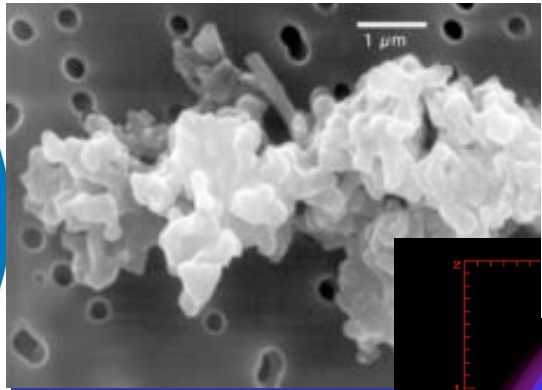
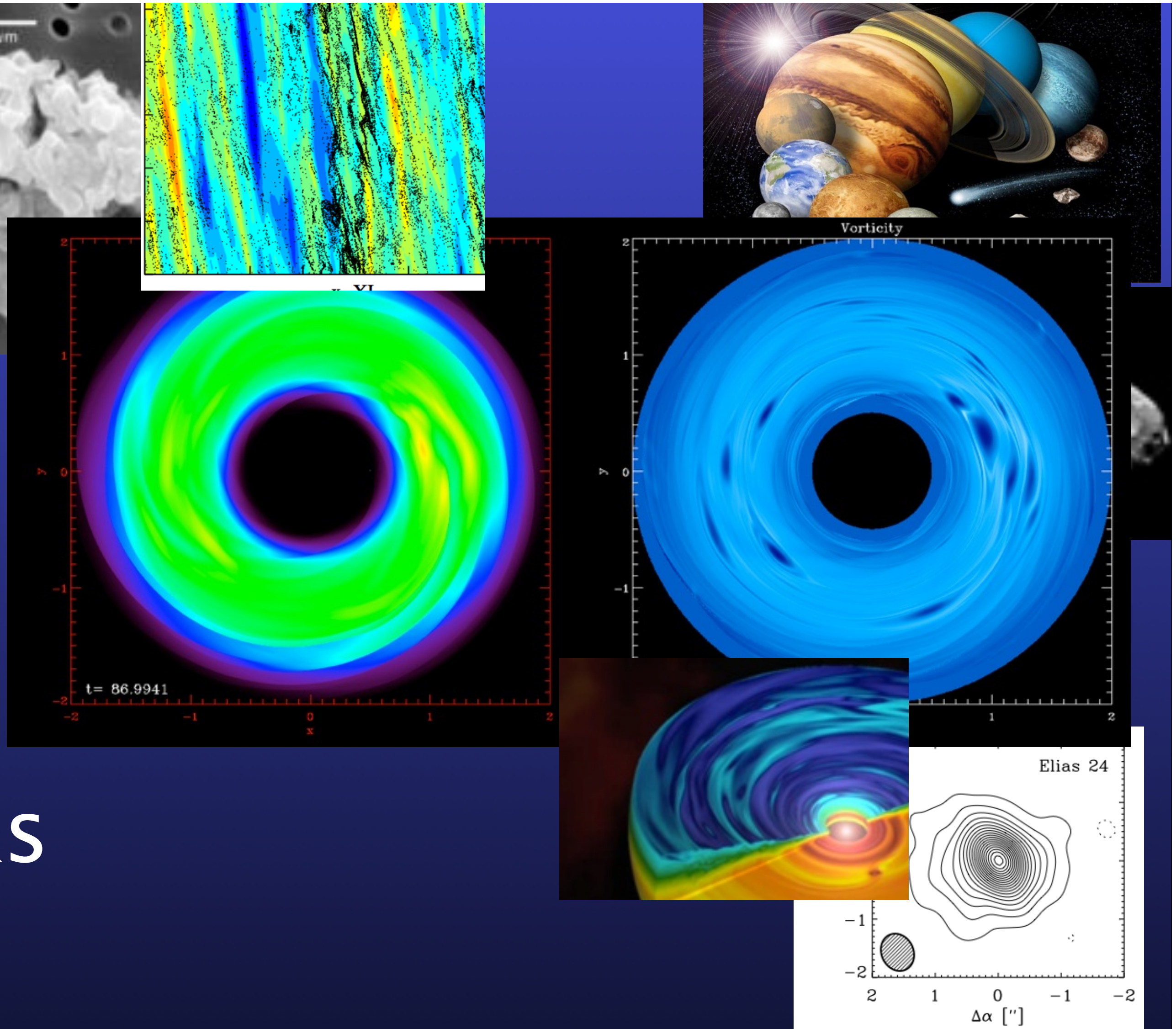


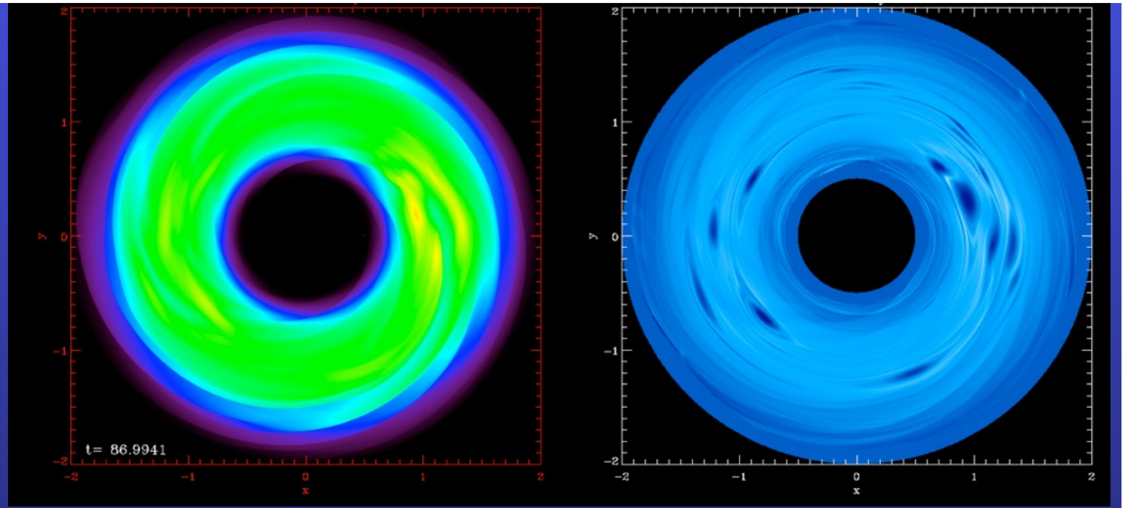


Kopenhagen 2014

Hydrodynamic Stability of Disks

Hubert Klahr, Alex Hubbard, Lingsong Ge, Wlad Lyra, Richard Nelson

Outline:

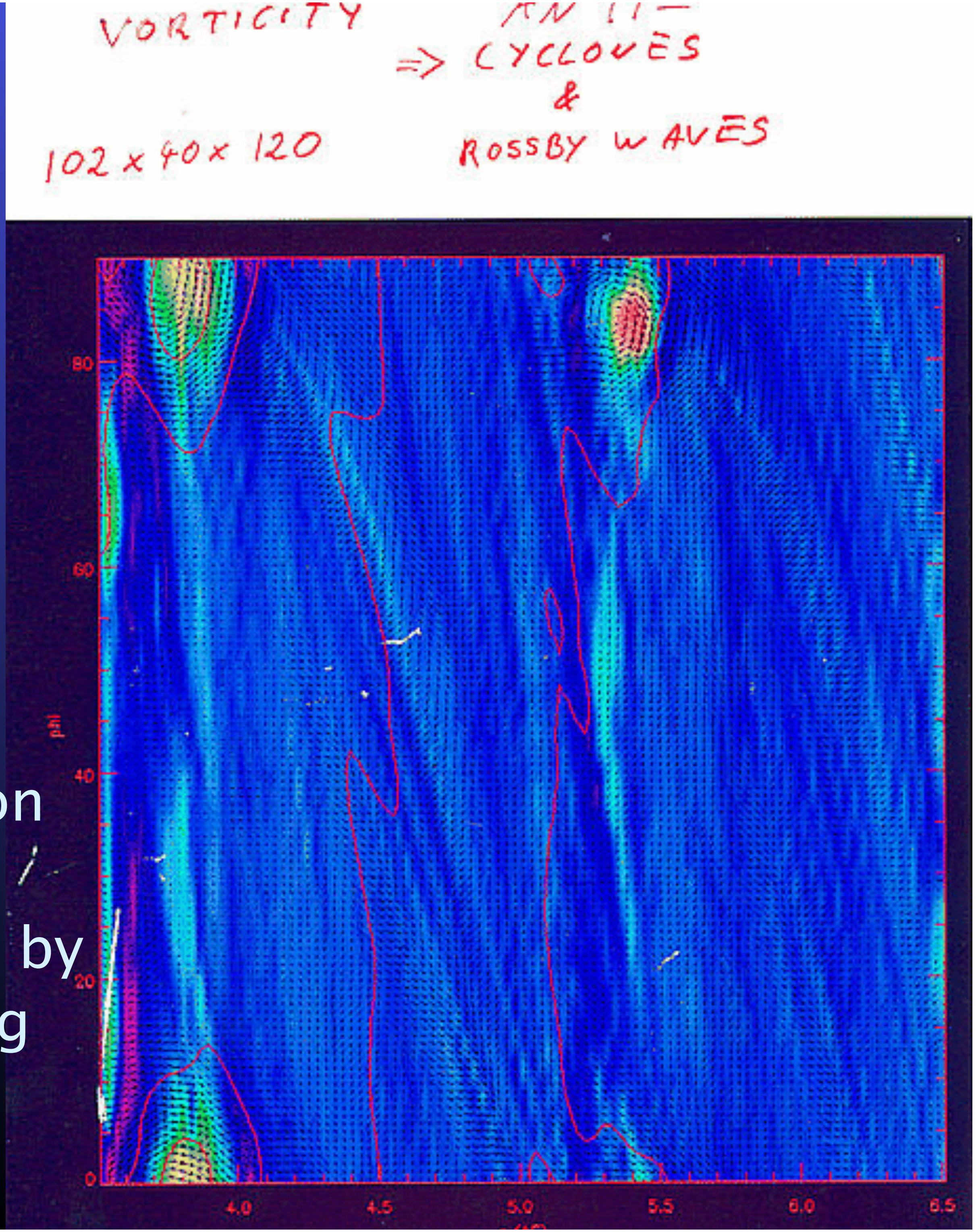


- Thermal Convection
- Global Baroclinic Instability
- Subcritical Baroclinic Instability
- Goldreich-Schubert-Fricke Inst. (VSI -> Richard)
- Convective Overstability
- Linear theory and non-linear Simulations

Simulations of thermal convection in disks:

Large Scale
3D - Simulation
90 degree
3.5 - 6.5 AU
102 X 40 X 120 cells
=> Vortices

3D Global Disk Simulation
flux limited Diffusion
temperature maintained by artificial (viscous) heating



Klahr & Bodenheimer 2003

Radial Entropy Gradient leads to vortices somehow…

Klahr 2004, Johnson & Gammie 2005, etc. :

not a linear instability

Because:

$$Ri^2 = -\frac{2}{3\gamma}\left(\frac{H}{R}\right)^2 \beta_p \beta_s$$

$-0.001 > Ri > -0.01$

Then a lot of discussion started…

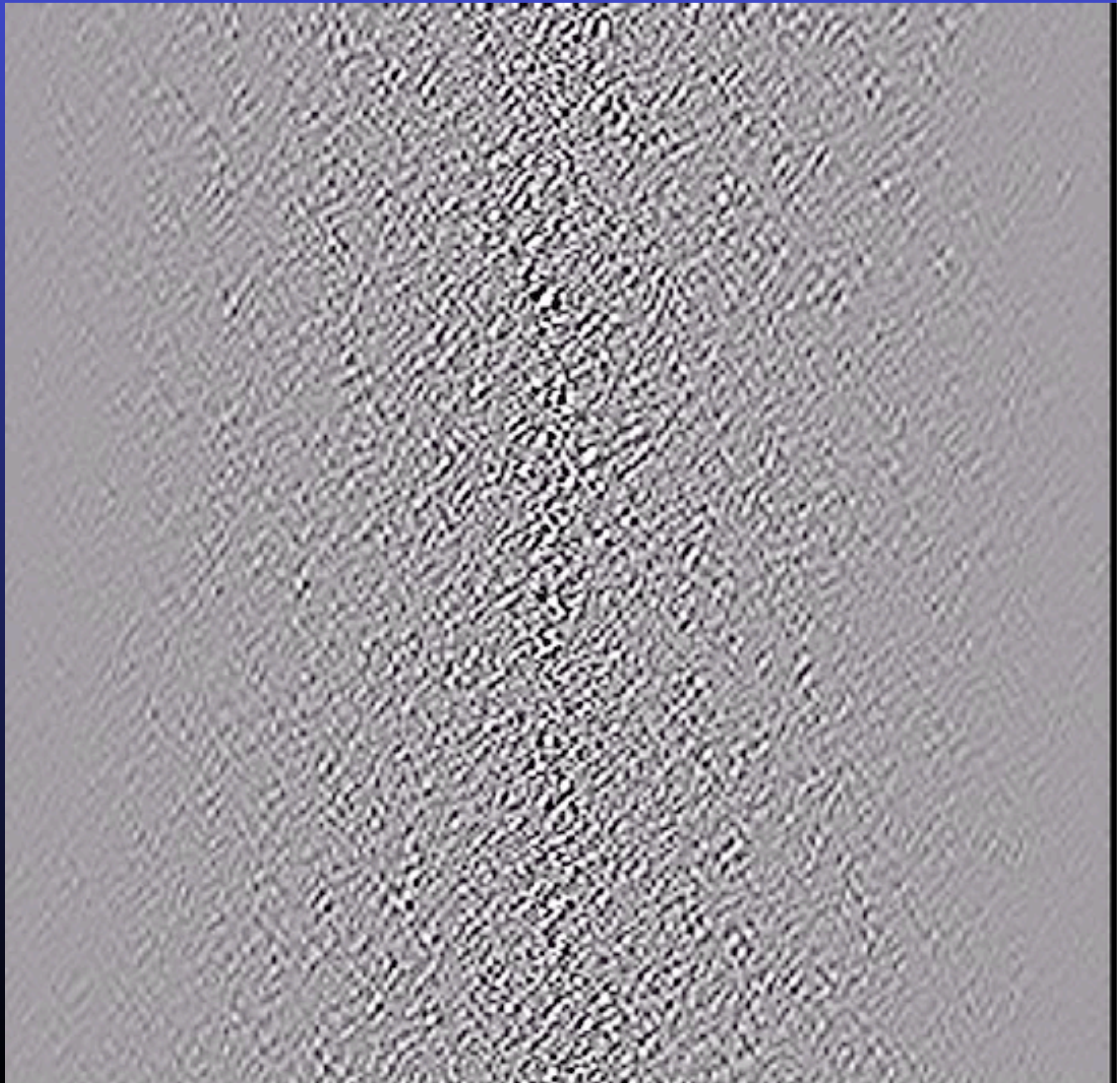
…but 4 years later:

Petersen, Stewart and Julien 2007:

“Works with the right amount of thermal relaxation!”

