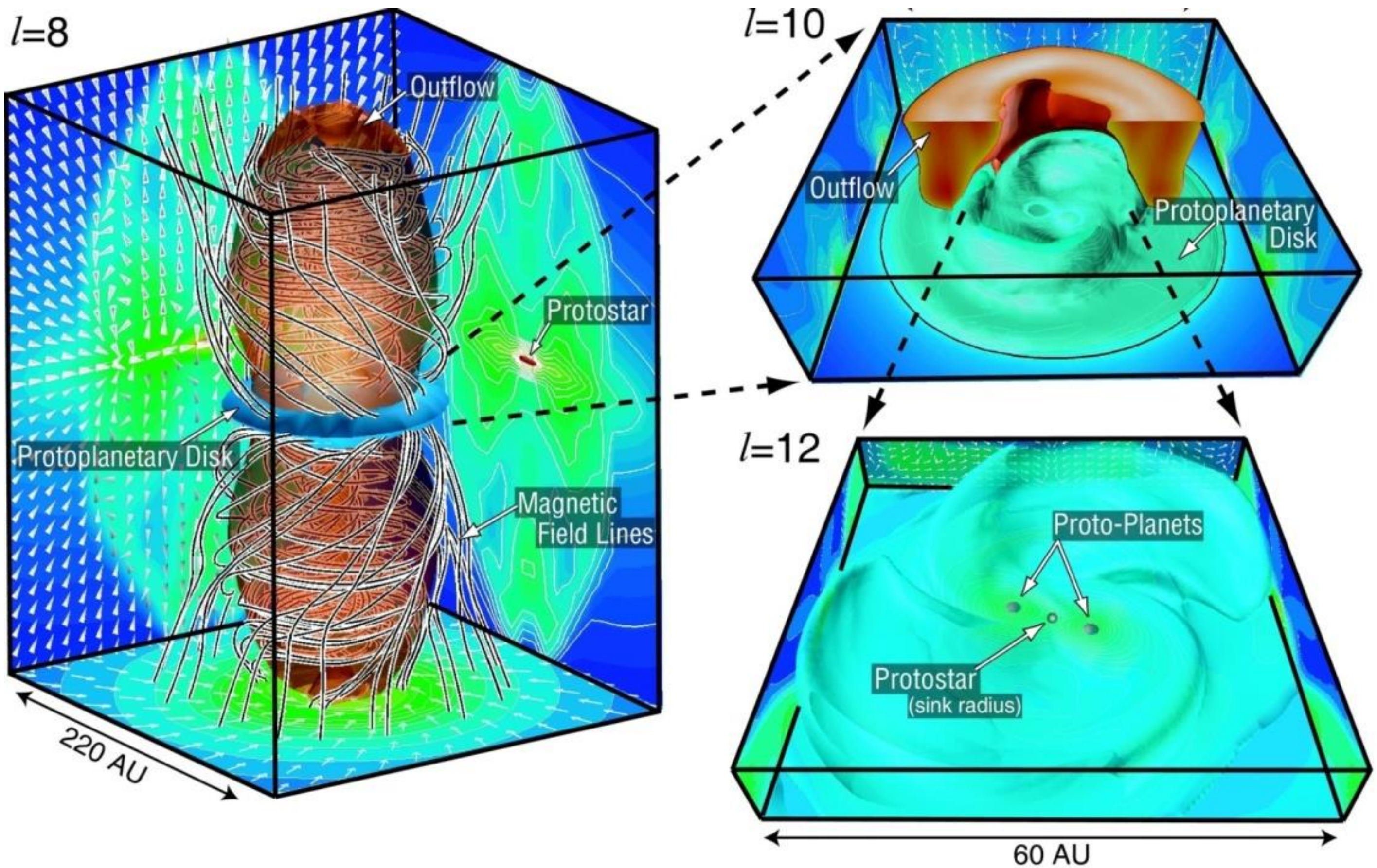
The Formation and Evolution of Protoplanetary Disks: The Critical Effects of Non-Ideal MHD

Shu-ichiro Inutsuka (Nagoya Univ.)



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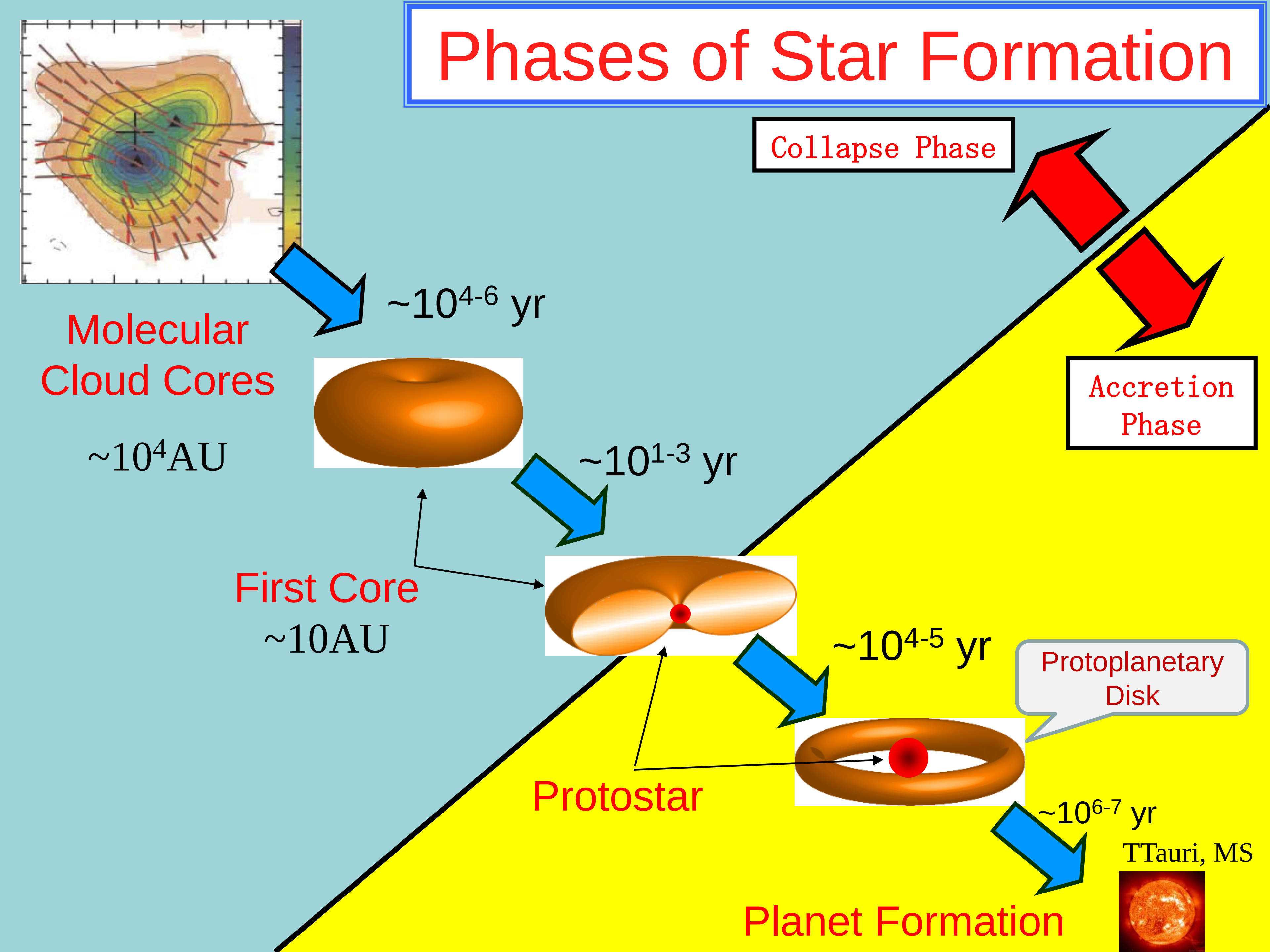
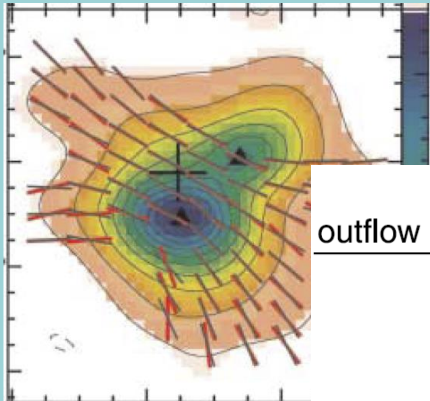
T. Matsumoto (Hosei Univ.),

S. Takahashi, Y. Tsukamoto, K. Iwasaki (Nagoya Univ.)

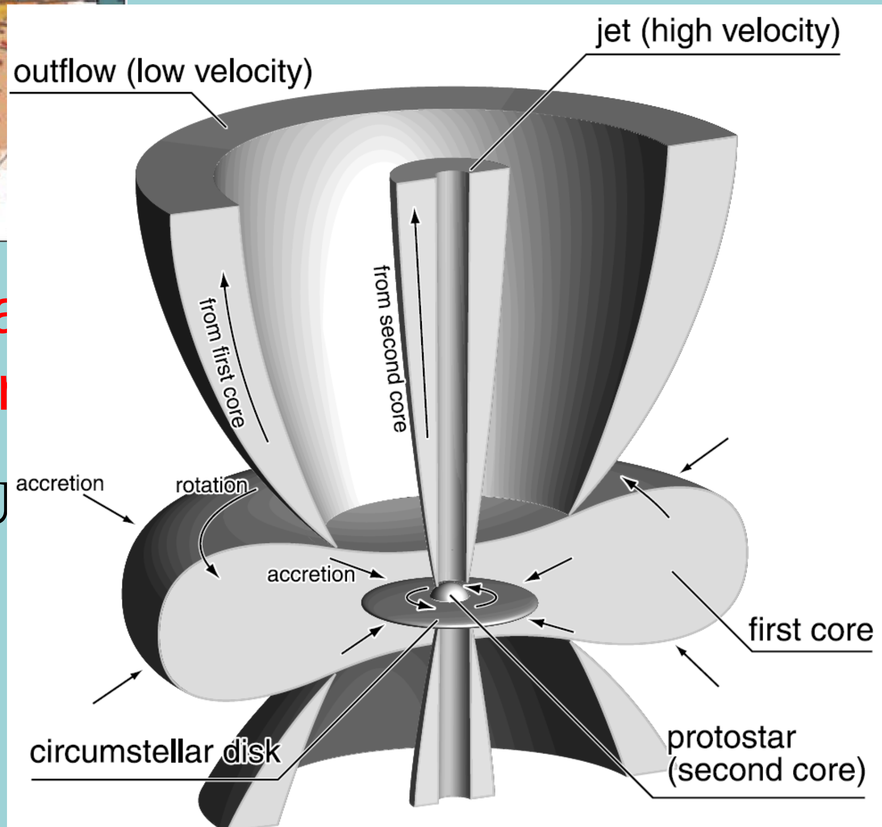

Phases of Star Formation
Collapse Phase
Accretion Phase
~10$^{4-6}$ yr
Molecular Cloud Cores
~10^{4}AU
~10$^{1-3}$ yr
First Core
~10AU
~10$^{4-5}$ yr
Protoplanetary Disk
Protostar
~10$^{6-7}$ yr
TTauri, MS
Planet Formation

Phases of Star Formation



Molecula
Cloud Cor

$\sim 10^4$AU

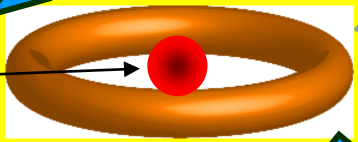


Collapse Phase

Accretion Phase

$\sim 10^{4\text{-}5}$ yr

Protoplanetary Disk



Protostar

$t = t_*$

$\sim 10^{6\text{-}7}$ yr

TTauri, MS

Planet Formation

Outline of Part 1

- Basic Problems of Star Formation
- 1st Collapse ➔ 1st Core

  2nd Collapse ➔ 2nd Core=Protostar
- Outflows vs. Jets

  Properties & Driving Mechanism
- Formation of Magnetized PPD and Fragmentation

  Dead Zone and Envelope Dispersal

Focus on Formation of Single Stars

Basic Problems in Star Formation

1. Angular Momentum Problem:

   Protostar:

   $$h_* = \Omega_* R_*^2 \sim (10^{11}\mathrm{cm})^2/(10^5\mathrm{s}) \sim 10^{17}\ \mathrm{cm}^2/\mathrm{s}$$

   Molecular Cloud:

   $$h_{\mathrm{core}} = \delta v_{\mathrm{core}} R_{\mathrm{core}} \sim 0.1\mathrm{km/s} \times 10^{17}\mathrm{cm} \sim 10^{21}\ \mathrm{cm}^2/\mathrm{s}$$

   $$\Rightarrow\ h_* \sim 10^{-4}\ h_{\mathrm{core}}$$

When?

2. Magnetic Flux Problem

   Protostar: $\Phi_* \sim B_* R_*^2 \sim \mathrm{kG}\times(10^{11}\mathrm{cm})^2$

   Molecular Cloud: $\Phi_{\mathrm{core}} \sim B_{\mathrm{core}} R_{\mathrm{core}}^2 \sim 10\mu\mathrm{G}\times(10^{17}\mathrm{cm})^2$

   $$\Rightarrow\ \Phi_* \sim 10^{-4}\ \Phi_{\mathrm{core}}$$

Self-Gravitational Collapse

Homologous Collapse

$P \propto \rho^{\gamma}$, $C_S^2 \propto \rho^{\gamma-1}$

$\rho \propto 1/R^3$, $M = \text{const.}$

$F_P \equiv (1/\rho) dP/dR \propto C_S^2/R$

$F_G \equiv GM/R^2 \propto 1/R^2$

$F_P / F_G \propto R^{-(3\gamma-4)}$

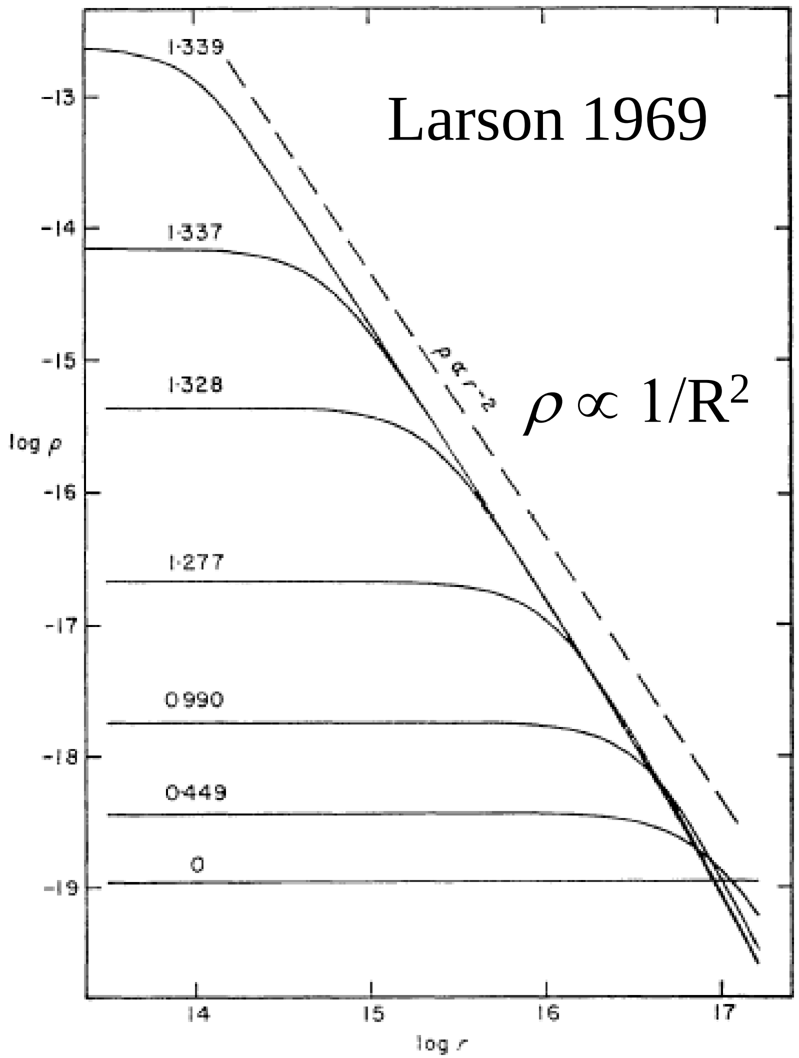
$$\gamma_{crit} = 4/3$$

if $\gamma < 4/3$ ➔ unstable

($\gamma \approx 1$ in Molecular Clouds)

$t_{ff} \sim (G\rho)^{-0.5}$ ➔ Run-Away

Run-Away Collapse



Rotating Run-Away Collapse

Isothermal Run-Away Collapse

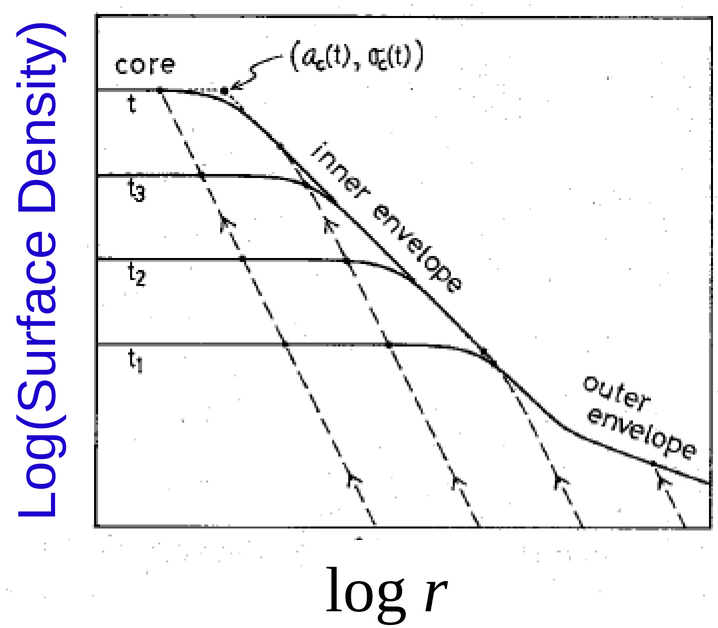
$\Omega(t) \propto (G\rho)^{1/2}$

$F_C \equiv r\,\Omega(t)^2 \propto r\,\rho \propto 1/r^2$

$F_G \equiv \pi G \rho\, r\, z/r_c \propto r\,\rho \propto 1/r^2$

$F_C / F_G = const. < 1$ @center

➔ Self-Similar Collapse



Narita, Miyama, Hayashi 1984

See also

Norman, Wilson, Burton 1980

Tomisaka, Basu, Matsumoto, etc.

Saigo, Matsumoto & Hanawa 2000

No Run-Away Collapse if $\gamma > 1$.

$\gamma_{\mathrm{crit}} = 1$ ... isothermal

Convergence to Self-Similar Solution?

Isothermal Rotating Run-Away Collapse ➔ Self-Similar Collapse

Convergence of Time-Evolution without Dissipation?

Same Protostars with Same Disks?

Unique Initial Condition of Planet Formation?

Answer:

Isothermal (Barotropic) Hydrodynamics is Hamiltonian.

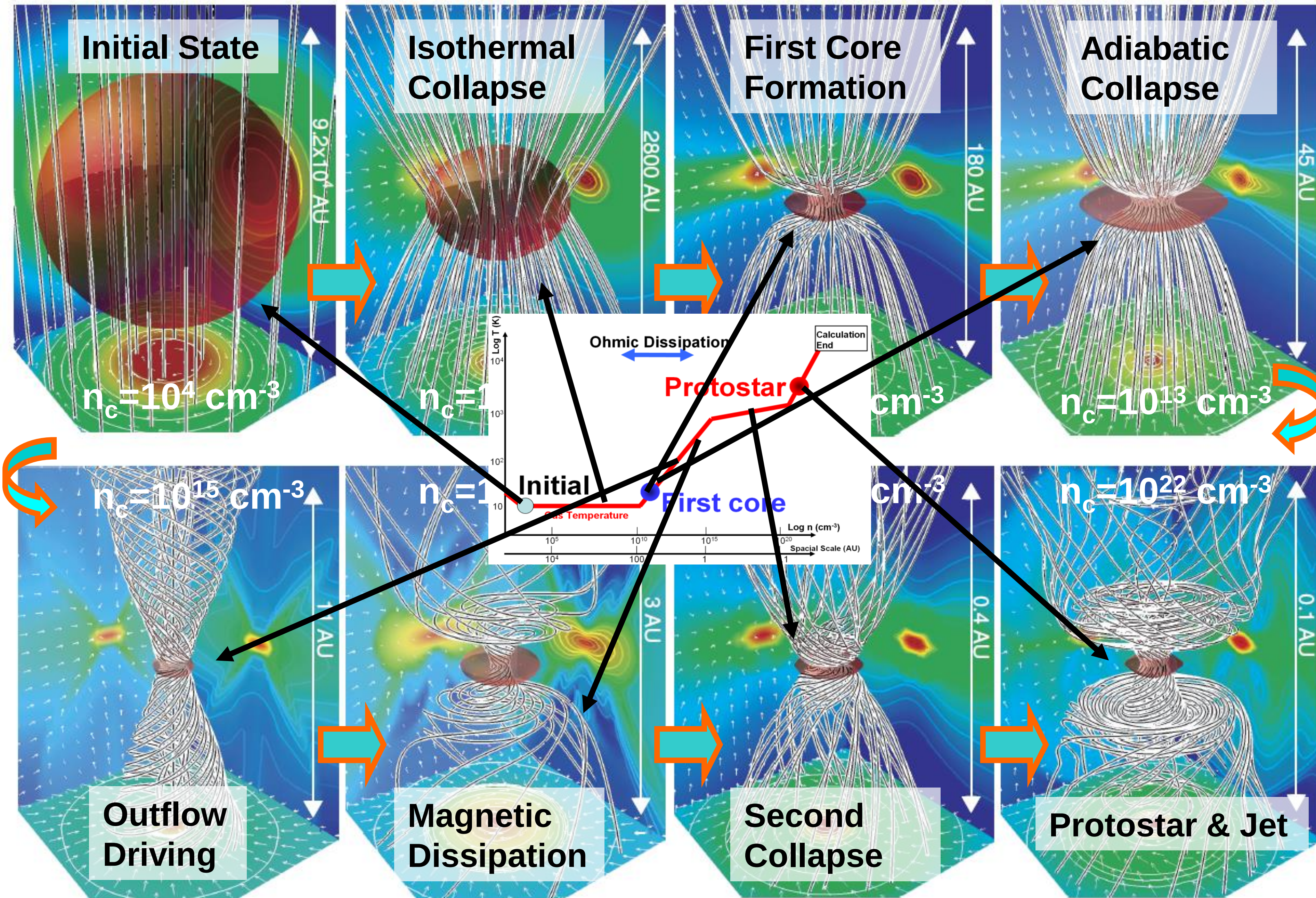
$$L = (1/2)\rho v^2 - \rho e$$

➔Liouville's Theorem

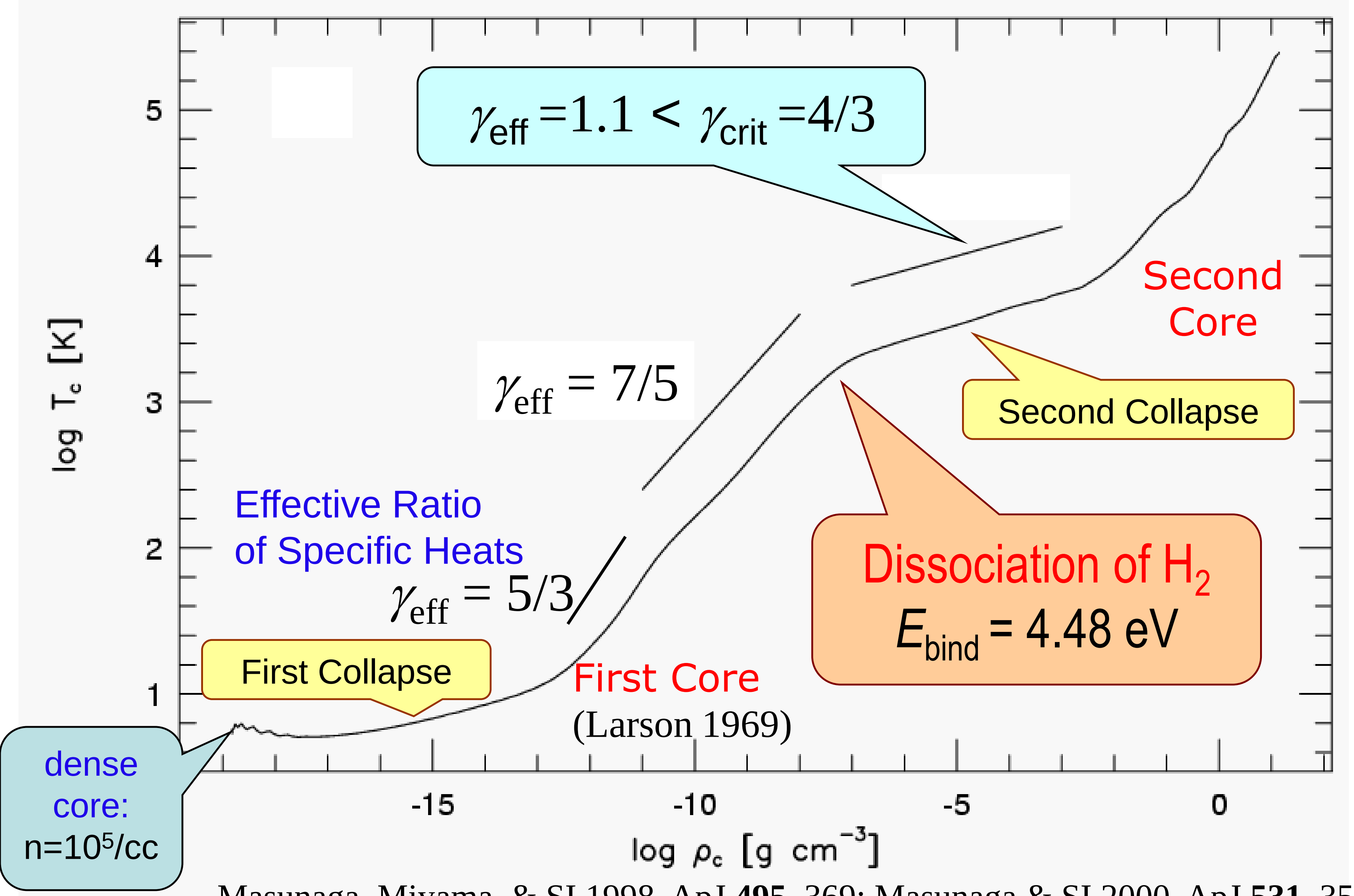
Conserved Phase Space Density = No Convergence

