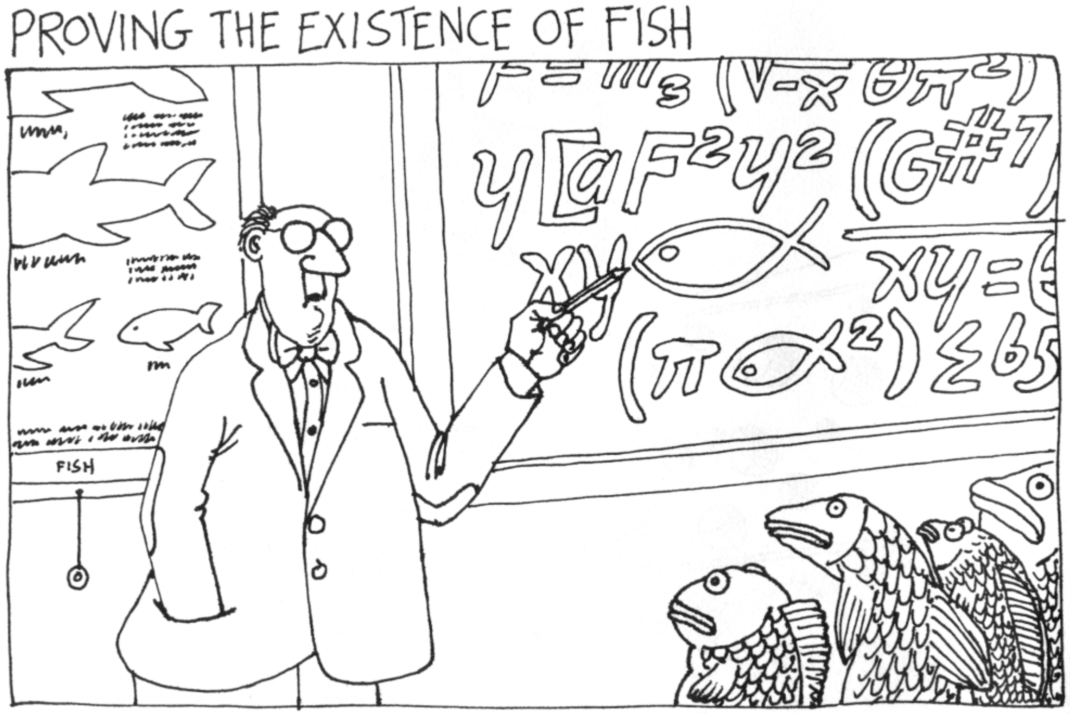
Magnetic drift in molecular cloud cores, and in protoplanetary and circumplanetary disks

Mark Wardle

Department of Physics & Astronomy

MACQUARIE UNIVERSITY
ASTRONOMY, ASTROPHYSICS AND ASTROPHOTONICS RESEARCH CENTRE

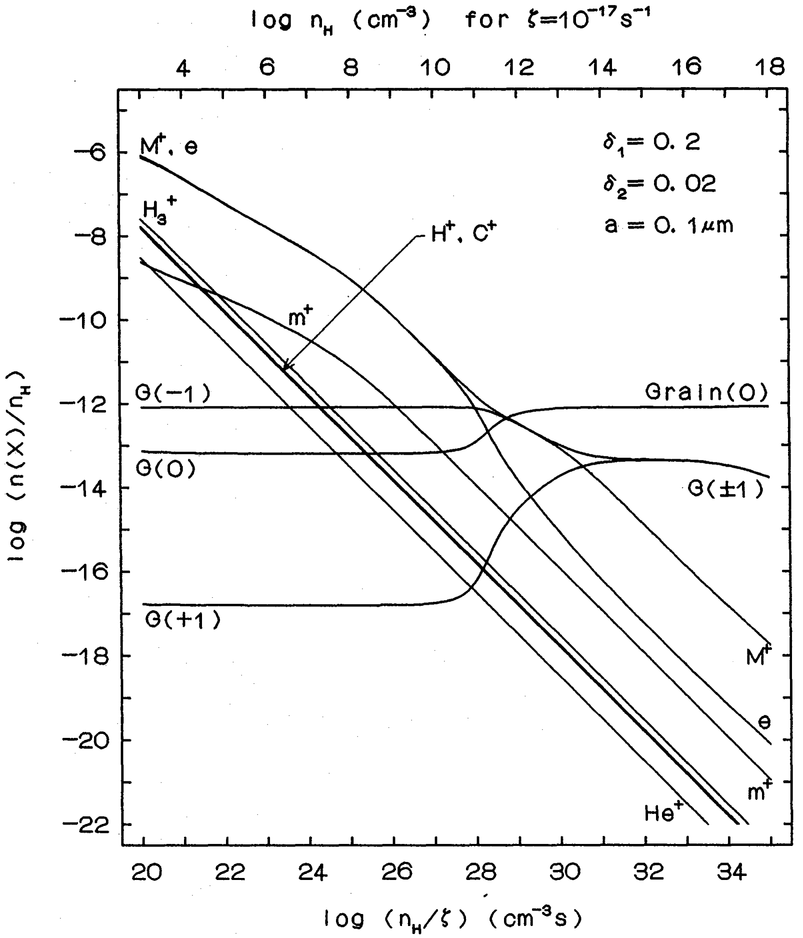


Kliban 1976

Magnetic diffusion and drift
Magnetorotational instability
Gravitational collapse
Jet launching

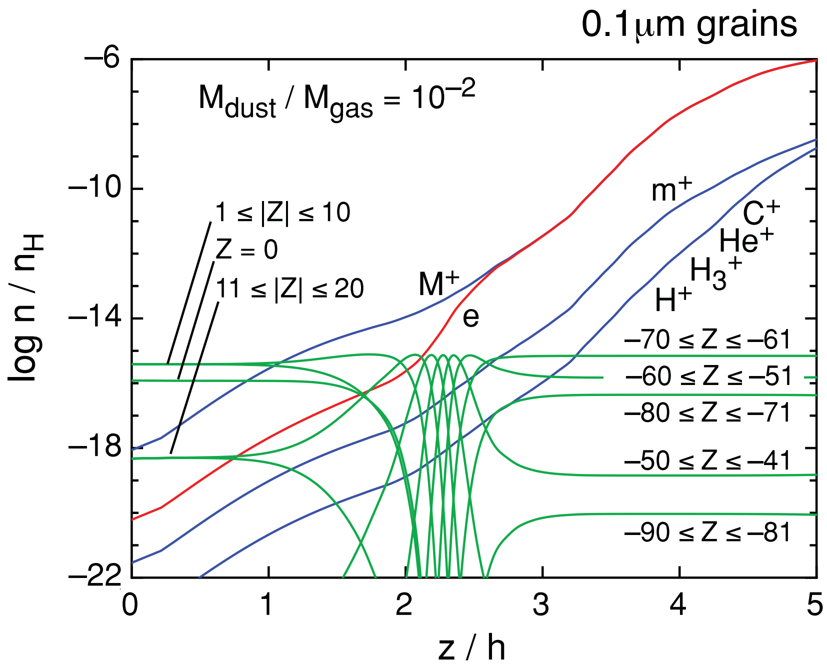
Molecular clouds and protoplanetary disks are weakly ionized

Molecular cloud

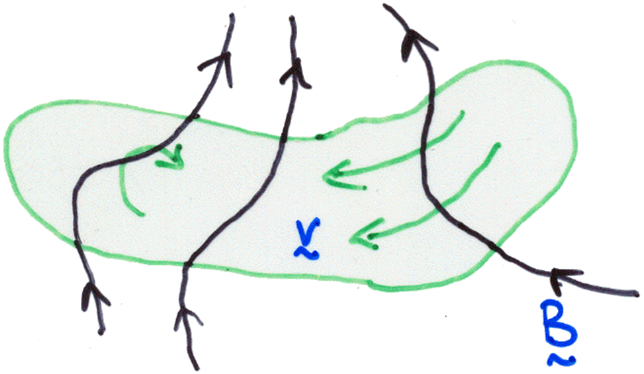


Umebayashi & Nakano 1990

MMSN @ 1 AU



Wardle & Salmeron 2012



Given $\rho, \underset{\sim}{v}, \underset{\sim}{B}$ how does the fluid evolve?

