# Locality and not-locality in accretion flows


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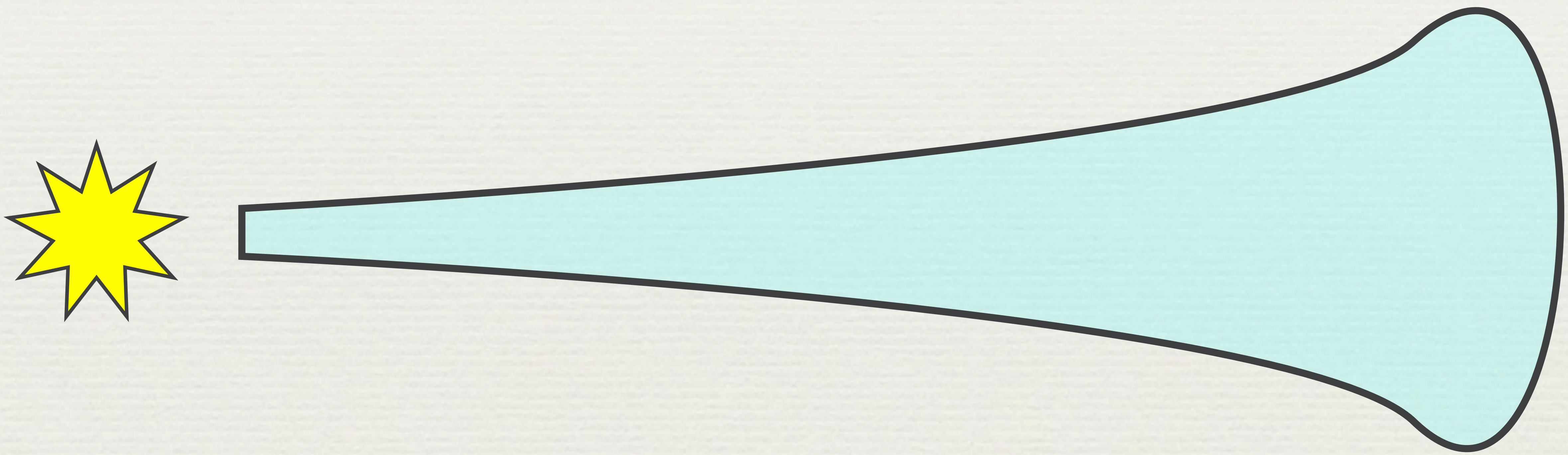
# To what extent can we treat accretion disks locally?

- Gravity ties disks together on nearly instantaneous timescales
  - Cannot treat self gravitating disks locally on anything but the shortest of timescales



- Let us try to consider a not-self-gravitating disk locally.
- We can do so only to the extent that we can ignore the inner and outer disk.

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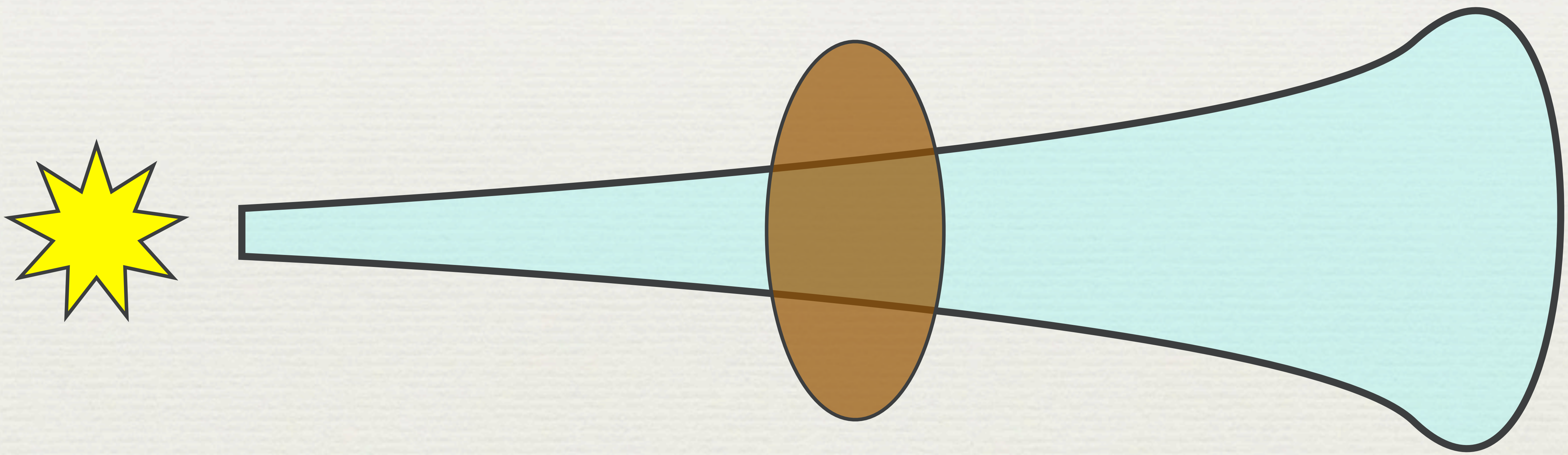

