Global Multifluid Simulations of the Magnetorotational Instability in Protoplanetary Disks

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Introduction

Motivation for global simulations

What is the weakly ionised approximation?

Simulations:

HYDRA – multifluid code

Computational setup

Angular momentum transport

Effect of magnetic field orientation

Resolution study

Future Work

Motivation for Global Simulations

Ohmic dissipation & ambipolar diffusion are generally bad news for the MRI

Investigation of Hall effect:
Wardle & Ng (1999), Sano & Stone (2002a,b), Wardle (2007), Salmeron & Wardle (2012), Kunz & Lesur (2013), Lesur et al. (2014), Bai (2014)

No **global** simulations of Hall dominated regime

HYDRA has excellent stability properties for the Hall dominated regime due to the Hall Diffusion Scheme (O'Sullivan & Downes, 2006, 2007)

First fully multifluid global simulations of protoplanetary disks which include all 3 non-ideal effects

The Weakly Ionised Approximation

Equations for weakly ionised multifluid MHD:

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \boldsymbol{v}_i) = 0\,;\,(1 \leqslant i \leqslant N)$$

Mass Continuity

$$\frac{\partial \rho_1 \boldsymbol{v}_1}{\partial t} + \nabla \cdot (\rho_1 \boldsymbol{v}_1 \boldsymbol{v}_1 + p_1 \mathbf{I}) + \nabla \mathbf{\Phi} = \mathbf{J} \times \mathbf{B}$$

Momentum Continuity

$$\alpha_i \rho_i (\mathbf{E} + \boldsymbol{v}_i \times \mathbf{B}) + \rho_i \rho_1 K_{i1} (\boldsymbol{v}_1 - \boldsymbol{v}_i) = 0;$$

→ Inertia and pressure of ionised species is negligible $(2 \leqslant i \leqslant N)$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\boldsymbol{v}_1 \mathbf{B} - \mathbf{B} \boldsymbol{v}_1) = \boxed{-\nabla \times \mathbf{E}'}$$

Induction Equation

Equations for multifluid MHD cont'd:

$$\mathbf{E}' = \mathbf{E}_o + \mathbf{E}_H + \mathbf{E}_A$$

$$\mathbf{E}' = r_o \frac{(\mathbf{J}\cdot\mathbf{B})\mathbf{B}}{B^2} + r_H \frac{\mathbf{J}\times\mathbf{B}}{B} - r_A \frac{(\mathbf{J}\times\mathbf{B})\times\mathbf{B}}{B^2}$$

$$\nabla \times \mathbf{B} = \mathbf{J}$$ Ampère's Law

$$\nabla \cdot \mathbf{B} = 0$$ Solenoidal Constraint, Dedner et al. (2002)

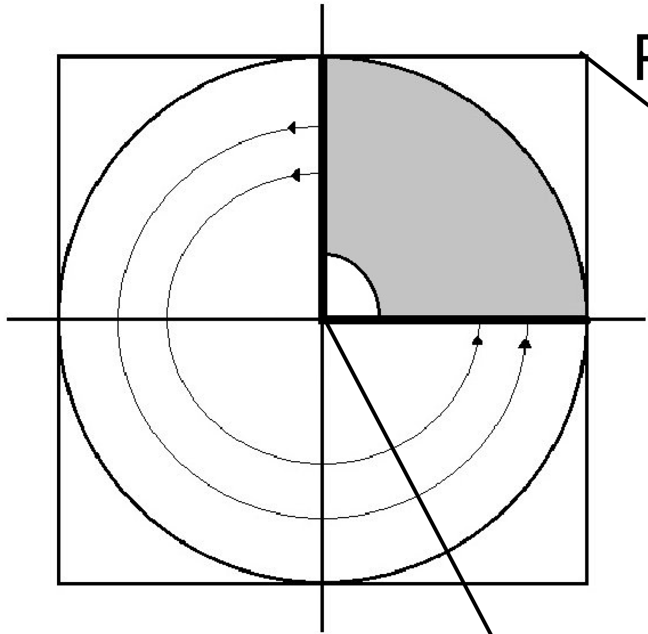
$$\sum_{i=2}^{N} \alpha_i \rho_i = 0$$ Charge Neutrality

$$\sum_{i=2}^{N} \alpha_i \rho_i \boldsymbol{v}_i = \mathbf{J}$$

HYDRA

- Shock-capturing multifluid MHD code: O'Sullivan & Downes (2006, 2007)
- Models weakly ionised plasmas
- Explicit, finite volume scheme - 2nd order in time & space
- Based on a cartesian grid
- Molecular cloud turbulence (Downes, 2012)
- Magnetic field amplification in SN remnants (Downes & Drury, 2014, Drury & Downes, 2012)
- Accretion disks (O'Keeffe & Downes, 2014)