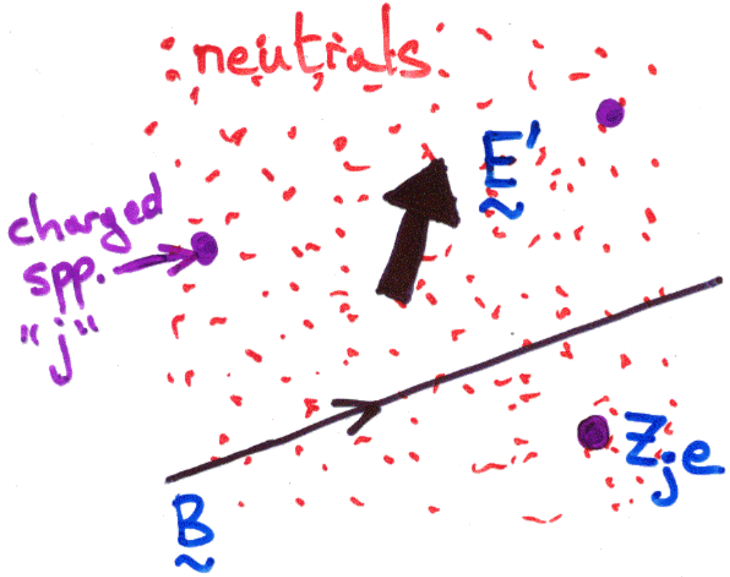
$\nabla\times\underset{\sim}{B} \Rightarrow \underset{\sim}{J}$ $\qquad$ $\underset{\sim}{J} = \frac{c}{4\pi}\nabla\times\underset{\sim}{B}$

$\Rightarrow \underset{\sim}{E}'$ (fluid) $\qquad$ $\boxed{\underset{\sim}{J} = \underset{\approx}{\sigma}\,\underset{\sim}{E}'}$

$\Rightarrow \underset{\sim}{E}$ (observer) $\qquad$ $\underset{\sim}{E} = \underset{\sim}{E}' - \frac{\underset{\sim}{v}}{c}\times\underset{\sim}{B}$

$\Rightarrow \frac{\partial\underset{\sim}{B}}{\partial t}$ $\qquad$ $\frac{\partial\underset{\sim}{B}}{\partial t} = -c\,\nabla\times\underset{\sim}{E}$

$$= \nabla\times(\underset{\sim}{v}\times\underset{\sim}{B}) - \frac{c^2}{4\pi}\nabla\times\left(\underset{\approx}{\sigma}^{-1}\nabla\times\underset{\sim}{B}\right)$$



$$Z_j e \boldsymbol{E}' + Z_j e \frac{\boldsymbol{v}_j}{c} \times \boldsymbol{B} - \gamma_j m_j \rho \boldsymbol{v}_j = 0$$

$$\beta_j = \frac{Z_j e B}{m_j c} \frac{1}{\gamma_j \rho} = \frac{\text{gyrofrequency}}{\text{collision frequency}}$$

$|\beta_j| \gg 1 : \quad Z_j e \boldsymbol{E}' \approx -Z_j e \frac{\boldsymbol{v}_j}{c} \times \boldsymbol{B}$ particles tied to field

$|\beta_j| \ll 1 : \quad Z_j e \boldsymbol{E}' \approx \gamma_j m_j \rho \boldsymbol{v}_j$ particles tied to neutral fluid

$$\boldsymbol{J} = \sum_j n_j e Z_j \boldsymbol{v}_j$$

$$= \frac{ec}{B} \sum_j n_j Z_j \beta_j \boldsymbol{E}'_\parallel + \frac{ec}{B} \sum_j \frac{n_j Z_j \beta_j^2}{1+\beta_j^2} \boldsymbol{E}' \times \hat{\boldsymbol{B}} + \frac{ec}{B} \sum_j \frac{n_j Z_j \beta_j}{1+\beta_j^2} \boldsymbol{E}'_\perp$$

Cowling 1957

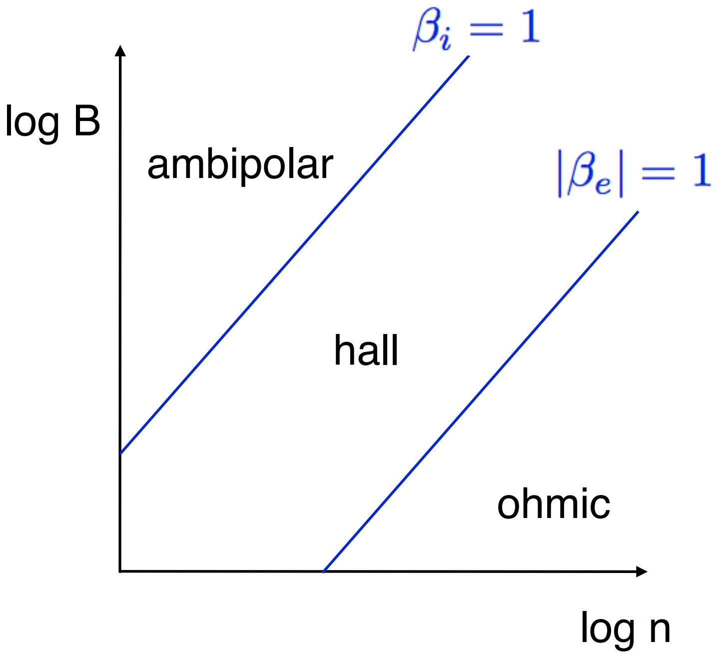
$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{v}\times\boldsymbol{B}) - \nabla\times\left[\eta\ \nabla\times\boldsymbol{B} + \eta_{\mathrm{H}}(\nabla\times\boldsymbol{B})\times\hat{\boldsymbol{B}} + \eta_{A}(\nabla\times\boldsymbol{B})_{\perp}\right]$$

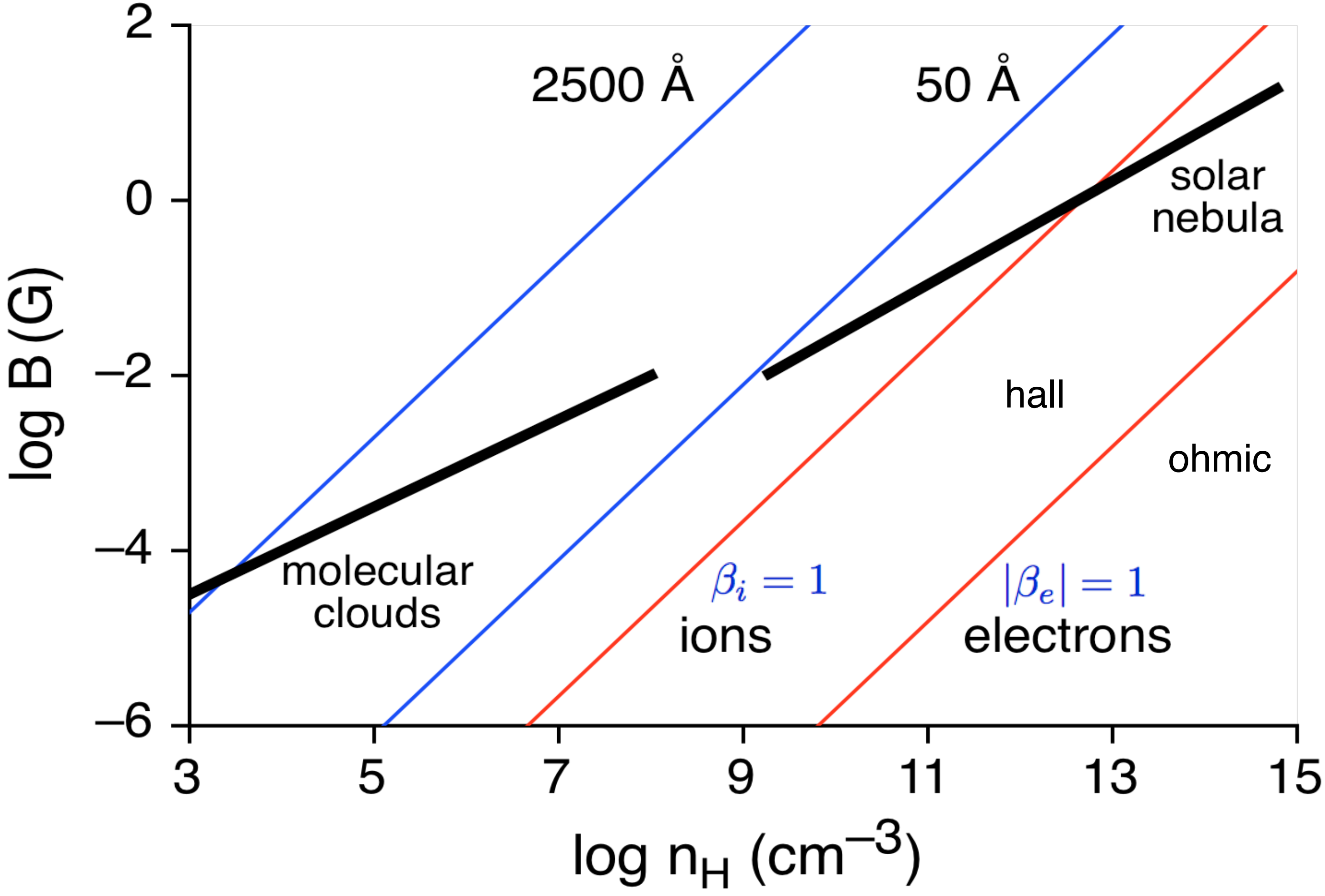
- If the only charged species are ions and electrons,

$$\eta_{\mathrm{H}} = |\beta_e|\,\eta$$
$$\eta_{\mathrm{A}} = \beta_i|\beta_e|\,\eta$$

