Nonlinear Ohm's Law:
Plasma Heating by Strong Electric Fields and the Ionization Balance in PPDs

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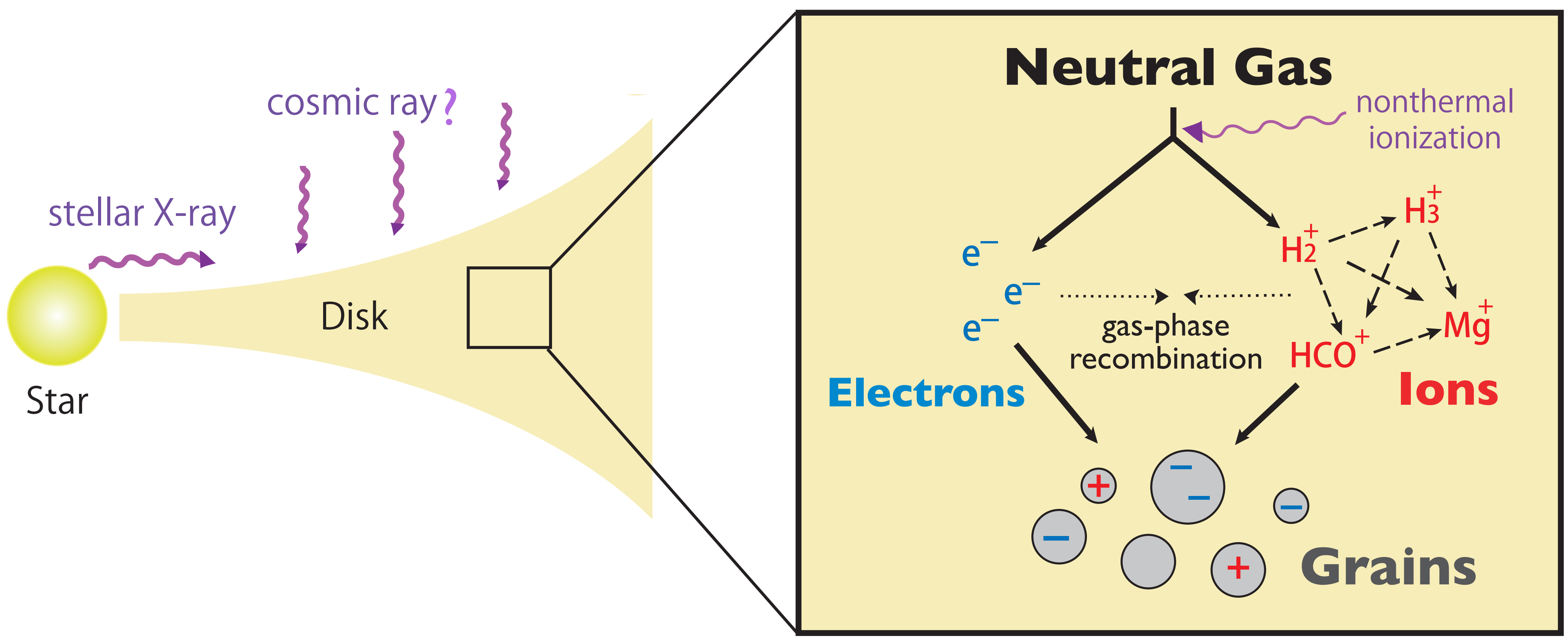
Ref: Okuzumi & Inutsuka, submitted (arXiv:1407.8110)

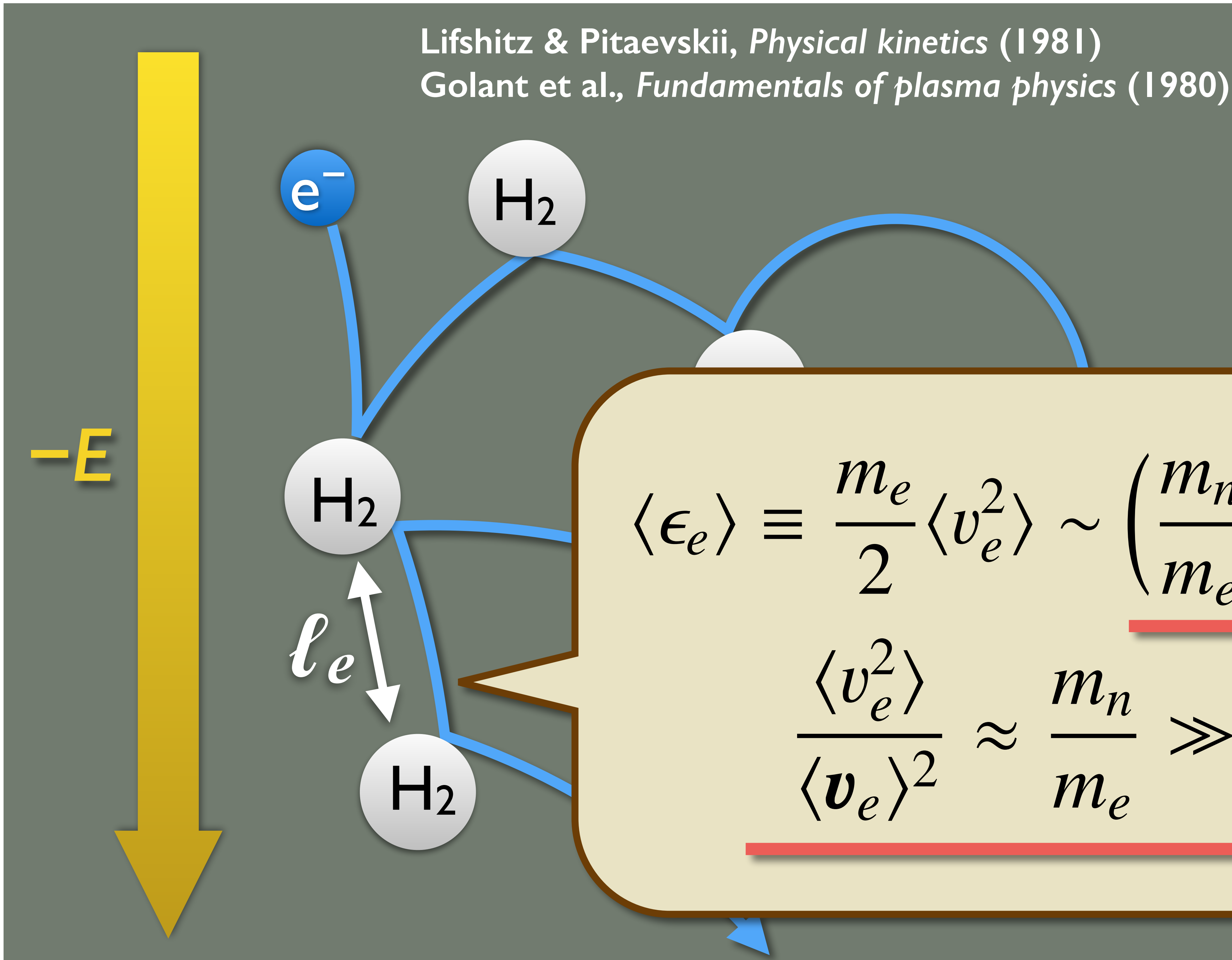
Disk Ionization and MRI

- Ionization by external high-energy sources
- Recombination in gas phase and in “solid phase”
- MRI turbulence if gas is sufficiently ionized
- How MRI turbulence changes ionization state?



Electron Heating in Weakly Ionized Gas

Electrons cannot move straight because they frequently collide with neutrals.
$\Rightarrow$ **Random velocity >> drift velocity**



$$\langle \epsilon_e \rangle \equiv \frac{m_e}{2}\langle v_e^2 \rangle \sim \underline{\left(\frac{m_n}{m_e}\right)}^{1/2} eE\ell_e$$

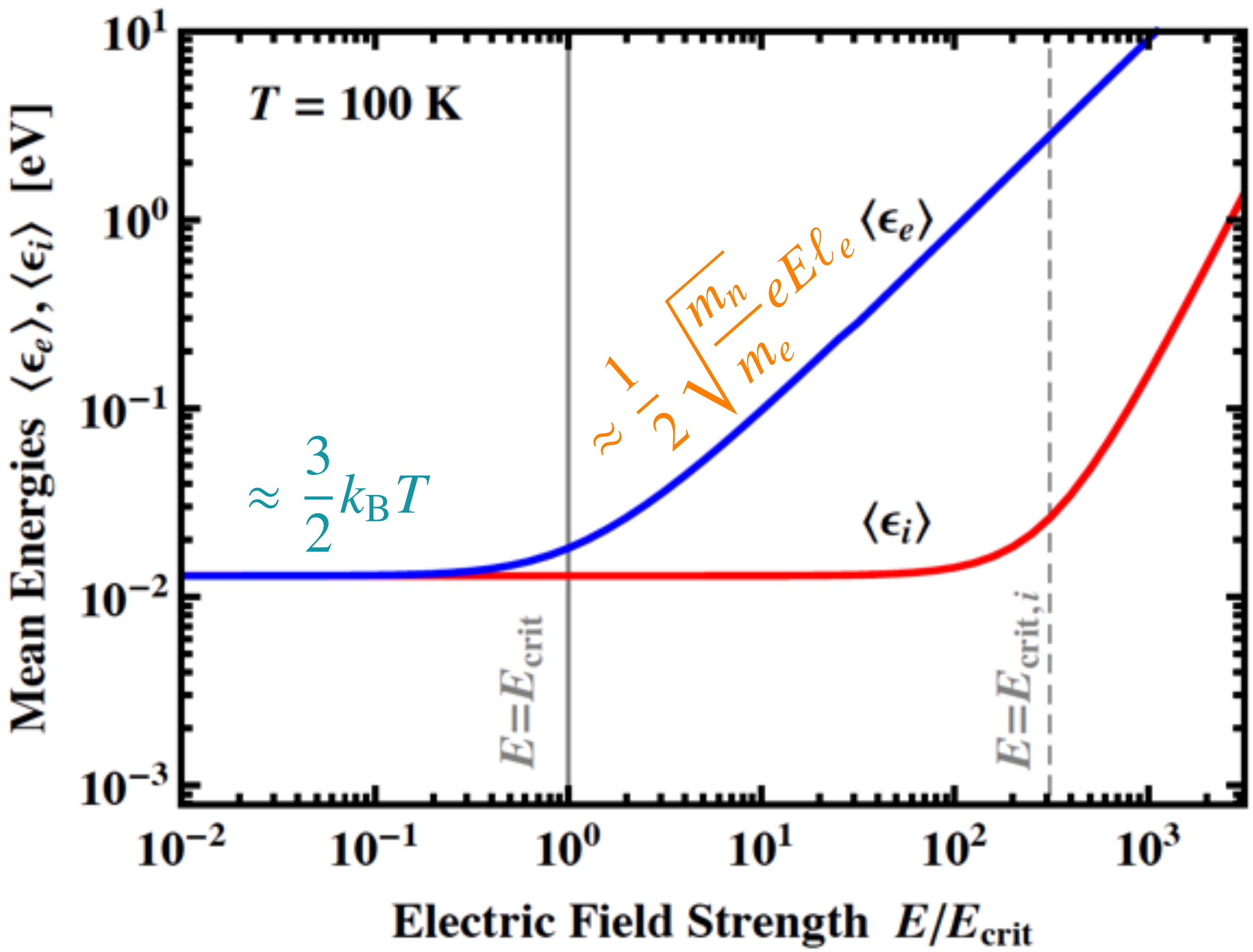
$$\underline{\frac{\langle v_e^2 \rangle}{\langle \boldsymbol{v}_e \rangle^2} \approx \frac{m_n}{m_e} \gg 1}$$