➔Convergence in Region of Vanishing Volume & Mass

Evolution from **Molecular Cloud Core** to **Protostar**

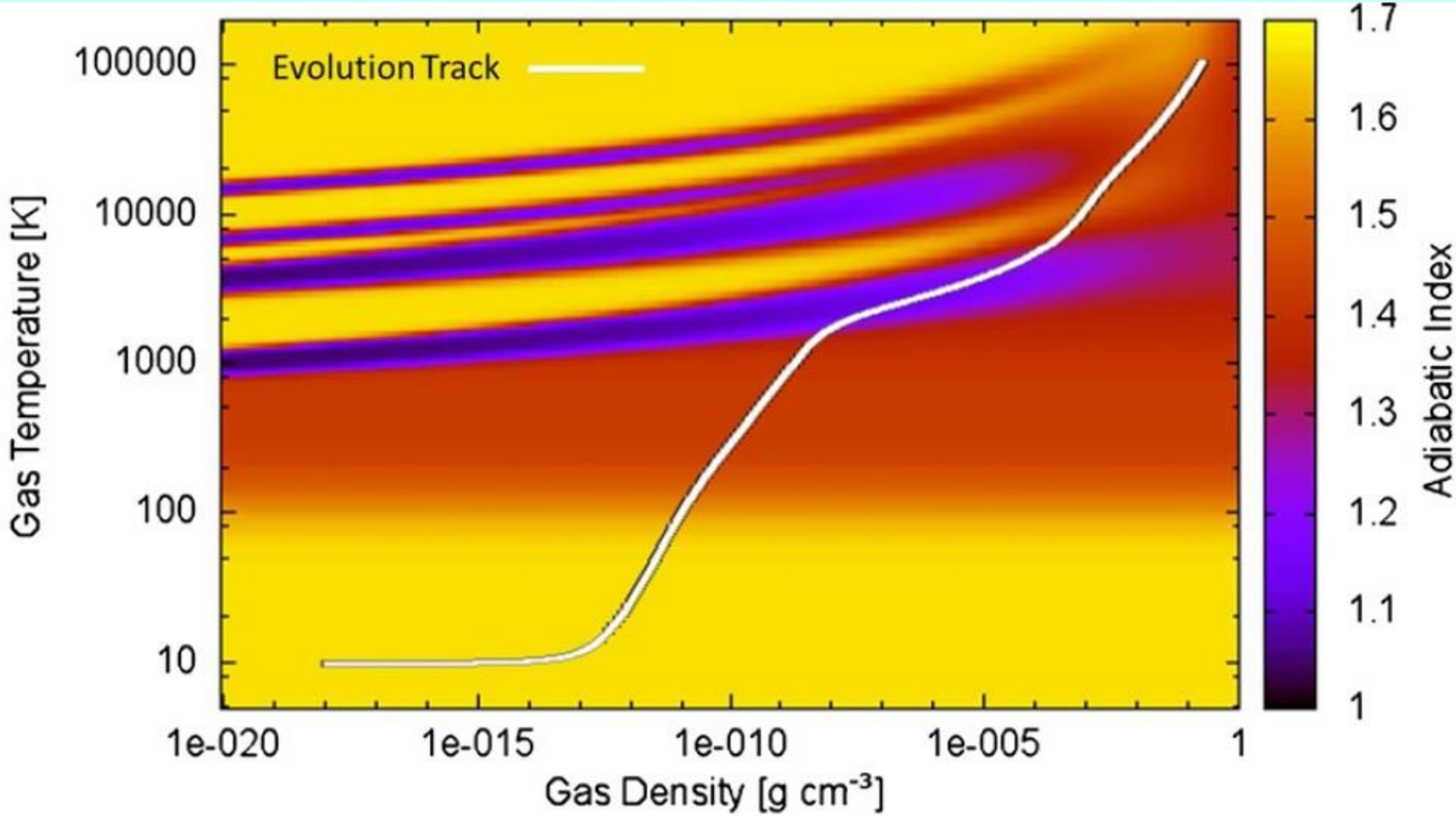


Temperature Evolution at Center



Masunaga, Miyama, & SI 1998, ApJ **495**, 369; Masunaga & SI 2000, ApJ **531**, 350

Temperature Evolution at Center



Tomida et al. 2013, ApJ **763**, 1

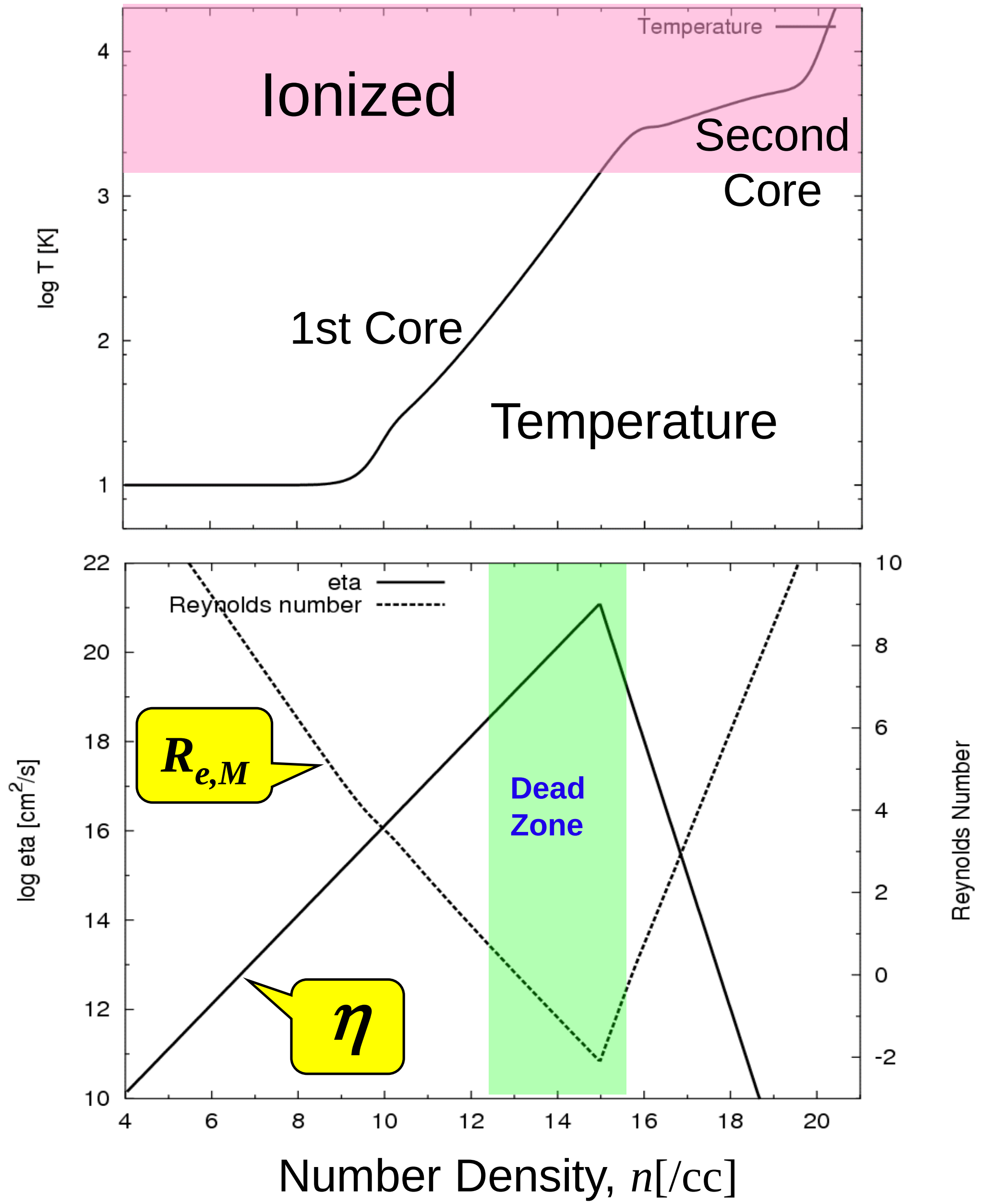
See also Commerçon & Hennebelle

Effect of ***Non-Ideal* MHD**

Weakly Ionized Gas

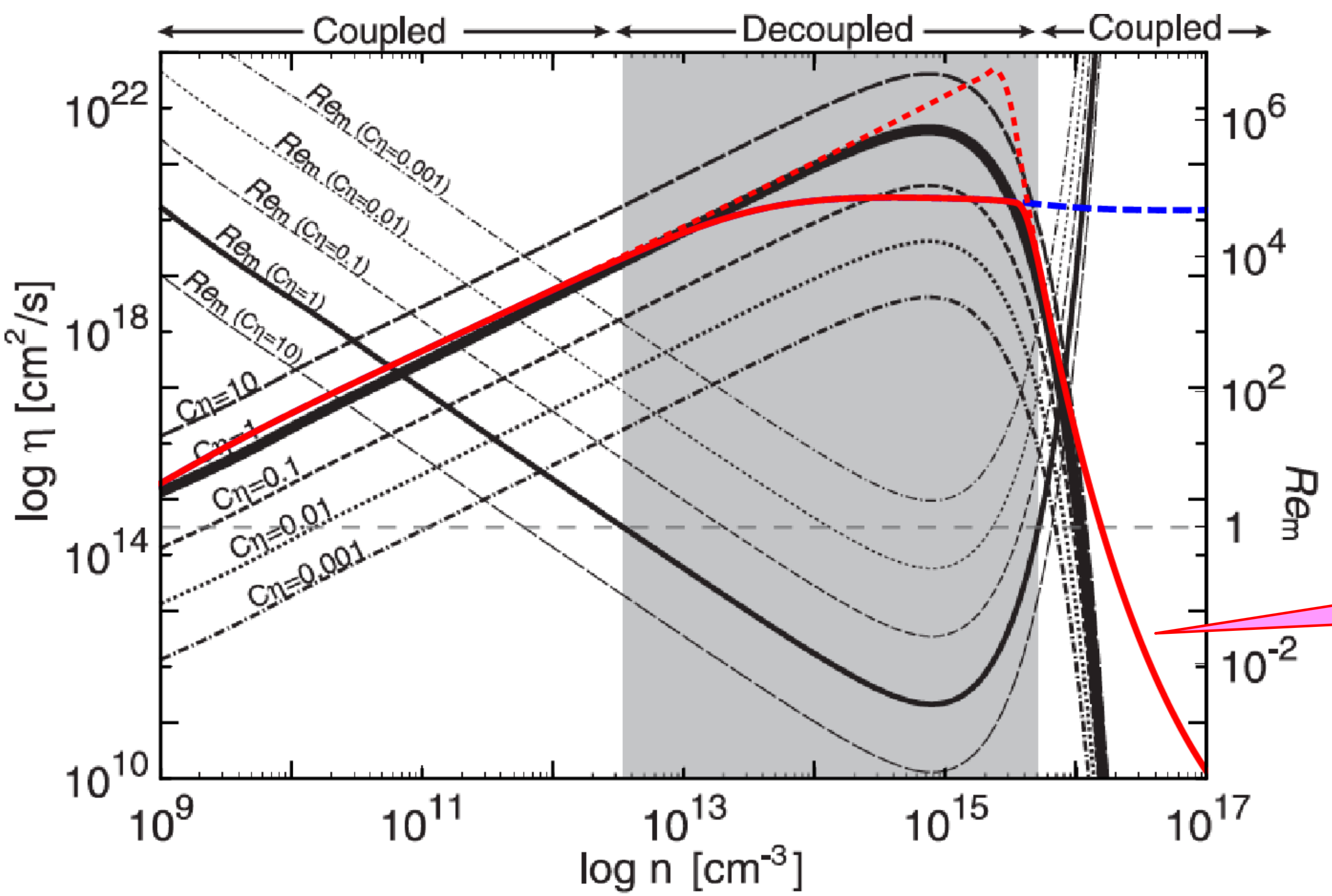
- Low density… <u>Ambipolar Diffusion</u>
- Intermediate… <u>Hall Current Effect</u>
- High density… <u>Ohmic Dissipation</u>

e.g., *Nakano, Mouchouvias, Wardle, Tassis, Galli,…*



Evolution of Ionization Degree

Because of uncertainty of dust grain properties, we have parameterized resistivity.



Machida, SI, & Matsumoto (2007) ApJ **670**, 1198

Stage 1: **Outflow** driven from the **first core**

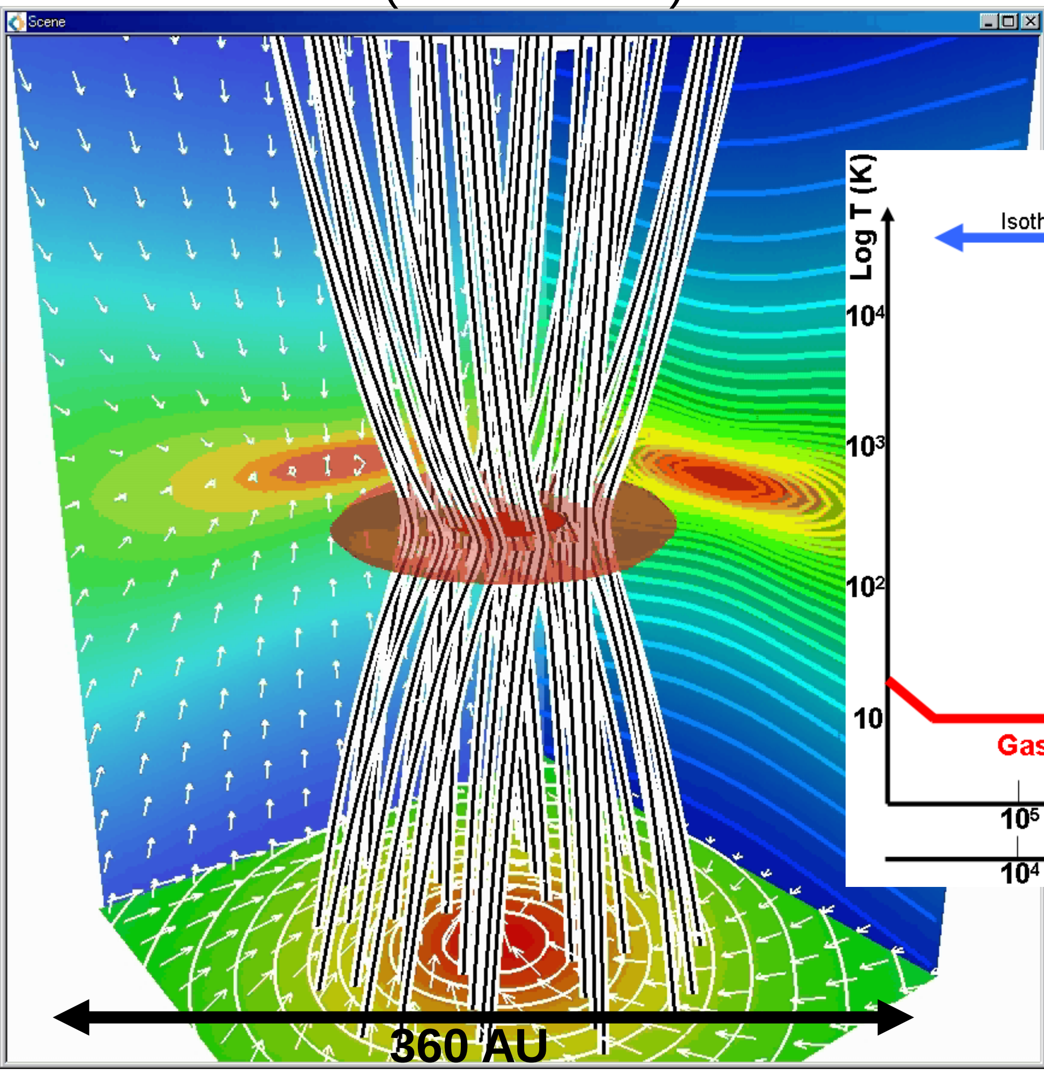
The evolution of the Outflow around the first core

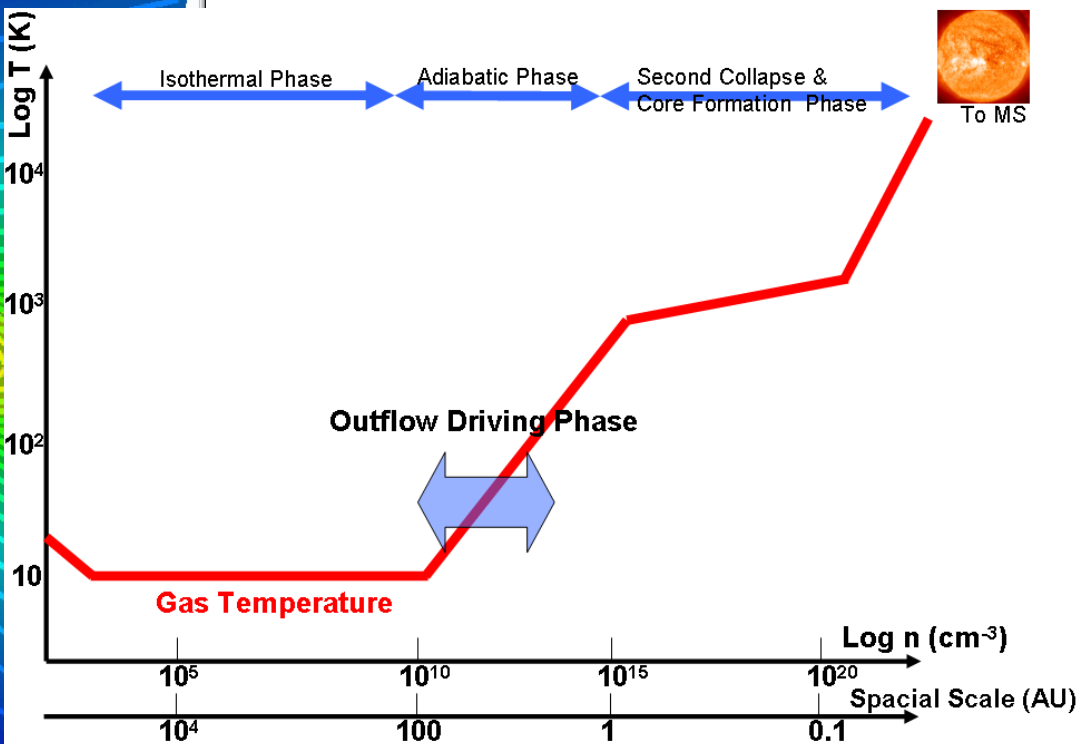
- This animation start after the first core is formed at n~10^{10} cm^{-3}

Model for
(α, ω)=1, 0.3

Grid level L=12 (Side on view)

Grid level L=12 (Top on view)





Same as in Tomisaka 2002

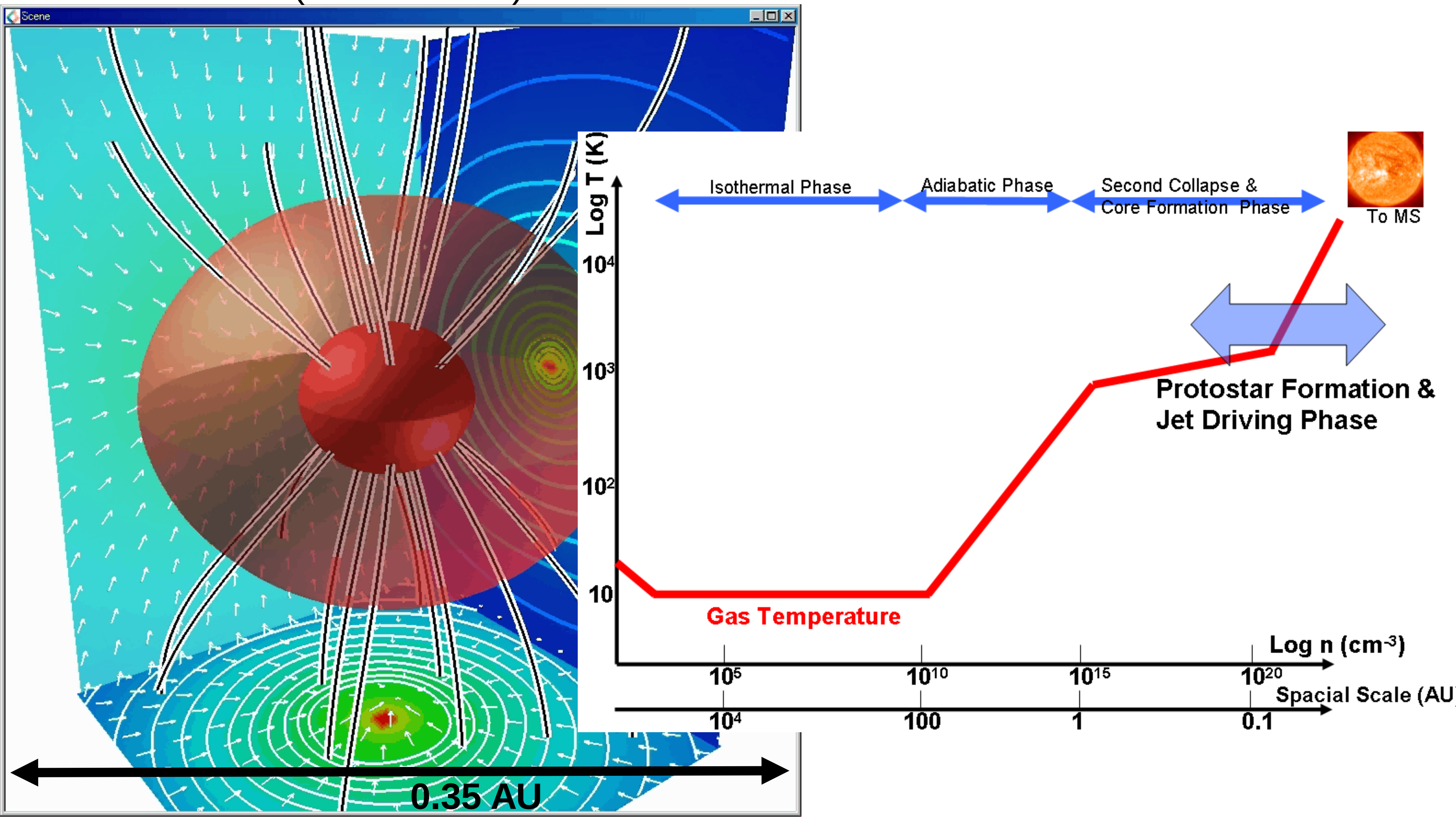
Stage 3: **Jet** driven from the **protostar**

The evolution of the Jet around the protostar

- This animation start before the protostar is formed at n~10^{19} cm^{-3}

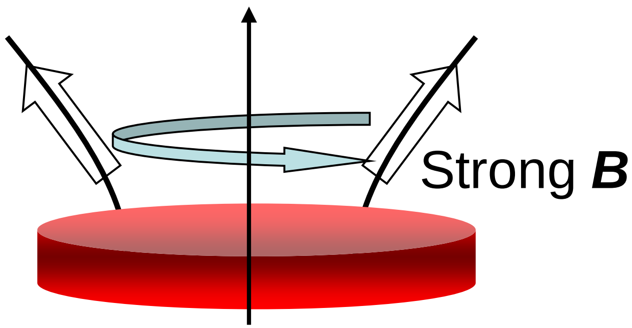
Model for
(α, ω)=1, 0.003

Grid level L=21 (Side on view)



Difference in Driving Mechanism

Magnetocentrifugally driven Wind



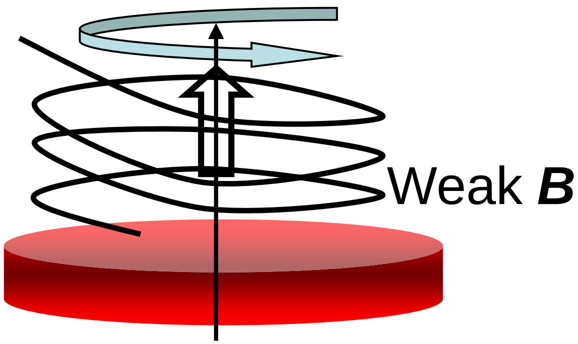
Wide Opening Angle

outflow around first core

$B_r \approx B_z \approx B_\phi$

only at launching region, not in distant region

Magnetic Pressure driven Wind



Narrow Opening Angle

jet around protostar

$B_z << B_\phi$

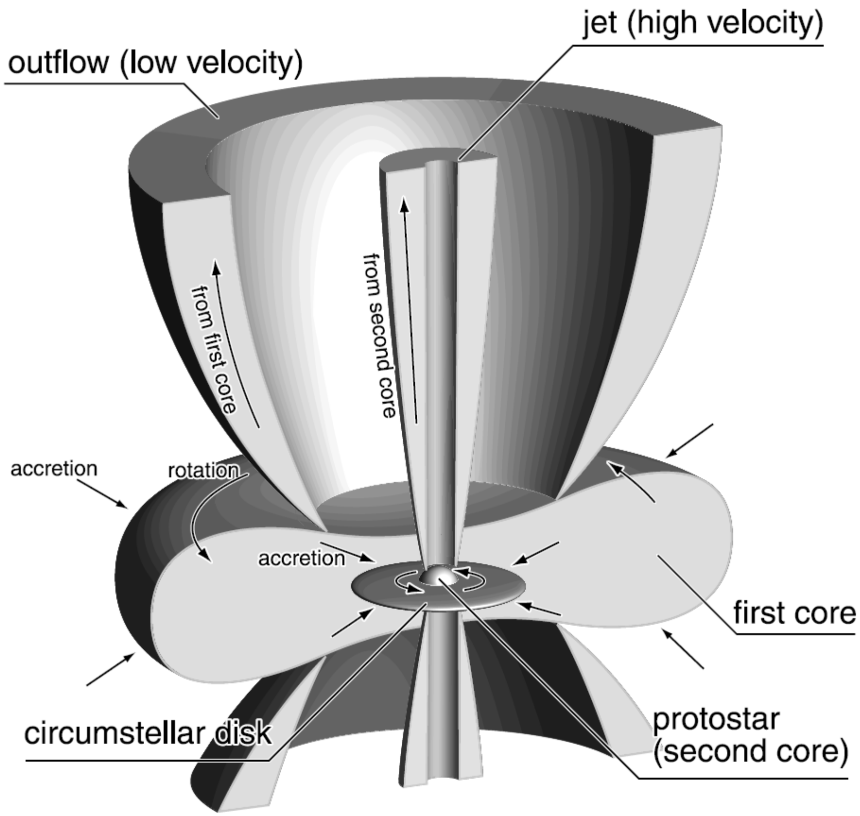
Machida, SI, & Matsumoto (2008) ApJ **676**, 1088

Summary of Machida et al.

Stiffening of EoS ➔ 2 different flows (outflow/jet)

- Outflow driven by the first core has wide opening angle and slow speed.

- Jet driven by the protostar has well-collimated structure and high speed.



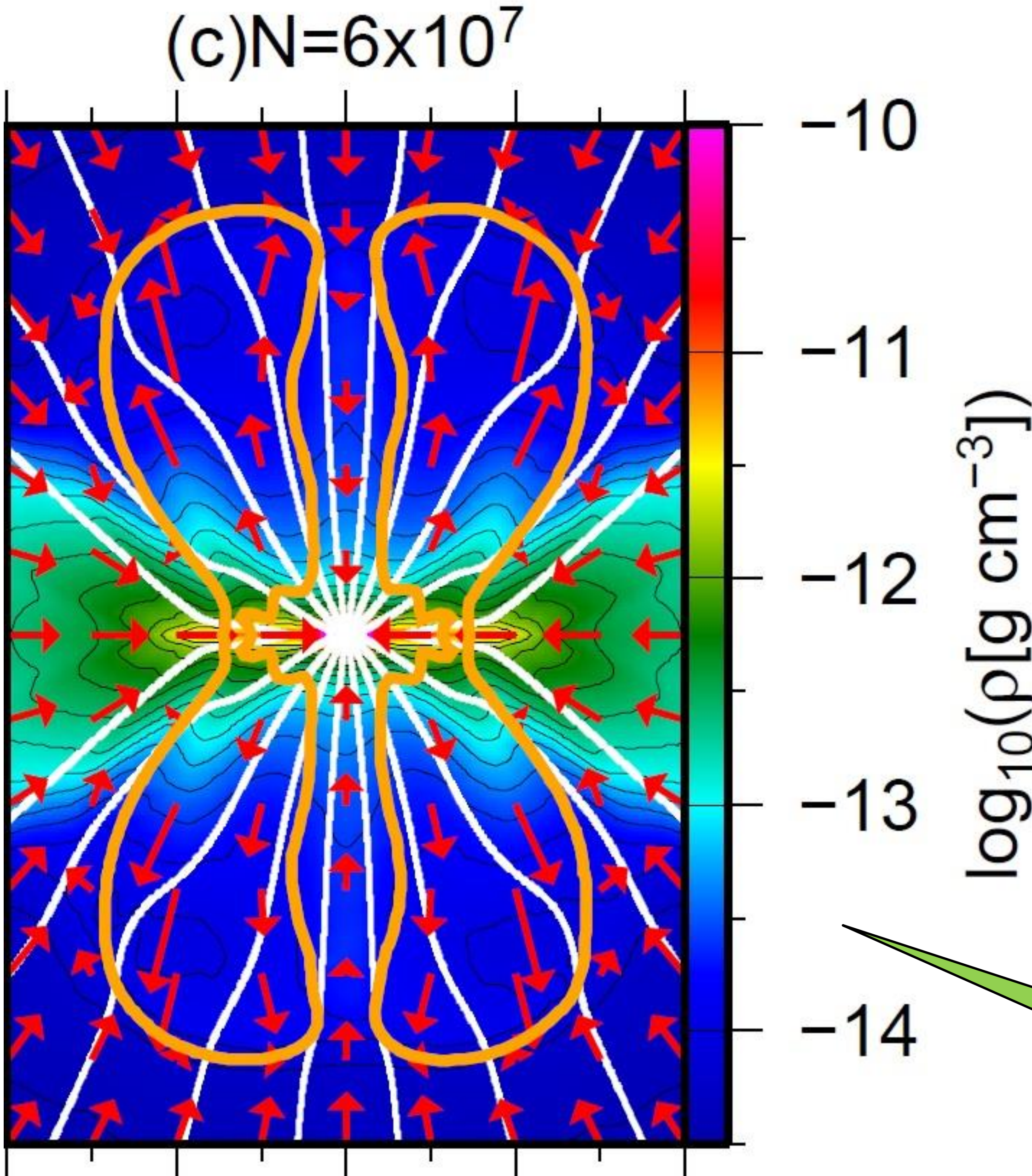
Machida, SI, Matsumoto (2008) ApJ **676**, 1088

Velocities = Escape speeds from the first core & protostar.