- Three distinct diffusion regimes:

$\beta_i \ll |\beta_e| \ll 1$ – Ohmic (resistive)
$\beta_i \ll 1 \ll |\beta_e|$ – Hall
$1 \ll \beta_i \ll |\beta_e|$ – Ambipolar

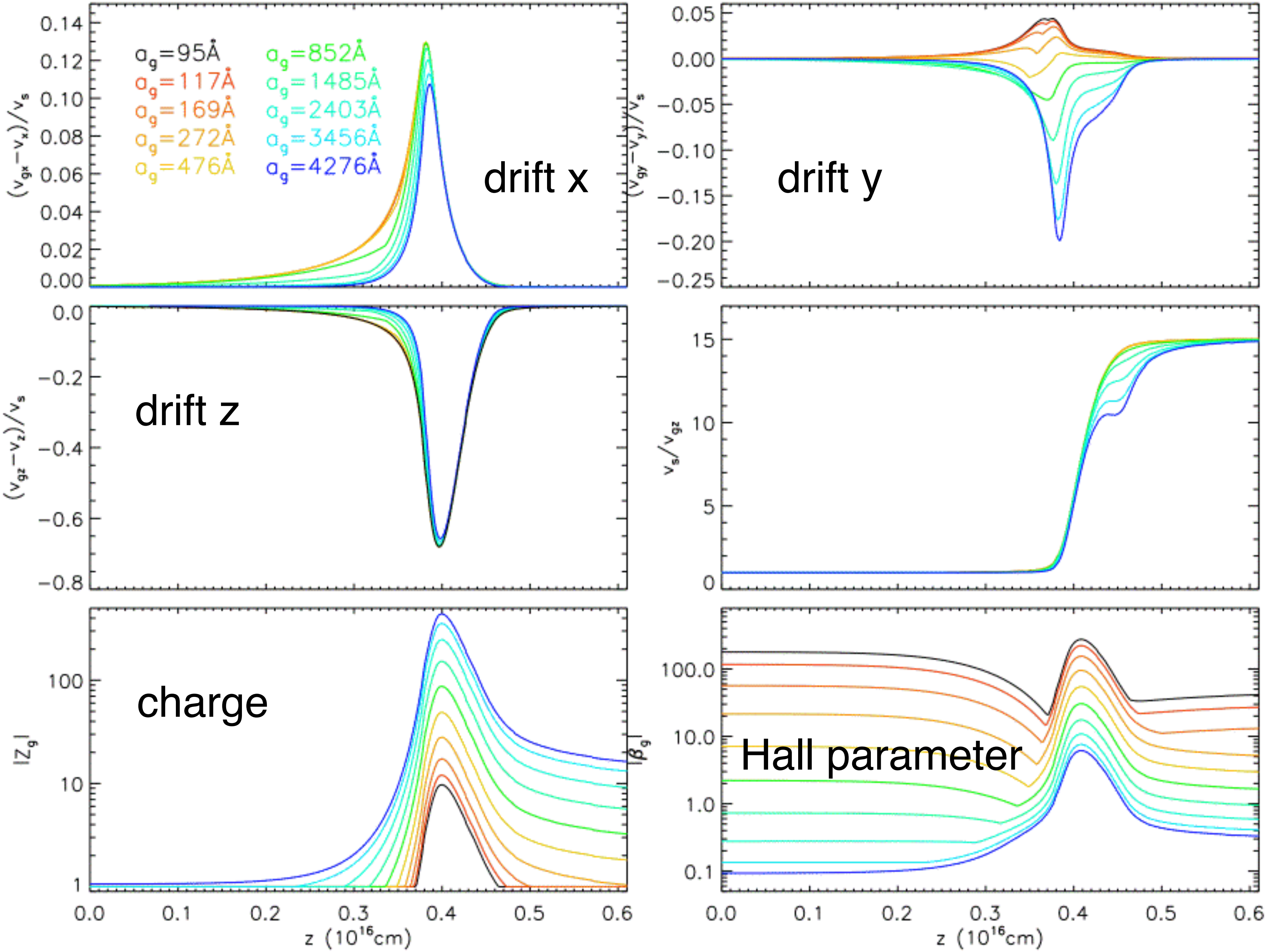



2500 Å
50 Å
solar
nebula
hall
ohmic
molecular
clouds
$\beta_i = 1$
ions
$|\beta_e| = 1$
electrons
log B (G)
2
0
−2
−4
−6
3
5
7
9
11
13
15
log n_H (cm^{-3})

Wardle 2007

Magnetic drift

regime	magnetised component	unmagnetised component	B drift through neutrals
Ideal MHD	neutrals, ions, electrons	–	0
Ambipolar	ions, electrons	neutrals	$\mathbf{v_i} - \mathbf{v_n} = \dfrac{\mathbf{J} \times \mathbf{B}}{c\gamma\rho_i\rho}$
Hall	electrons	neutrals, ions	$\mathbf{v_e} - \mathbf{v_i} = -\dfrac{\mathbf{J}}{en_e}$
Ohmic	–	neutrals, ions, electrons	$c\dfrac{\mathbf{E}' \times \mathbf{B}}{B^2} = \dfrac{4\pi\eta}{c}\dfrac{\mathbf{J} \times \mathbf{B}}{B^2}$



drift x

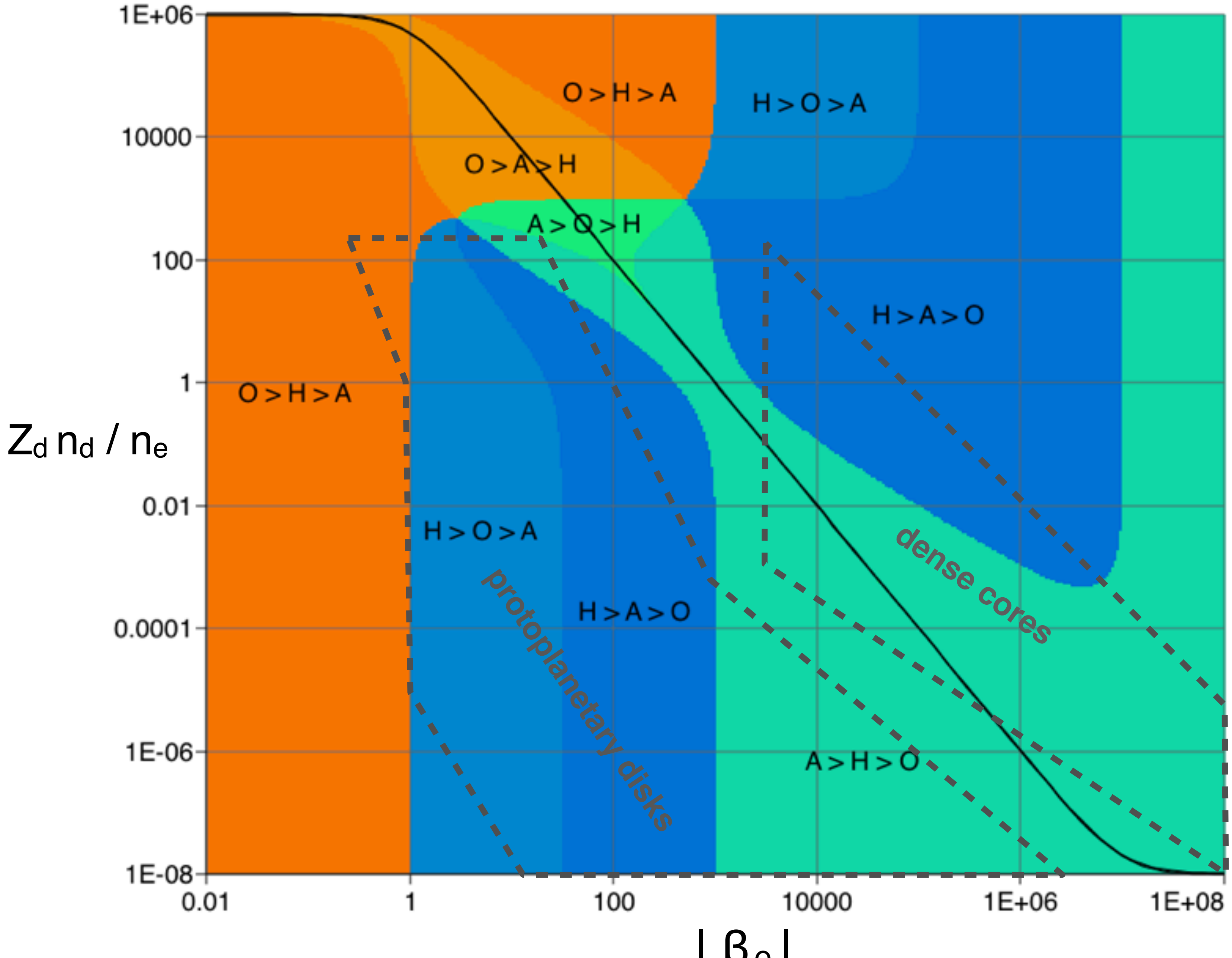
drift y

drift z

charge

Hall parameter

Chapman & Wardle 2006



Wardle & Pandey in prep

$$\eta_A = \frac{1+\beta_g^2+(1+\beta_i\beta_g)P}{(1+P/P_0)^2+(\beta_g+\beta_i P)^2}\,\frac{B^2}{4\pi\gamma\rho_i\rho}$$

