# Neutrino Theory and Phenomenology

## PART 2

**Carlo Giunti**

INFN, Sezione di Torino, and Dipartimento di Fisica Teorica, Università di Torino

carlo.giunti@to.infn.it

Neutrino Unbound: http://www.nu.to.infn.it

NBIA PhD School
Neutrinos underground & in the heavens

The Niels Bohr International Academy, Copenhagen, Denmark

23-27 June 2014

# Two-Component Theory of a Massless Neutrino

[L. Landau, Nucl. Phys. 3 (1957) 127], [T.D. Lee, C.N. Yang, Phys. Rev. 105 (1957) 1671], [A. Salam, Nuovo Cim. 5 (1957) 299]

- Dirac Equation: $(i\gamma^{\mu}\partial_{\mu} - m)\,\psi = 0$
- Chiral decomposition of a Fermion Field: $\psi = \psi_L + \psi_R$
- Equations for the Chiral components are coupled by mass:

$$i\gamma^{\mu}\partial_{\mu}\psi_L = m\,\psi_R$$
$$i\gamma^{\mu}\partial_{\mu}\psi_R = m\,\psi_L$$

- They are decoupled for a massless fermion: Weyl Equations (1929)

$$i\gamma^{\mu}\partial_{\mu}\psi_L = 0$$
$$i\gamma^{\mu}\partial_{\mu}\psi_R = 0$$

- A massless fermion can be described by a single chiral field $\psi_L$ or $\psi_R$ (Weyl Spinor).

- $\psi_L$ and $\psi_R$ have only two independent components: in the chiral representation

$$\psi_L = \begin{pmatrix} 0 \\ \chi_L \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 0 \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix} \qquad \psi_R = \begin{pmatrix} \chi_R \\ 0 \end{pmatrix} \equiv \begin{pmatrix} \chi_{R1} \\ \chi_{R2} \\ 0 \\ 0 \end{pmatrix}$$

- The possibility to describe a physical particle with a Weyl spinor was rejected by Pauli in 1933 because it leads to parity violation ($\psi_L \overset{P}{\rightleftharpoons} \psi_R$)

- The discovery of parity violation in 1956-57 invalidated Pauli's reasoning, opening the possibility to describe massless particles with Weyl spinor fields $\Longrightarrow$ Two-component Theory of a Massless Neutrino (1957)

- $V - A$ Charged-Current Weak Interactions $\Longrightarrow \nu_L$

- In the 1960s, the Two-component Theory of a Massless Neutrino was incorporated in the SM through the assumption of the absence of $\nu_R$

# Majorana Equation

- Can a two-component spinor describe a massive fermion?

  Yes! (E. Majorana, 1937)

- Trick: $\psi_R$ and $\psi_L$ are not independent: $\boxed{\psi_R = \psi_L^c = \mathcal{C}\,\overline{\psi_L}^T}$

  charge-conjugation matrix: $\mathcal{C}\,\gamma_\mu^T\,\mathcal{C}^{-1} = -\gamma_\mu$

- $\psi_L^c$ is right-handed: $P_R \psi_L^c = \psi_L^c \qquad P_L \psi_L^c = 0$

- $i\gamma^\mu \partial_\mu \psi_L = m\,\psi_R \quad \to \quad \boxed{i\gamma^\mu \partial_\mu \psi_L = m\,\psi_L^c}$ Majorana Equation

- Majorana Field: $\psi = \psi_L + \psi_R = \psi_L + \psi_L^c$

  $\boxed{\psi = \psi^c}$ Majorana Condition

- $\psi = \psi^c$ implies the equality of particle and antiparticle
- Only neutral fermions can be Majorana particles
- For a Majorana field, the electromagnetic current vanishes identically:

$$\overline{\psi}\gamma^\mu\psi = \overline{\psi^c}\gamma^\mu\psi^c = -\psi^T\mathcal{C}^\dagger\gamma^\mu\mathcal{C}\overline{\psi}^T = \overline{\psi}\mathcal{C}\gamma^{\mu T}\mathcal{C}^\dagger\psi = -\overline{\psi}\gamma^\mu\psi = 0$$

- Only two independent components: $\psi = \begin{pmatrix} i\sigma^2\chi_L^* \\ \chi_L \end{pmatrix} = \begin{pmatrix} \chi_{L2}^* \\ -\chi_{L1}^* \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix}$