Observational Proof ➔ Velusamy, T., et al. 2007 ApJ 668, L159,

Velusamy, T. et al. 2011 ApJ 741, 60

How About SPH?



Godunov SPH:
SI (2002) JCP **179**, 238

Godunov SPH for
Ideal MHD:
Iwasaki & SI (2011) MN **418**, 1668

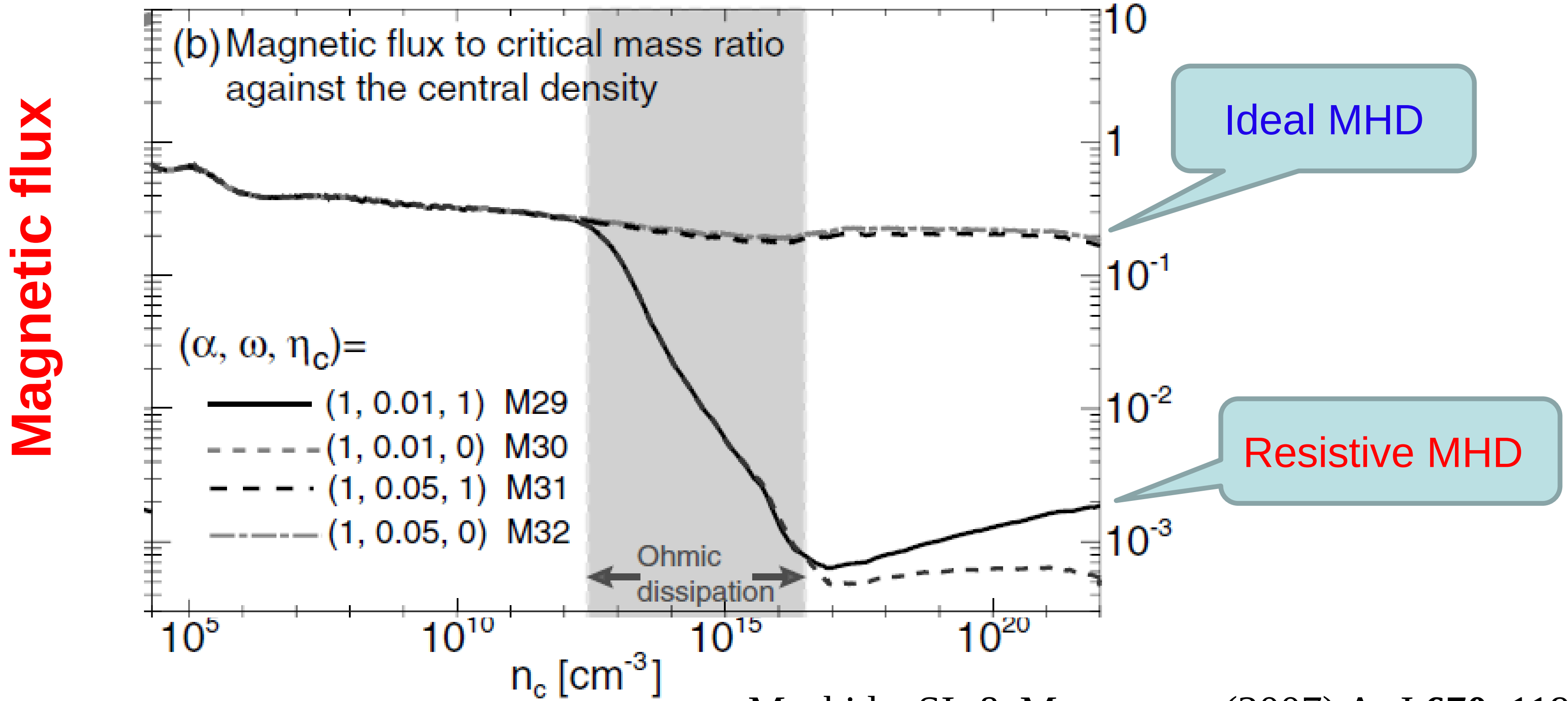
Godunov SPH for
Resistive MHD:
Tsukamoto, Iwasaki & SI (arXiv:1305.4436)

See also Bate & Price

Iwasaki 2013

Magnetic Flux Loss

Magnetic flux largely removed from First Core
when $n = 10^{12} \sim 10^{16}\ \mathrm{cm}^{-3}$ ➔ B = kG or less



Machida, SI, & Matsumoto (2007) ApJ **670**, 1198

Sufficient Flux Loss?

Machida, SI, & Matsumoto (2008)

$\boldsymbol{B}_{\text{protostar}}$ ~ kGauss in standard model of $\boldsymbol{\eta}$

resistivity

my guess:

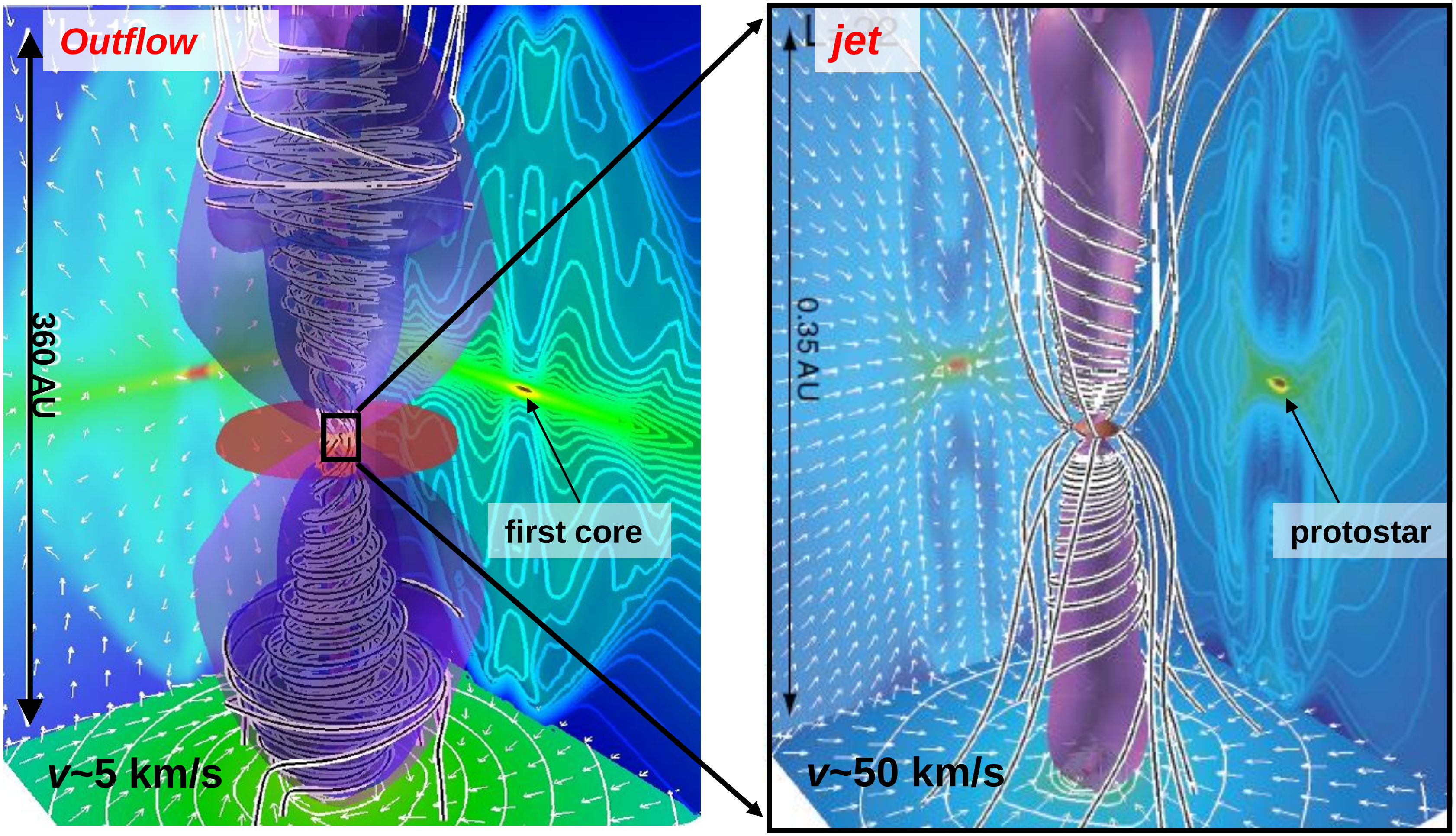
Turbulent Diffusion in Convection on Hayashi-Track

➔ Decrease of $\boldsymbol{B}$ in T-Tauri Phase（kG➔G?）

For Turbulent Diffusion, see Lazarian & Vishniac (1999)

Formation & Gravitational Evolution of Disks

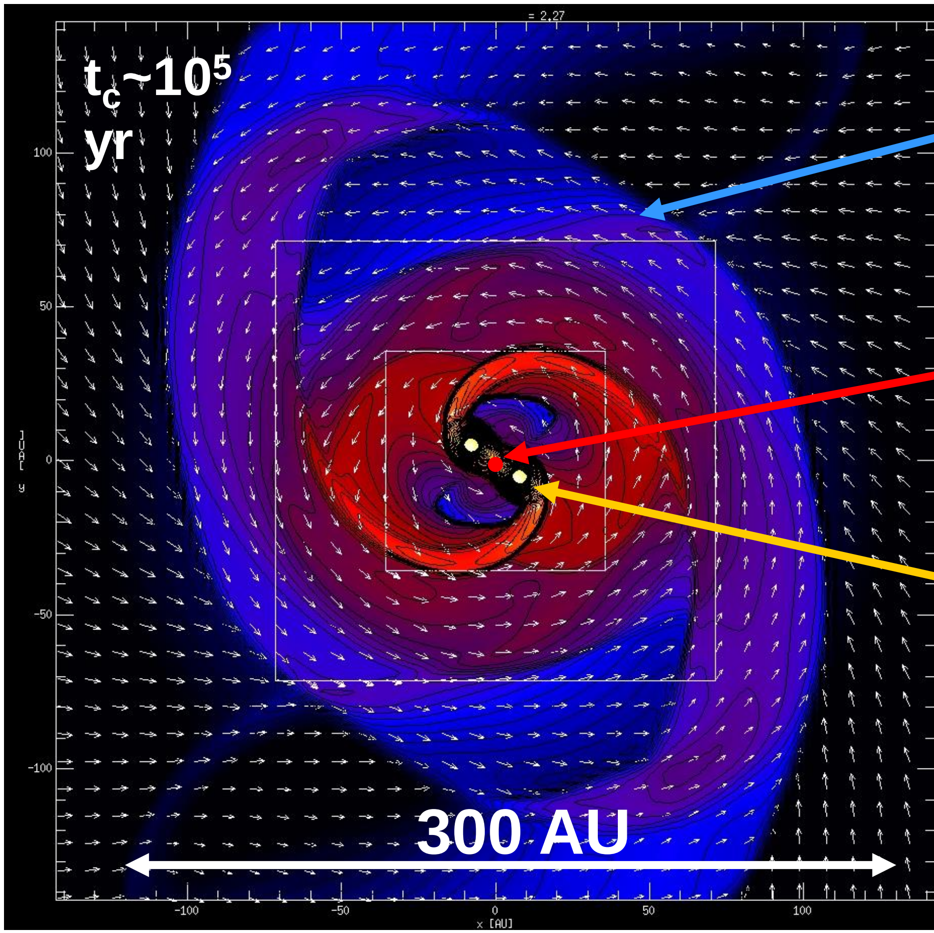
Formation of a Protostar



Machida et al. (2006-2012), Banerjee & Pudritz (2006), Hennebelle et al. (2008), Duffin & Pudritz (2011), Commerçon et al. (2011), Tomida et al. (2011)

Outflows & Jets are Natural By-Products!

Formation of Planetary Mass Companions in Protoplanetary Disk



Protoplanetary Disk

Protostar

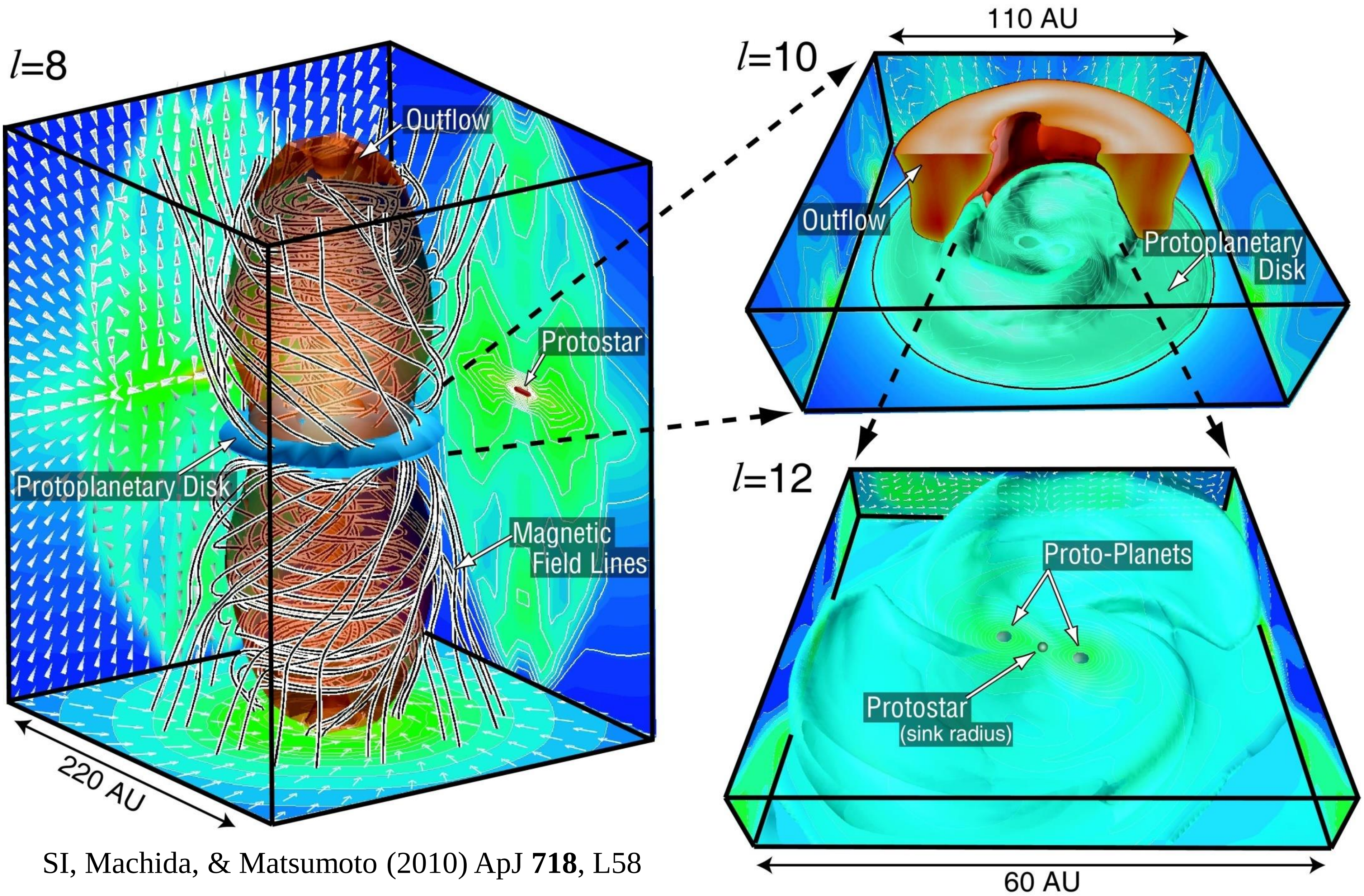
- ~0.1 M_{sun}

Protoplanet

- M~8 M_{Jup}
- R_{sep}~10-20 AU

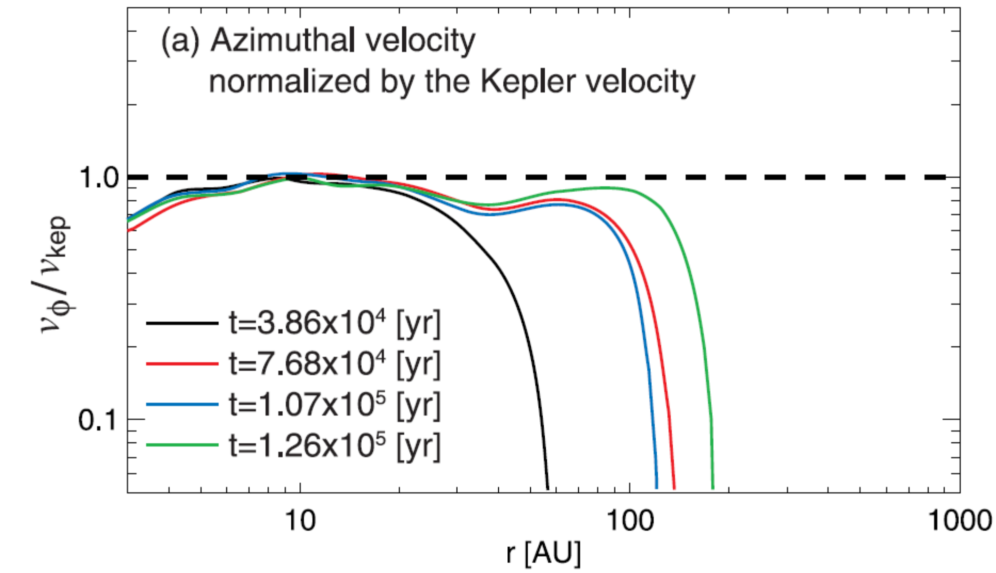
Machida, SI, Matsumoto (2009)

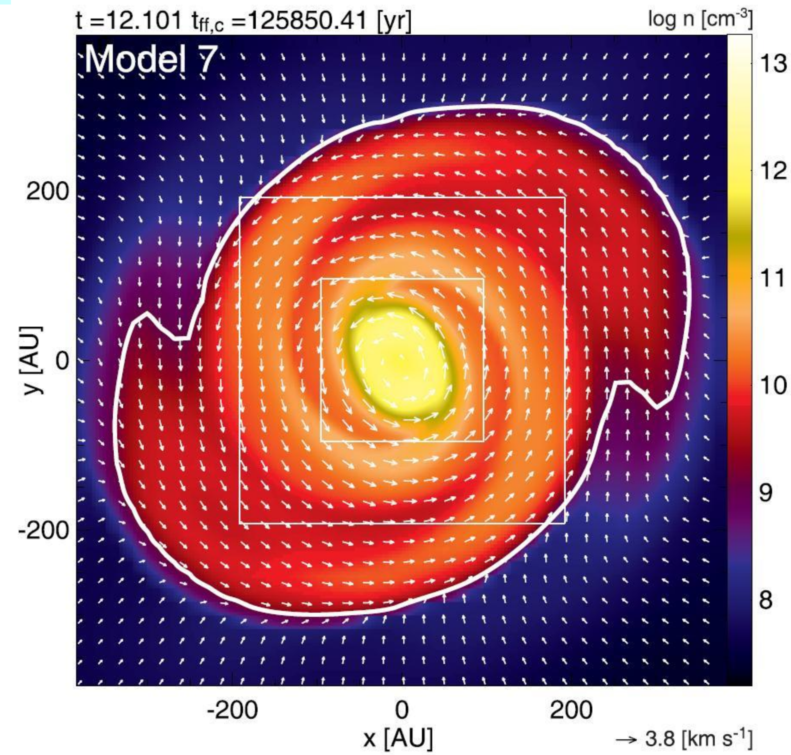
Resistive MHD Calc. from Mole. Cloud Core

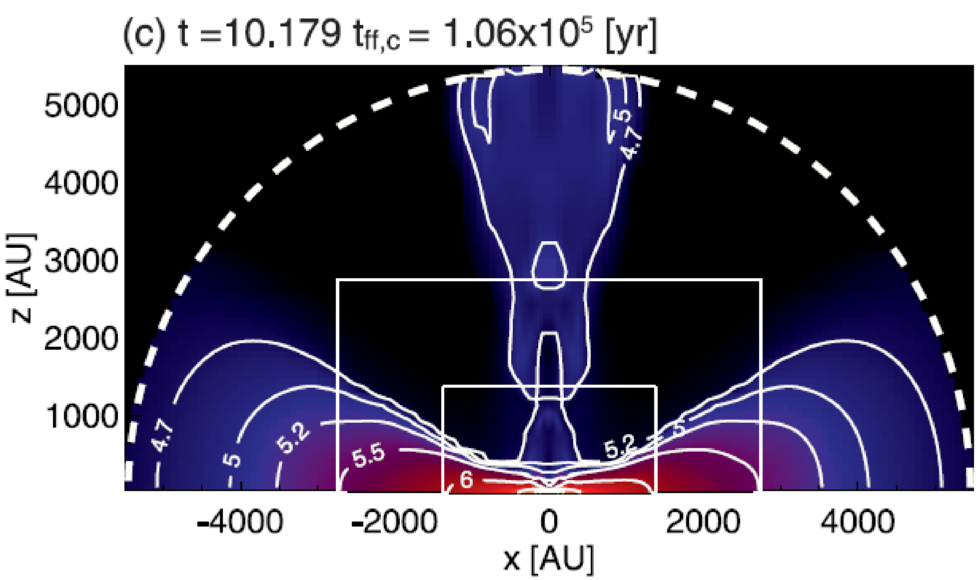


SI, Machida, & Matsumoto (2010) ApJ **718**, L58

End of Formation Phase



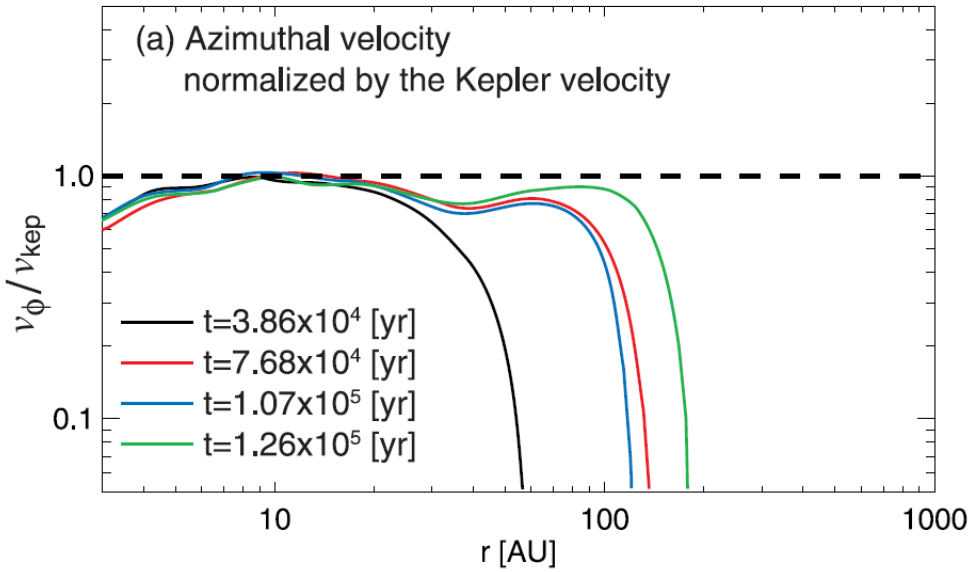


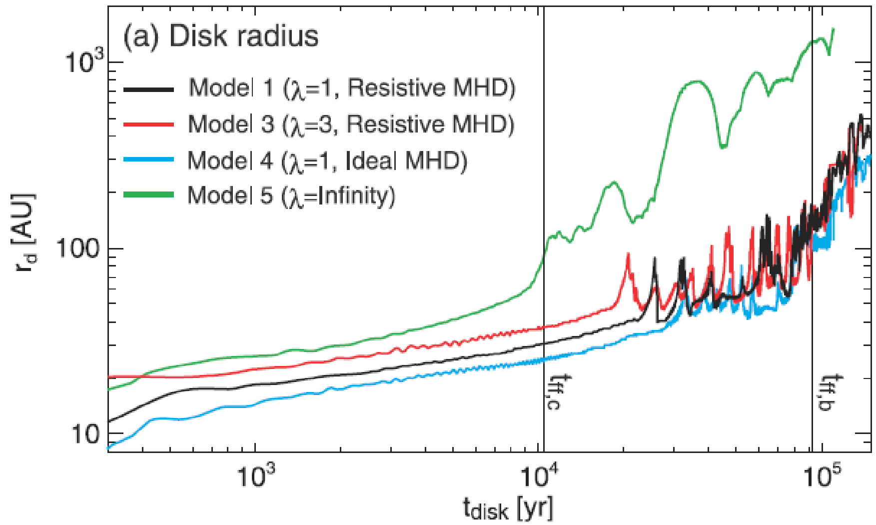


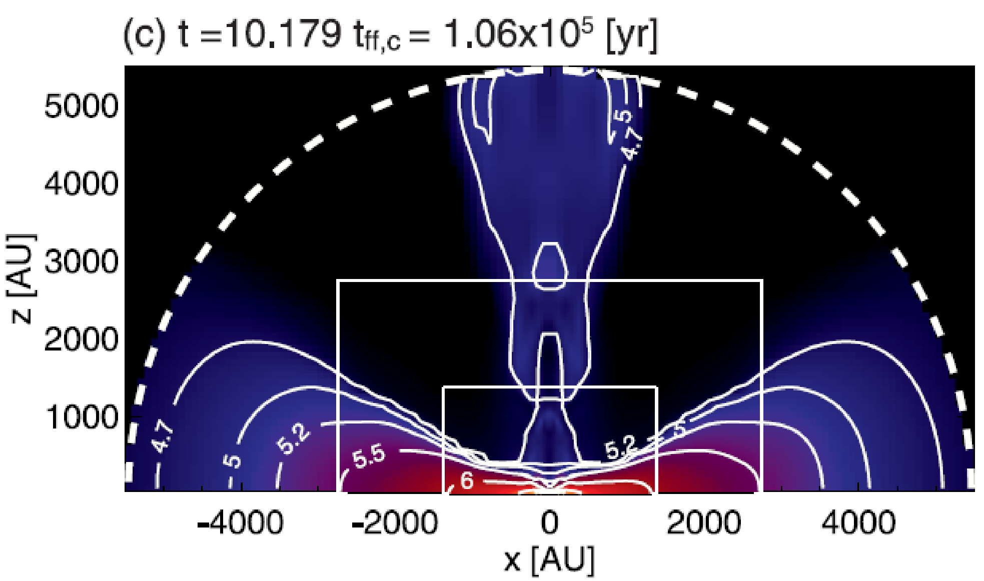
Spiral Structure
with/without a Planet

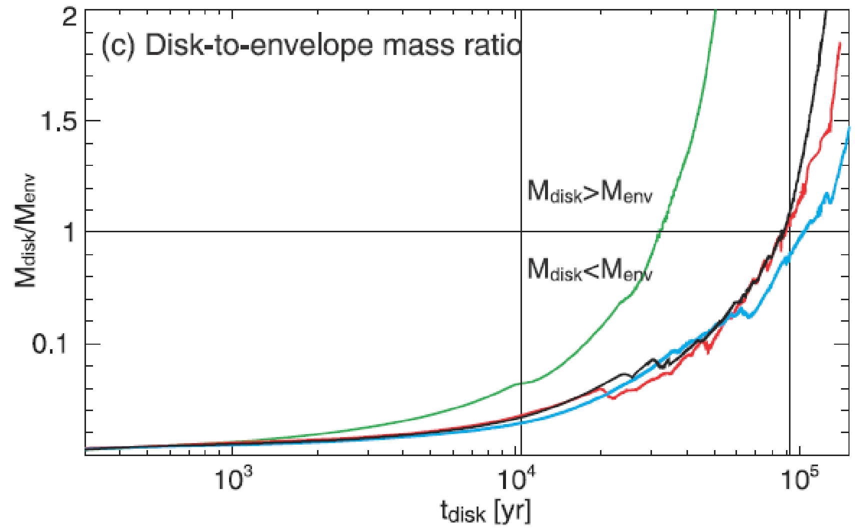
Machida, SI & Matsumoto (2011)