$$\eta_H = \frac{1+\beta_g^2-\beta_i^2 P}{(1+P/P_0)^2+(\beta_g+\beta_i P)^2}\,\frac{cB}{4\pi e n_e}$$

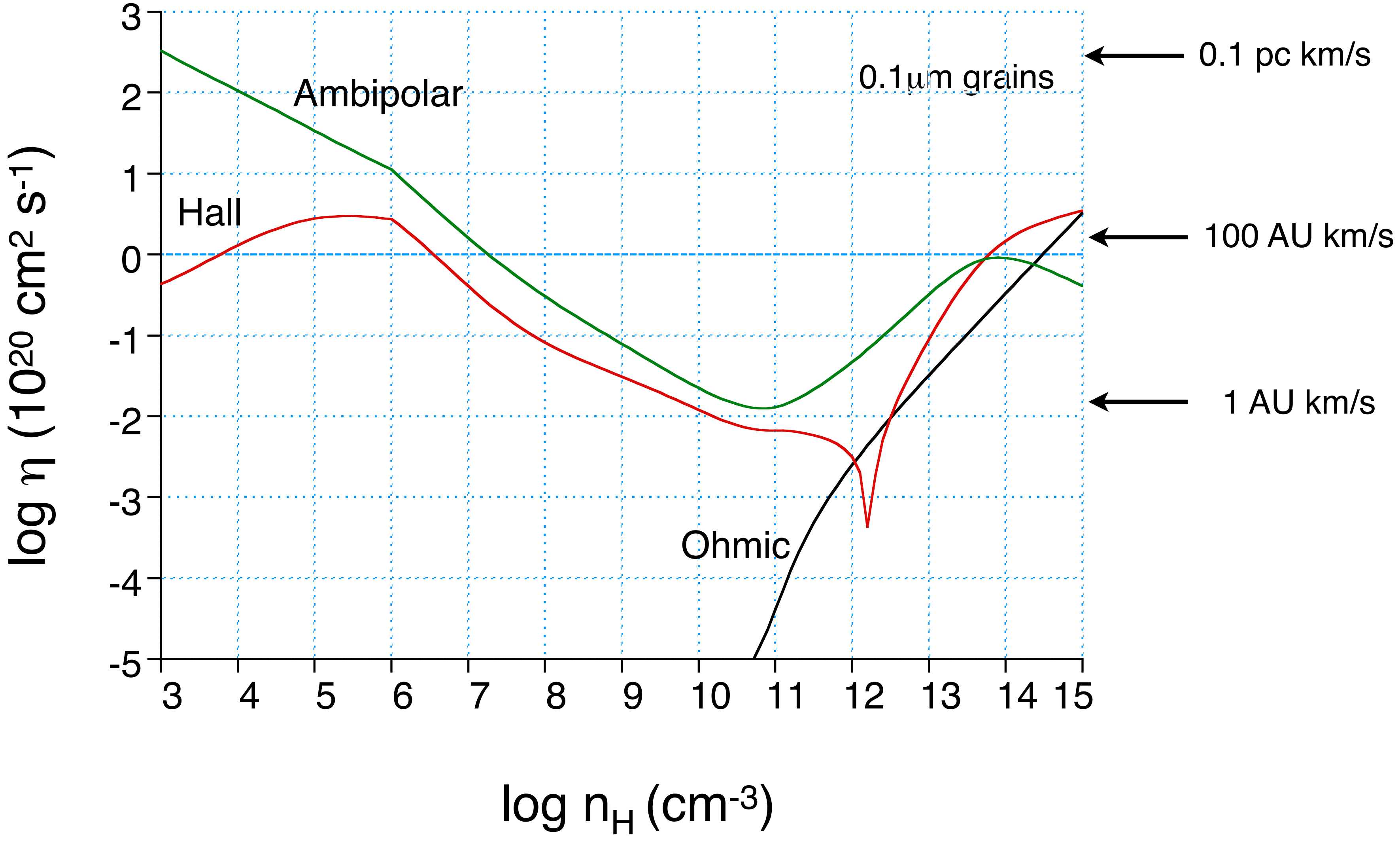
$$\eta = \frac{1}{1+P/P_0}\,\frac{c^2}{4\pi\sigma_e}$$