Critical E-Field Strength

Electrons are significantly heated when E is higher than

$$E_{\rm crit} = \sqrt{\frac{6m_e}{m_n}}\frac{k_{\rm B}T}{e\ell_e} \sim 10^{-9}\left(\frac{T}{100\ {\rm K}}\right)\left(\frac{n_n}{10^{12}\ {\rm cm^{-3}}}\right)\ {\rm esu\ cm^{-2}}$$

T: neutral gas temperature $\ell_e=(\sigma_{en}n_n)^{-1}$: electron m.f.p.

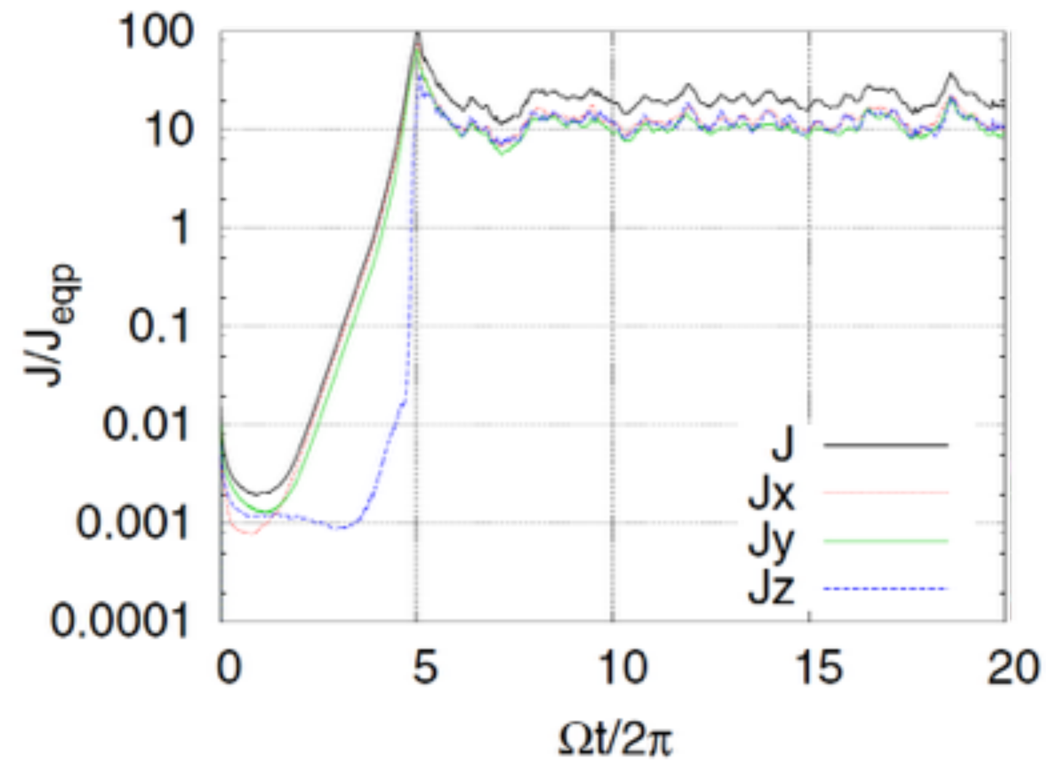


E-field Strength in MRI Turbulence

- MHD simulations show RMS current density is insensitive to ohmic resistivity:

$$\boxed{J_{\rm MRI} \approx 10 J_{\rm eqp}} \qquad J_{\rm eqp} = \sqrt{\frac{\rho}{8\pi}}c\Omega$$

(Muranushi, Okuzumi, & Inutsuka 2012)



- By using Ohm's law $E' = (4\pi\eta/c^2)J$, we obtain

$$\boxed{E'_{\rm MRI} \sim 10^{-7}\Lambda_z^{-1}\left(\frac{10^2}{\beta_z}\right)\left(\frac{T}{100\ {\rm K}}\right)\left(\frac{n_n}{10^{-12}\ {\rm cm^{-3}}}\right)^{1/2}\ {\rm esu\ cm^{-2}}}$$

$\Lambda_z \equiv \langle v_{Az}^2\rangle/\eta\Omega$ (≳1 for MRI to be active)

$\beta_z \equiv 2c_s^2/\langle v_{Az}^2\rangle$ (~100–1000 for fully saturated turbulence)

Criterion for Electron Heating in MRI Turbulence

- Critical E-field strength for electron heating

$$E_{\rm crit} = \sqrt{\frac{6m_e}{m_n}}\frac{k_{\rm B}T}{e\ell_e} \sim 10^{-9}\left(\frac{T}{100\ {\rm K}}\right)\left(\frac{n_n}{10^{12}\ {\rm cm}^{-3}}\right)\ {\rm esu\ cm}^{-2}$$

- RMS strength of *comoving* electric field in MRI turbulence

$$E'_{\rm MRI} \sim 10^{-7}\Lambda_z^{-1}\left(\frac{10^2}{\beta_z}\right)\left(\frac{T}{100\ {\rm K}}\right)\left(\frac{n_n}{10^{-12}\ {\rm cm}^{-3}}\right)^{1/2}\ {\rm esu\ cm}^{-2}$$

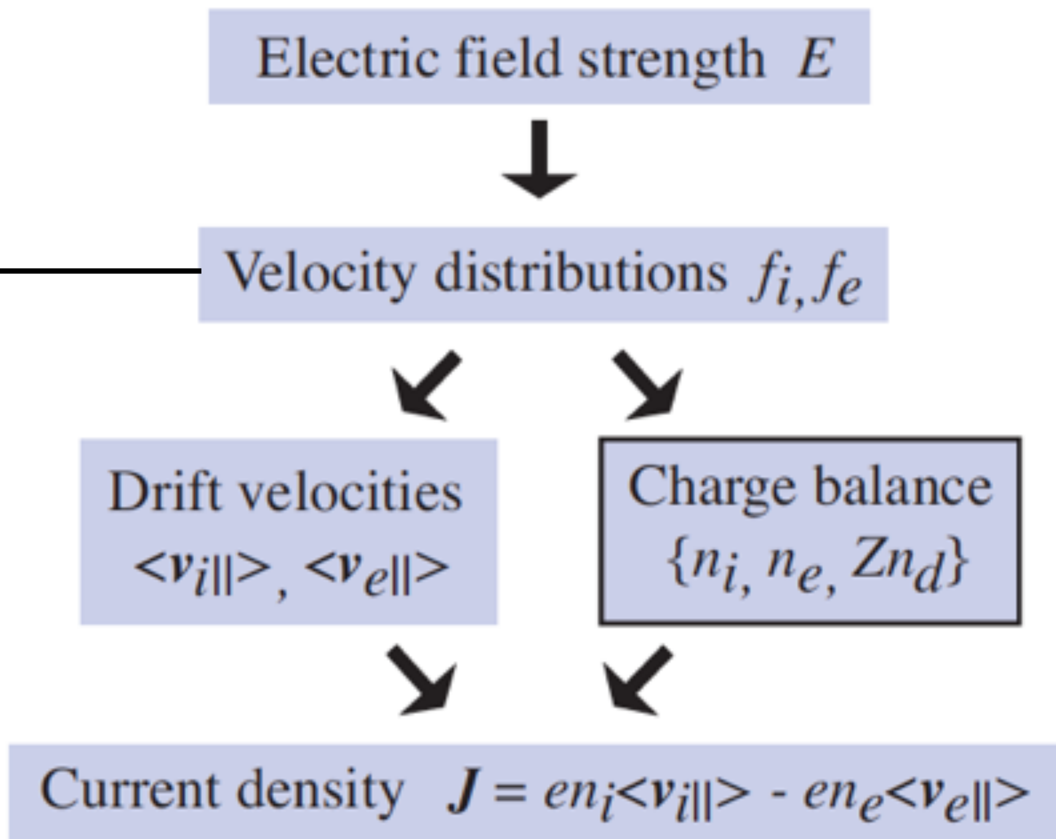
(Okuzumi & Inutsuka, 2014; based on Muranushi, Okuzumi, & Inutsuka 2012)

