Neutrinos

in Astrophysics and Cosmology

Cosmological Neutrinos 2

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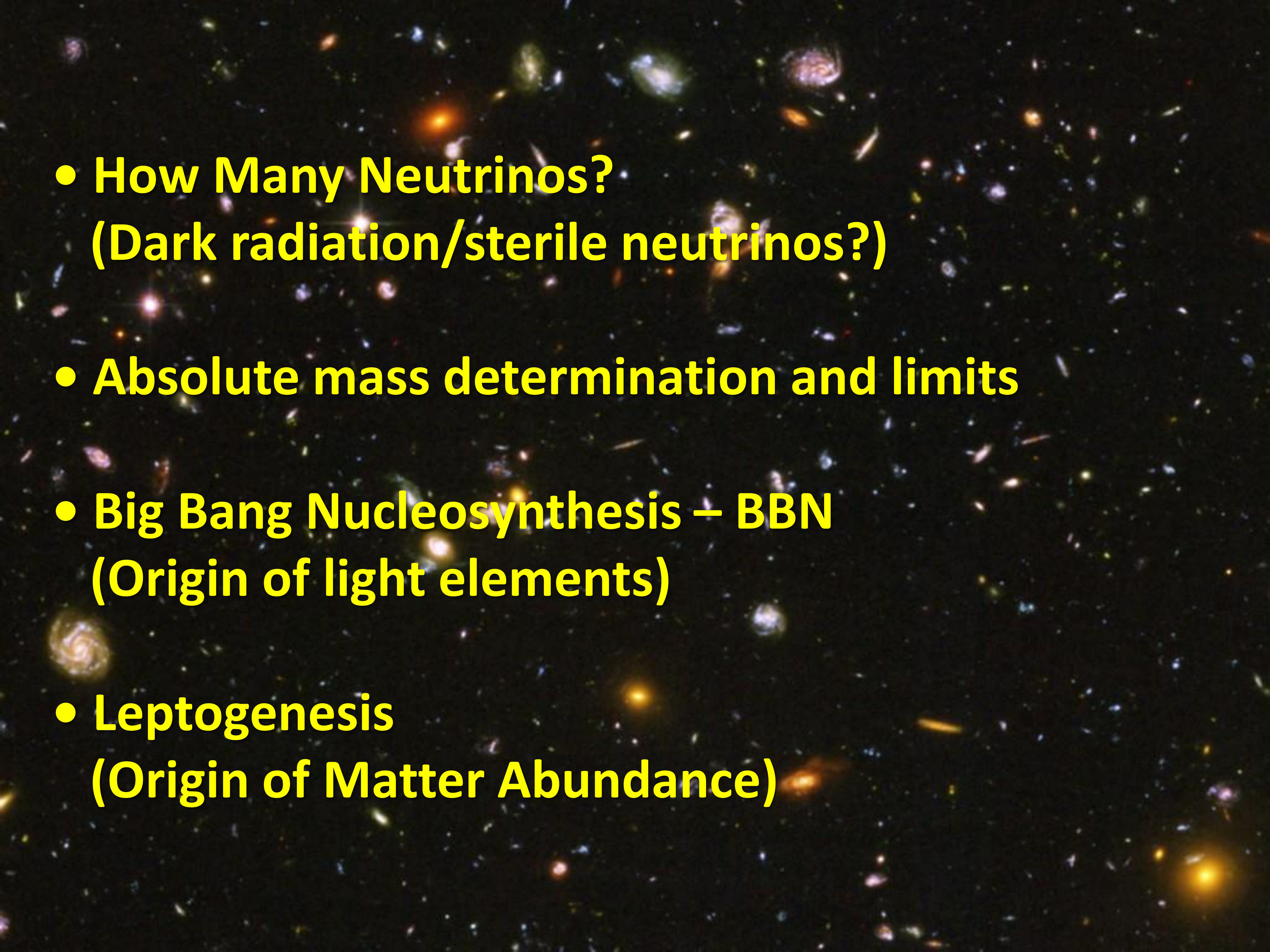
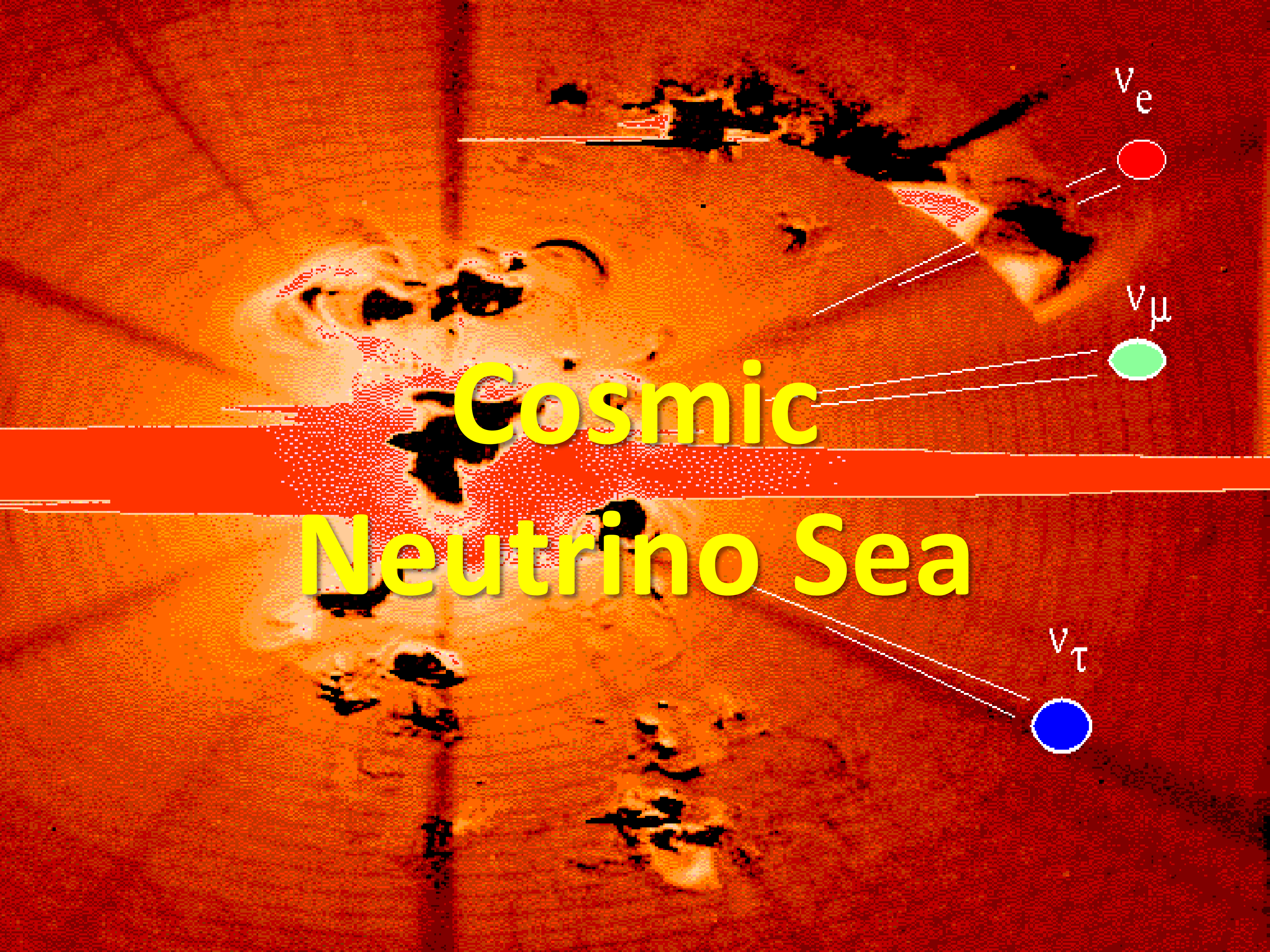