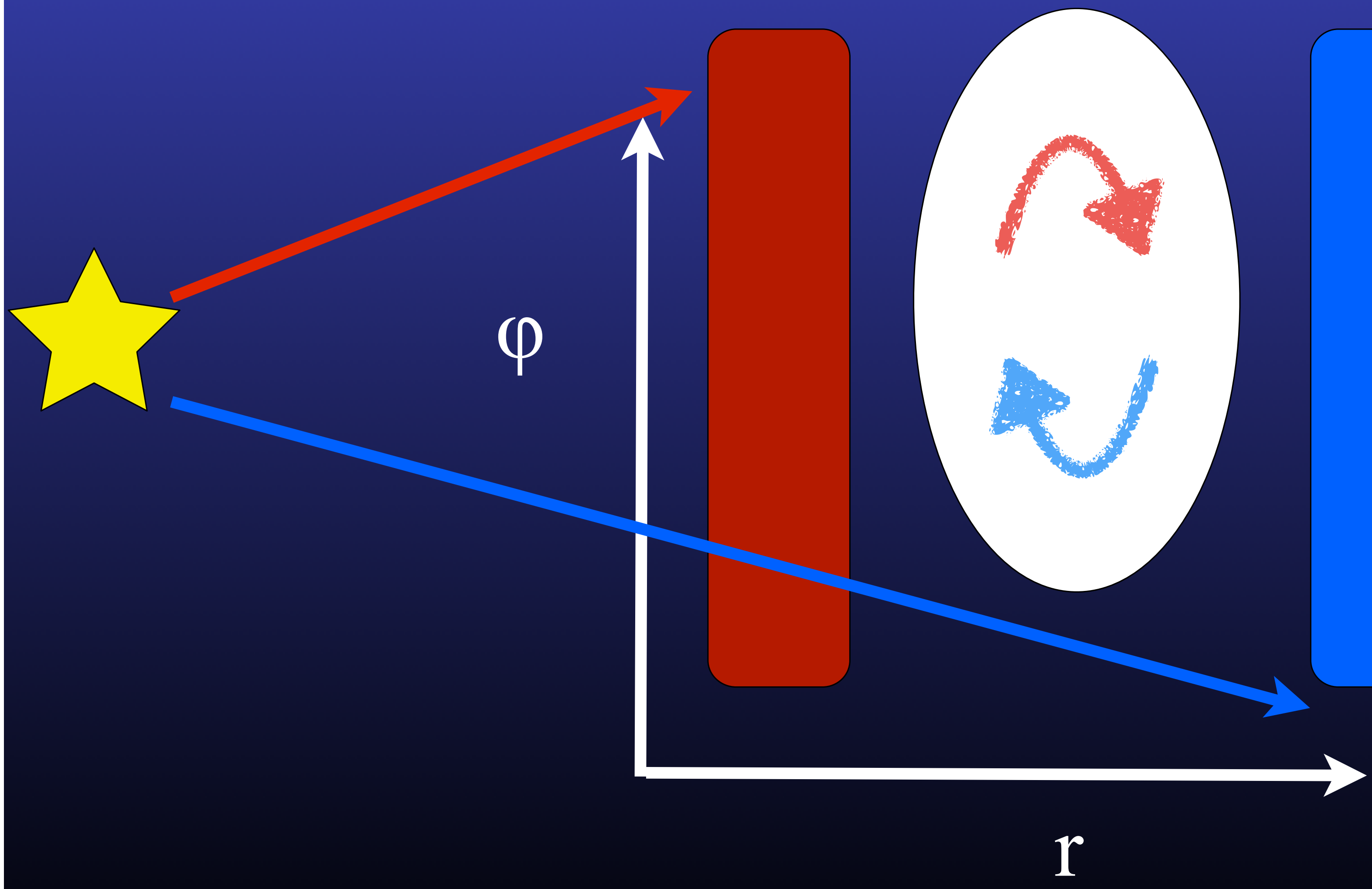
Vorticity

Vorticity: Pencil Code: Lyra and Klahr 2011; $\beta = 2$; $N = 256$; $\tau_c = 1$



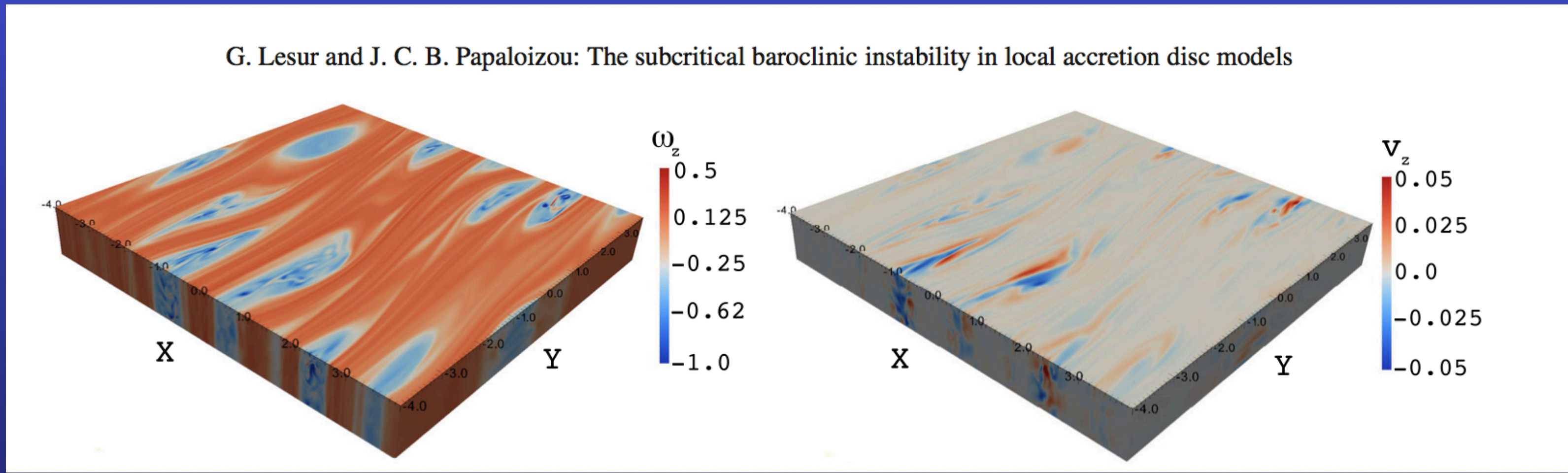
Lesur and Palaloizou 2010:”Subcritical Baroclinic Instability”

Like Convection Cells:

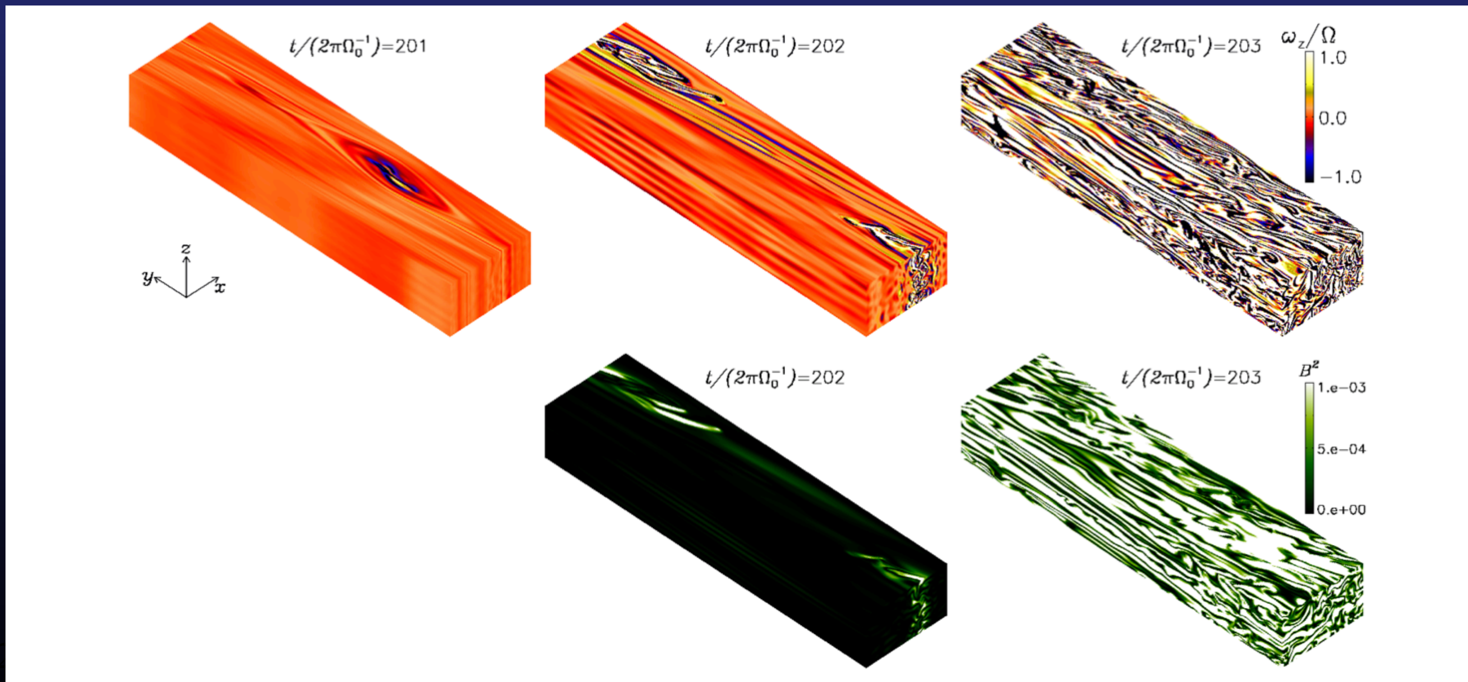


Convective Overstability!

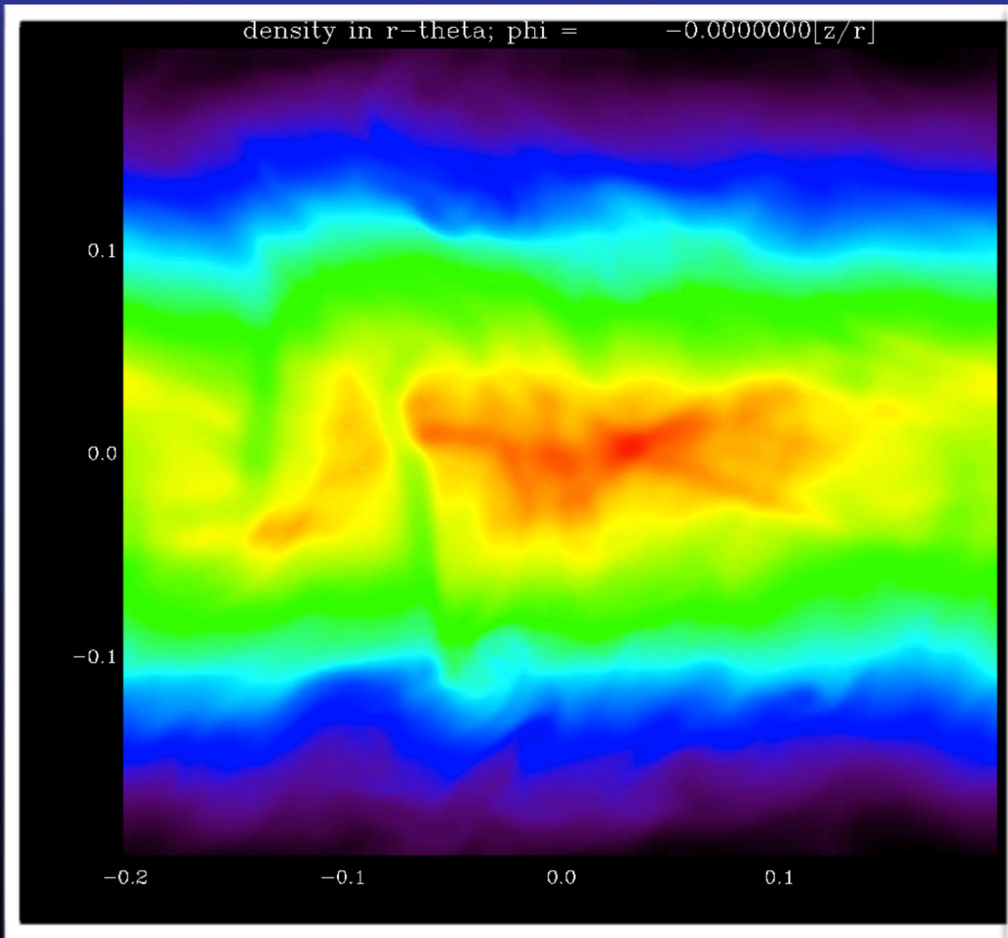
Lesur and Palaloizou 2010: 3D Unstratified Boussinesq



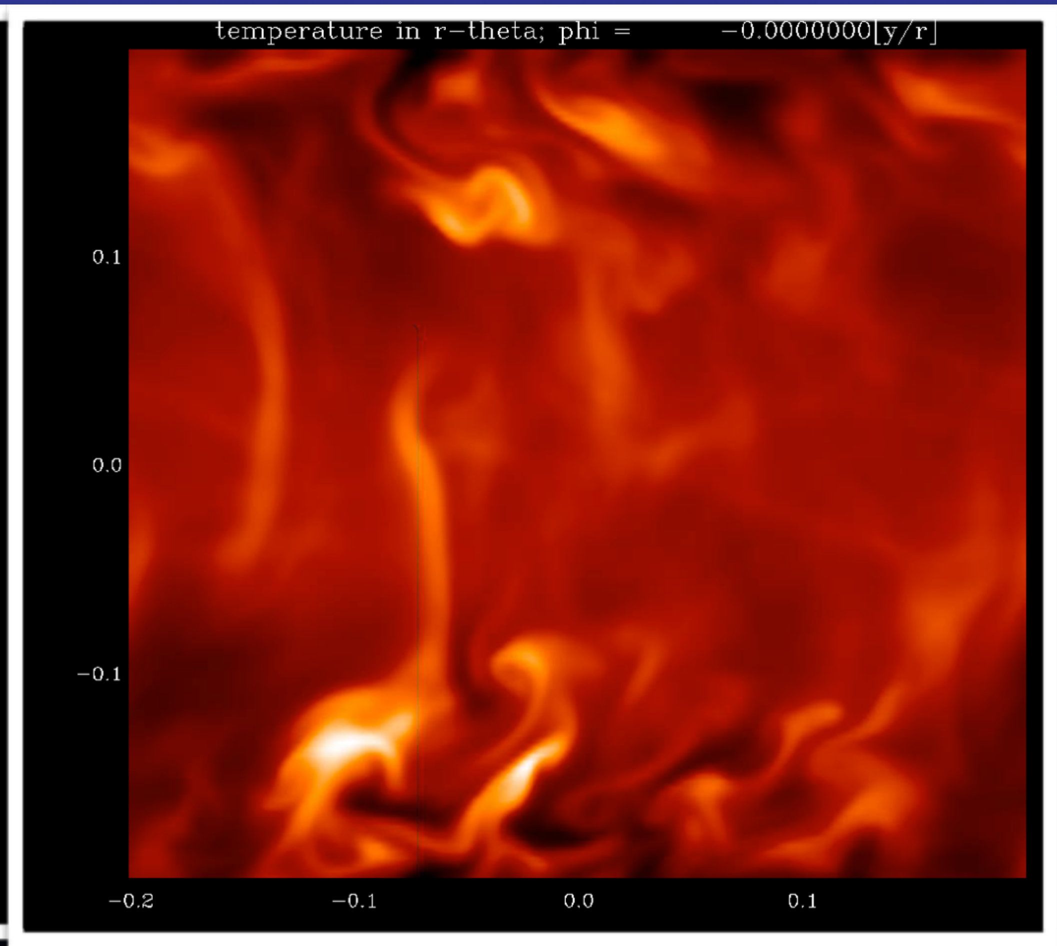
Lyra and Klahr 2011: 3D Unstratified Compressible + MHD



2D Local (radial - vertical), Including thermal wind / vertical shear! How Come?

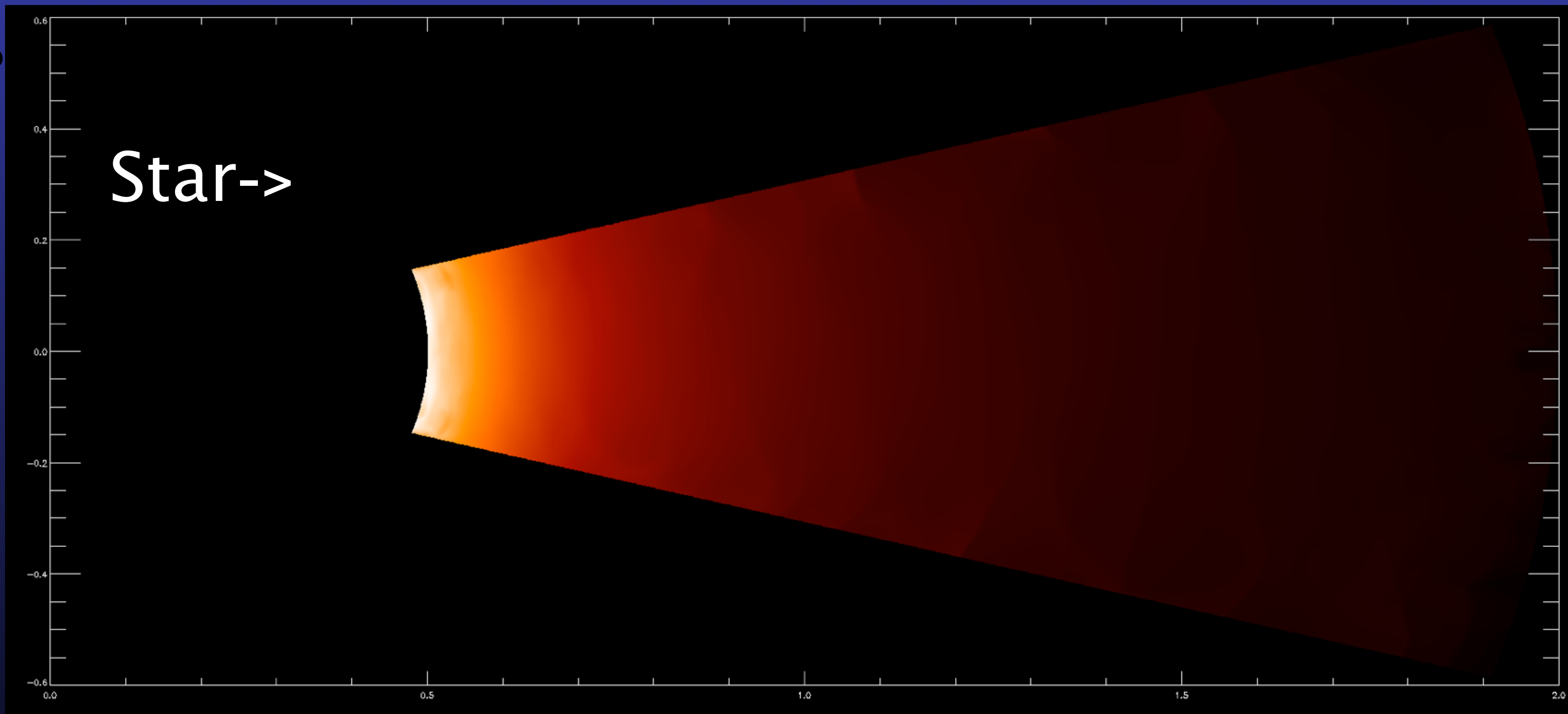


density



temperature pert.