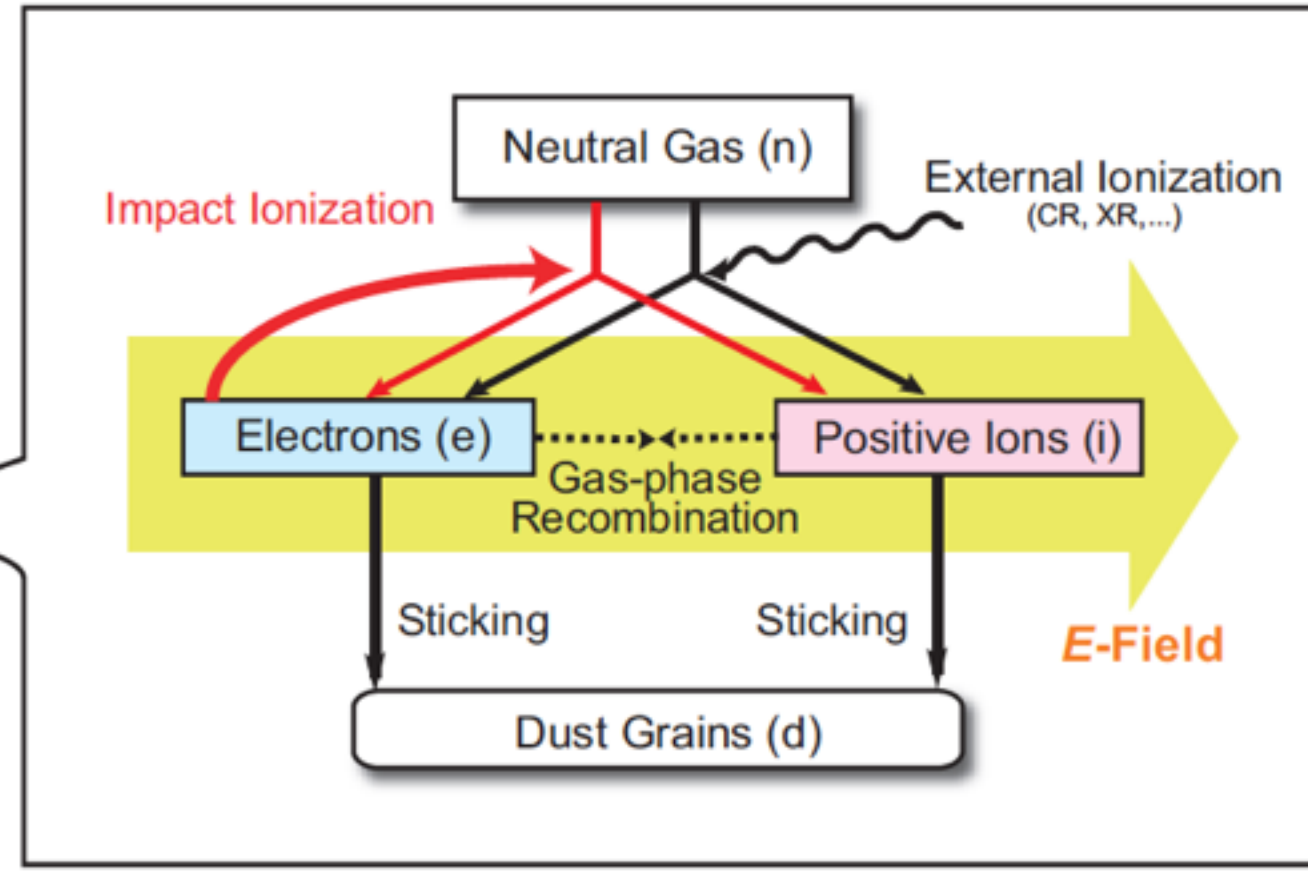
If $1 \lesssim \Lambda_z \lesssim 100$, MRI-induced E-fields heat up free electrons (up to ~ 1 eV)

(see also Inutsuka & Sano 2005)

Ionization Model with Plasma Heating

Okuzumi & Inutsuka (2014)





- f_e : **Davydov–Druyvesteyn**
- f_i : **Offset Maxwellian**

- Analytic functions of E
- Inelastic energy losses neglected

sticking to grain	gas-phase rec.	impact ionization

$$\frac{dn_i}{dt} = \zeta n_n - K_{di}(Z) n_d n_i - K_{\rm rec} n_i n_e + K_* n_n n_e$$

$$\frac{dn_e}{dt} = \zeta n_n - K_{de}(Z) n_d n_e - K_{\rm rec} n_i n_e + K_* n_n n_e$$

$$\frac{dZ}{dt} = K_{di}(Z) n_i - K_{de}(Z) n_e$$

Model Parameters

Model	ζ (s^{-1})	f_{dg}	Impact ionization?
A	10^{-17}	10^{-6}	No
B, B*	10^{-17}	10^{-4}	No (B), Yes (B*)
C, C*	10^{-17}	10^{-2}	No (C), Yes (C*)
D	10^{-19}	10^{-2}	No

Note. — The other parameters are fixed to $m_n = 2.3$ amu, $m_i = 29$ amu, $T = 100$ K, $n_n = 10^{12}$ cm^{-3}, IP $= 15.4$ eV, $a = 1$ μm, and $\rho_\bullet = 2$ g cm^{-3}.