End of Formation Phase





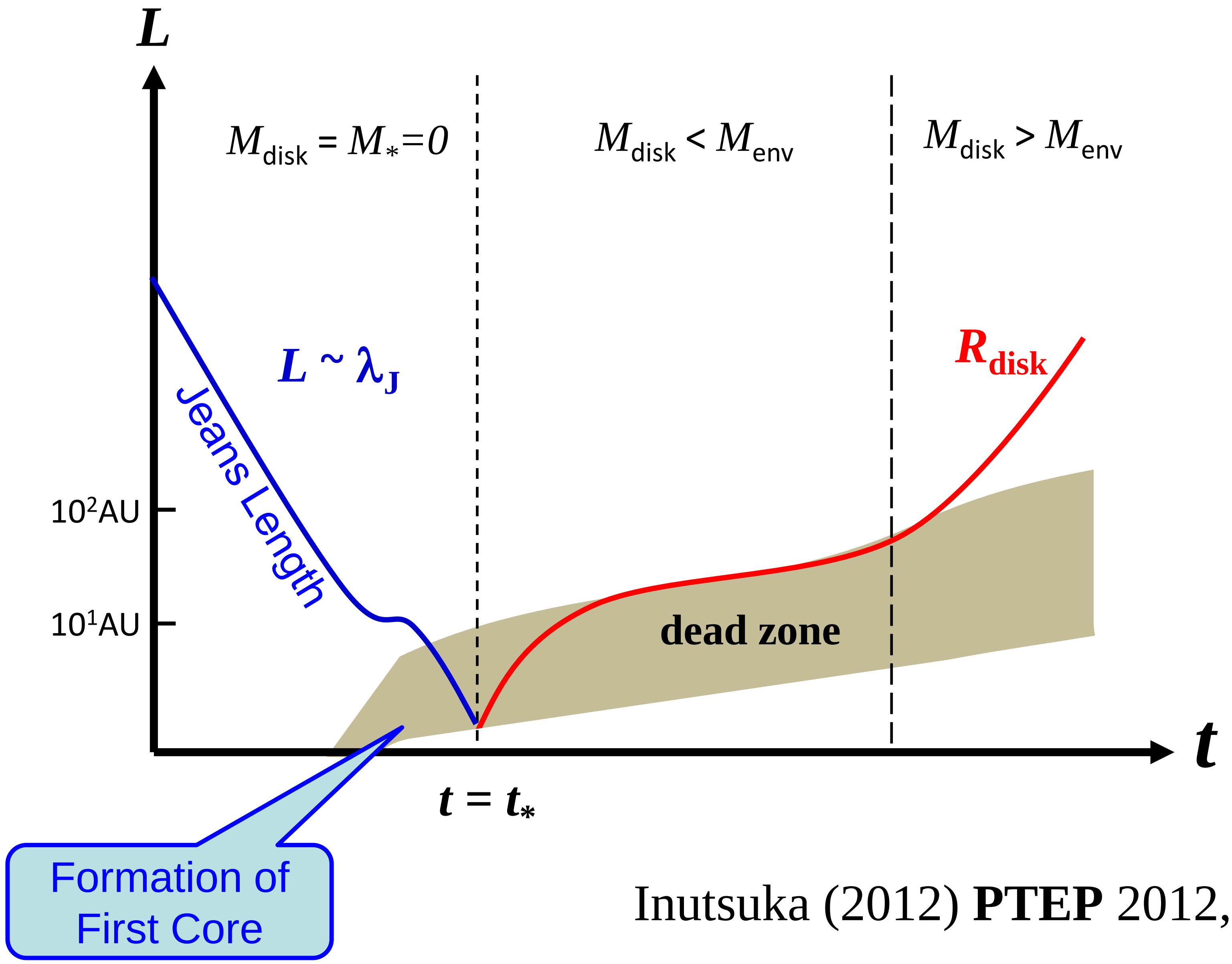




Machida, SI & Matsumoto (2011) PASJ **63**, 555

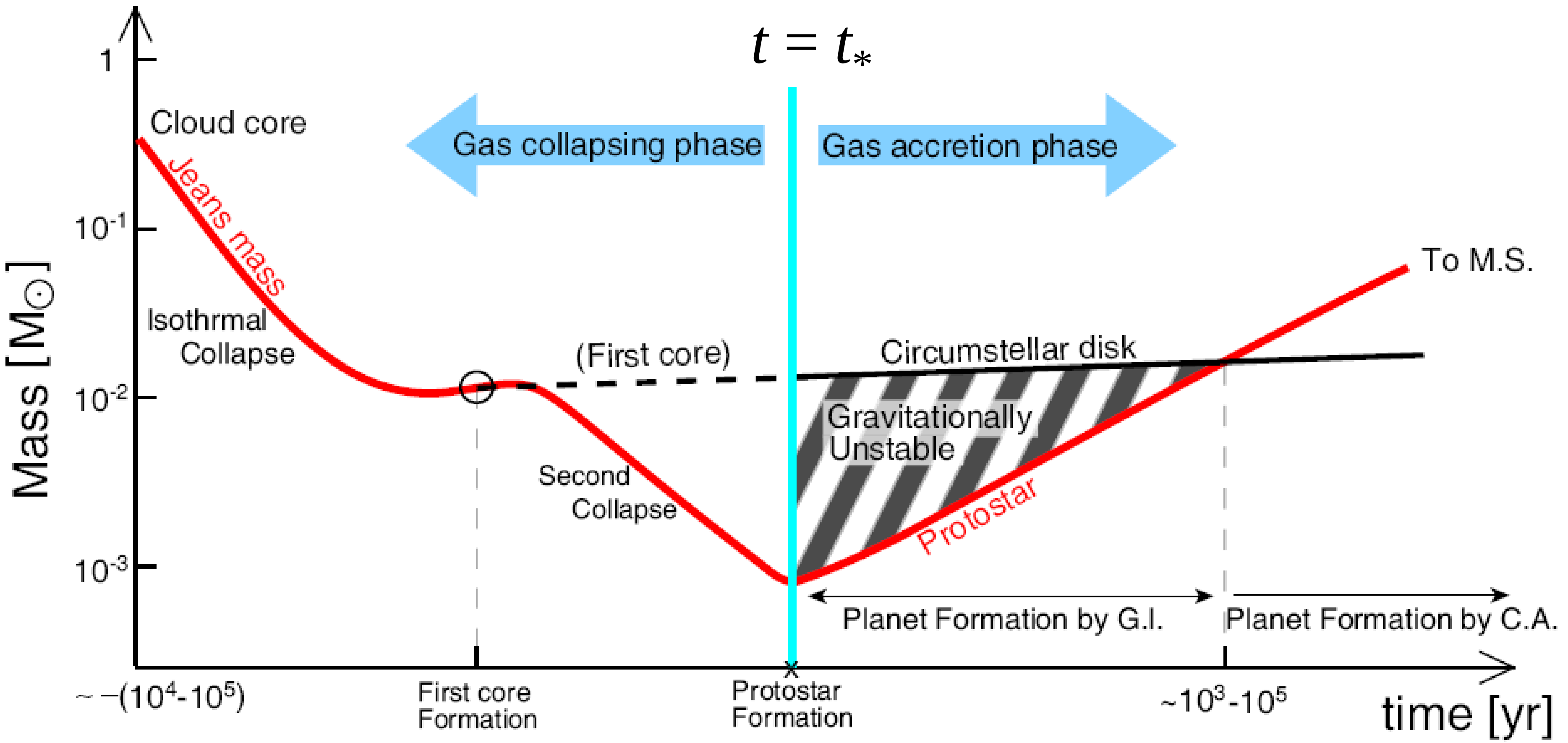
Disk Growth Correlated with Depletion of Envelope Mass

Formation of Protoplanetary Disk



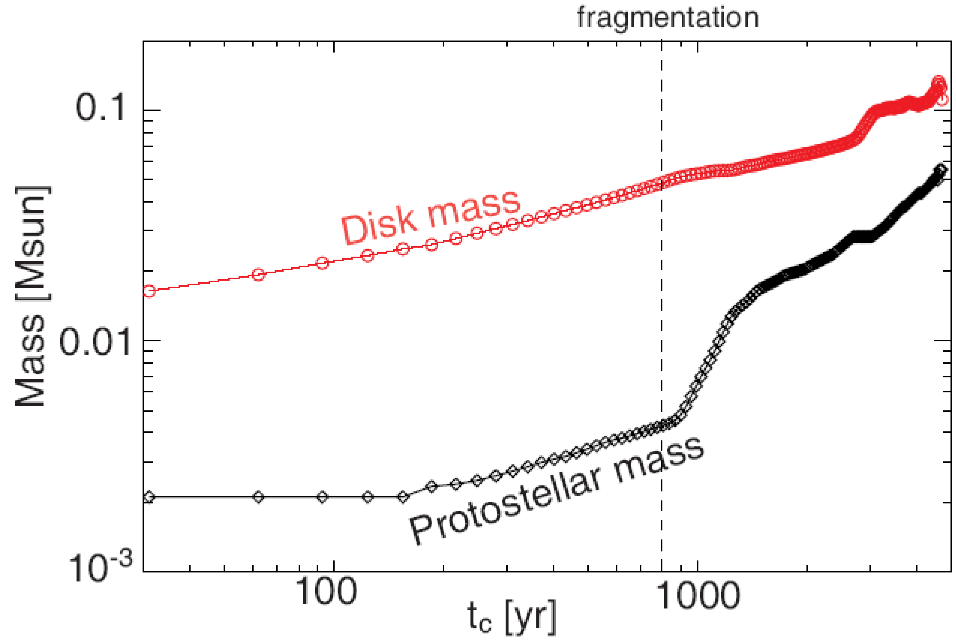
Inutsuka (2012) **PTEP** 2012, 01A307

Formation of Protoplanetary Disk



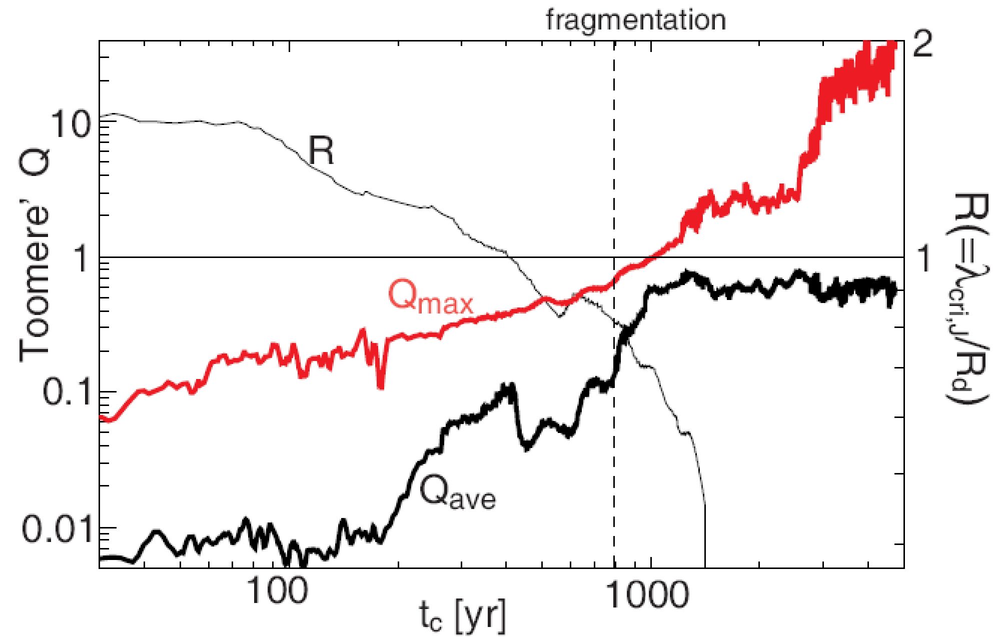
SI, Machida, & Matsumoto (2010) ApJ **718**, L58

Evolution of Stellar Mass & Disk Mass



Local Criterion for Gravitational Instability:

$$Q \equiv \kappa\, C_s /(\pi G \Sigma)$$



SI, Machida, & Matsumoto (2010) ApJ **718**, L58

Can planets cool and collapse?

if cooling make $\gamma_{\mathrm{eff}} < 4/3$ ➔ Gravitational Collapse

Energy that should be radiated

$$\Delta E = \frac{M_{\mathrm{p}}}{\gamma - 1}\left(\frac{P_{\mathrm{d}}}{\rho_{\mathrm{p}}}\right)\left[\left(\frac{\rho_{\mathrm{p}}}{\rho_{\mathrm{d}}}\right)^{\gamma} - \left(\frac{\rho_{\mathrm{p}}}{\rho_{\mathrm{d}}}\right)^{\gamma_{\mathrm{eff}}}\right]$$

Resultant Luminosity

$$\langle L \rangle \equiv \frac{\Delta E}{\Delta t} \approx 1.5 \times 10^{-1}\left(\frac{M_{\mathrm{p}}}{10^{-3} M_{\odot}}\right)\left(\frac{T_{\mathrm{d}}}{10^{2} K}\right)\left(\frac{10^{2}\mathrm{yr}}{\Delta t}\right)\left(\frac{\rho_{\mathrm{p}}}{10^{5}\rho_{\mathrm{d}}}\right)^{2/5} L_{\odot}$$

Luminosity of Planet at $T = T_{\mathrm{p}}$

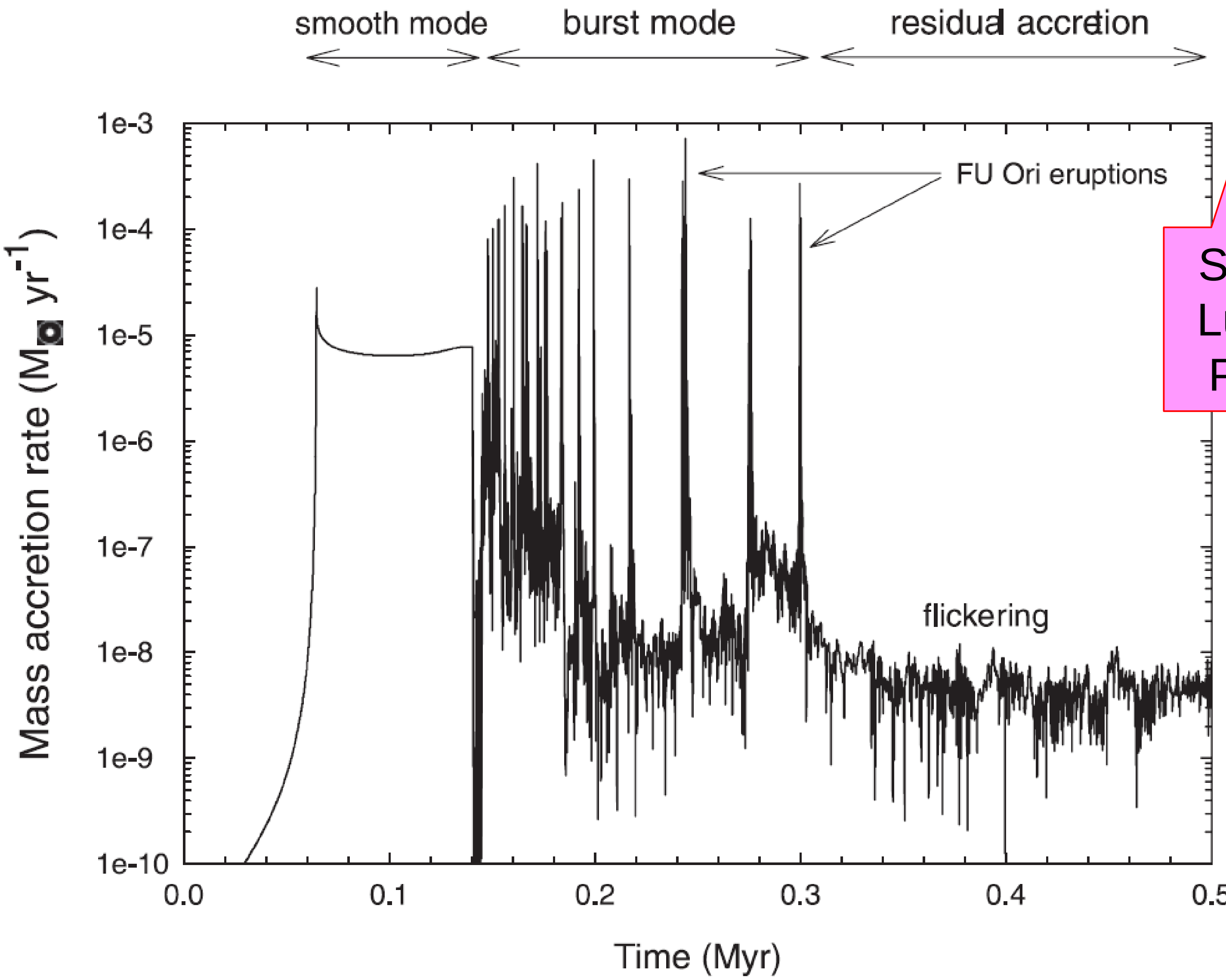
$$L_{\mathrm{p}} = 4\pi R_{\mathrm{p}}^{2}\sigma_{\mathrm{SB}}T_{\mathrm{p}}^{4} = 1.9 \times 10^{-1}\left(\frac{R_{\mathrm{p}}}{10^{12}\mathrm{cm}}\right)^{2}\left(\frac{T_{\mathrm{p}}}{10^{3} K}\right)^{4} L_{\odot}$$

Cooling with $T = 10^{3}$K ➔ Collapse by 10^{-5} in 10^{2} yr

Luminosity ~ $0.2 L_{\odot}$

SI, Machida, & Matsumoto (2010) ApJ **718**, L58

2D Modeling by Vorobyov & Basu 2006



Solution to Luminosity Problem?

2D Simulations of Infinitesimally Thin Disk

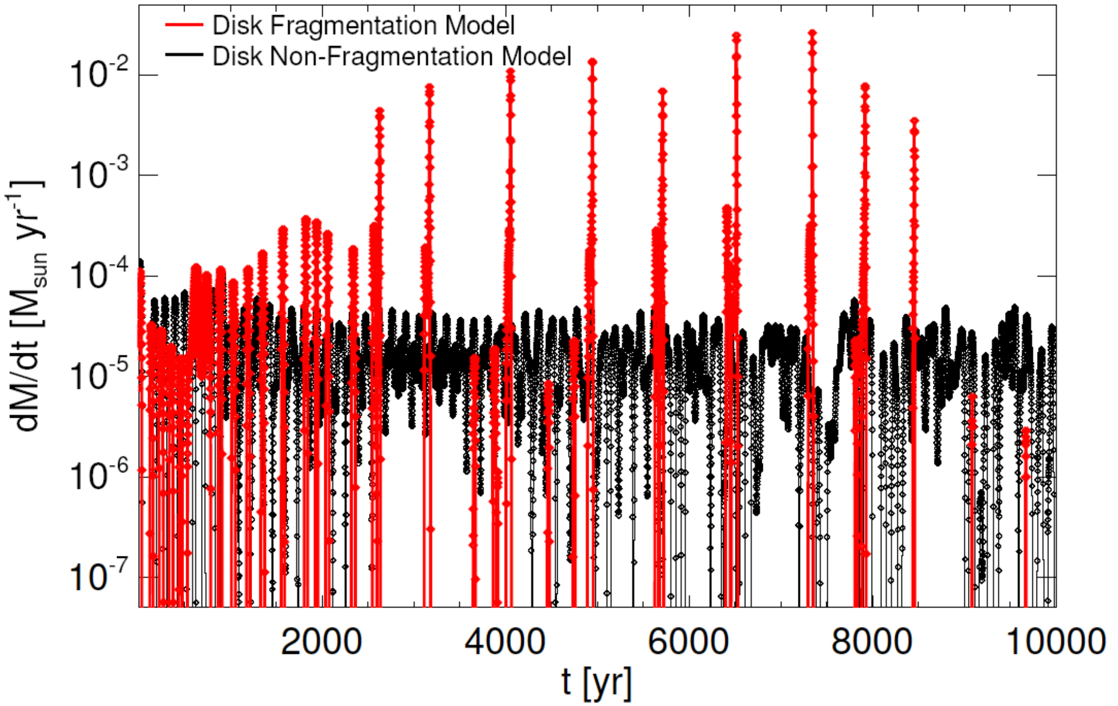
Time-Variability

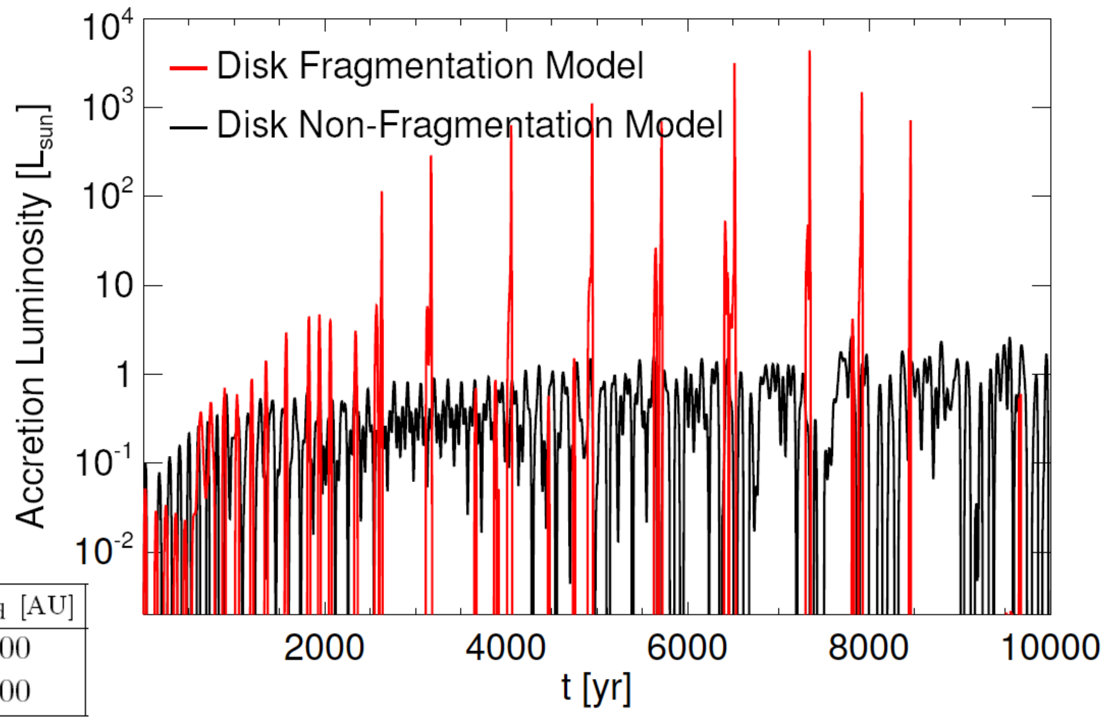
Vorobyov & Basu 2006

Observational Clue:

Variability (FU Ori?) in deeply embedded protostar!

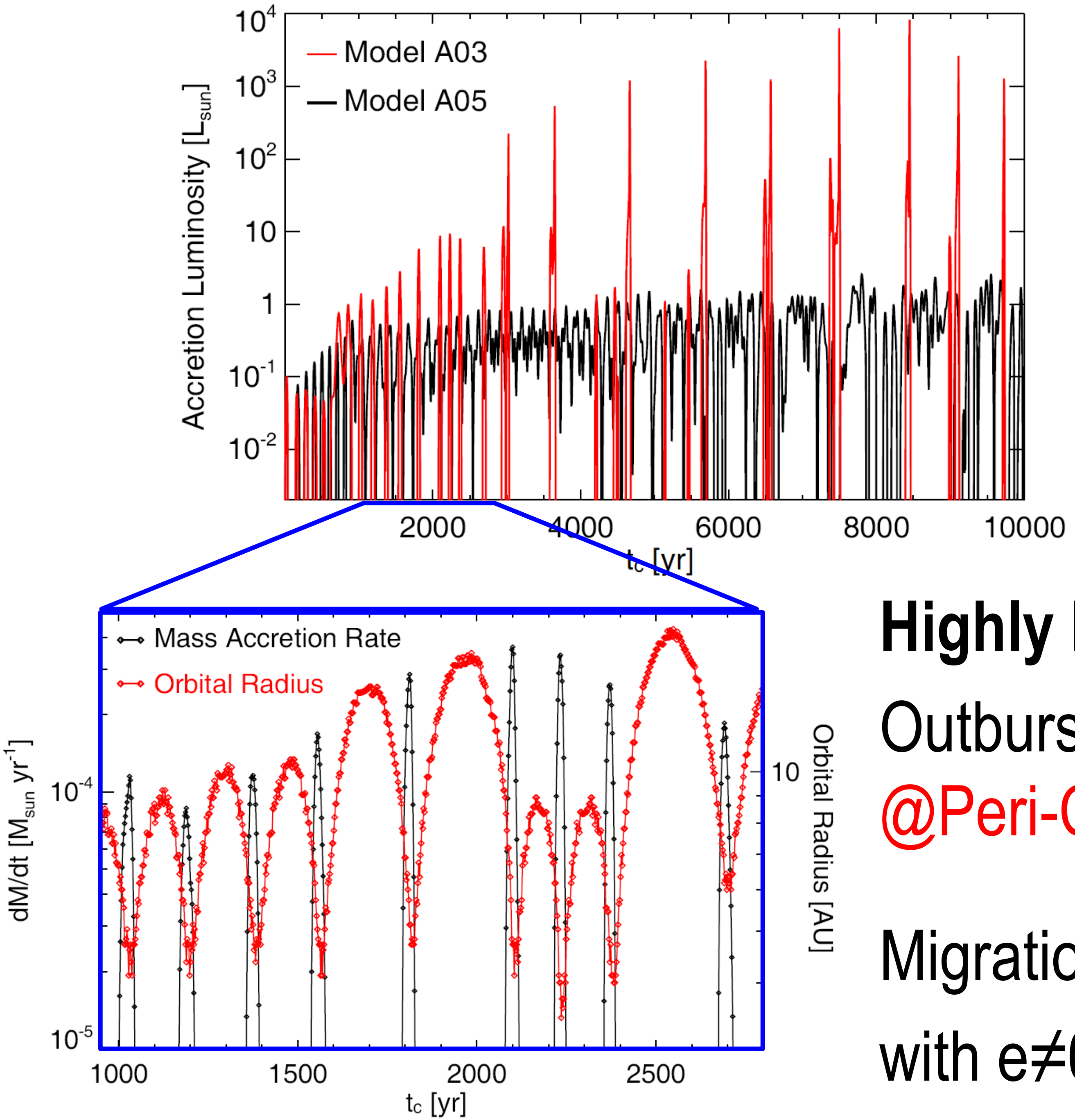
Machida, SI & Matsumoto (2011) ApJ **729**, 42





Model	α_0	f	Ω_0 [s^{-1}]	B_{ini} [μG]	M_{cl} [$M_\odot$]	R_{cloud} [AU]
A03	0.3	2.8	1.1×10^{-13}	37	1.3	4700
A05	0.5	1.7	1.1×10^{-13}	37	0.8	4700

When?