# Majorana Lagrangian

Dirac Lagrangian

$$\begin{aligned} \mathscr{L}^{\mathrm{D}} &= \overline{\nu}\,(i\partial\!\!\!/ - m)\,\nu \\ &= \overline{\nu_L} i\partial\!\!\!/ \nu_L + \overline{\nu_R} i\partial\!\!\!/ \nu_R - m\,(\overline{\nu_R}\,\nu_L + \overline{\nu_L}\,\nu_R) \end{aligned}$$

$$\nu_R \quad \to \quad \nu_L^c = \mathcal{C}\,\overline{\nu_L}^T$$

$$\frac{1}{2}\,\mathscr{L}^{\mathrm{D}} \quad \to \quad \overline{\nu_L}\,i\partial\!\!\!/\,\nu_L - \frac{m}{2}\left(-\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \overline{\nu_L}\,\mathcal{C}\,\overline{\nu_L}^T\right)$$

Majorana Lagrangian

$$\begin{aligned} \mathscr{L}^{\mathrm{M}} &= \overline{\nu_L}\,i\partial\!\!\!/\,\nu_L - \frac{m}{2}\left(-\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \overline{\nu_L}\,\mathcal{C}\,\overline{\nu_L}^T\right) \\ &= \overline{\nu_L}\,i\partial\!\!\!/\,\nu_L - \frac{m}{2}\left(\overline{\nu_L^c}\,\nu_L + \overline{\nu_L}\,\nu_L^c\right) \end{aligned}$$

# Lepton Number

$$\cancel{L = +1} \quad \longleftarrow \quad \boxed{\nu = \nu^c} \quad \longrightarrow \quad \cancel{L = -1}$$

$$\nu_L \quad \Longrightarrow \quad L = +1 \qquad\qquad \nu_L^c \quad \Longrightarrow \quad L = -1$$

$$\mathscr{L}^{\text{M}} = -\frac{m}{2}\left(\overline{\nu_L^c}\,\nu_L + \overline{\nu_L}\,\nu_L^c\right)$$

Total Lepton Number is not conserved: $\boxed{\Delta L = \pm 2}$

Best process to find violation of Total Lepton Number:

Neutrinoless Double-$\beta$ Decay

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + 2e^- + \cancel{2\bar{\nu}_e} \qquad (\beta\beta^-_{0\nu})$$
$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z-2) + 2e^+ + \cancel{2\nu_e} \qquad (\beta\beta^+_{0\nu})$$

# No Majorana Neutrino Mass in the SM

- Majorana Mass Term $\propto \left[\nu_L^T\,\mathcal{C}^\dagger\,\nu_L - \overline{\nu_L}\,\mathcal{C}\,\overline{\nu_L}^T\right]$ involves only the neutrino left-handed chiral field $\nu_L$, which is present in the SM
- Eigenvalues of the weak isospin $I$, of its third component $I_3$, of the hypercharge $Y$ and of the charge $Q$ of the lepton and Higgs multiplets:

| | $I$ | $I_3$ | $Y$ | $Q = I_3 + \frac{Y}{2}$ |
|---|---|---|---|---|
| lepton doublet $L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$ | $1/2$ | $1/2$<br>$-1/2$ | $-1$ | $0$<br>$-1$ |
| lepton singlet $\ell_R$ | $0$ | $0$ | $-2$ | $-1$ |
| Higgs doublet $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$ | $1/2$ | $1/2$<br>$-1/2$ | $+1$ | $1$<br>$0$ |

- $\nu_L^T\,\mathcal{C}^\dagger\,\nu_L$ has $I_3 = 1$ and $Y = -2 \Longrightarrow$ needed $Y = 2$ Higgs triplet ($I = 1$, $I_3 = -1$)
- Compare with Dirac Mass Term $\propto \overline{\nu_R}\nu_L$ with $I_3 = 1/2$ and $Y = -1$ balanced by $\phi_0 \to v$ with $I_3 = -1/2$ and $Y = +1$

# Confusing Majorana Antineutrino Terminology

- A Majorana neutrino is the same as a Majorana antineutrino

- Neutrino interactions are described by the CC and NC Lagrangians

$$\mathscr{L}_{\mathrm{I,L}}^{\mathrm{CC}} = -\frac{g}{\sqrt{2}} \left( \overline{\nu_L} \, \gamma^{\mu} \, \ell_L \, W_{\mu} + \overline{\ell_L} \, \gamma^{\mu} \, \nu_L \, W_{\mu}^{\dagger} \right)$$

$$\mathscr{L}_{\mathrm{I},\nu}^{\mathrm{NC}} = -\frac{g}{2 \cos \vartheta_{\mathrm{W}}} \, \overline{\nu_L} \, \gamma^{\mu} \, \nu_L \, Z_{\mu}$$

- Dirac: $\nu_L$ { destroys left-handed neutrinos; creates right-handed antineutrinos }