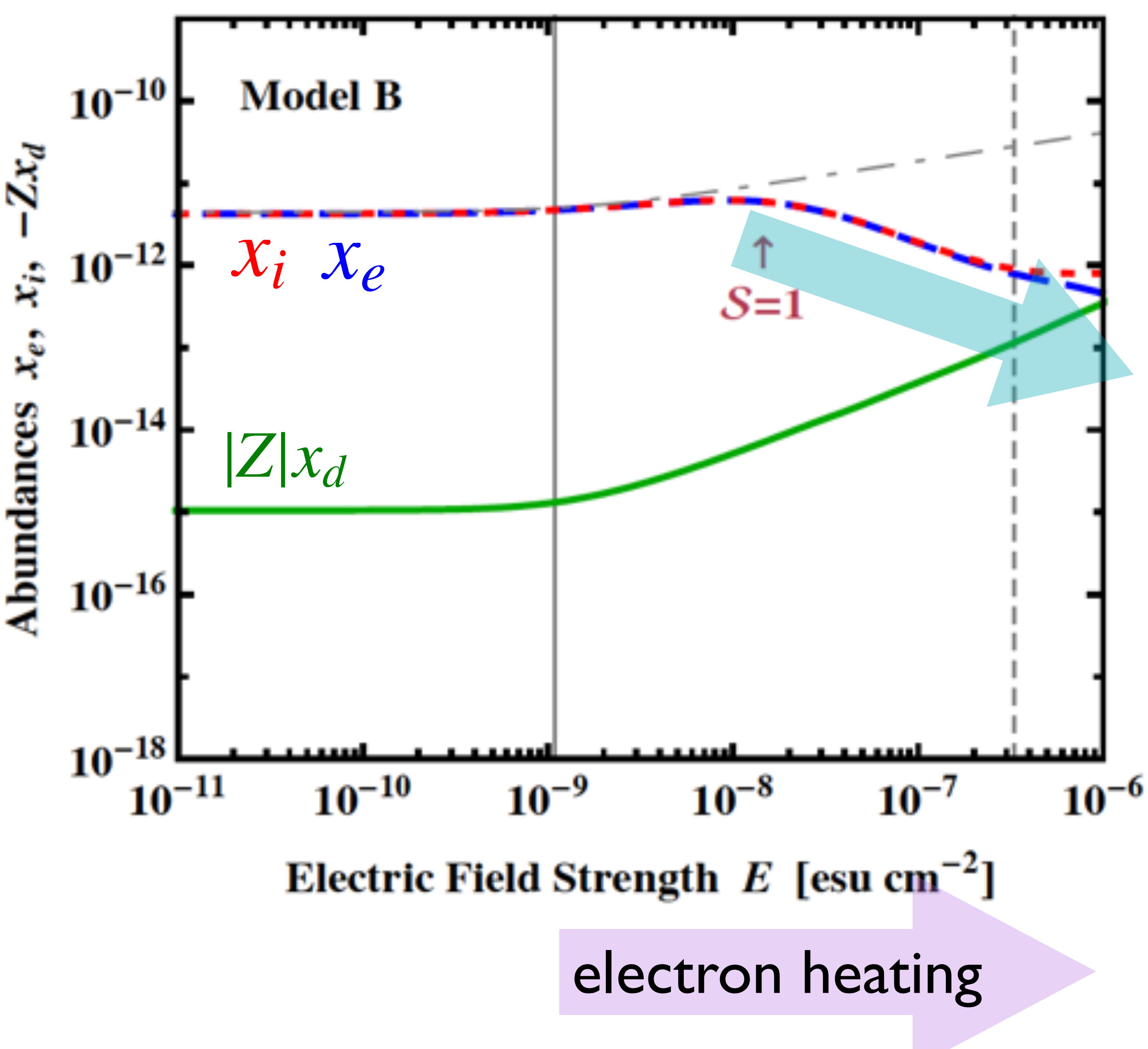
Ionization Balance

Electron heating ➡ Electron–grain collision freq. ↑

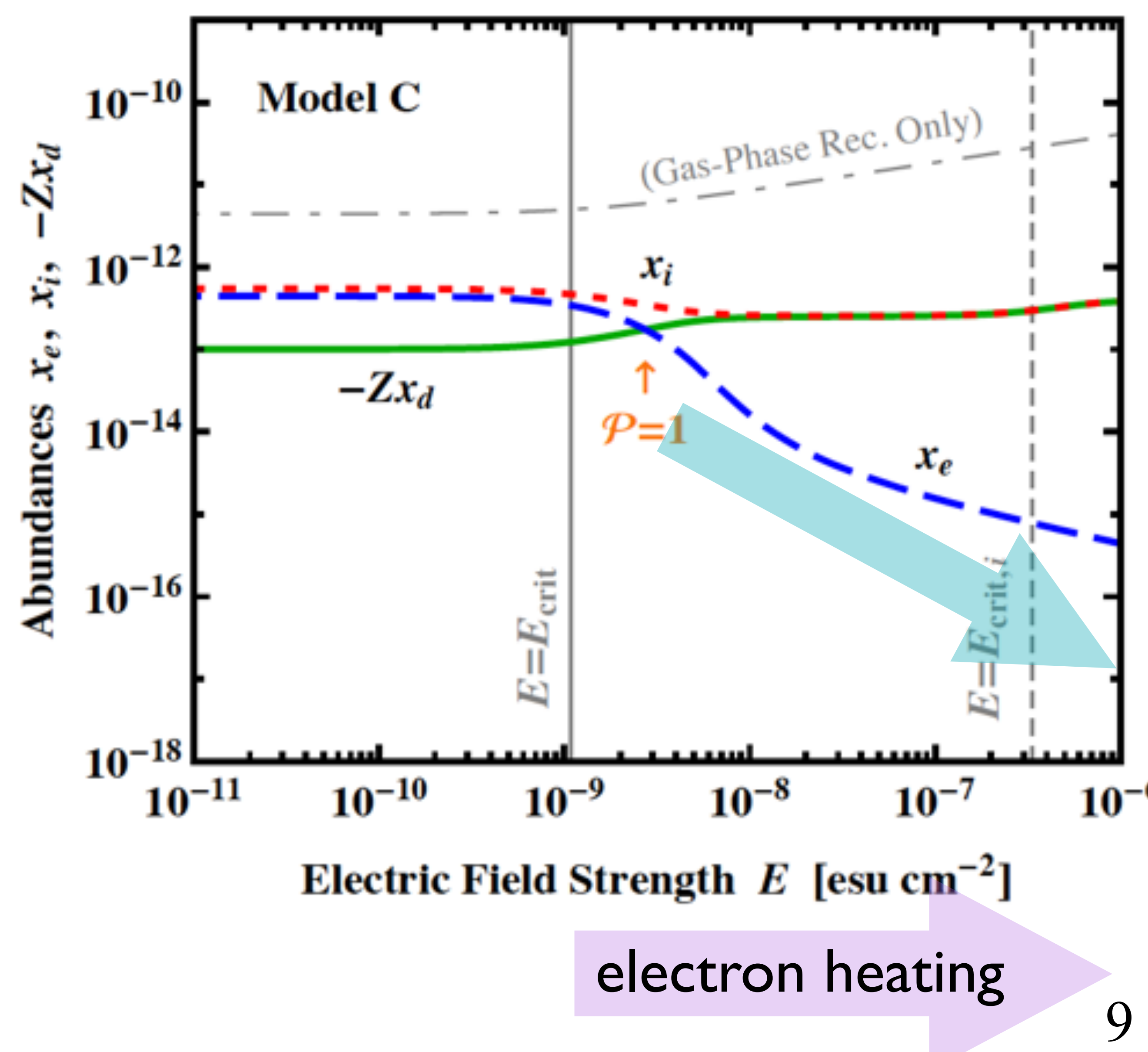
➡ **Electron abundance ↓**, Grain charge ↑

(if grain charging dominates over gas-phase rec.)

Model B (d/g=10^{-4})



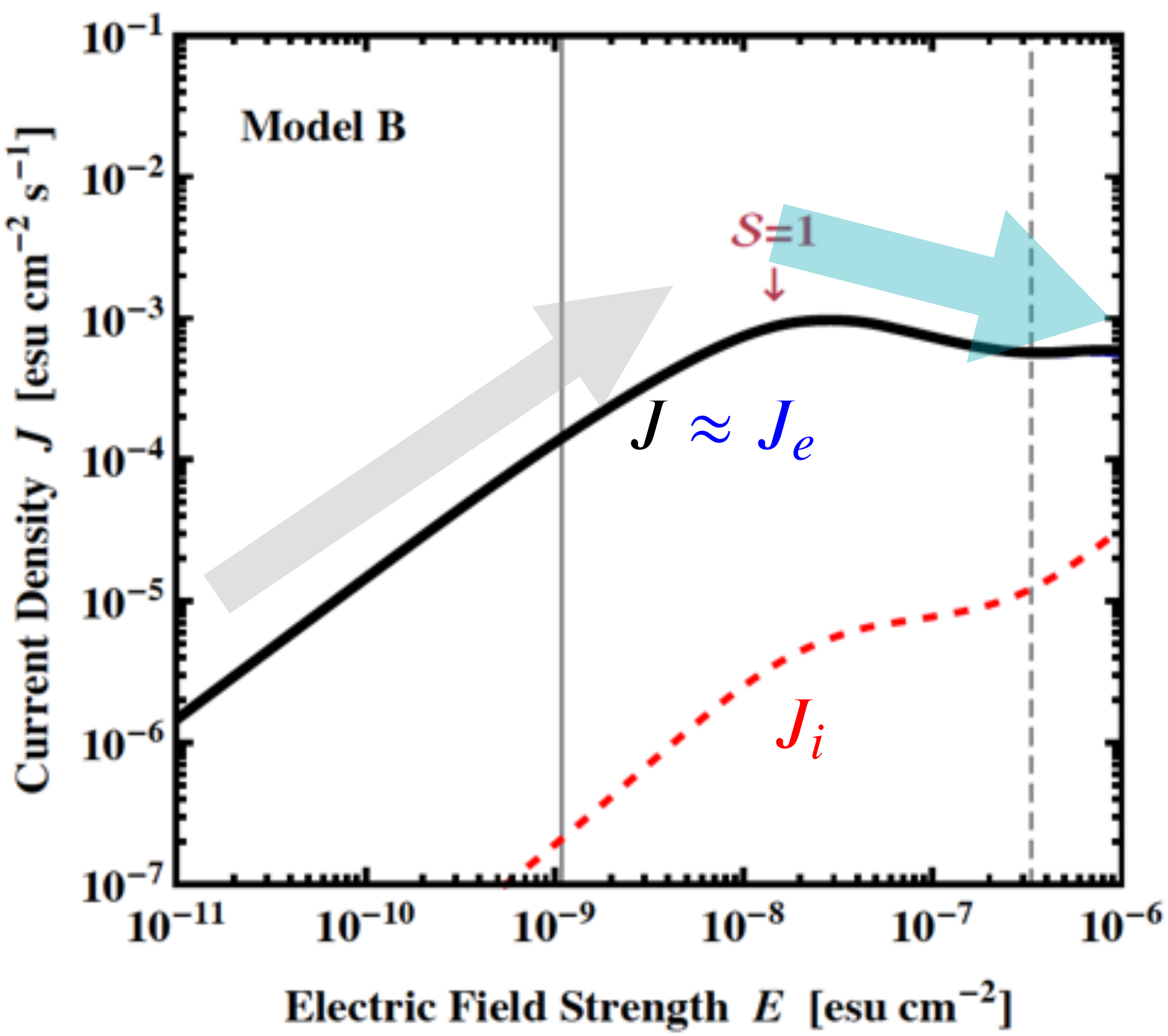
Model C (d/g=10^{-2})



J–E Relation

$$J = J_e + J_i = en_e|\langle \boldsymbol{v}_{e\|}\rangle| + en_i|\langle \boldsymbol{v}_{i\|}\rangle|$$

Model B (d/g=10^{-4})



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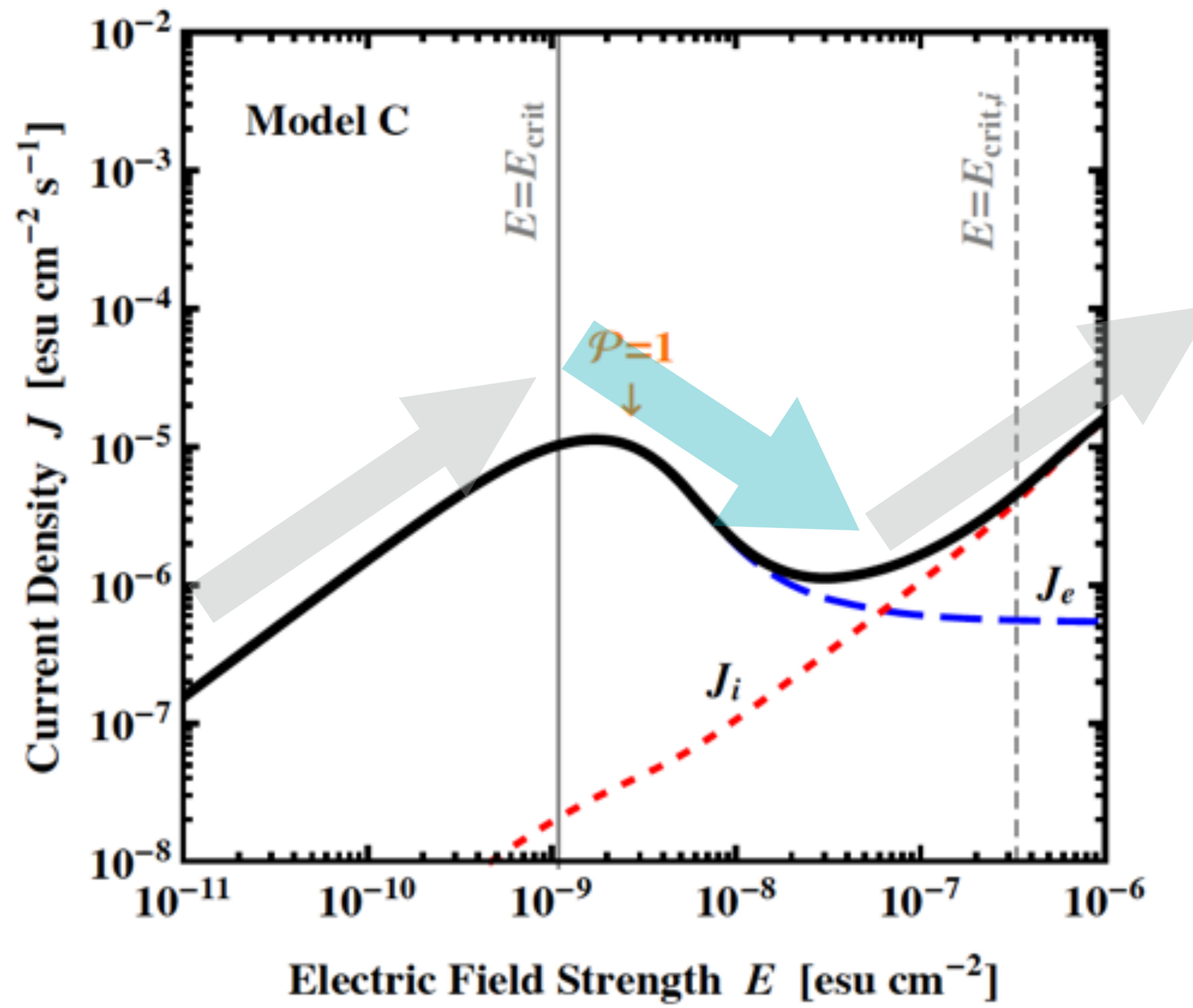
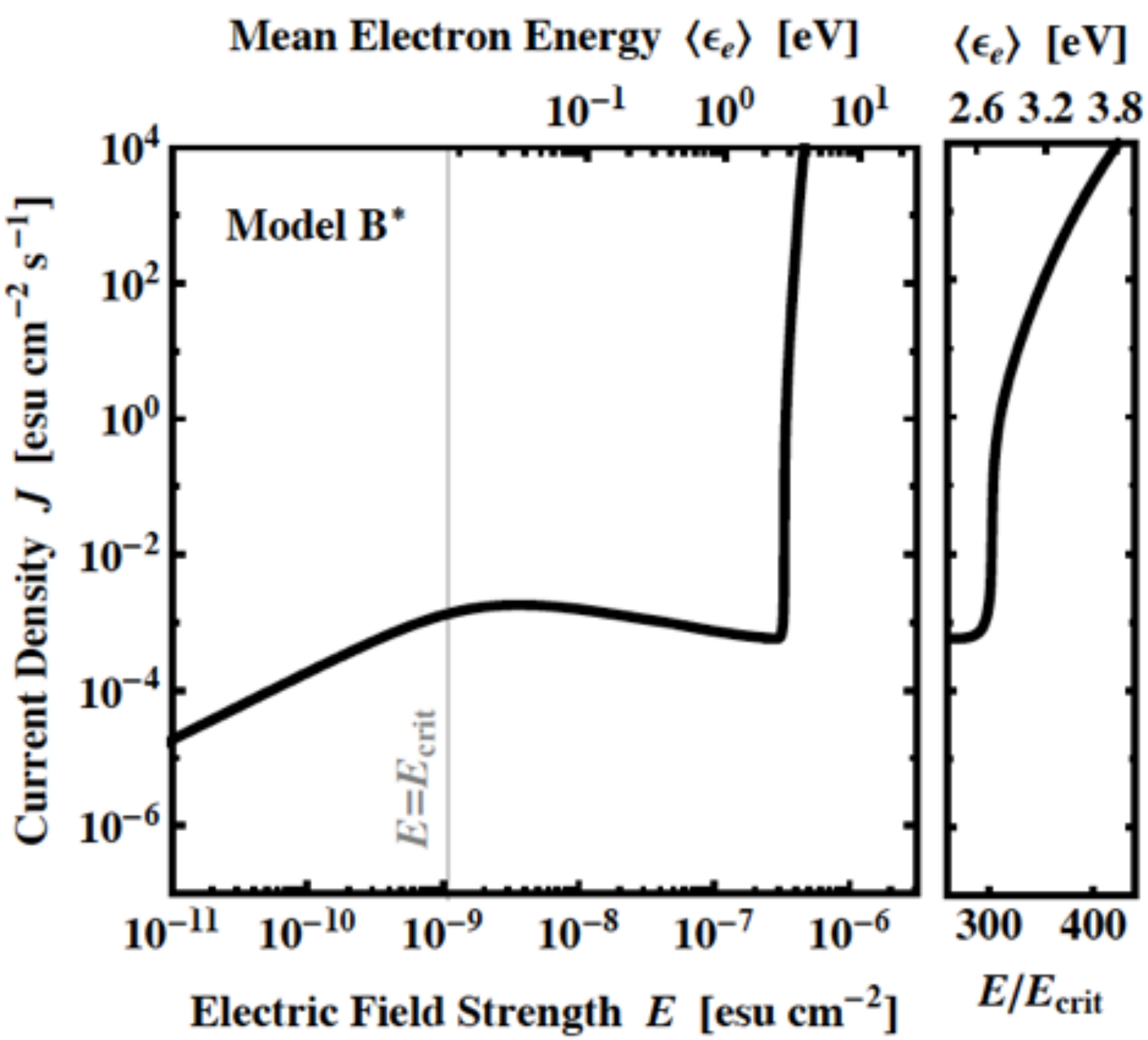