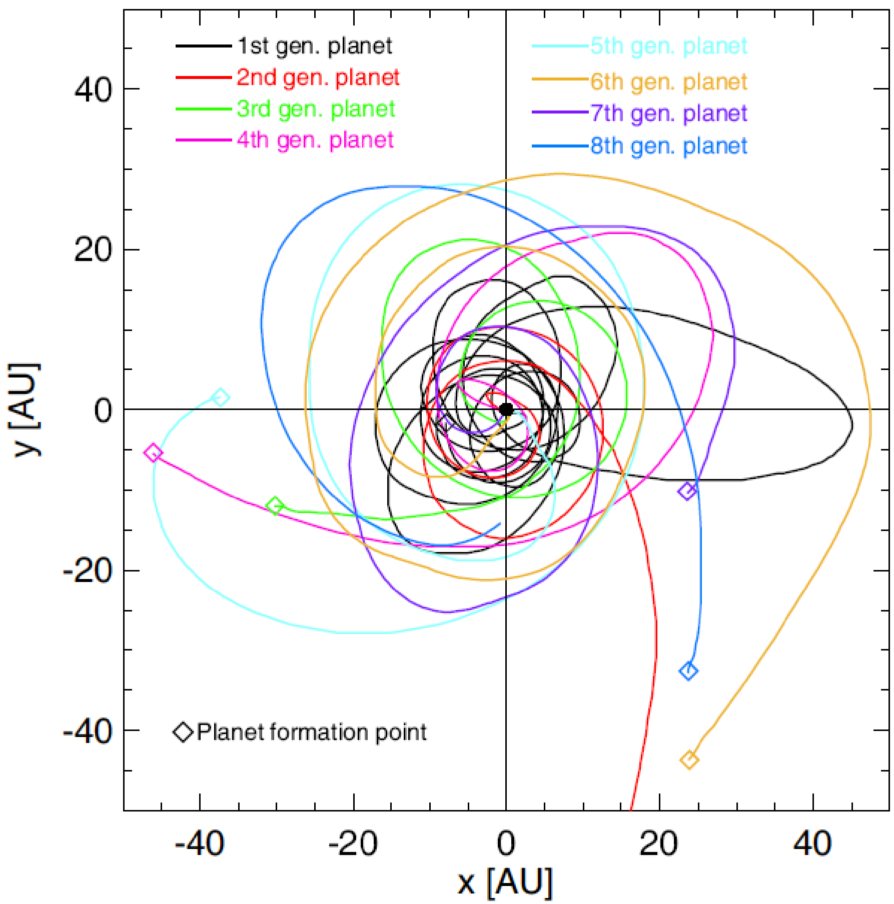
Machida, SI & Matsumoto (2011) ApJ **729**, 42

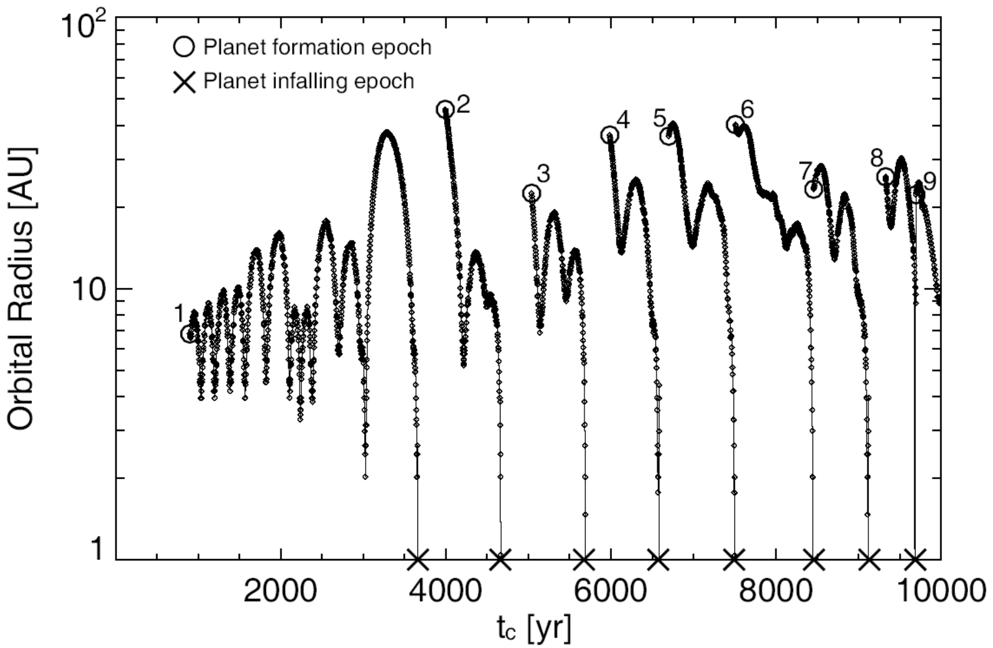
Highly Eccentric Orbit!

Outburst due to Planet @Peri-Center

Migration Theory with e≠0

Orbits of Planets & Their Fates

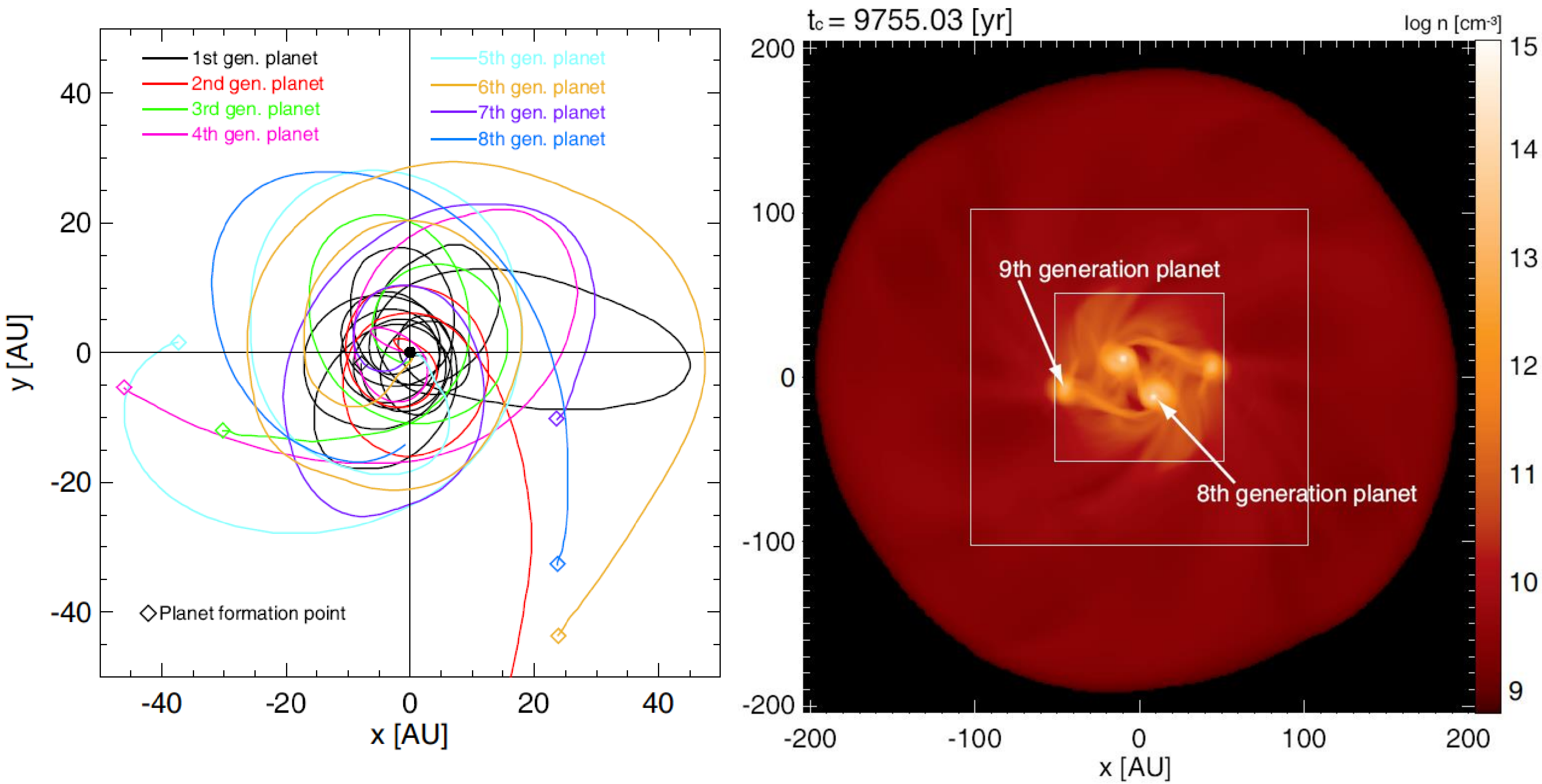




Multiple Episodes of Planet Formation and Orbital Decay

Machida, SI & Matsumoto (2011) ApJ **729**, 42

Final Outcome?



Model	α_0	f	Ω_0 [s^{-1}]	B_{ini} [μG]	M_{cl} [$M_\odot$]	R_{cloud} [AU]	M_{ps} [$M_\odot$]	M_{disk} [$M_\odot$]	N_{pl}
A03	0.3	2.8	1.1×10^{-13}	37	1.3	4700	0.59	0.15	16

Machida, SI & Matsumoto (2011) ApJ **729**, 42

Summary of Part 1

Ang. Mom. Problem

- Outflows from First Core & Jets from Protostar
  - Ang. Mom. & Mag. Flux Problem

Mag Flux Problem

- Disk Emerges in Dead Zone and Both Expand Together!
- Disk Grows Rapidly after Dispersal of Envelope!
- The First Core becomes Protoplanetary Disk!

➔ $M_{\rm disk} > M_*$ in disk formation phase

➔ Successive Formation of Planetary-Mass Objects

- Episodic Accretion and Planet Falling

➔ Outburst of Protostellar Luminosity

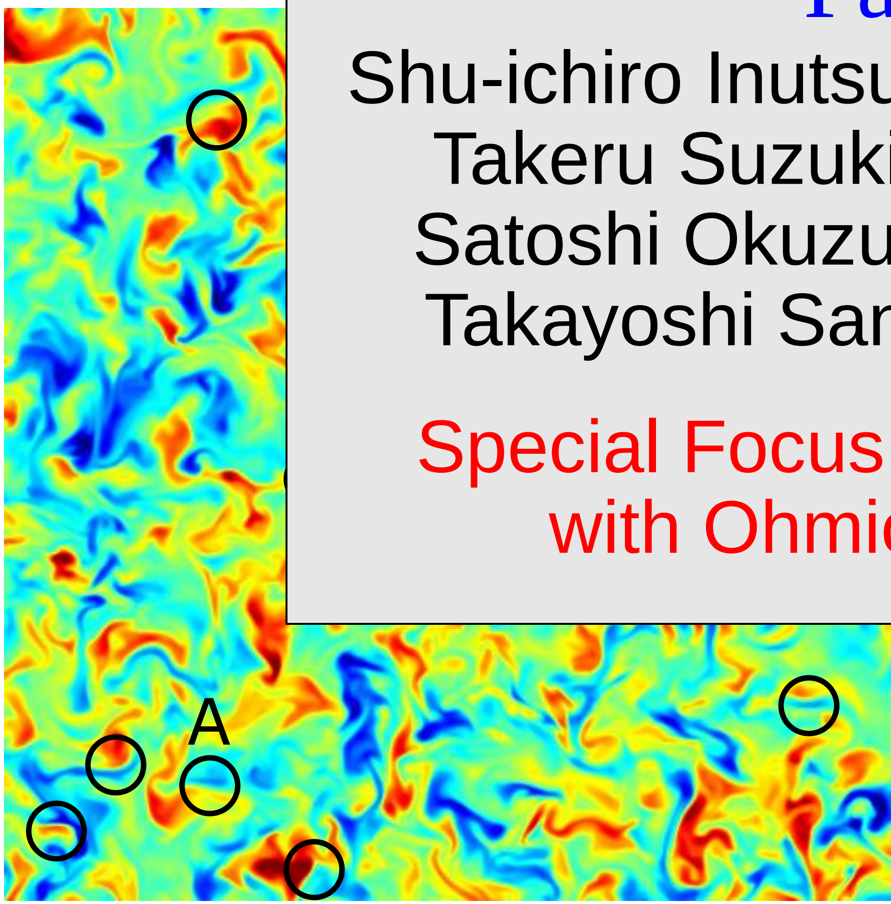
Summarized in *Inutsuka (2012)* ***PTEP*** *2012, 01A307*

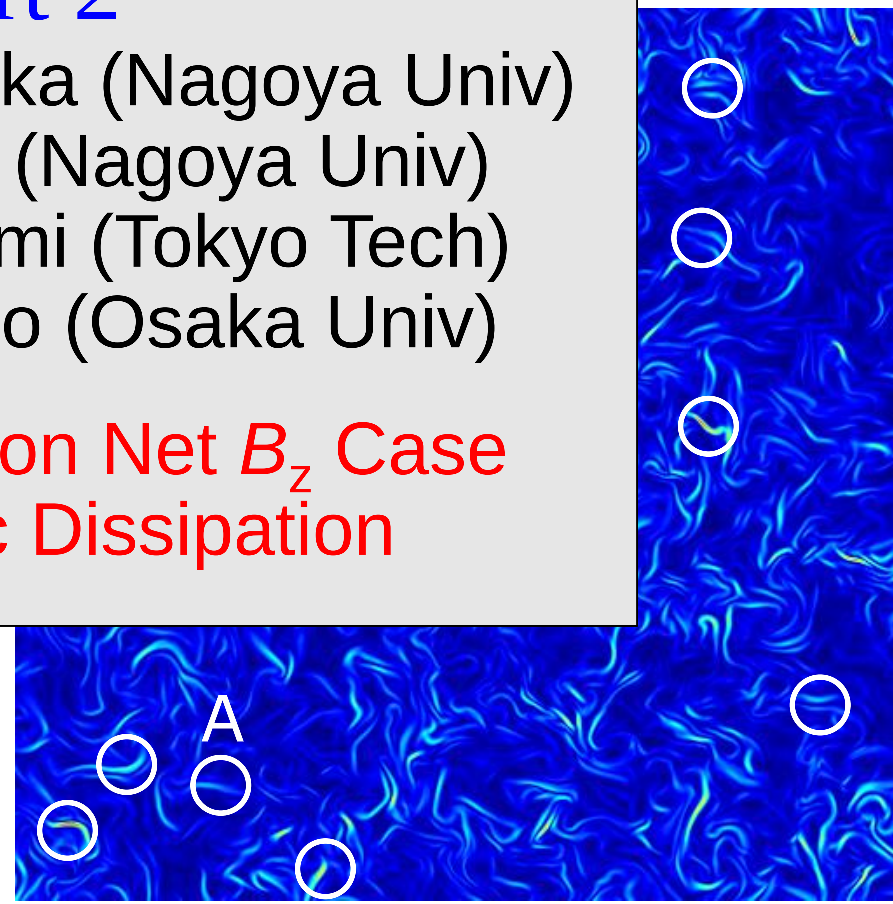
The Formation and Evolution of Protoplanetary Disks: The Critical Effects of Non-Ideal MHD

Part 2

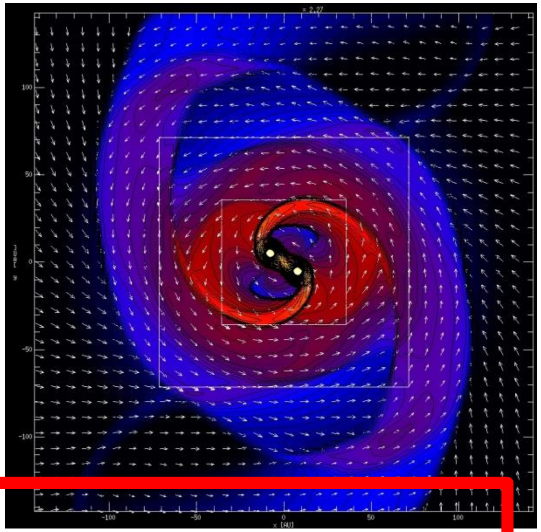
Shu-ichiro Inutsuka (Nagoya Univ)
Takeru Suzuki (Nagoya Univ)
Satoshi Okuzumi (Tokyo Tech)
Takayoshi Sano (Osaka Univ)

Special Focus on Net B_z Case with Ohmic Dissipation





Evolution of Circumstellar Discs



Early Phase

Rapid Gas Accretion due to Gravitational Torque of “m=2” Spiral Modes

Typically $\alpha > 0.1$

Later Phase

Slow Accretion due to Magnetorotational Instability (MRI)

Balbus & Hawley 1991 (Velikhov 1959, Chandrasekhar 1961)

Remarkable Difference from Protostellar Collapse Phase

Rotationally Supported & $t_{\mathrm{evo}} > 10^3\, t_{\mathrm{dyn}}$!

➔ Need for Good Codes and Theories

Basic Energetics 1

Lynden-Bell & Pringle 1974

specific angular momentum:

$$h = r\,\mathrm{v}_\phi$$

specific energy:

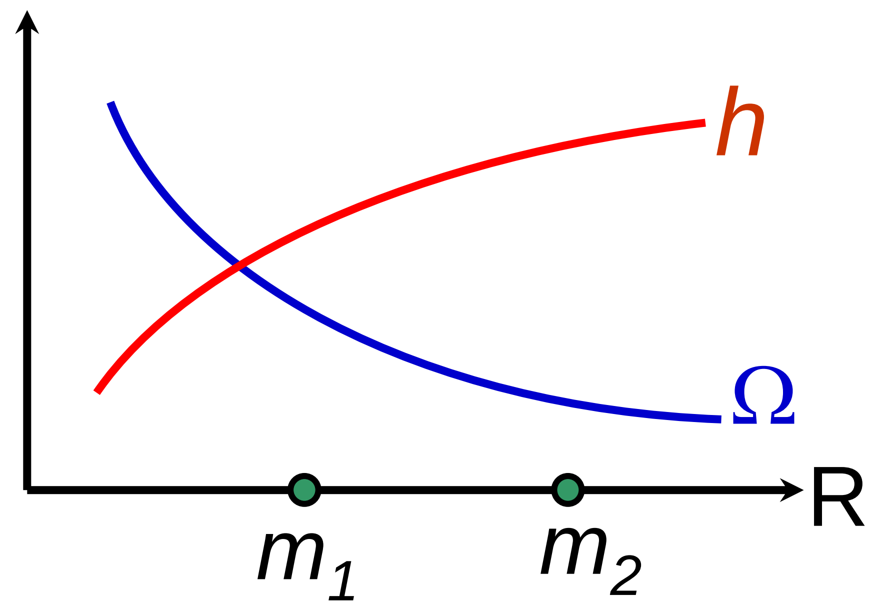
$$e = \mathrm{v}_\phi^2/2 + \psi = (h/r)^2/2 + \psi(\mathrm{r})$$

total energy:

$$E = m_1 e_1 + m_2 e_2$$

total angular momemtum:

$$H = m_1 h_1 + m_2 h_2$$



Transfer angular momentum (dh) between 1 & 2 :

$$dH = m_1\,dh_1 + m_2\,dh_2 = 0$$

$$dE = m_1(\partial e/\partial h)\,dh_1 + m_2(\partial e/\partial h)\,dh_2$$

$$= m_1\,dh_1\,(\Omega_1 - \Omega_2) \quad < 0 \quad \text{for } dh_1 < 0$$

i.e., Outward transfer of angular momentum should be unstable.

Basic Energetics 2

Lynden-Bell & Pringle 1974

Next, transfer mass & angular mom.

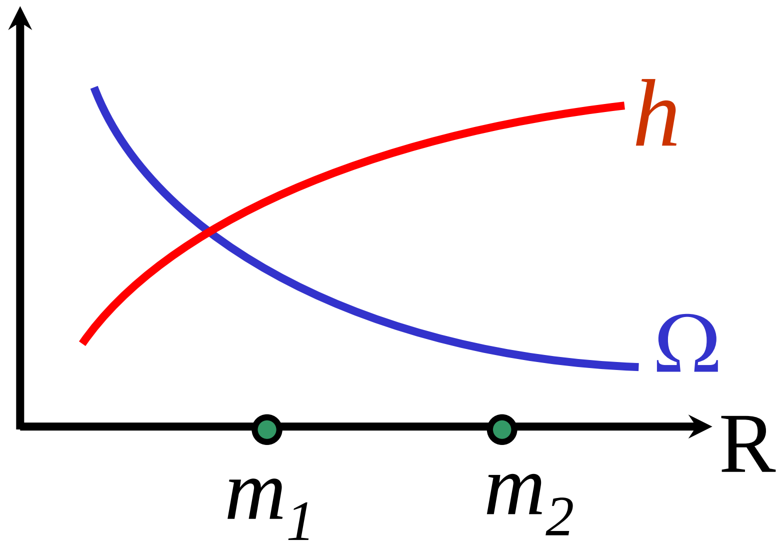
$$dM = dm_1 + dm_2 = 0$$

$$dH = d(m_1 h_1) + d(m_2 h_2) = 0$$

$$dE = d(m_1 e_1) + d(m_2 e_2)$$

$$= dm_1 \{ (e_1 - h_1 \Omega_1) - (e_2 - h_2 \Omega_2) \}$$

$$+ d(m_1 h_1) (\Omega_1 - \Omega_2)$$



where

$$d\,(e - h\,\Omega)\,/dr = d(-\mathrm{v}_\phi^{\,2}/2 + \psi\,)/dr = -r\,\mathrm{v}_\phi\; d\Omega\,/dr \;> 0$$

Thus, $(e_1 - h_1 \Omega_1) - (e_2 - h_2 \Omega_2) < 0$

$dE < 0$ for $dm_1 > 0$ and $d(m_1 h_1) < 0$

Mass Inward — Ang Mom Outward

Important only in rotationally supported case!

How to transfer of Angular Momentum and Mass?

Basic Eq.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \overrightarrow{v}\right) = 0$$

$$\frac{d\overrightarrow{v}}{dt} + \frac{1}{\rho}\nabla\left(P + \frac{B^2}{8\pi}\right) - \frac{1}{4\pi\rho}\left(\overrightarrow{B}\cdot\nabla\right)\overrightarrow{B} + \nabla\Phi = 0$$

$$\frac{\partial \overrightarrow{B}}{\partial t} - \nabla \times \left(\overrightarrow{v} \times \overrightarrow{B}\right) = \eta \nabla^2 \overrightarrow{B}$$

$$\rho T \frac{ds}{dt} = \frac{\eta}{4\pi}\left(\nabla \times \overrightarrow{B}\right)^2$$

No Cooling!

where,

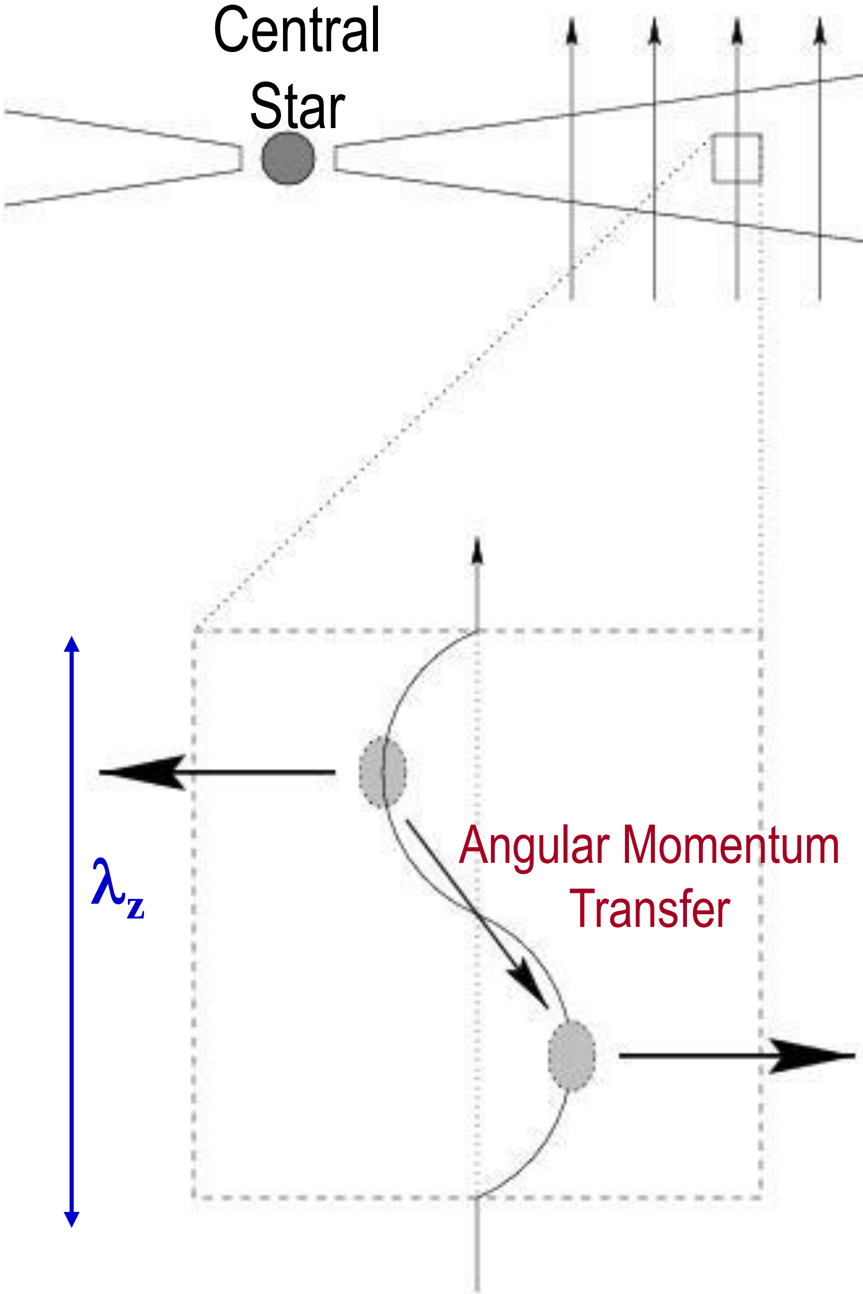
d/dt : Lagrangian Derivative

Φ : Gravitational Potential

η : Magnetic Diffusivity

s : Enthoropy per Unit Mass

Magneto-Rotational Instability with Vertical Mag Flux



Local Linear Analysis with Bousinesq approx.

$\delta \propto e^{i(kz + \omega t)}$, $k = 2\pi/\lambda_z$

Dispersion Relation in Ideal MHD ($\eta=0$) Case

$$\omega^4 - \omega^2[\,\kappa^2 + 2(k \cdot v_A)^2\,] + (k \cdot v_A)^2\,[\,(k \cdot v_A)^2 + R\, d\Omega^2/dR\,] = 0$$

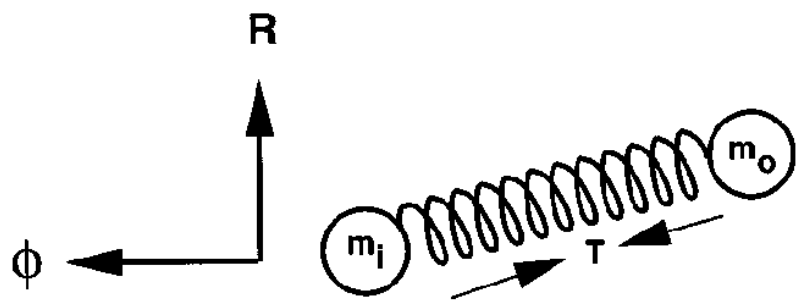
Balbus & Hawley 1991

Simple Explanation for Instability

Equivalent Model with a Spring!

