Neutrinos

in Astrophysics and Cosmology

Neutrinos and the Stars 2

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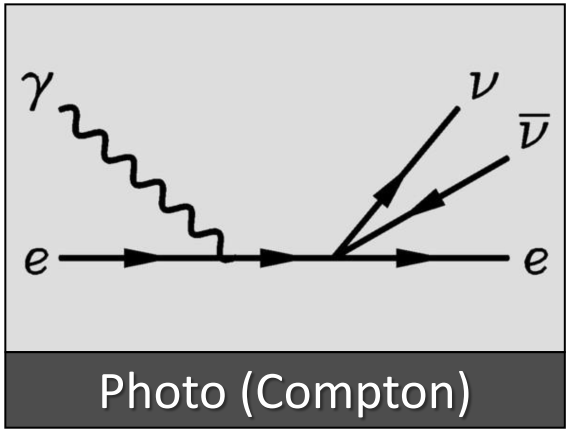
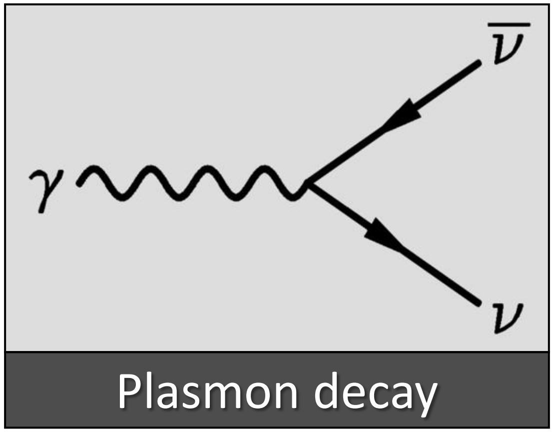
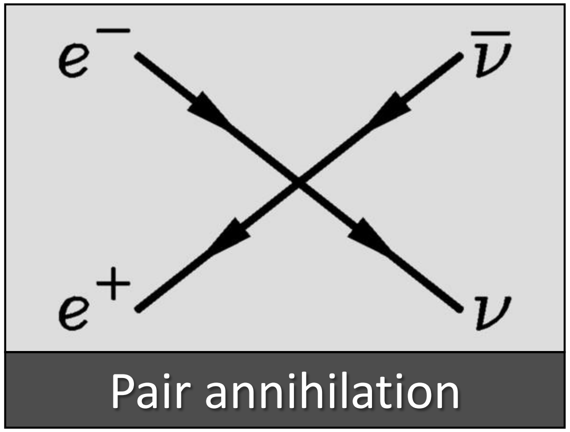
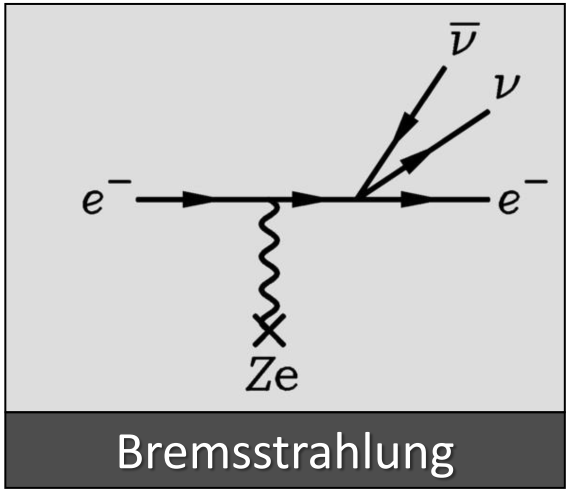
Neutrinos from Thermal Processes

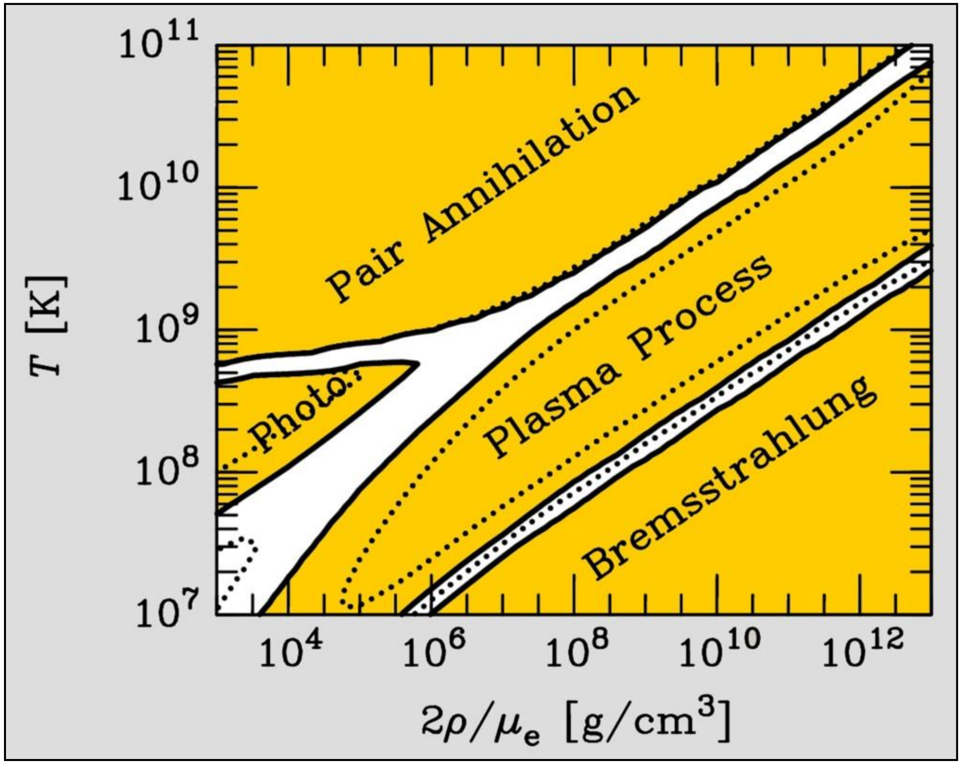






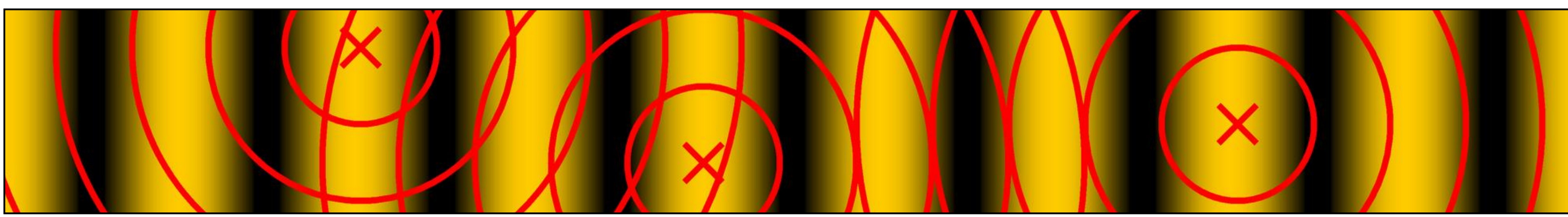


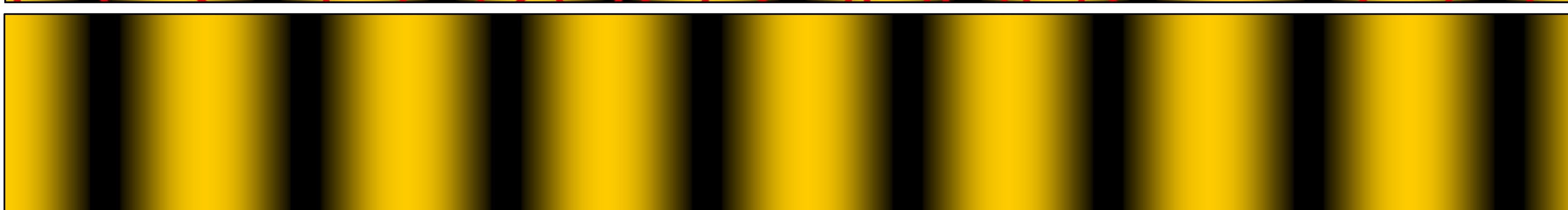
These processes were first discussed in 1961–63 after V–A theory