Periodic boundary conditions:
low x,y and z axes

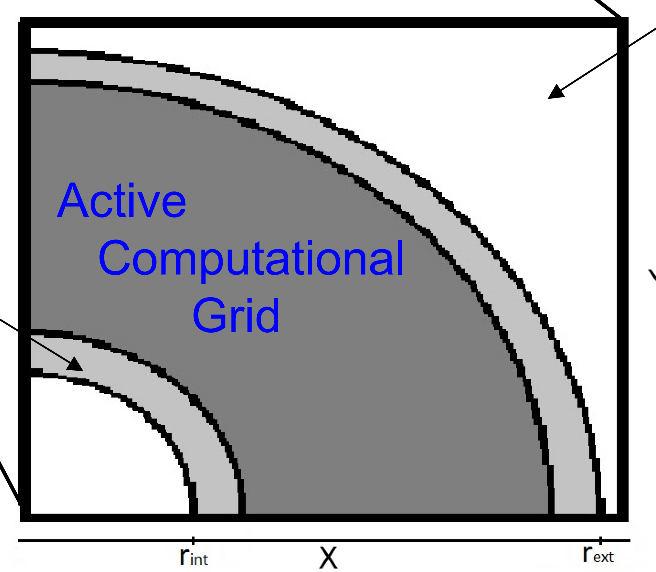


Frozen zone

Wavekilling region
(Lyra et al. 2008)

Simulating $\frac{\pi}{2}$ disk

Stone (2000),
O'Keeffe (2013)



Physical Setup

Isothermal simulations: $P_1 = c_s^2 \rho_1$

Gravity: $$\phi = -\frac{GM_*}{\sqrt{x^2 + y^2}}$$ ← No vertical component of gravity

