Neutrinos

in Astrophysics and Cosmology

Neutrinos and the Stars 1

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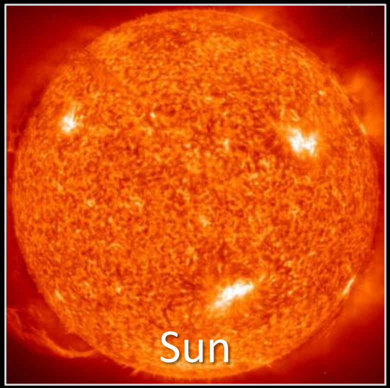
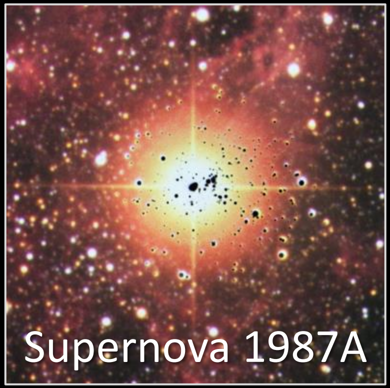
Neutrinos and the Stars



- Strongest local neutrino flux
- Long history of detailed measurements
- Crucial for flavor oscillation physics
- Resolve solar metal abundance problem in future?
- Use Sun as source for other particles (especially axions)

- Neutrino energy loss crucial in stellar evolution theory
- Backreaction on stars provides limits, e.g. neutrino magnetic dipole moments



- Collapsing stars most powerful neutrino sources
- Once observed from SN 1987A
- Provides well-established particle-physics constraints
- Next galactic supernova: learn about astrophyiscs of core collapse
- Diffuse Supernova Neutrino Background (DSNB) is detectable

Basics of Stellar Evolution

Equations of Stellar Structure

Assume spherical symmetry and static structure (neglect kinetic energy)
Excludes: Rotation, convection, magnetic fields, supernova-dynamics, …

Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{G_N M_r \rho}{r^2}$$

Energy conservation

$$\frac{dL_r}{dr} = 4\pi r^2 \epsilon \rho$$

Energy transfer

$$L_r = \frac{4\pi r^2}{3\kappa\rho} \frac{d(aT^4)}{dr}$$

Literature

- Clayton: Principles of stellar evolution and nucleosynthesis (Univ. Chicago Press 1968)
- Kippenhahn & Weigert: Stellar structure and evolution (Springer 1990)

r Radius from center

P Pressure

G_N Newton's constant

ρ Mass density

M_r Integrated mass up to r

L_r Luminosity (energy flux)

ϵ Local rate of energy generation $[\mathrm{erg\,g^{-1}s^{-1}}]$

$$\epsilon = \epsilon_{\rm nuc} + \epsilon_{\rm grav} - \epsilon_\nu$$

κ Opacity

$$\kappa^{-1} = \kappa_\gamma^{-1} + \kappa_{\rm c}^{-1}$$

κ_γ Radiative opacity

$$\kappa_\gamma \rho = \langle \lambda_\gamma \rangle_{\rm Rosseland}^{-1}$$

$\kappa_{\rm c}$ Electron conduction

Convection in Main-Sequence Stars

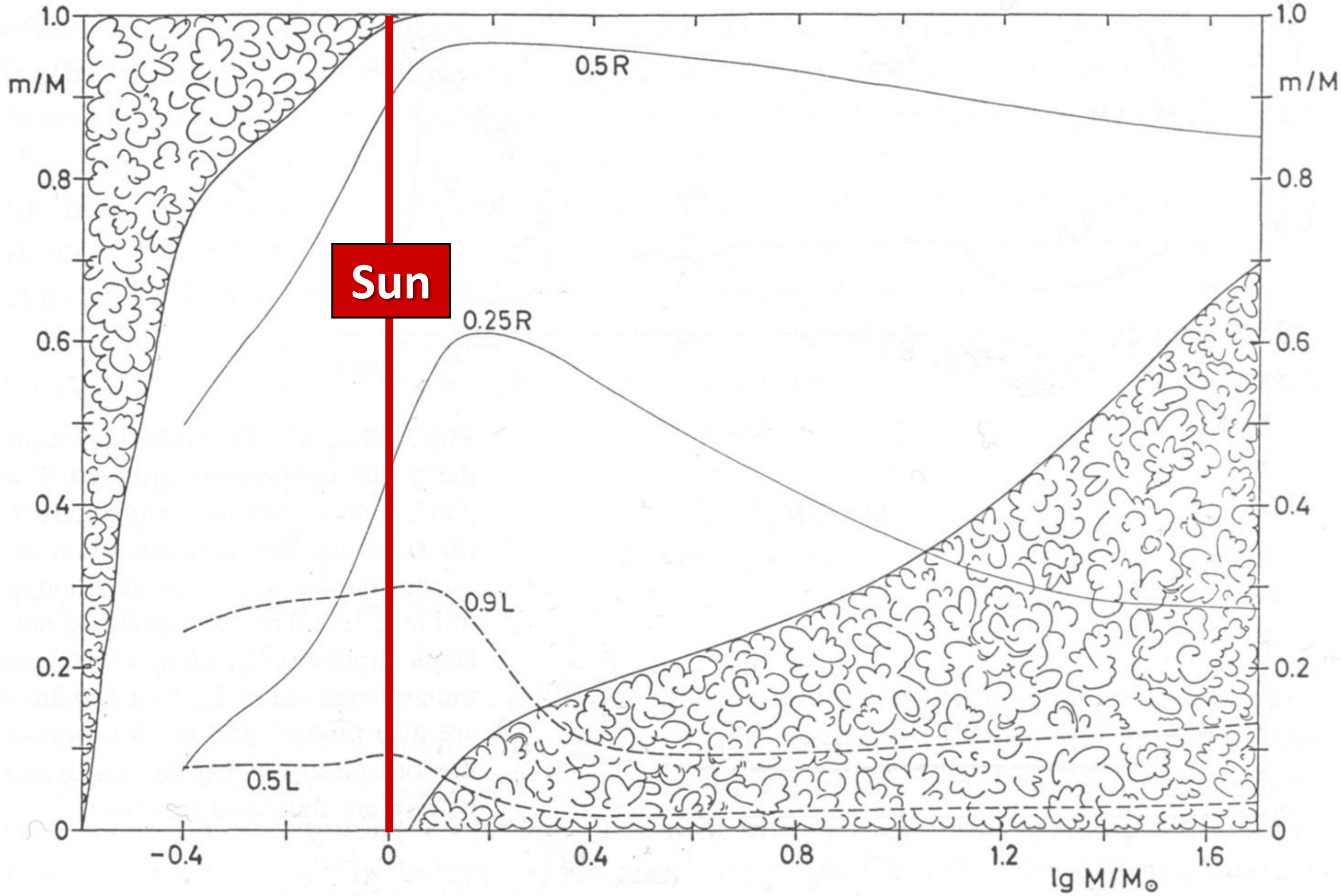


Fig. 22.7. The mass values m from centre to surface are plotted against the stellar mass M for the same zero-age main-sequence models as in Fig. 22.1. "Cloudy" areas indicate the extension of convective zones inside the models. Two solid lines give the m values at which r is 1/4 and 1/2 of the total radius R. The dashed lines show the mass elements inside which 50% and 90% of the total luminosity L are produced

Kippenhahn & Weigert, Stellar Structure and Evolution

Virial Theorem and Hydrostatic Equilibrium

Hydrostatic equilibrium

$$\frac{dP}{dr} = \frac{G_N M_r \rho}{r^2}$$

Integrate both sides

$$\int_0^R dr\, 4\pi r^3 P' = -\int_0^R dr\, 4\pi r^3 \frac{G_N M_r \rho}{r^2}$$

L.h.s. partial integration with $P = 0$ at surface R

$$-3\int_0^R dr\, 4\pi r^2 P = E_{\rm grav}^{\rm tot}$$

Monatomic gas: $P = \frac{2}{3}U$
(U density of internal energy)

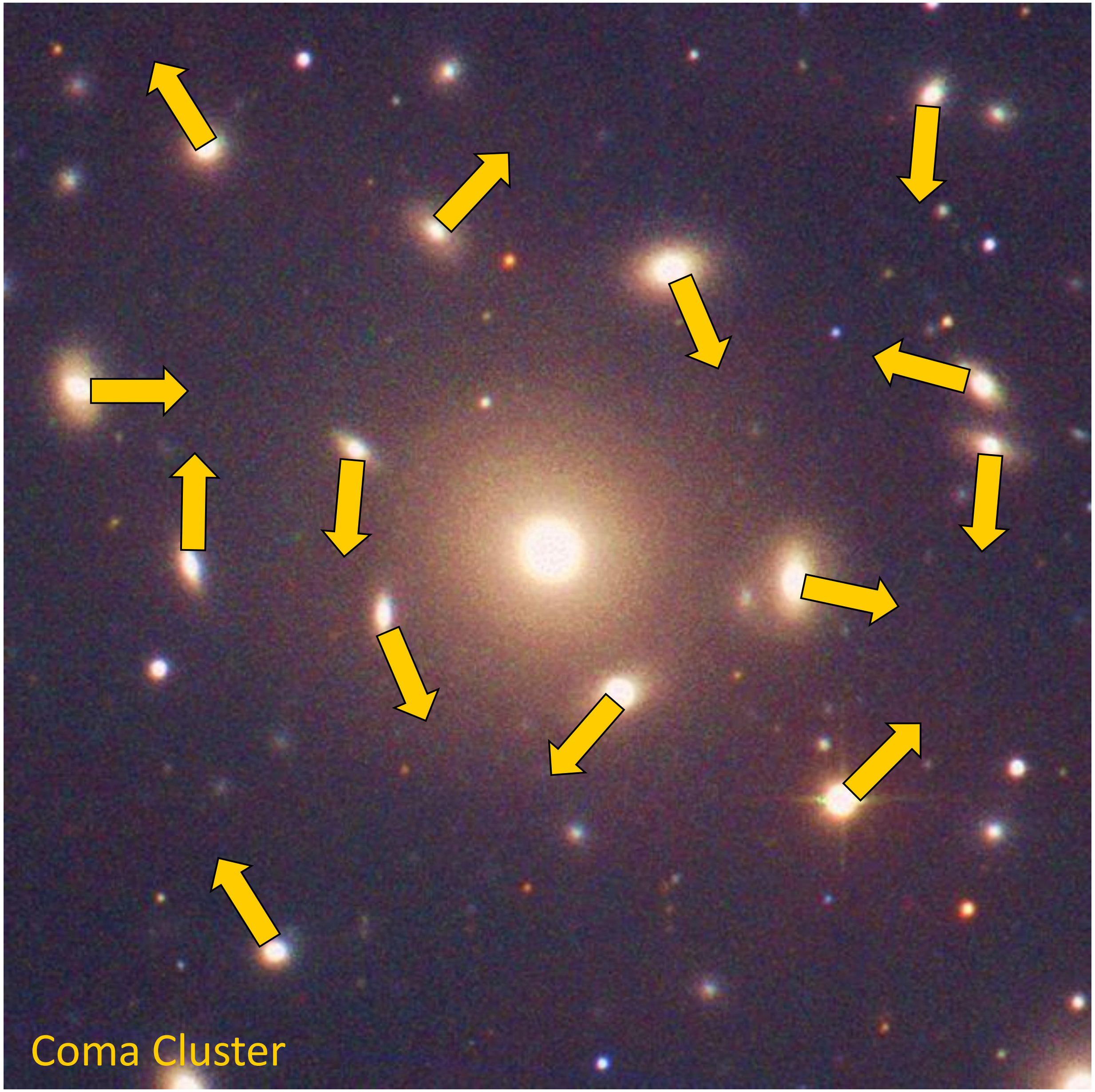
$$U^{\rm tot} = -\frac{1}{2}E_{\rm grav}^{\rm tot}$$

Average energy of single "atoms" of the gas

$$\langle E_{\rm kin}\rangle = -\frac{1}{2}\langle E_{\rm grav}\rangle$$

Virial Theorem:
Most important tool to study self-gravitating systems

Dark Matter in Galaxy Clusters



A gravitationally bound system of many particles obeys the virial theorem

$$2\langle E_{\text{kin}}\rangle = -\langle E_{\text{grav}}\rangle$$

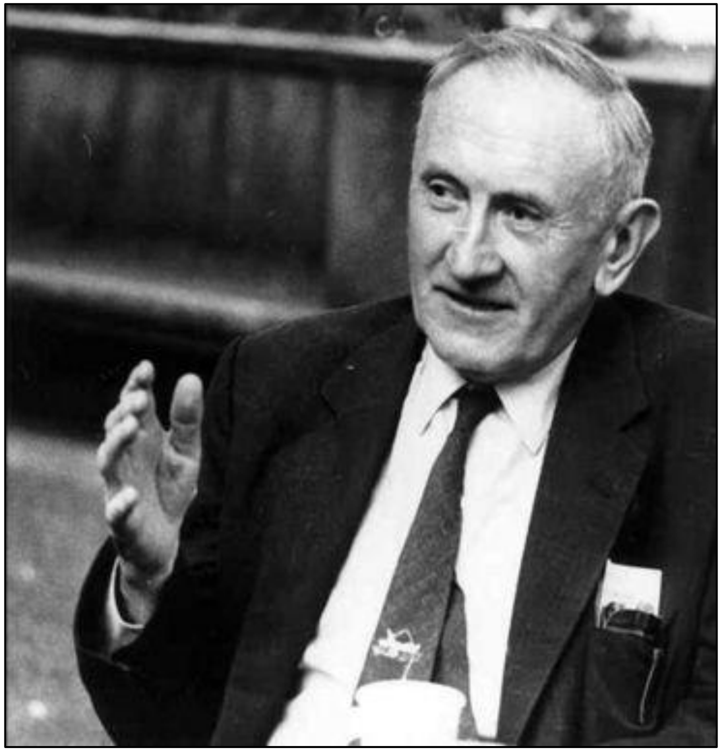
$$2\left\langle\frac{mv^2}{2}\right\rangle = \left\langle\frac{G_N M_r m}{r}\right\rangle$$

$$\langle v^2\rangle \approx G_N M_r \langle r^{-1}\rangle$$

Velocity dispersion from Doppler shifts and geometric size

Total Mass

Dark Matter in Galaxy Clusters



Fritz Zwicky:
Die Rotverschiebung von Extragalaktischen Nebeln
(The redshift of extragalactic nebulae)
Helv. Phys. Acta 6 (1933) 110

In order to obtain the observed average Doppler effect of 1000 km/s or more, the average density of the Coma cluster would have to be at least 400 times larger than what is found from observations of the luminous matter. Should this be confirmed one would find the surprising result that dark matter is far more abundant than luminous matter.