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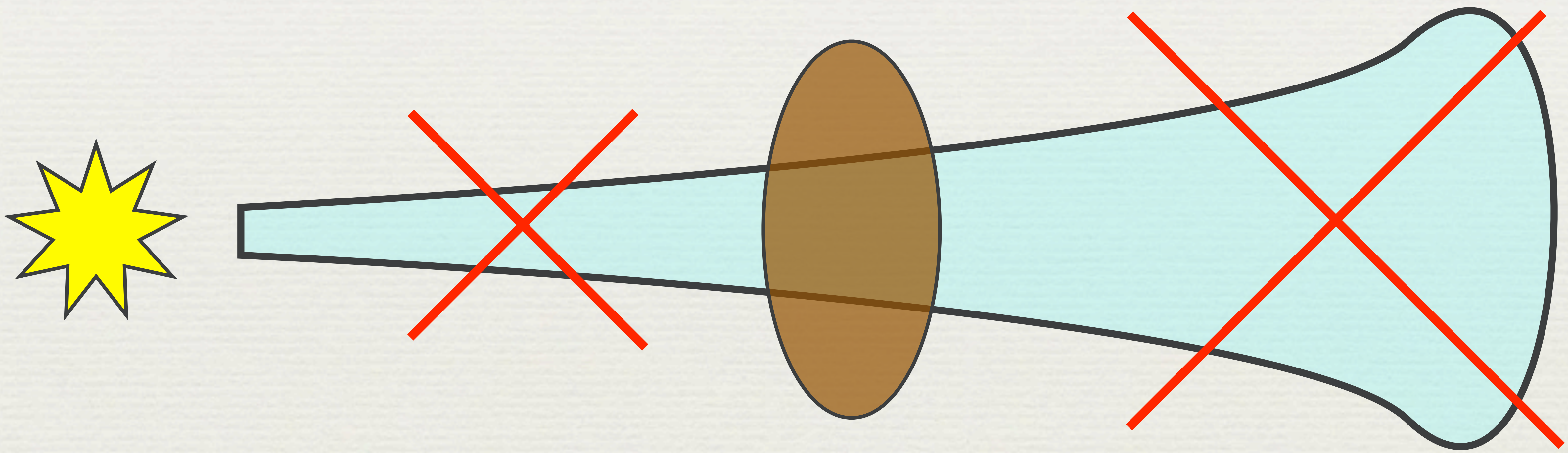

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# Quick refresher

Reynolds stress

Total stress $$T_{ij} = \rho v_i v_j - \frac{B_i B_j}{\mu_0}$$ Maxwell stress

$$W_{r\phi} \equiv \Sigma^{-1} \int dz \langle T_{\phi r} \rangle_\phi$$

This talk assumes horizontal stresses (MRI) are significant and that the stresses are powered by accretion (not irradiation etc...)

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$$\partial_t \langle \rho r v_\phi \rangle_\phi + \frac{1}{r} \partial_r \langle r^2 T_{\phi r} \rangle_\phi + r \partial_z \langle T_{\phi z} \rangle_\phi = 0$$

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$$\frac{1}{r} \partial_r \langle r^2 T_{\phi r} \rangle_\phi < 0 \quad \Rightarrow \quad \text{Decretion}$$

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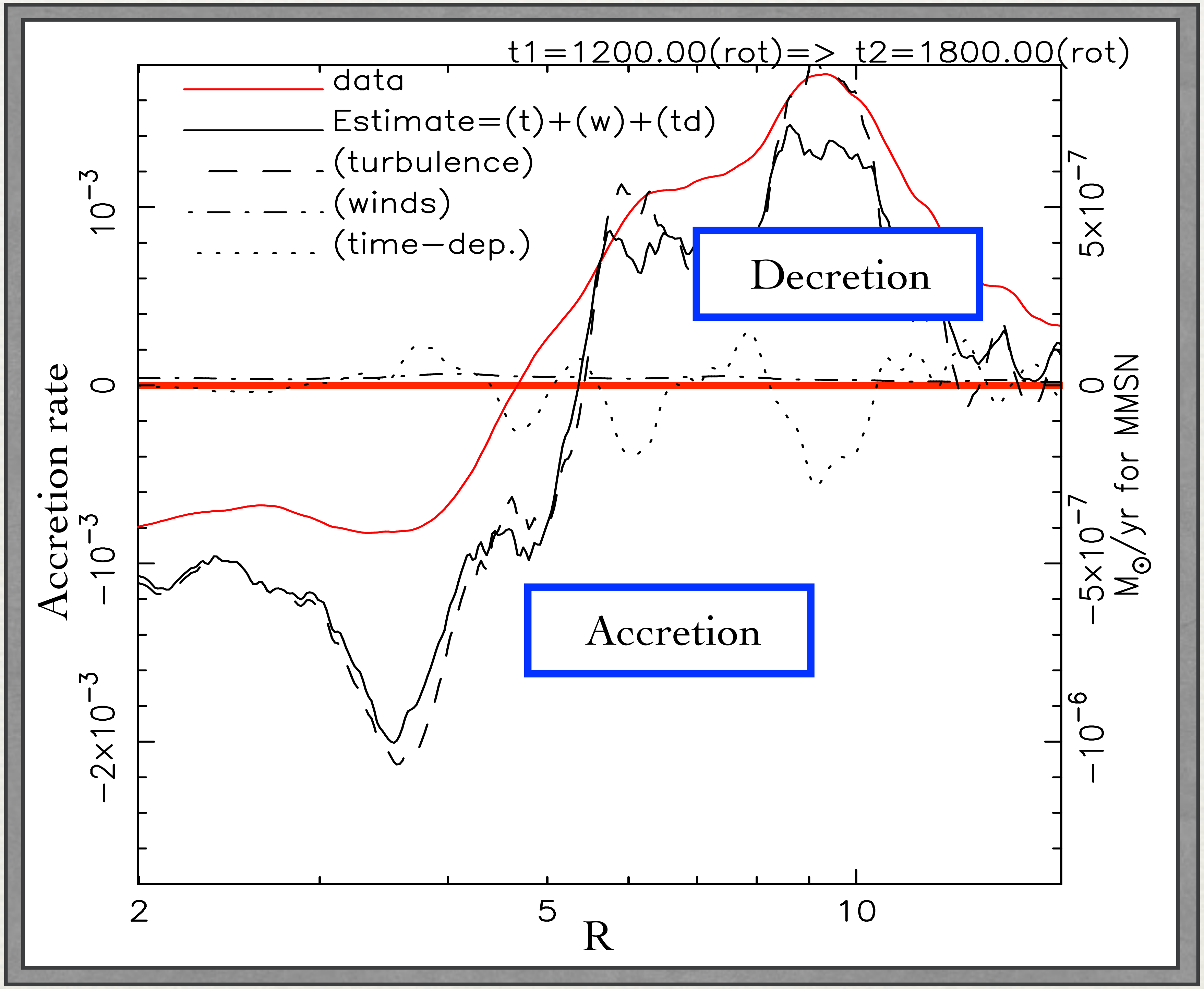
# Alpha disks have decreting power law regions

