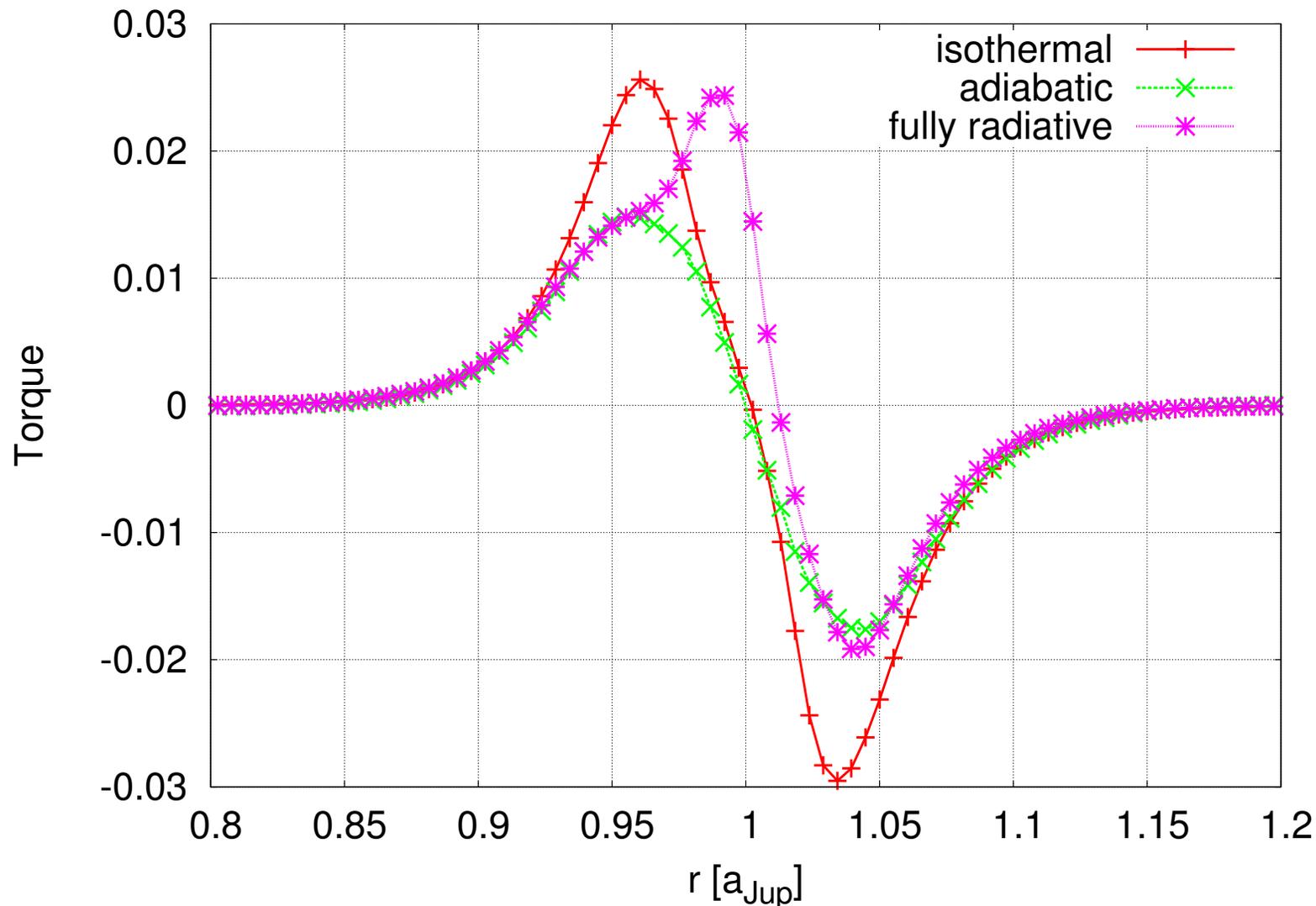
(Kley & Nelson 2012)

\Rightarrow Need viscosity to prevent saturation !