- Majorana: $\nu_L$ { destroys left-handed neutrinos; creates right-handed neutrinos }

- Common implicit definitions:

  left-handed Majorana neutrino $\equiv$ neutrino
  right-handed Majorana neutrino $\equiv$ antineutrino

# Mixing of Three Majorana Neutrinos

- $\boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix}$

$$\mathscr{L}^{\text{M}} = \frac{1}{2} \boldsymbol{\nu}'^{T}_{L} \, \mathcal{C}^{\dagger} \, M^{L} \, \boldsymbol{\nu}'_{L} + \text{H.c.}$$
$$= \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu'^{T}_{\alpha L} \, \mathcal{C}^{\dagger} \, M^{L}_{\alpha\beta} \, \nu'_{\beta L} + \text{H.c.}$$

- In general, the matrix $M^L$ is a complex symmetric matrix

$$\sum_{\alpha,\beta} \nu'^{T}_{\alpha L} \, \mathcal{C}^{\dagger} \, M^{L}_{\alpha\beta} \, \nu'_{\beta L} = \sum_{\alpha,\beta} \left( \nu'^{T}_{\alpha L} \, \mathcal{C}^{\dagger} \, M^{L}_{\alpha\beta} \, \nu'_{\beta L} \right)^{T}$$
$$= -\sum_{\alpha,\beta} \nu'^{T}_{\beta L} \, M^{L}_{\alpha\beta} \, (\mathcal{C}^{\dagger})^{T} \, \nu'_{\alpha L} = \sum_{\alpha,\beta} \nu'^{T}_{\beta L} \, \mathcal{C}^{\dagger} \, M^{L}_{\alpha\beta} \, \nu'_{\alpha L}$$
$$= \sum_{\alpha,\beta} \nu'^{T}_{\alpha L} \, \mathcal{C}^{\dagger} \, M^{L}_{\beta\alpha} \, \nu'_{\beta L}$$

$$M^{L}_{\alpha\beta} = M^{L}_{\beta\alpha} \quad \Longleftrightarrow \quad M^{L} = {M^{L}}^{T}$$

# Diagonalization of Majorana Mass Matrix

- $\mathscr{L}^{\text{M}} = \frac{1}{2} \boldsymbol{\nu}_L'^T \, \mathcal{C}^\dagger \, M^L \, \boldsymbol{\nu}_L' + \text{H.c.}$

- $\boldsymbol{\nu}_L' = V_L^\nu \, \mathbf{n}_L \qquad \Longrightarrow \qquad \mathscr{L}^{\text{M}} = \frac{1}{2} \boldsymbol{\nu}_L'^T \, (V_L^\nu)^T \, \mathcal{C}^\dagger \, M^L \, V_L^\nu \, \boldsymbol{\nu}_L' + \text{H.c.}$

- $(V_L^\nu)^T \, M^L \, V_L^\nu = M \,, \qquad M_{kj} = m_k \, \delta_{kj} \qquad (k, j = 1, 2, 3)$

- Neutrino fields with definite mass: $\mathbf{n}_L = V_L^{\nu\dagger} \, \boldsymbol{\nu}_L' = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$

$$\mathscr{L}^{\text{M}} = \frac{1}{2} \sum_{k=1}^{3} m_k \left( \nu_{kL}^T \, \mathcal{C}^\dagger \, \nu_{kL} - \overline{\nu_{kL}} \, \mathcal{C} \, \nu_{kL}^T \right)$$

# Mixing Matrix

- Leptonic Weak Charged Current:

$$j_{W,\mathrm{L}}^{\rho\dagger} = 2\,\overline{\boldsymbol{\ell}_L}\,\gamma^\rho\,U\,\mathbf{n}_L \qquad \text{with} \qquad U = V_L^{\ell\dagger}\,V_L^{\nu}$$

- As in the Dirac case, we define the left-handed flavor neutrino fields as

$$\boldsymbol{\nu}_L = U\,\mathbf{n}_L = V_L^{\ell\dagger}\,\boldsymbol{\nu}'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- In this way, as in the Dirac case, the Leptonic Weak Charged Current has the SM form

$$j_{W,\mathrm{L}}^{\rho\dagger} = 2\,\overline{\boldsymbol{\ell}_L}\,\gamma^\rho\,\boldsymbol{\nu}_L = 2\sum_{\alpha=e,\mu,\tau}\overline{\ell_{\alpha L}}\,\gamma^\rho\,\nu_{\alpha L}$$

- Important difference with respect to Dirac case:
  Two additional CP-violating phases: Majorana phases