$$\frac{1}{r}\partial_r \langle r^2 T_{\phi r}\rangle_\phi < 0 \quad \Rightarrow \quad \text{Decretion}$$

| Viscous disk midplane | Vertically integrated |
|---|---|
| $r^2 T_{\phi r} \propto r^2 \rho c_s^2 \propto H\Sigma r^{-1}$ | $r^2 \int_z dz T_{\phi r} \propto H^2 \Sigma r^{-1}$ |
| decretion $\leftarrow H\Sigma \propto r^{\epsilon}$ | $H^2\Sigma \propto r^{1+\delta} \rightarrow$ accretion |

$$\rho(r,z) = \rho_0(r) e^{-z^2/2H^2(r)} \qquad \frac{1}{\rho(r,z)}\partial_r \rho(r,z) = \frac{1}{\rho_0}\partial_r \rho_0 + \frac{z^2}{H^2(r)}\frac{1}{H(r)}\partial_r H(r)$$

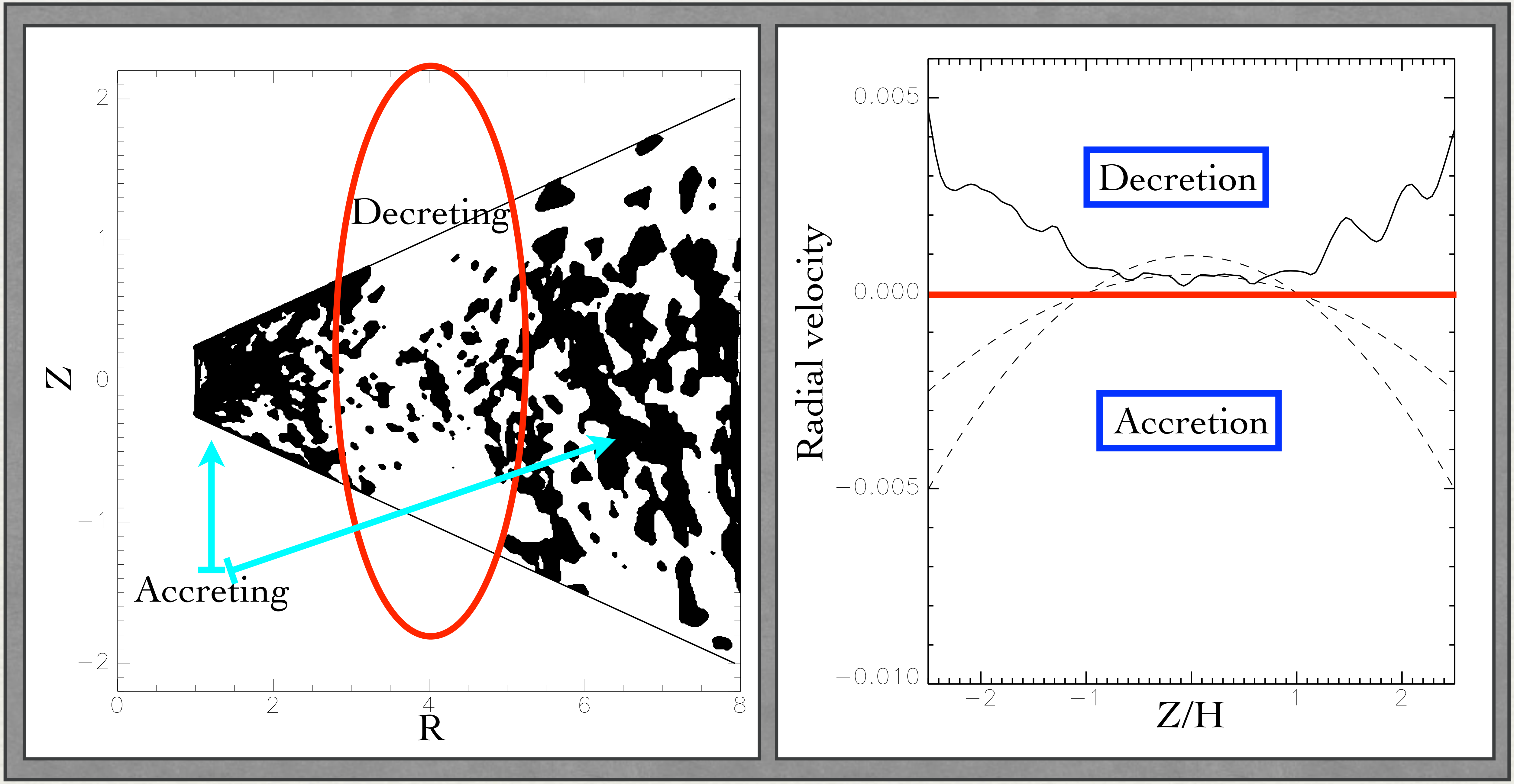
# Simulations see decretion



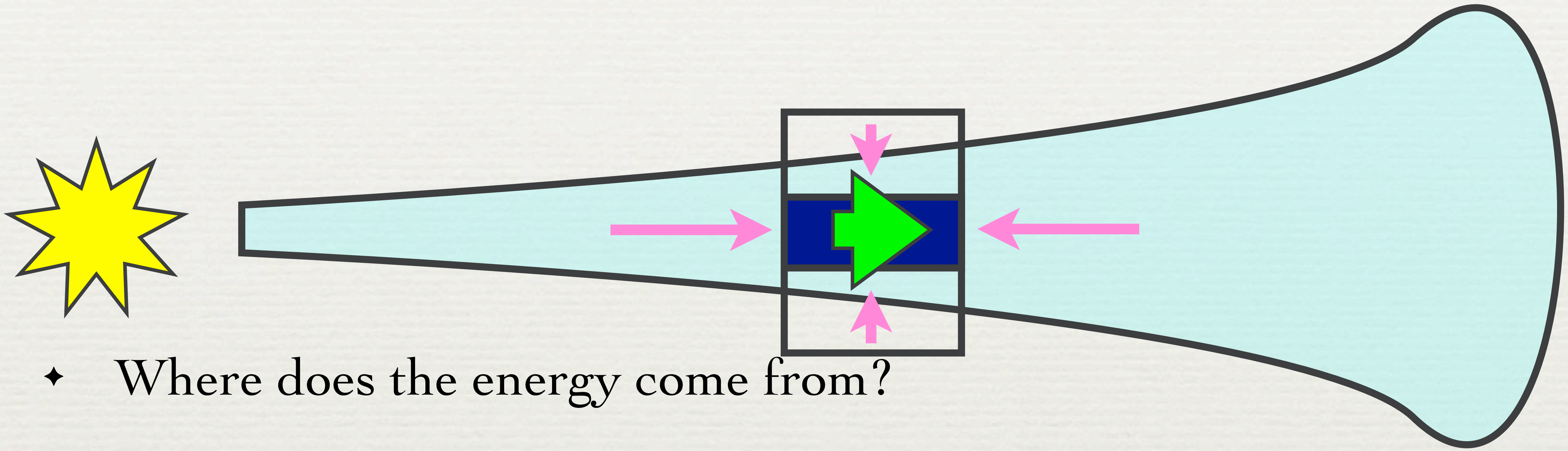

Suzuki & Inutsuka (2014)

# Simulations see decretion

Fromang, Lyra & Masset (2011)



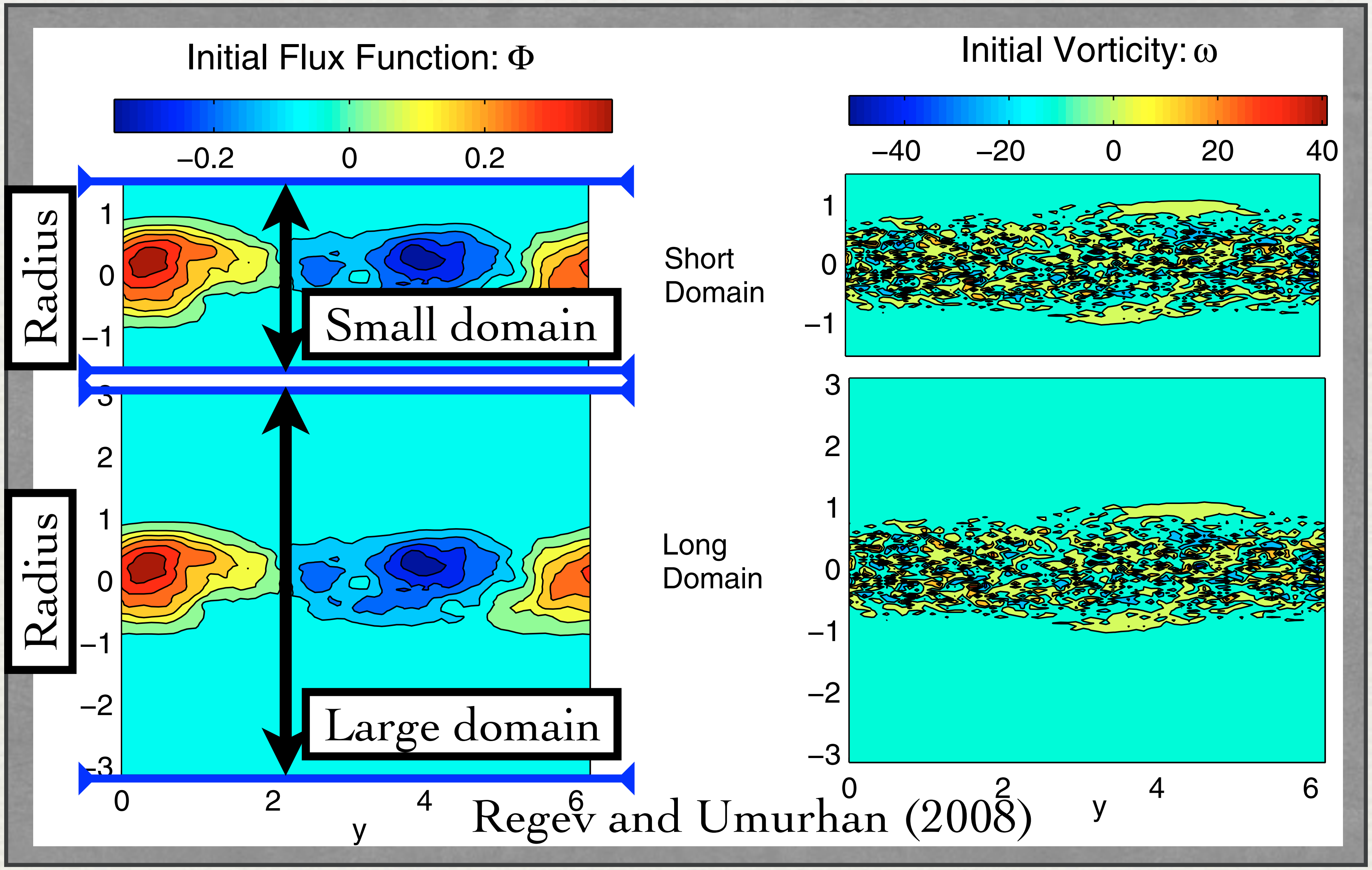
# Decretion costs energy.  What powers it?