Virial Theorem Applied to the Sun

Virial Theorem $\langle E_{\rm kin}\rangle = -\frac{1}{2}\langle E_{\rm grav}\rangle$

Approximate Sun as a homogeneous sphere with

Mass $M_{\rm sun} = 1.99 \times 10^{33}\,{\rm g}$

Radius $R_{\rm sun} = 6.96 \times 10^{10}\,{\rm cm}$

Gravitational potential energy of a proton near center of the sphere

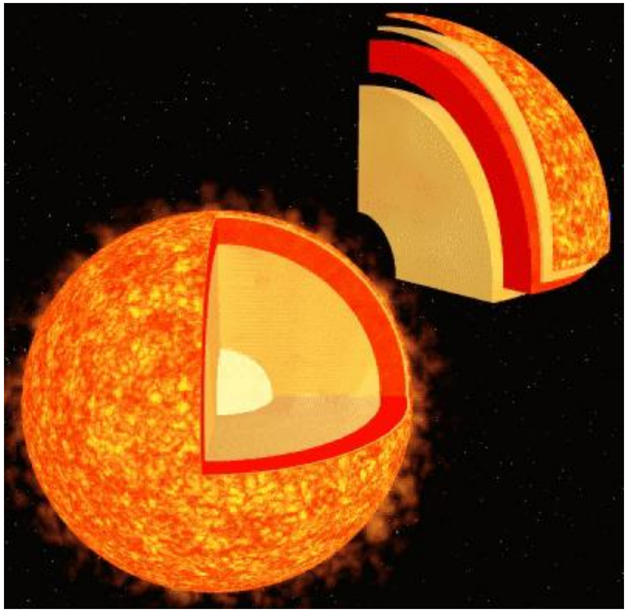
$$\langle E_{\rm grav}\rangle = -\frac{3}{2}\frac{G_N M_{\rm sun} m_p}{R_{\rm sun}} = -3.2\ {\rm keV}$$

Thermal velocity distribution

$$\langle E_{\rm kin}\rangle = \frac{3}{2}k_{\rm B}T = -\frac{1}{2}\langle E_{\rm grav}\rangle$$

Estimated temperature

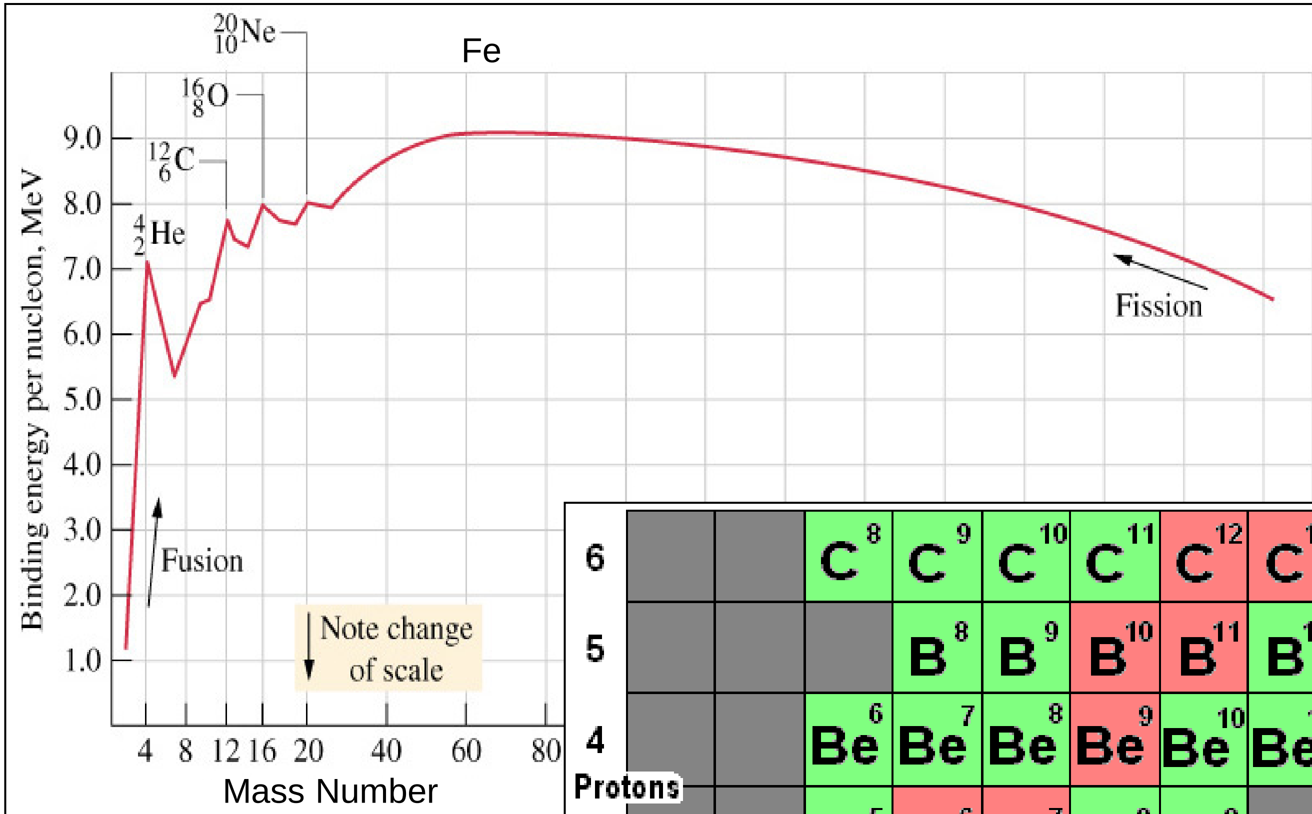
$T = 1.1\ {\rm keV}$

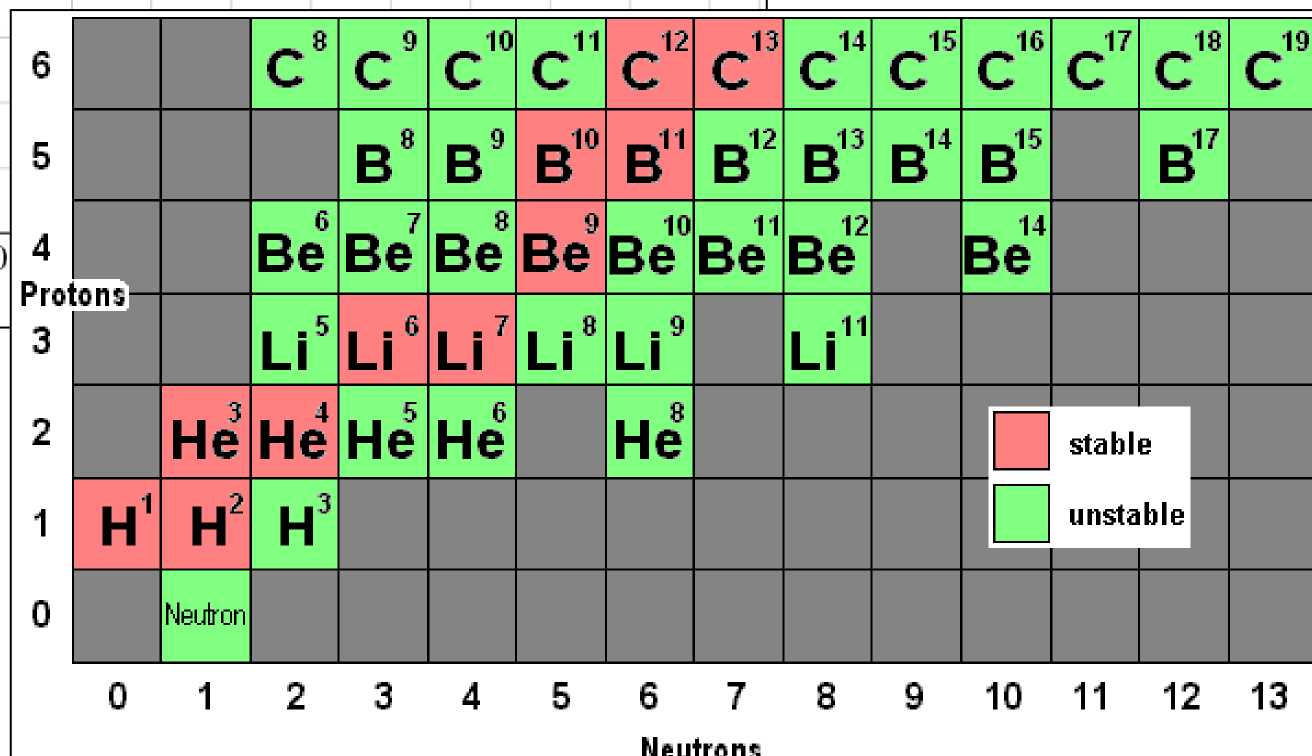


Central temperature from standard solar models

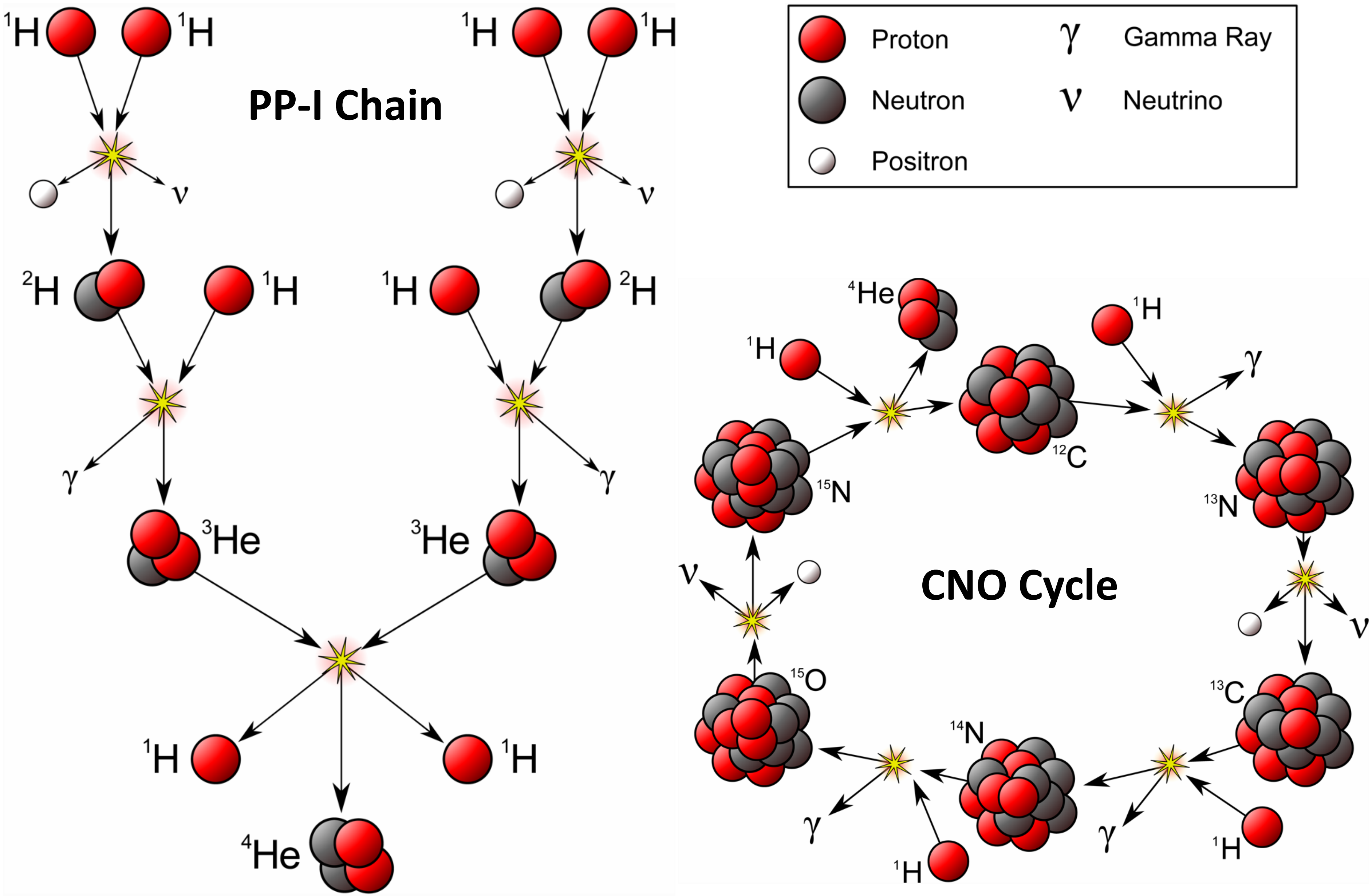
$T_{\rm c} = 1.56 \times 10^{7}\,{\rm K} = 1.34\ {\rm keV}$