Constant density

3 fluid simulations: neutrals, electrons and ions

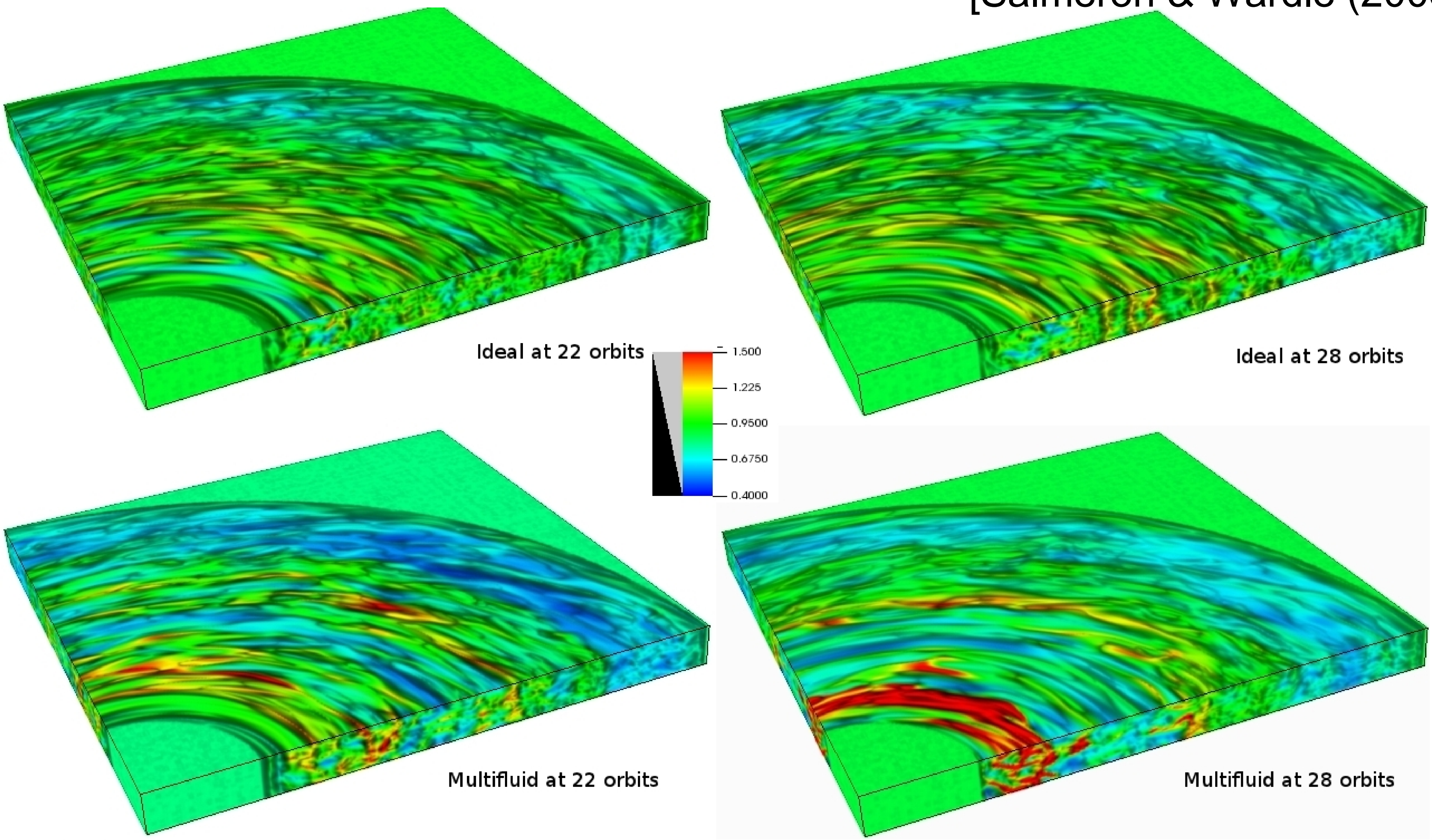
Highest resolution: Nx, Ny, Nz = 480,480,36

Active computational domain ~0.6 – 2.48 au

Initial setup:

$$B_z = 50\mathrm{mG}\ ; \quad n_H \approx 7 \times 10^{12}\mathrm{cm}^{-3};\ x_e = 4 \times 10^{-11}$$