- Where does the energy come from?
  - If there is net accretion, there is net power
  - Maybe boundary work?
  - Balbus & Papaloizou 1999:

$$\boldsymbol{\mathcal{F}}_E = \boldsymbol{v}\left(\frac{1}{2}\rho v^2 + \rho\Phi + P\right) + \frac{\boldsymbol{B}}{\mu_0} \times (\boldsymbol{v} \times \boldsymbol{B})$$

# Should we think about it as boundary work?



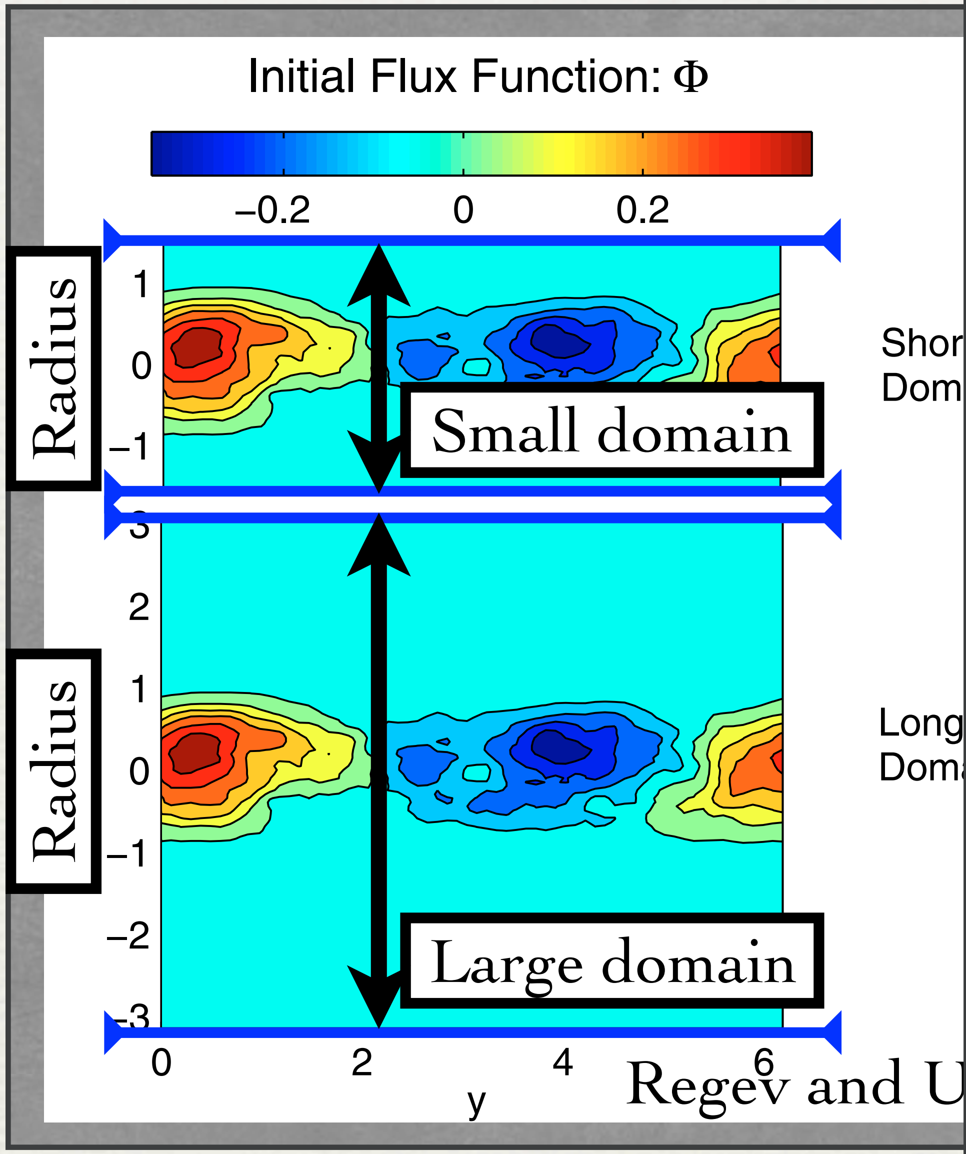

Regev and Umurhan (2008)