Nuclear Binding Energy





Hydrogen Burning



Thermonuclear Reactions and Gamow Peak

Coulomb repulsion prevents nuclear reactions, except for Gamow tunneling

Tunneling probability

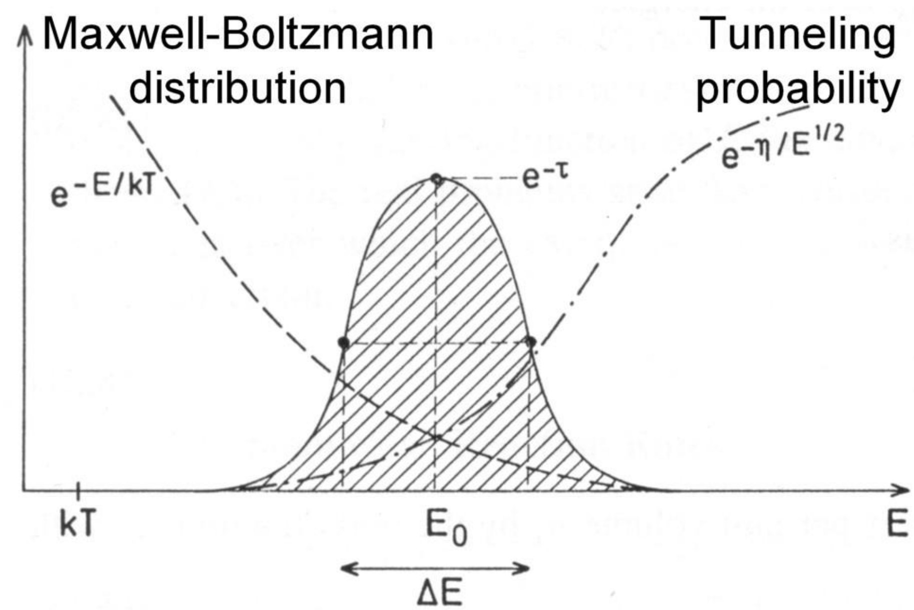
$$p \propto E^{-1/2} e^{-2\pi\eta}$$

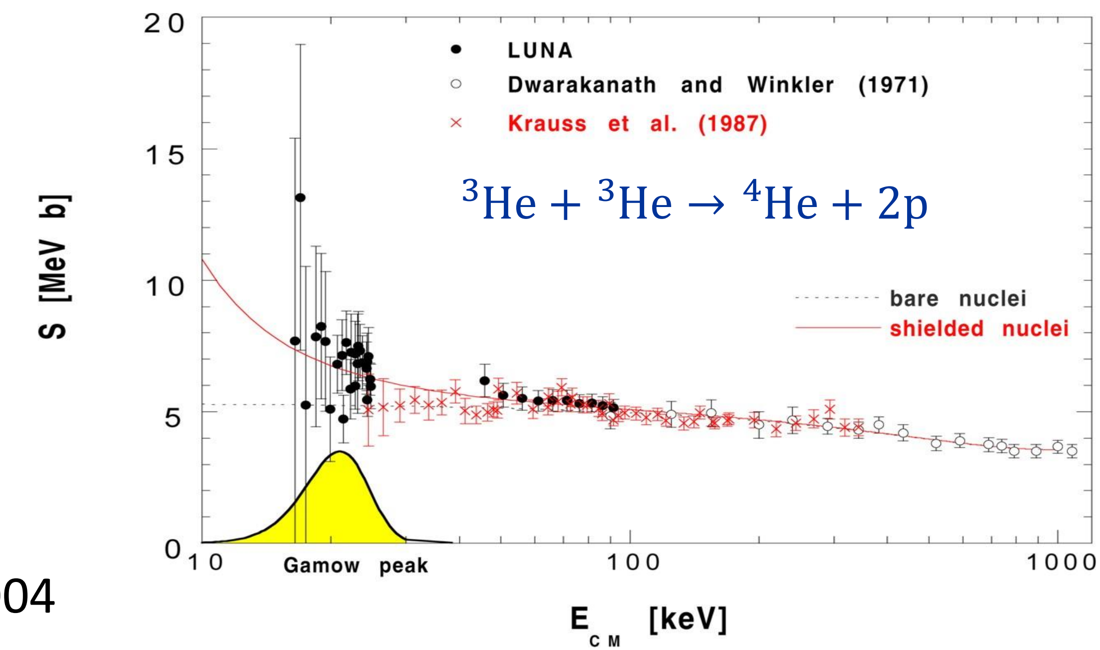
where the Sommerfeld parameter is

$$\eta = \left(\frac{m}{2E}\right)^{1/2} Z_1 Z_2 e^2$$

Parameterize cross section with astrophysical S-factor

$$S(E) = \sigma(E)\, E\, e^{2\pi\eta(E)}$$





LUNA Collaboration, nucl-ex/9902004

Main Nuclear Burning Stages

Hydrogen burning $4p + 2e^- \rightarrow {}^4He + 2\nu_e$

- Proceeds by pp chains and CNO cycle
- No higher elements are formed because no stable isotope with mass number 8
- Neutrinos from $p \rightarrow n$ conversion
- Typical temperatures: 10^7 K (~1 keV)

Helium burning

$${}^4He + {}^4He + {}^4He \leftrightarrow {}^8Be + {}^4He \rightarrow {}^{12}C$$

"Triple alpha reaction" because ${}^8Be$ unstable, builds up with concentration $\sim 10^{-9}$

$${}^{12}C + {}^4He \rightarrow {}^{16}O$$
$${}^{16}O + {}^4He \rightarrow {}^{20}Ne$$

Typical temperatures: 10^8 K (~10 keV)

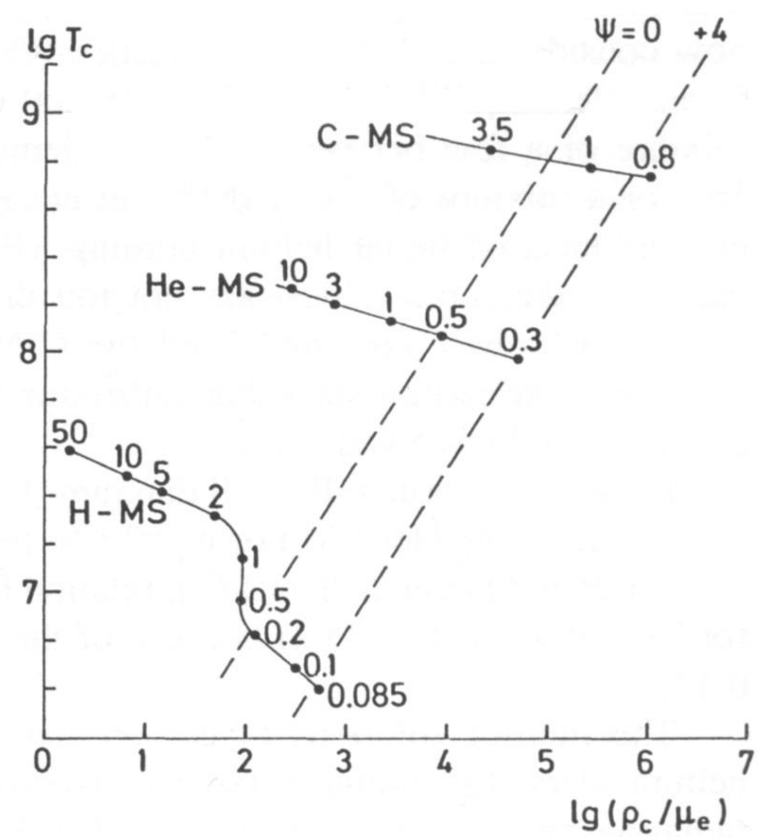
Carbon burning

Many reactions, for example

${}^{12}C + {}^{12}C \rightarrow {}^{23}Na + p$ or ${}^{20}Ne + {}^4He$ etc

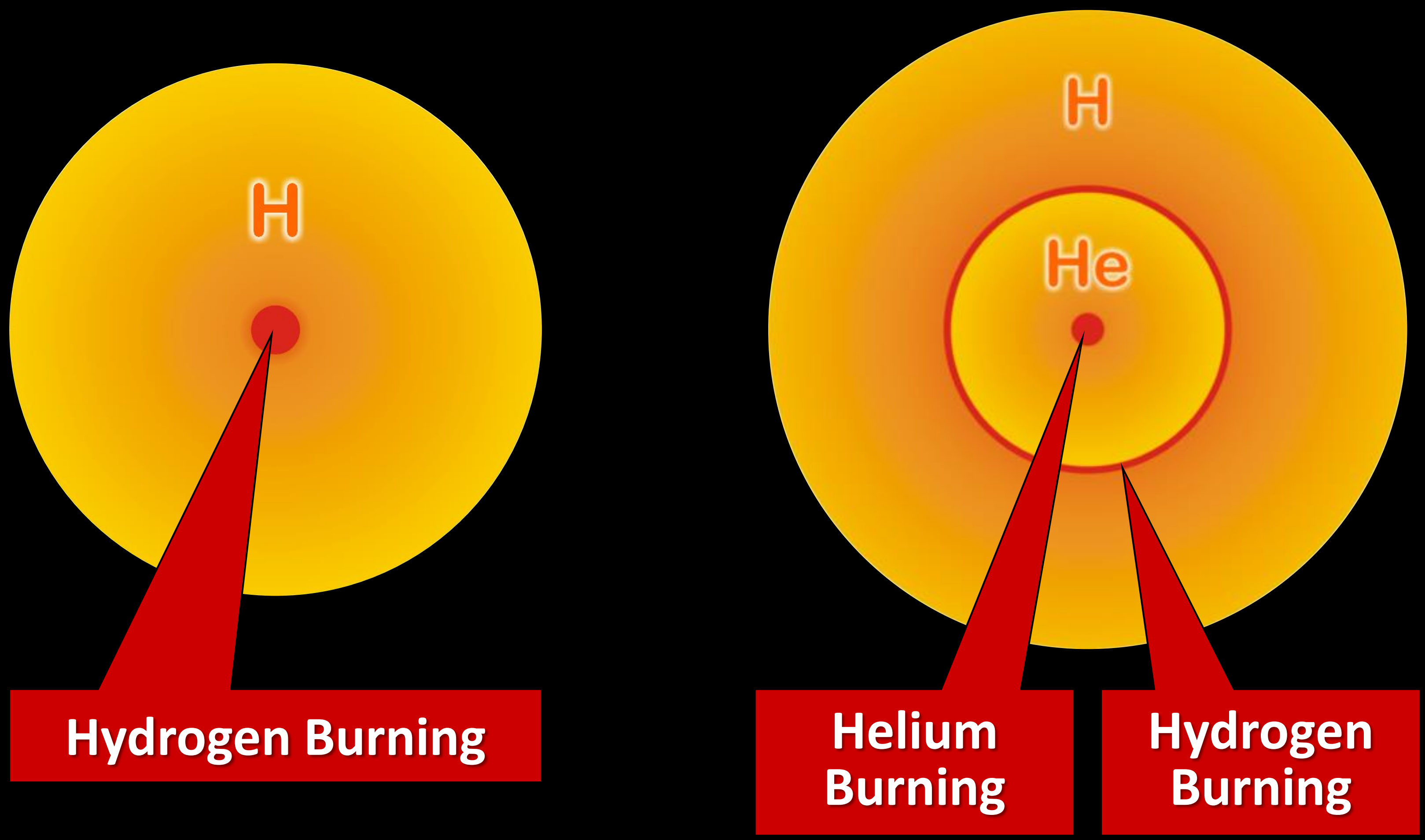
Typical temperatures: 10^9 K (~100 keV)

- Each type of burning occurs at a very different T but a broad range of densities
- Never co-exist in the same location

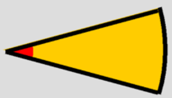
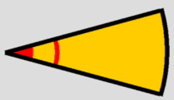
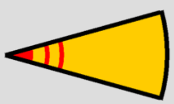
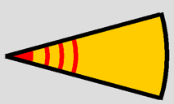
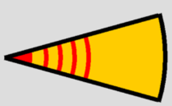
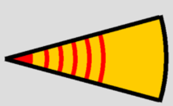


Hydrogen Exhaustion

Main-sequence star | **Helium-burning star**



Burning Phases of a 15 Solar-Mass Star

Burning Phase		Dominant Process	T_c [keV]	ρ_c [g/cm^3]	L_γ [10^4 L_{sun}]	L_ν/L_γ	Duration [years]
	Hydrogen	H → He	3	5.9	2.1	–	1.2×10^7
	Helium	He → C, O	14	1.3×10^3	6.0	1.7×10^{-5}	1.3×10^6
	Carbon	C → Ne, Mg	53	1.7×10^5	8.6	1.0	6.3×10^3
	Neon	Ne → O, Mg	110	1.6×10^7	9.6	1.8×10^3	7.0
	Oxygen	O → Si	160	9.7×10^7	9.6	2.1×10^4	1.7
	Silicon	Si → Fe, Ni	270	2.3×10^8	9.6	9.2×10^5	6 days

Neutrinos from Thermal Processes

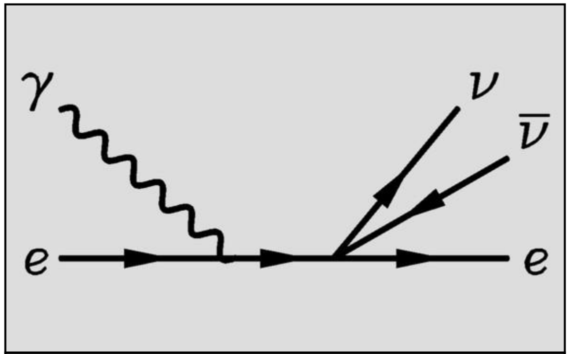
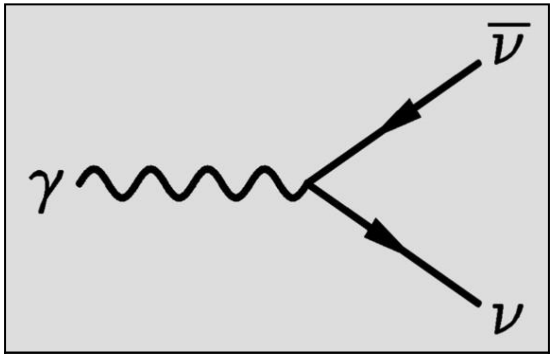
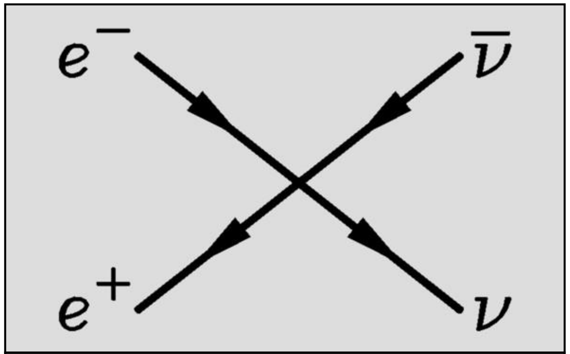


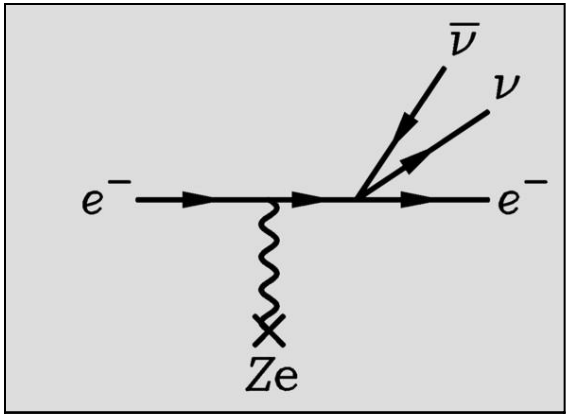
Photo (Compton)