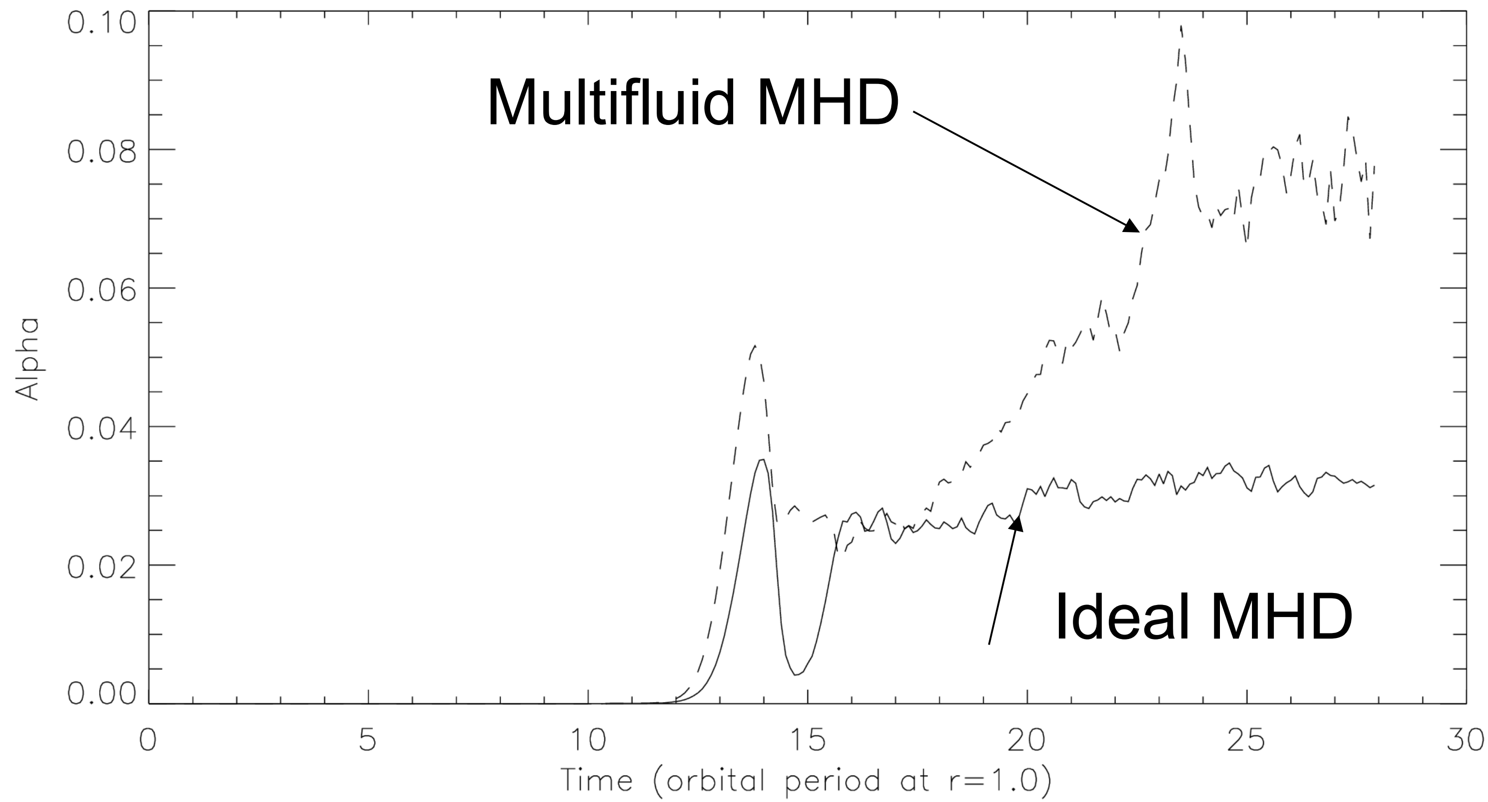
[Salmeron & Wardle (2003)]



O'Keeffe & Downes (2014)

$$\alpha = \alpha_R + \alpha_M = \frac{(\overline{\rho \delta v_r \delta v_\phi} - \overline{\delta B_r \delta B_\phi})}{\overline{P}}$$