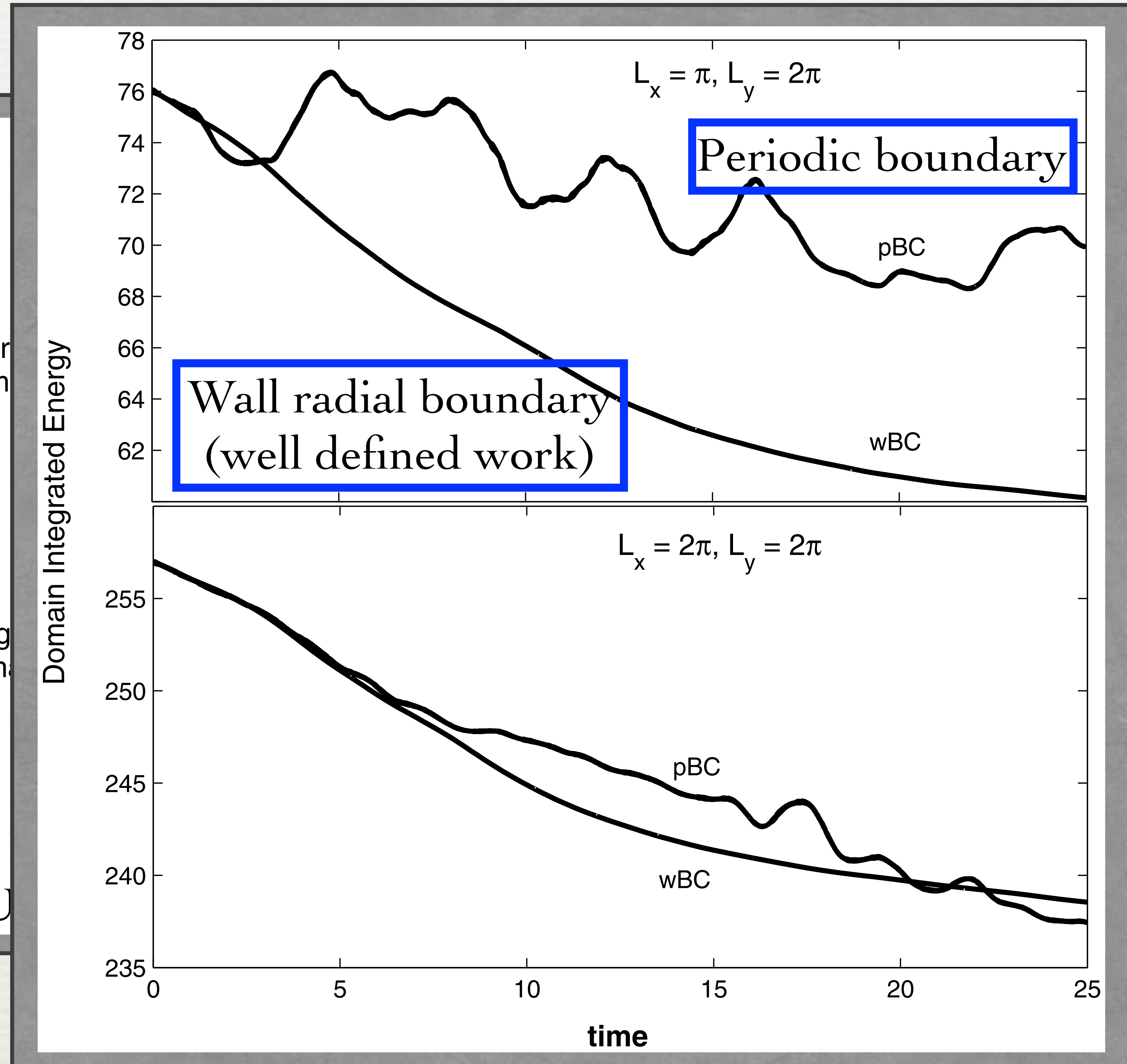
Performed two sets of numerical simulations

Same initial condition

- One with periodic boundaries (shearing box)
- One with wall boundary conditions

# Should we think about it as boundary work?

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# Energy is transported radially

$$\boldsymbol{\mathcal{F}}_E = \boldsymbol{v}\left(\frac{1}{2}\rho v^2 + \rho\Phi + P\right) + \frac{\boldsymbol{B}}{\mu_0} \times (\boldsymbol{v} \times \boldsymbol{B})$$

$$\boldsymbol{v} = \boldsymbol{v}_K + \boldsymbol{w} \to \boldsymbol{\mathcal{F}}_E = \boldsymbol{w}\left(\frac{1}{2}\rho|\boldsymbol{v}_K + \boldsymbol{w}|^2 + \rho\Phi + P\right) + \frac{\boldsymbol{B}}{\mu_0} \times (\boldsymbol{v}_K \times \boldsymbol{B})$$

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Drop the accretion flow

$$\boldsymbol{\mathcal{F}}_E = R\Omega\left[\rho w_r w_\phi - \frac{B_\phi B_r}{\mu_0}\right]\hat{\boldsymbol{r}} + R\Omega\left[\rho w_\phi w_z - \frac{B_\phi B_z}{\mu_0}\right]\hat{\boldsymbol{z}}$$

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$\boldsymbol{E} = -\boldsymbol{U} \times \boldsymbol{B} \to \boldsymbol{S}_R = (\boldsymbol{E} \times \boldsymbol{B})_R = -R\Omega\frac{B_\phi B_r}{\mu_0}$ is the radial Poynting flux