- Majorana Mass Term $\mathscr{L}^{\mathrm{M}} = \frac{1}{2} \sum_{k=1}^{3} m_k \, \nu_{kL}^T \, \mathcal{C}^\dagger \, \nu_{kL} + \text{H.c.}$ is not invariant under the global U(1) gauge transformations

$$\nu_{kL} \to e^{i\varphi_k} \, \nu_{kL} \quad (k = 1, 2, 3)$$

- For eliminating some of the 6 phases of the unitary mixing matrix we can use only the global phase transformations (3 arbitrary phases)

$$\ell_\alpha \to e^{i\varphi_\alpha} \, \ell_\alpha \quad (\alpha = e, \mu, \tau)$$

- Weak Charged Current: $j^{\rho\dagger}_{W,\mathrm{L}} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, \gamma^{\rho} \, U_{\alpha k} \, \nu_{kL}$

- Performing the transformation $\ell_{\alpha} \to e^{i\varphi_{\alpha}} \, \ell_{\alpha}$ we obtain

$$j^{\rho\dagger}_{W,\mathrm{L}} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, e^{-i\varphi_{\alpha}} \, \gamma^{\rho} \, U_{\alpha k} \, \nu_{kL}$$

$$j^{\rho\dagger}_{W,\mathrm{L}} = 2 \, \underbrace{e^{-i\varphi_{e}}}_{1} \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, \underbrace{e^{-i(\varphi_{\alpha}-\varphi_{e})}}_{2} \, \gamma^{\rho} \, U_{\alpha k} \, \nu_{kL}$$

- We can eliminate 3 phases of the mixing matrix: one overall phase and two phases which can be factorized on the left.

- In the Dirac case we could eliminate also two phases which can be factorized on the right.

- In the Majorana case there are two additional physical Majorana phases which can be factorized on the right of the mixing matrixd:

$$\boxed{U = U^{\mathrm{D}}\, D^{\mathrm{M}}} \qquad D^{\mathrm{M}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- $U^{\mathrm{D}}$ is a Dirac mixing matrix, with one Dirac phase

- Standard parameterization:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

# One-Generation Dirac-Majorana Mass Term

If $\nu_R$ exists, the most general mass term is the

Dirac-Majorana Mass Term

$$\mathscr{L}^{\text{D+M}} = \mathscr{L}^{\text{D}} + \cancel{\mathscr{L}^{L}} + \mathscr{L}^{R}$$

$$\mathscr{L}^{\text{D}} = -m_{\text{D}}\,\overline{\nu_R}\,\nu_L + \text{H.c.}$$ Dirac Mass Term

$$\mathscr{L}^{L} = \frac{1}{2}\,m_L\,\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \text{H.c.}$$

$\nu_L$ Majorana Mass Term forbidden by SM Symmetries

$$\mathscr{L}^{R} = \frac{1}{2}\,m_R\,\nu_R^T\,\mathcal{C}^\dagger\,\nu_R + \text{H.c.}$$

New $\nu_R$ Majorana Mass Term allowed by SM Symmetries!

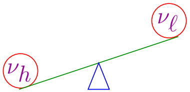
# Seesaw Mechanism

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2} \left( \overline{\nu_L^c} \quad \overline{\nu_R} \right) \begin{pmatrix} 0 & m_{\mathrm{D}} \\ m_{\mathrm{D}} & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \mathrm{H.c.}$$

$m_R$ can be arbitrarily large (not protected by SM symmetries)

$m_R \sim$ scale of new physics beyond Standard Model $\Rightarrow$ $m_R \gg m_{\mathrm{D}}$

diagonalization of $\begin{pmatrix} 0 & m_{\mathrm{D}} \\ m_{\mathrm{D}} & m_R \end{pmatrix} \Longrightarrow \quad m_\ell \simeq \dfrac{m_{\mathrm{D}}^2}{m_R} \qquad m_h \simeq m_R$



seesaw mechanism

natural explanation of smallness
of light neutrino masses

massive neutrinos are Majorana!

$\nu_\ell \simeq -i \left( \nu_L - \nu_L^c \right) \qquad \nu_h \simeq \nu_R + \nu_R^c$

3-GEN $\Rightarrow$ effective low-energy 3-$\nu$ mixing

[Minkowski, PLB 67 (1977) 42]

[Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

# Effective Majorana Mass from New Physics BSM

- Dimensional analysis: Fermion Field $\sim [E]^{3/2}$ Boson Field $\sim [E]$
- Dimensionless action: $I = \int \mathrm{d}^4x \, \mathscr{L}(x) \Longrightarrow \mathscr{L}(x) \sim [E]^4$
- Kinetic terms: $\overline{\psi} i \partial\!\!\!/ \psi \sim [E]^4$, $(\partial_\mu \phi)^\dagger \partial^\mu \phi \sim [E]^4$
- Mass terms: $m \overline{\psi} \psi \sim [E]^4$, $m^2 \phi^\dagger \phi \sim [E]^4$
- CC weak interaction: $g \, \overline{\boldsymbol{\nu}_L} \gamma^\rho \boldsymbol{\ell}_L W_\rho \sim [E]^4$
- Yukawa couplings: $y \, \overline{L_L} \Phi \ell_R \sim [E]^4$
- Product of fields $\mathscr{O}_d$ with energy dimension $d \equiv$ dim-$d$ operator
- $\mathscr{L}_{(\mathscr{O}_d)} = C_{(\mathscr{O}_d)} \mathscr{O}_d \quad \Longrightarrow \quad C_{(\mathscr{O}_d)} \sim [E]^{4-d}$
- $\mathscr{O}_{d>4}$ are not renormalizable

- SM Lagrangian includes all $\mathscr{O}_{d \leq 4}$ invariant under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
- SM cannot be considered as the final theory of everything
- SM is an effective low-energy theory
- It is likely that SM is the low-energy product of the symmetry breaking of a high-energy unified theory
- It is plausible that at low-energy there are effective non-renormalizable $\mathscr{O}_{d>4}$ [S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566]
- All $\mathscr{O}_d$ must respect $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, because they are generated by the high-energy theory which must include the gauge symmetries of the SM in order to be effectively reduced to the SM at low energies

- $\mathscr{O}_{d>4}$ is suppressed by a coefficient $\mathcal{M}^{4-d}$, where $\mathcal{M}$ is a heavy mass characteristic of the symmetry breaking scale of the high-energy unified theory:

$$\mathscr{L} = \mathscr{L}_{\text{SM}} + \frac{g_5}{\mathcal{M}}\,\mathscr{O}_5 + \frac{g_6}{\mathcal{M}^2}\,\mathscr{O}_6 + \ldots$$

- Analogy with $\quad \mathscr{L}_{\text{eff}}^{(\text{CC})} \propto G_{\text{F}}\,(\overline{\nu_{eL}}\gamma^{\rho}e_L)\,(\overline{e_L}\gamma_{\rho}\nu_{eL}) + \ldots$

$$\mathscr{O}_6 \to (\overline{\nu_{eL}}\gamma^{\rho}e_L)\,(\overline{e_L}\gamma_{\rho}\nu_{eL}) + \ldots \qquad \frac{g_6}{\mathcal{M}^2} \to \frac{G_{\text{F}}}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

- $\mathcal{M}^{4-d}$ is a strong suppression factor which limits the observability of the low-energy effects of the new physics beyond the SM

- The difficulty to observe the effects of the effective low-energy non-renormalizable operators increase rapidly with their dimensionality

- $\mathscr{O}_5 \quad \Longrightarrow \quad$ Majorana neutrino masses (Lepton number violation)

- $\mathscr{O}_6 \quad \Longrightarrow \quad$ Baryon number violation (proton decay)

- Only one dim-5 operator:

$$\mathscr{O}_5 = (L_L^T \, \sigma_2 \, \Phi) \, \mathcal{C}^\dagger \, (\Phi^T \, \sigma_2 \, L_L) + \text{H.c.}$$
$$= \frac{1}{2} \, (L_L^T \, \mathcal{C}^\dagger \, \sigma_2 \, \vec{\sigma} \, L_L) \cdot (\Phi^T \, \sigma_2 \, \vec{\sigma} \, \Phi) + \text{H.c.}$$

$$\mathscr{L}_5 = \frac{g_5}{2\mathcal{M}} \, (L_L^T \, \mathcal{C}^\dagger \, \sigma_2 \, \vec{\sigma} \, L_L) \cdot (\Phi^T \, \sigma_2 \, \vec{\sigma} \, \Phi) + \text{H.c.}$$

- Electroweak Symmetry Breaking: $\Phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \xrightarrow[\text{Breaking}]{\text{Symmetry}} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

- $\mathscr{L}_5 \xrightarrow[\text{Breaking}]{\text{Symmetry}} \mathscr{L}_{\text{mass}}^{\text{M}} = \frac{1}{2} \, \frac{g_5 \, v^2}{\mathcal{M}} \, \nu_L^T \, \mathcal{C}^\dagger \, \nu_L + \text{H.c.} \qquad \Longrightarrow \qquad \boxed{m = \frac{g_5 \, v^2}{\mathcal{M}}}$

- The study of Majorana neutrino masses provides the most accessible low-energy window on new physics beyond the SM