3D-simulations, radiative diffusion, 20 M_{Earth} planet (Kley, Bitsch & Klahr 2009)

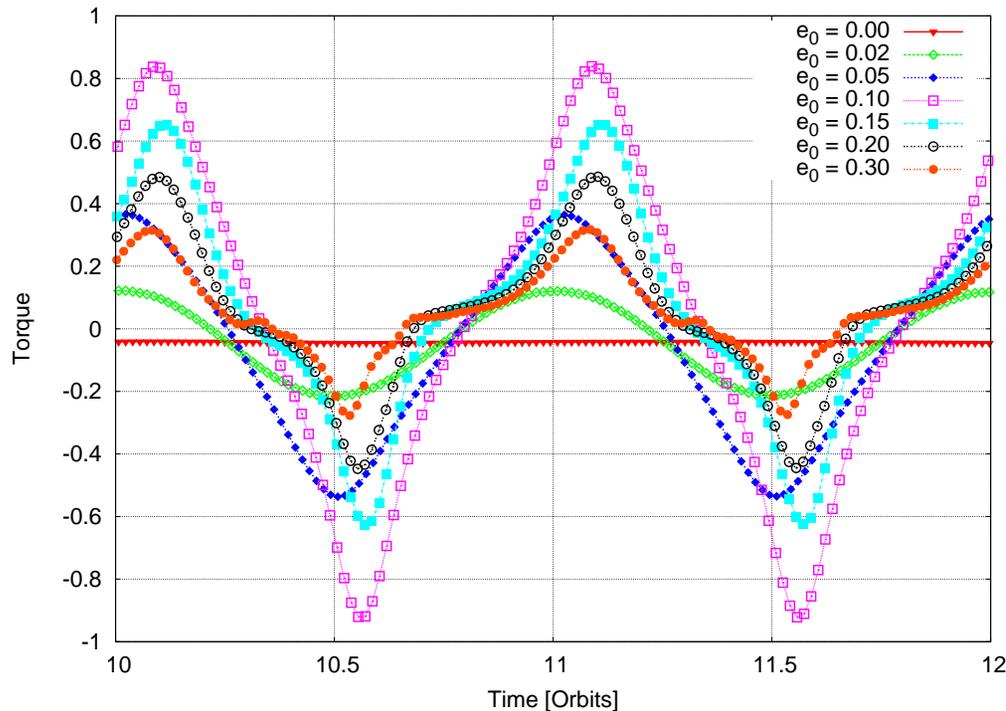
$d\Gamma/dm$, with $\Gamma_{\text{tot}} = 2\pi \int (d\Gamma/dm) \Sigma dr$ Radiative: \Rightarrow additional positive contrib.





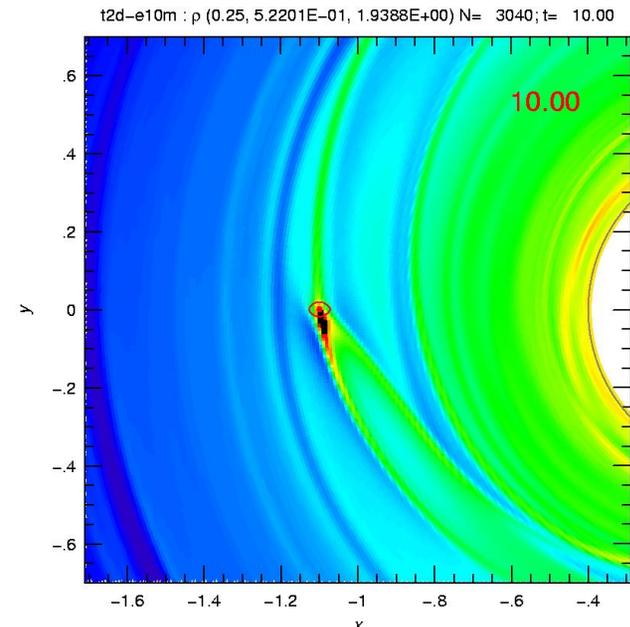
Torque on planet due to disk

$$T_{\text{disk}} = - \int_{\text{disk}} (\vec{r} \times \vec{F}) \Big|_z df$$



Power: Energy loss of planet

$$P_{\text{disk}} = \int_{\text{disk}} \dot{\vec{r}}_p \cdot \vec{F} df$$



$$L_p = m_p \sqrt{GM_* a} \sqrt{1 - e^2}$$

$$\frac{\dot{L}_p}{L_p} = \frac{1}{2} \frac{\dot{a}}{a} - \frac{e^2}{1 - e^2} \frac{\dot{e}}{e} = \frac{T_{\text{disk}}}{L_p}$$