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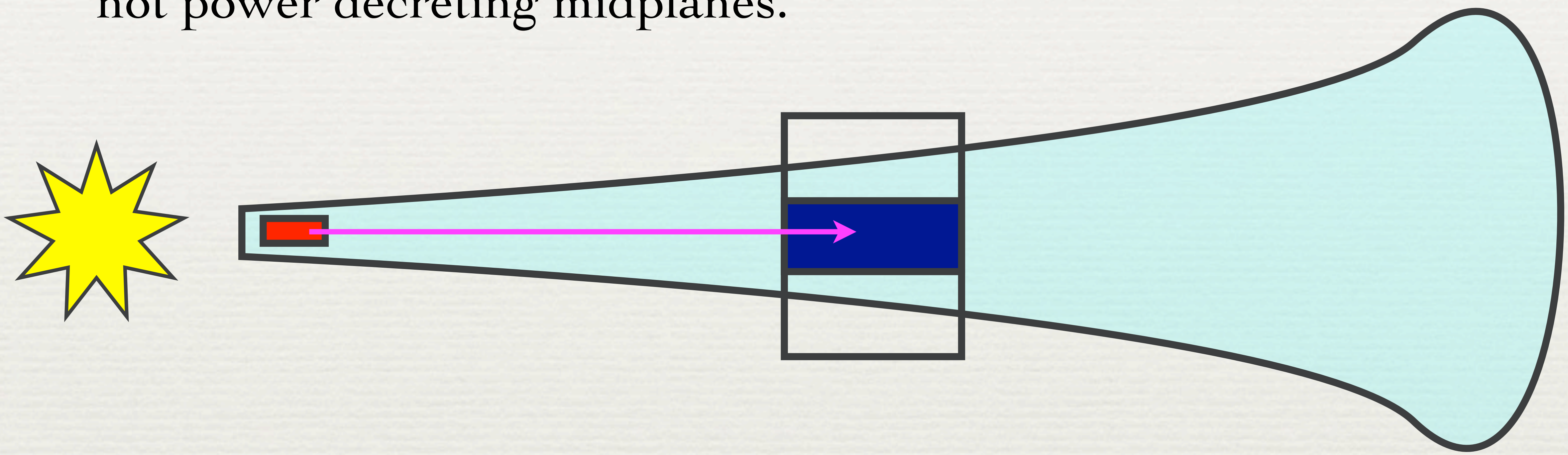
$$T_{ij} = \rho v_i v_j - \frac{B_i B_j}{\mu_0}$$

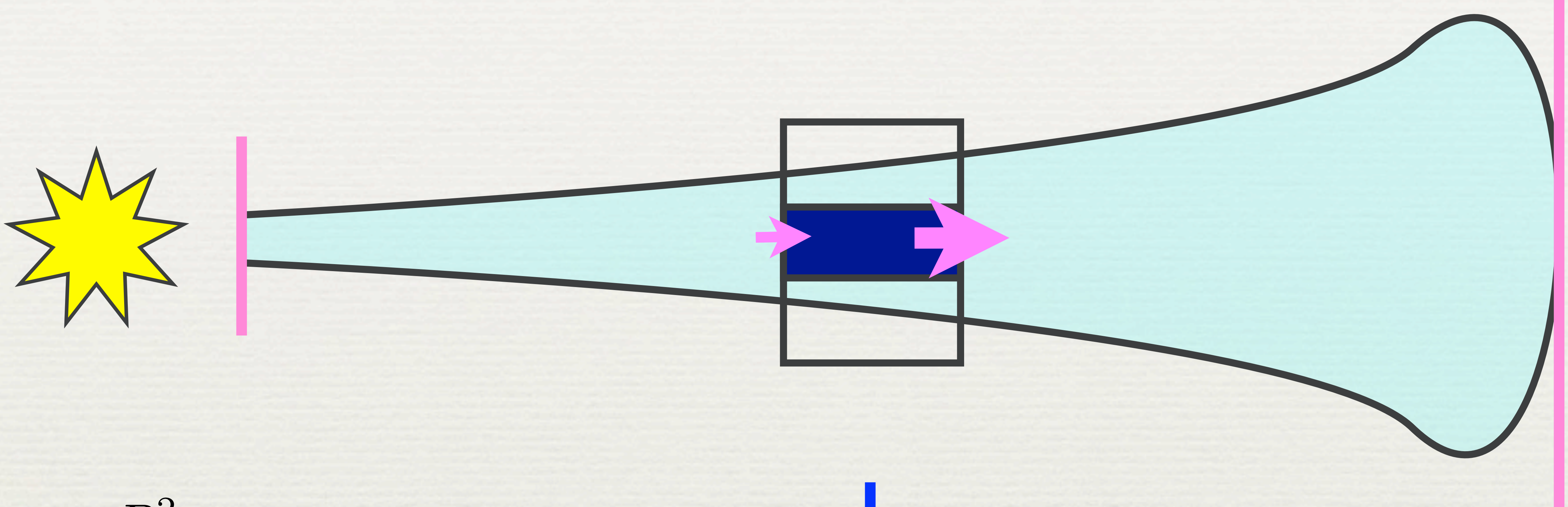
# Energy comes from the inner disk

- Radial energy transport means that accreting surface layers do not power decreting midplanes.



- Energy comes from the inner disk (gravitational potential energy scales as 1/r)
    - If the power law regions decrete (e.g. stress proportional to pressure), then the boundary between accretion and decretion must be close enough to the inner edge that the disk is locally not a power law.