2D axissymetric Pluto Simulation: Temperature due to irradiation from star and thermal relaxation tau = 0.1 (also works for flux limited diffusion in irradiated disks)

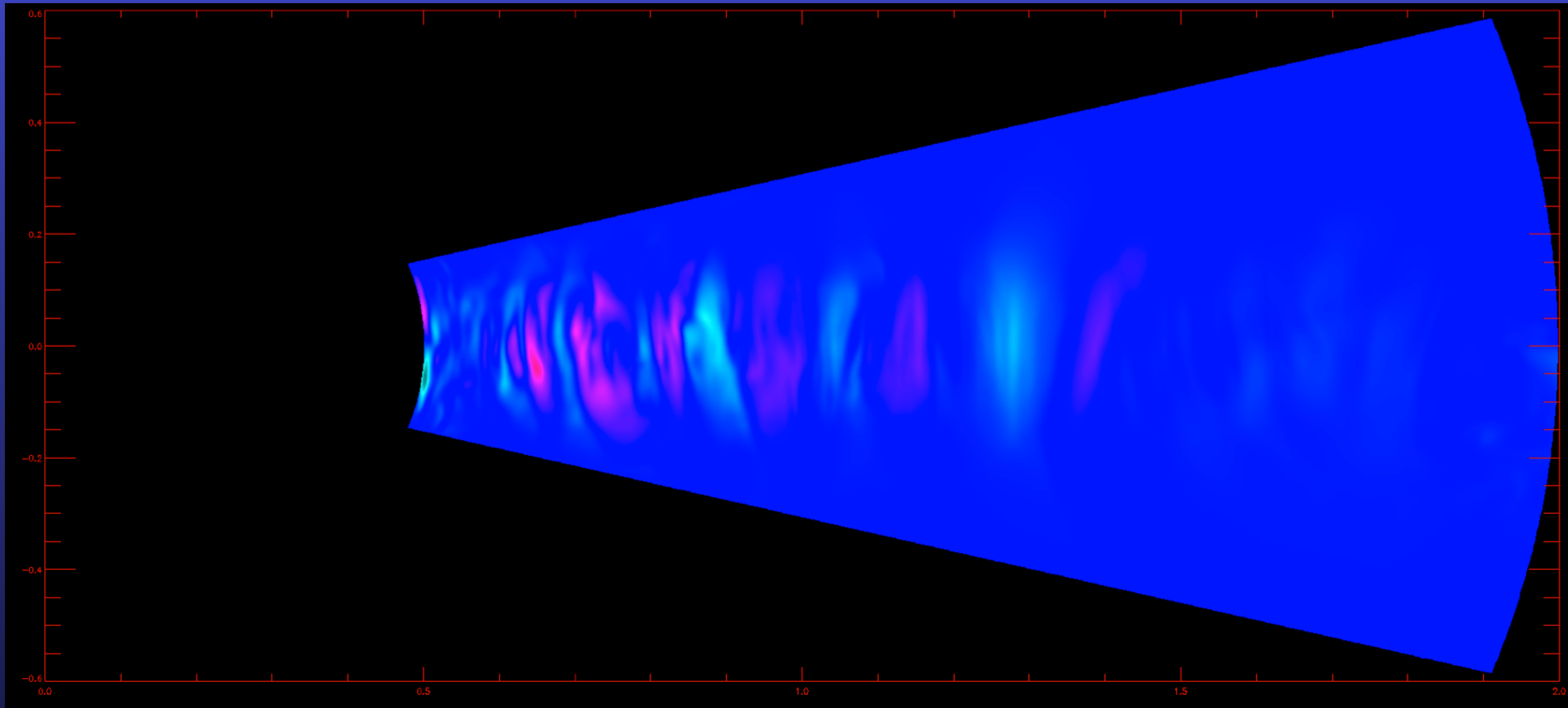
- 

Thermal wind:

$$\Omega_K \left[1 + \frac{1}{2}\left(\frac{H}{R}\right)^2 \left(p + q + \frac{q}{2}\frac{Z^2}{H^2}\right)\right]$$

See Nelson, Gressel & Umurhan 2013

2D axissymetric Pluto Simulation:
Overstability due to thermal wind leads to convection like motion:
Convective Overstability



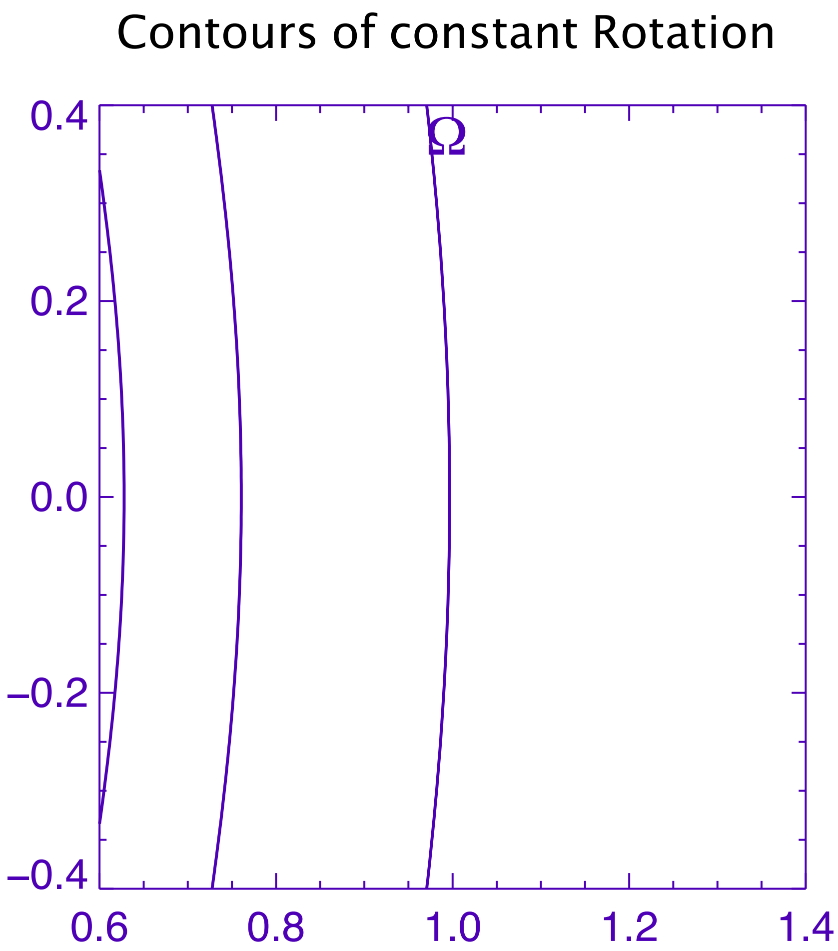
Modification of Solberg-Hoiland Criterion, including thermal relaxation:
In collaboration with Alexander Hubbard
Or instantanous cooling: Goldreich & Schubert 1967 - Fricke 1968 Instability

Linear and nonlinear evolution of the vertical shear instability in accretion discs

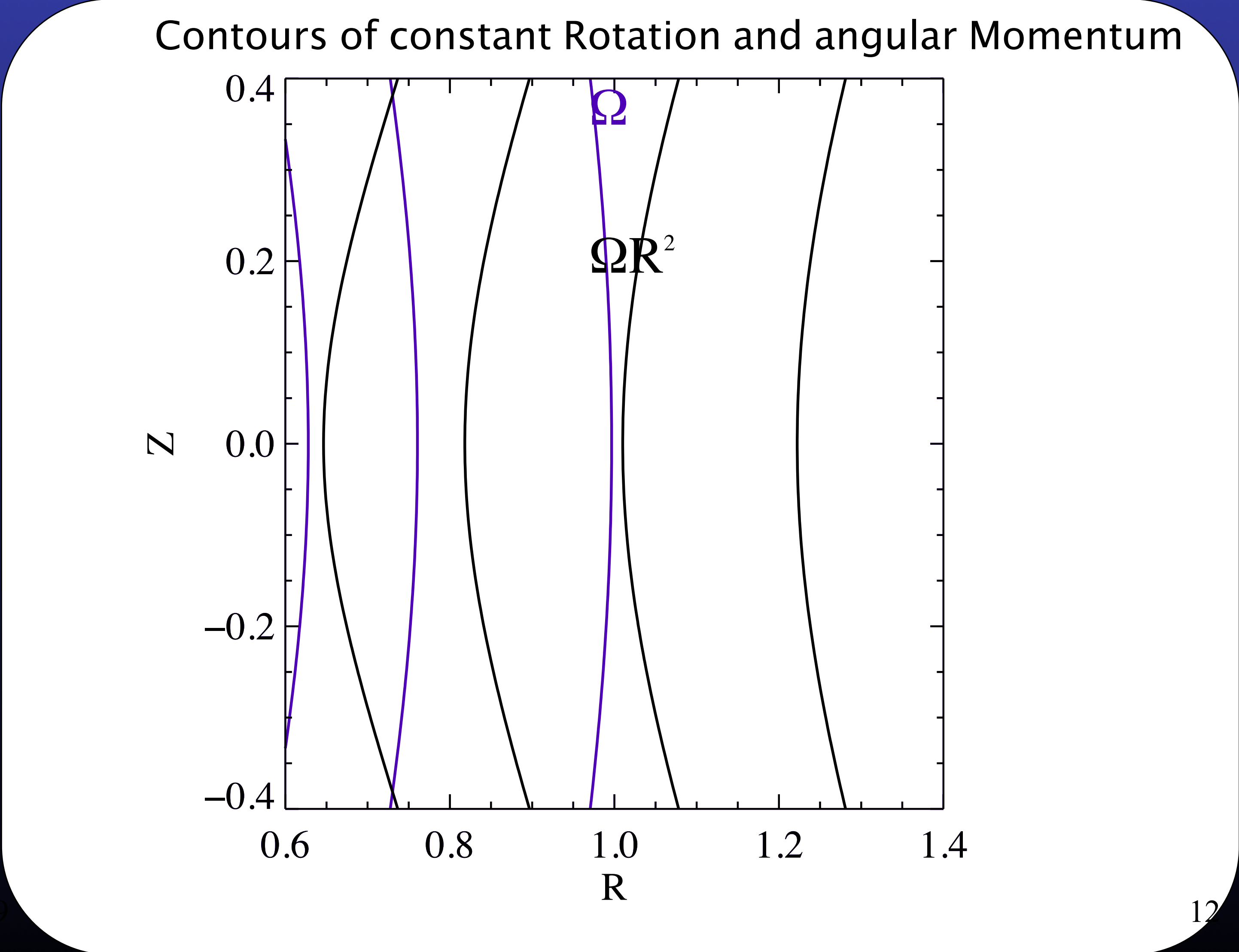
Richard P. Nelson[1⋆], Oliver Gressel[1,2⋆] and Orkan M. Umurhan[1,3⋆]
[1] *Astronomy Unit, Queen Mary University of London, Mile End Road, London E1 4NS*
[2] *NORDITA, KTH Royal Institute of Technology and Stockholm University, Roslagstullsbacken 23, 106 91 Stockholm, Sweden*
[3] *School of Natural Sciences, University of California, Merced, 5200 North Lake Rd, Merced, CA 95343, USA*

$$\frac{\partial j^2}{\partial R} - \frac{k_R}{k_Z}\frac{\partial j^2}{\partial Z} < 0.$$

Disk with radial temp. profile: $T \sim R^{-q}$ with $q = 1$

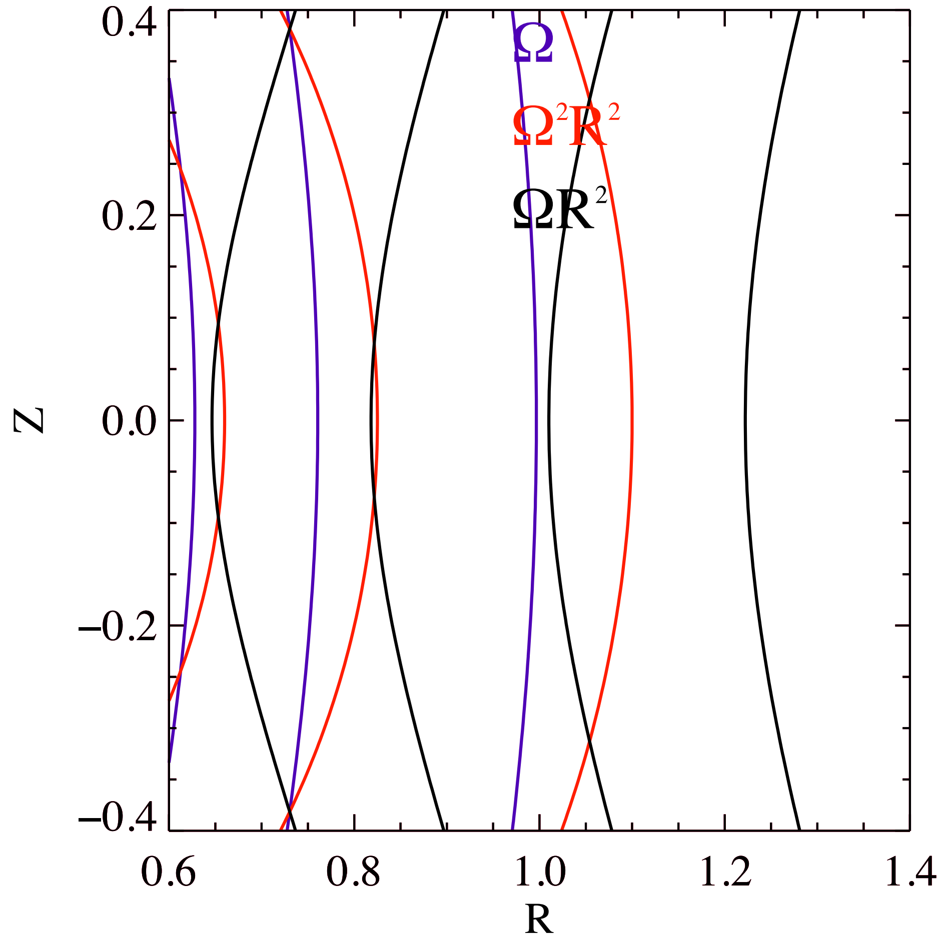


Disk with radial temp. profile: $T \sim R^{-q}$ with $q = 1$



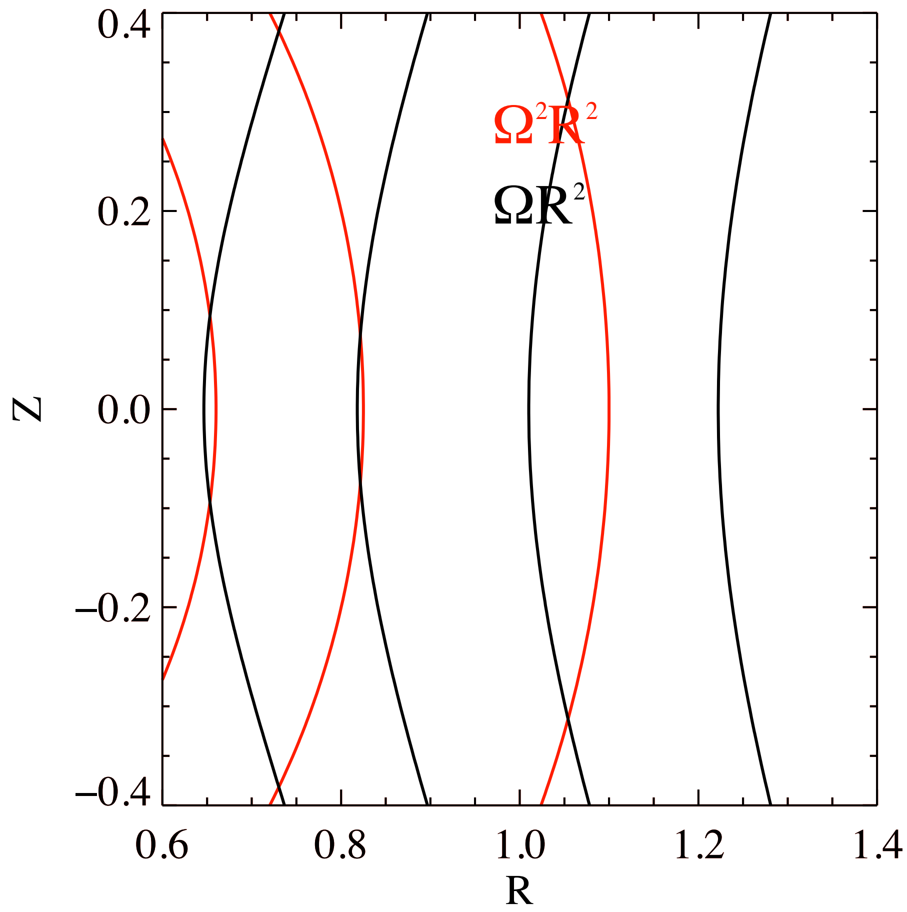
Disk with radial temp. profile: $T \sim R^{-q}$ with $q = 1$