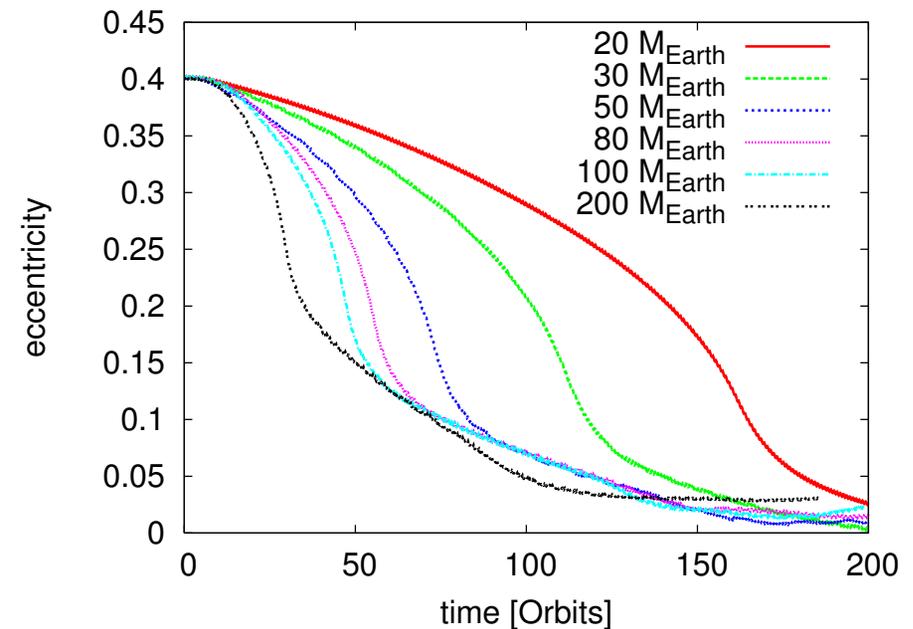
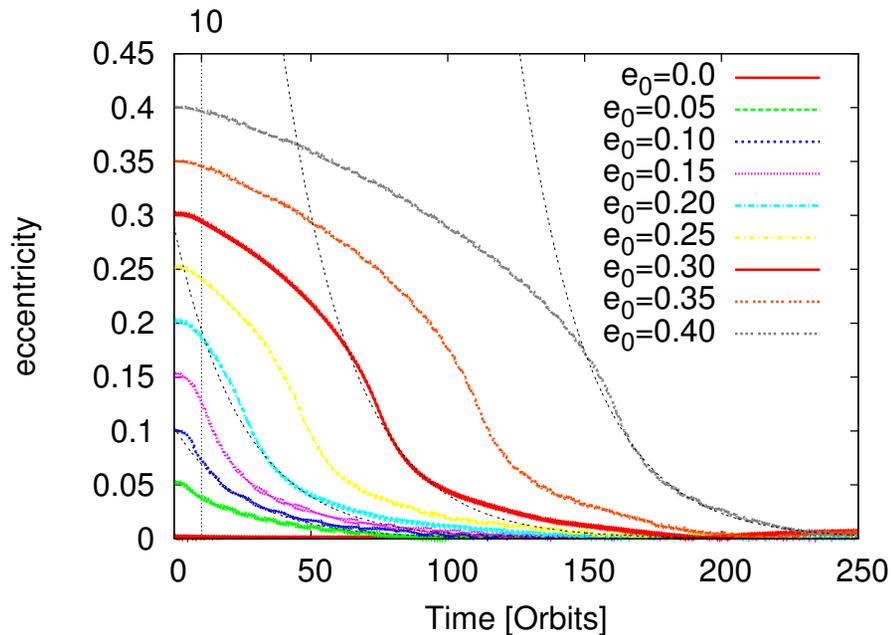
$$E_p = - \frac{1}{2} \frac{GM_* m_p}{a}$$