Connect two bodies

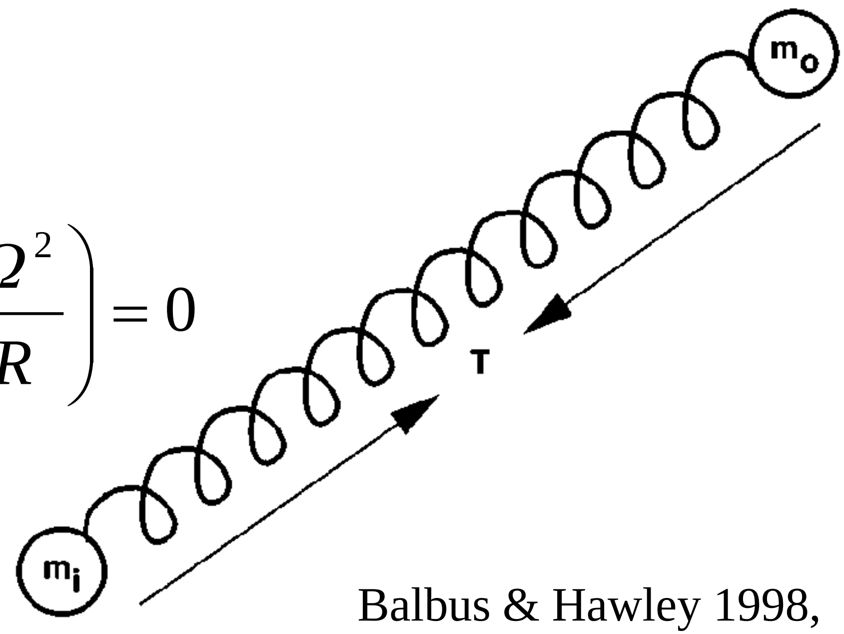
with a spring $K_s=(k\,v_A)^2$.



$$\ddot{x} - 2\Omega\,\dot{y} = -xR\frac{d\Omega^2}{dR} - K_s\,x$$

$$\ddot{y} + 2\Omega\,\dot{x} = -K_s\,y$$

$$\rightarrow \omega^4 - \omega^2\left(\kappa^2 + 2K_s\right) + K_s\left(K_s + R\frac{d\Omega^2}{dR}\right) = 0$$



If $K_s=(k\,v_A)^2$, this is equiv. to

$$\omega^4 - \omega^2[\,\kappa^2+2(k\cdot v_A)^2\,] + (k\cdot v_A)^2\,[\,(k\cdot v_A)^2 + R\,d\Omega^2/dR\,] = 0$$

Balbus & Hawley 1998,
Rev. Mod. Phys. **70**, 1

Basics of MRI

In Ideal MHD

Linear Growth Rate:

$\omega_{max} \approx (3/4)\ \Omega_{Kepler}$

Exponential Growth from Small Field ➔

~~Kinetic Dynamo for Rot.~~ Supported System

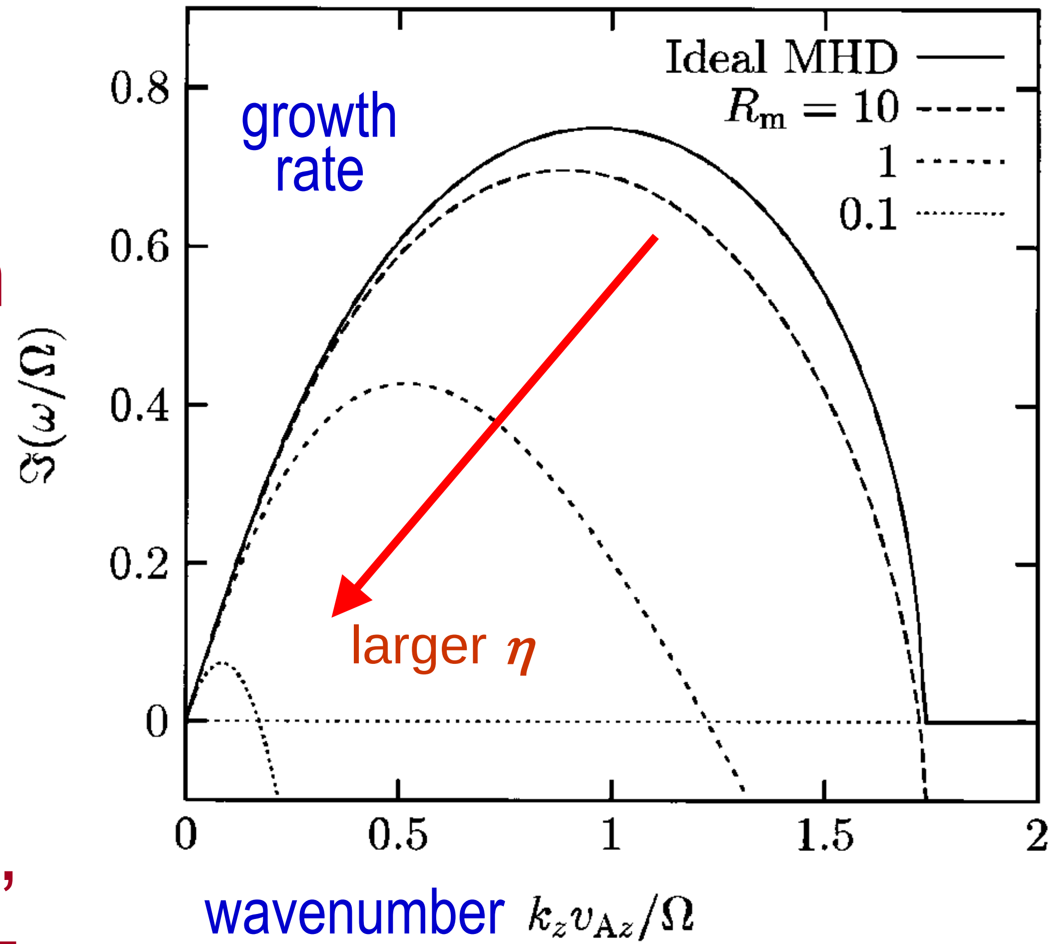
$\lambda_{max} \approx 2\pi\ v_a/\Omega_{kepler}$

⇒"Inverse Cascade"

Lucky for Simulations

k_x=0 axisymmetric (m=0) mode

$R_m \equiv v_A (v_A/\Omega) / \eta$



Sano & Miyama 1999, ApJ **515**, 776

Non-Linear Stage of MRI

- Hawley & Balbus (1991)
- Hawley, Gammie & Balbus (1995, 1996)
- Matsumoto & Tajima (1995)
- Brandenburg et al. (1995)

….

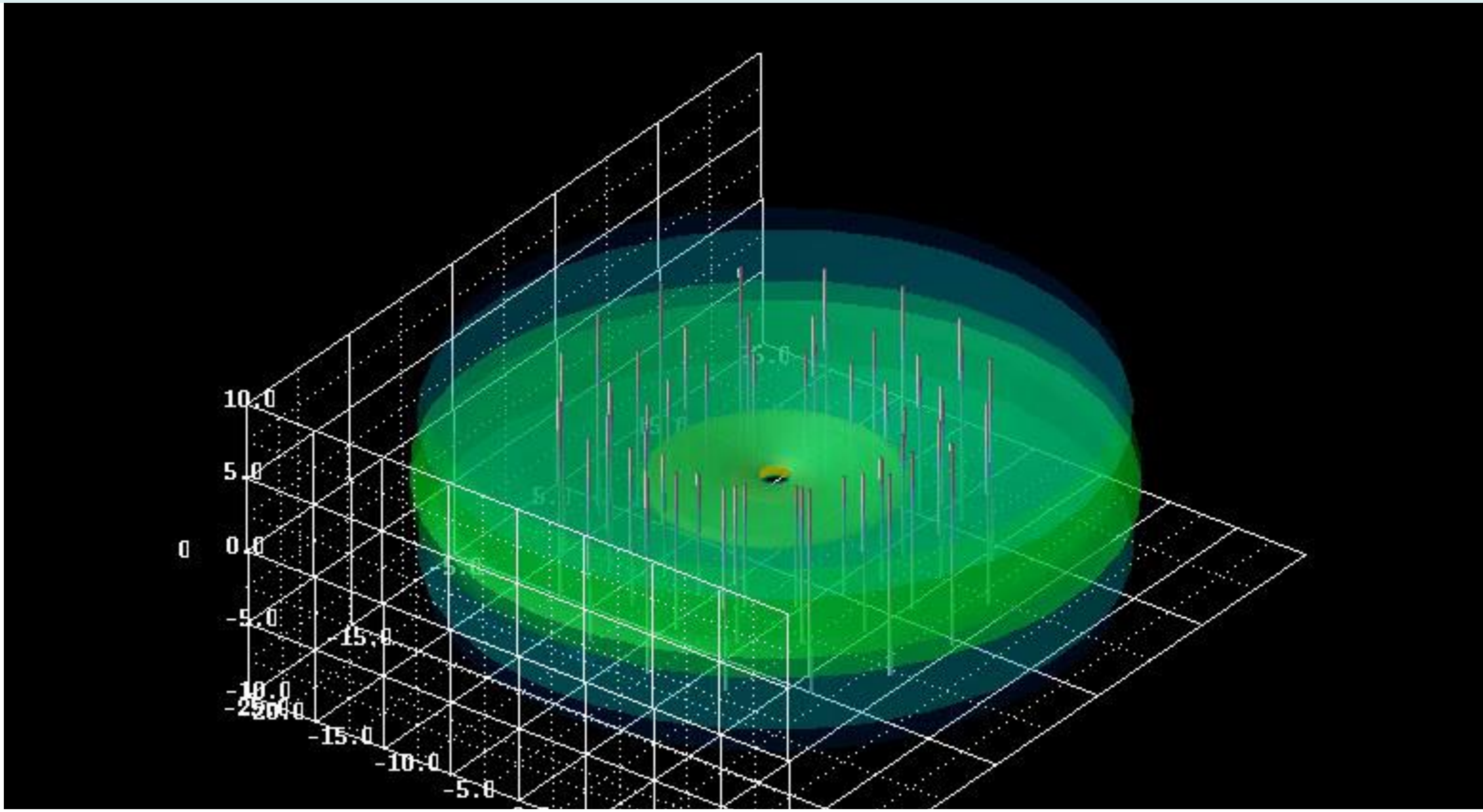
- Balbus & Hawley (1998) Rev. Mod. Phys. **70**, 1

Sorry for not citing your contribution…

Analytical Efforts:

- Pessah+(2007), Pessah (2010), Vishniac (2009)…
- Okuzumi+(2014), Takeuchi+(2014)…

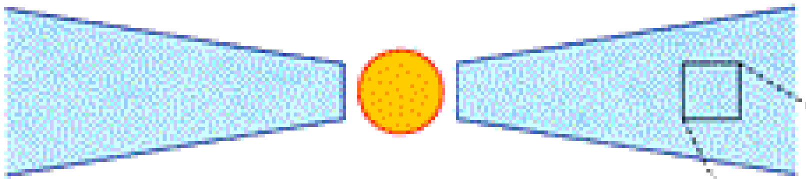
Global Disk Simulation by T. Suzuki



Wait for other talks on global simulations!
Focus on local simulations!

MHD Simulations including Ohmic Dissipation

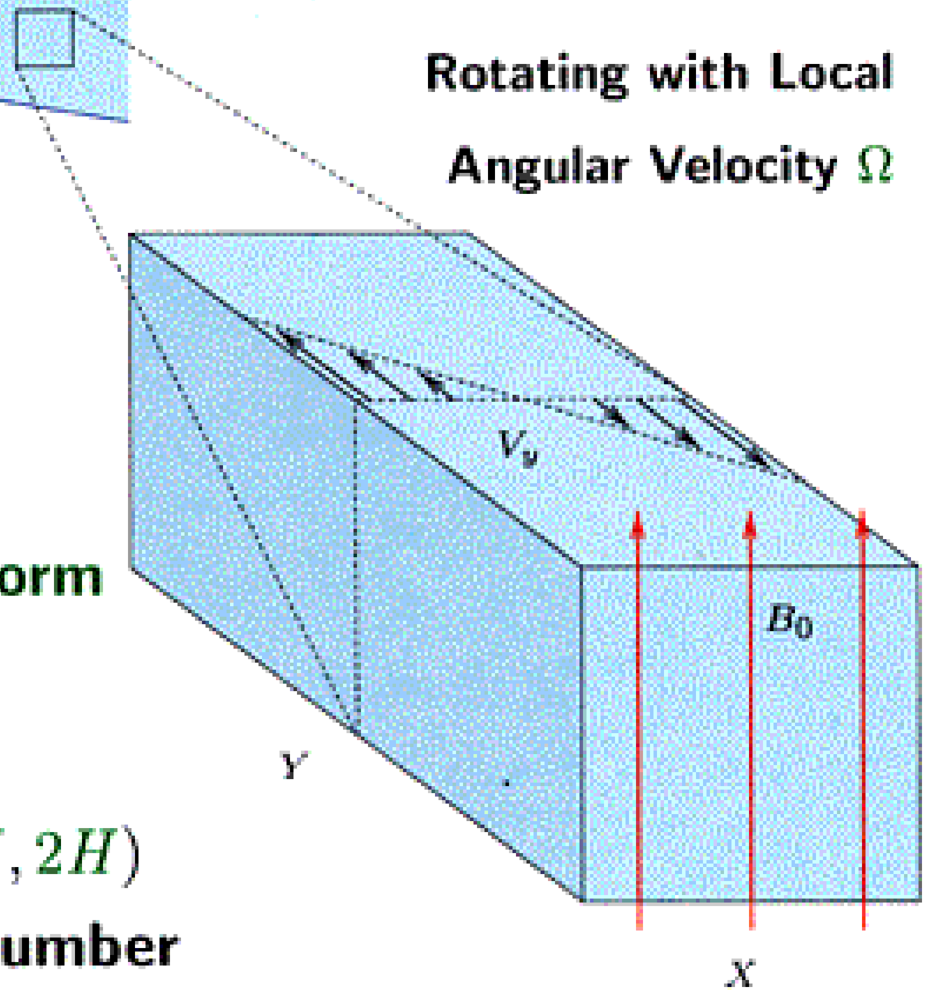
A Keplerian Disk + Uniform Vertical Fields B_0



On the Flame

Rotating with Local

Angular Velocity Ω



Local Approximation:

Box < Disk Thickness H

Density ρ_0, Pressure P_0,

Magnetic Diffusivity η are Uniform

Boundary Conditions: Periodic

Size: $(x, y, z) =$

$$(0.5H, 2H, 0.5H) \sim (2H, 8H, 2H)$$

$$= (64, 256, 64)$$: Grid Number

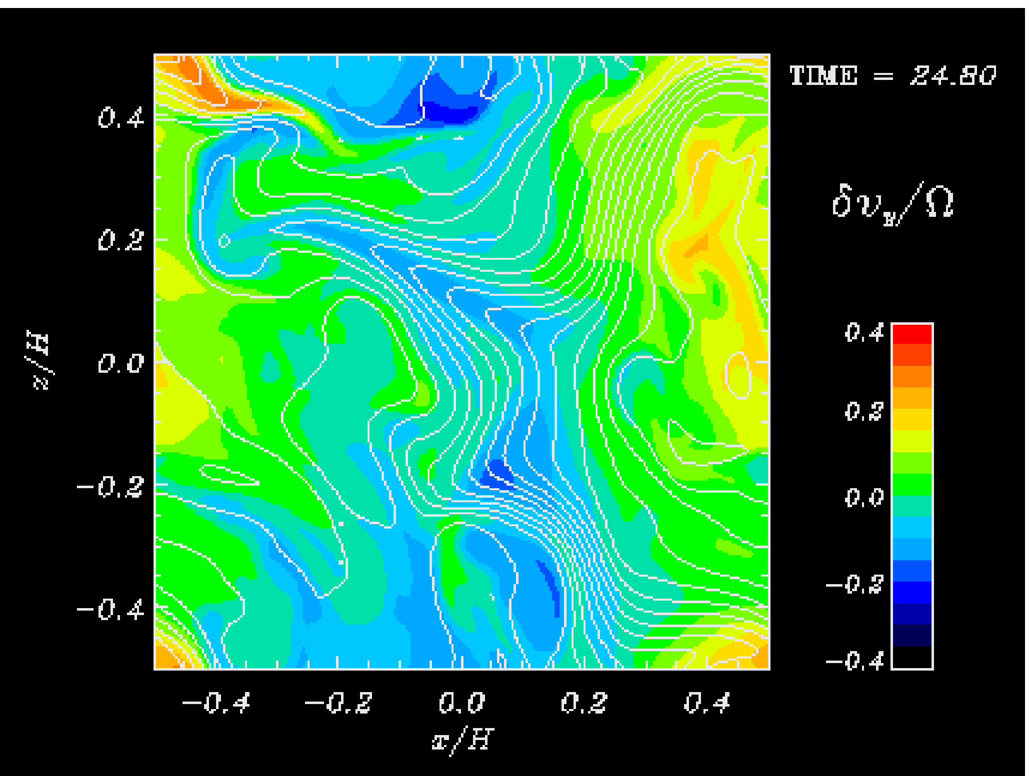
ex.) Sano+2004:2nd-order Godunov Method + MoC-CT

2D Axisymmetric Calculation

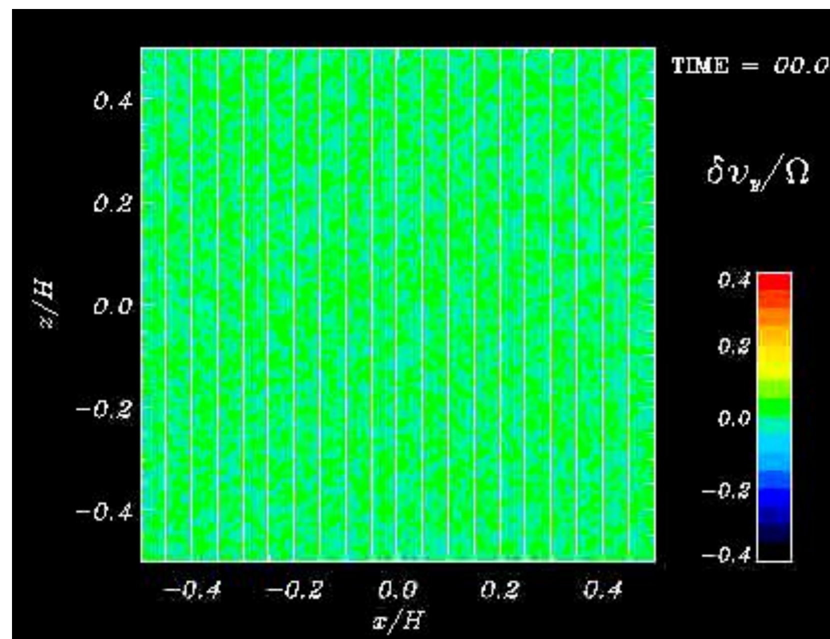
Magnetic Raynolds Number: $R_{\mathrm{M}} < 1$

"Uniformly Random" Turbulent State

$\Rightarrow$ η-Dependent Saturation Level (Sano et al. 1998)



$\beta_0 = 3200,\ R_{\mathrm{m}} = 0.5$

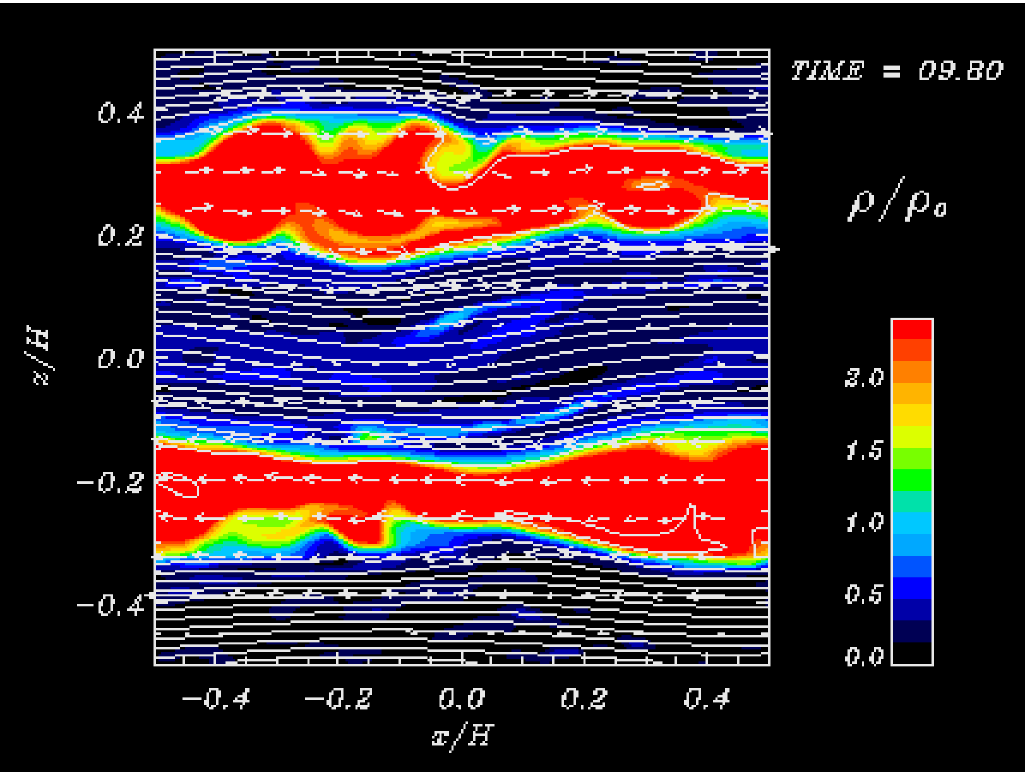


2D Axisymmetric Calculation

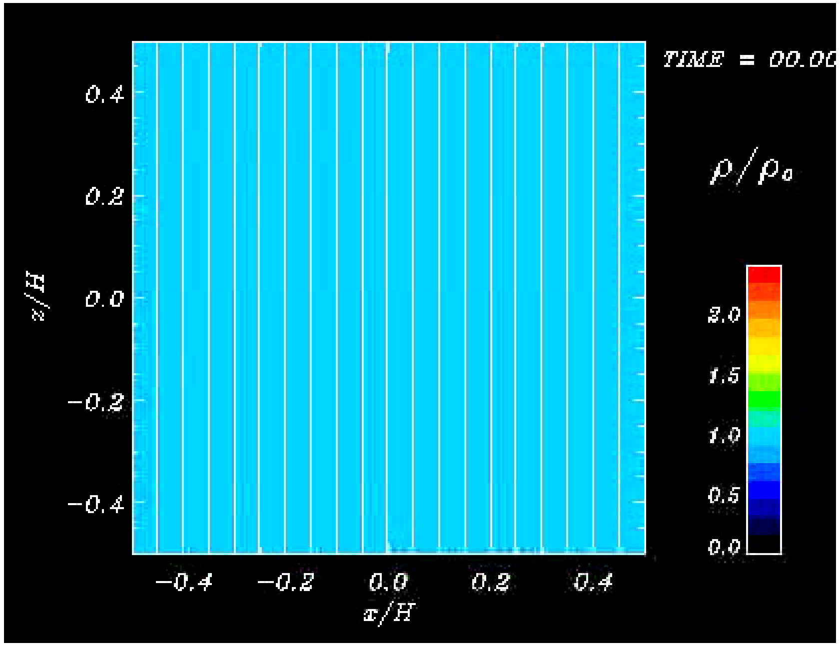
Magnetic Raynolds Number: $R_M > 1$

simple growth of the most unstable mode

$\Rightarrow$ Channel Flow… indefinite growth of B



$\beta_0 = 3200$, $R_m = 1.5$

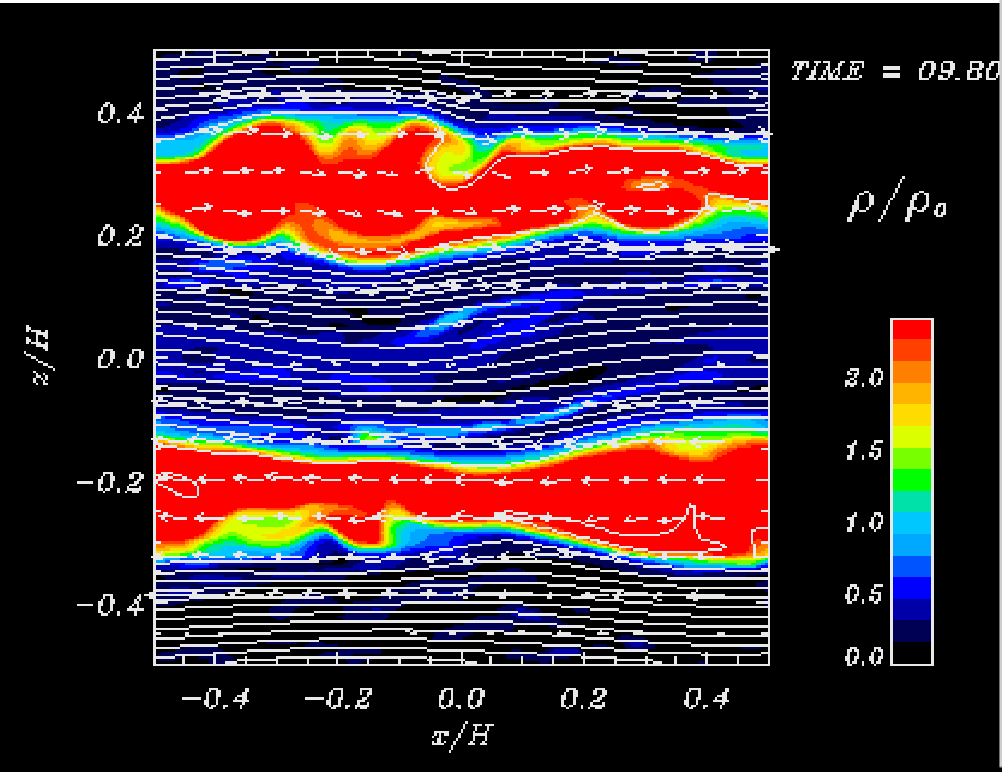


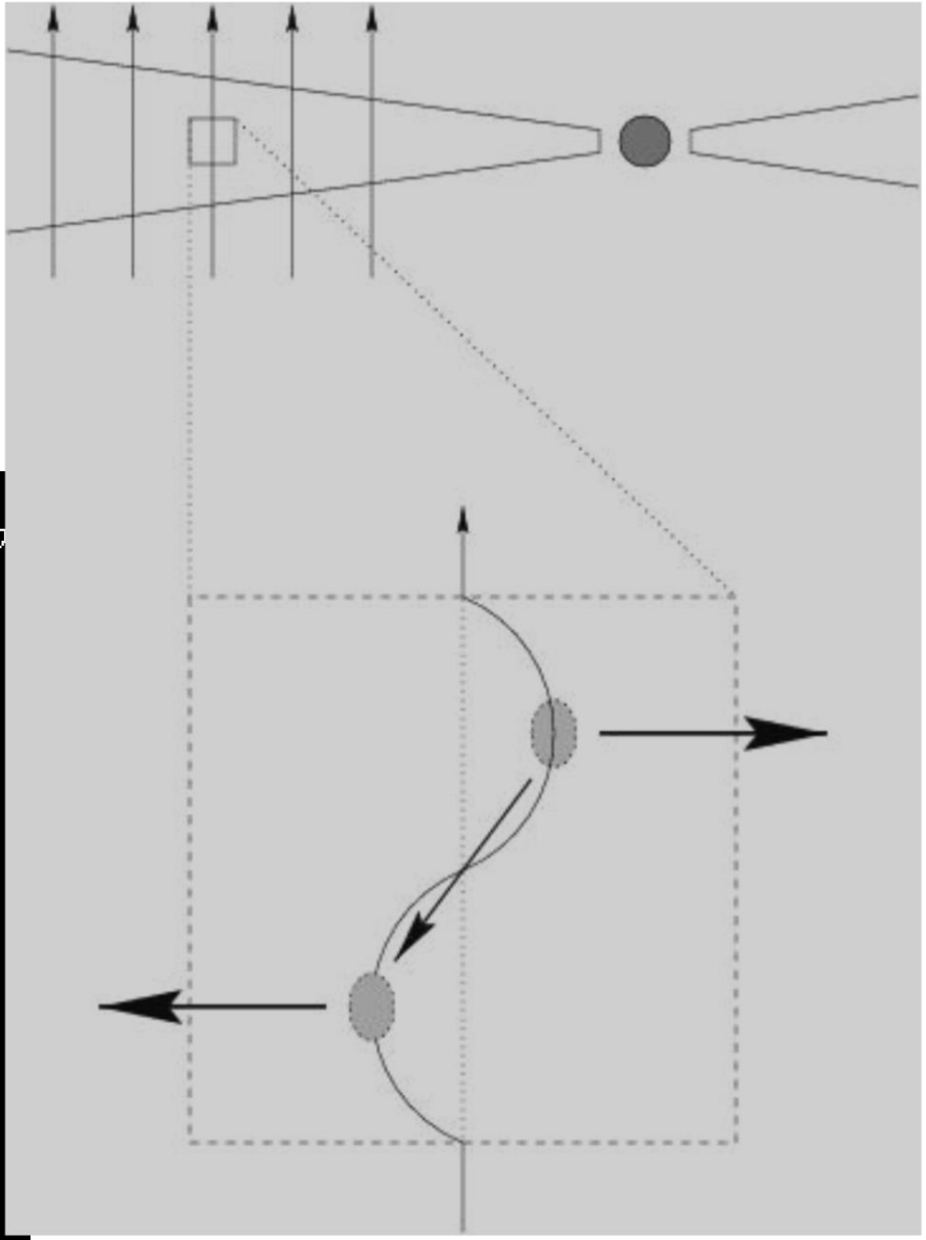
2D Axisymmetric Calculation

$\underline{R_M > 1}$

simple growth of the most unstable mode

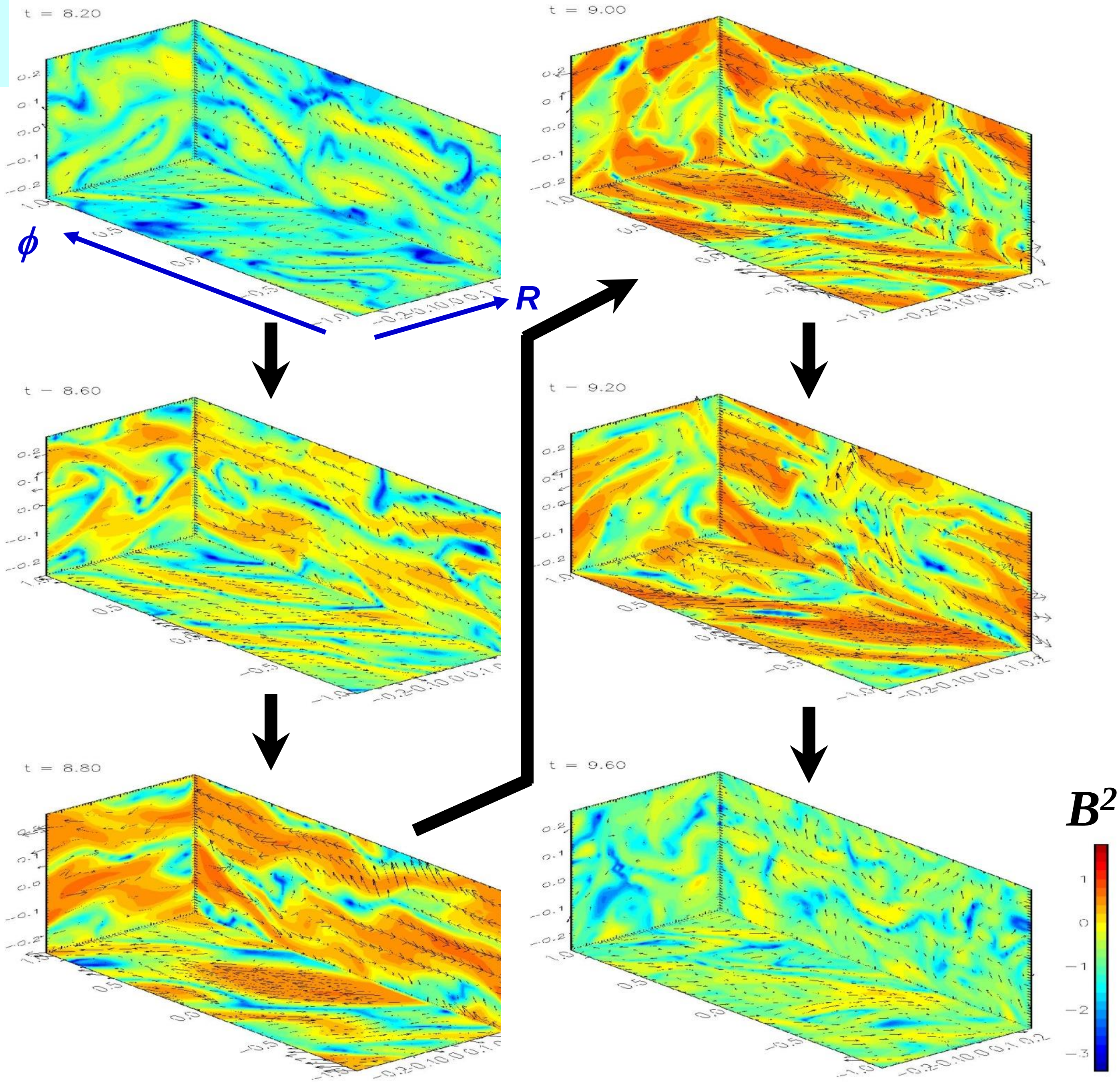
⇒ Channel Flow… indefinite growth of B





3D Calculations
$Re_M > 1$

Exponential Growth of Most Unstable Mode
⇒ channel flow
⇒ dissipation due to reconnection