- How Many Neutrinos?
  (Dark radiation/sterile neutrinos?)
- Absolute mass determination and limits
- Big Bang Nucleosynthesis – BBN
  (Origin of light elements)
- Leptogenesis
  (Origin of Matter Abundance)



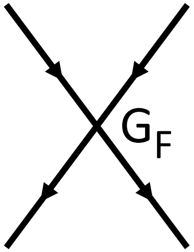
Cosmic Neutrino Sea

Neutrino Thermal Equilibrium

Neutrino reaction rate

Examples for neutrino processes

$$e^+ + e^- \leftrightarrow \overline{\nu} + \nu$$
$$\overline{\nu} + \nu \leftrightarrow \overline{\nu} + \nu$$
$$\nu + e^\pm \leftrightarrow \nu + e^\pm$$



Dimensional analysis of reaction rate in a thermal medium for $T \ll m_{W,Z}$

$$\Gamma \sim G_F^2 T^5$$

Cosmic expansion rate

Friedmann equation (flat universe)

$$H^2 = \frac{8\pi}{3}\frac{\rho}{m_{Pl}^2} \qquad \left(G_N = \frac{1}{m_{Pl}^2}\right)$$

Radiation dominates

$$\rho \sim T^4$$

Expansion rate

$$H \sim \frac{T^2}{m_{Pl}}$$

Condition for thermal equilibrium: $\Gamma > H$

$$T > \left(m_{Pl} G_F^2\right)^{-1/3} \sim \left[10^{19}\mathrm{GeV}\,\left(10^{-5}\mathrm{GeV}^{-2}\right)^2\right]^{-1/3} = 1\ \mathrm{MeV}$$

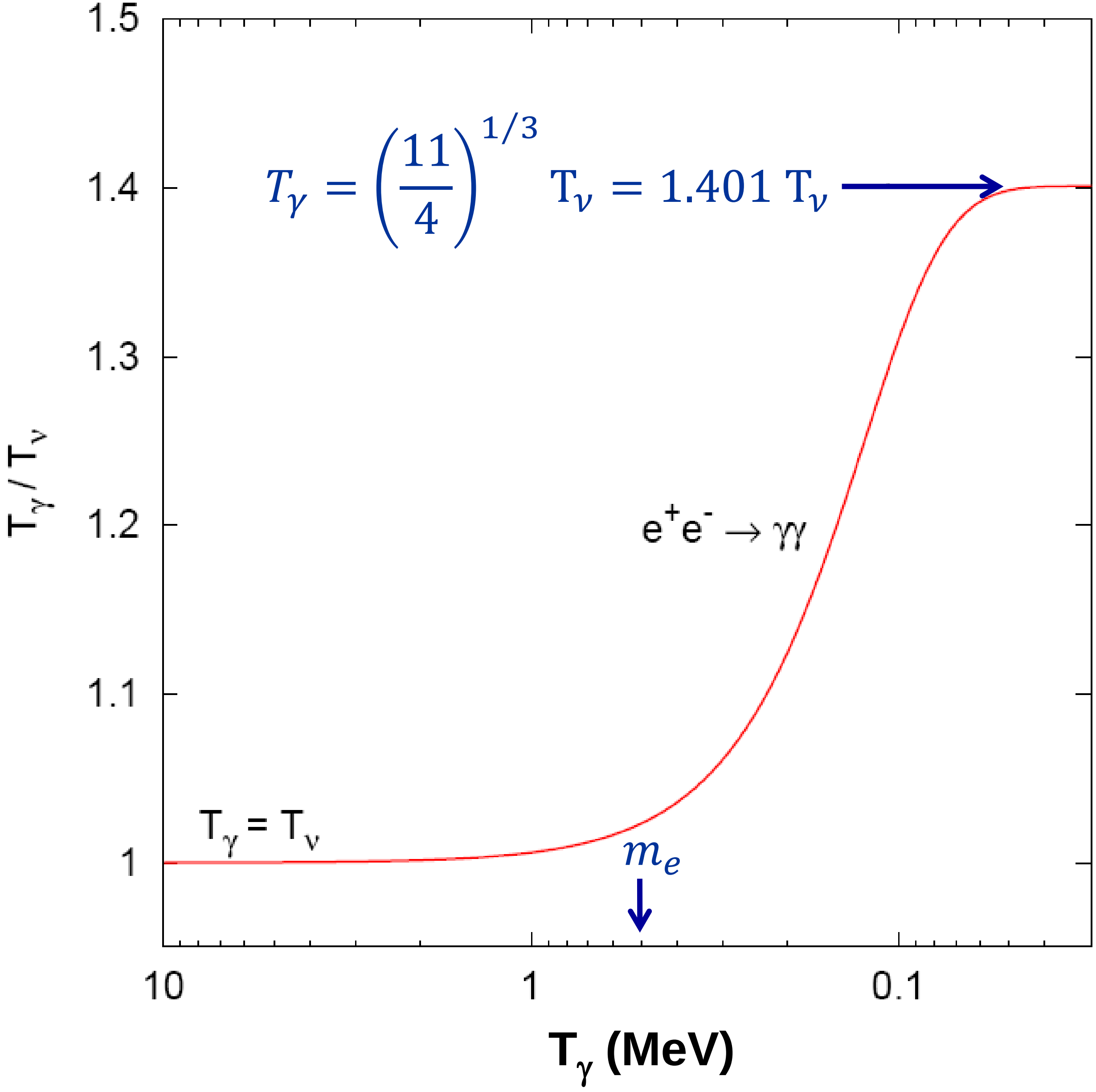
Neutrinos are in thermal equilibrium for $T \gtrsim 1$ MeV
corresponding to $t \lesssim 1$ sec