$$\frac{\dot{E}_p}{E_p} = \frac{\dot{a}}{a} = \frac{P_{\text{disk}}}{E_p}$$



Planet mass $M_p = 20M_{\text{Earth}}$
- Vary Eccentricity

Vary Planet Mass $10 - 200M_{\text{Earth}}$
- Fixed $e_0 = 0.40$

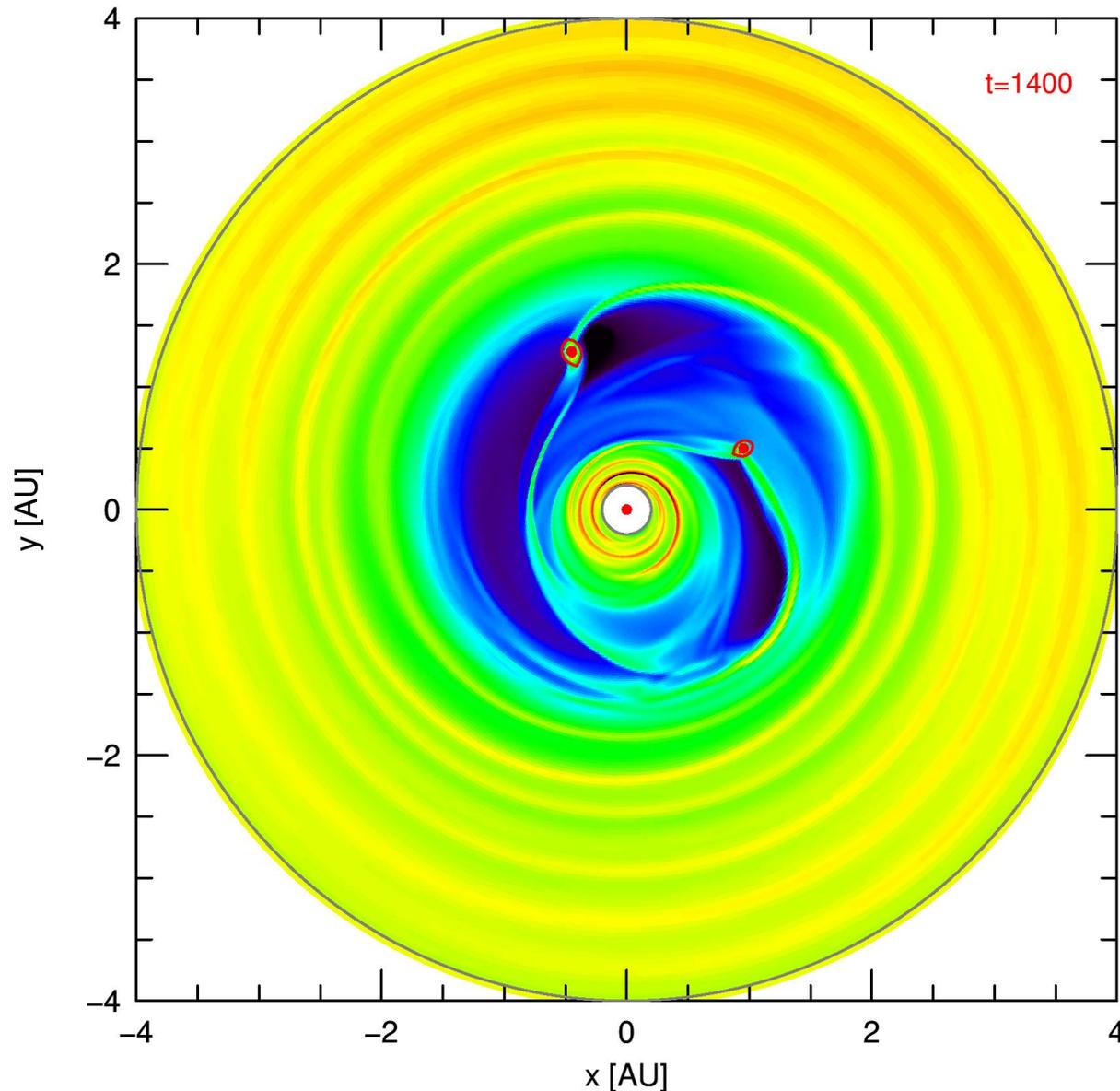


(Bitsch & Kley 2010)

- **e -damping for all planet masses.**
Small e : exponential damping, **large e : $\dot{e} \propto e^{-2}$**
- Need $e < 0.01 - 0.02$ for outward migration to work (radiative disks)
 \implies Need multiple objects ! (Scattering)



2 massive planets in disk

(HD 73526 $\approx 2.5 M_{\text{Jup}}$ each)