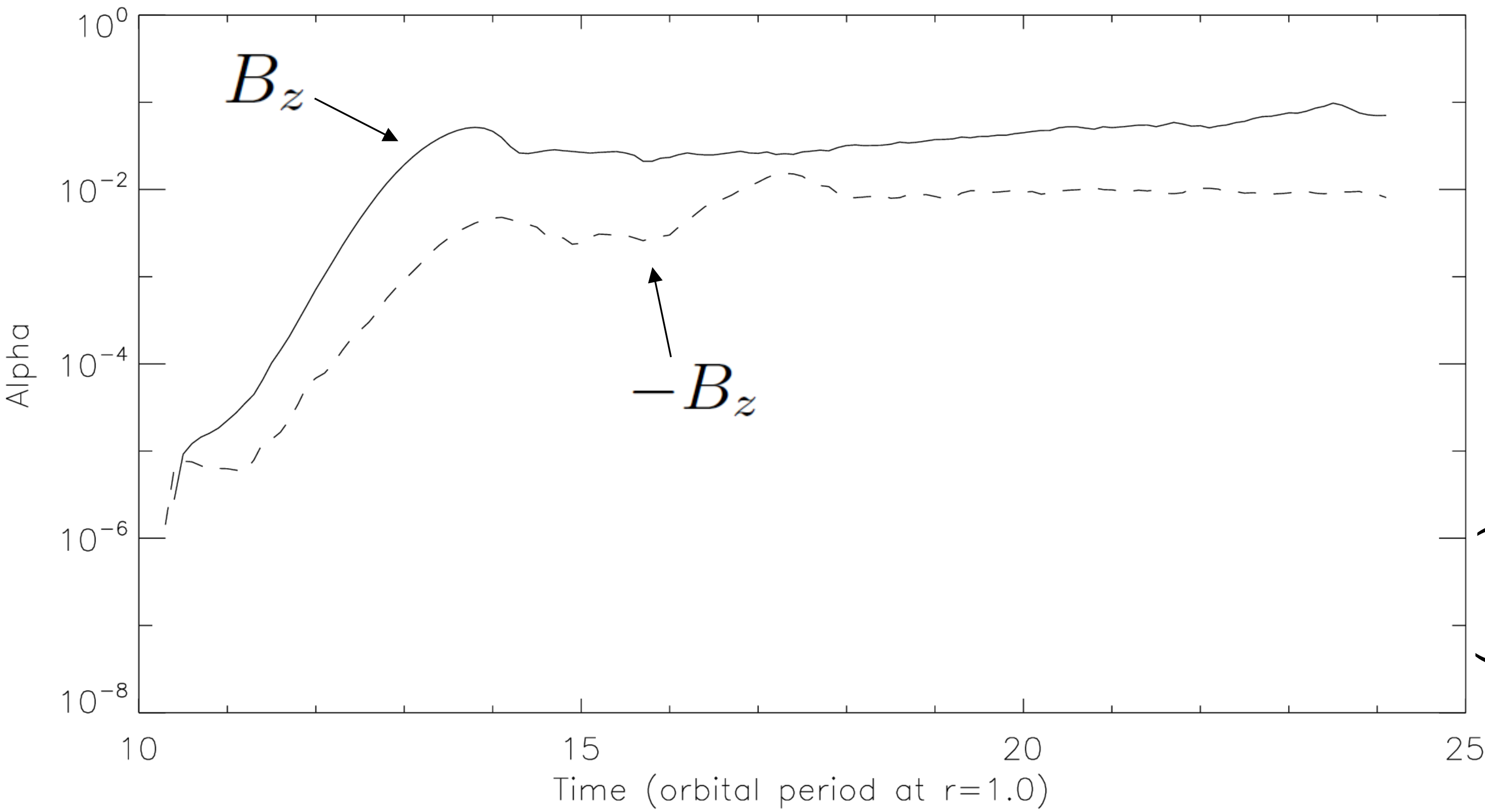
The Hall Effect *enhances* angular momentum transport



O'Keeffe & Downes (2014)