Thermal Radiations

	General	Bosons	Fermions
Number density n	$g\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{e^{E_{\boldsymbol{p}}/T}\pm 1}$	$g_B\frac{\zeta_3}{\pi^2}T^3$	$\frac{3}{4}\,g_F\frac{\zeta_3}{\pi^2}T^3$
Energy density ρ	$g\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{E_{\boldsymbol{p}}}{e^{E_{\boldsymbol{p}}/T}\pm 1}$	$g_B\frac{\pi^2}{30}T^4$	$\frac{7}{8}\,g_F\frac{\pi^2}{30}T^4$
Pressure P	$\frac{\rho}{3}$		
Entropy density s	$\frac{\rho+P}{T}=\frac{4}{3}\,\frac{\rho}{T}$	$g_B\frac{2\pi^2}{45}T^3$	$\frac{7}{8}\,g_F\frac{2\pi^2}{45}T^3$

Riemann Zeta Function
$\zeta_3 = 1.2020569\ldots$

Photon Heating by Electron-Positron Annihilation



Present-Day Neutrino Density

Neutrino decoupling (freeze out)	$H \sim \Gamma$ $T \approx 2.4\ \mathrm{MeV}$ (electron flavor) $T \approx 3.7\ \mathrm{MeV}$ (other flavors)
Redshift of Fermi-Dirac distribution ("nothing changes at freeze-out")	$\frac{dn_{\nu\bar\nu}}{dE} = \frac{1}{\pi^2} \frac{E^2}{e^{E/T}+1}$ Temperature scales with redshift $T_\nu = T_\gamma \propto (z+1)$
Electron-positron annihilation beginning at T ≈ m_e = 0.511 MeV	- QED plasma is "strongly" coupled - Stays in thermal equilibrium (adiabatic process) - Entropy of e^+e^- transfered to photons $\left.\begin{matrix}\underbrace{g_* T_\gamma^3}_{2+\frac{7}{8}4=\frac{11}{2}}\Big\|_{\text{before}} = \underbrace{g_* T_\gamma^3}_{2}\Big\|_{\text{after}}\end{matrix}\right\\} T_\gamma^3\Big\|_{\text{before}} = \frac{\mathbf{4}}{\mathbf{11}} T_\gamma^3\Big\|_{\text{after}}$
Redshift of neutrino and photon thermal distributions so that today we have	$n_{\nu\bar\nu}(\text{1 flavor}) = \frac{4}{11} \times \frac{3}{4} \times n_\gamma = \frac{3}{11} n_\gamma \approx 112\ \mathrm{cm}^{-3}$ $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \approx 1.95\ \mathrm{K}$ for massless neutrinos

Present-Day Neutrino Distribution

	Normal	Inverted
Minimal neutrino masses from oscillation experiments	$m_3 \gtrsim 50$ meV $m_2 \gtrsim 8$ meV $m_1 \geq 0$	$m_1 \approx m_2 \gtrsim 50$ meV $m_3 \geq 0$
Temperature of massless cosmic background neutrinos	T = 1.95 K = 0.17 meV	
Cosmic redshift of **momenta** (not energies)	$\frac{dn_{\nu\bar{\nu}}}{dp} = \frac{1}{\pi^2} \frac{p^2}{e^{p/T} + 1}$	Not a thermal distribution unless $T \gg m$
Average velocity for $m \gg T$	$\langle v \rangle \approx \frac{3T}{m}$	
Normal hierarchy neutrinos	$\langle v_3 \rangle < 1 \times 10^{-2} c$	$\langle v_2 \rangle < 6 \times 10^{-2} c$