Plasmon decay

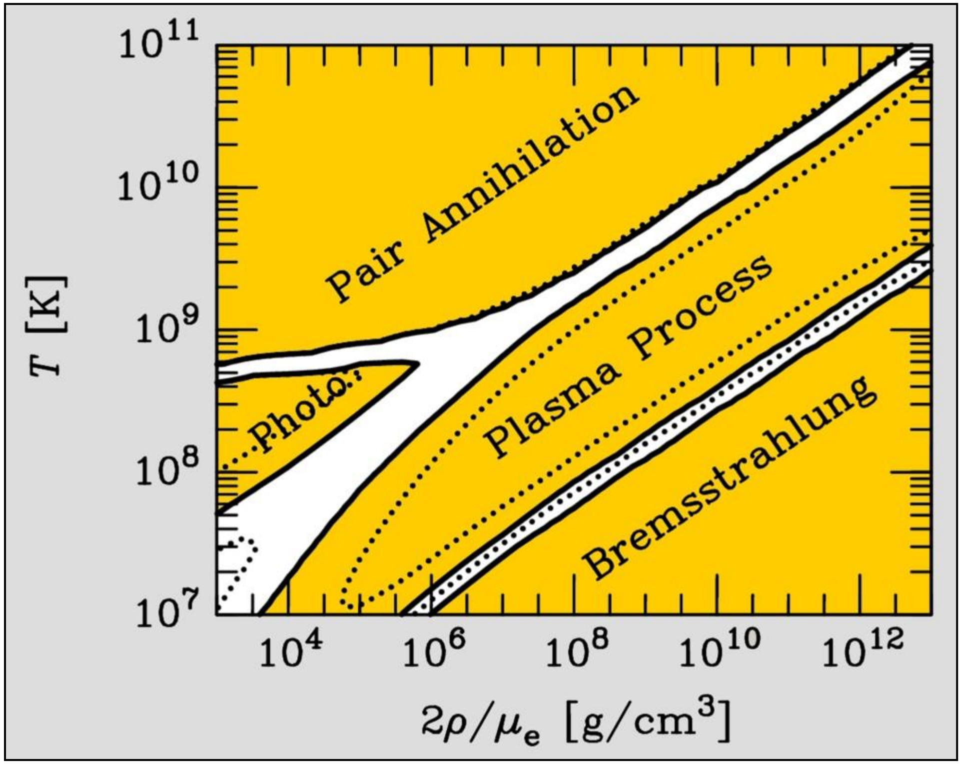


Pair annihilation

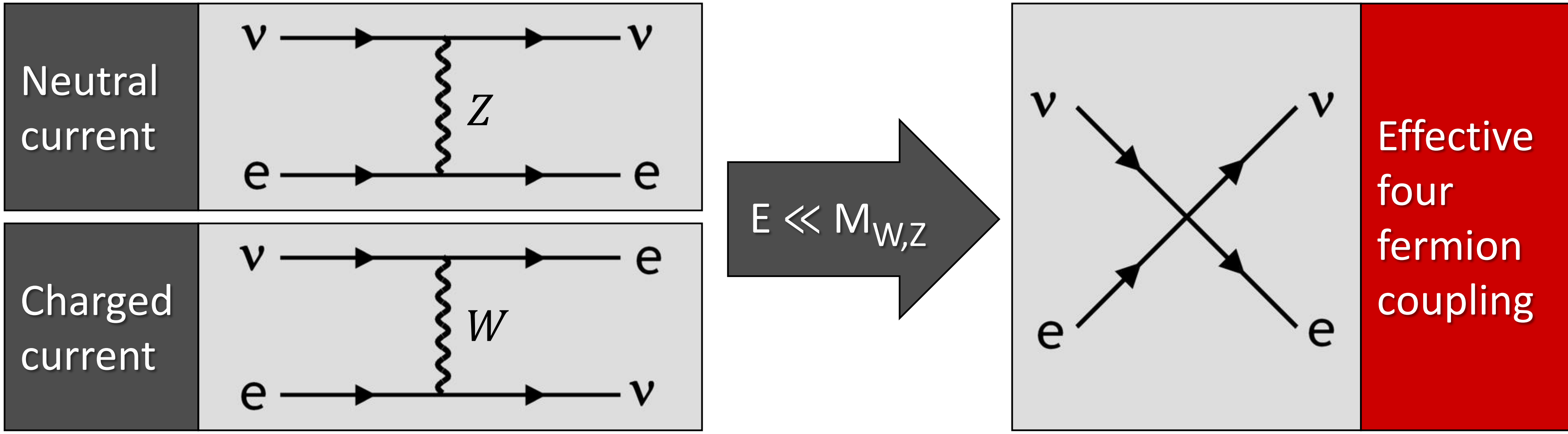


Bremsstrahlung

These processes were first discussed in 1961–63 after V–A theory



Effective Neutrino Neutral-Current Couplings



$$H_{\rm int} = \frac{G_{\rm F}}{\sqrt{2}} \overline{\Psi}_f \gamma_\mu (C_{\rm V} - C_{\rm A}\gamma_5)\Psi_f \, \overline{\Psi}_\nu \gamma^\mu (1-\gamma_5)\Psi_\nu$$

Neutrino	Fermion	C_V	C_A
ν_e	Electron	$+\frac{1}{2} + 2\sin^2\Theta_{\rm W} \approx 1$	$+\frac{1}{2}$
$\nu_\mu \;\; \nu_\tau$		$-\frac{1}{2} + 2\sin^2\Theta_{\rm W} \approx 0$	$-\frac{1}{2}$
$\nu_e \;\; \nu_\mu \;\; \nu_\tau$	Proton	$+\frac{1}{2} - 2\sin^2\Theta_{\rm W} \approx 0$	$+\frac{1.26}{2}$
	Neutron	$-\frac{1}{2}$	$-\frac{1.26}{2}$

Fermi constant $G_{\rm F}$
$1.166 \times 10^{-5}\,{\rm GeV}^{-2}$

Weak mixing angle
$\sin^2\Theta_{\rm W} = 0.231$

ASTROPHYSICAL DETERMINATION OF THE COUPLING CONSTANT FOR THE ELECTRON-NEUTRINO WEAK INTERACTION

Richard B. Stothers*
Goddard Institute for Space Studies, National Aeronautics and Space Administration, New York, New York 10025
(Received 22 December 1969)

The existence of the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ weak interaction is confirmed by the results of nine astrophysical tests. The value of the coupling constant is equal to, or close to, the coupling constant of beta decay, namely, $g^2 = 10^{0\pm2} g_\beta{}^2$.

Of all the astrophysical tests applied so far for the inference of a direct electron-neutrino interaction in nature, none has unambiguously provided a useful upper limit on the coupling constant, which in the $V-A$ theory of Feynman and Gell-Mann[1] is taken to be equal to the "universal" weak-interaction coupling constant measured from beta decays (called g_β hereafter). However, it is important to point out that these tests, made by the author and his colleagues during the past eight years, do provide a nonzero lower limit, and therefore establish at least the existence of the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ interaction. It should be emphasized, nonetheless, that all of these tests rely on the validity of various stellar model calculations. These models, while not subject to scrutiny in the same sense as a laboratory experiment, are based on reasonable physical assumptions, so that the input physics is presumed relative theoretical lifetimes, calculated with and without the inclusion of neutrino emission. In this Letter, the unmodified term "luminosity" will mean the photon luminosity L radiated by the star. The "neutrino luminosity" will be designated L_ν. Quantities referring to the sun are subscripted with an encircled dot.

The most accurate available data on white dwarfs are those collected by Eggen[7] for the two clusters Hyades and Pleiades and for the nearby general field. Of chief interest here are the hot white dwarfs, for which the observational data[7,8] have been reduced following the procedure of Van Horn.[9] The resulting luminosities are estimated to have a statistical accuracy of ±0.1 in $\log(L/L_\odot)$, which is adequate here.

Models of cooling white dwarfs have been constructed in great detail by a large number of authors. Fortunately, the stellar structure is

Neutrinos from Thermal Processes

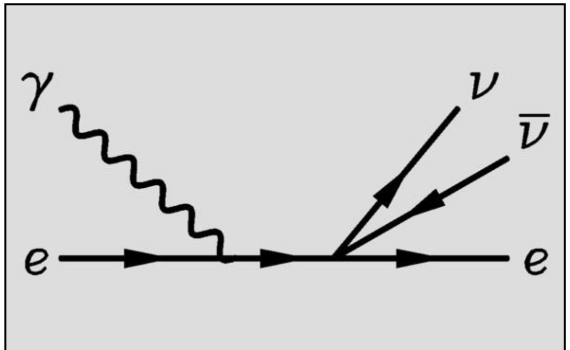
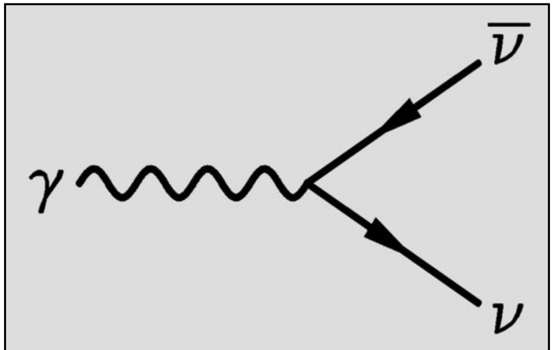
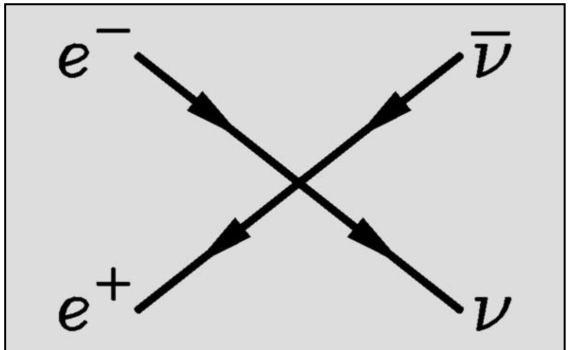


Photo (Compton)



Plasmon decay



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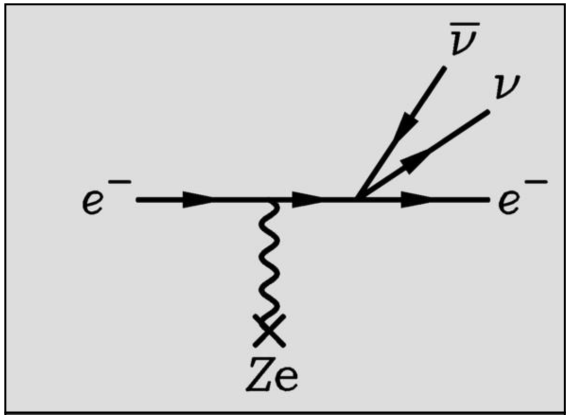

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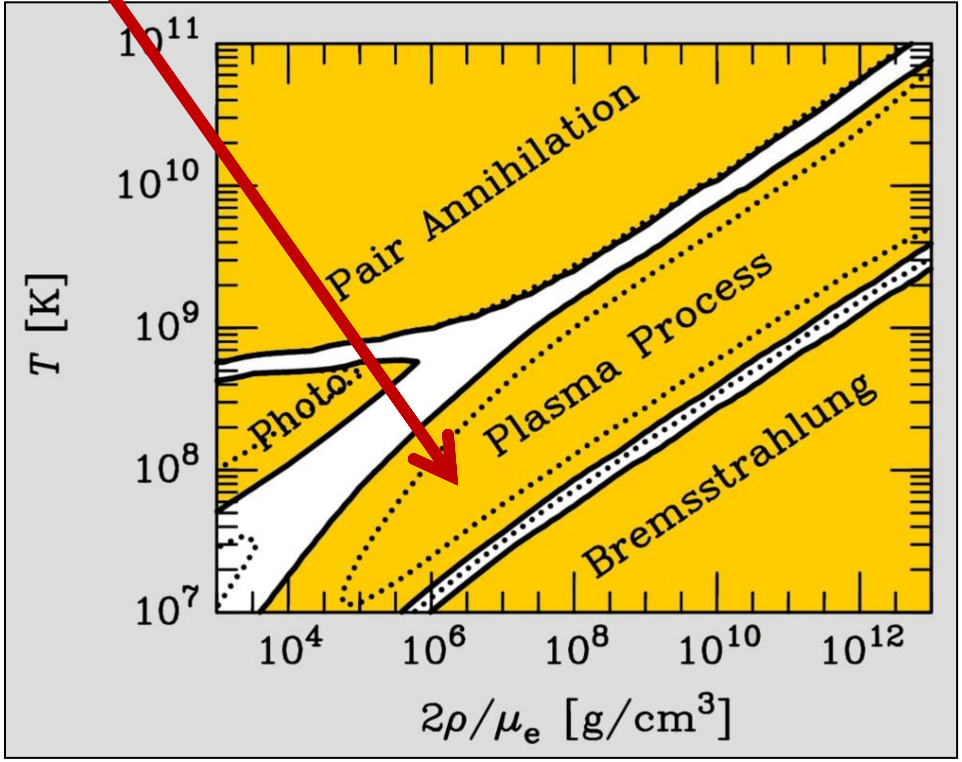
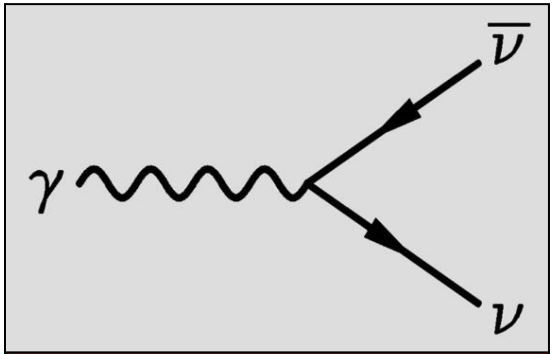


Plasmon Decay in Neutrinos

Propagation in vacuum:
- Photon massless
- Can not decay into other particles, even if they themselves are massless



Plasmon decay

Interaction in vacuum:
- Massless neutrinos do not couple to photons
- May have dipole moments or even “millicharges”

Propagation in a medium:
- Photon acquires a “refractive index”
- In a non-relativistic plasma (e.g. Sun, white dwarfs, core of red giant before helium ignition, …) behaves like a massive particle:

  $\omega^2 - k^2 = \omega_{\rm pl}^2$

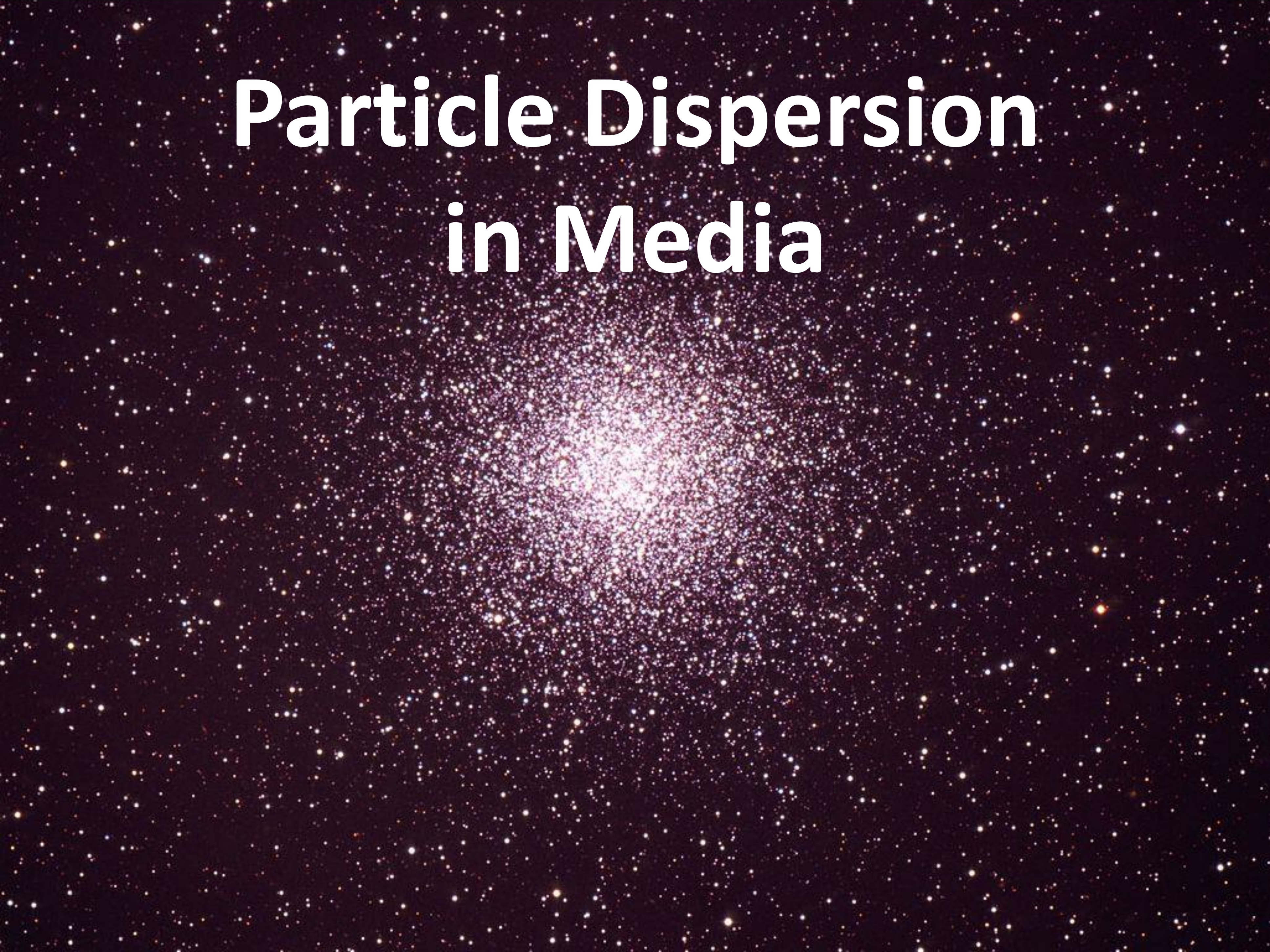
  Plasma frequency $\omega_{\rm pl}^2 = 4\pi\alpha n_e/m_e$ (electron density n_e)
- Degenerate helium core $\omega_{\rm pl} = 18\ {\rm keV}$ ($\rho = 10^6 {\rm g\ cm^{-3}},\ T = 8.6\ {\rm keV}$)

Interaction in a medium:
- Neutrinos interact coherently with the charged particles which themselves couple to photons
- Induces an “effective charge”
- In a degenerate plasma (electron Fermi energy E_F and Fermi momentum p_F)

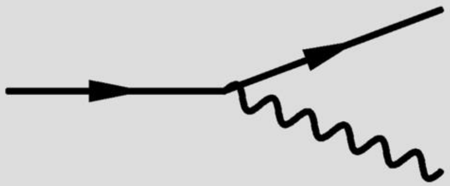
  $$\frac{e_\nu}{e} = 16\sqrt{2}\, {\rm C_V\, G_F\, E_F\, p_F}$$
- Degenerate helium core (and $C_V = 1$)

  $e_\nu = 6 \times 10^{-11} e$

Particle Dispersion in Media



Plasmon Decay vs. Cherenkov Effect

Photon dispersion in a medium can be	"Time-like" $\omega^2 - k^2 > 0$	"Space-like" $\omega^2 - k^2 < 0$
Refractive index n $(k = n\,\omega)$	$n < 1$	$n > 1$
Example	• Ionized plasma • Normal matter for large photon energies	Water ($n \approx 1.3$), air, glass for visible frequencies
Allowed process that is forbidden in vacuum	Plasmon decay to neutrinos $\gamma \to \nu\bar{\nu}$ 	Cherenkov effect $e \to e + \gamma$ 

Particle Dispersion in Media

Vacuum	Most general Lorentz-invariant dispersion relation $\omega^2 - k^2 = m^2$ ω = frequency, k = wave number, m = mass Gauge invariance implies $m = 0$ for photons and gravitons
Medium	Particle interaction with medium breaks Lorentz invariance so that $\omega^2 - k^2 = \pi(\omega, k)$ Implies a relationship between ω and k (dispersion relation) Often written in terms of • Refractive index n $k = n\,\omega$ • Effective mass (note that $m_{\rm eff}$ can be negative) $\omega^2 - k^2 = m_{\rm eff}^2$ • Effective potential (natural for neutrinos with m the vacuum mass) $(\omega - V)^2 - k^2 = m^2$ Which form to use depends on convenience

Refraction and Forward Scattering

Plane wave in vacuum

$$\Phi(\boldsymbol{r},t) \propto e^{-i\omega t + i\boldsymbol{k}\cdot\boldsymbol{r}}$$

With scattering centers

$$\Phi(\boldsymbol{r},t) \propto e^{-i\omega t}\left[e^{ik\cdot r} + f(\omega,\theta)\,\frac{e^{ik\cdot r}}{r}\right]$$

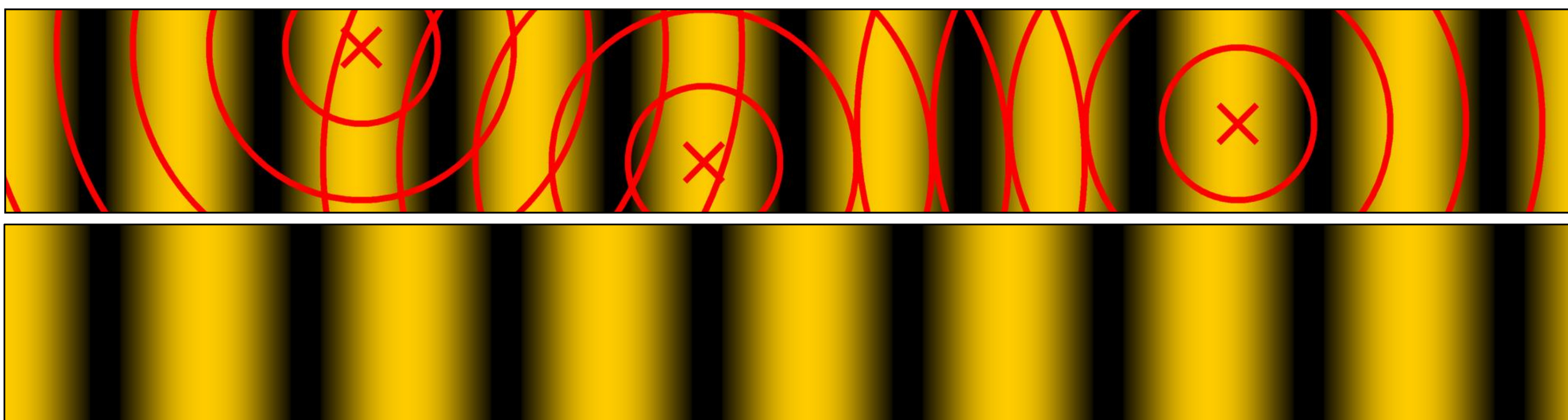
In forward direction, adds coherently to a plane wave with modified wave number

$$k = n_{\rm refr}\,\omega$$

$$n_{\rm refr} = 1 + \frac{2\pi}{\omega^2}\,N\,f(\omega,0)$$

N = number density of scattering centers

$f(\omega,0)$ = forward scattering amplitude



Electromagnetic Polarization Tensor

Klein-Gordon-Equation in Fourier space	$(-K^2 g^{\mu\nu} + K^\mu K^\nu + \Pi^{\mu\nu})A_\nu = 0$
Polarization tensor (self-energy of photon)	
Gauge invariance and current conservation	$\Pi^{\mu\nu}K_\mu = \Pi^{\mu\nu}K_\nu = 0$ Vacuum: $\Pi^{\mu\nu} = a g^{\mu\nu} + b K^\mu K^\nu$ $\rightarrow a = b = 0$ (photon massless) Medium: Four-velocity U available to construct Π

QED Plasma

$$\Pi^{\mu\nu}(K) = 16\pi\alpha \int \frac{d^3\boldsymbol{p}}{2E(2\pi)^3} f_e(\boldsymbol{p}) \frac{(PK)^2 g^{\mu\nu} + K^2 P^\mu K^\nu - PK(P^\mu K^\nu + K^\mu P^\nu)}{(PK)^2 - \frac{1}{4}(K^2)^2}$$

Photon: $K = (\omega, \boldsymbol{k})$ Electron/positron: $P = (E, \boldsymbol{p})$ with $E = \sqrt{\boldsymbol{p}^2 + m_e^2}$

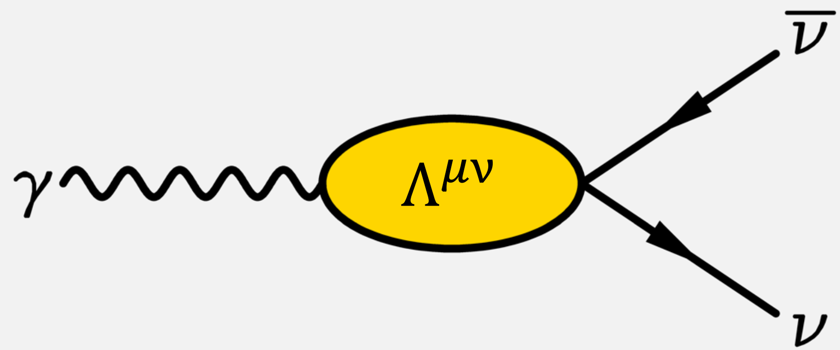
Electron/positron phase-space distribution with chemical potenial μ_e

$$f_e(\boldsymbol{p}) = \frac{1}{e^{\frac{E-\mu_e}{T}} + 1} + \frac{1}{e^{\frac{E+\mu_e}{T}} + 1}$$

Neutrino-Photon-Coupling in a Plasma

Neutrino effective in-medium coupling

$$L_{\rm eff} = -\sqrt{2}G_{\rm F}\overline{\Psi}\gamma_\alpha \frac{1}{2}(1-\gamma_5)\Psi\Lambda^{\alpha\beta}A_\beta$$



For vector current it is analogous to photon polarization tensor

$$\Lambda_{\rm V}^{\mu\nu}(K) = 4eC_{\rm V}\int\frac{d^3\mathbf{p}}{2E(2\pi)^3}[f_e(\mathbf{p})+f_{\overline{e}}(\mathbf{p})]\,\frac{(PK)^2g^{\mu\nu}+K^2P^\mu P^\nu - PK(P^\mu K^\nu + K^\mu P^\nu)}{(PK)^2-\frac{1}{4}(K^2)^2}$$

$$= \frac{C_{\rm V}}{e}\,\Pi_{\rm V}^{\mu\nu}(K)$$

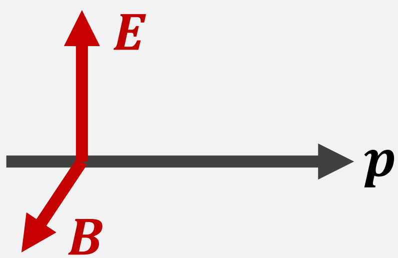
$$\Lambda_{\rm A}^{\mu\nu}(K) = 2ieC_{\rm A}\epsilon^{\mu\nu\alpha\beta}\int\frac{d^3\mathbf{p}}{2E(2\pi)^3}[f_e(\mathbf{p})-f_{\overline{e}}(\mathbf{p})]\,\frac{K^2P_\alpha K_\beta}{(PK)^2-\frac{1}{4}(K^2)^2}$$

Usually negligible

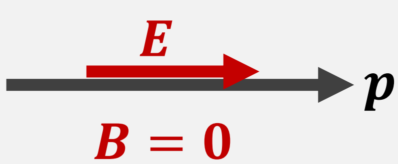
Transverse and Longitudinal “Plasmons”