- $m \propto \frac{v^2}{\mathcal{M}} \propto \frac{m_D^2}{\mathcal{M}}$ natural explanation of smallness of neutrino masses

  (special case: Seesaw Mechanism)

- Example: $m_D \sim v \sim 10^2\,\text{GeV}$ and $\mathcal{M} \sim 10^{15}\,\text{GeV} \implies m \sim 10^{-2}\,\text{eV}$

# Seesaw Mechanism from Effective Lagrangian

- Dirac–Majorana neutrino mass term with $m_L = 0$:

$$\mathscr{L}^{\text{D+M}} = -m_{\text{D}} \left( \overline{\nu_R} \, \nu_L + \overline{\nu_L} \, \nu_R \right) + \frac{1}{2} \, m_R \left( \nu_R^T \, \mathcal{C}^\dagger \, \nu_R + \nu_R^\dagger \, \mathcal{C} \, \nu_R^* \right)$$

- Above the electroweak symmetry-breaking scale:

$$\mathscr{L}^{\text{D+M}} = -y^\nu \left( \overline{\nu_R} \, \widetilde{\Phi}^\dagger \, L_L + \overline{L_L} \, \widetilde{\Phi} \, \nu_R \right) + \frac{1}{2} \, m_R \left( \nu_R^T \, \mathcal{C}^\dagger \, \nu_R + \nu_R^\dagger \, \mathcal{C} \, \nu_R^* \right)$$

- If $m_R \gg v \Longrightarrow \nu_R$ is static $\Longrightarrow$ kinetic term in equation of motion can be neglected:

$$0 \simeq \frac{\partial \mathscr{L}^{\text{D+M}}}{\partial \nu_R} = m_R \, \nu_R^T \, \mathcal{C}^\dagger - y^\nu \, \overline{L_L} \, \widetilde{\Phi}$$

$$\nu_R \simeq -\frac{y^\nu}{m_R} \, \widetilde{\Phi}^T \, \mathcal{C} \, \overline{L_L}^T$$

$$\mathscr{L}^{\text{D+M}} \to \mathscr{L}_5^{\text{D+M}} \simeq -\frac{1}{2} \, \frac{(y^\nu)^2}{m_R} (L_L^T \, \sigma_2 \, \Phi) \, \mathcal{C}^\dagger \, (\Phi^T \, \sigma_2 \, L_L) + \text{H.c.}$$

$$\mathscr{L}_5 = \frac{g}{\mathcal{M}} \left( L_L^T \, \sigma_2 \, \Phi \right) \mathcal{C}^\dagger \left( \Phi^T \, \sigma_2 \, L_L \right) + \text{H.c.}$$

$$\mathscr{L}_5^{\text{D+M}} \simeq -\frac{1}{2} \frac{(y^\nu)^2}{m_R} \left( L_L^T \, \sigma_2 \, \Phi \right) \mathcal{C}^\dagger \left( \Phi^T \, \sigma_2 \, L_L \right) + \text{H.c.}$$

$$g = -\frac{(y^\nu)^2}{2} \qquad\qquad \mathcal{M} = m_R$$

- Seesaw mechanism is a particular case of the effective Lagrangian approach.
- Seesaw mechanism is obtained when dimension-five operator is generated only by the presence of $\nu_R$ with $m_R \sim \mathcal{M}$.
- In general, other terms can contribute to $\mathscr{L}_5$.

# Generalized Seesaw

- General effective Dirac-Majorana mass matrix:

$$M = \begin{pmatrix} m_L & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix}$$

- $m_L$ generated by dim-5 operator:

$$m_L \ll m_{\text{D}} \ll m_R$$

- Eigenvalues:

$$\begin{vmatrix} m_L - \mu & m_{\text{D}} \\ m_{\text{D}} & m_R - \mu \end{vmatrix} = 0$$

$$\mu^2 - (\cancel{m_L} + m_R)\,\mu + m_L m_R - m_{\text{D}}^2 = 0$$

$$\mu = \frac{1}{2}\left[m_R \pm \sqrt{m_R^2 - 4\left(m_L m_R - m_{\text{D}}^2\right)}\right]$$

$$\mu = \frac{1}{2}\left[m_R \pm \sqrt{m_R^2 - 4\left(m_L m_R - m_{\text{D}}^2\right)}\right]$$
$$= \frac{1}{2}\left[m_R \pm m_R\left(1 - 4\frac{m_L m_R - m_{\text{D}}^2}{m_R^2}\right)^{1/2}\right]$$
$$\simeq \frac{1}{2}\left[m_R \pm m_R\left(1 - 2\frac{m_L m_R - m_{\text{D}}^2}{m_R^2}\right)\right]$$

$$+ \quad \to \quad m_{\text{heavy}} \simeq m_R$$
$$- \quad \to \quad m_{\text{light}} \simeq m_L - \frac{m_{\text{D}}^2}{m_R}$$