# Energy fluxes ~ Accretion Power



$$\partial_t \frac{B^2}{2\mu_0} = -\boldsymbol{\nabla}\cdot\boldsymbol{S} - \eta \boldsymbol{J}^2 - \boldsymbol{F}_L \cdot \boldsymbol{v}$$

$$\langle M_{\phi r}\rangle_{\phi,t} \propto r^{-s}$$

$$\downarrow$$

$$\langle \boldsymbol{\nabla}\cdot\boldsymbol{S}\rangle_{\phi,t} = (1/2 - s)\Omega\langle M_{\phi r}\rangle_{\phi,t}$$

## Accretion power

(Balbus & Papaloizou (1999) Eq 28)

$$\Sigma R^2 W_{r\phi} \propto r^{-g}$$

$$\downarrow$$

$$P_{acc} = \Sigma R \Omega_K^2 \langle u_r\rangle_\rho \sim 2g\Omega\Sigma W_{r\phi}$$

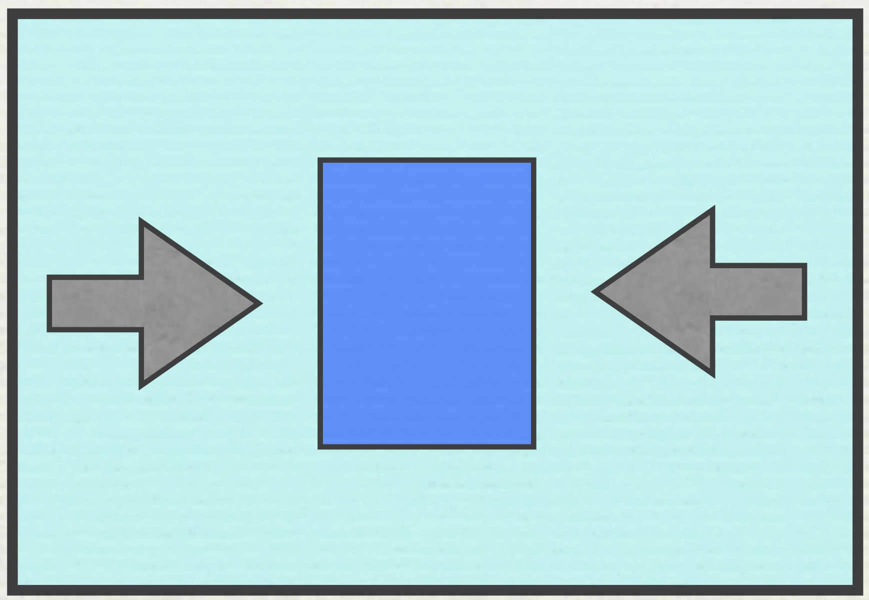
Poynting flux and accretion power have the same scale!

# Consequences:

- Poynting flux ties different radii in disks together
  - A local simulation from a seed field will not have the right energy flow.
    - **You cannot start a local simulation from a seed field and get the stresses or accretion rate!**
    - MRI saturation cannot be determined locally in an accretion disk because the saturated state depends on distant regions.
- Horizontal stresses mean horizontal energy fluxes
  - Accreting surface layers do not power decreting midplanes
  - Disks should be thought of as layered slabs, not adjacent annuli
  - Under the standard assumption of power law disks, the boundary between accretion and decretion **in each layer** is close to the **inner edge of that layer.**

# Can we extract a local patch from a (low resolution) global model?

- Disturbances will propagate inwards from the new boundaries.



- Viscous time scales are long (thousands of orbits), so maybe the inner region of the local patch is protected.

# The energy buffer and the stresses are comparable

$$E_T = \int_z \left( \frac{1}{2}\rho w^2 + \frac{B^2}{2\mu_0} \right)$$

- $E_T$ measures the energy in the fields that exert the stresses

$$\boxed{T_{ij} = \rho v_i v_i - \frac{B_i B_j}{\mu_0} \qquad\qquad E_T = \int_z f T_{ij}}$$

- Horizontal magnetic case:
  - Magnetic pitch angle

$$f = -\frac{B^2}{2B_\phi B_r} \simeq -\frac{B_\phi}{2B_r}$$

# Stresses are proportional to the accretion power

Balbus & Papaloizou (1999) Eq 28

$$P_{acc} = \Sigma R \Omega_K^2 \langle u_r \rangle_\rho \sim 2g\Omega\Sigma W_{r\phi}$$

$$\Sigma R^2 W_{r\phi} \propto r^{-g}$$

$$\boxed{\left|\frac{E_T}{P_{acc}}\right| = \left|\frac{f}{2g\Omega}\right|}$$

Energy buffer lasts at most a few orbits. Local patches are very vulnerable to exterior influences (i.e. changes in the order-unity Poynting flux).

Balbus & Papaloizou (1999) considered the time evolution of the orbital energy, but not the energy in the stresses! The orbital energy is vast, and varies on viscous time scales. Conversely, the stress energy is small and can vary on orbital time scales.

# Consequences

- Energy transport controls the stresses on short timescales
  - Large Poynting fluxes, small stress energy densities
- Inner disks have fast viscous timescales that control accretion
  - This allows for, but does not guarantee disk variability
  - Variability is seen EXors, FUors in protoplanetary disks
- Extracting local patches from global simulations is only reliable for short (orbital) times.
  - (AMR will work because it remains global)

# Conclusions

- Saturated stresses must be determined globally
  - Poynting flux deposits/extracts an order unity fraction of the accretion power, so tightly links the accreting inner disk to the outer (decreting) disk
  - Horizontal stresses ➔ horizontal energy fluxes ➔ layered slabs.
    - Transition between accretion and decretion occurs on a slab-by-slab basis.
    - Viscous disks: transition is at the inner edge of the power law region.
- The energy in the stresses is small
  - Only buffers for orbital time scales.
  - The evolution of the inner disk over short timescales controls the system.