$P = Z_d\, n_d \,/\, n_e$

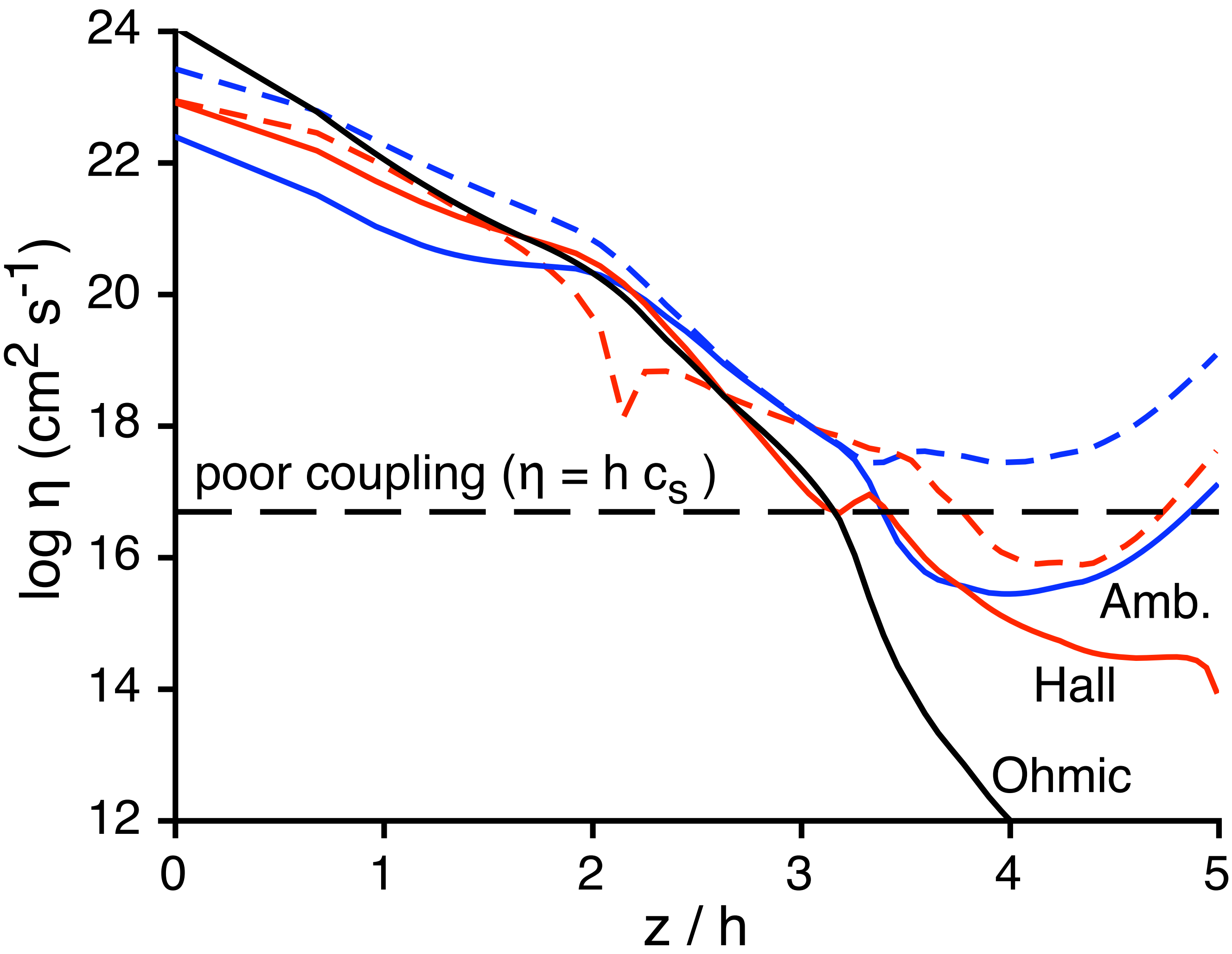
P_0 = ion/electron Hall parameter

Wardle & Pandey in prep

Clouds and cores

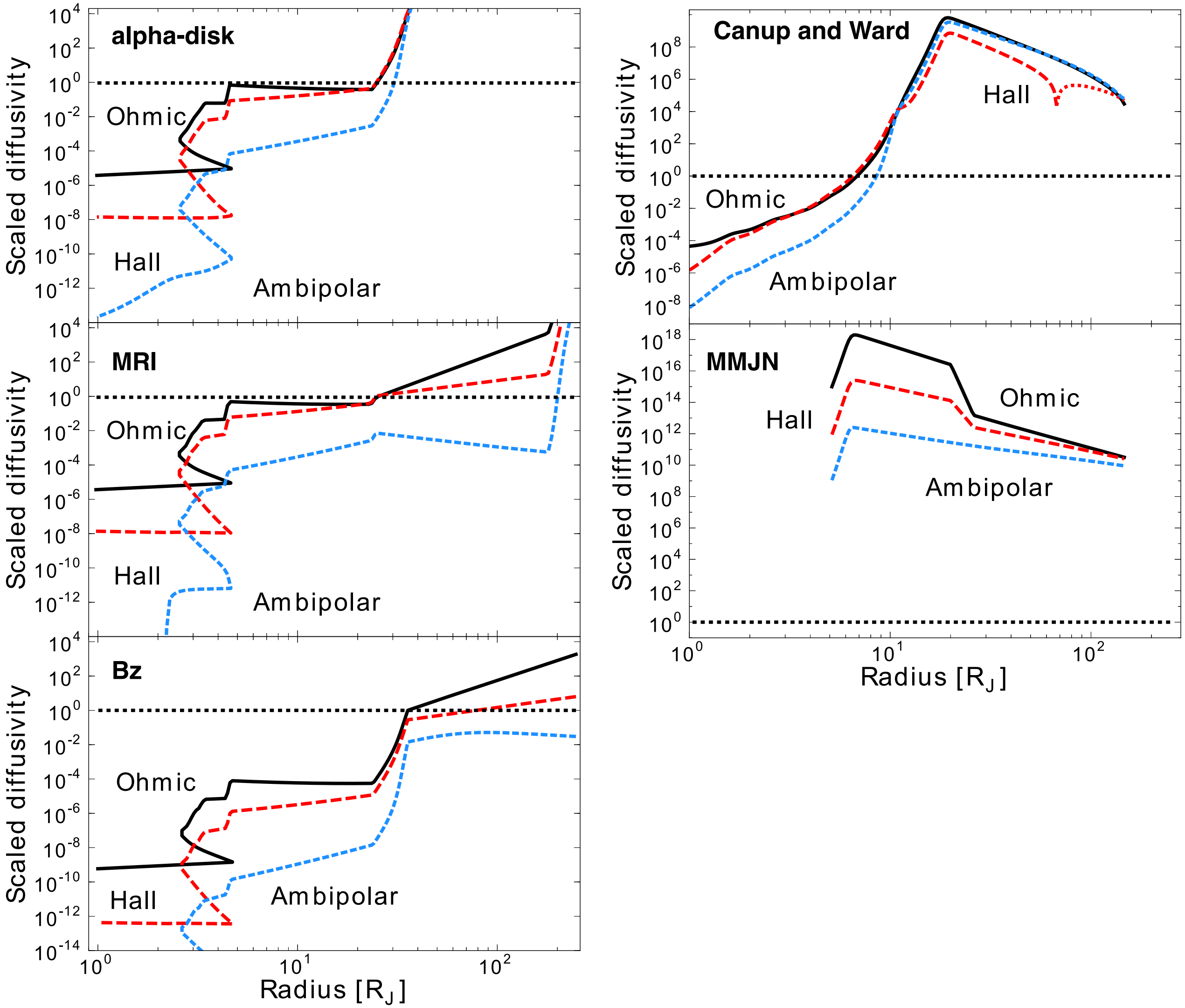


MMSN, 1AU



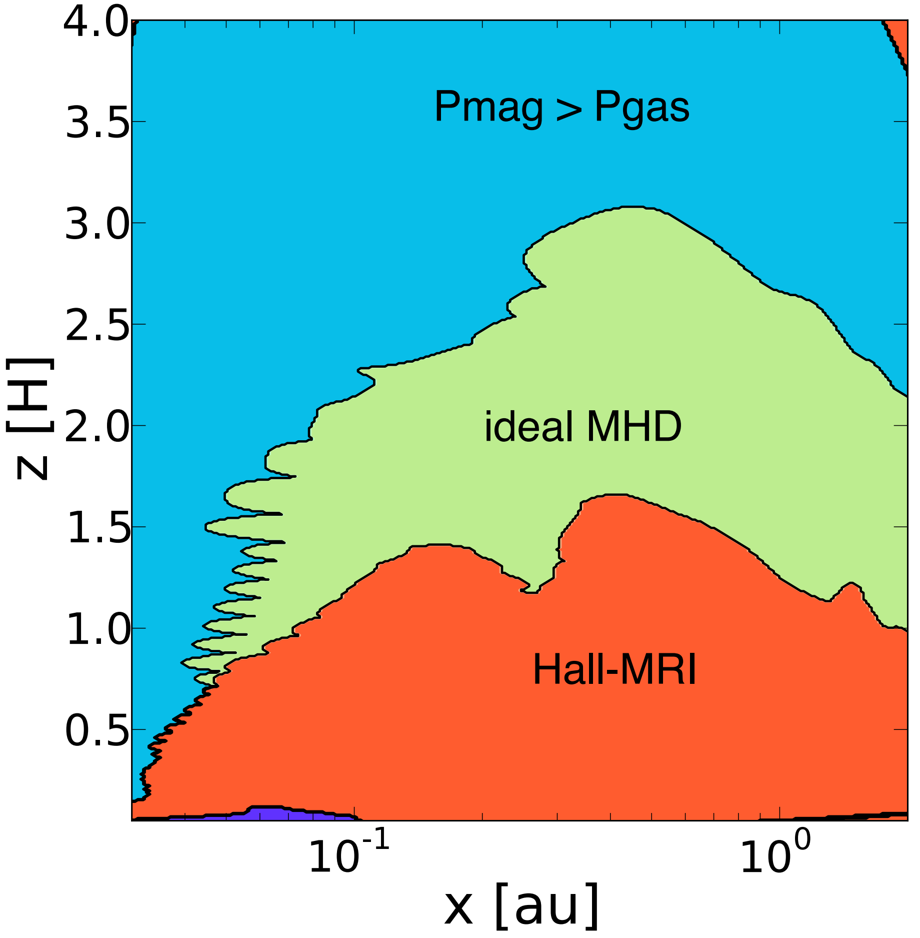
Wardle 2007

Circumplanetary disks



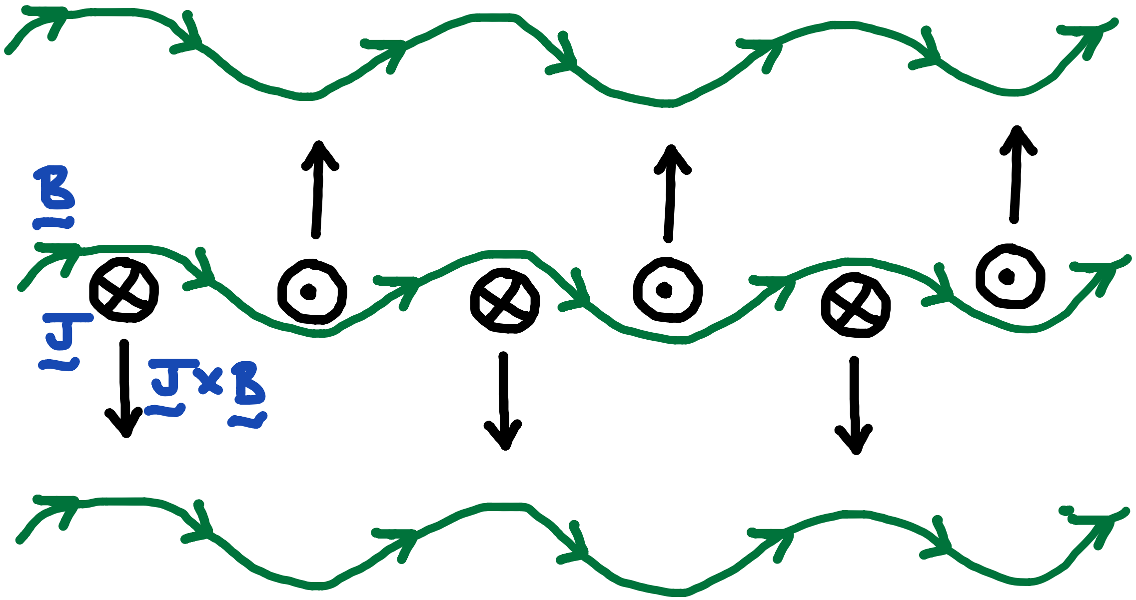
Keith & Wardle 2014

Circumplanetary disks



Keith & Wardle in preparation

Field line drift



Ambipolar: $\dfrac{\mathbf{J}\times\mathbf{B}}{c\gamma\rho_i\rho}$

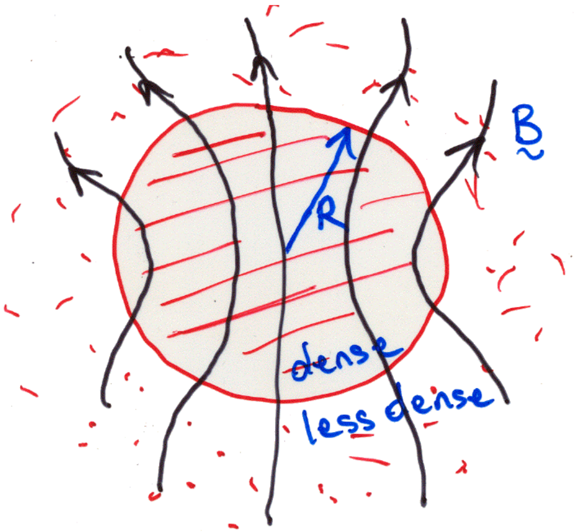
Ohmic: $\dfrac{4\pi\eta}{c}\,\dfrac{\mathbf{J}\times\mathbf{B}}{B^2}$

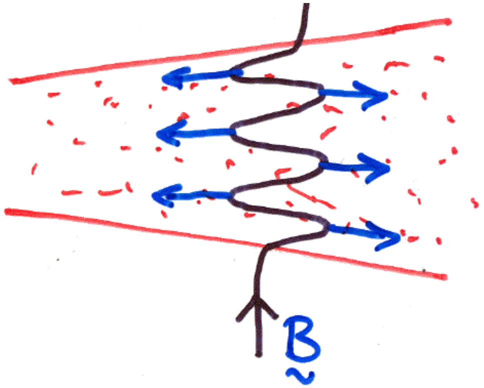
Hall: $-\dfrac{\mathbf{J}}{e\,n_e}$

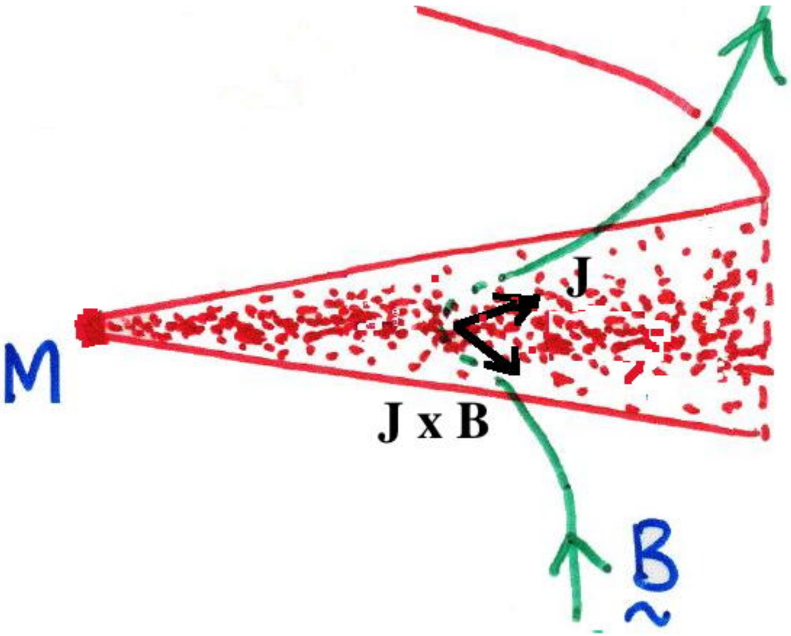
– Hall drift is in and out of plane of screen
tends to induce or reduce twisting in B

– sense depends on global direction of B

Field line drift: collapsing cores / protoplanetary disks







Ambipolar: $$\frac{\mathbf{J}\times\mathbf{B}}{c\gamma\rho_i\rho}$$