Cosmic radiation density after e^+e^- annihilation

Radiation density for $N_\nu = 3$ standard neutrino flavors

$$\rho_{\rm rad} = \rho_\gamma + \rho_\nu = \frac{\pi^2}{15}\left(T_\gamma^4 + N_\nu \frac{7}{8}\, T_\nu^4\right) = \left[1 + \mathrm{N}_\nu \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right]\rho_\gamma$$

Cosmic radiation density is expressed in terms of
"effective number of thermally excited neutrino species" $\mathrm{N}_{\rm eff}$

$$\rho_{\rm rad} = \left[1 + \mathrm{N}_{\rm eff} \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right]\rho_\gamma = [1 + \mathrm{N}_{\rm eff}\, 0.2271]\rho_\gamma$$

$\mathrm{N}_{\rm eff}$ is a measure for the radiation density, not necessarily related to neutrinos

Residual neutrino heating by e^+e^- annihilation and corrections for plasma effects and neutrino flavor oscillations implies

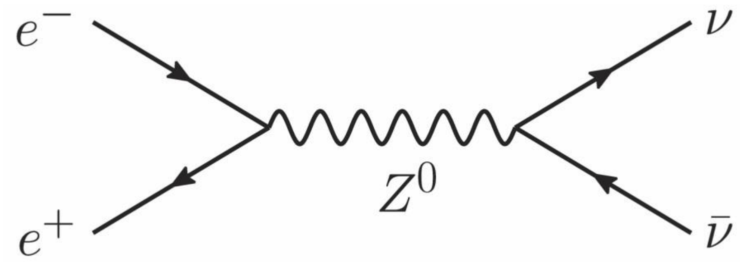
$N_{\rm eff} = 3.046$ Standard value

$$\rho_{\rm rad} = (1 + 0.6918 + 0.2271\, \Delta\mathrm{N}_{\rm eff})\, \rho_\gamma$$

Of course, the number of known neutrino species ν_e, ν_μ, ν_τ is exactly 3

Neutrino Counting in Particle Physics

Resonant Z-Boson production at LEP
(Electron-positron collider at CERN before LHC, in the same tunnel)



Cross section follows resonance curve

$$\frac{d\sigma}{dE} \propto \frac{\Gamma/2}{(E-m_Z)^2+(\Gamma/2)^2}$$

Or other final states

$$q\bar{q},\ e^+e^-,\ \mu^+\mu^-,\ \tau^+\tau^-$$

Measured properties

$$m_Z = (91.1876 \pm 0.0021)\ \mathrm{GeV}$$
$$\Gamma_{\mathrm{tot}} = (2.4952 \pm 0.0023)\ \mathrm{GeV}$$
$$\Gamma_{\mathrm{invis}} = (499.0 \pm 1.5)\ \mathrm{MeV}$$