Effect of field orientation

$$\Omega \cdot \mathbf{B} < 0$$

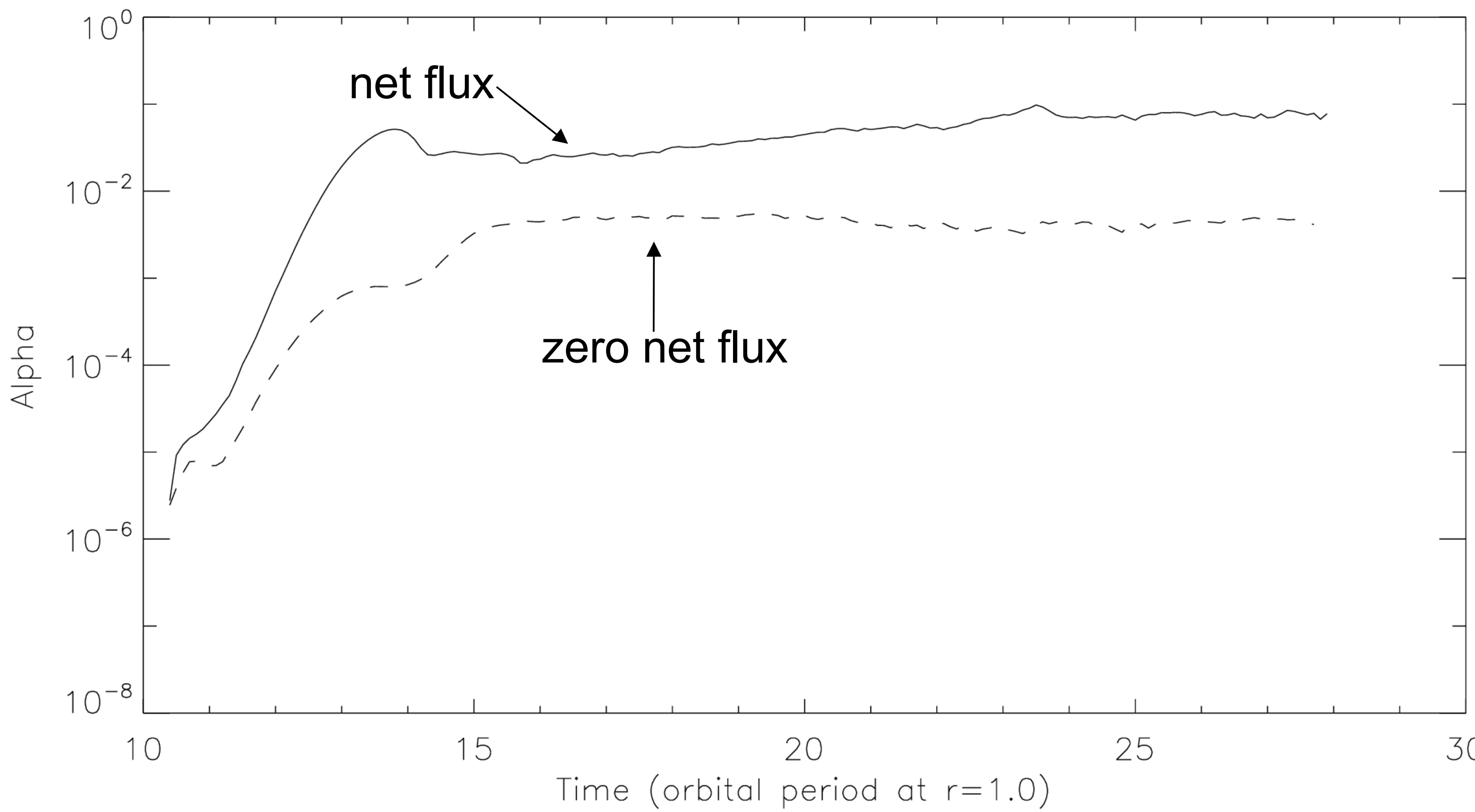


O'Keeffe & Downes (2014)

Angular momentum transport is suppressed