Refraction and Forward Scattering

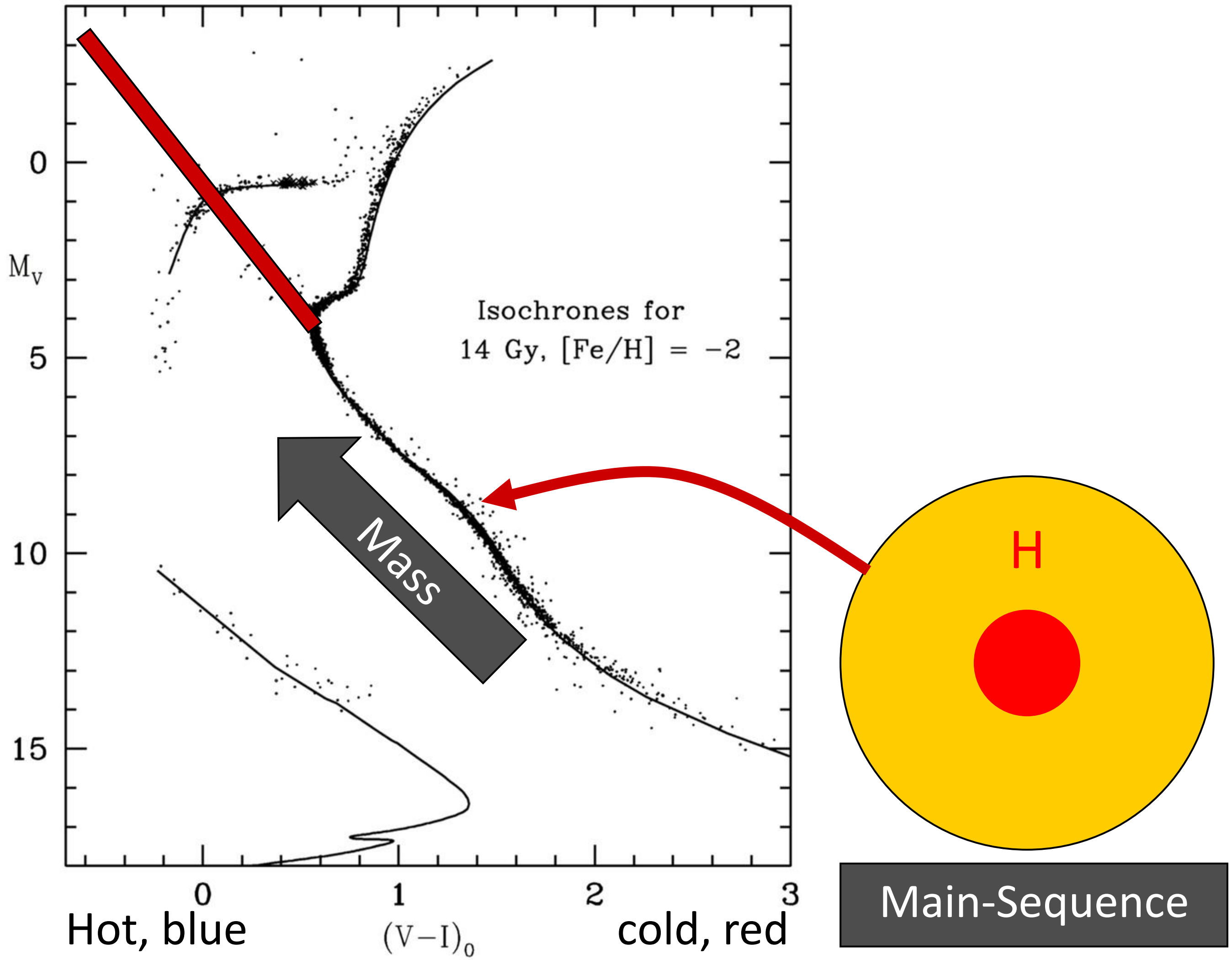
Plane wave in vacuum	$\Phi(\boldsymbol{r},t) \propto e^{-i\omega t + i\boldsymbol{k}\cdot\boldsymbol{r}}$
With scattering centers	$\Phi(\boldsymbol{r},t) \propto e^{-i\omega t}\left[e^{ik\cdot r} + f(\omega,\theta)\,\dfrac{e^{ik\cdot r}}{r}\right]$
In forward direction, adds coherently to a plane wave with modified wave number	$k = n_{\rm refr}\,\omega$ $n_{\rm refr} = 1 + \dfrac{2\pi}{\omega^2}\,N\,f(\omega,0)$ N = number density of scattering centers $f(\omega,0)$ = forward scattering amplitude





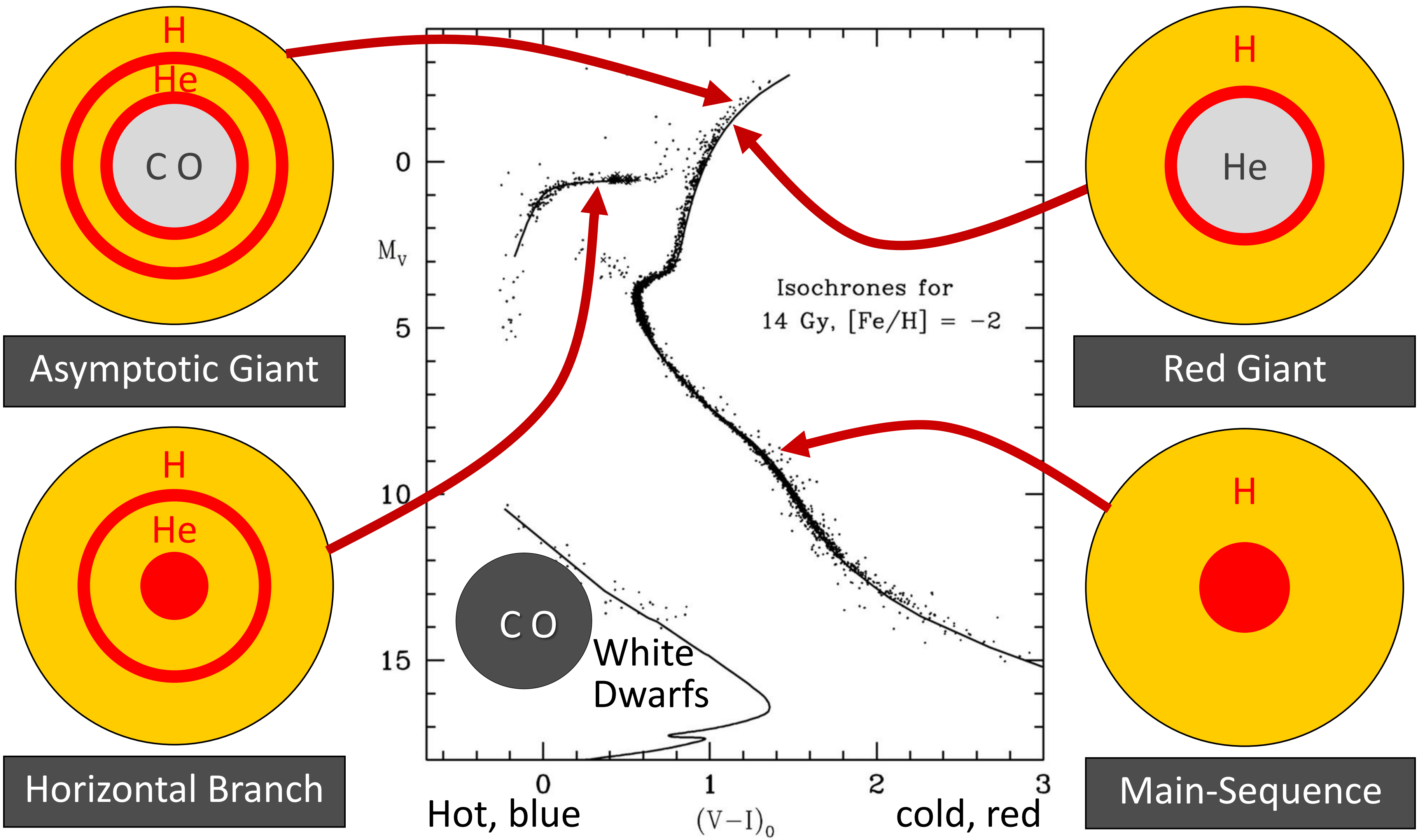
Color-Magnitude Diagram for Globular Clusters