Ohmic: $$\frac{4\pi\eta}{c}\,\frac{\mathbf{J}\times\mathbf{B}}{B^2}$$

Hall: $$-\frac{\mathbf{J}}{e\,n_e}$$

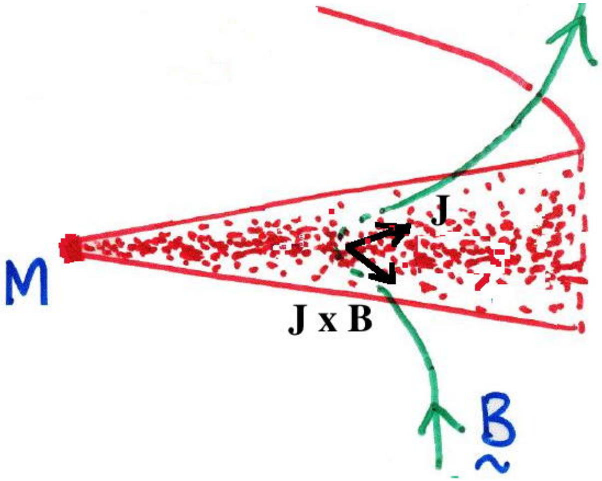
Momentum equation

$$\frac{\partial v}{\partial t} + \Omega \frac{\partial v}{\partial \phi} + (v \cdot \nabla) v - 2\Omega v_\phi \hat{r} + \frac{1}{2}\Omega v_r \hat{\phi}$$

$$= r^2 \Omega \hat{r} - \nabla \Phi - \frac{1}{\rho} \nabla P + \frac{J \times B}{\rho c}$$

<u>r cpt</u> $$-2\Omega v_\phi = \frac{(J \times B)_r}{\rho c} > 0$$

<u>ϕ cpt</u> $$\frac{1}{2}\Omega v_r = \frac{(J \times B)_\phi}{\rho c} < 0$$



Effect on the MRI

$\underline{\phi \text{ cpt}}$:

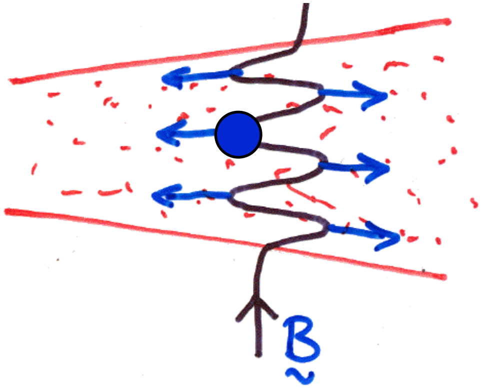
$$\frac{1}{2}\Omega v_r = \frac{(J\times B)_\phi}{\rho c} = -\frac{J_r B_z}{\rho c} = -en_e(\underset{\uparrow\, v_r}{v_{ir}} - v_{er})\frac{B_z}{\rho c}$$

$$v_{er} = \left(1 + \frac{\overset{\uparrow}{s}\,\eta_H\Omega}{2v_A^2}\right) v_r < 0 \quad \text{For MRI}$$

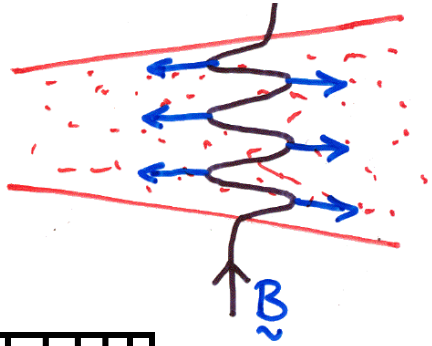
s: sign(B_z)