Contours of constant Rotation and angular Momentum and kinetic Energy



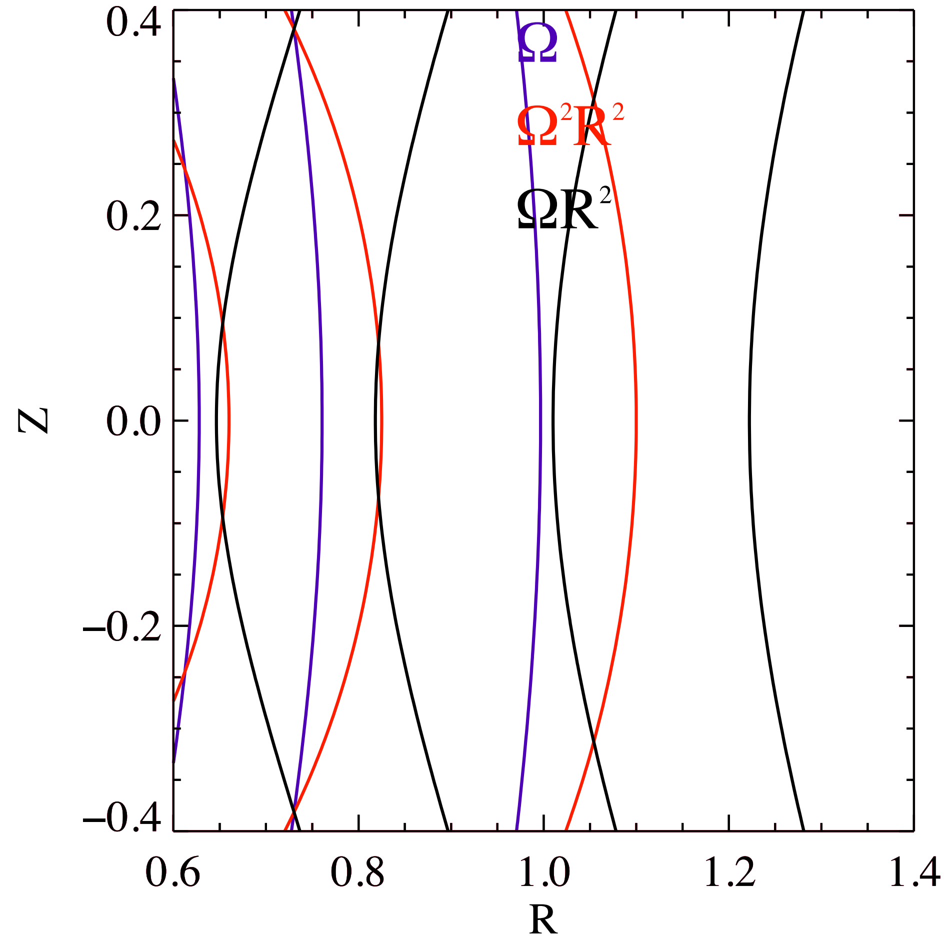
Disk with radial temp. profile: $T \sim R^{-q}$ with $q = 1$

Contours of constant Rotation and angular Momentum and kinetic Energy

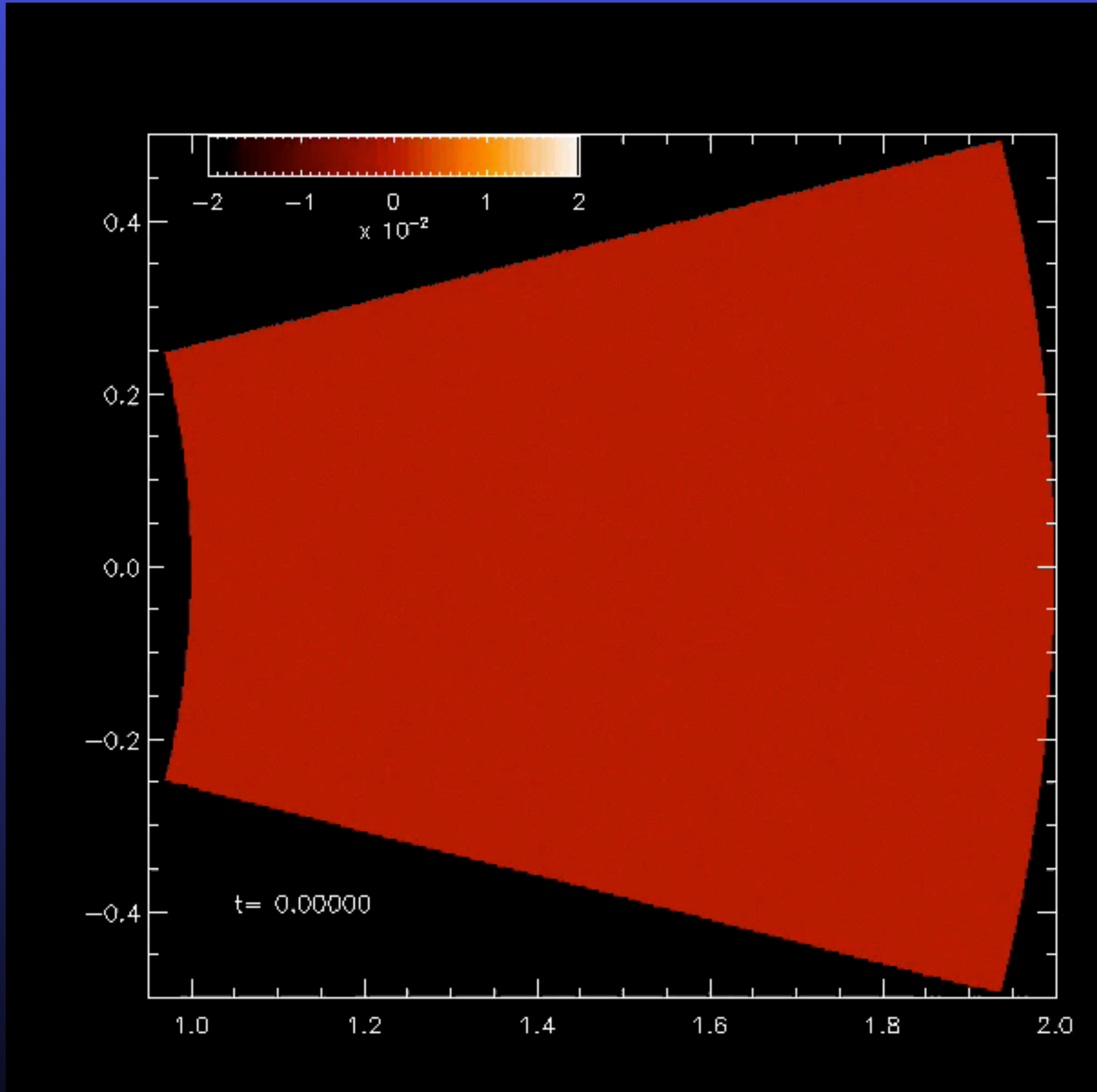


Disk with radial temp. profile: $T \sim R^{-q}$ with $q = 1$

Contours of constant Rotation and angular Momentum and kinetic Energy



$$\frac{\partial j^2}{\partial R} - \frac{k_R}{k_Z}\frac{\partial j^2}{\partial Z} < 0.$$



Movie by Richard Nelson

Is VSI / GSF already at stage 3?

We assume the grains are well mixed with the gas by either turbulent motion generated by convection, or effects like meridional circulation or Goldreich–Schubert–Fricke instabilities in radiative regions (Goldreich & Schubert 1967; Fricke 1968).

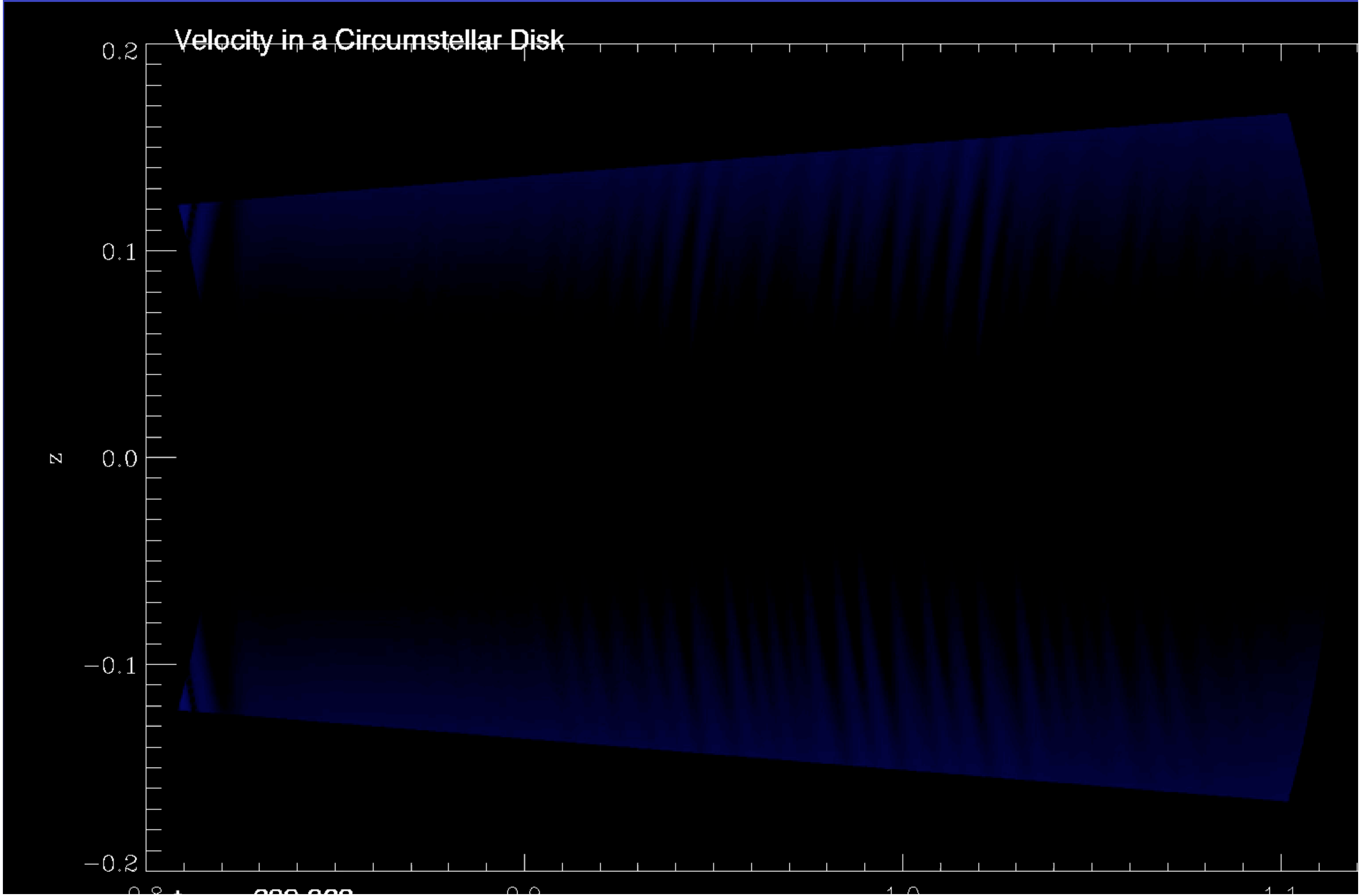
Mon. Not. R. astr. Soc. (1980) **191**, 37–48

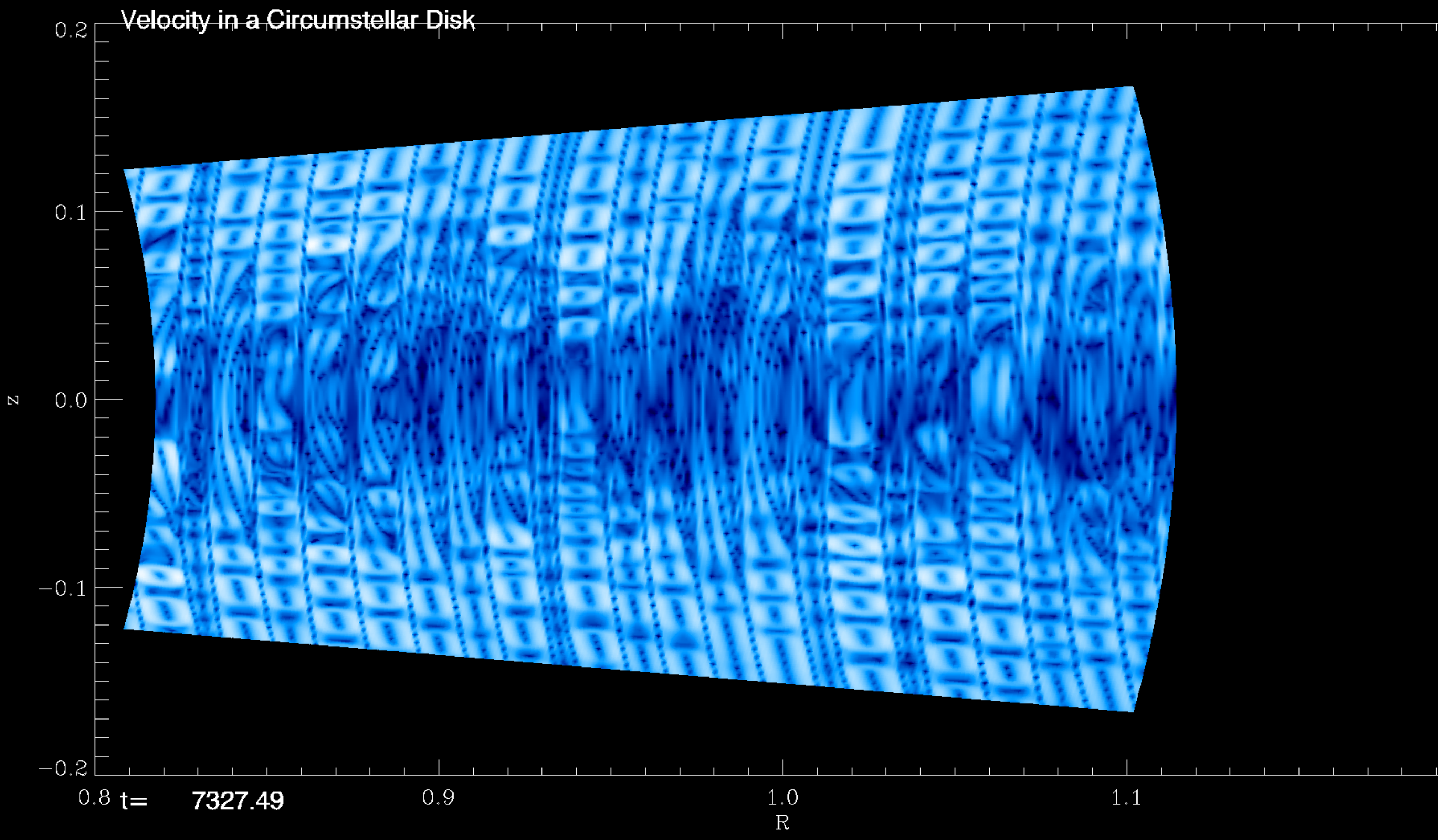
On the structure and evolution of the primordial solar nebula

D. N. C. Lin and J. Papaloizou *Institute of Astronomy, Madingley Road, Cambridge CB3 0HA and Board of Studies in Astronomy and Astrophysics, University of California, Santa Cruz, CA 95064, USA*

Received 1979 July 30; in original form 1979 April 20

G.S.F. also for $\Omega\tau \approx 10$?