- Stars with M so large that they have burnt out in a Hubble time
- No new star formation in globular clusters



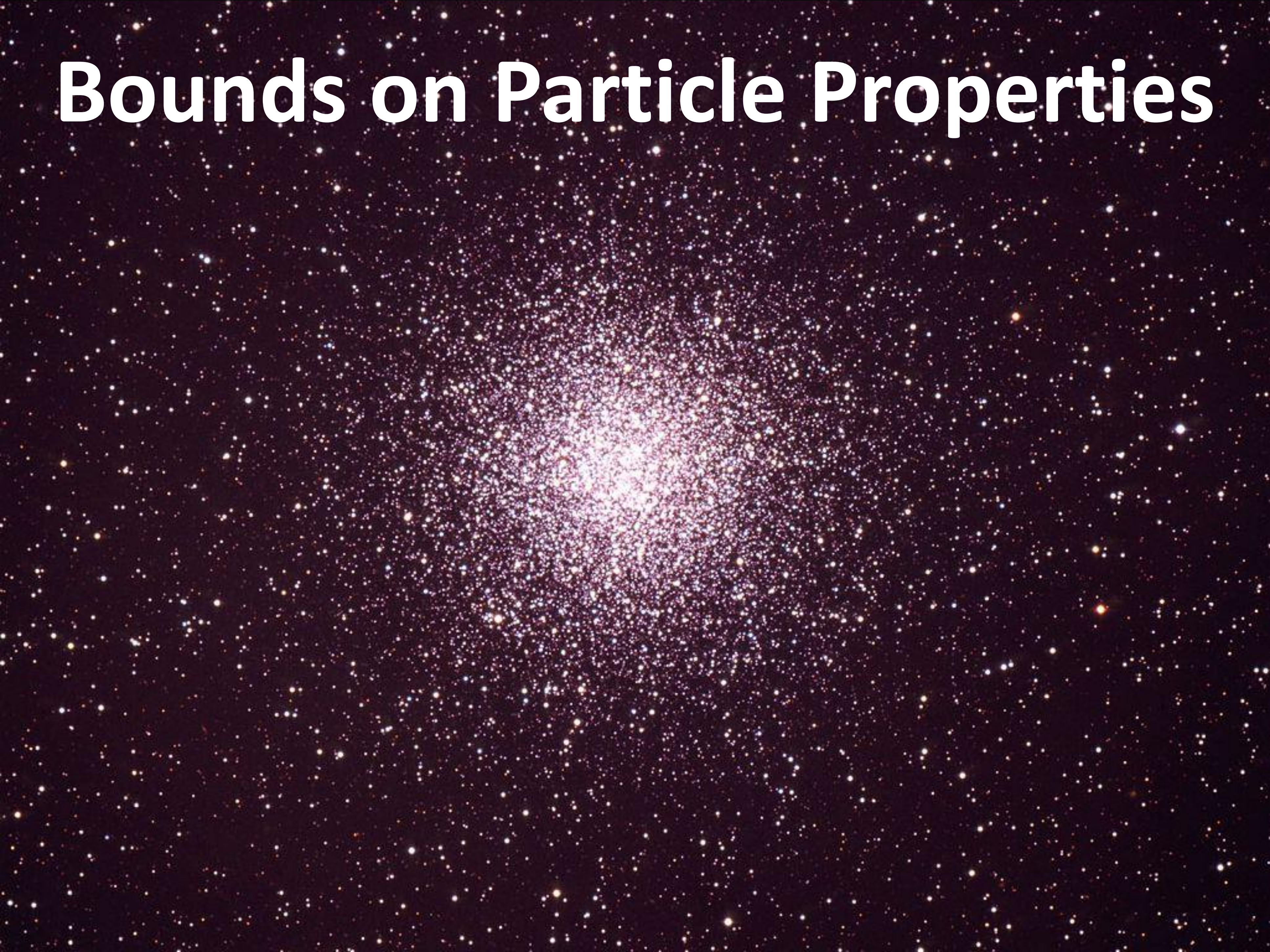
Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris, 2000)

Color-Magnitude Diagram for Globular Clusters

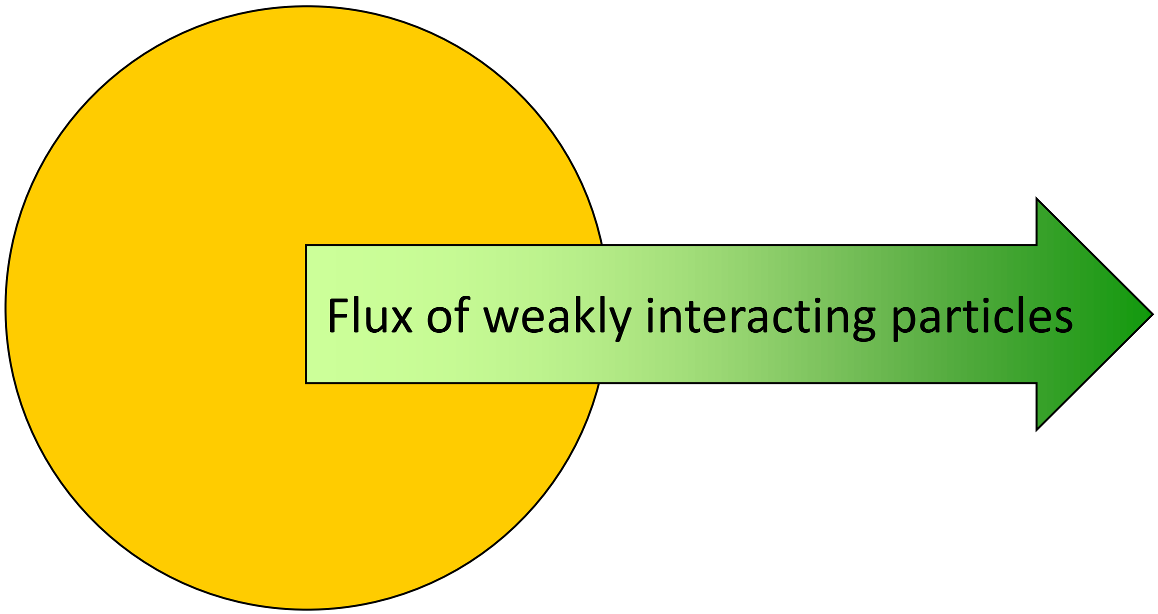


Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris, 2000)

Bounds on Particle Properties



Basic Argument: Stars as Bolometers



- Low-mass weakly-interacting particles can be emitted from stars
- New energy-loss channel
- Back-reaction on stellar properties and evolution

- What are the emission processes?
- What are the observable consequences?

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Electromagnetic Properties of the Neutrino

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(Received 11 June 1963)

In this note we make a detailed survey of the experimental information on the neutrino charge, charge radius, and magnetic moment. Both weak-interaction data and astrophysical results can be used to give precise limits to these quantities, independent of the supposition that the weak interactions are charge conserving.

I. INTRODUCTION

MOST physicists now accept the prospect that there are two neutrinos—ν_e and ν_μ—identical except for interaction (ν_e couples weakly with electrons and ν_μ with muons) and that these neutrinos have the simplest properties compatible with existing experimental evidence; i.e., zero mass, charge, electric, and magnetic dipole moments. However, the weak interactions have produced so many surprises that it is worthwhile, from time to time, to study the *experimental* limits that have been set on these quantities. In this note we present a systematic survey of the properties of the two neutrinos that can be inferred from experiment.

II. PROPERTIES

We begin by listing the properties of the neutrinos to

tritium experiments give

$$m_{\nu_e}<200\ \text{eV}, \tag{2}$$

and the experiments are consistent with $m_{\nu_e}=0$.