Two planets:
joint, large gap

Outer planet :
Pushed inward
by outer disk

Inner planet :
Pushed outward
by inner disk

Separation reduction:
Resonant capture

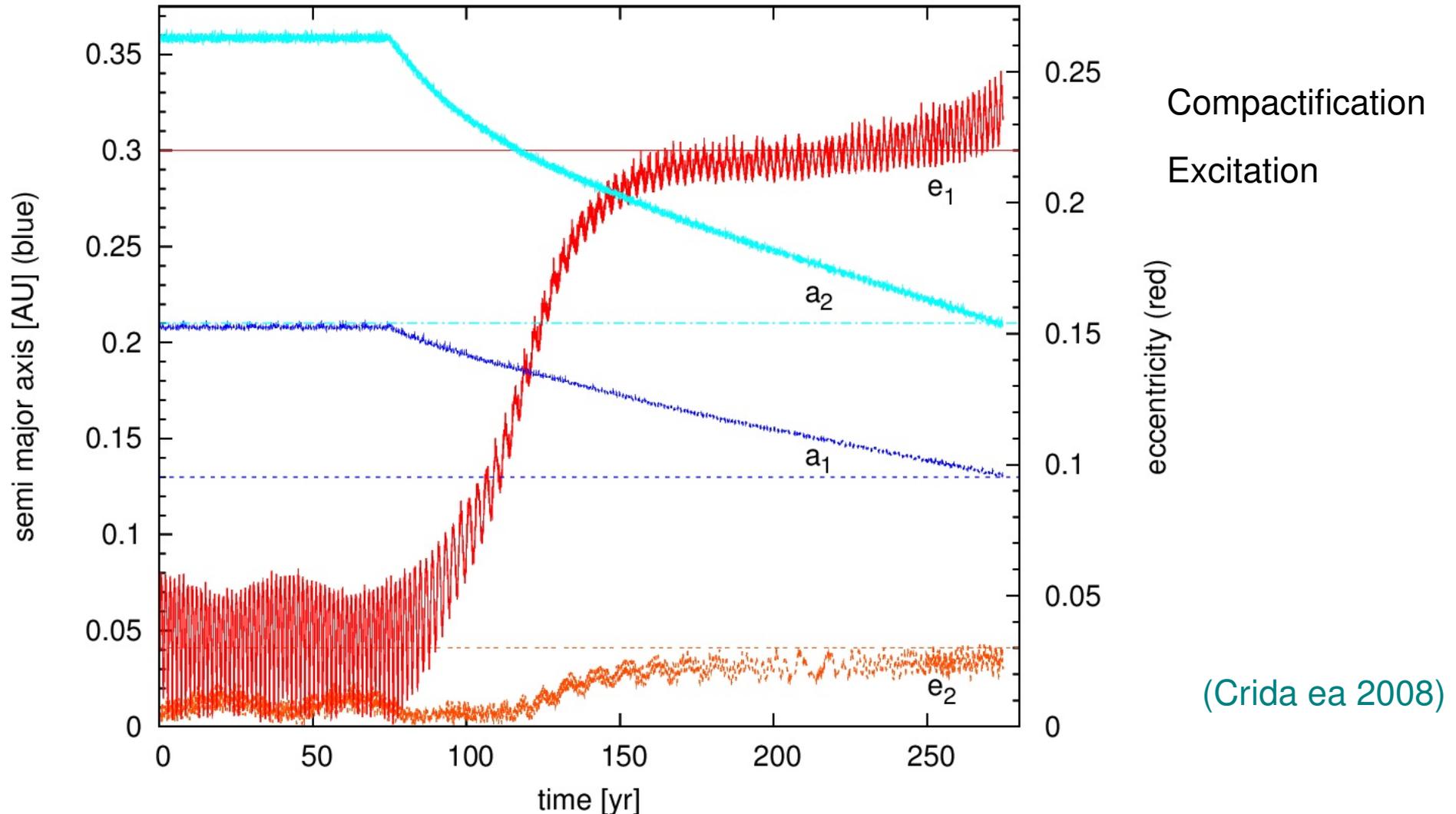
(Sandor, Kley & Klagyivik
2007)

compare: HD142527



Here: **System-parameter of GJ 876**

(damping of inner & outer disk)