Effect of zero net magnetic flux

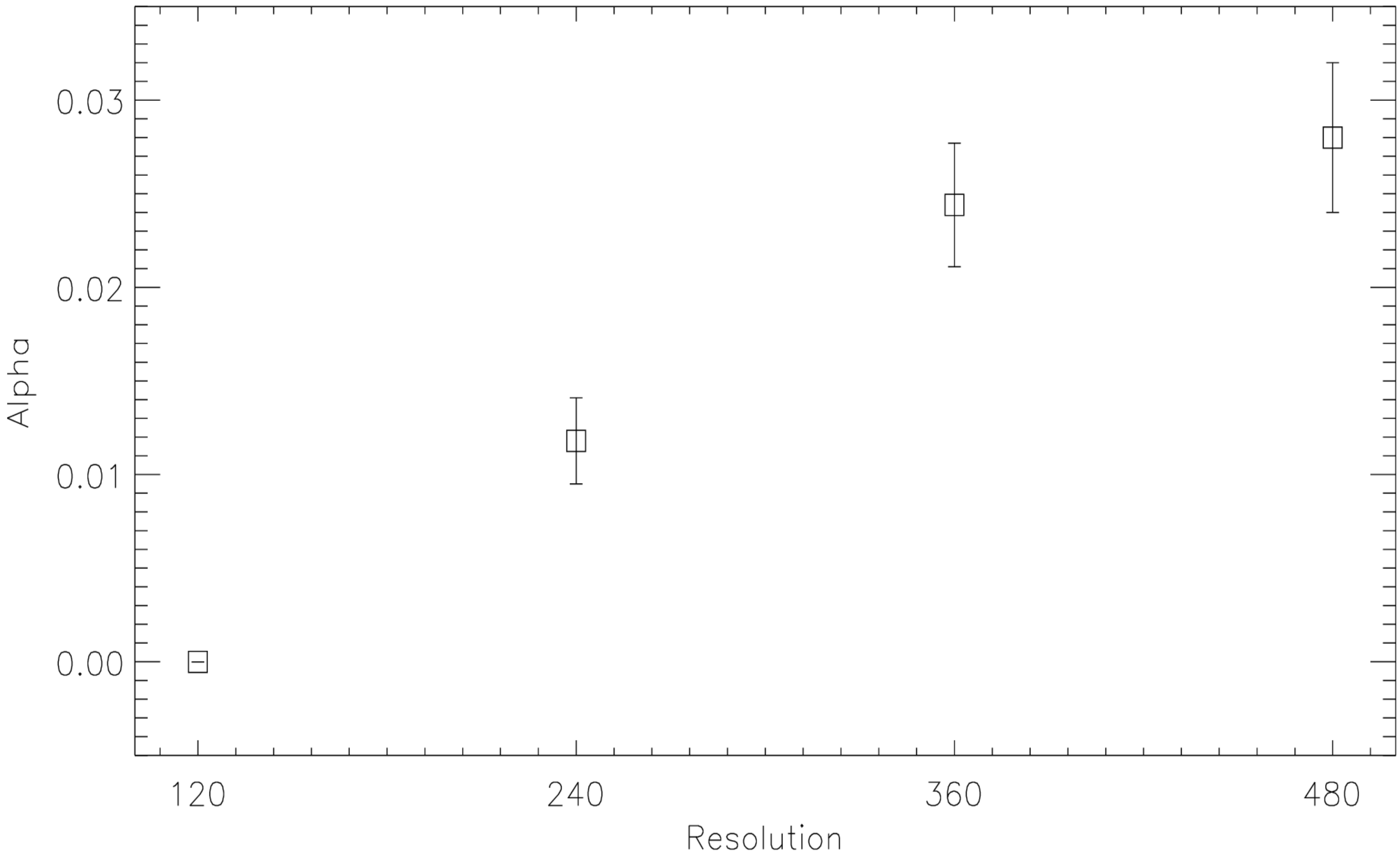
$$B_z(r) = B_0 sin\left(2n\pi\frac{(r-r_i)}{(r_o-r_i)}\right)$$