(2) ν_μ: The mass of the muon neutrino is the least well known of the parameters associated with either neutrino. The best measurements of it come from the energy-momentum balance in π decay. The experiment of Barkas *et al.*[3] gives[4]

$$m_{\nu_\mu}<3.5\ \text{MeV}. \tag{3}$$

The reason for this uncertainty lies in the kinematic fact that the small neutrino mass is given as the difference between measured quantities of order 1. In the $\pi\rightarrow\mu+\nu$ decay, the accuracy with which the neutrino mass can be determined is given by

Neutrino Electromagnetic Form Factors

Effective coupling of electromagnetic field to a neutral fermion

$$
\begin{aligned}
L_{\text{eff}} = & -F_1 \overline{\Psi} \gamma_\mu \Psi \, A^\mu \\
& -G_1 \overline{\Psi} \gamma_\mu \gamma_5 \Psi \, \partial_\nu F^{\mu\nu} \\
& -\frac{1}{2} F_2 \, \overline{\Psi} \sigma_{\mu\nu} \Psi \, F^{\mu\nu} \\
& -\frac{1}{2} G_2 \, \overline{\Psi} \sigma_{\mu\nu} \gamma_5 \Psi \, F^{\mu\nu}
\end{aligned}
$$

Charge $e_\nu = F_1(0) = 0$

Anapole moment $G_1(0)$

Magnetic dipole moment $\mu = F_2(0)$

Electric dipole moment $\varepsilon = G_2(0)$

- Charge form factor $F_1(q^2)$ and anapole $G_1(q^2)$ are short-range interactions if charge $F_1(0) = 0$
- Connect states of equal helicity
- In the standard model they represent radiative corrections to weak interaction

- Dipole moments connect states of opposite helicity
- Violation of individual flavor lepton numbers (neutrino mixing)
  $\rightarrow$ Magnetic or electric dipole moments can connect different flavors or different mass eigenstates ("Transition moments")
- Usually measured in "Bohr magnetons" $\mu_B = e/2m_e$

Plasmon Decay and Stellar Energy Loss Rates

Assume photon dispersion relation like a massive particle (nonrelativistic plasma)

$$E_\gamma^2 - p_\gamma^2 = \omega_{\rm pl}^2 = \frac{4\pi\alpha n_e}{m_e}$$

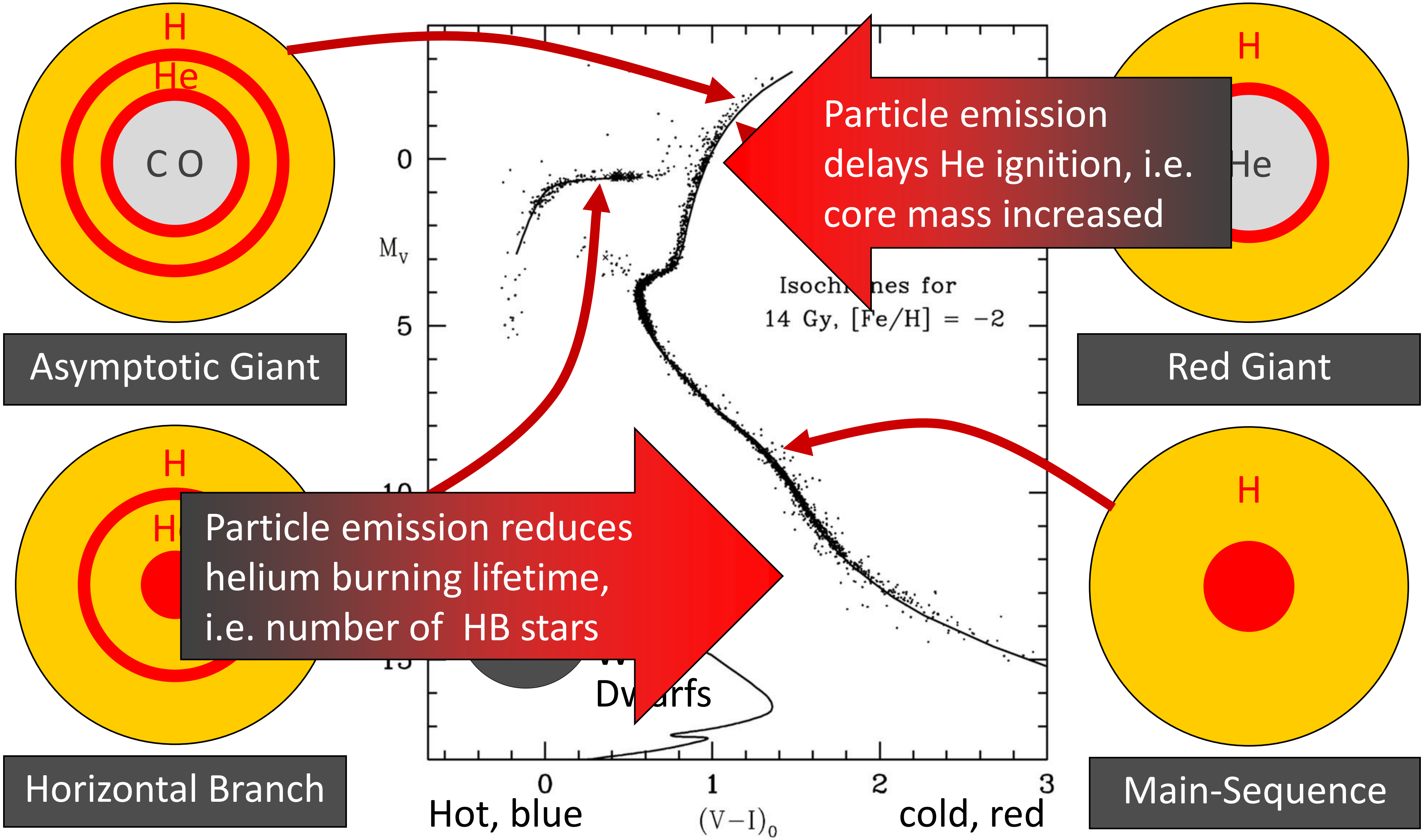
Photon decay rate (transverse plasmon) with energy E_γ

$$\Gamma(\gamma\to\nu\overline{\nu}) = \frac{4\pi}{3E_\gamma}\times\begin{cases}\alpha_\nu\left(\omega_{\rm pl}^2/4\pi\right) & \text{Millicharge}\\ (\mu_\nu^2/2)\left(\omega_{\rm pl}^2/4\pi\right)^2 & \text{Dipole moment}\\ \left(C_{\rm V}^2 G_{\rm F}^2/\alpha\right)\left(\omega_{\rm pl}^2/4\pi\right)^3 & \text{Standard model}\end{cases}$$

Energy-loss rate of stellar plasma

$$Q(\gamma\to\nu\overline{\nu}) = \int\frac{2d^3\mathbf{p}}{(2\pi)^3}\frac{E_\gamma\Gamma_{\gamma\to\nu\overline{\nu}}}{e^{E_\gamma/T}-1} = \frac{8\zeta_3 T^3}{3\pi}\times\begin{cases}\alpha_\nu\left(\omega_{\rm pl}^2/4\pi\right)\\ (\mu_\nu^2/2)\left(\omega_{\rm pl}^2/4\pi\right)^2\\ \left(C_{\rm V}^2 G_{\rm F}^2/\alpha\right)\left(\omega_{\rm pl}^2/4\pi\right)^3\end{cases}$$