Velocity in a Circumstellar Disk
0.2
0.1
0.0
−0.1
−0.2
z
0.8
0.9
1.0
1.1
R
t= 7327.49

Stability of vertically unstratified disk

(Klahr and Hubbard 2014)

Stability under the influence of thermal relaxation

$$\Gamma = \frac{1}{2} \frac{-\tau N_R^2}{1 + \tau^2 \left(\kappa_R^2 + N_R^2\right)}$$

$$\Gamma = \frac{1}{2} \frac{-\frac{l^2}{\mu} N_R^2}{1 + \left(\frac{l^2}{\mu}\right)^2 \left(\kappa_R^2 + N_R^2\right)} - \frac{\nu}{l^2}$$

2010 Similar to Lesur and Papaloizou 2010 for finite size vortices

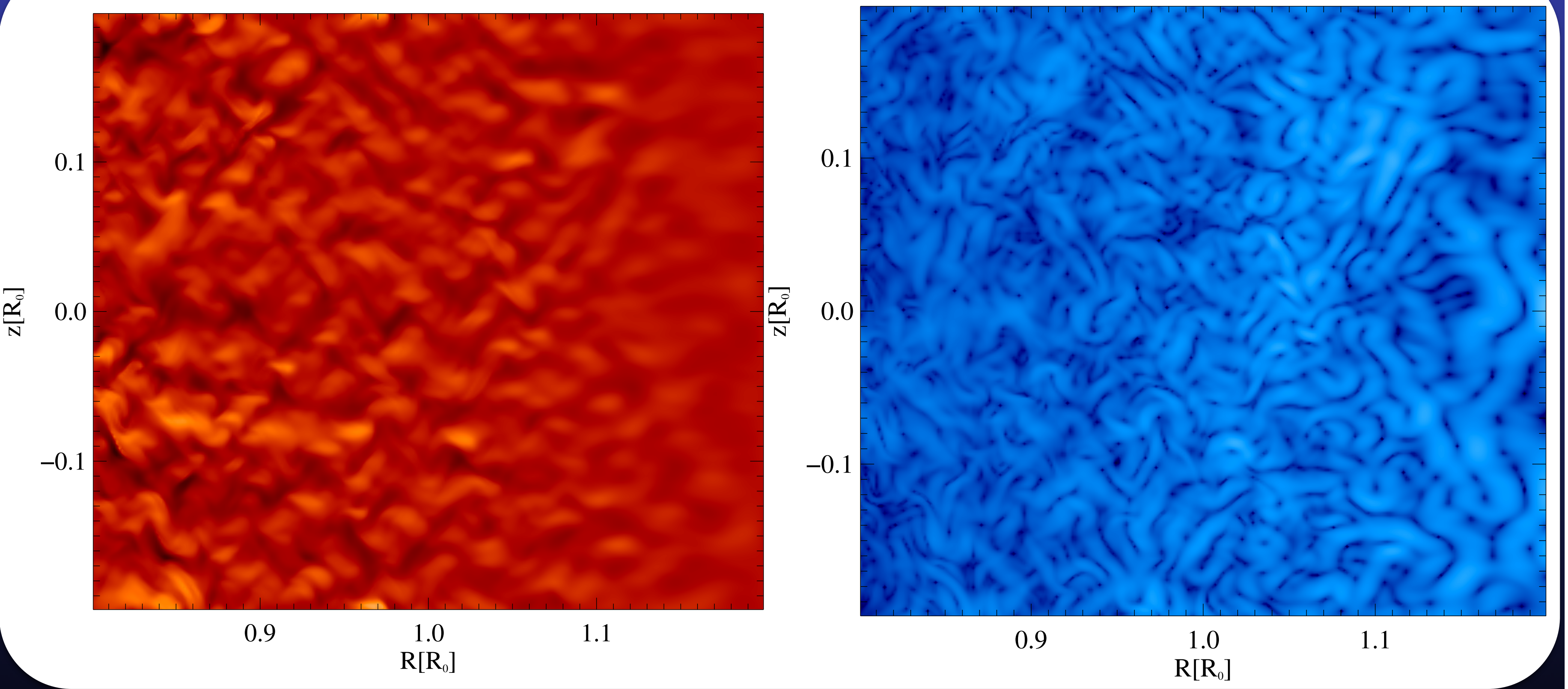
$$\gamma \sim \frac{(-N^2)\sigma^2}{\mu} \phi_\omega(S\sigma^2/\mu) - \frac{\nu}{\sigma^2}.$$

Convective Overstability in radially stratified accretion disks under thermal relaxation

Hubert Klahr[1] & Alexander Hubbard[2]

klahr@mpia.de

ApJ in press



Convective Overstability in radially stratified accretion disks under thermal relaxation

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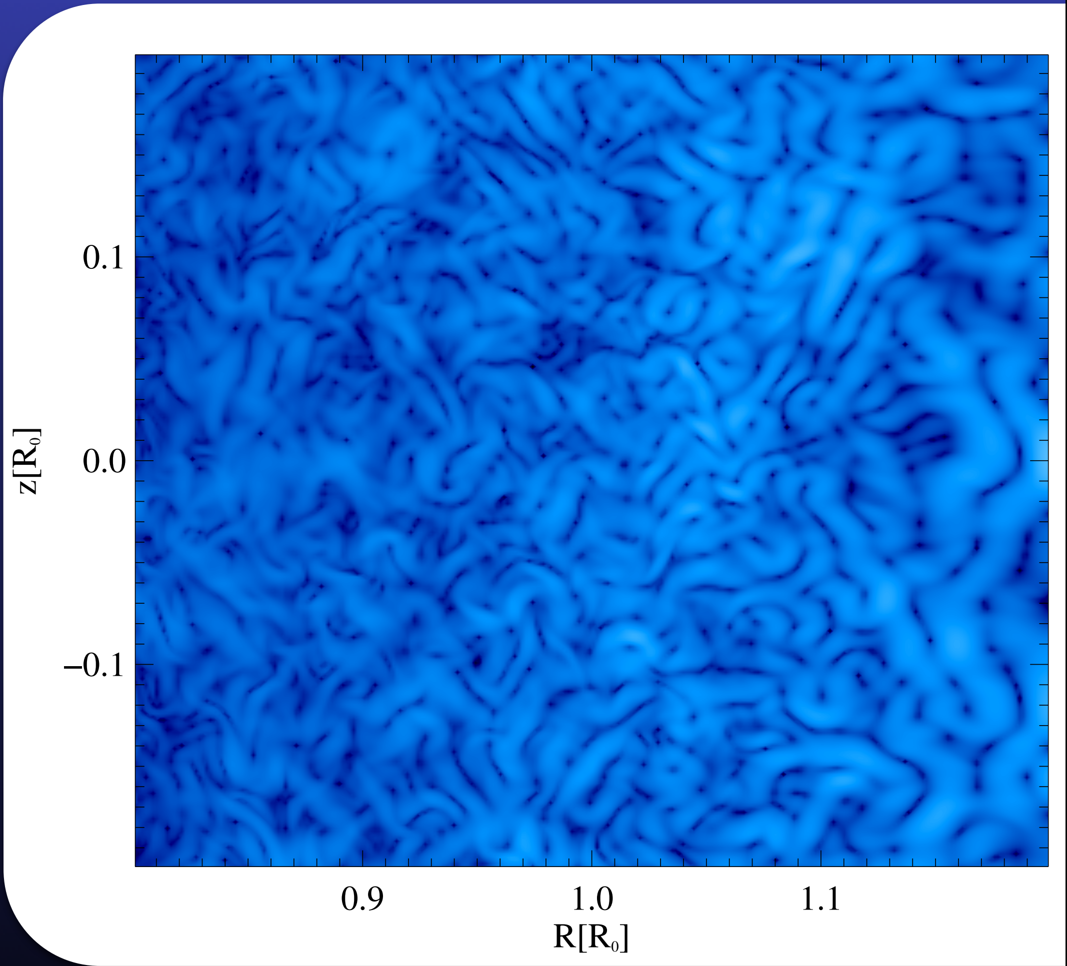
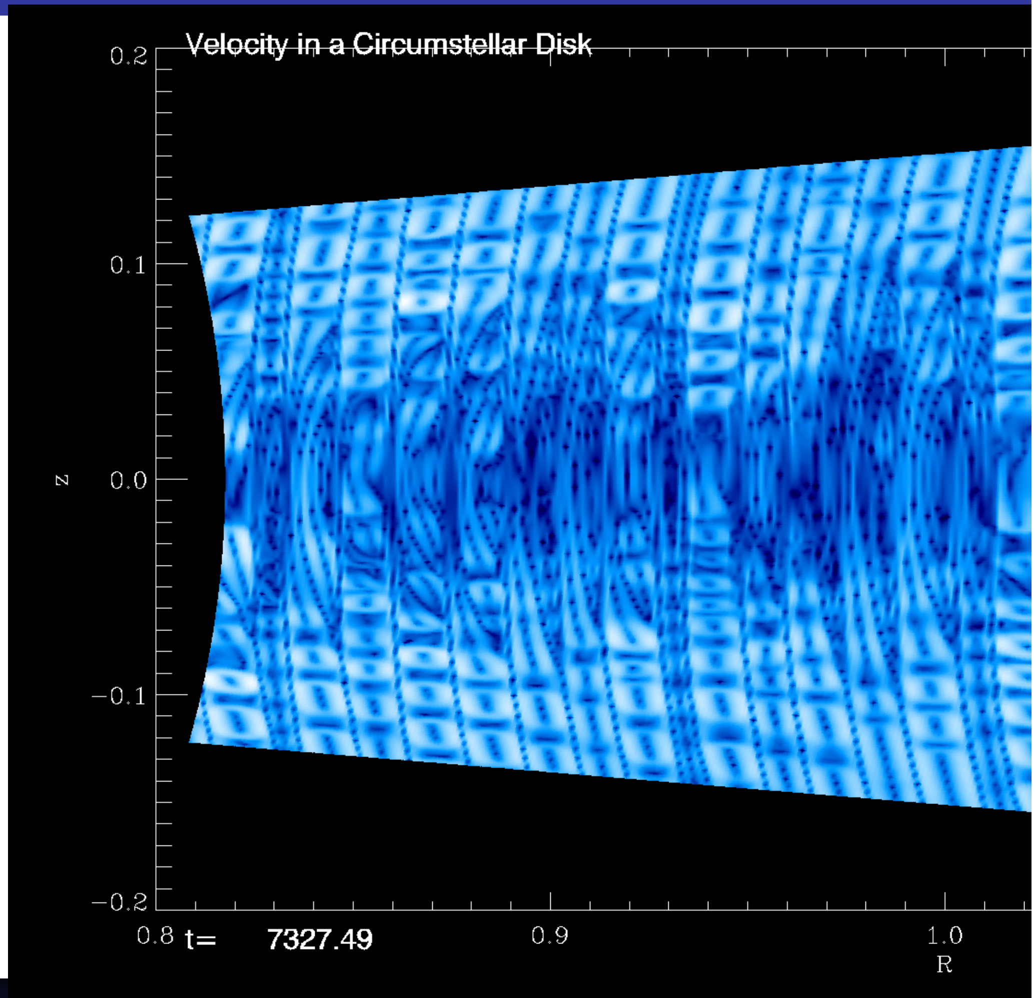
ApJ in press



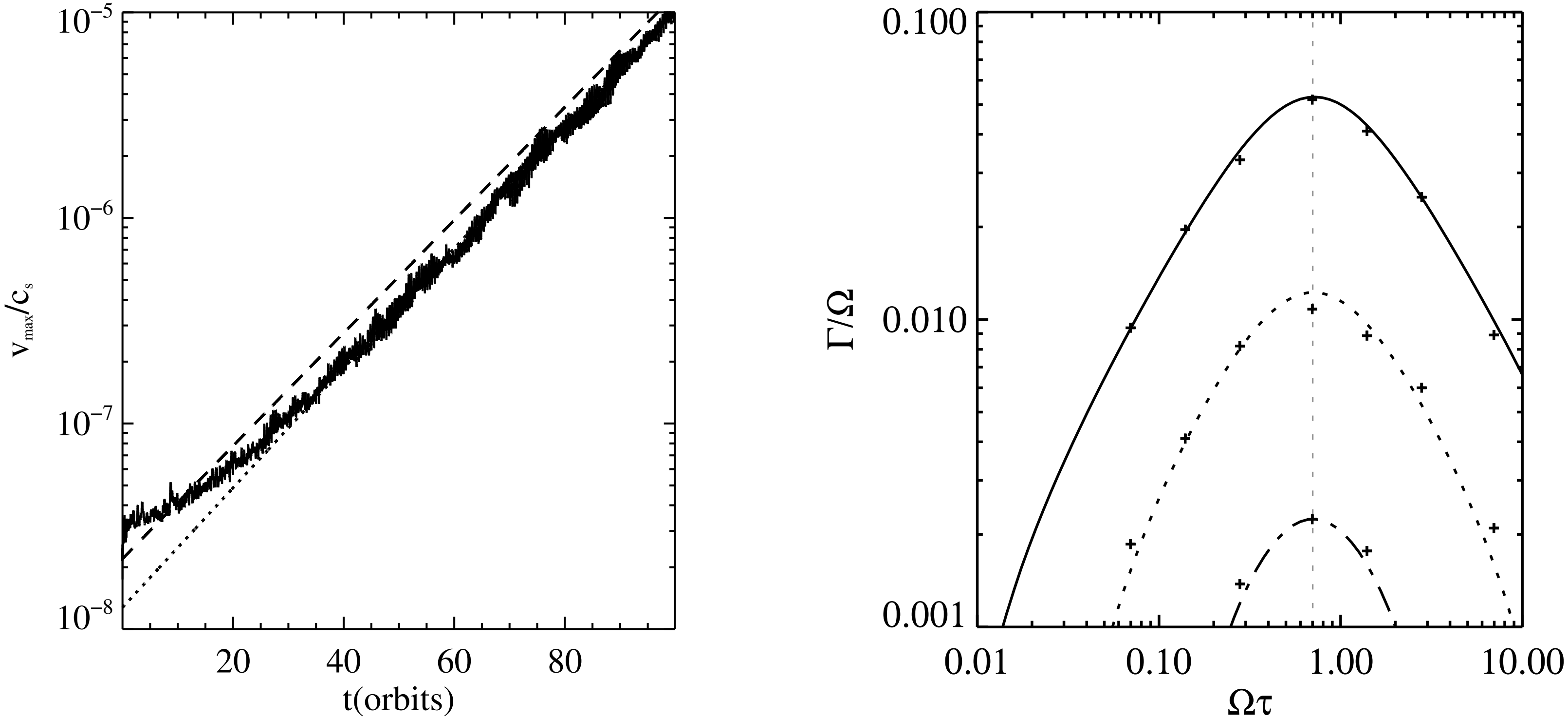


Unstratified

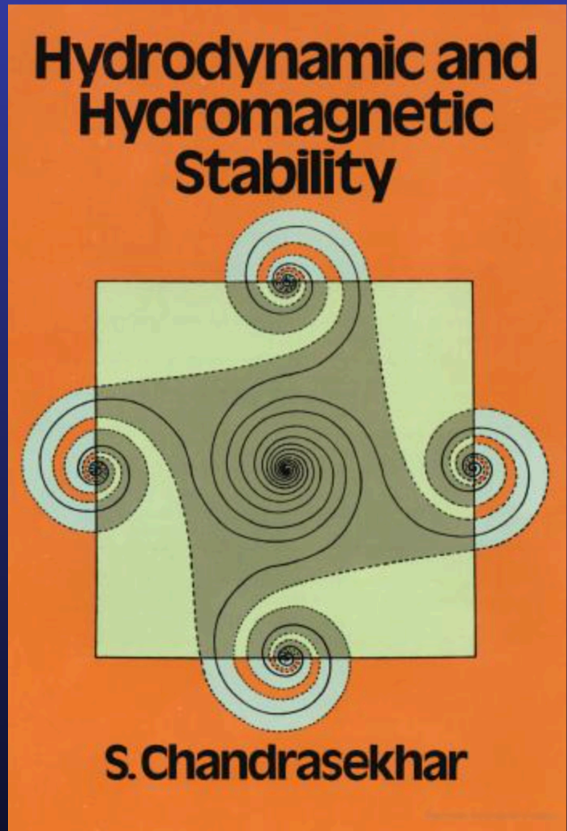
Stratified

Analytic and Numerical results:

Klahr and Hubbard 2014



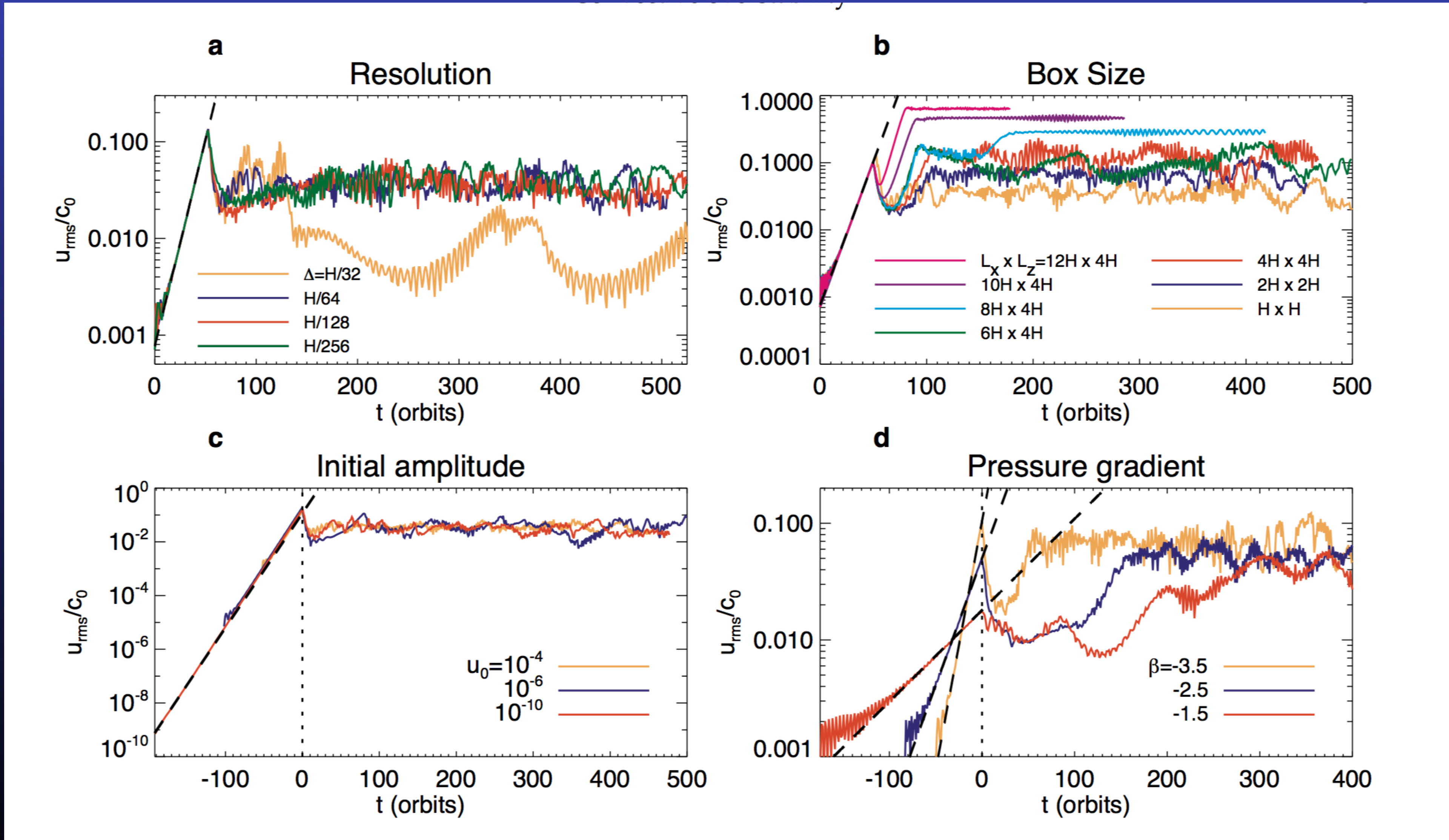
This guy knew it, but only investigated the $\hat{g}$ parallel Ω case.