O'Keeffe & Downes (2014)

Angular momentum transport is suppressed

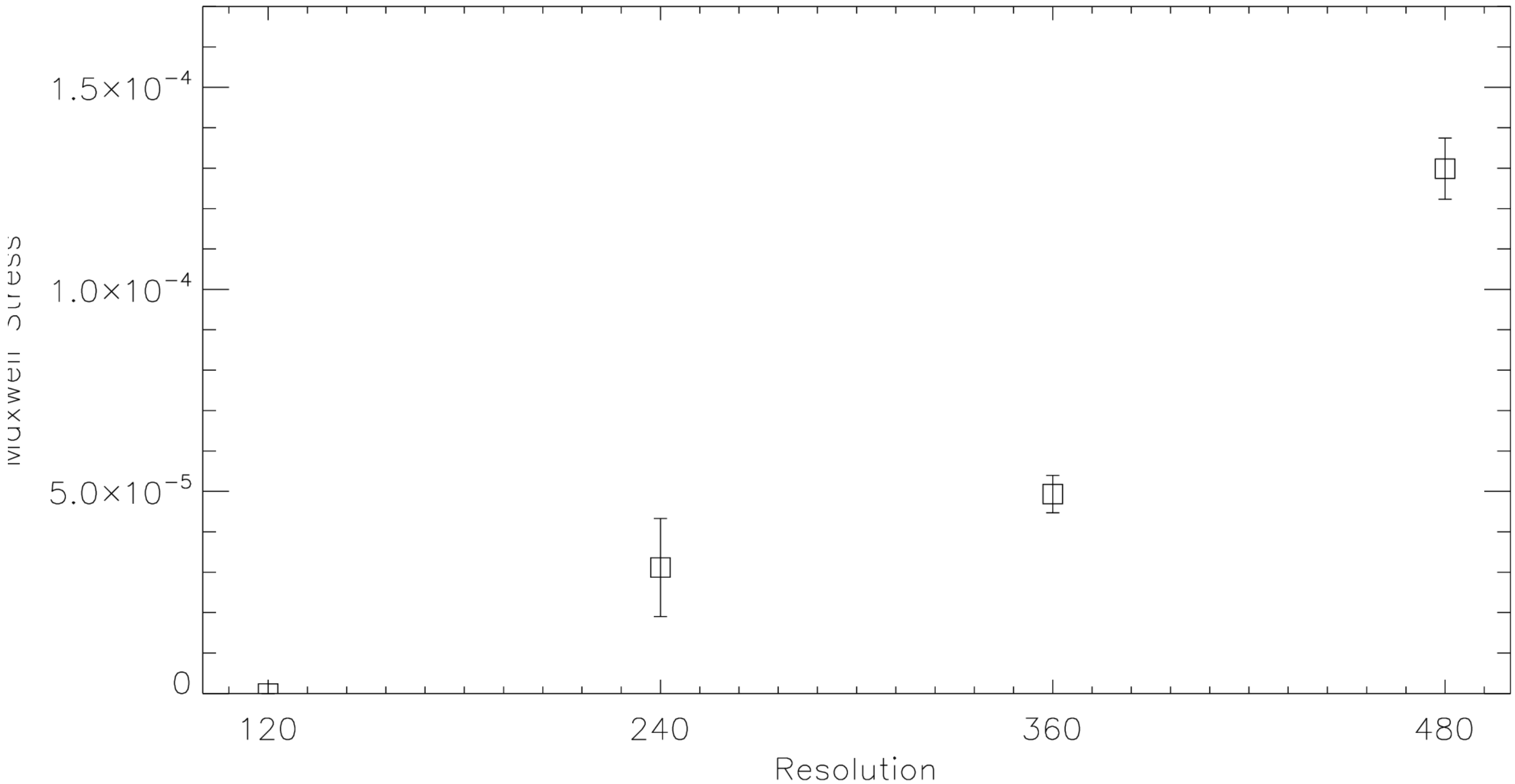
Resolution Study: Ideal MHD



O'Keeffe & Downes (2014)

α converges with increasing resolution

Resolution Study: Multifluid MHD



O'Keeffe & Downes (2014)

α diverges with increasing resolution

Resolution Study: Multifluid MHD

α diverges with increasing resolution:

Alfvén speed

$$L_H = \frac{c}{\omega_{pi}} \frac{v_A}{v_0}$$

Ion plasma frequency

Orbital speed

At r = 2 au, $L_H = 6.0\times10^{-5}$ au

Whereas grid spacing is 0.1 – 0.03 au

$\therefore$ The Hall effect is initially unresolved

But once the MRI becomes active the magnetic field strength amplifies and L_H increases

Future Work

- Higher resolution, greater radial extent (1-6 au), longer runs – how dead is the dead zone?
- Comparison with observations-turbulent velocity [Simon et al. (2011), Hughes et al. (2011), Guilloteau et al. (2012)]
- Radially stratified disks (Steinacker & Papaloizou, 2002)
- Inclusion of 4th species (dust)
- Vertically stratified accretion disks (Fromang & Nelson, 2006, Suzuki et al., 2013)
- MRI + Magnetocentrifugal disk winds

Conclusions

First fully multifluid global simulations of protoplanetary disks

The simulations of O'Keeffe & Downes (2014) show an enhancement of angular momentum transport in comparison to ideal MHD simulations

The Hall effect, with correct magnetic field orientation, is likely to be responsible for this increase in the α parameter

Resolution study: Higher resolution runs are needed to check for convergence

Definitions of Resistivities

$$r_o = \frac{1}{\sigma_o}\ ; \qquad r_H = \frac{\sigma_H}{\sigma_H^2 + \sigma_A^2}\ ; \qquad r_A = \frac{\sigma_A}{\sigma_H^2 + \sigma_A^2}$$

Resistivities

$$\sigma_o = \frac{1}{B}\sum_{n=2}^{N} \alpha_n \rho_n \beta_n\ ; \qquad \sigma_H = \frac{1}{B}\sum_{n=2}^{N} \frac{\alpha_n \rho_n}{1+\beta_n^2}$$

Conductivites

$$\sigma_A = \frac{1}{B}\sum_{n=2}^{N} \frac{\alpha_n \rho_n \beta_n}{1+\beta_n^2}\ ; \qquad \beta_n = \frac{\alpha_n B}{K_{1n}\rho_1}$$

Hall Parameter