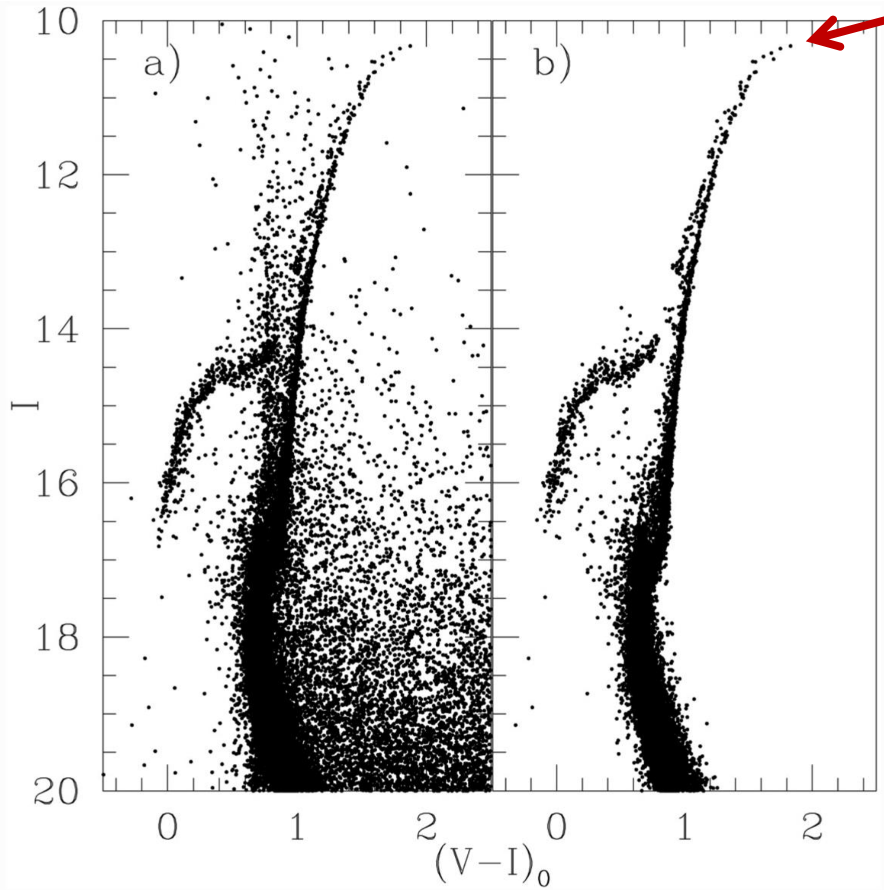
Color-Magnitude Diagram for Globular Clusters



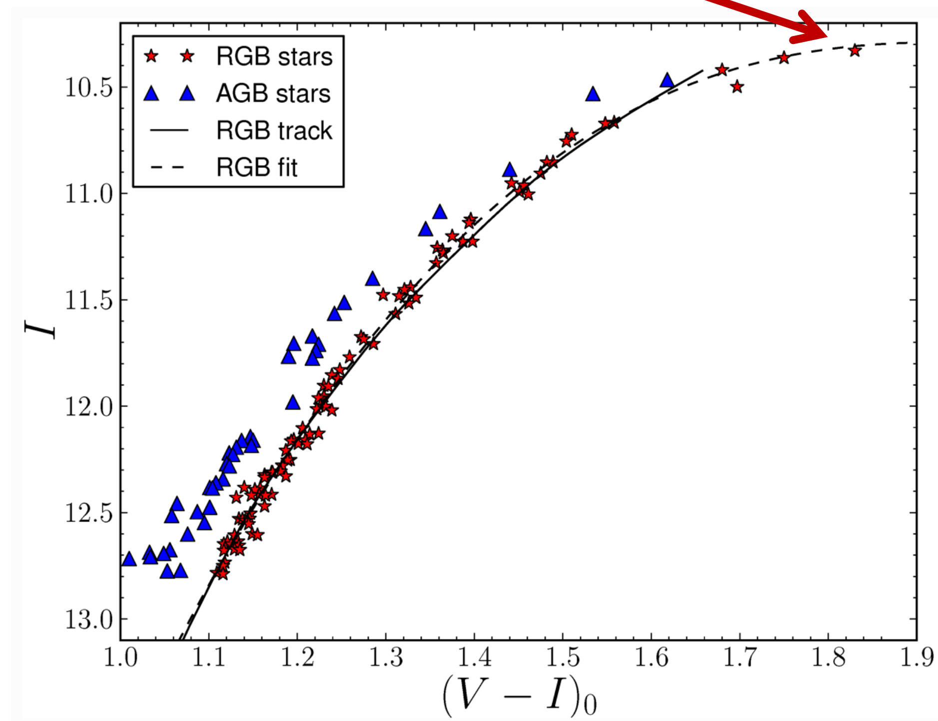
Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris, 2000)