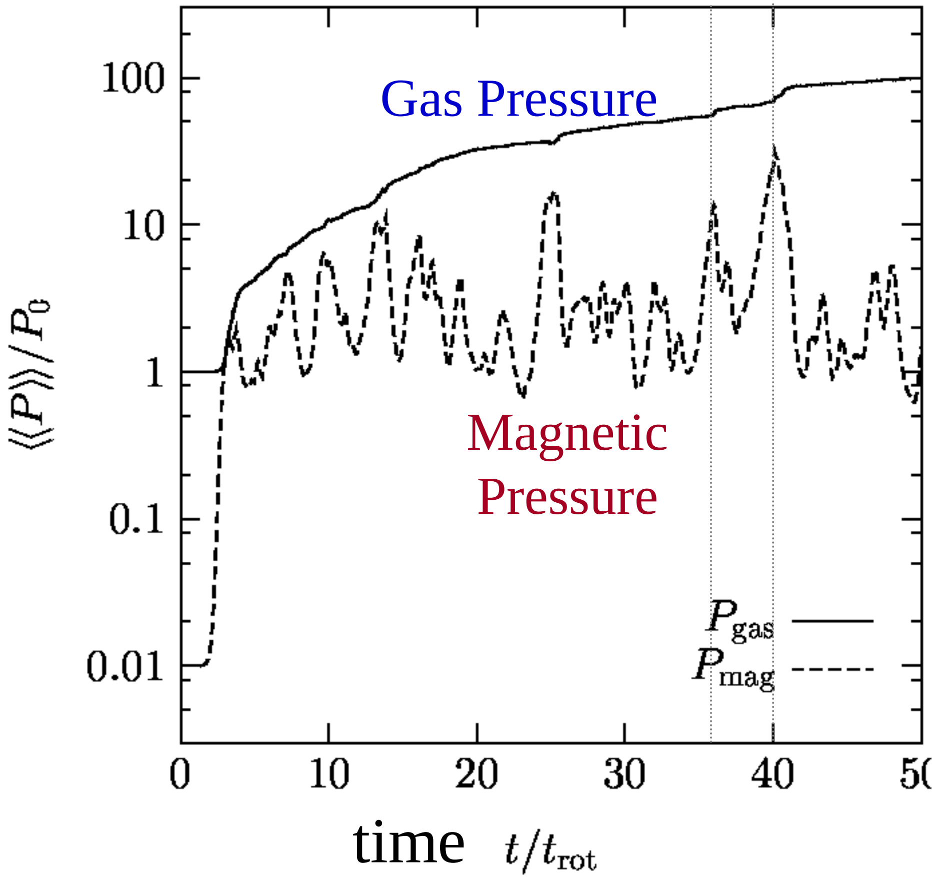
Sano, SI,
Turner, & Stone
2004, ApJ **605**, 321

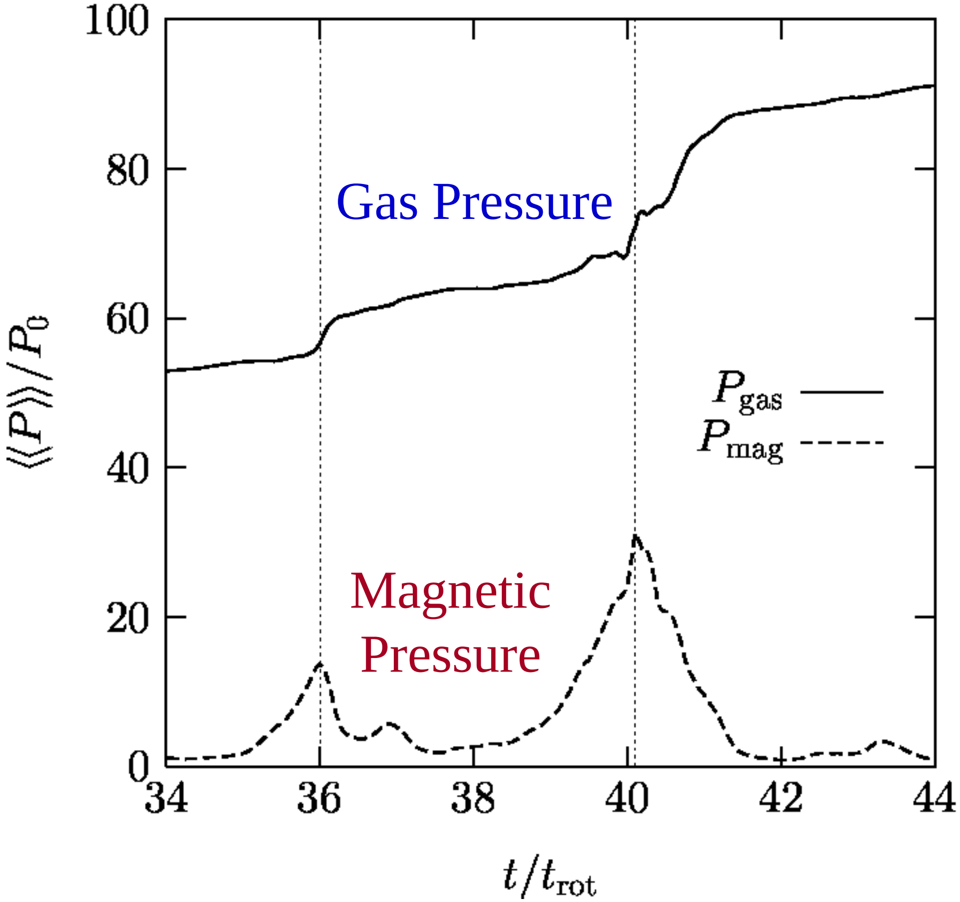
Nonlinear Time Evolution

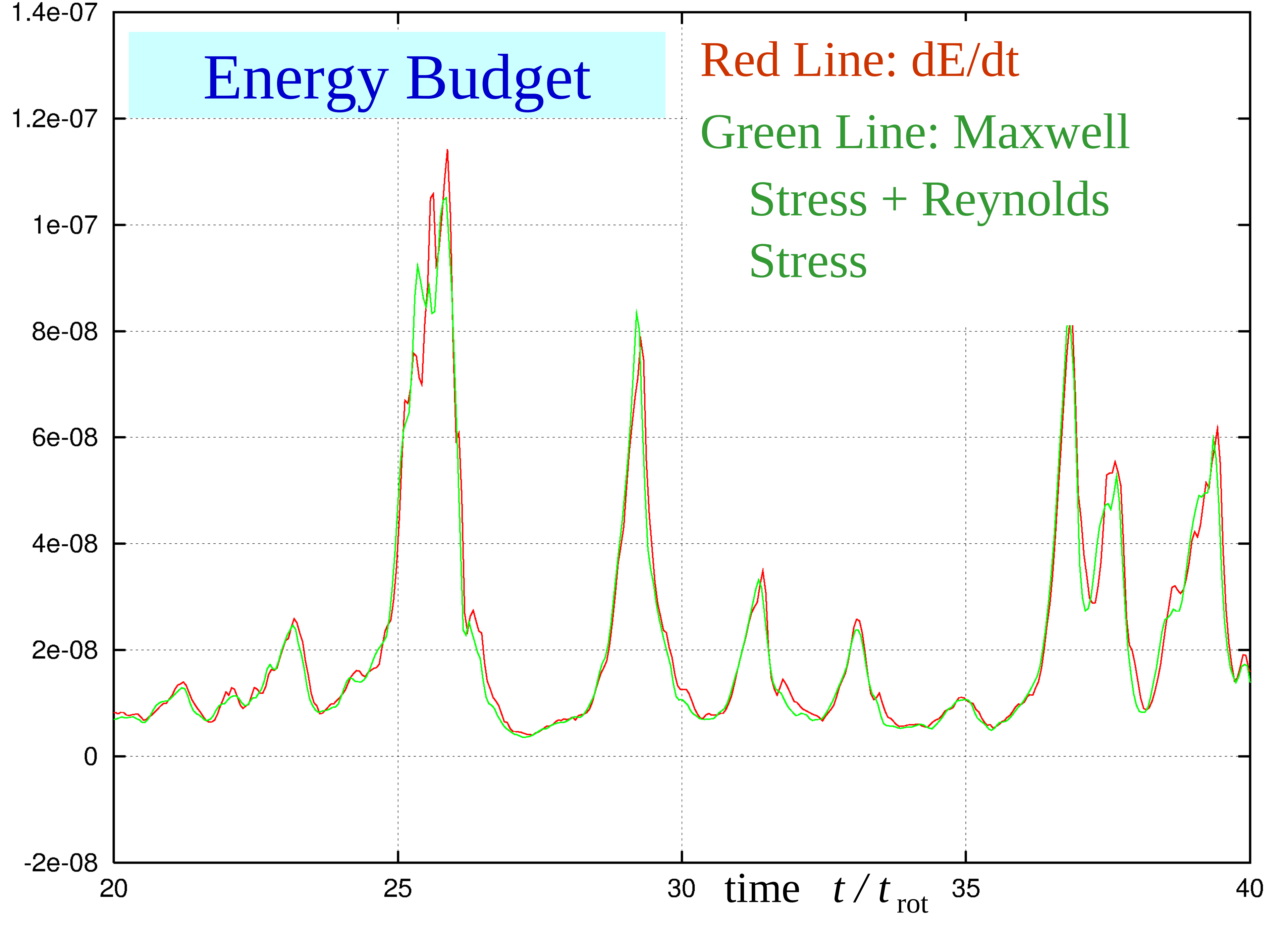
When $Re_M > 1$,

Spicky Feature in Time Evolution of Energy

= Recurrence of Exponential Growth and Magnetic Reconnection







Energy Budget
Red Line: dE/dt
Green Line: Maxwell Stress + Reynolds Stress
1.4e-07
1.2e-07
1e-07
8e-08
6e-08
4e-08
2e-08
0
-2e-08
20
25
30
35
40
time t / t_{rot}

Fluctuation vs Dissipation

$$\Gamma \equiv \iiint \left[\rho \left(\frac{1}{2} v^2 + u + \psi \right) + \frac{B^2}{8\pi} \right] dV$$

Thermal Energy

Hawley et al. 1995

$$\frac{d\Gamma}{dt} \equiv \iint \left[\rho \ \vec{v} \left(\frac{1}{2} v^2 + u + \frac{P}{\rho} + \psi \right) + \vec{S} \right] \cdot \overrightarrow{dA} = \frac{3}{2} \Omega \ \ L_x \iint_{yz} \left(\rho \, \mathrm{v}_x \delta \mathrm{v}_y \ - \frac{B_x B_y}{4\pi} \right) dA$$

Poynting Flux

Stress Tensor, *Wxy*

$$\dot{M} \propto W_{R\phi} \equiv \rho \mathrm{v}_R \delta \mathrm{v}_\phi - \frac{B_R B_\phi}{4\pi} \propto \frac{d\Gamma}{dt} \ .$$

If saturated, $\left\langle\left\langle \frac{\partial \mathrm{v}^2}{\partial t} \right\rangle\right\rangle = \left\langle\left\langle \frac{\partial B^2}{\partial t} \right\rangle\right\rangle = 0$, then , $\left\langle \frac{d\Gamma}{dt} \right\rangle = \left\langle\left\langle \frac{\partial \rho u}{\partial t} \right\rangle\right\rangle = \frac{3\Omega}{2} \left\langle\left\langle \rho \mathrm{v}_R \delta \mathrm{v}_\phi - \frac{B_R B_\phi}{4\pi} \right\rangle\right\rangle$,

where $\langle \ \rangle$ denotes time-average , and $\langle\langle \ \rangle\rangle$ denotes time- and spatial- average.

Note that $\langle \mathrm{v}_R \rangle = \langle \delta \mathrm{v}_\phi \rangle = \langle B_R \rangle = \langle B_\phi \rangle = 0$.

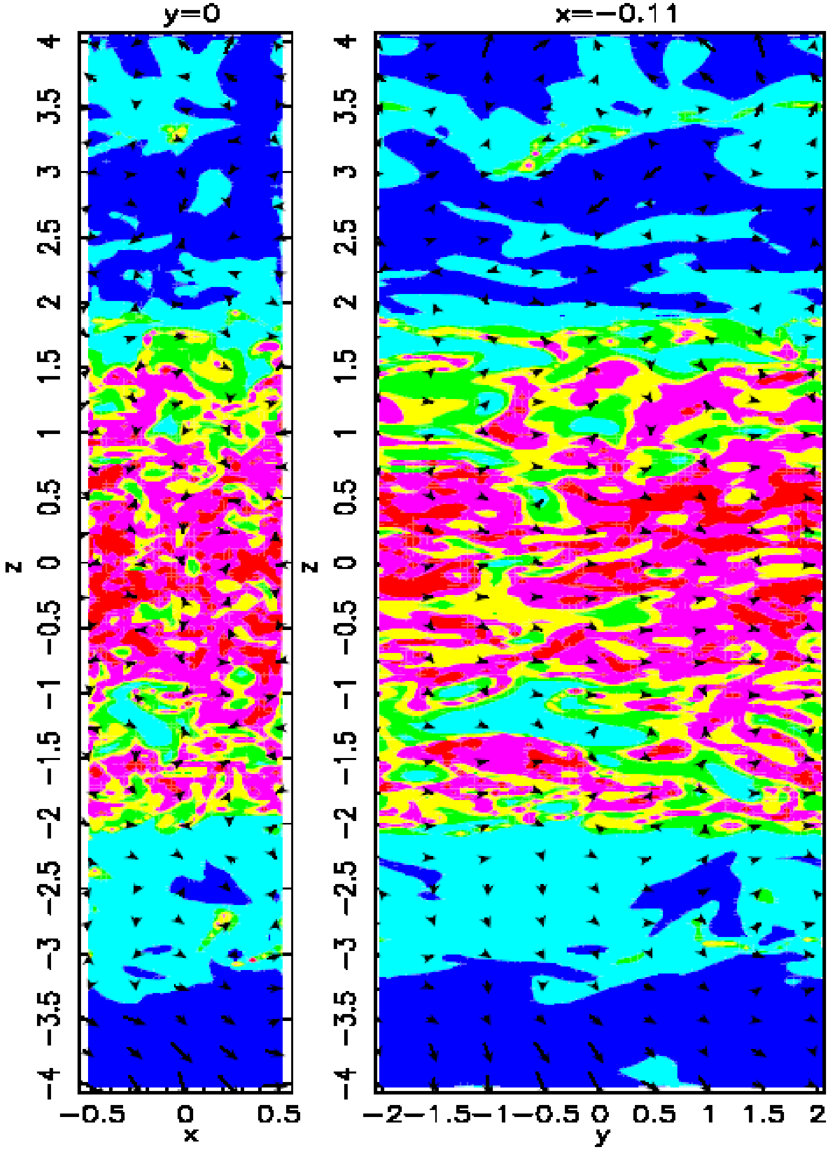
Sano & SI (2001) ApJ **561**, L179

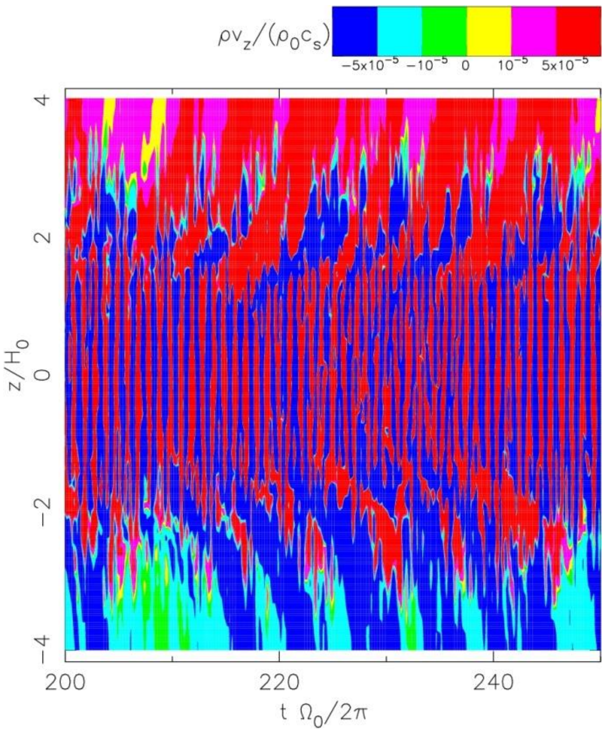
Saturation Value of $\langle\langle B^2 \rangle\rangle \Rightarrow$ Dissipation Rate $\approx 0.03\Omega \langle\langle B^2 \rangle\rangle$

SI & Sano (2005) ApJL **628**, L155

Powerful Quasi-Steady "Disk Wind"