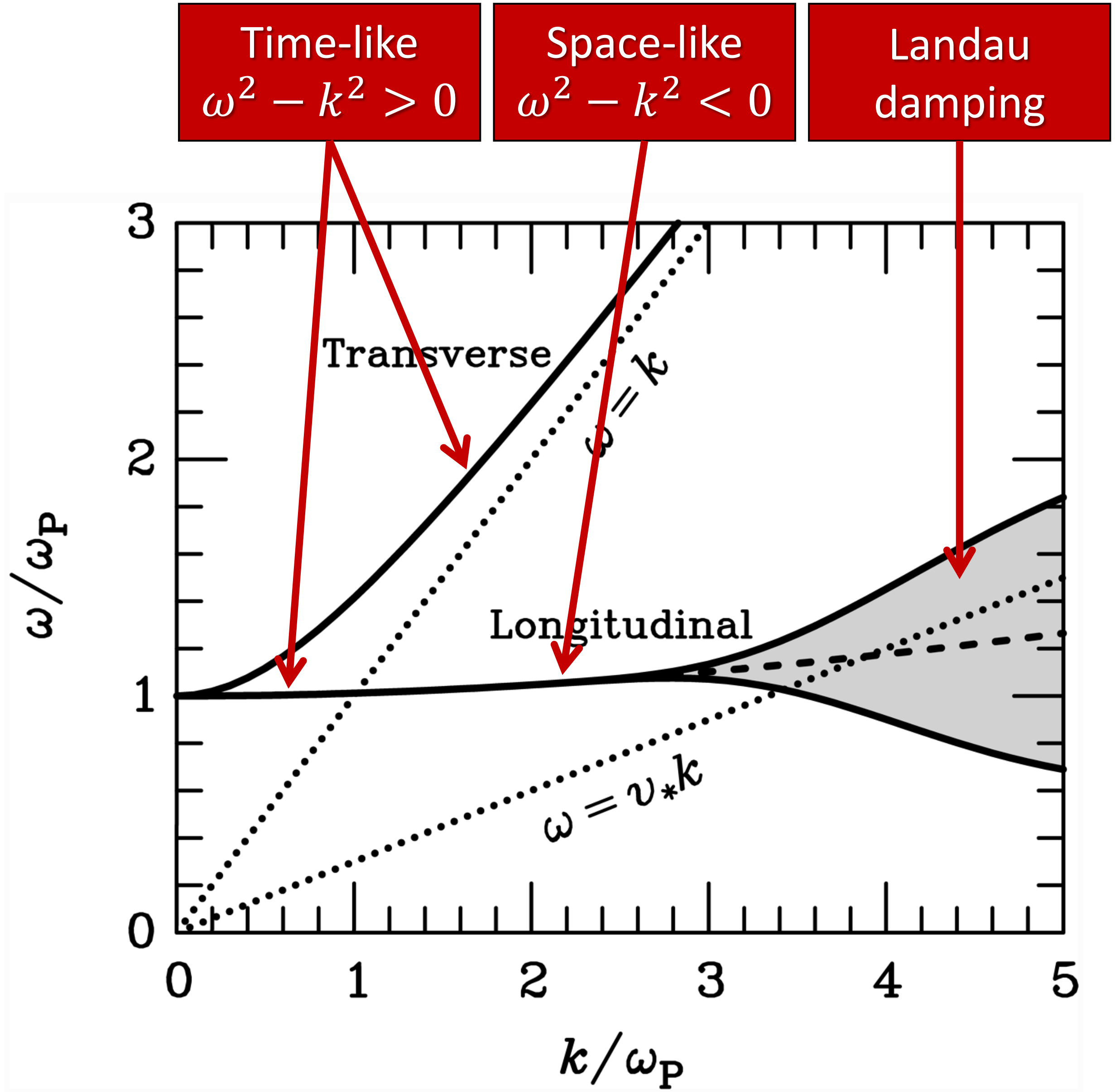
Dispersion relation in a non-relativistic, non-degenerate plasma

Transverse Excitation



Longitudinal Excitation (oscillation of electrons against positive charges)





Electron (Positron) Dispersion Relation

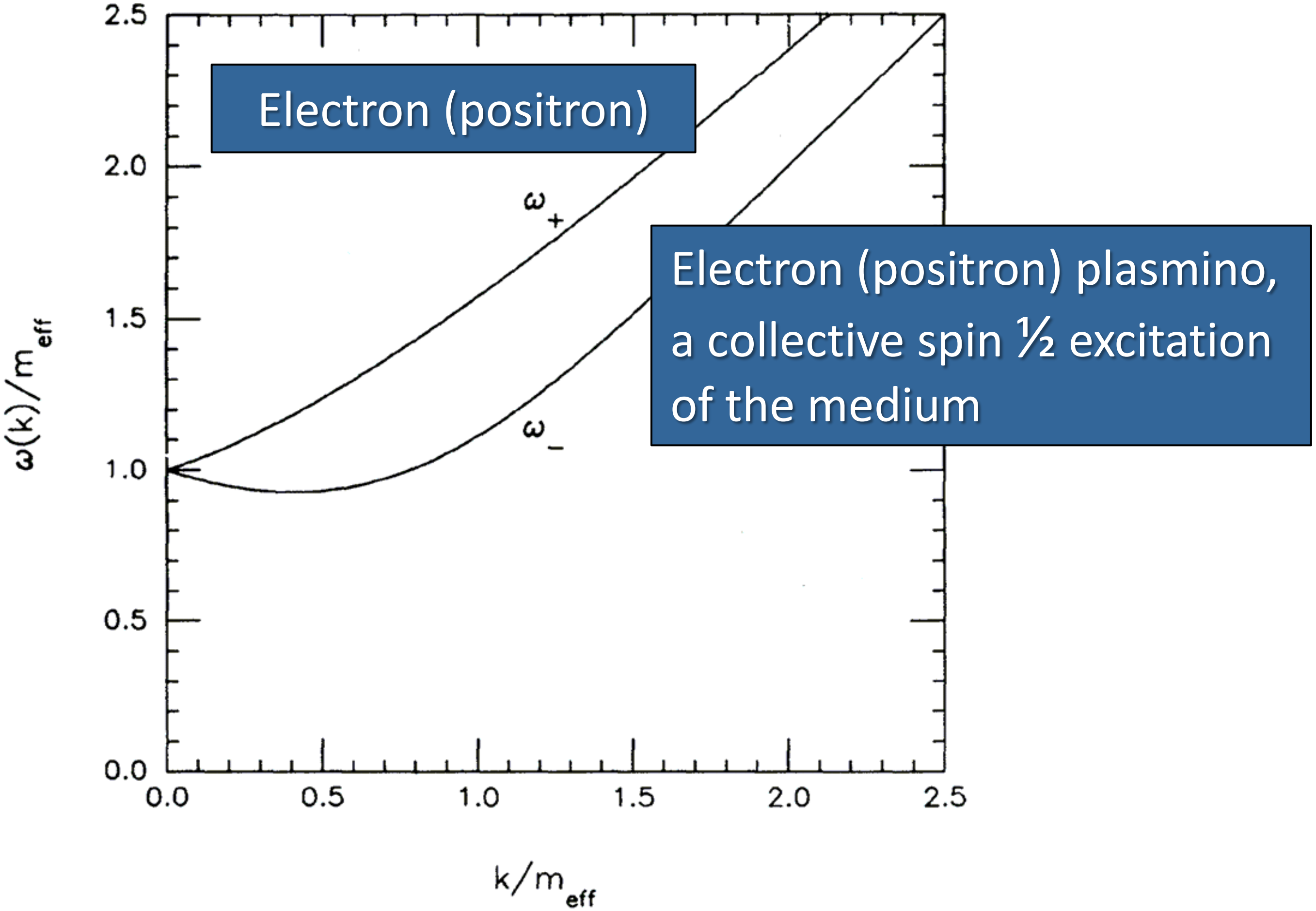


FIG. 1.—Ultrarelativistic dispersion relations for the electron or positron [$\omega_+(k)$] and for the electron plasmino or positron plasmino [$\omega_-(k)$].

E. Braaten, Neutrino emissivity of an ultrarelativistic plasma from positron and plasmino annihilation, Astrophys. J. 392 (1992) 70

PHYSICAL REVIEW D VOLUME 17, NUMBER 9 1 MAY 1978

3700 citations

Neutrino oscillations in matter

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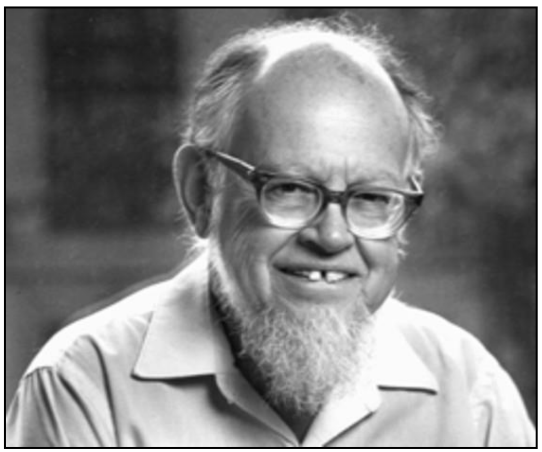
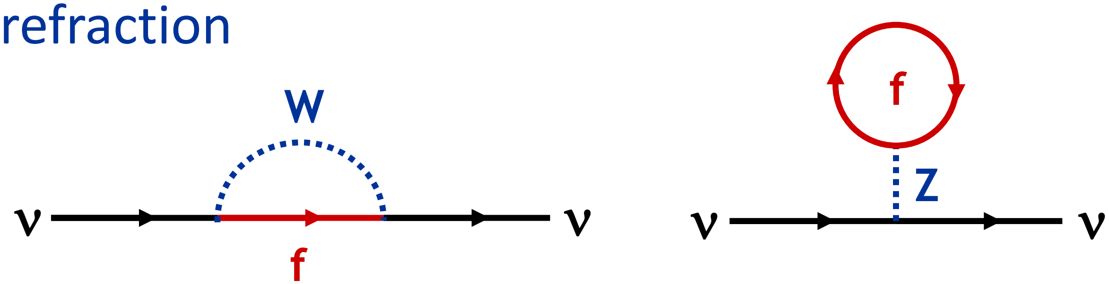
(Received 6 October 1977; revised manuscript received 5 December 1977)

The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.



Lincoln Wolfenstein

Neutrinos in a medium suffer flavor-dependent refraction



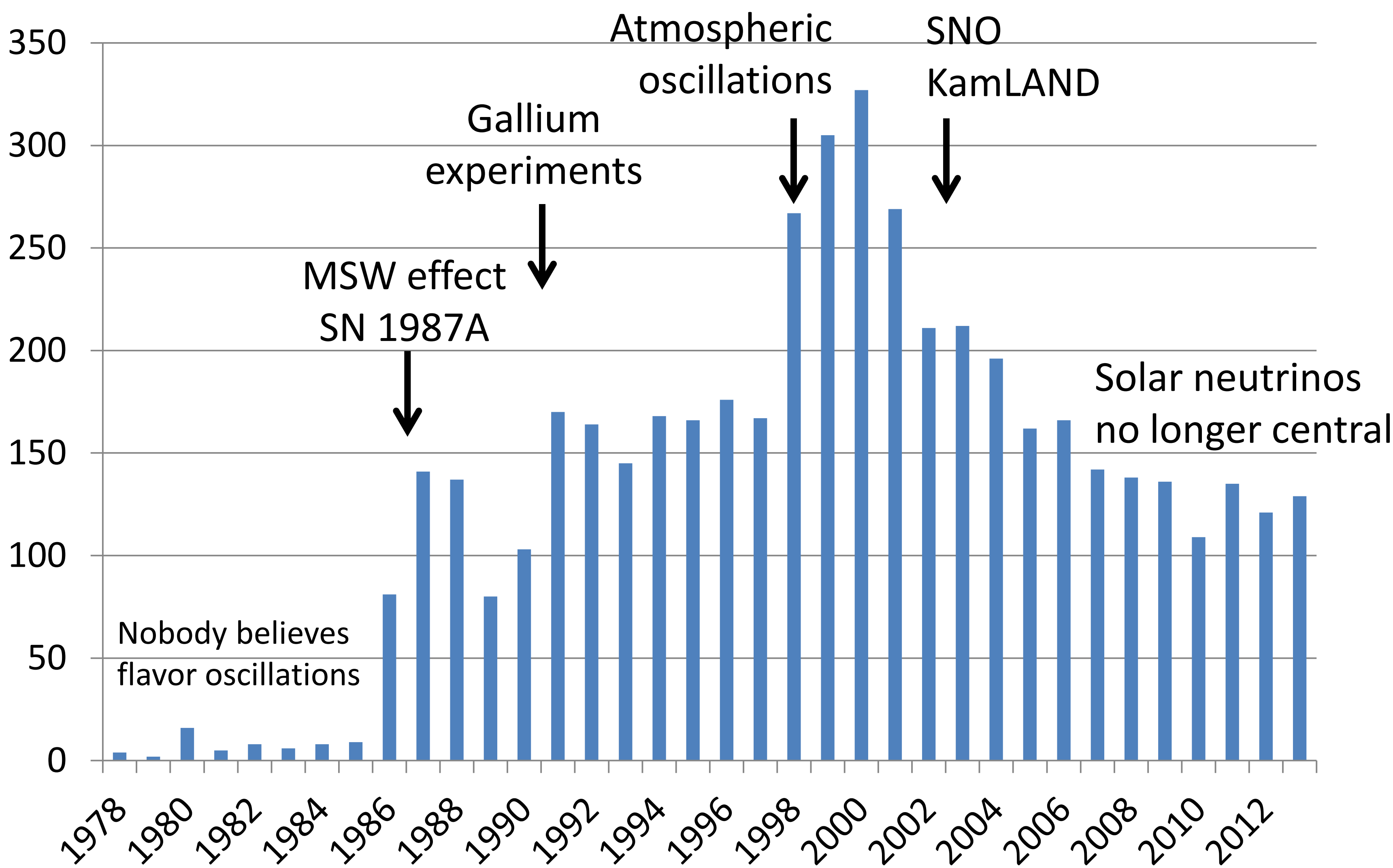
$$V_{\rm weak} = \sqrt{2} G_{\rm F} \times \begin{cases} N_{\rm e} - N_{\rm n}/2 & \text{for } \nu_{\rm e} \\ -N_{\rm n}/2 & \text{for } \nu_{\mu} \end{cases}$$