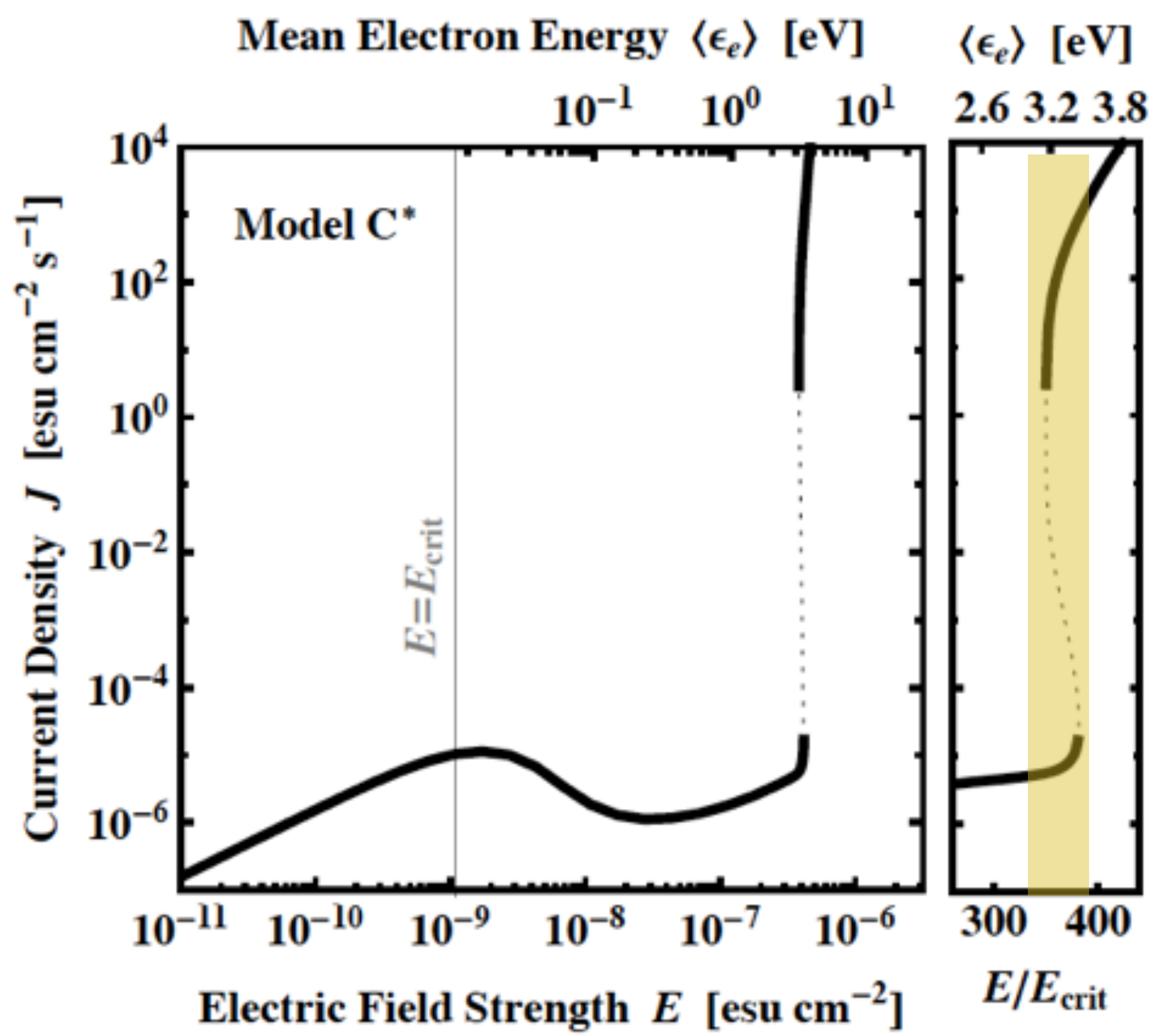
J decreases with increasing E until J_i takes over J_e

➡ **N-shaped *J–E* curve!**

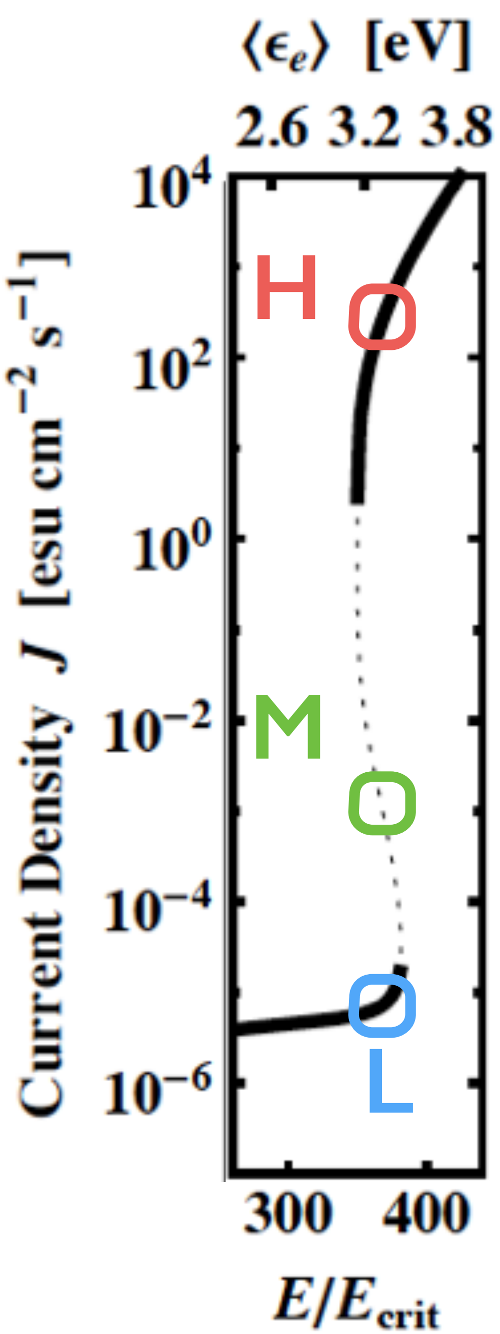
Effect of Impact Ionization

- “**Electric Discharge**” at $\langle \epsilon_e \rangle \approx 3\mathrm{eV}$ (*cf.* IP $=15\mathrm{eV}$)
- For grain-rich cases, the discharge current becomes *triple-valued* (S-shaped J-E curve)