Color-Magnitude Diagram of Globular Cluster M5

Brightest red giant
measures nonstandard energy loss



CMD (a) before and (b) after cleaning

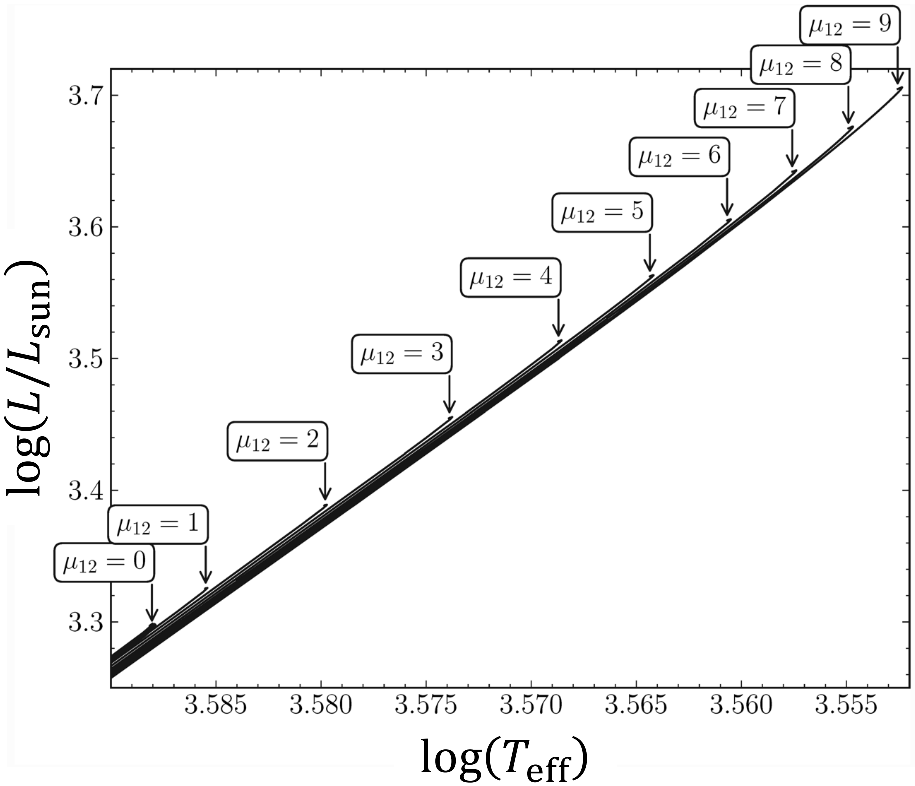


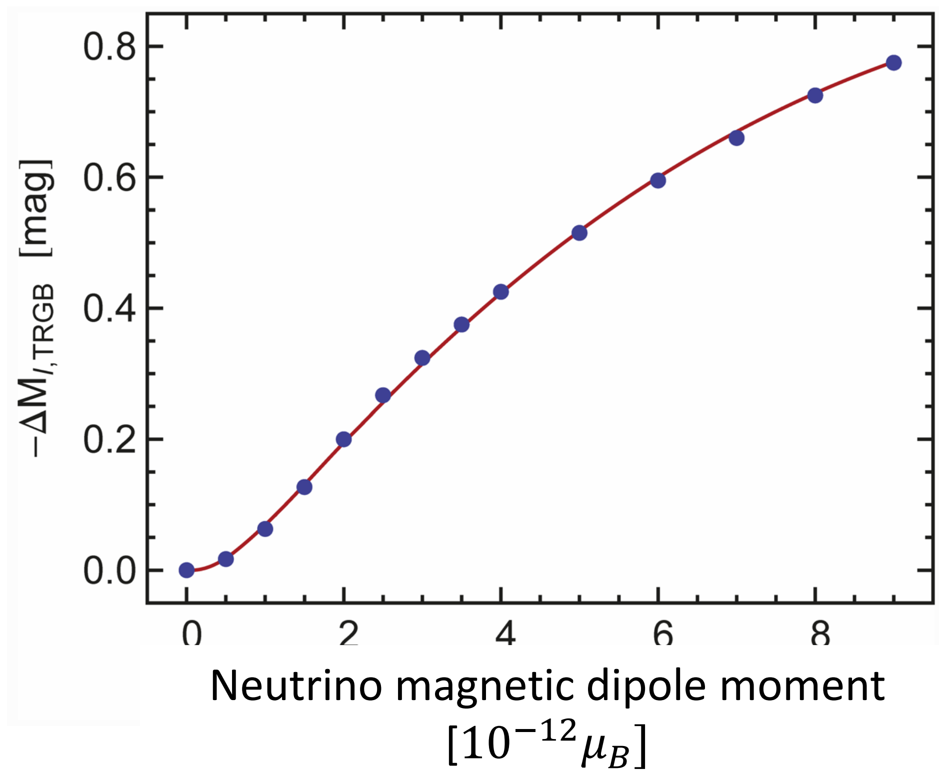
CMD of brightest 2.5 mag of RGB

Viaux, Catelan, Stetson, Raffelt, Redondo, Valcarce & Weiss, arXiv:1308.4627

Helium Ignition for Low-Mass Red Giants

Brightness increase at He ignition by nonstanderd neutrino losses

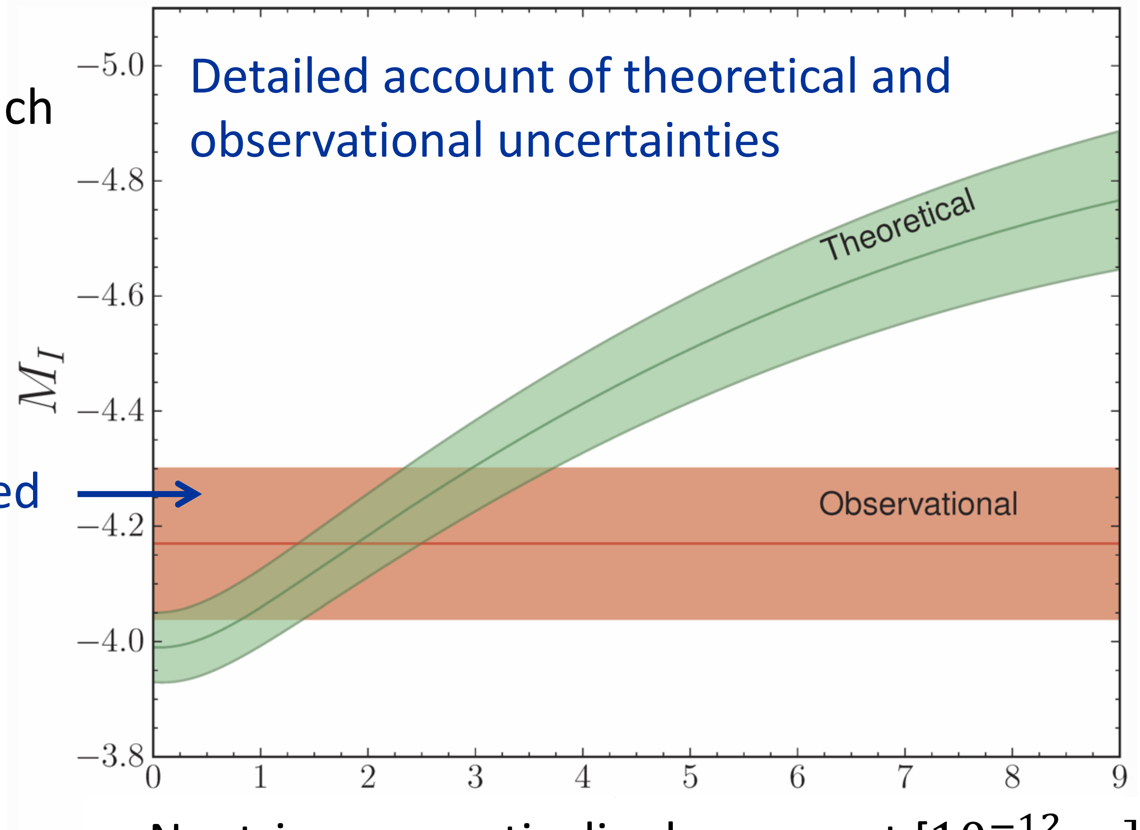




Viaux, Catelan, Stetson, Raffelt, Redondo, Valcarce & Weiss, arXiv:1308.4627

Neutrino Dipole Limits from Globular Cluster M5

I-band brightness
of tip of red-giant brach
[magnitudes]



- Uncertainty dominated by distance
- Can be improved in future (GAIA mission)

Neutrino magnetic dipole moment $[10^{-12}\mu_B]$

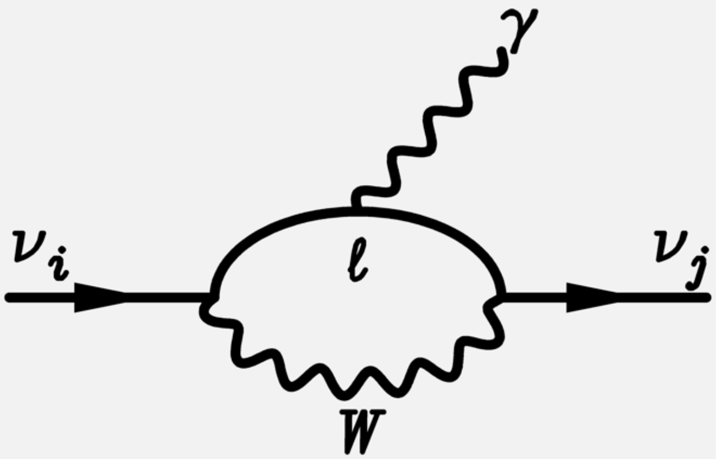
Most restrictive limit on neutrino electromagnetic properties

$$\mu_\nu < \begin{cases} 2.6 \times 10^{-12}\mu_B & (68\%\ \text{CL}) \\ 4.5 \times 10^{-12}\mu_B & (95\%\ \text{CL}) \end{cases}$$

Viaux, Catelan, Stetson, Raffelt, Redondo, Valcarce & Weiss, arXiv:1308.4627

Standard Dipole Moments for Massive Neutrinos

Standard electroweak model:
Neutrino dipole and
transition moments
are induced at higher order



Massive neutrinos ν_i $(i = 1,2,3)$
mixed to form weak eigenstates

$$\nu_\ell = \sum_{i=1}^{3} U_{\ell i} \nu_i$$

Explicitly for Dirac neutrinos
Magnetic moments μ_{ij}
Electric moments ϵ_{ij}

$$\mu_{ij} = \frac{e\sqrt{2}G_\mathrm{F}}{(4\pi)^2}(m_i + m_j) \sum_{\ell=e,\mu,\tau} U_{\ell j} U_{\ell i}^* f\left(\frac{m_\ell}{m_W}\right)$$

$$\epsilon_{ij} = \ldots (m_i - m_j) \ \ldots$$

$$f\left(\frac{m_\ell}{m_W}\right) = -\frac{3}{2} + \frac{3}{4}\left(\frac{m_\ell}{m_W}\right)^2 + \mathcal{O}\left(\frac{m_\ell}{m_W}\right)^4$$

Standard Dipole Moments for Massive Neutrinos

Diagonal case:
Magnetic moments
of Dirac neutrinos

$$\mu_{ii} = \frac{3e\sqrt{2}G_\mathrm{F}}{(4\pi)^2} m_i = 3.20 \times 10^{-19} \mu_\mathrm{B} \frac{m_i}{\mathrm{eV}} \qquad \mu_\mathrm{B} = \frac{e}{2m_e}$$

$$\epsilon_{ii} = 0$$

Off-diagonal case
(Transition moments)

First term in $f(m_\ell/m_W)$
does not contribute:
“GIM cancellation”