System ends in: **apsidal corotation**, with **correct eccentricities**

Less disk damping: \Rightarrow much higher $e \Rightarrow$ possible Instability

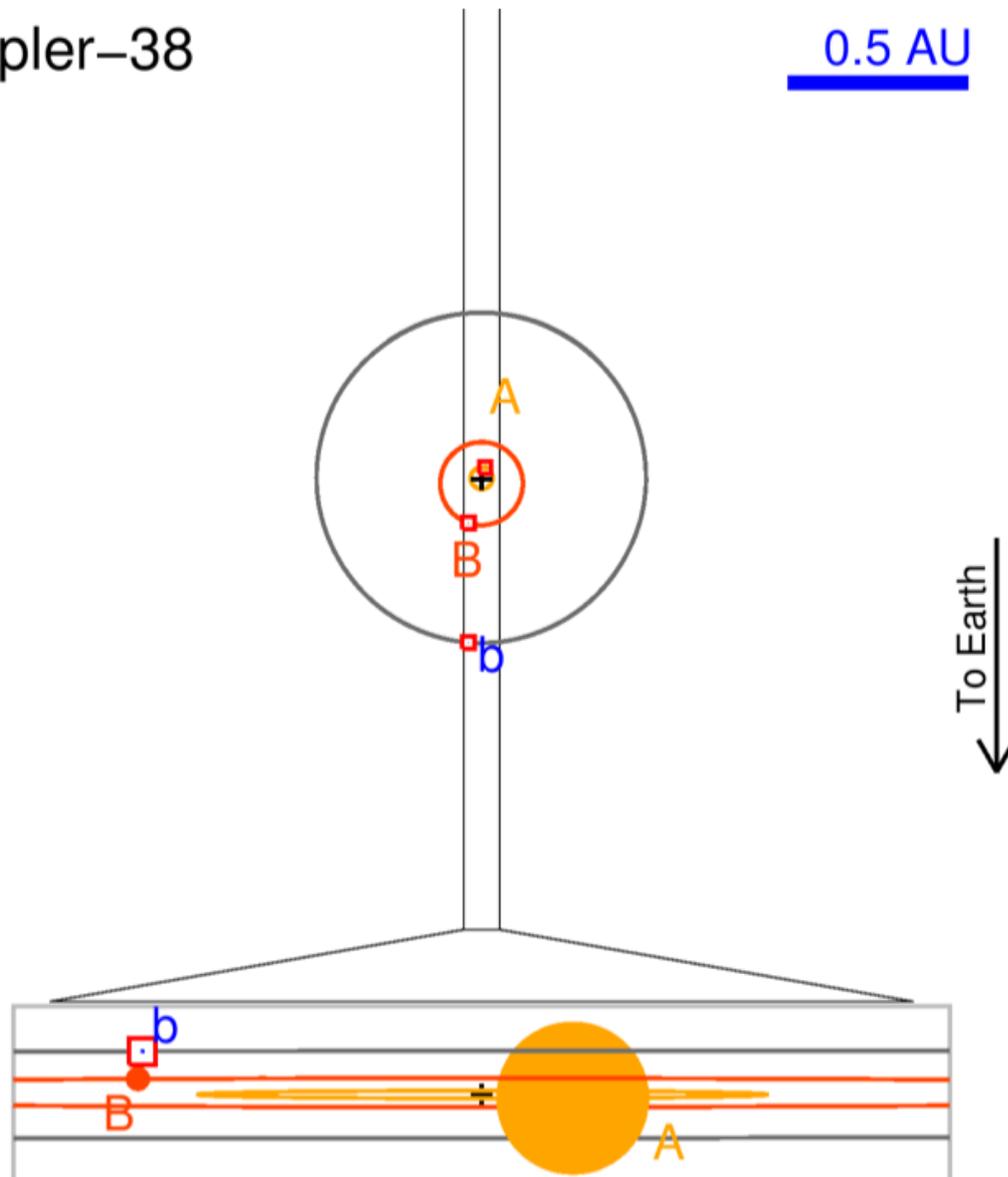


-
- RV-Obs.: ≈ 50 multi-planet extrasolar planetary systems
 $\approx 1/4$ contain planets in a low-order **mean-motion resonance** (MMR)
In **Solar System**: 3:2 between Neptune and Pluto (plutinos)
 - Resonant capture through convergent migration process
dissipative forces due to disk-planet interaction
 - Existence of resonant systems
 - **Clear evidence for planetary migration**
 - Hot Jupiters (Neptunes) & Kepler systems
 - **Clear evidence for planetary migration**