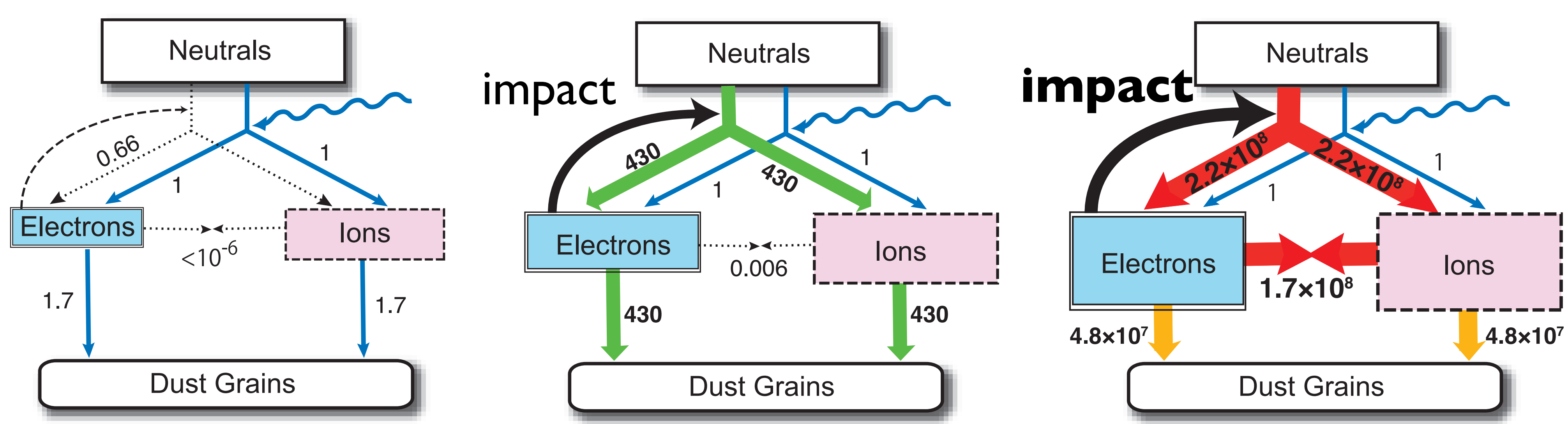




Nature of Multiple Equilibria



- **Low State** (stable)
  (external ionization) = (grain charging)
- **Middle State** (unstable)
  (impact ionization) = (grain charging)
- **High State** (stable)
  (impact ionization) = (gas-phase recombination)

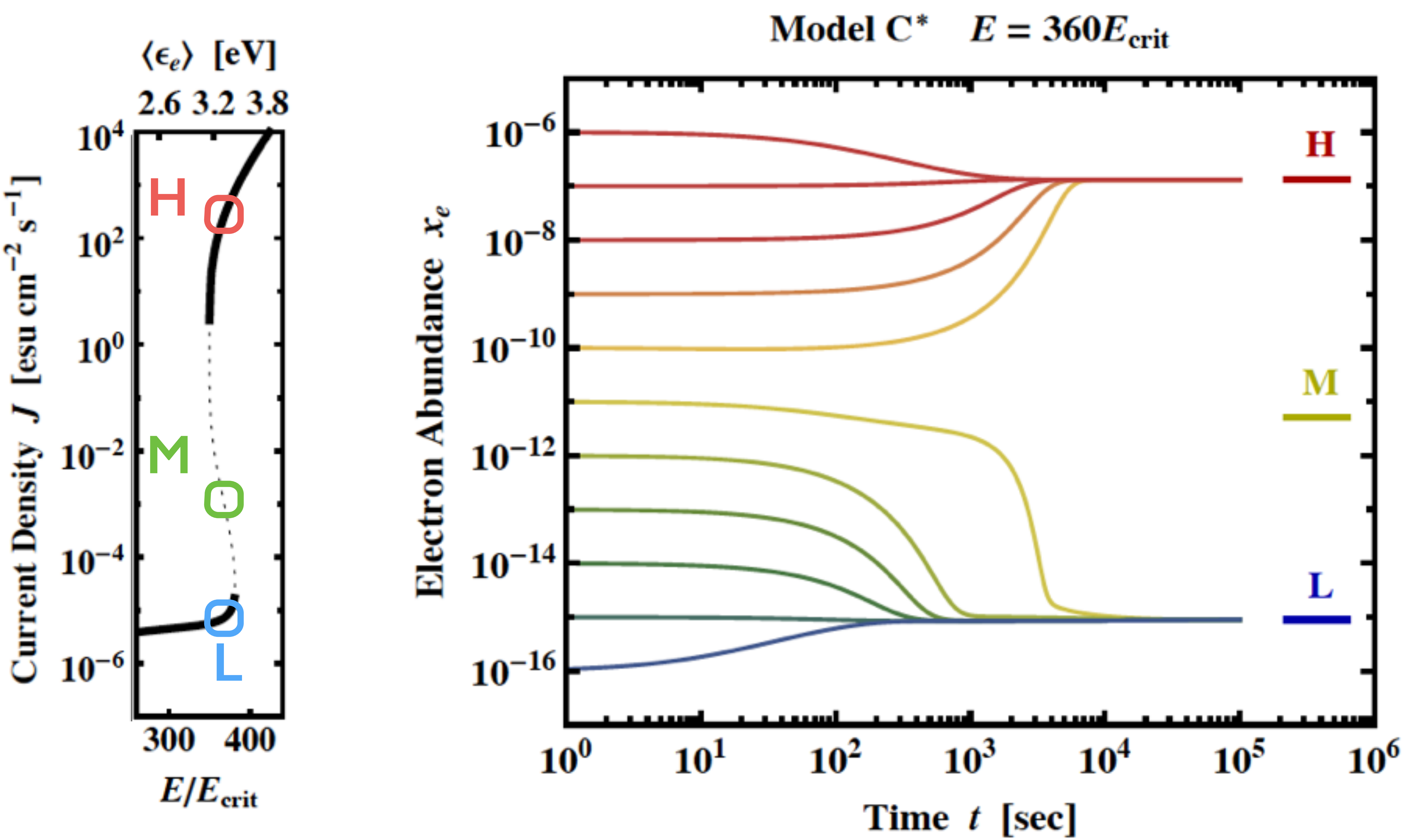


(a) Low State (STABLE)

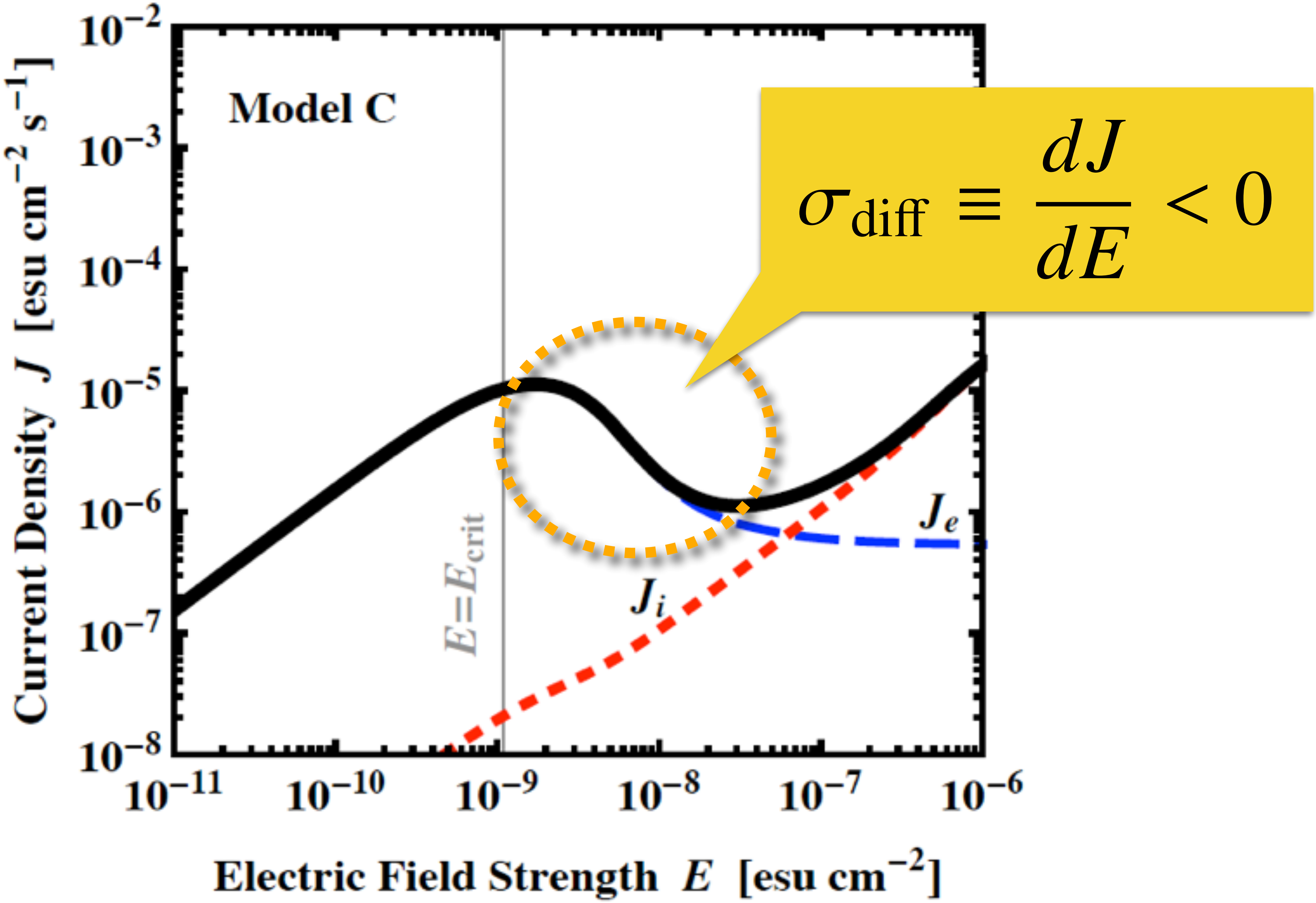
(b) Middle State (UNSTABLE)

(c) High State (STABLE)

Nature of Multiple Equilibria



Negative Differential Resistance (NDR)



NDR Destabilizes E-Field!