...the other angles will be similar to the MHD case...

CONVECTIVE OVERSTABILITY IN ACCRETION DISKS
3D LINEAR ANALYSIS AND NONLINEAR SATURATION

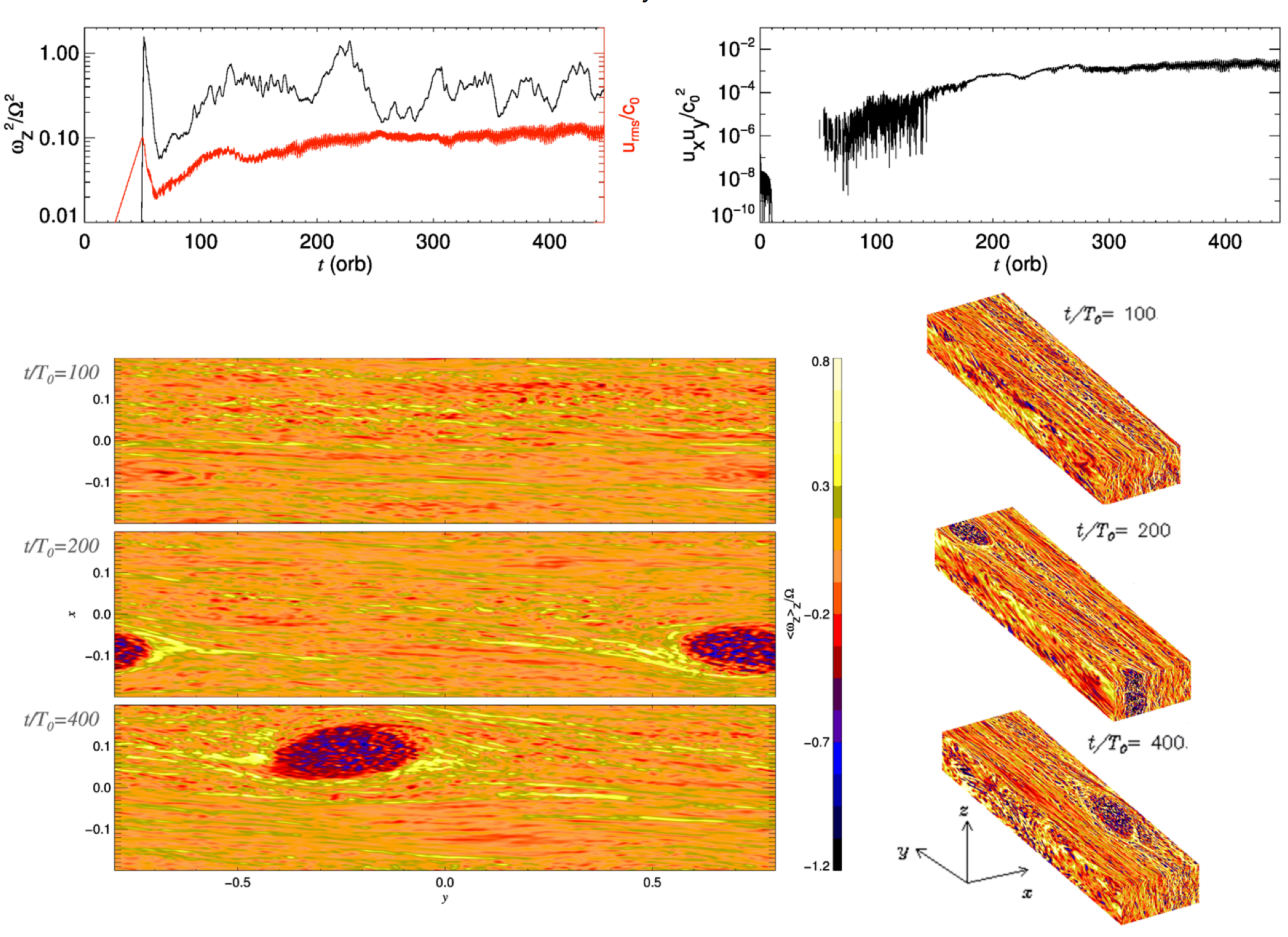
WLADIMIR LYRA[1,2,3] ApJ 2014



CONVECTIVE OVERSTABILITY IN ACCRETION DISKS
3D LINEAR ANALYSIS AND NONLINEAR SATURATION

WLADIMIR LYRA[1,2,3] ApJ 2014

Lyra



Stability in vertically and radially stratified accretion disks under thermal relaxation

Hubert Klahr[1] Lingsong Ge[1] & Alexander Hubbard [2]

$$\partial_t \rho + \frac{1}{R}\partial_R R\rho u_R + \frac{1}{R}\partial_\phi \rho u_\phi + \partial_z \rho u_z = 0.$$

$$\partial_t S + u_R \partial_R S + \frac{u_\phi}{R}\partial_\phi S + u_z \partial_z S = -\frac{c_v}{T_0}\frac{T - T_0}{\tau}.$$

$$\partial_t u_R + u_R \partial_R u_R + \frac{u_\phi}{R}\partial_\phi u_R + u_z \partial_z u_R - \frac{u_\phi^2}{R} = -\frac{1}{\rho}\partial_R p + g_R$$

$$\partial_t u_z + u_R \partial_R u_z + \frac{u_\phi}{R}\partial_\phi u_z + u_z \partial_z u_z = -\frac{1}{\rho}\partial_z p + g_z$$

$$\partial_t u_\phi + u_R \partial_R u_\phi + \frac{u_\phi}{R}\partial_\phi u_\phi + u_z \partial_z u_\phi + \frac{u_\phi u_R}{R} = -\frac{1}{R\rho}\partial_\phi p,$$

plain waves $\exp[i(k_r R + k_z z + m\phi - \omega t)]$.

$$-i(\omega - m\Omega)u_R - 2\Omega u_\phi + ik_R\frac{p_1}{\rho_0} - \frac{\rho_1}{\rho_0^2}\partial_R p_0 = 0$$

$$-i(\omega - m\Omega)u_\phi + \frac{1}{R}\partial_R(R^2\Omega)u_R + R\partial_z(\Omega)u_z = 0$$

$$-i(\omega - m\Omega)u_z + ik_z\frac{p_1}{\rho_0} - \frac{\rho_1}{\rho_0^2}\partial_z p_0 = 0$$

$$-i(\omega - m\Omega)\frac{\rho_1}{\rho_0} + \partial_R \log\rho_0 u_R + \partial_z \log\rho_0 u_z + ik_R u_R + ik_z u_z = 0$$

and

$$\left(-i(\omega - m\Omega) + \frac{1}{\tau}\right)\frac{p_1}{p_0} - \left(-i(\omega - m\Omega) + \frac{1}{\gamma\tau}\right)\gamma\frac{\rho_1}{\rho_0} + u_R\frac{1}{c_v}\partial_R S_0 + u_z\frac{1}{c_v}\partial_z S_0 = 0$$

A&A 391, 781–787 (2002)
DOI: 10.1051/0004-6361:20020853

Astronomy & Astrophysics

Hydrodynamic stability in accretion disks under the combined influence of shear and density stratification

G. Rüdiger[1], R. Arlt[1], and D. Shalybkov[1,2]

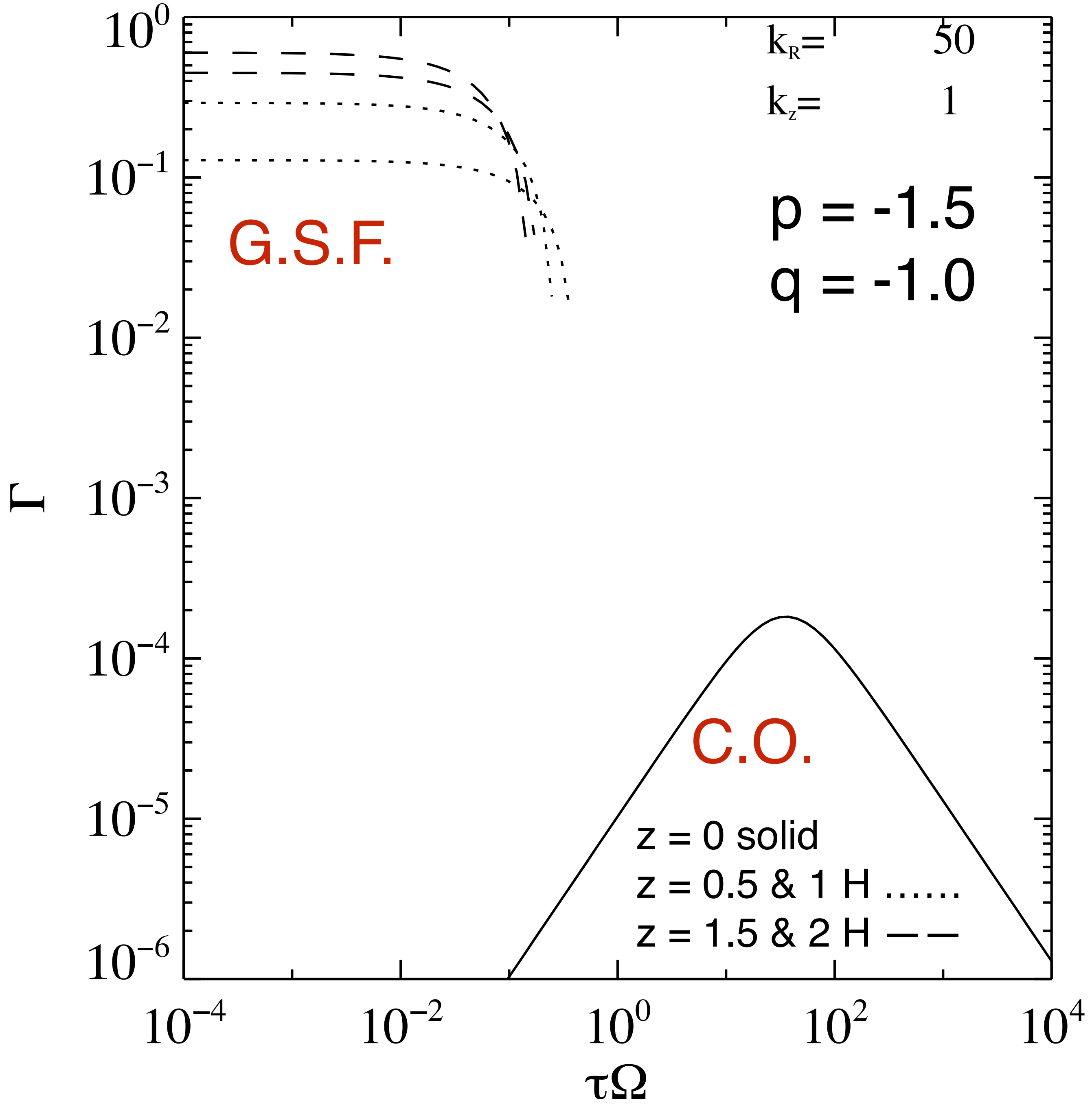
$$\omega_m^5 + \frac{i}{\tau}\omega_m^4 - A\omega_m^3 + B\frac{i}{\tau}\omega_m^2 + C\omega_m + \frac{i}{\tau}D = 0$$

$$A = k^2 c_s^2 + \frac{1}{\rho_0^2}\frac{\partial p_0}{\partial R}\frac{\partial \rho_0}{\partial R} + \frac{1}{\rho_0^2}\frac{\partial p_0}{\partial z}\frac{\partial \rho_0}{\partial z} + \kappa_R^2$$

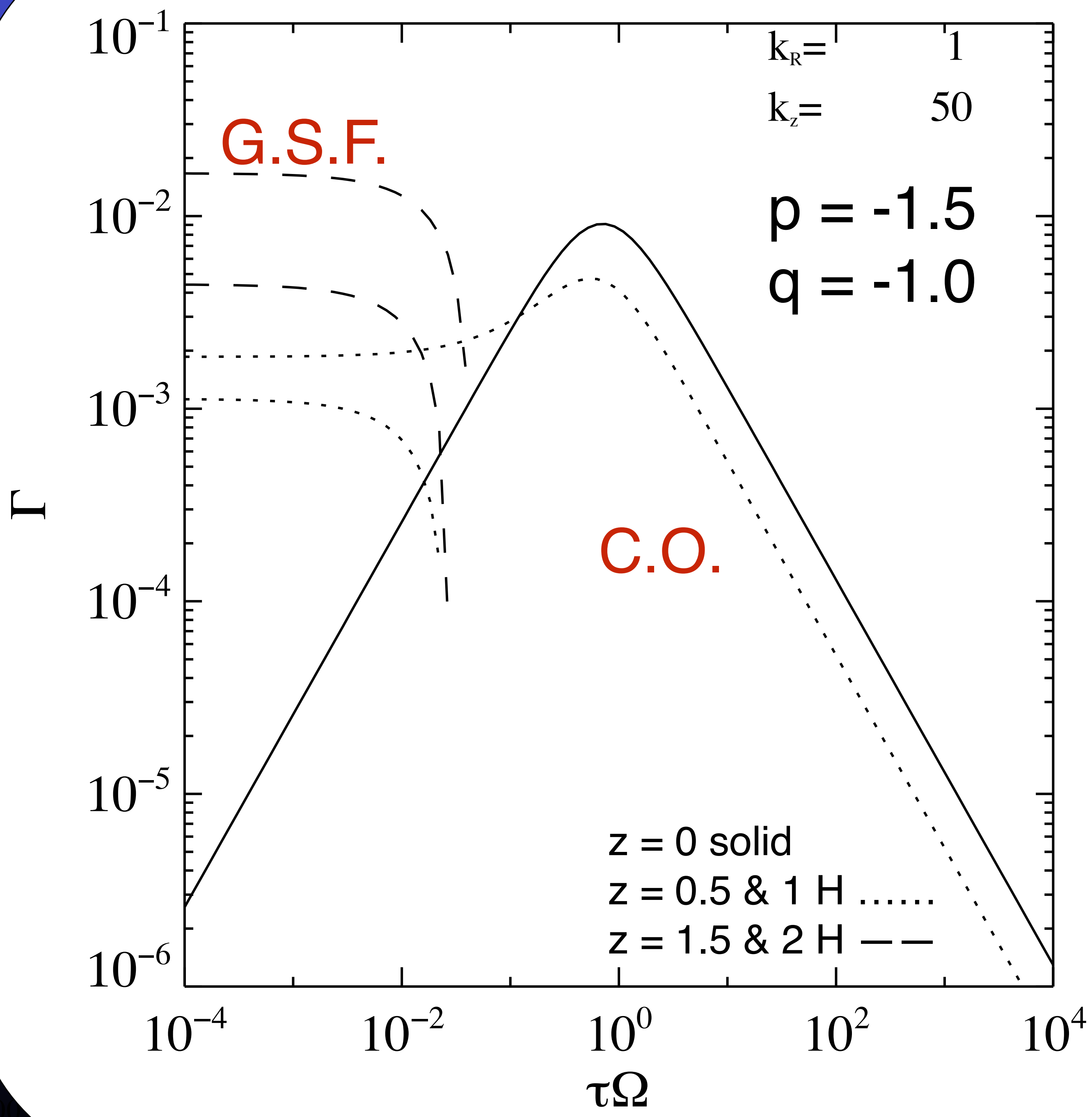
$$\begin{aligned} B =& - i\left(\frac{k_z}{\rho_0}\frac{\partial p_0}{\partial z} + \frac{k_R}{\rho_0}\frac{\partial p_0}{\partial R} - \frac{k_z c_s^2}{\rho_0 \gamma}\frac{\partial \rho_0}{\partial z} - \frac{k_R c_s^2}{\rho_0 \gamma}\frac{\partial \rho_0}{\partial R}\right) \\ & - \left(\frac{k^2 c_s^2}{\gamma} + \frac{1}{\rho_0^2}\frac{\partial p_0}{\partial R}\frac{\partial \rho_0}{\partial R} + \frac{1}{\rho_0^2}\frac{\partial p_0}{\partial z}\frac{\partial \rho_0}{\partial z} + \kappa_R^2\right) \end{aligned}$$