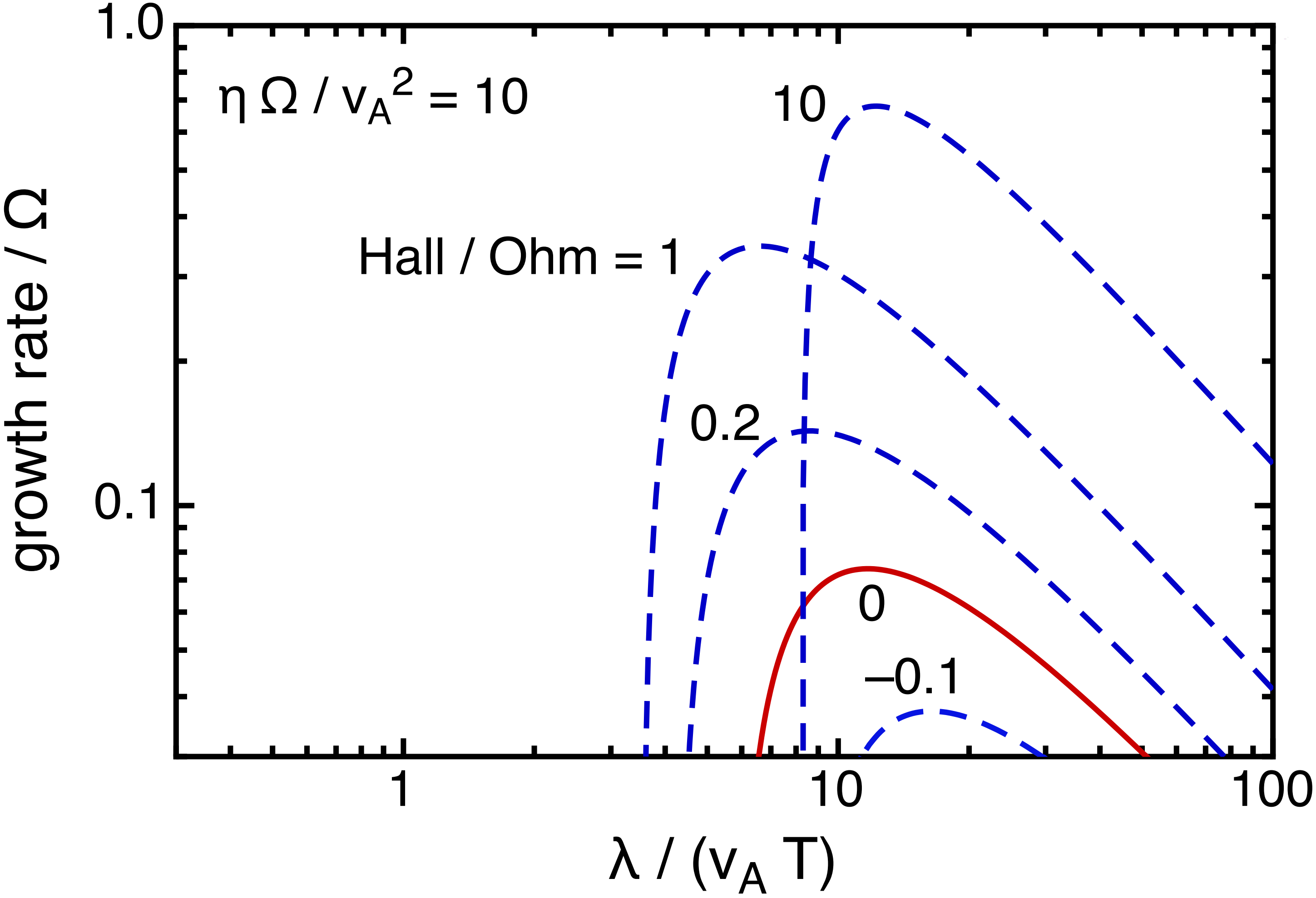
$\eta_H = \frac{cB}{4\pi e n_e}$

$\Rightarrow$ need $\frac{s\eta_H\Omega}{v_A^2} > -2$



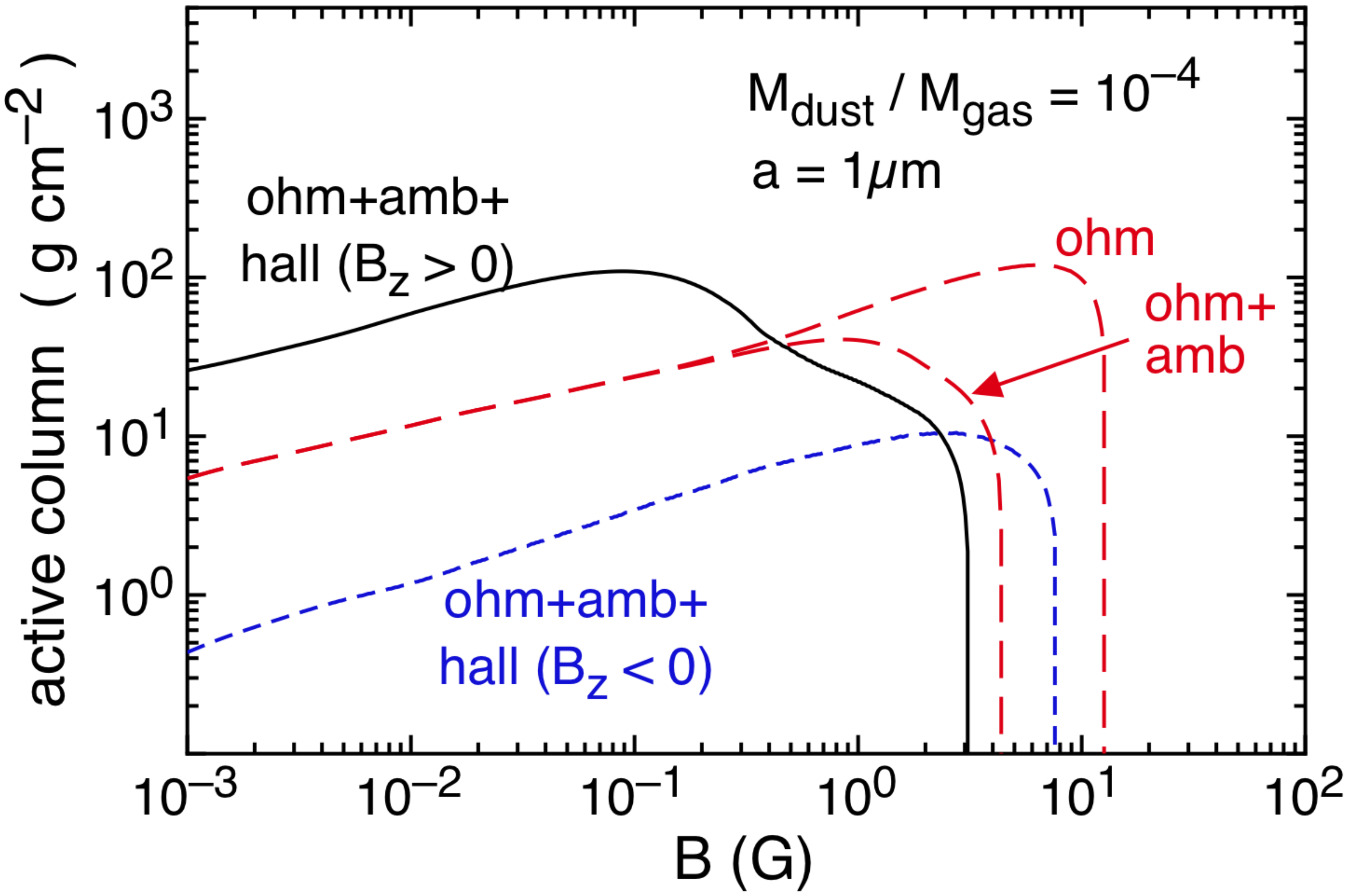
MRI growth rate - ohmic vs Hall





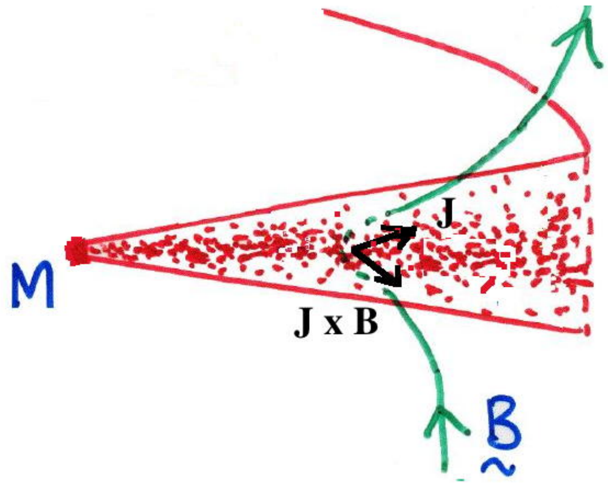
Wardle 1999 ; Balbus & Terquem 2001

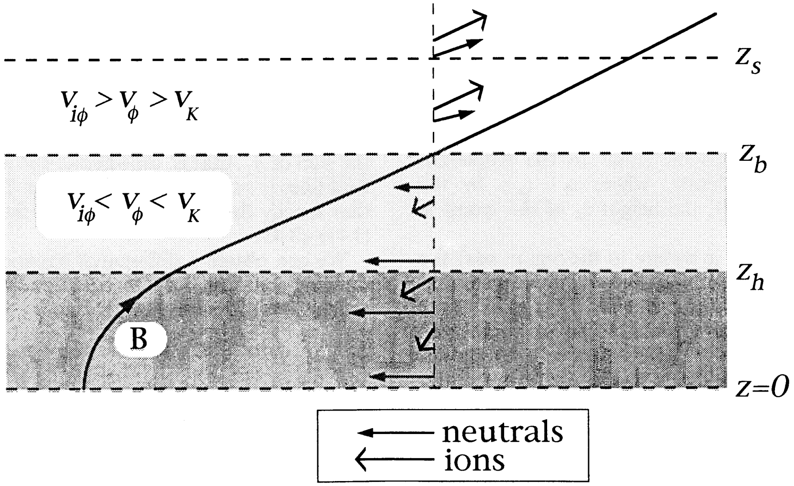
Column density of active layer



Wardle & Salmeron 2012

Effect on disk winds





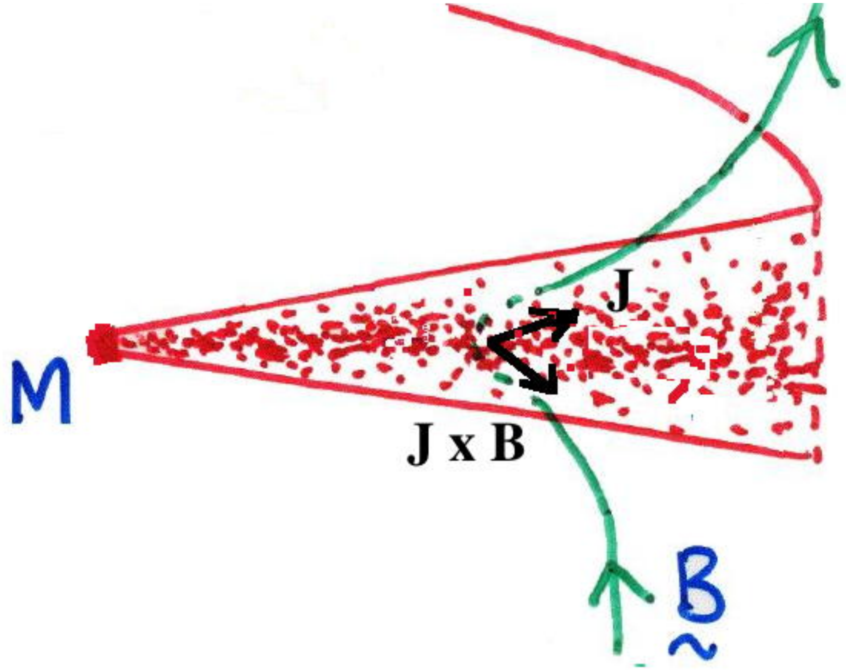
Wardle & Konigl 1993

r cpt: $-2\Omega v_\phi = \frac{(J\times B)_r}{\rho c} = \frac{J_\phi B_z}{\rho c} = e n_e (\underbrace{v_{i\phi}}_{v_\phi} - v_{e\phi}) \frac{B_z}{\rho c}$

$v_{e\phi} = \left(1 + 2\frac{s\eta_H \Omega}{v_A^2}\right)\underbrace{v_\phi}_{-ve} < 0$ for normal behaviour

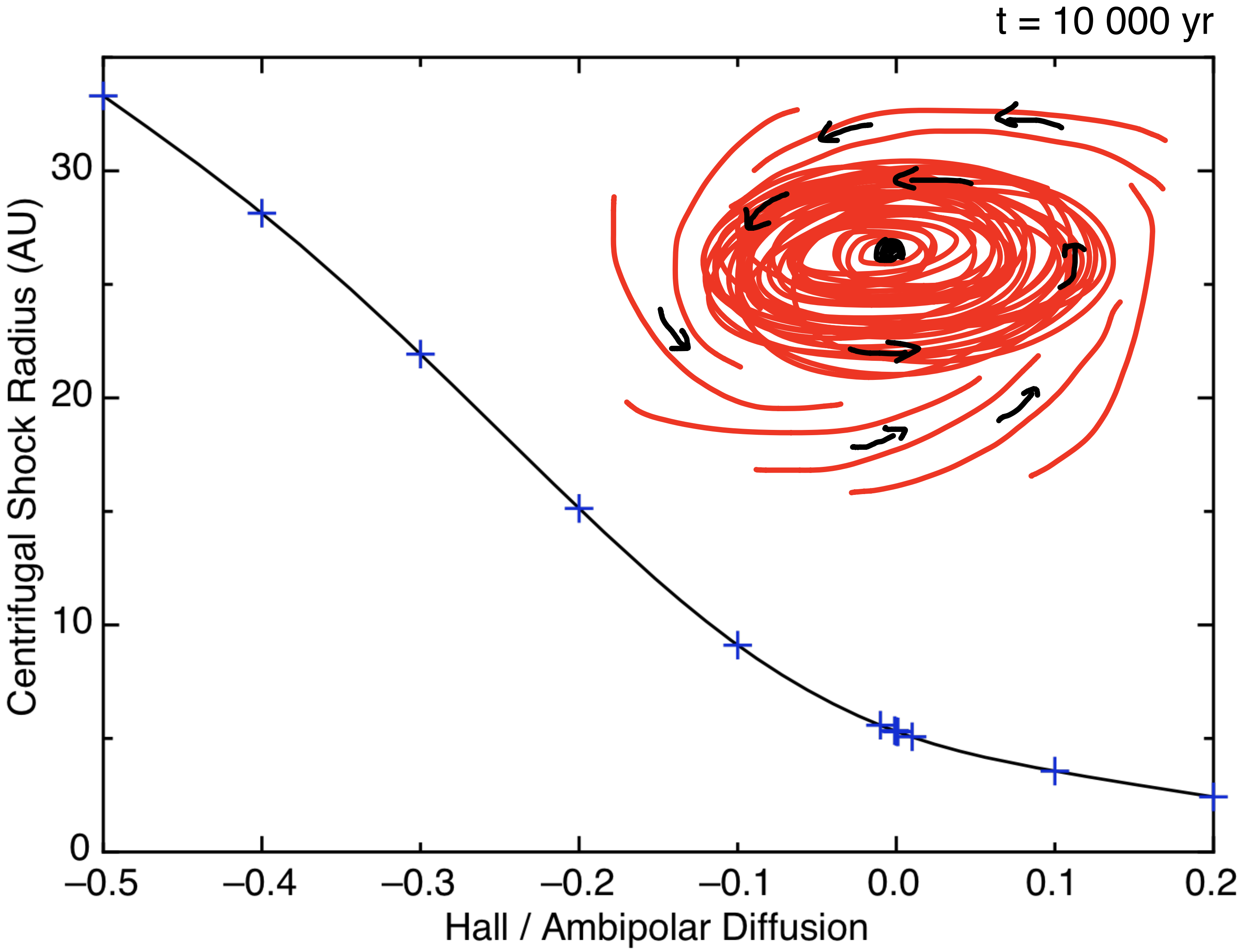
i.e. $\frac{s\eta_H\Omega}{v_A^2} > -\frac{1}{2}$

Transport of magnetic flux vs angular momentum



Hall drift: magnetic torque => radial drift of field through gas

Self-similar collapse with non-ideal MHD



Braiding & Wardle 2012

Summary

- Ideal MHD breaks down on scale of cloud cores and protoplanetary disks
  - core collapse: angular momentum, magnetic flux
  - protoplanetary disks: distribution and nature of MHD turbulence
  - disk-driven jets: launching, coupling between jet and disk
- Field line drift is a critical part of MRI / wind launching / flux transport problems
  - "low" density (clouds, cores): ambipolar diffusion (Mestel & Spitzer 1956)
  - high density (disks): ohmic resistivity (e.g. Hayashi 1981)
  - Hall effect (collapsing cores and disks) (Wardle 1999, Wardle & Ng 1999)

    AD: $v_B \sim JxB \sim B^2$

    Hall: $v_B \sim \pm J \sim B$; depends on sign of B ; no dissipation

    Ohm: $v_B \sim JxB / B^2 \sim B^0$; important only for high density, weak fields
- Figure of merit for Hall is NOT given by ratio of diffusivities
  - dissipationless => compare v_B (Hall) with v e.g. $\eta_H \Omega / v_A^2$