Maxwell-Ampère Eq. $$\boxed{\frac{\partial \boldsymbol{E}}{\partial t}} = c\nabla \times \boldsymbol{B} - 4\pi J(E)\hat{\boldsymbol{E}}$$

displacement current

- Equilibrium: $c\nabla \times \boldsymbol{B}_0 = 4\pi \boldsymbol{J}(\boldsymbol{E}_0)$. (Ampère's law)
- Perturbation: $\boldsymbol{E} = \boldsymbol{E}_0 + \delta \boldsymbol{E}$

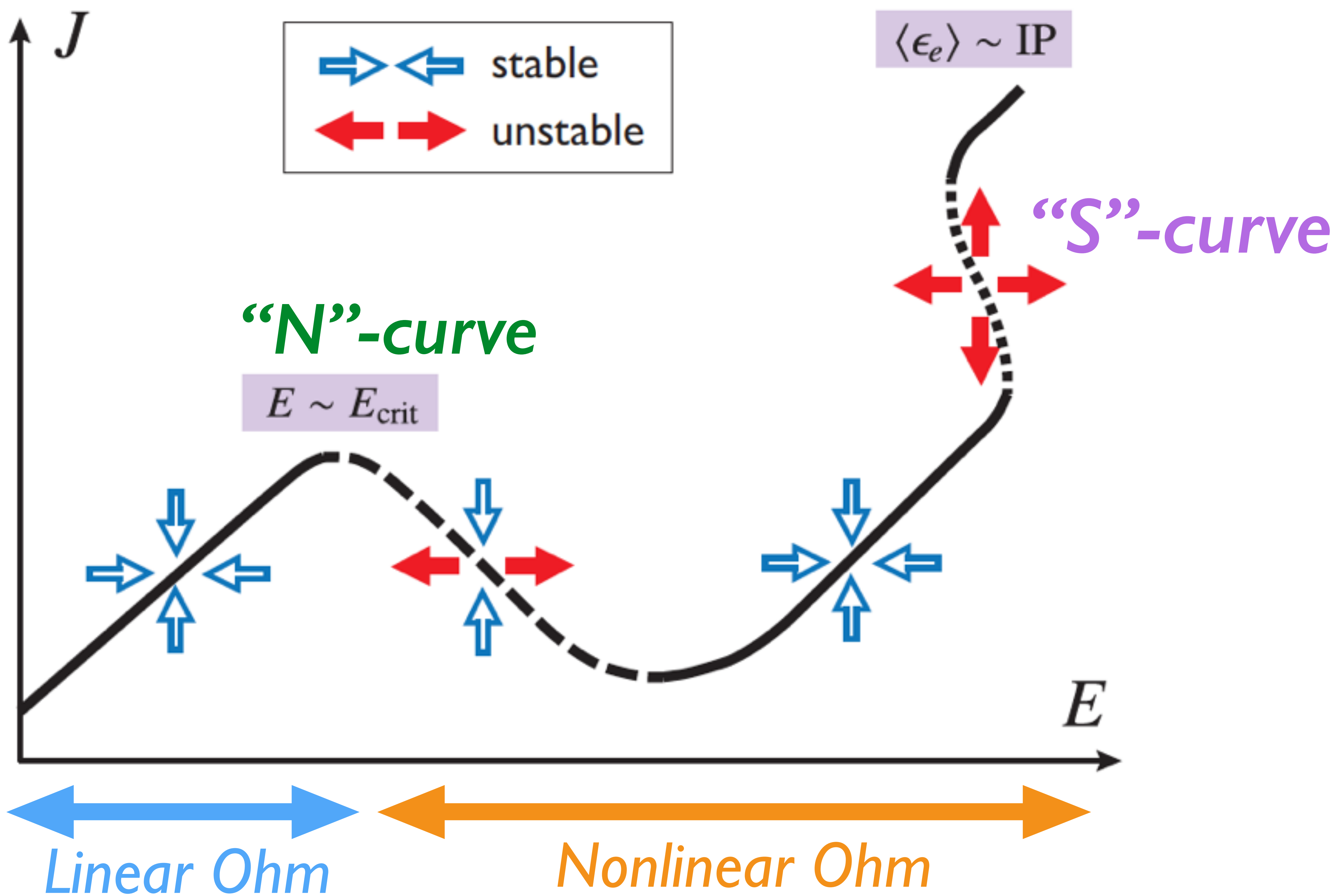
In the long-wavelength limit (just for simplicity),

$$\frac{d}{dt}\delta \boldsymbol{E} = -4\pi\sigma_{\text{diff}}\delta \boldsymbol{E} \qquad \sigma_{\text{diff}} \equiv \frac{dJ}{dE}(|\boldsymbol{E}_0|)$$

If $\sigma_{\text{diff}} < 0$ (NDR), δE grows.

Quasi-steady Ampère's law is no longer valid!