Type I seesaw: $\quad m_L \ll \frac{m_{\text{D}}^2}{m_R} \quad \Longrightarrow \quad m_{\text{light}} \simeq \frac{m_{\text{D}}^2}{m_R}$

Type II seesaw: $\quad m_L \gg \frac{m_{\text{D}}^2}{m_R} \quad \Longrightarrow \quad m_{\text{light}} \simeq m_L$

# Right-Handed Neutrino Mass Term

Majorana mass term for $\nu_R$ respects the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ Standard Model Symmetry!

$$\mathcal{L}_R^{\mathrm{M}} = -\frac{1}{2}\, m \left( \overline{\nu_R^c}\, \nu_R + \overline{\nu_R}\, \nu_R^c \right)$$

Majorana mass term for $\nu_R$ breaks Lepton number conservation!

Three possibilities:

- Lepton number can be explicitly broken
- Lepton number is spontaneously broken locally, with a massive vector boson coupled to the lepton number current
- Lepton number is spontaneously broken globally and a massless Goldstone boson appears in the theory (Majoron)

# Singlet Majoron Model

[Chikashige, Mohapatra, Peccei, Phys. Lett. B98 (1981) 265, Phys. Rev. Lett. 45 (1980) 1926]

$$\mathcal{L}_\Phi = -y_d \left( \overline{L_L}\, \Phi\, \nu_R + \overline{\nu_R}\, \Phi^\dagger\, L_L \right) \xrightarrow[\langle\Phi\rangle\neq 0]{} -m_{\text{D}} \left( \overline{\nu_L}\, \nu_R + \overline{\nu_R}\, \nu_L \right)$$

$$\mathcal{L}_\eta = -y_s \left( \eta\, \overline{\nu_R^c}\, \nu_R + \eta^\dagger\, \overline{\nu_R}\, \nu_R^c \right) \xrightarrow[\langle\eta\rangle\neq 0]{} -\frac{1}{2}\, m_R \left( \overline{\nu_R^c}\, \nu_R + \overline{\nu_R}\, \nu_R^c \right)$$

$$\eta = 2^{-1/2} \left( \langle\eta\rangle + \rho + i\,\chi \right) \qquad \mathcal{L}_{\text{mass}} = -\frac{1}{2} \begin{pmatrix} \overline{\nu_L^c} & \overline{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \text{H.c.}$$

$$\underset{\text{scale of } L \text{ violation}}{m_R} \gg \underset{\text{EW scale}}{m_{\text{D}}} \Longrightarrow \text{Seesaw:} \quad \boxed{m_\ell \simeq \frac{m_{\text{D}}^2}{m_R}}$$

$\rho$ = massive scalar, $\chi$ = Majoron (massless pseudoscalar Goldstone boson)

The Majoron is weakly coupled to the light neutrino

$$\mathcal{L}_{\chi-\nu} = \frac{i y_s}{\sqrt{2}}\, \chi \left[ \overline{\nu_h}\gamma^5\nu_h - \frac{m_{\text{D}}}{m_R} \left( \overline{\nu_h}\gamma^5\nu_\ell + \overline{\nu_\ell}\gamma^5\nu_h \right) + \left( \frac{m_{\text{D}}}{m_R} \right)^2 \overline{\nu_\ell}\gamma^5\nu_\ell \right]$$

# Three-Generation Mixing

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = \mathscr{L}_{\text{mass}}^{\text{D}} + \mathscr{L}_{\text{mass}}^{L} + \mathscr{L}_{\text{mass}}^{R}$$

$$\mathscr{L}_{\text{mass}}^{\text{D}} = - \sum_{s=1}^{N_S} \sum_{\alpha=e,\mu,\tau} \overline{\nu'_{sR}} \, M_{s\alpha}^{\text{D}} \, \nu'_{\alpha L} + \text{H.c.}$$

$$\mathscr{L}_{\text{mass}}^{L} = \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu'^{T}_{\alpha L} \, \mathcal{C}^{\dagger} \, M_{\alpha\beta}^{L} \, \nu'_{\beta L} + \text{H.c.}$$

$$\mathscr{L}_{\text{mass}}^{R} = \frac{1}{2} \sum_{s,s'=1}^{N_S} \nu'^{T}_{sR} \, \mathcal{C}^{\dagger} \, M_{ss'}^{R} \, \nu'_{s'R} + \text{H.c.}$$

$$\mathbf{N}'_L \equiv \begin{pmatrix} \boldsymbol{\nu}'_L \\ \boldsymbol{\nu}'^{C}_R \end{pmatrix} \qquad \boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \qquad \boldsymbol{\nu}'^{C}_R \equiv \begin{pmatrix} \nu'^{C}_{1R} \\ \vdots \\ \nu'^{C}_{N_S R} \end{pmatrix}$$