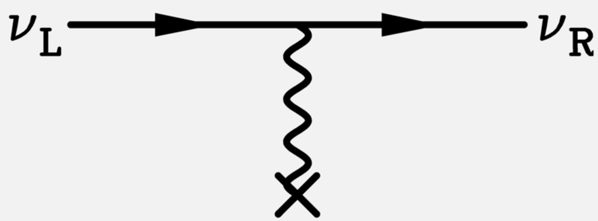
$$\mu_{ij} = \frac{3e\sqrt{2}G_\mathrm{F}}{4(4\pi)^2}(m_i+m_j)\left(\frac{m_\tau}{m_W}\right)^2 \sum_{\ell=e,\mu,\tau} U_{\ell j}U^*_{\ell i}\left(\frac{m_\ell}{m_\tau}\right)^2$$

$$= 3.96 \times 10^{-23} \mu_\mathrm{B} \frac{m_i + m_j}{\mathrm{eV}} \sum_{\ell=e,\mu,\tau} U_{\ell j}U^*_{\ell i}\left(\frac{m_\ell}{m_\tau}\right)^2$$

Largest neutrino mass eigenstate $0.05~\mathrm{eV} < m < 0.2~\mathrm{eV}$
For Dirac neutrino expect
$1.6 \times 10^{-20}\mu_B < \mu_\nu < 6.4 \times 10^{-20}\mu_B$

Consequences of Neutrino Dipole Moments

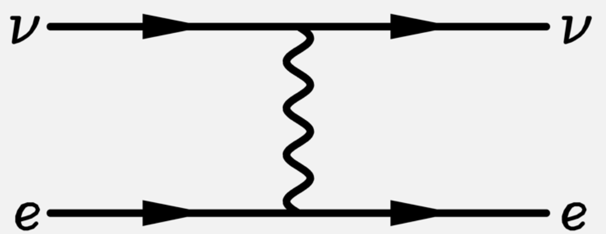
Spin precession in external E or B fields



$$\mathrm{i}\,\frac{\partial}{\partial t}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix}=\begin{pmatrix}0 & \mu_\nu B_\perp\\ \mu_\nu B_\perp & 0\end{pmatrix}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix}$$

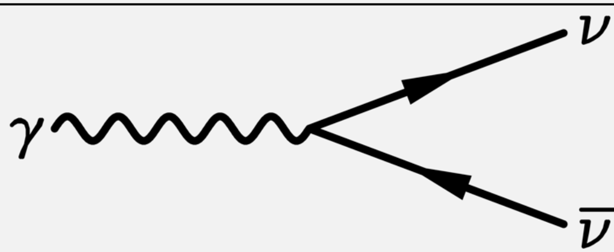
Scattering

$$\frac{d\sigma}{dT}=\frac{G_F^2 m_e}{2\pi}\left[(C_\mathrm{V}+C_\mathrm{A})^2+(C_\mathrm{V}-C_\mathrm{A})^2\left(1-\frac{T}{E}\right)^2+(C_\mathrm{V}^2-C_\mathrm{A}^2)\frac{m_e T}{E^2}\right]+\alpha\mu_\nu^2\left(\frac{1}{T}+\frac{1}{E}\right)$$



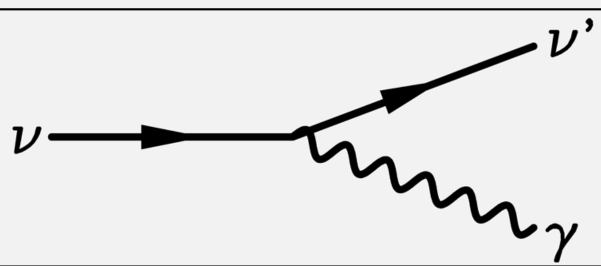
T electron recoil energy

Plasmon decay in stars



$$\Gamma=\frac{\mu_\nu^2}{24\pi}\,\omega_\mathrm{pl}^3$$

Decay or Cherenkov effect

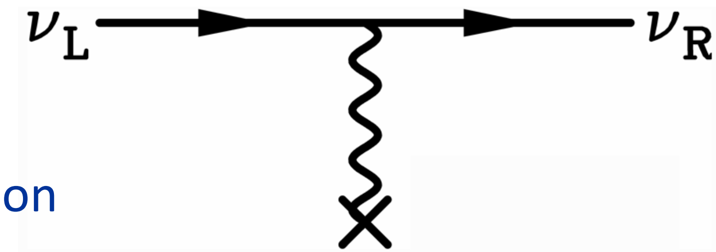


$$\Gamma=\frac{\mu_\nu^2}{8\pi}\left(\frac{m_2^2-m_1^2}{m_2}\right)^3$$

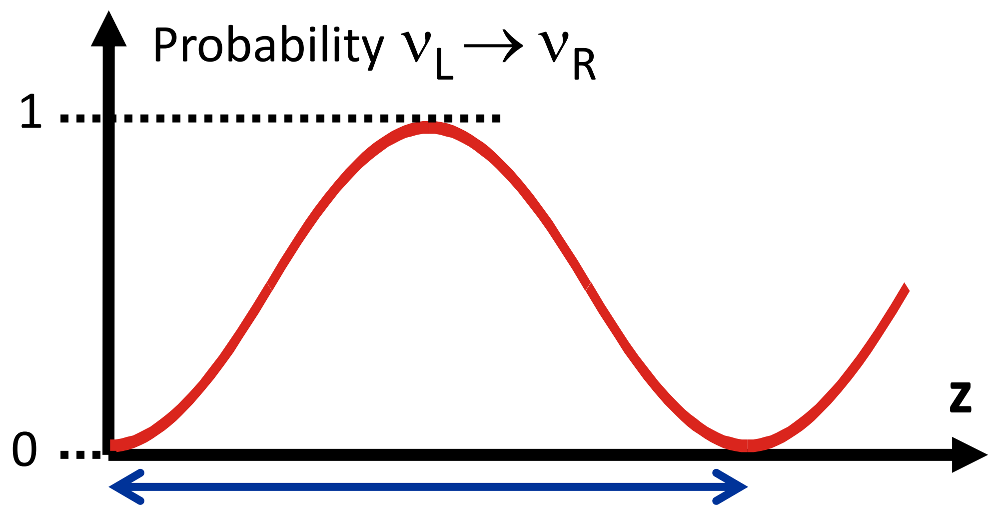
Neutrino Spin Oscillations

Spin Precession in external E or B fields

$$i\partial_t \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} = \begin{pmatrix} 0 & \mu_\nu B_T \\ \mu_\nu B_T & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$



For relativistic neutrinos the oscillation equation
- is independent of energy
- involves only the transverse B field



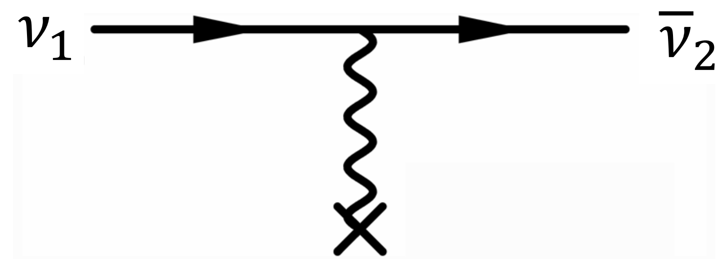
Oscillation Length $\dfrac{\pi}{\mu_\nu B_T}$

Distance for helicity reversal

$$\frac{\pi}{2\mu_\nu B_\mathrm{T}} = 5.36 \times 10^{13}\,\mathrm{cm}\,\frac{10^{-10}\mu_\mathrm{B}}{\mu_\nu}\,\frac{1\mathrm{G}}{B_T}$$