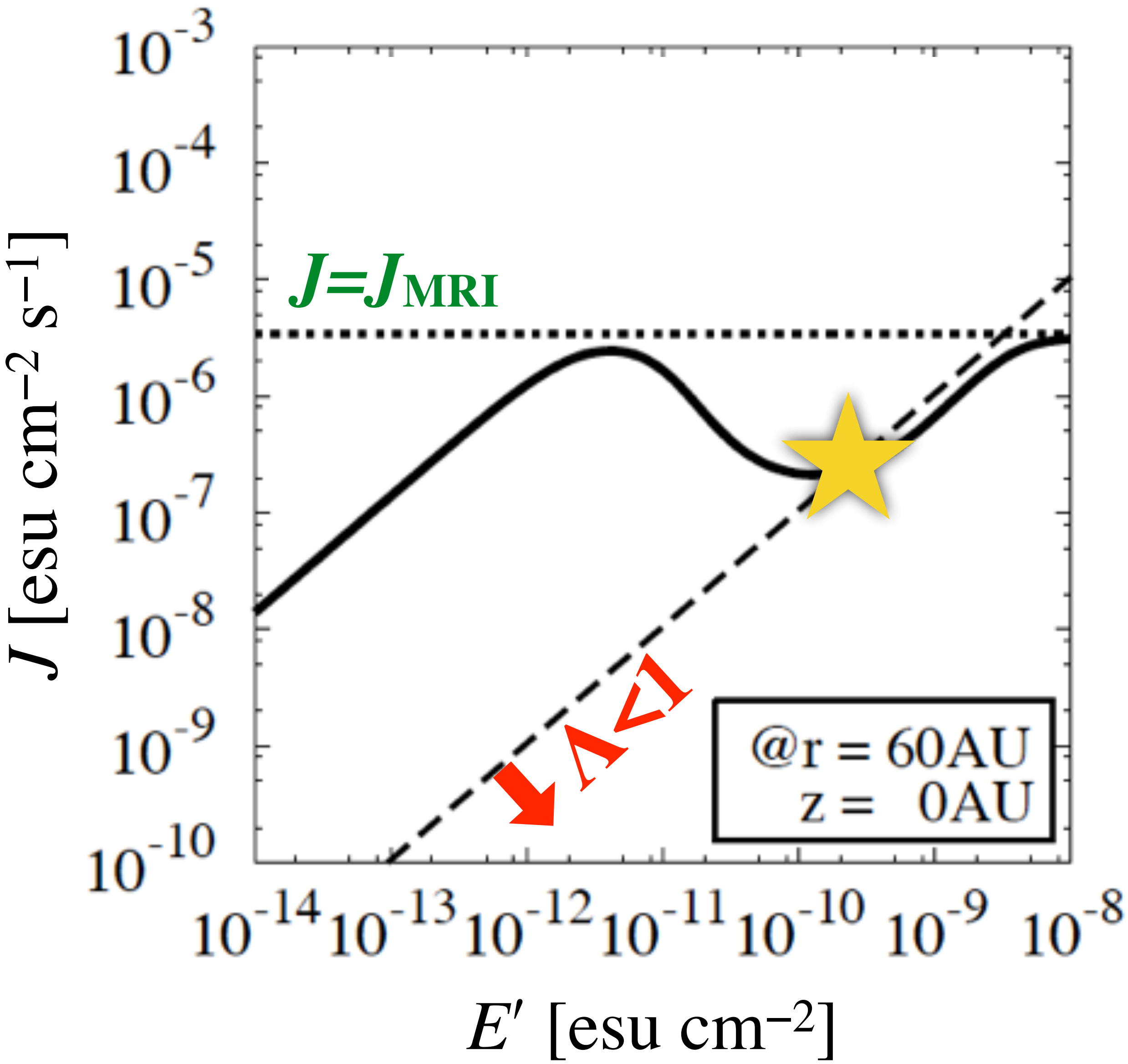
Nonlinear Ohm's Law: Summary

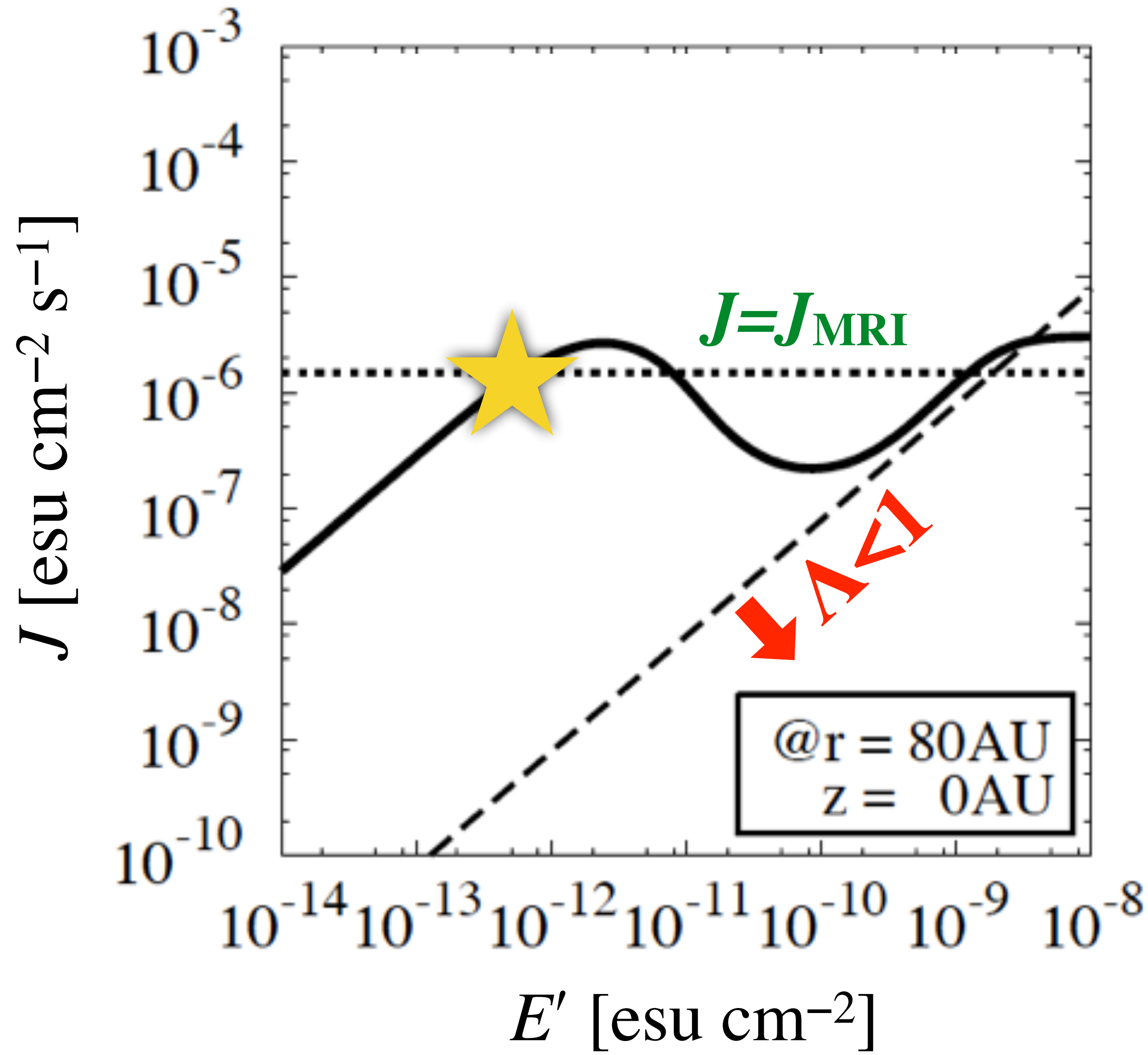


Application to PPDs

(Mori & Okuzumi, in prep.)

Example: MMSN, d/g = 0.01, *a* = 0.1μm





- At < 60 AU, Λ falls below 1 in nonlinear regime (self-regulated MRI?)
- **Caveat: non-ohmic effects** (gyromotion of i & e) **not included here**

Conclusions

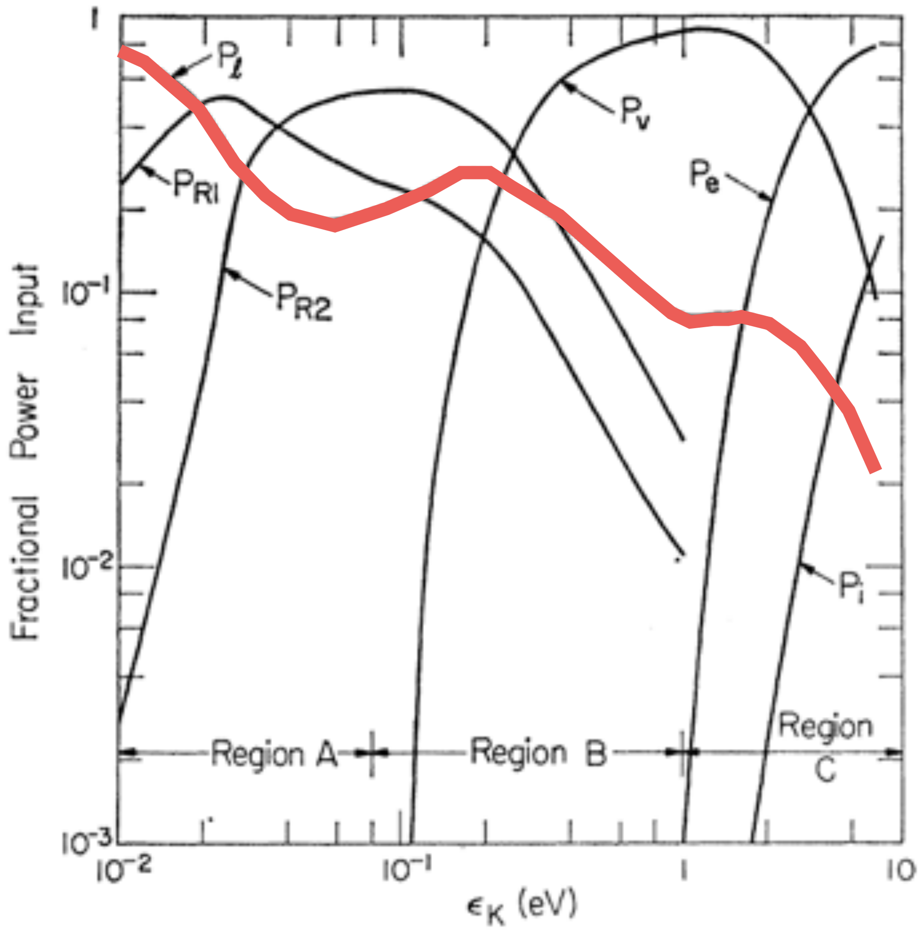
- MRI-induced electric fields heat up electrons in PPDs (in particular when $1<\Lambda<100$).

- Under electron heating, the conductivity decreases with increasing *E*. Even the electric current *J* decreases ("*N*-shaped" *J*–*E* curve).

- Discharge current can have an unstable intermediate branch ("*S*-shaped" *J*–*E* curve) when dust is abundant.

- A ≪generalized≫ nonlinear Ohm's law (including AD & Hall drift) is also coming soon!

Effect of Inelastic Energy Losses



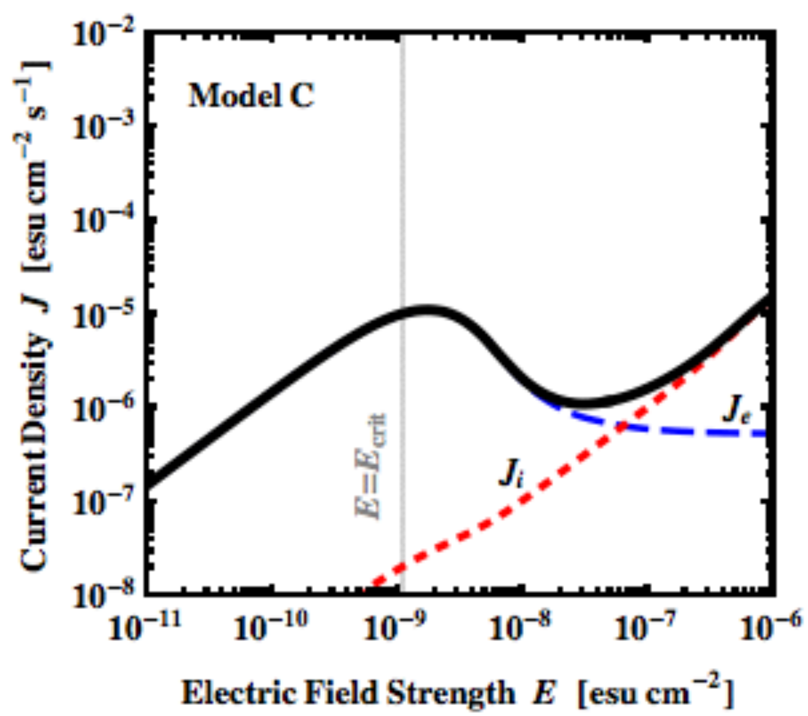
Engelhardt & Phelps (1963)

With inelastic losses, the electron kinetic energy is

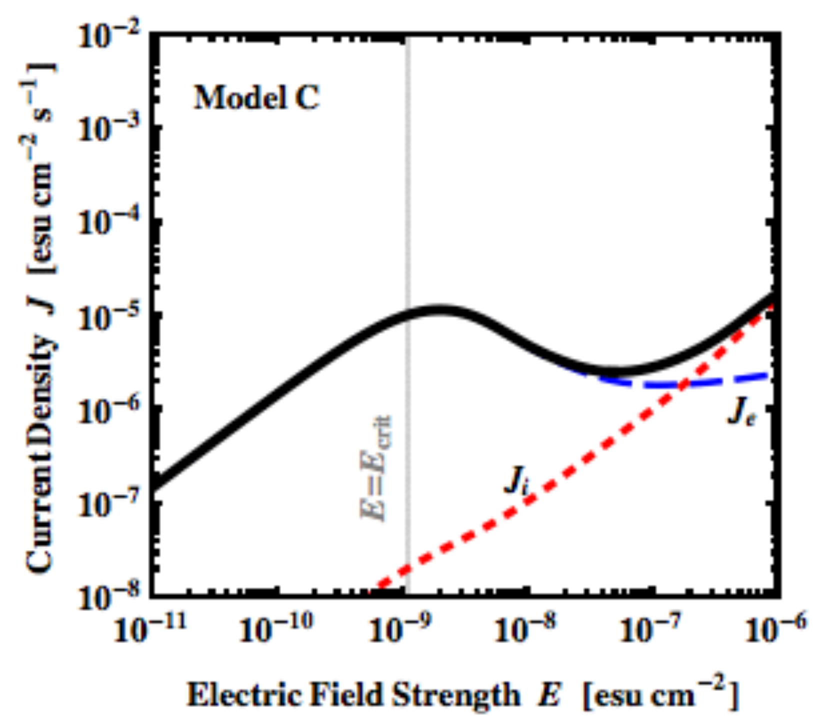
$$\langle \epsilon_e \rangle \approx 0.4 \sqrt{\frac{m_n}{m_e P_\ell}} eE\ell_e$$

- P_ℓ: **elastic energy loss**
- P_R: rotational excitation
- P_V: vibrational excitation
- P_I: ionization
- $P_\ell + P_R + P_V + P_I = 1$

elastic only (P_ℓ=1)



w/ inelastic loss



Toward a Generalized Nonlinear Ohm's Law

Gyromotion suppresses heating by $\boldsymbol{E}$ perp. to $\boldsymbol{B}$:

$$E_{\rm eff}^2 = E_{\|}^2 + E_{\perp}^2 \frac{1}{1 + (\Omega_{c,\alpha} t_{s,\alpha})^2}$$

(Golant et al. 1980)

- $\Omega_{\mathrm{c},\alpha}$: gyrofrequency of species α
- $t_{\mathrm{s},\alpha}$: stopping time of species α