Kepler-38

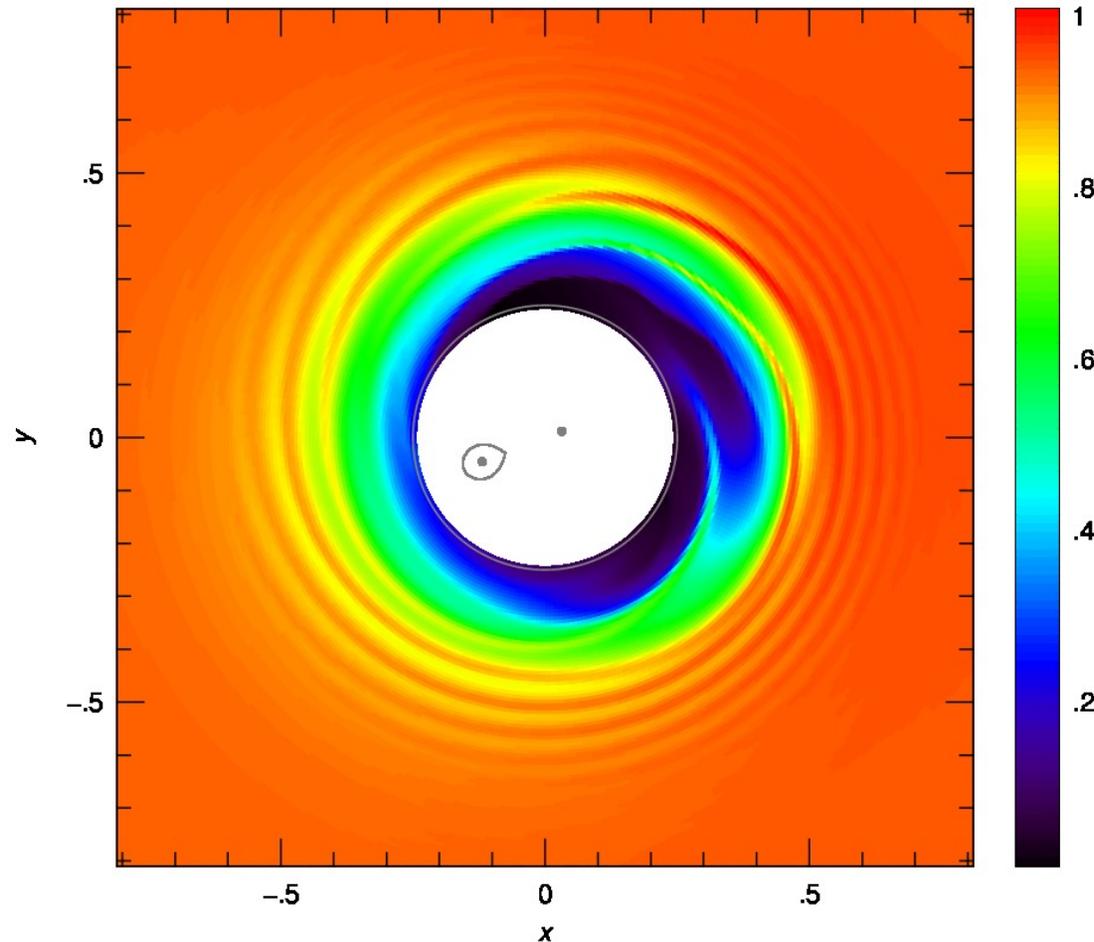
0.5 AU





Animation of disk around Binary Star
(shown is the surface density)

k38a : ρ (0.25, 3.0546E-09, 1.0357E+00) N=3500000; t= 3057.829



Binary Parameter:

$$M_1 = 0.95 M_{\odot},$$

$$M_2 = 0.25 M_{\odot}$$

$$a_B = 0.15 \text{ AU}$$

$$e_B = 0.10$$

Planet Parameter:

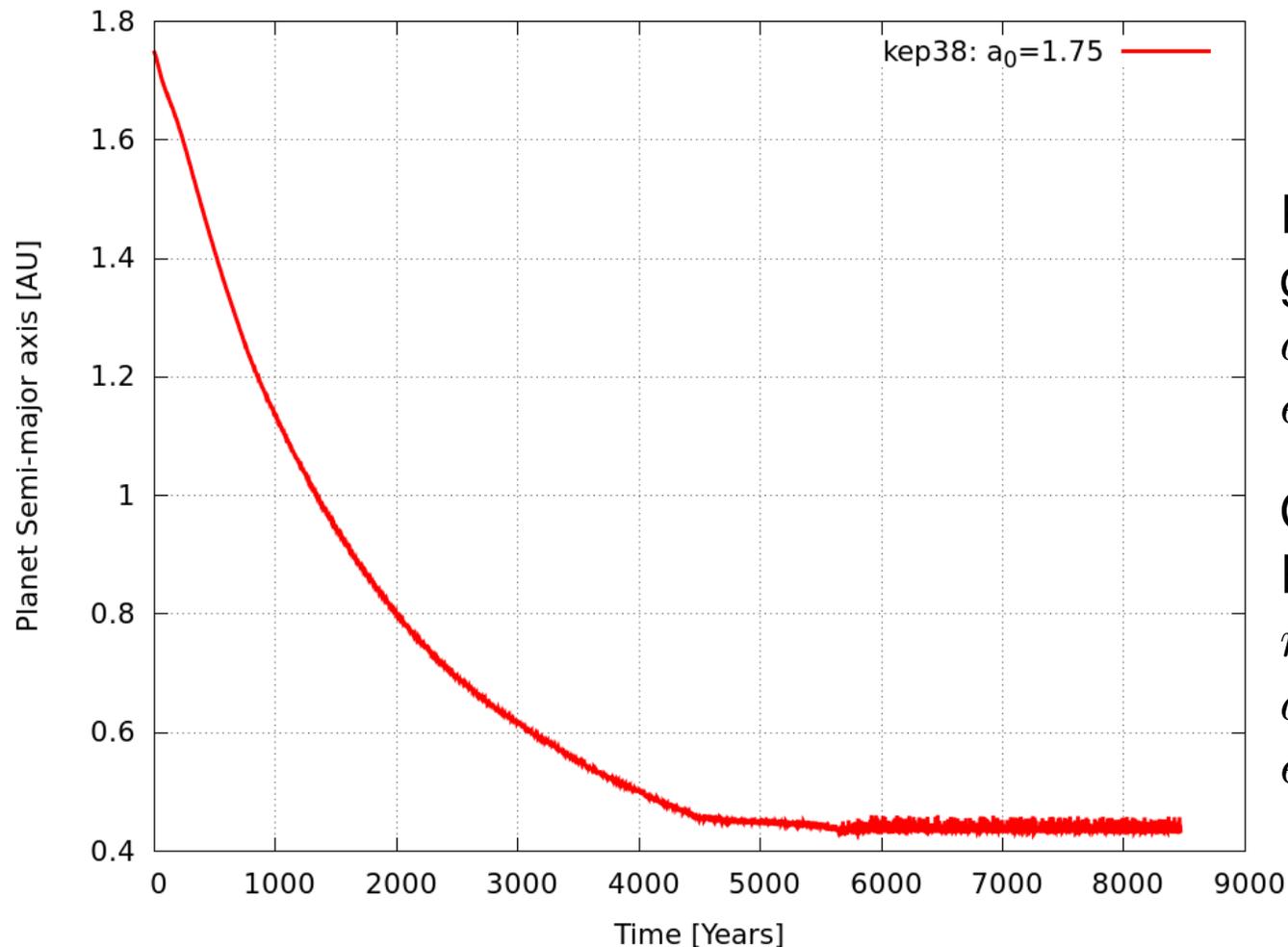
$$m_p = 0.36 M_{Jup}$$

$$a_p = 0.46 \text{ AU}$$

$$e_p = 0.03$$



Migration of planet in disk



Planet stops at
gap edge with:

$$a_p = 0.43 \text{ AU}$$

$$e_p = 0.15 \text{ AU}$$

Observed

Planet Parameter:

$$m_p = 0.36 M_{Jup}$$

$$a_p = 0.46 \text{ AU}$$

$$e_p = 0.03$$

(Kley & Haghighipour, in prep.)

(see also: Pierens & Nelson, 2013)



New properties of extrasolar Planets

- High Masses
- Large eccentricities
- Close distances

Planet-disk simulations

- Coord. System
- Angular momentum conservation
- FARGO-Accelaration
- Grid-Refinement
- Thermodynamics

Accurate accretion & migration history of planets requires

- Full 3D simulations
- High resolution (locally)
- Radiation transport
- Self-Gravity
- Magnetic Fields



Thank you for your attention !

(A. Crida)