Spin-Flavor Oscillations

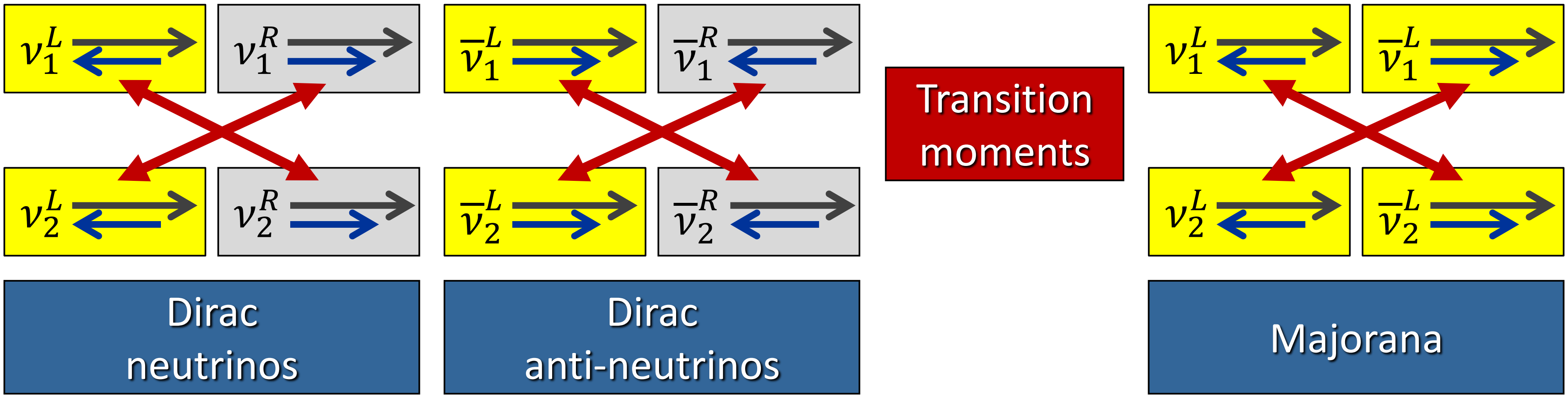
Spin-flavor precession in external E or B fields



$$i\partial_t \begin{pmatrix} \nu_1 \\ \bar{\nu}_2 \end{pmatrix} = \begin{pmatrix} 0 & \mu_\nu B_T \\ -\mu_\nu B_T & 0 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \bar{\nu}_2 \end{pmatrix}$$

Majorana neutrinos:

- Diagonal dipole moments vanish
- Transition moments inevitably exist, couple neutrinos with anti-neutrinos
- Standard model calculation ~ Dirac case



Neutrino Spin-Flavor Oscillations in a Medium

Two-flavor oscillations of Majorana neutrinos with a transition magnetic moment μ and ordinary flavor mixing in a medium

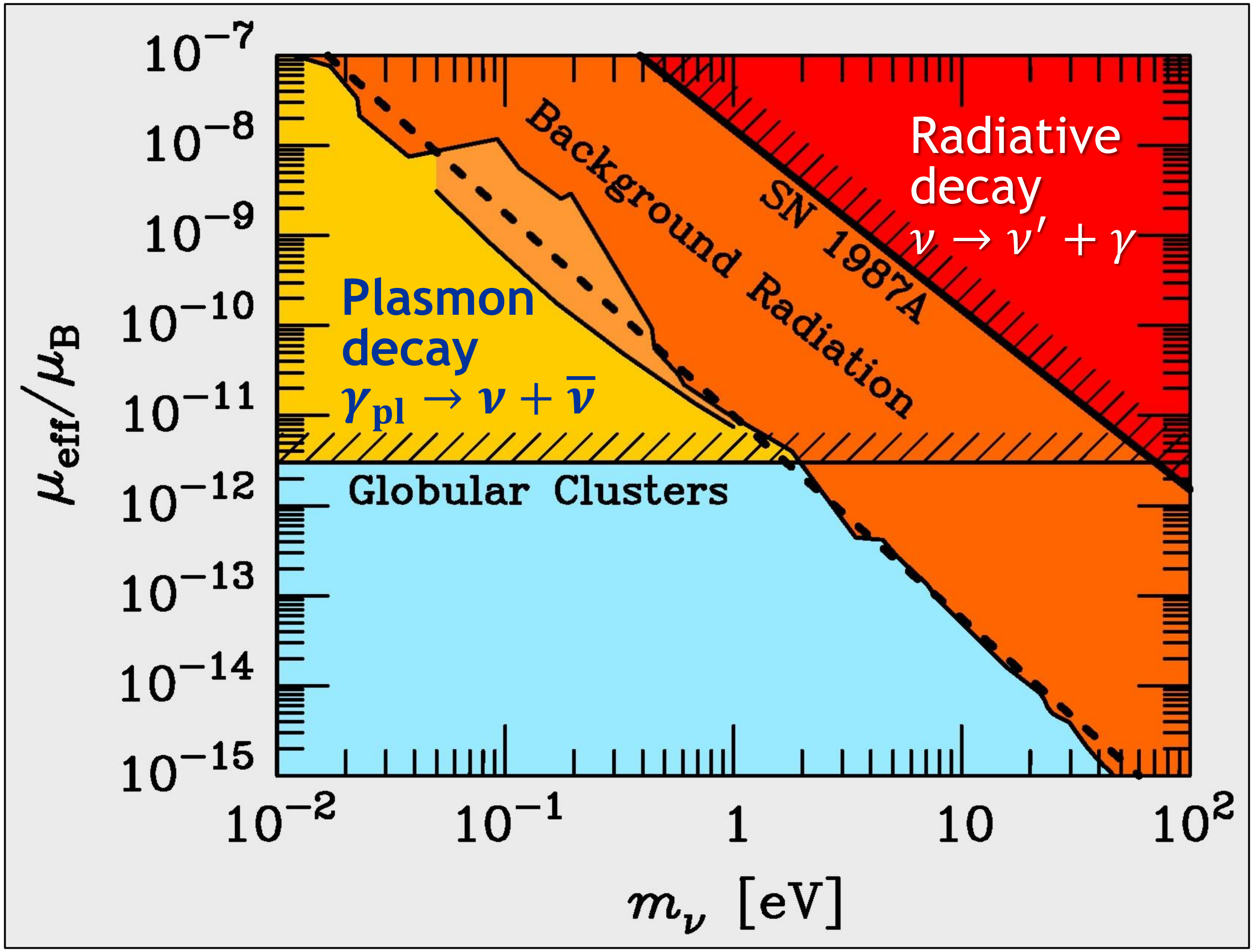
$$i\partial_r \begin{pmatrix} \nu_e \\ \nu_\mu \\ \overline{\nu}_e \\ \overline{\nu}_\mu \end{pmatrix} = \begin{pmatrix} c\Delta + a_e & s\Delta & 0 & \mu B \\ s\Delta & -c\Delta + a_\mu & -\mu B & 0 \\ 0 & -\mu B & c\Delta - a_e & s\Delta \\ \mu B & 0 & s\Delta & -c\Delta - a_\mu \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \overline{\nu}_e \\ \overline{\nu}_\mu \end{pmatrix}$$

with $c = \cos(2\Theta)$, $s = \sin(2\Theta)$,

$\Delta = (m_2^2 - m_1^2)/4E,\ a_e = \sqrt{2}G_F\left(n_e - \frac{1}{2}n_n\right)$ and $a_\mu = \sqrt{2}G_F\left(-\frac{1}{2}n_n\right)$

- Resonant spin-flavor precession (RSFP) can be a subdominant effect for solar neutrino conversion and can produce a small solar anti-neutrino flux
- Can be important for supernova neutrinos

Neutrino Radiative Lifetime Limits



$$\Gamma_{\nu\to\nu'\gamma} = \frac{\mu_{\rm eff}^2}{8\pi}\, m_\nu^3$$

$$\Gamma_{\gamma\to\nu\bar{\nu}} = \frac{\mu_{\rm eff}^2}{24\pi}\, \omega_{\rm pl}^3$$

For low-mass neutrinos, plasmon decay in globular cluster stars yields the most restrictive limits

Further Reading on Particle Limits from Stars

Georg Raffelt:

Astrophysical Methods to Constrain Axions
and Other Novel Particle Phenomena
Phys. Rept. 198 (1990) 1–113

Stars as Laboratories for Fundamental Physics
(University of Chicago Press, 1996)
http://wwwth.mpp.mpg.de/members/raffelt/mypapers/199613.pdf

Neutrinos and the Stars
Proc. ISAPP 2012 "Neutrino Physics and Astrophysics"
(26 July–5 August 2011, Varenna, Lake Como, Italy)
arXiv:1201.1637