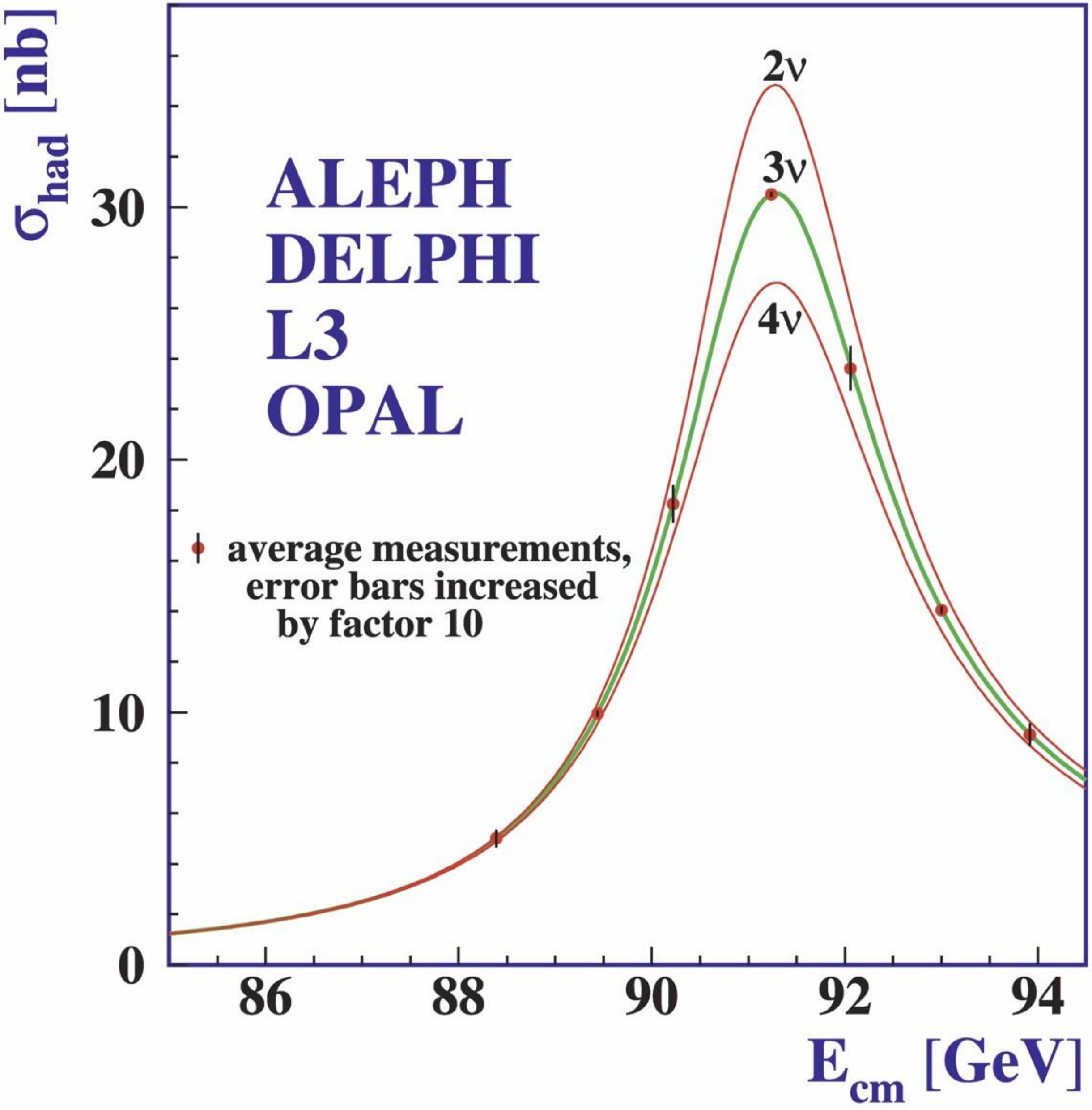
Contribution to Γ_{invis} of one standard neutrino family

$$\Gamma_{\nu\bar{\nu}} = 167.2\ \mathrm{MeV}$$

$$\frac{\Gamma_{\mathrm{invis}}}{\Gamma_{\nu\bar{\nu}}} = 2.984 \pm 0.008$$

No room for weakly interacting particles that couple to the Z unless m > $m_Z/2$ or strongly reduced coupling strength

Measured Z^0 Width at LEP (ca 1990)



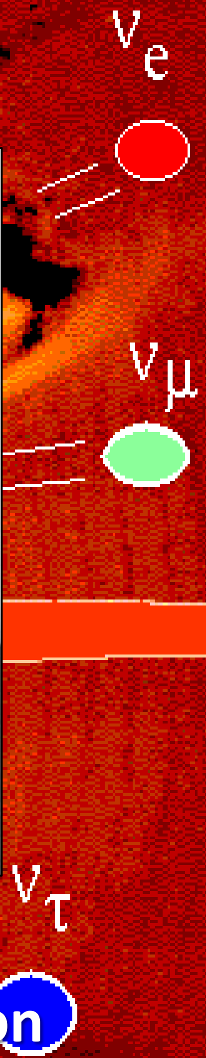
Cosmological consequences:

- **Standard radiation density fixed**
- **No “trivial” weakly interacting dark matter particles**

Dirac vs Majorana Neutrinos

Paul Dirac

Ettore Majorana



4 states per flavor
Twice the radiation density?

2 states per flavor
Standard assumption in cosmology

How many neutrino degrees of freedom?