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = \frac{1}{2} \, \mathbf{N}'^{T}_L \, \mathcal{C}^{\dagger} \, M^{\text{D+M}} \, \mathbf{N}'_L + \text{H.c.} \qquad M^{\text{D+M}} = \begin{pmatrix} M^L & M^{\text{D}\,T} \\ M^{\text{D}} & M^R \end{pmatrix}$$

- Diagonalization of the Dirac-Majorana Mass Term $\Longrightarrow$ massive Majorana neutrinos

- Seesaw Mechanism $\Longrightarrow$ right-handed neutrinos have large Majorana masses and are decoupled from the low-energy phenomenology.

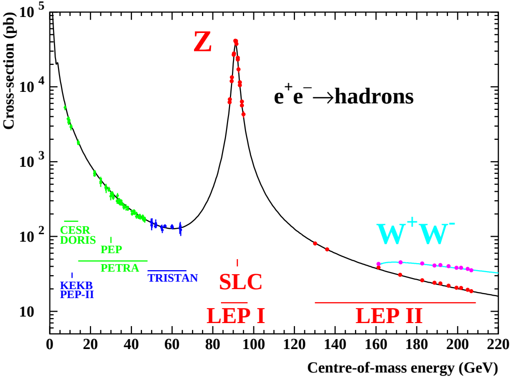
- If all right-handed neutrinos have large Majorana masses, at low energy we have an effective mixing of three Majorana neutrinos.

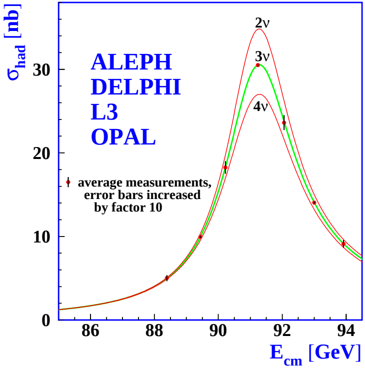
- It is possible that not all right-handed neutrinos have large Majorana masses: some right-handed neutrinos may correspond to low-energy Majorana particles which belong to new physics beyond the Standard Model.

- Light anti-$\nu_R$ are called sterile neutrinos

$$\nu_R^c \to \nu_{sL} \qquad \text{(left-handed)}$$

# Number of Flavor and Massive Neutrinos?




[LEP, Phys. Rept. 427 (2006) 257, arXiv:hep-ex/0509008]

$$\Gamma_Z = \sum_{\ell=e,\mu,\tau} \Gamma_{Z\to\ell\bar{\ell}} + \sum_{q\neq t} \Gamma_{Z\to q\bar{q}} + \Gamma_{\text{inv}} \qquad \Gamma_{\text{inv}} = N_\nu \, \Gamma_{Z\to\nu\bar{\nu}}$$

$$N_\nu = 2.9840 \pm 0.0082$$

$$e^+ e^- \to Z \xrightarrow{\text{invisible}} \sum_{a=\text{active}} \nu_a \bar{\nu}_a \quad \Longrightarrow \quad \nu_e \; \nu_\mu \; \nu_\tau$$

3 light active flavor neutrinos

mixing $\Rightarrow$ $$\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau$$

$N \geq 3$
no upper limit!

| | | | | | | |
|---|---|---|---|---|---|---|
| Mass Basis: | $\nu_1$ | $\nu_2$ | $\nu_3$ | $\nu_4$ | $\nu_5$ | $\cdots$ |
| Flavor Basis: | $\nu_e$ | $\nu_\mu$ | $\nu_\tau$ | $\nu_{s_1}$ | $\nu_{s_2}$ | $\cdots$ |
| | ACTIVE | | | STERILE | | |

$$\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau, s_1, s_2, \ldots$$

# Sterile Neutrinos

- Sterile means no standard model interactions
- Obviously no electromagnetic interactions as normal active neutrinos
- Thus sterile means no standard weak interactions
- But sterile neutrinos are not absolutely sterile:
  - Gravitational Interactions
  - New non-standard interactions of the physics beyond the Standard Model which generates the masses of sterile neutrinos
- Active neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ can oscillate into sterile neutrinos $(\nu_s)$
- Observables:
  - Disappearance of active neutrinos
  - Indirect evidence through combined fit of data
- Powerful window on new physics beyond the Standard Model