Typical density of Earth: 5 g/cm^3

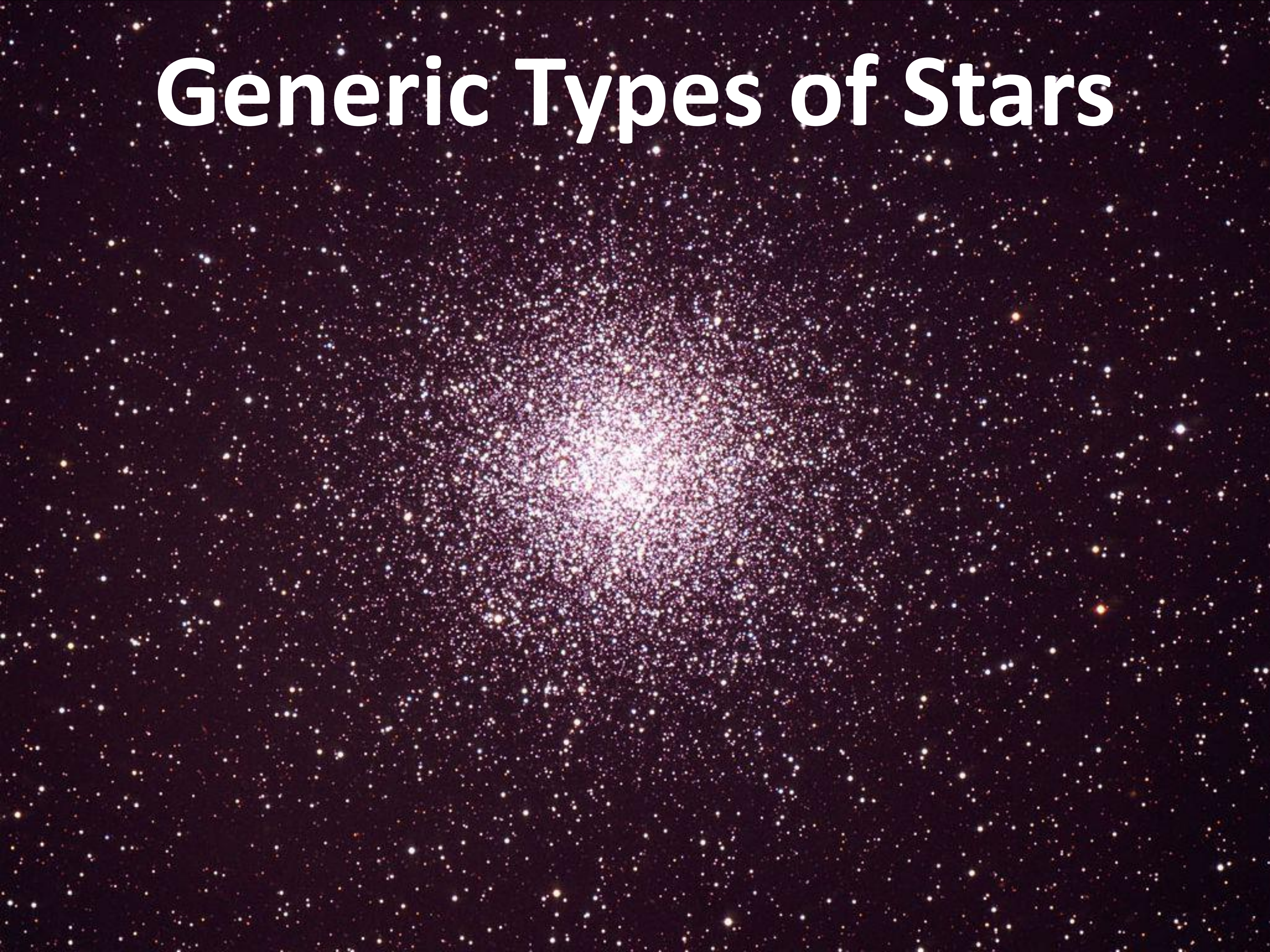
$$\Delta V_{\rm weak} \approx 2 \times 10^{-13}\ {\rm eV} = 0.2\ {\rm peV}$$

Citations of Wolfenstein's Paper on Matter Effects

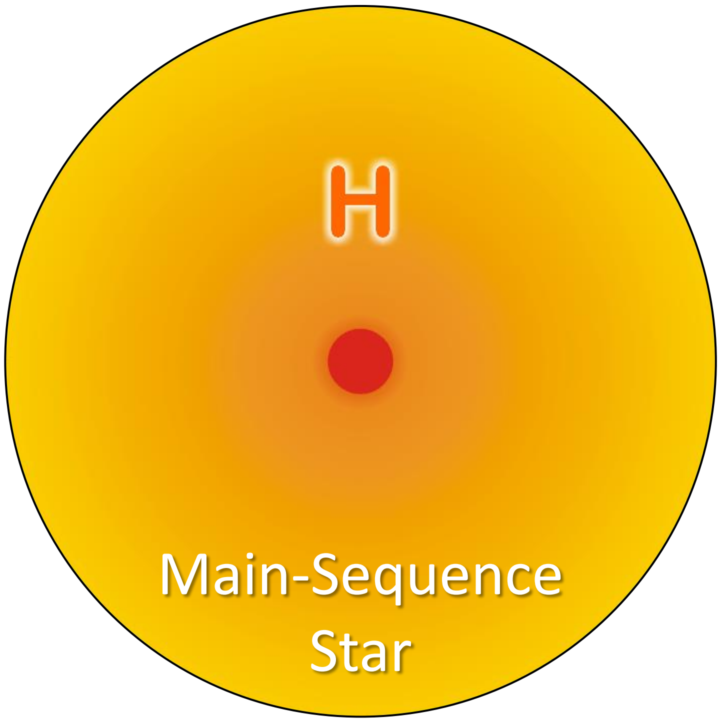
inSPIRE: 3700 citations of Wolfenstein, Phys. Rev. D17 (1978) 2369



Generic Types of Stars



Self-Regulated Nuclear Burning



Virial Theorem: $\langle E_{\rm kin}\rangle = -\frac{1}{2}\langle E_{\rm grav}\rangle$

Small Contraction
$\rightarrow$ Heating
$\rightarrow$ Increased nuclear burning
$\rightarrow$ Increased pressure
$\rightarrow$ Expansion

Additional energy loss ("cooling")
$\rightarrow$ Loss of pressure
$\rightarrow$ Contraction
$\rightarrow$ Heating
$\rightarrow$ Increased nuclear burning

Hydrogen burning at nearly fixed T
$\rightarrow$ Gravitational potential nearly fixed:
$G_{\rm N}M/R \sim \text{constant}$
$\rightarrow$ $R \propto M$ (More massive stars bigger)

Modified Stellar Properties by Particle Emission

Assume that some small perturbation (e.g. axion emission) leads to a "homologous" modification: Every point is mapped to a new position $r' = yr$

Requires power-law relations for constitutive relations

- Nuclear burning rate $\epsilon \propto \rho^n T^m$
- Mean opacity $\kappa \propto \rho^s T^t$

Implications for other quantities

- Density $\rho'(r') = y^{-3}\rho(r)$
- Pressure $p'(r') = y^{-4}p(r)$
- Temperature gradient $dT'(r')/dr' = y^{-2}\, dT(r)/dr$

Impact of small novel energy loss

- Modified nuclear burning rate $\epsilon = (1-\delta_x)\,\epsilon_{\rm nuc}$
- Assume Kramers opacity law $\mathrm{s} = 1$ and $\mathrm{t} = -3.5$
- Hydrogen burning $\mathrm{n} = 1$ and $\mathrm{m} = 4\text{–}6$
- Star contracts, heats, and shines brighter in photons:

$$\frac{\delta R}{R} = -\frac{2\delta_x}{2m+5} \qquad \frac{\delta T}{T} = \frac{\delta_x}{2m+5} \qquad \frac{\delta L_\gamma}{L_\gamma} = \frac{\delta_x}{2m+5}$$

Degenerate Stars (“White Dwarfs”)

Assume temperature very small
$\rightarrow$ No thermal pressure
$\rightarrow$ Electron degeneracy is pressure source

Pressure ~ Momentum density × Velocity

- Electron density $n_e = p_\mathrm{F}^3/(3\pi^3)$
- Momentum p_F (Fermi momentum)
- Velocity $v \propto p_\mathrm{F}/m_e$
- Pressure $P \propto p_\mathrm{F}^5 \propto \rho^{5/3} \propto M^{5/3}R^{-5}$
- Density $\rho \propto MR^{-3}$

Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{G_\mathrm{N} M_r \rho}{r^2}$$

With $dP/dr \sim -P/R$ we have

$$P \propto G_\mathrm{N} M \rho R^{-1} \propto G_\mathrm{N} M^2 R^{-4}$$

Inverse mass radius relationship

$$R \propto M^{-1/3}$$

$$R = 10{,}500\ \mathrm{km} \left(\frac{0.6\ M_\odot}{M}\right)^{1/3} (2Y_e)^{5/3}$$

(Y_e electrons per nucleon)

For sufficiently large stellar mass M, electrons become relativistic

- Velocity = speed of light
- Pressure

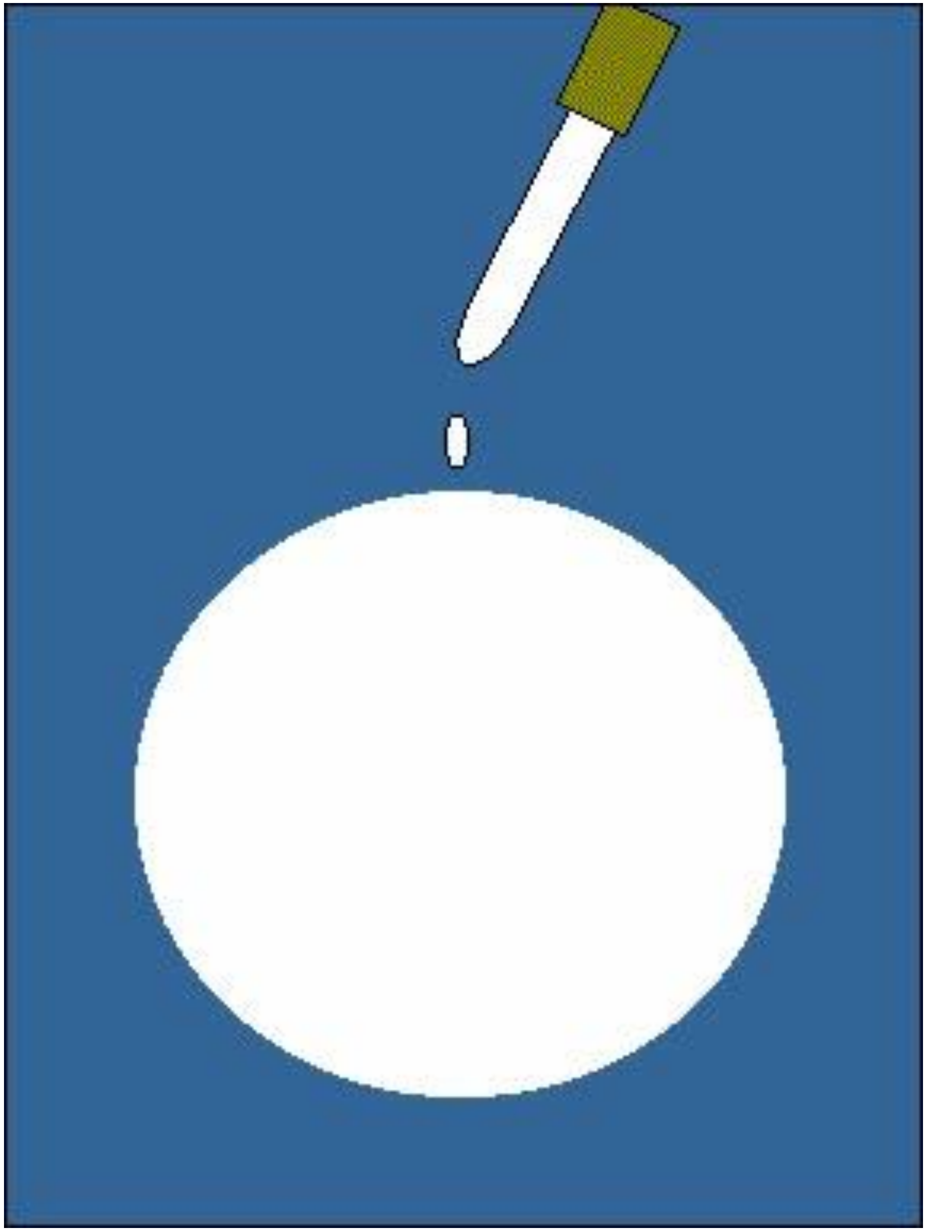
$$P \propto p_\mathrm{F}^4 \propto \rho^{4/3} \propto M^{4/3}R^{-4}$$

No stable configuration

Chandrasekhar mass limit

$$M_\mathrm{Ch} = 1.457\ M_\odot\ (2Y_e)^2$$

Degenerate Stars ("White Dwarfs")