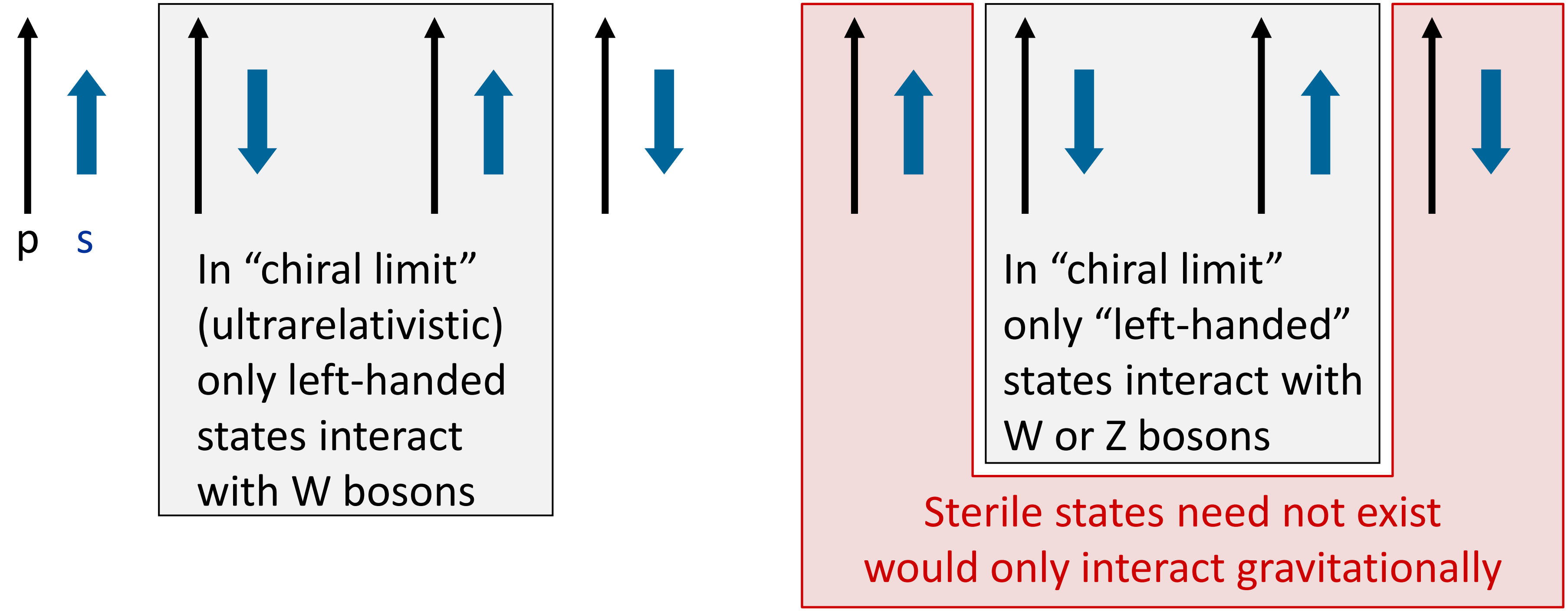
Charged fermions (leptons, quarks) each have four degrees of freedom:

2 spin states
2 charge states

Neutrinos may have only two degrees of freedom

2 spin states
2 charge conjugate states?

Electron	Positron	Neutrino	Antineutrino



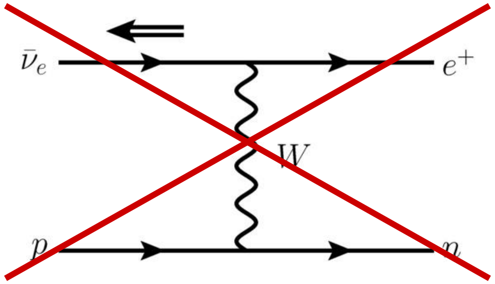
In "chiral limit" (ultrarelativistic) only left-handed states interact with W bosons

In "chiral limit" only "left-handed" states interact with W or Z bosons

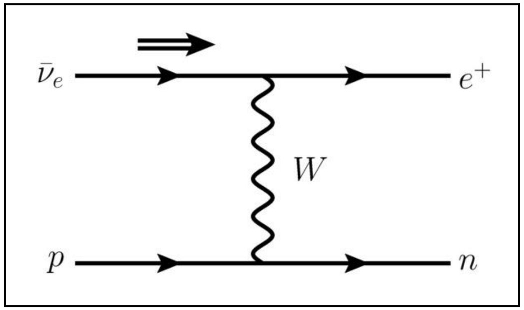
Sterile states need not exist would only interact gravitationally