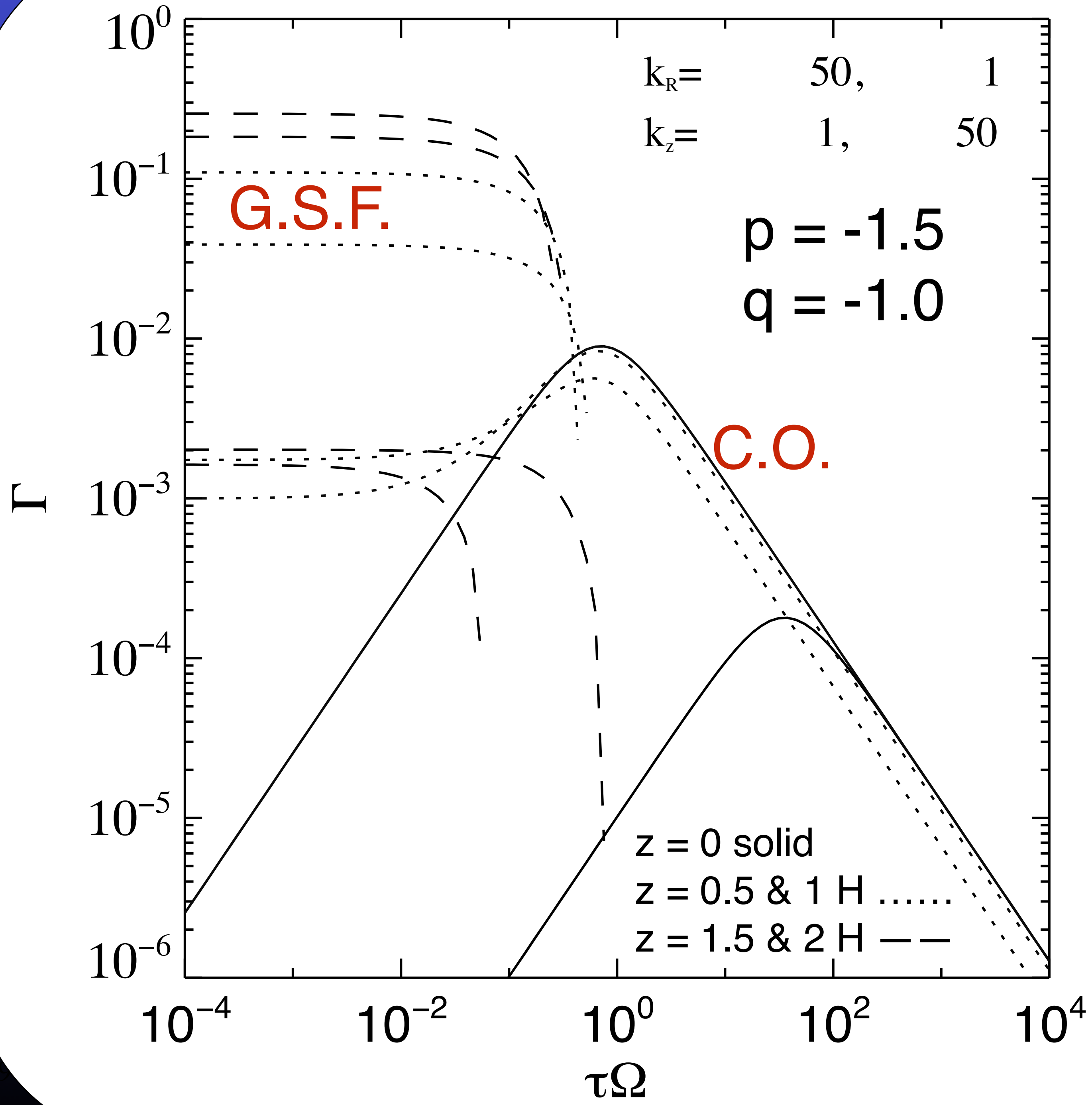
$$\begin{aligned} C =& (\frac{k_R}{\rho_0}\frac{\partial p_0}{\partial z} - \frac{k_z}{\rho_0}\frac{\partial p_0}{\partial R})\left[\frac{k_R c_s^2}{c_v \gamma}\frac{\partial S_0}{\partial z} - \frac{k_z c_s^2}{c_v \gamma}\frac{\partial S_0}{\partial R} + \frac{i}{\rho_0^2}(\frac{\partial p_0}{\partial z}\frac{\partial \rho_0}{\partial R} - \frac{\partial p_0}{\partial R}\frac{\partial \rho_0}{\partial z})\right] \\ & + \kappa_R^2(k_z^2 c_s^2 + \frac{1}{\rho_0^2}\frac{\partial \rho_0}{\partial z}\frac{\partial p_0}{\partial z}) - \kappa_z^2\left[k_R k_z c_s^2 + \frac{1}{\rho_0^2}\frac{\partial \rho_0}{\partial z}\frac{\partial p_0}{\partial z} + i(\frac{k_R}{\rho_0}\frac{\partial p_0}{\partial z} - \frac{k_z}{\rho_0}\frac{\partial p_0}{\partial R})\right] \end{aligned}$$

$$\begin{aligned} D =& \kappa_R^2\left[\frac{c_s^2 k_z^2}{\gamma} + \frac{1}{\rho_0^2}\frac{\partial \rho_0}{\partial z}\frac{\partial p_0}{\partial z} - i\frac{k_z c_s^2}{\rho_0 \gamma}\frac{\partial \rho_0}{\partial z} + i\frac{k_z}{\rho_0}\frac{\partial p_0}{\partial z}\right] \\ & + \kappa_z^2\left[\frac{c_s^2 k_z k_R}{\gamma} + \frac{1}{\rho_0^2}\frac{\partial \rho_0}{\partial R}\frac{\partial p_0}{\partial z} - i\frac{k_z c_s^2}{\rho_0 \gamma}\frac{\partial \rho_0}{\partial R} + i\frac{k_R}{\rho_0}\frac{\partial p_0}{\partial z}\right] \end{aligned}$$



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
Γ
10^{-4}
10^{-2}
10^0
10^2
10^4
$\tau\Omega$
k_R= 50
k_z= 1
p = -1.5
q = -1.0
G.S.F.
C.O.
z = 0 solid
z = 0.5 & 1 H ......
z = 1.5 & 2 H — —


10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
Γ
$k_R = 1$
$k_z = 50$
G.S.F.
p = -1.5
q = -1.0
C.O.
z = 0 solid
z = 0.5 & 1 H ......
z = 1.5 & 2 H — —
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}
$\tau\Omega$



k_R= 50, 1
k_z= 1, 50
G.S.F.
p = -1.5
q = -1.0
C.O.
z = 0 solid
z = 0.5 & 1 H ......
z = 1.5 & 2 H — —
Γ
$\tau\Omega$
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}

Incompressible limit:

$$A'\omega_m^3 - B'\frac{1}{\tau}\omega_m^2 - C'\omega_m - \frac{1}{\tau}D' = 0$$

If $k_R \gg k_z$

$$\omega_m^2 = \frac{k_z^2}{k_R^2}\left(\kappa_R^2 - \frac{k_R}{k_z}\kappa_z^2\right)$$

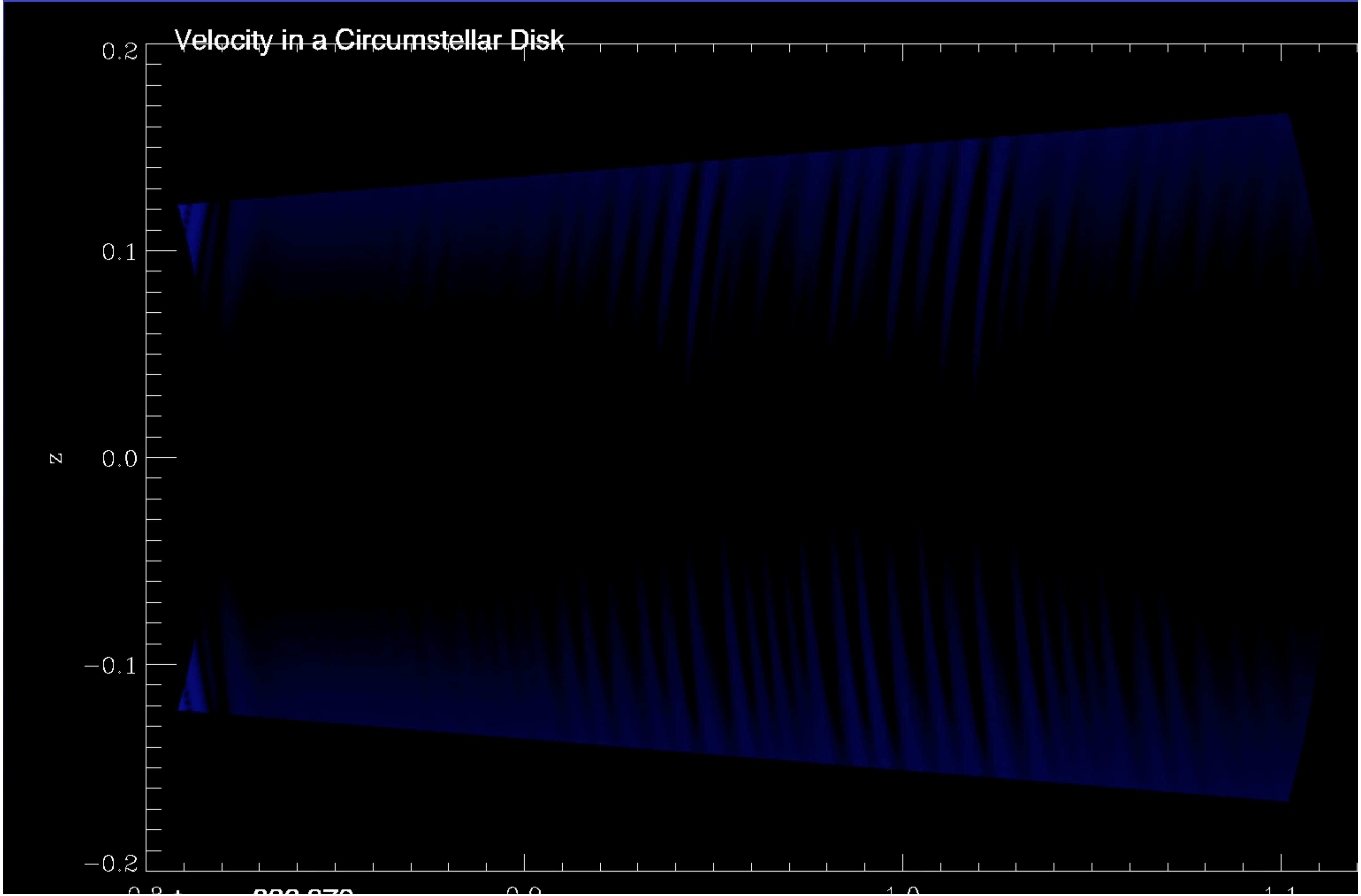
G.S.F or V.S. instability

If $k_R \ll k_z$

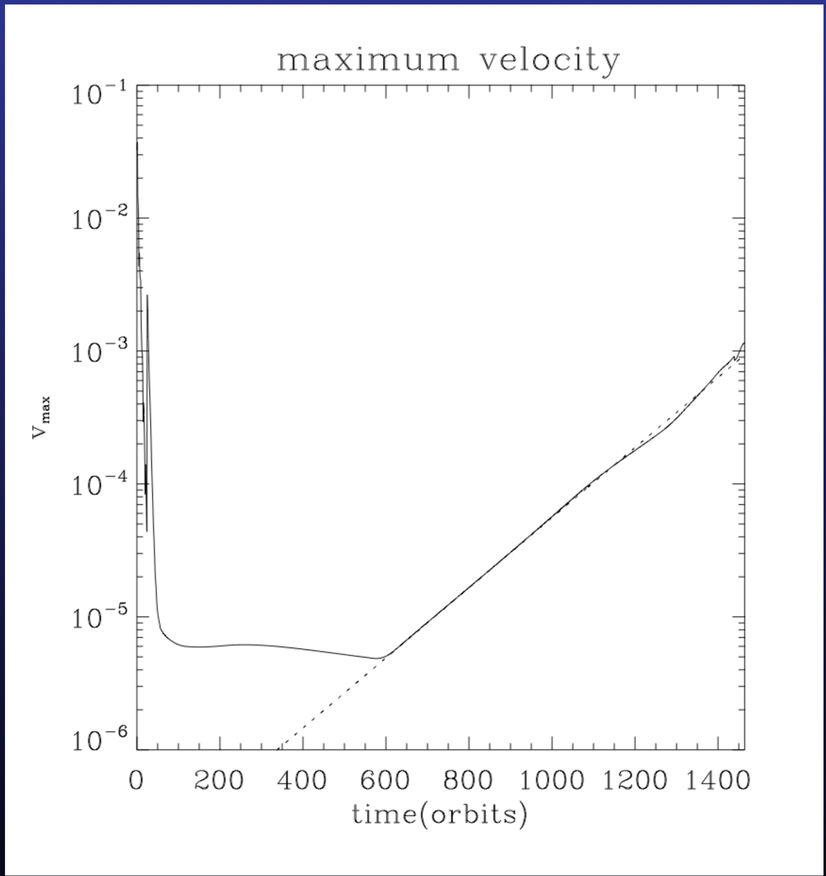
$$\omega_m^3 + \frac{i}{\gamma\tau}\omega_m^2 - (N_R^2 + \kappa_R^2)\omega_m - \frac{i\kappa_R^2}{\gamma\tau} = 0$$