after 210 rotations



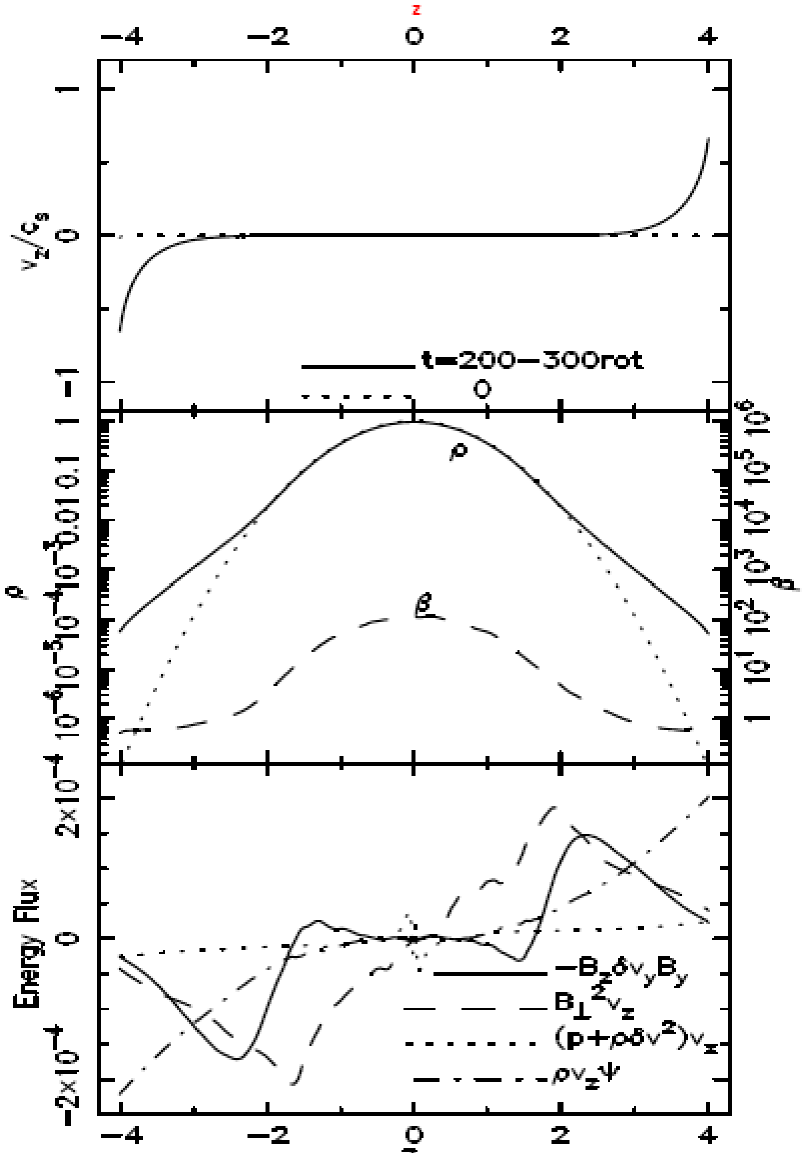


Powerful MHD Wind from Disk

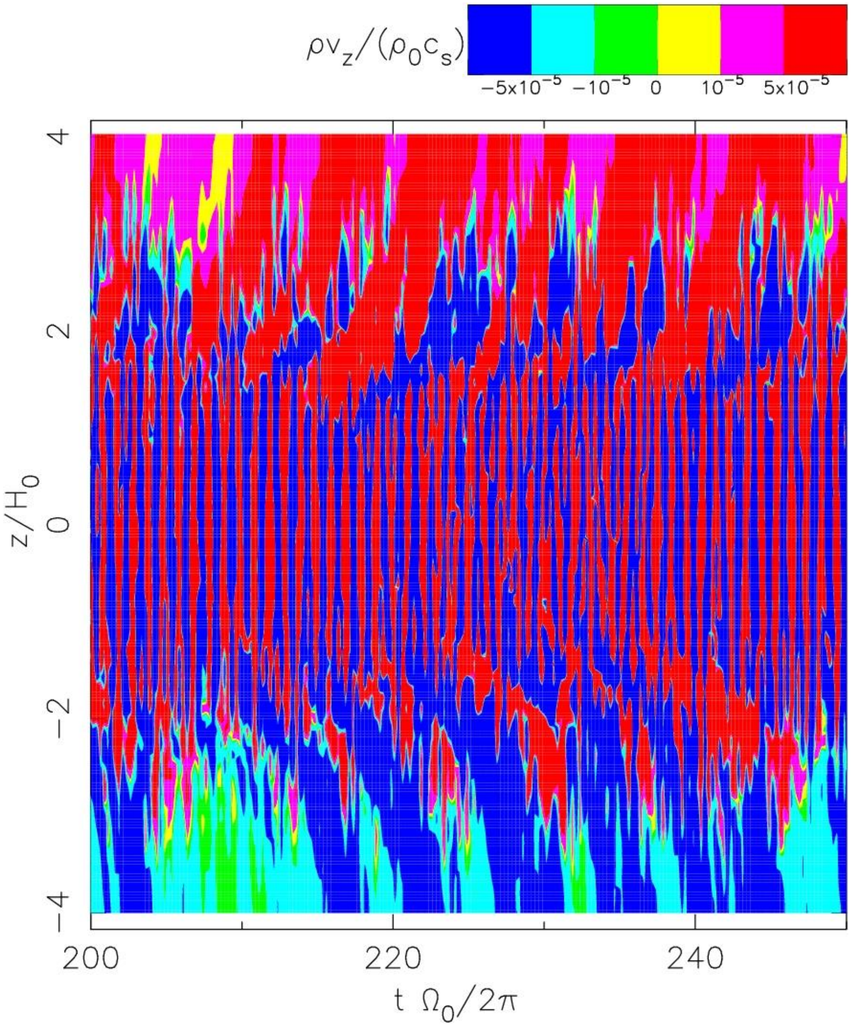
just like Solar Wind

Suzuki & SI (2009) ApJ **691**, L49

Powerful Quasi-Steady Disk Wind



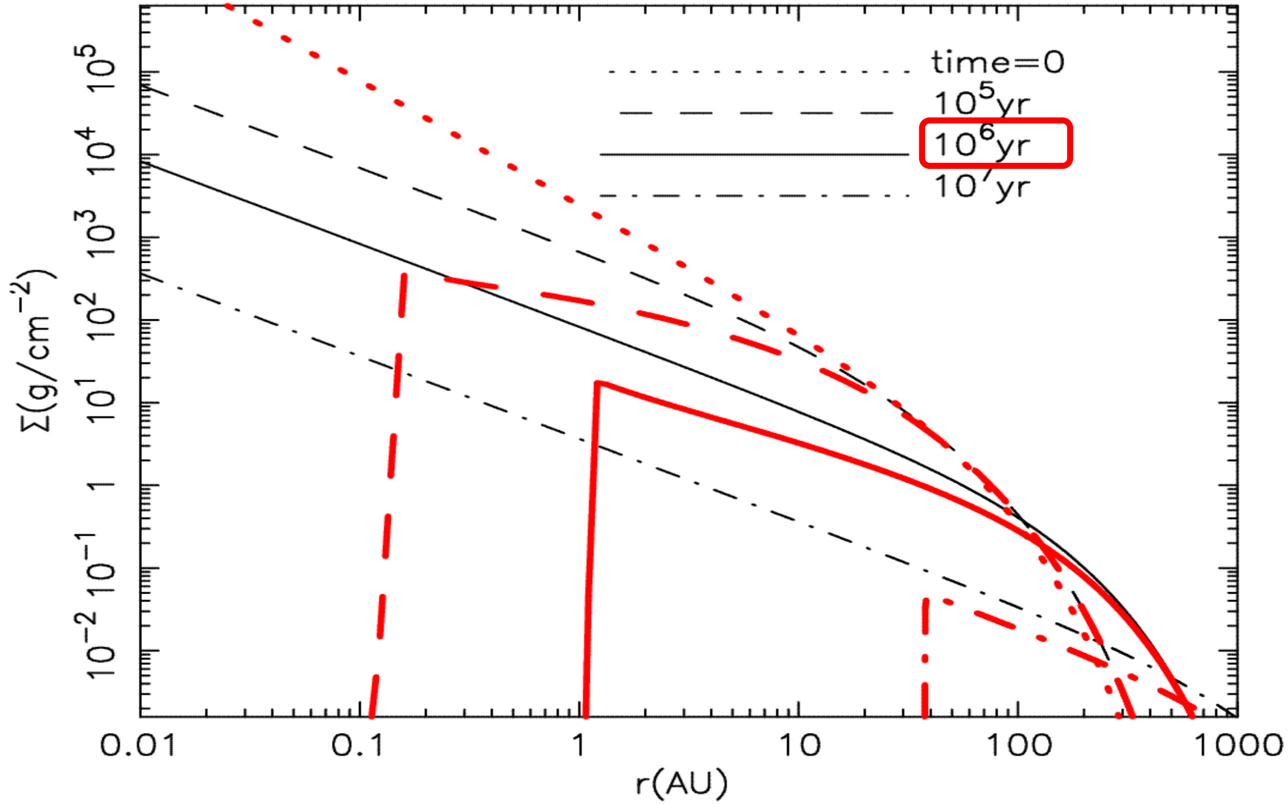
Distance from Midplane, Z



Suzuki & SI (2009) ApJ **691**, L49

Suzuki, Muto, & SI (2010) ApJ **718**, 1289

Inner Hole Creation and Dispersal



Distance from Central Star

Dispersal Timescale ~ a few Myr

for typical disk models in Ideal MHD

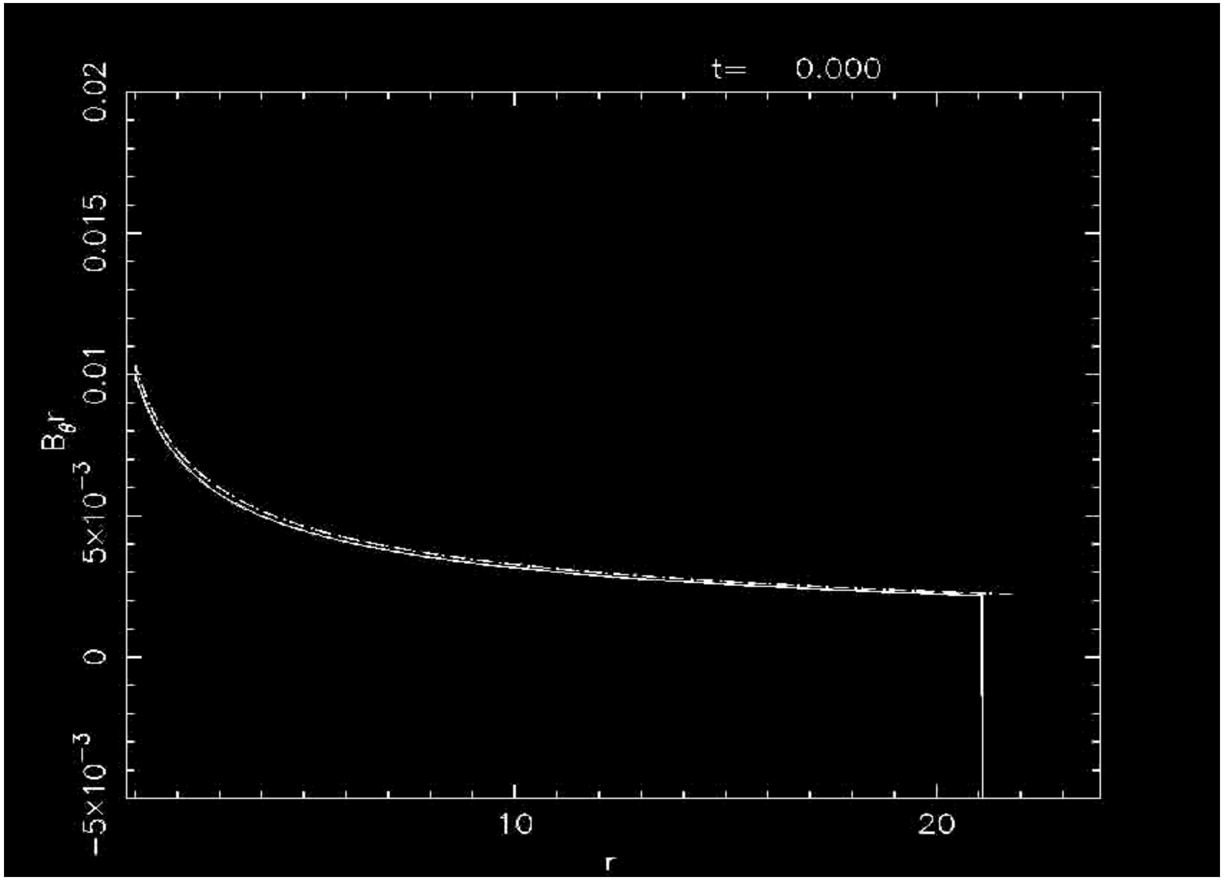
Suzuki, Muto, & SI (2009) ApJ **718**, 1289

Transport of Vertical Flux?

Vertical magnetic flux should leak outward!

Evolution of
Vertical Flux
by T. Suzuki

Preliminary!



Radius

Std Model of Protoplanetary Disks

$$\Sigma_e(r) = 1.7\times10^3 f_\Sigma \left(\frac{r}{1\mathrm{AU}}\right)^{-\frac{3}{2}} \frac{\mathrm{g}}{\mathrm{cm}^2}$$

$$T(r) = 280 \left(\frac{r}{1\mathrm{AU}}\right)^{-\frac{1}{2}} \mathrm{K}$$

$$C_S(r) \approx 10^5 \left(\frac{r}{1\mathrm{AU}}\right)^{-\frac{1}{4}} \left(\frac{\mu}{2.34}\right)^{-\frac{1}{2}} \frac{\mathrm{cm}}{\mathrm{s}}$$

$$\rho(r,z) \approx 1.4\times10^{-9} f_\Sigma \left(\frac{r}{1\mathrm{AU}}\right)^{-\frac{11}{4}} \left(\frac{M_*}{M_O}\right)^{\frac{1}{2}} \left(\frac{\mu}{2.34}\right)^{\frac{1}{2}} \mathrm{g/cm^3}$$

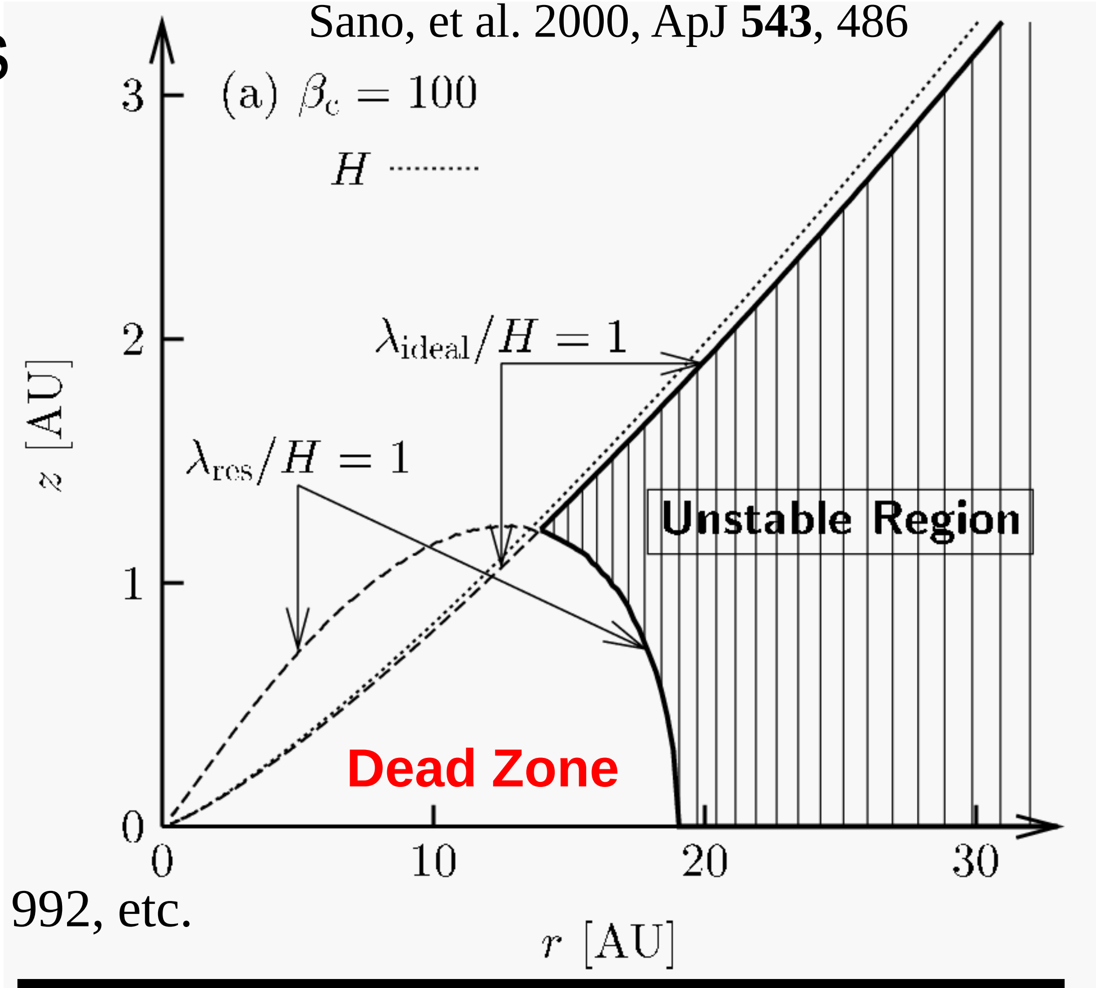
$$B(r) \approx 1.9 f_\Sigma^{\frac{1}{2}} \left(\frac{r}{1\mathrm{AU}}\right)^{-\frac{13}{8}} \left(\frac{\beta_c}{100}\right)^{-\frac{1}{2}} \mathrm{G}, \quad \beta_c \equiv \left(\frac{2C_S^2}{v_A^2}\right)$$

Ionization Degree in PP Disks

neutral gas + ionized gas
+ dust grains

$\zeta_{CR} = 10^{-17}$ s^{-1}

cosmic ray ionization
$\Rightarrow$ resistivity



"Classical" Models:

Sano et al. 2000, ApJ **543**, 486

Glassgold et al. 2000, PPIV

Fromang et al. 2002, MN **329**, 18

Salmeron & Wardle 2003, MN **345**, 992, etc.

The site of planet formation **Highly Uncertain**

But this depends on dust grain properties.

Dependence on Density and Ionization Source

Resistivity:

$$\eta = c^2/(4\pi\sigma_c) = 2\cdot 10^2\,(T)^{1/2} / x_e$$

Magnetic Reynolds Number:

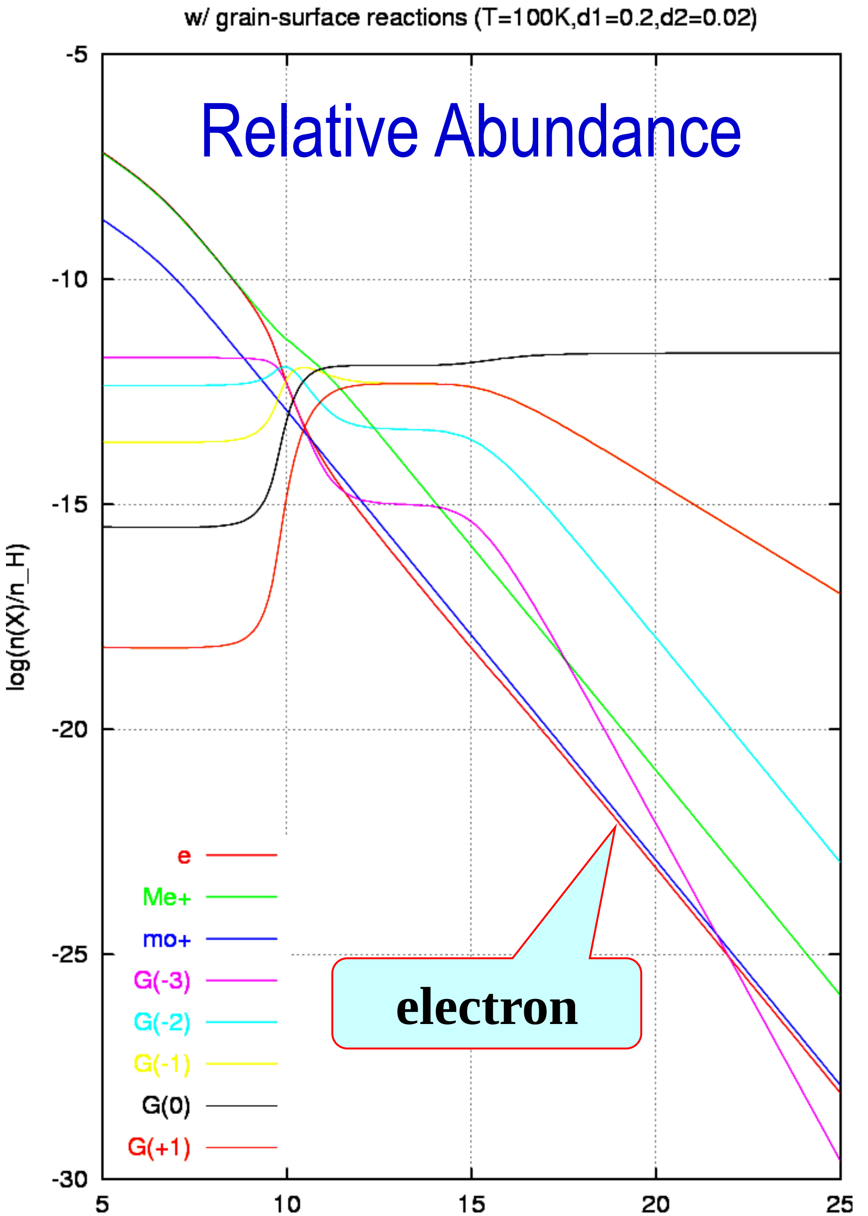
$$Re_{,M} = v_A{}^2/\eta\,\Omega \approx C_s H/\beta\eta$$

$$= (10/\beta)\,(x_e/10^{-13})\,(H/0.1AU)$$

$$\beta = (2C_s{}^2)/v_a{}^2$$

Required Electron Ionization:

$$x_e \sim 10^{-13}$$



$\log(n_H/\zeta_{-17})$

Ionization Degree Required for MRI

resistivity: $\eta = c^2/(4\pi\sigma_c) = 2\cdot10^2\,(T)^{1/2} / x_e$

$Re,_M = v_A{}^2/\eta\Omega = (10/\beta)\,(x_e/10^{-13})\,(H/0.1AU) > 1$

Required Electron Ionization: $x_e \sim 10^{-13}$

Scaling Relation in High Density Regime with Dust grains

For $x_e \sim 10^{-13}$, $10^7 < n_H/\zeta_{-17} < 10^{10}$

$$x_e \approx 3\cdot10^{15}\,(\zeta/n_H)$$

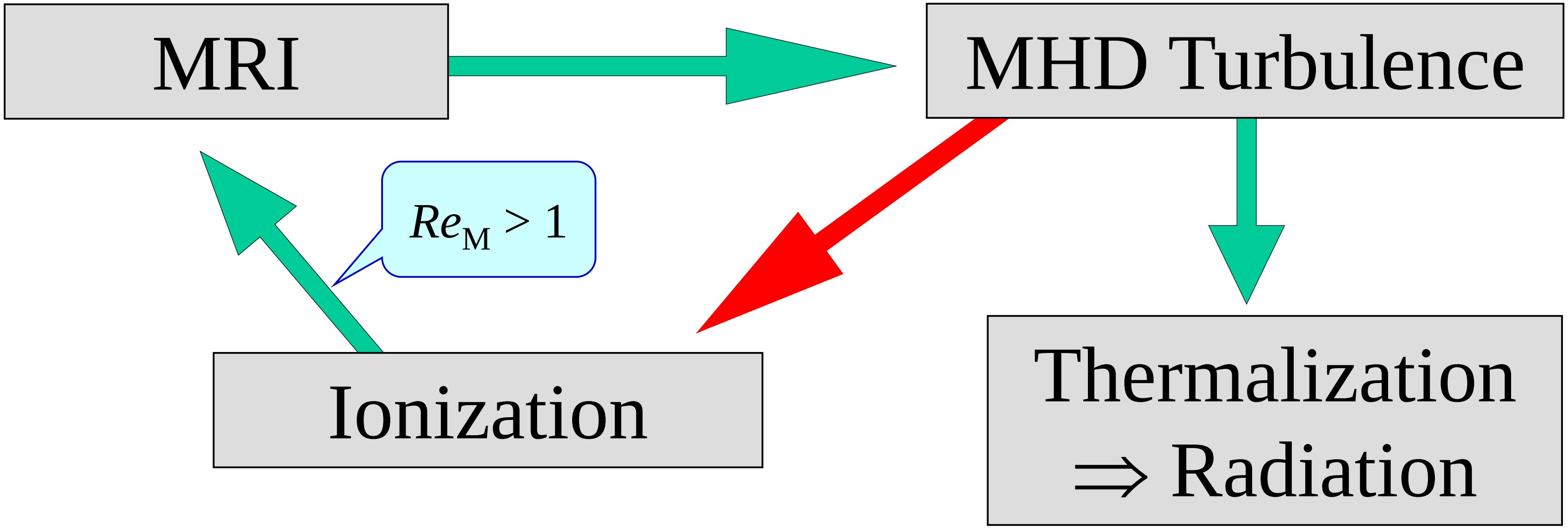
i.e., $(x_e / 10^{-13}) \approx (3\cdot10^4)^{-1}(\zeta/ 10^{-17}s^{-1})\,(10^{15}cm^{-3}/n_H)$

requires an ionization process **30,000 times higher** than the standard cosmic rays rate $\zeta_{CR}=10^{-17}/s$.

For $n_H=10^{15}cm^{-3}$, $n_H\,\zeta = 300\ cm^{-3}s^{-1}$

Is it possible?

Feedback from MRI Turbulence



Energy Budget for Sustaining Ionization

The ratio $\boldsymbol{f}_{\mathrm{ionize}}$ of energy required for the ionization to the energy available in the MRI driven turbulence,

$$\boldsymbol{f}_{\mathrm{ionize}} = \varepsilon_{\mathrm{ionize}}\, \zeta\, n_H / [dE/dt]$$

$$= \underline{\mathbf{0.03}}\ (n_H / 10^{15}\mathrm{cm}^{-3})^2 \cdot (2\pi \mathrm{yr}^{-1}/\Omega) \cdot (6\mathrm{G}/B)^2 \cdot (\varepsilon / 13.6\mathrm{eV})$$

is sufficiently **small**.

SI & Sano (2005) ApJL **628**, L155

Muranushi, Okuzumi & SI (2013)

Thermal Ionization ?

Saturation State of MRI driven Turbulence = High β plasma

Magnetic Energy < Thermal Energy of Gas

Magnetic dissipation does not result in thermal ionization.

NB.

- Magnetic Reconnection with $Re_{\mathrm{M}} > 1$?
  What is the mechanism of saturation?
- thermal ionization of Alkali metal @1000K
  (see, eg., Umebayashi 1983)

⇒ <u>not promising</u>

Microphysics (1)

Electric Currents $4\pi \boldsymbol{j} = c\, \nabla \times \boldsymbol{B} \Rightarrow\ j \approx c\, B/(4\pi\, L)$

$$\boldsymbol{j} = \Sigma_i\, e\, q_i\, n_i\, \boldsymbol{v}_i \ \Rightarrow\ e\, q_i\, n_i\, v_i \equiv f_i\, j$$

We can estimate the average velocities of charged species.

$$v_e \approx \frac{cB}{4\pi L e n_e} = 42 \left(\frac{B}{6\mathrm{G}}\right) \left(\frac{0.03\mathrm{AU}}{L}\right) \left(\frac{10^{15}\,\mathrm{cm}^{-3}}{n_\mathrm{H}}\right) \left(\frac{10^{-13}\,\mathrm{cm}^{-3}}{x_e}\right) \mathrm{km/s}$$

$$v_{M+} \approx \frac{f_{M+} cB}{4\pi L e n_{M+}} = 42\, f_{M+} \left(\frac{B}{6\mathrm{G}}\right) \left(\frac{0.03\mathrm{AU}}{L}\right) \left(\frac{10^{15}\,\mathrm{cm}^{-3}}{n_\mathrm{H}}\right) \left(\frac{10^{-13}\,\mathrm{cm}^{-3}}{x_{M+}}\right) \mathrm{km/s}$$

The electron bulk velocity is surprizingly large !

Inutsuka & Sano (2005) ApJL **628**, L155

Microphysics (2)

electron distribution function $\boldsymbol{f}(\boldsymbol{p})$ in weakly ionized plasma

very small change of **ε** at each collision with H_2 because $m_e/M < 3670^{-1} \ll 1 \Rightarrow$ Fokker-Planck approx. for ε or *p*.

$$\frac{\partial f}{\partial t} - e\boldsymbol{E}\frac{\partial f}{\partial \boldsymbol{p}} = \frac{1}{p^2}\frac{\partial f}{\partial p} p^2 B\left(\frac{v}{T} f + \frac{\partial f}{\partial p}\right) + Nv\int [f(t,p,\theta') - f(t,p,\theta)]d\sigma$$

Let $\boldsymbol{E} = Ee_z, \quad f = f_0(p) + f_1(p)\cos(\theta),$ Druyvesteyn 1930

$$f_0(p) = A\exp\left(-\frac{3\varepsilon^2}{T^2\gamma^2}\right), \quad f_1(p) = 6\sqrt{\frac{m_e}{M}}\frac{\varepsilon}{T\gamma} f_0, \quad \gamma \equiv \frac{eEl}{T}\sqrt{\frac{M}{m_e}}.$$

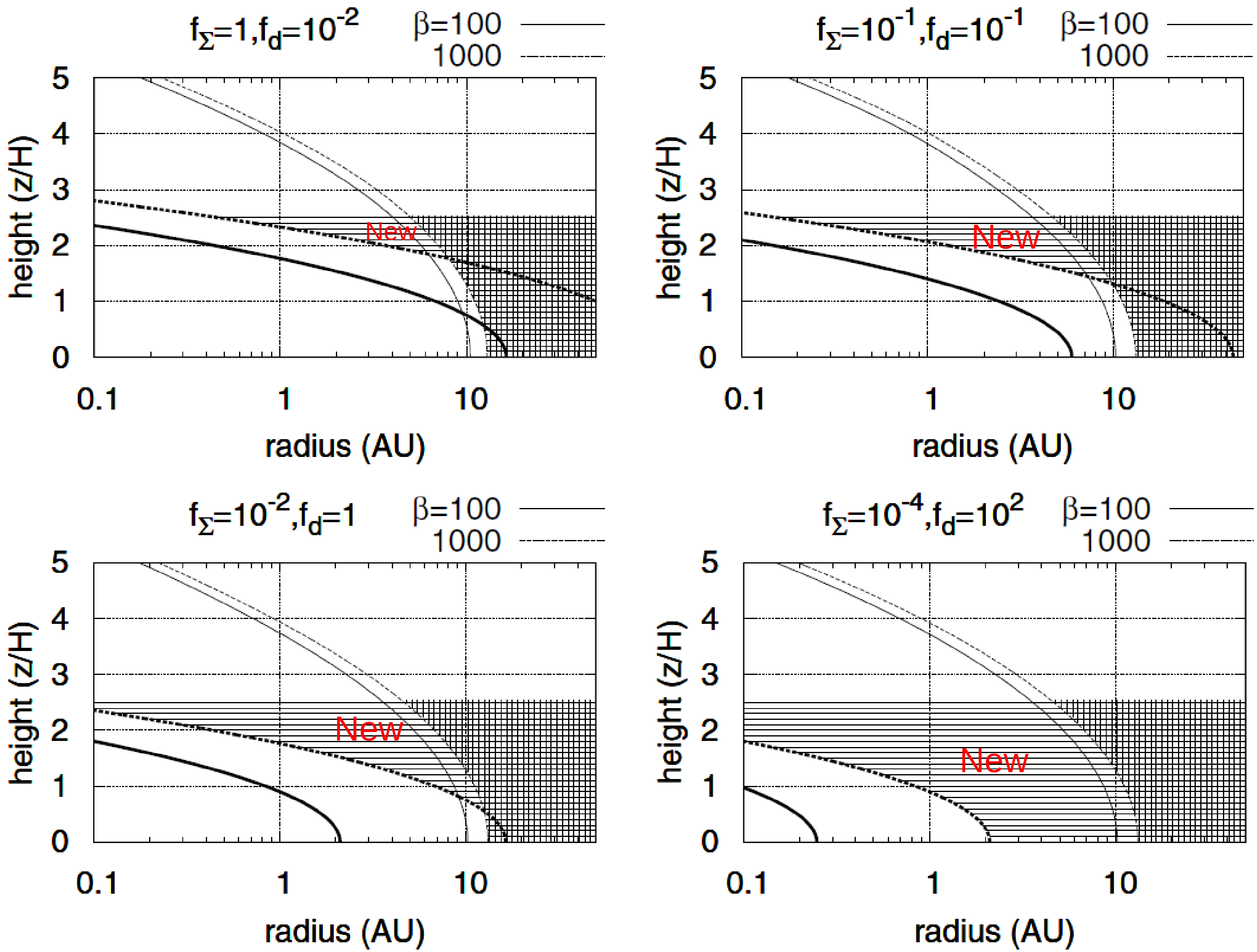
$$\bar{\varepsilon} = 0.43\, eEl\sqrt{\frac{M}{m_e}}, \quad \bar{v}_z = 0.897\sqrt{\frac{eEl}{m_e}}\left(\frac{m_e}{M}\right)^{\frac{1}{4}} \Rightarrow \bar{\varepsilon} = 0.534\, M\bar{v}_z^{\,2}$$

Electrons have large velocity dispersion: $\varepsilon \approx 19$ eV

Inutsuka & Sano (2005) ApJL **628**

This is expected to provide sufficient ionization.

Simulation of MRI with Discharge



Muranushi, Okuzumi & SI (2012) ApJ **760**, 56

Wait for Talk by Okuzumi!

Turbulent Mixing of Ionization

In region with no dust grains, recombination happens in gas phase: $dx_e/dt = \zeta - \Sigma_j \beta_j x_e n_j = \zeta - \beta' n x_e^2$

where $\beta' = \Sigma_j \beta_j (x_j/x_e)$

Time evolution of ionization degree can be solved as

$$x_e(\boldsymbol{t})^{-1} = x_{e,0}^{-1} + 10^{11} (\beta'/3\cdot10^{-12}\mathrm{s}^{-1}) (n_H/10^{15}\mathrm{cm}^{-3})(\boldsymbol{t}/1\mathrm{yr}).$$

i.e., within an eddy turn-over time (~1yr @1AU), the ionization degree keeps the level of $x_e > 10^{-13}$.

Thus, the ionized region penetrates into the neutral region by the turbulent motions and homogenize the ionization.

⇒ the most of the region in the protoplanetary disk will be sufficiently ionized.

SI & Sano (2005) ApJL **628**, L155

See also Ilgner & Nelson (2006), Turner+

Summary

Results of 3D Resistive MHD Calculation

When Magnetic Reynolds Number (Re_m) > 1

Exponential Growth from very small *B*

- Growth Rate = $(4/3)\Omega$… independent on *B* Field Strength

cf. Kinematic Dynamo

- $\lambda_{\text{maximum growth}}$ becomes larger as *B* becomes greater.

➔ Inverse Cascade of Energy

Saturated States…≠ Energy Equipartition

Classified by Re_m

- $Re_m < 1$…quasi-steady saturation similar to 2D results
- $Re_m > 1$… recurrence of Channel Flow & Reconnection

Fluctuation-Dissipation Relation

$$\langle\langle \text{Energy Dissipation Rate} \rangle\rangle \propto \langle\langle \rho v_x \delta v_y - B_x B_y / 4\pi \rangle\rangle$$

$$\propto \text{Mass Accretion Rate}$$

To Do List

Saturation Level α with Net Vertical Flux

- Vertical & Radial Stratification ➔ Global Simulation

Disk Winds

- Driven even in Local Simulation ➔ Suzuki,…
- Magneto-Centrifugally Driven only in Global Simulation

Non-Ideal MHD Effects in Disks

- Non-Linear Ohms' Law & Impact Ionization ➔ Okuzumi
- Hall Term ➔ Wardle,…
- Ambipolar Diffusion ➔ Gressel, Bai,…

<u>Magnetic Flux Loss from Disks!</u>

- Transport Mass inward, Angular Momentum Outward, Flux Outward! ➔ Who?