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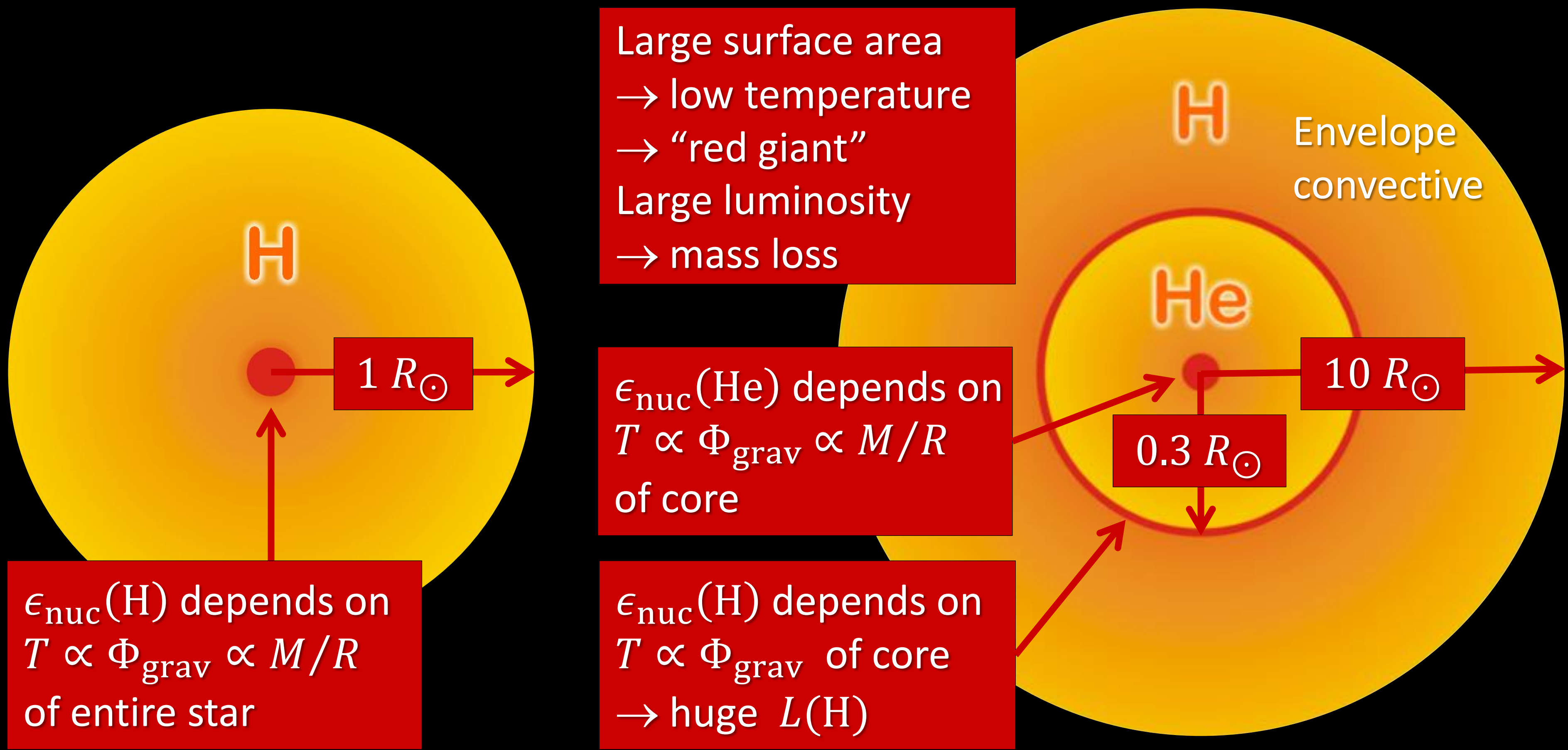
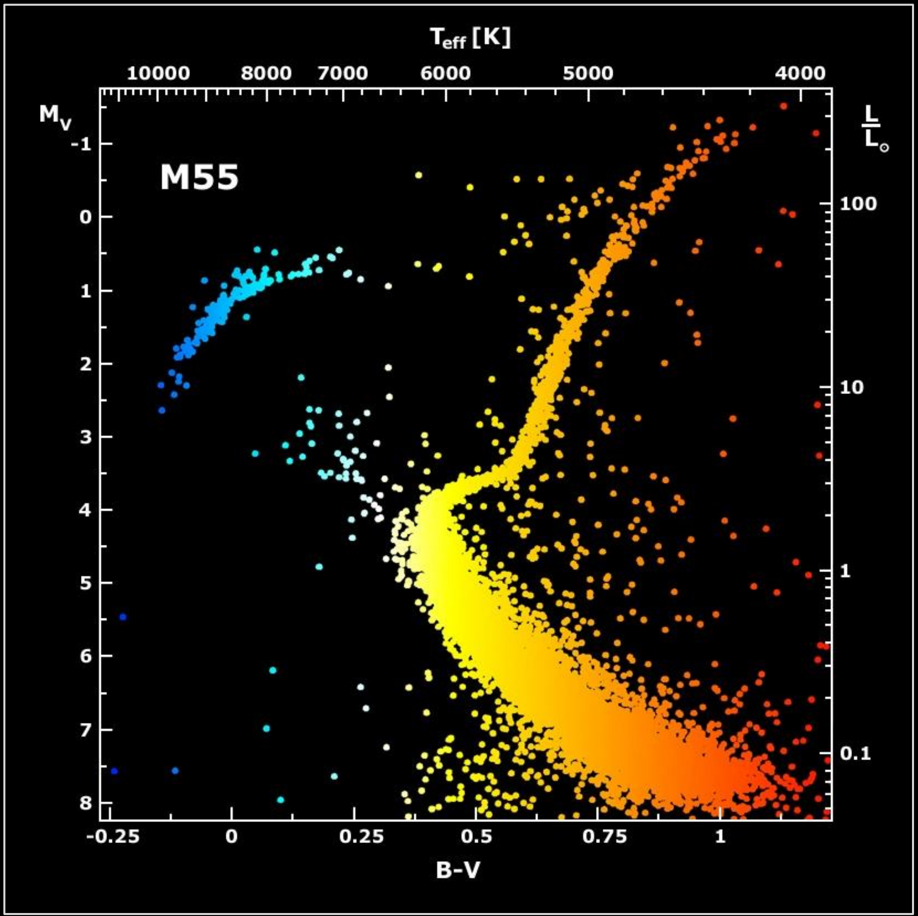
Giant Stars

Main-sequence star $1M_\odot$ (Hydrogen burning)

Helium-burning star $1M_\odot$

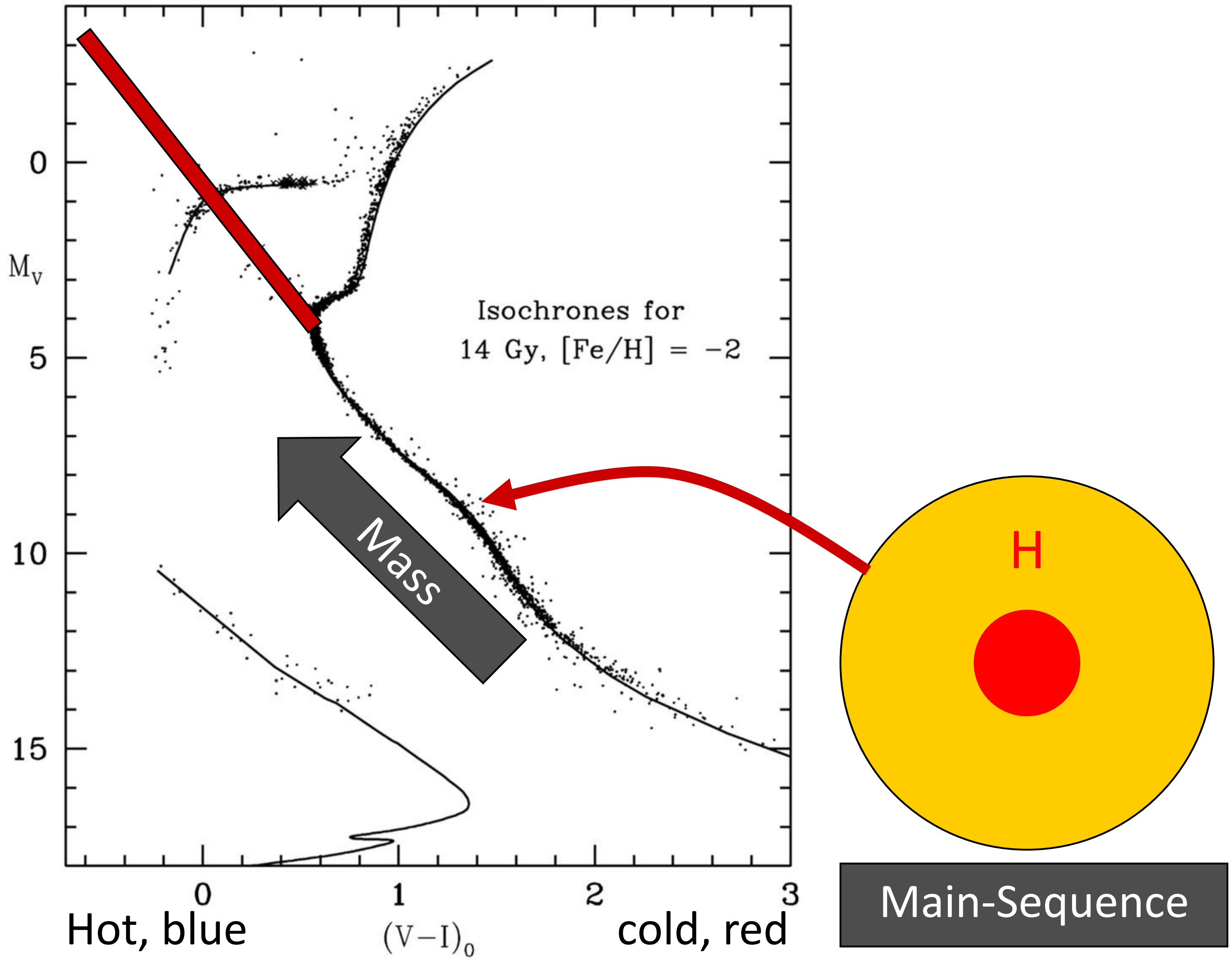


Galactic Globular Cluster M55



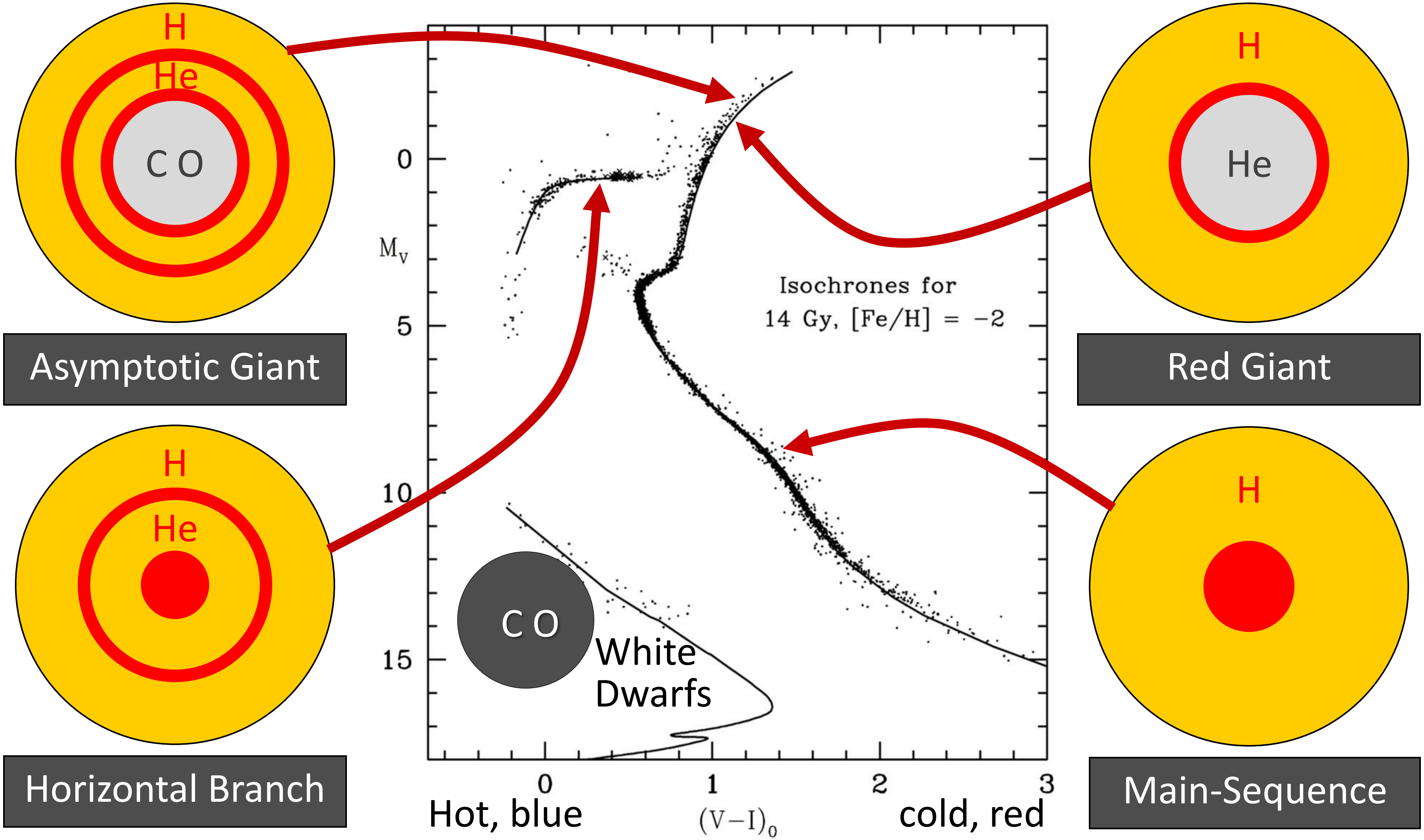
Color-Magnitude Diagram for Globular Clusters

- Stars with M so large that they have burnt out in a Hubble time
- No new star formation in globular clusters



Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris, 2000)

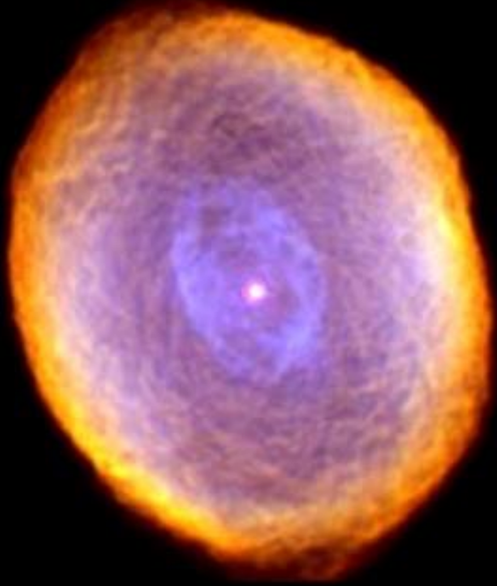
Color-Magnitude Diagram for Globular Clusters



Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris, 2000)

Planetary Nebulae

Hour Glass Nebula



Planetary Nebula IC 418

Planetary Nebula NGC 3132

Eskimo Nebula

Evolution of Stars

$M < 0.08\,M_{sun}$	Never ignites hydrogen → cools ("hydrogen white dwarf")		Brown dwarf
$0.08 < M \lesssim 0.8\,M_{sun}$	Hydrogen burning not completed in Hubble time		Low-mass main-sequence star
$0.8 \lesssim M \lesssim 2\,M_{sun}$	Degenerate helium core after hydrogen exhaustion		• Carbon-oxygen white dwarf • Planetary nebula
$2 \lesssim M \lesssim 5\text{–}8\,M_{sun}$	Helium ignition non-degenerate		
$8\,M_{sun} \lesssim M < ???$	All burning cycles → Onion skin structure with degenerate iron core	Core collapse supernova	• Neutron star (often pulsar) • Sometimes black hole? • Supernova remnant (SNR), e.g. crab nebula