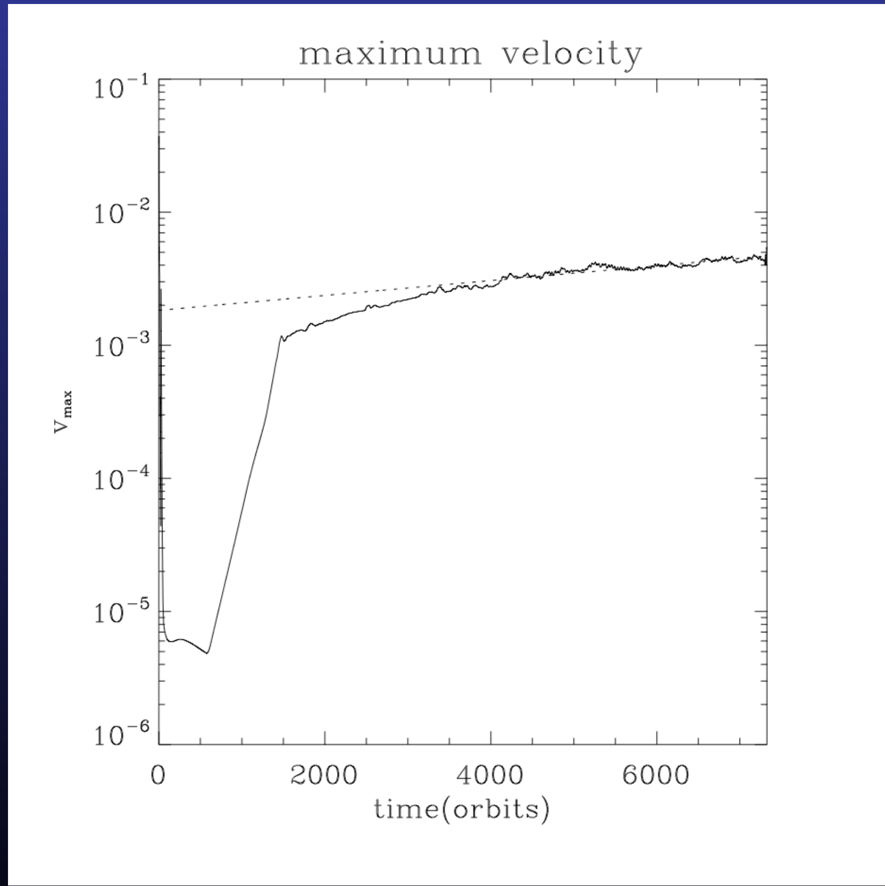
Convective Overstability

C.O. $\Omega\tau$ = 10; p = -0.66; q = 1; H/R = 0.1

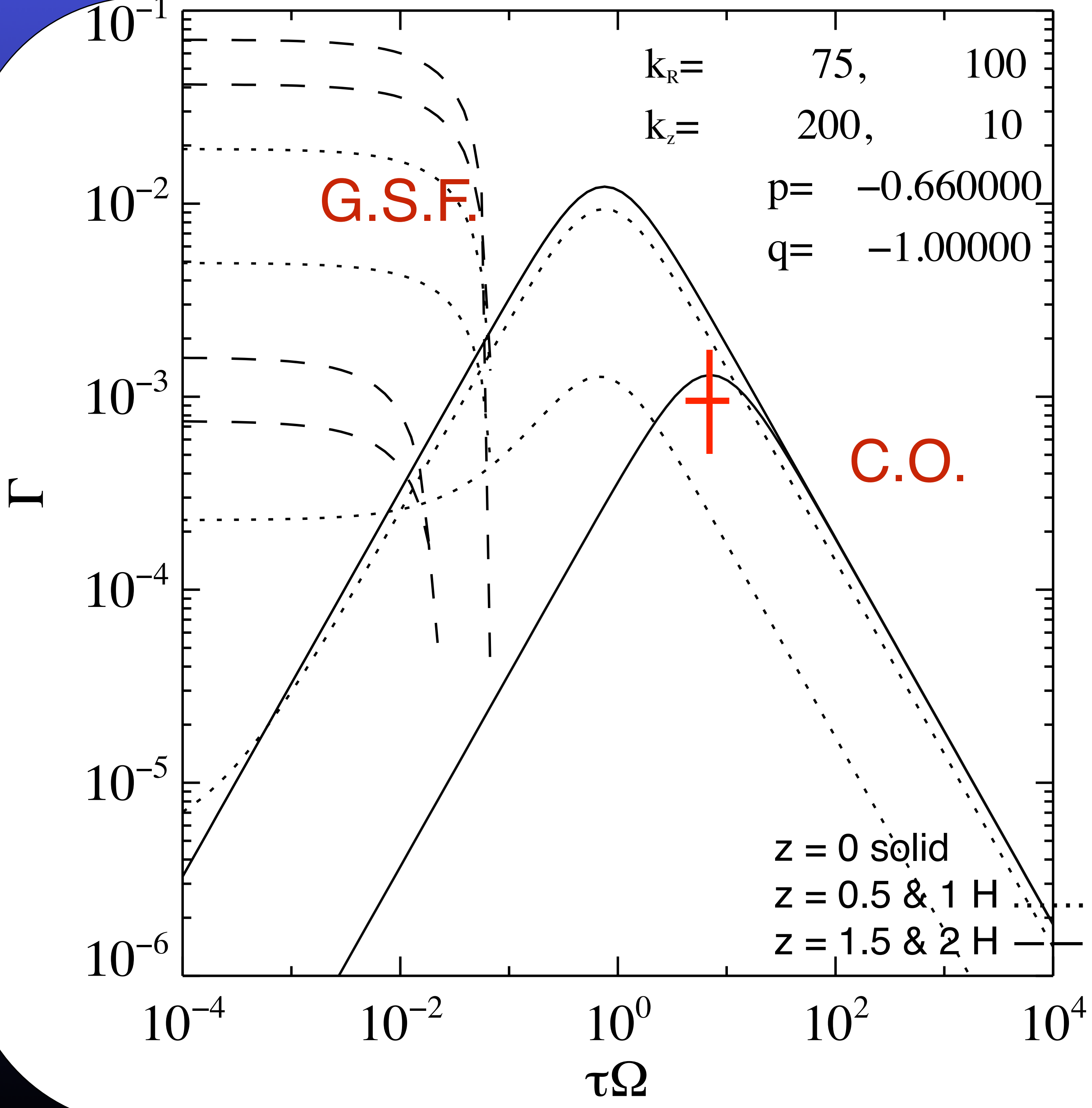


Evolution of largest velocity in simulation domain:





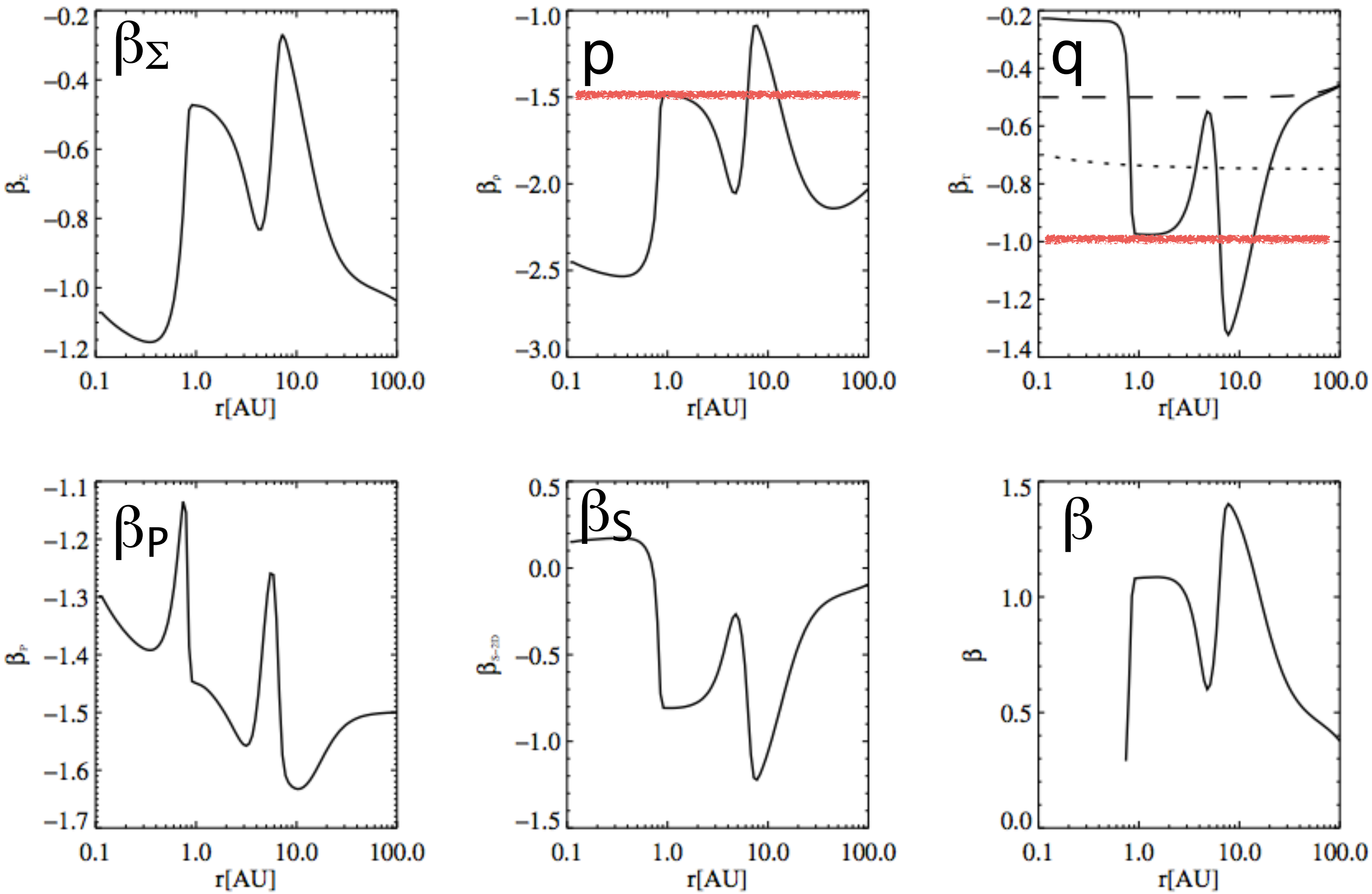
Numerical Result vs. linear theory

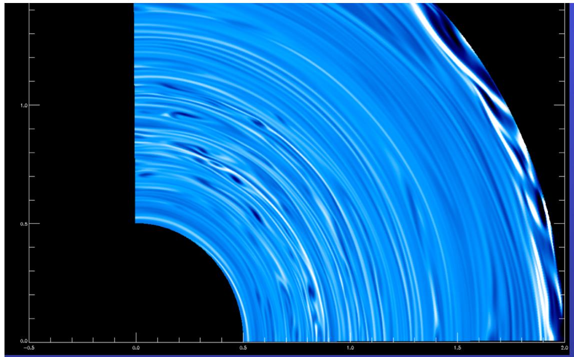


Radial Stratification of acc. disks: Using data from Sean Andrews

alpha = 0.001; Mdot = 1E-7 Msol/yr;

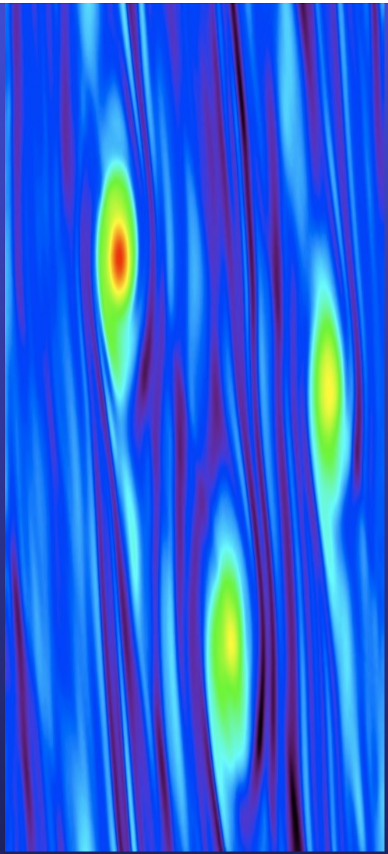
Plus irradiation: Tstar = 4300; Rstar = 2 Rsol



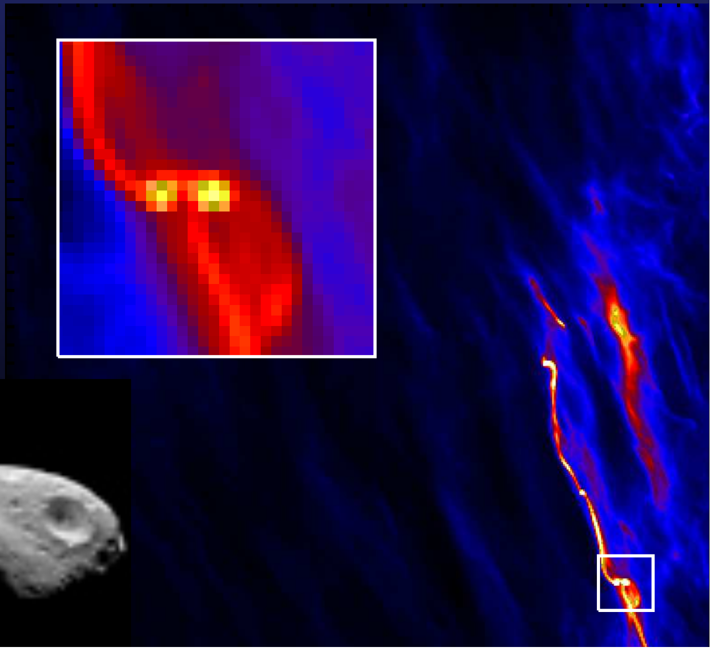


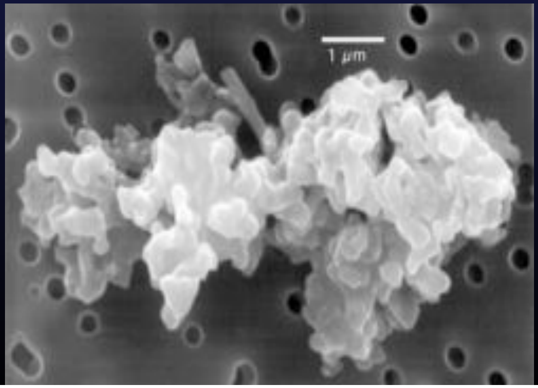
Conclusions

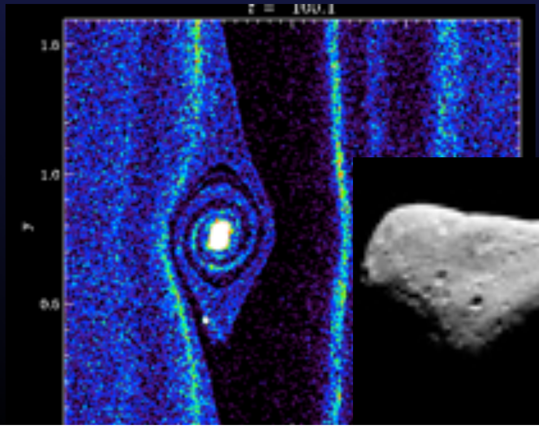
GSF: Nelson, Umurhan & Gressel et al . 2013
CO: Klahr & Hubbard 2014, Lyra 2014

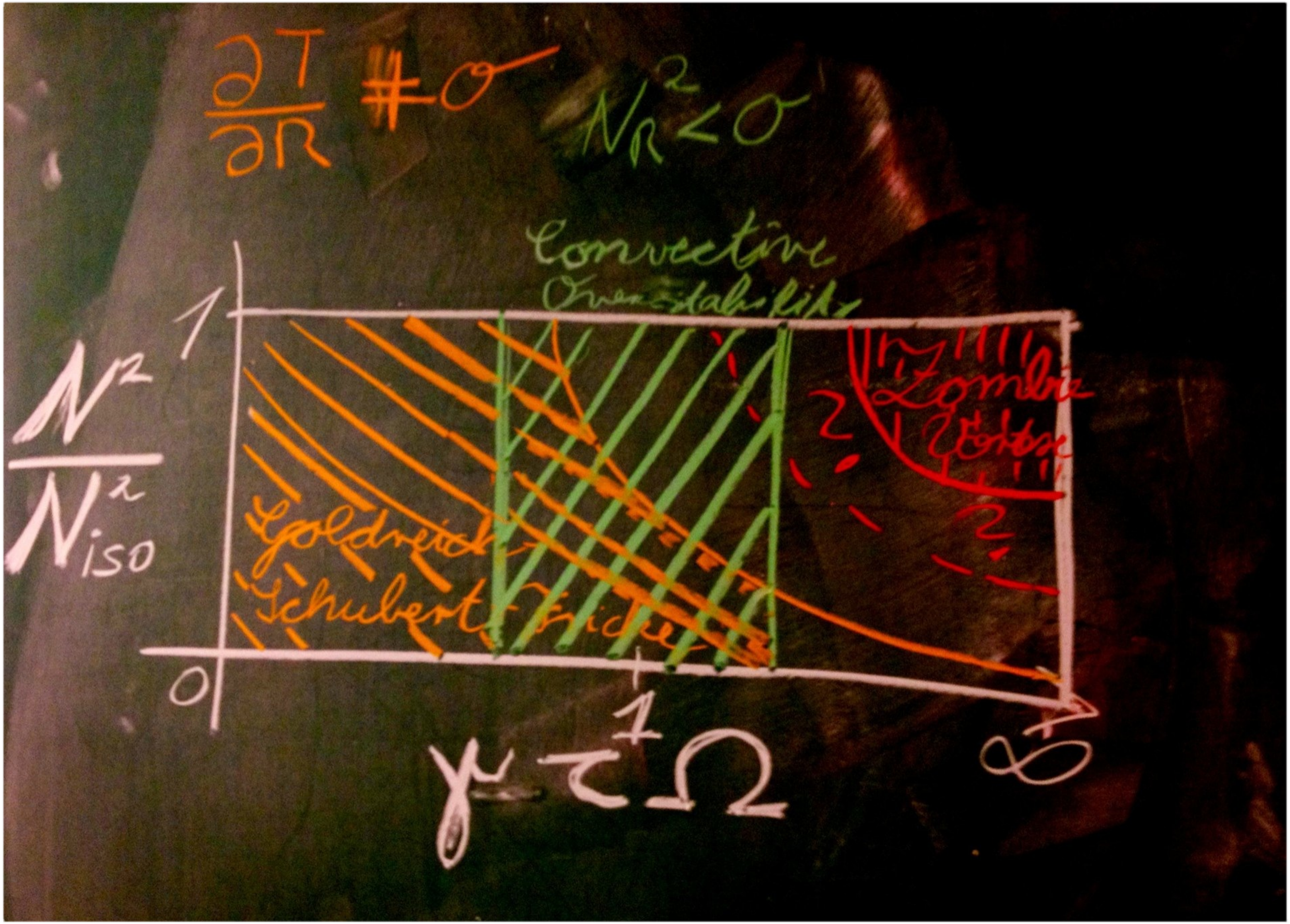


- 2 new / rediscovered instabilities that should occur in sufficiently dead zones.
- Many open questions - you name them.
- Properties and fate of vortices?
- 3D full radiation hydro runs… revisiting: Klahr & Bodenheimer 2003








$\frac{\partial T}{\partial R} \neq 0$
$N_R^2 < 0$
Convective Overstability
Zombie Vortex
Goldreich-Schubert-Fricke
$\frac{N^2}{N^2_{iso}}$
1
0
$\gamma \tau \Omega$
1
∞

Richardson number & thermal diffusion time

$$N^2 = -\frac{1}{\gamma}\left(\frac{H}{R}\right)^2 \beta_s \beta_p \Omega^2$$

$$Ri = -\frac{2}{3\gamma}\left(\frac{H}{R}\right)^2 \beta_p \beta_s$$

$$D = \frac{\lambda c 4 a_R T^3}{\rho(\kappa + \sigma)},$$

$$\tau_{therm} = H^2 / \frac{D}{\rho c_v}$$