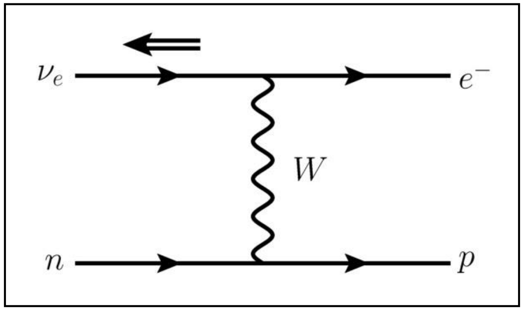
For massless particles, helicity is a relativistic invariant ("chirality")

Role of Handedness



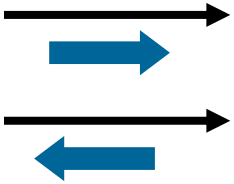
Allowed by
left-handedness of
weak interactions

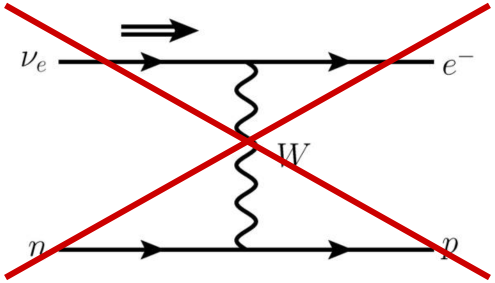




Interpret as polarization
states of one “neutrino”

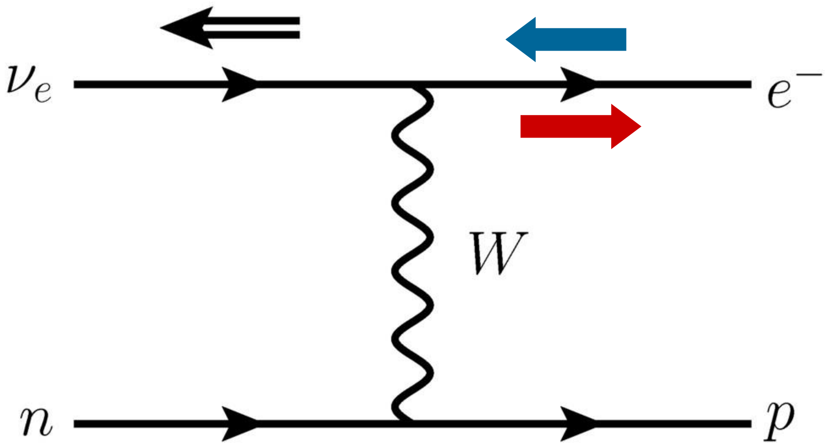
Handedness distinguishes
between “particles”
and “antiparticles”





Role of Mass

Massless,
left-handed
neutrino with
$E \gg m_e$

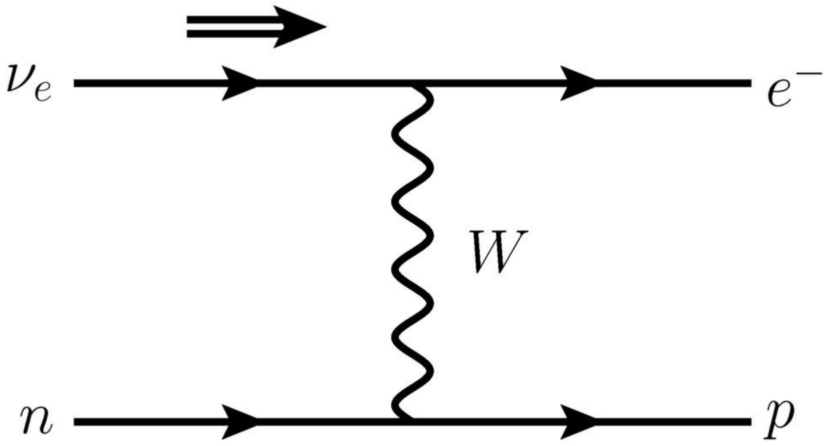


Relativistic electron
is strongly, but not
perfectly polarized

“Wrong” helicity
state with probability
$\sim (m_e/E)^2$

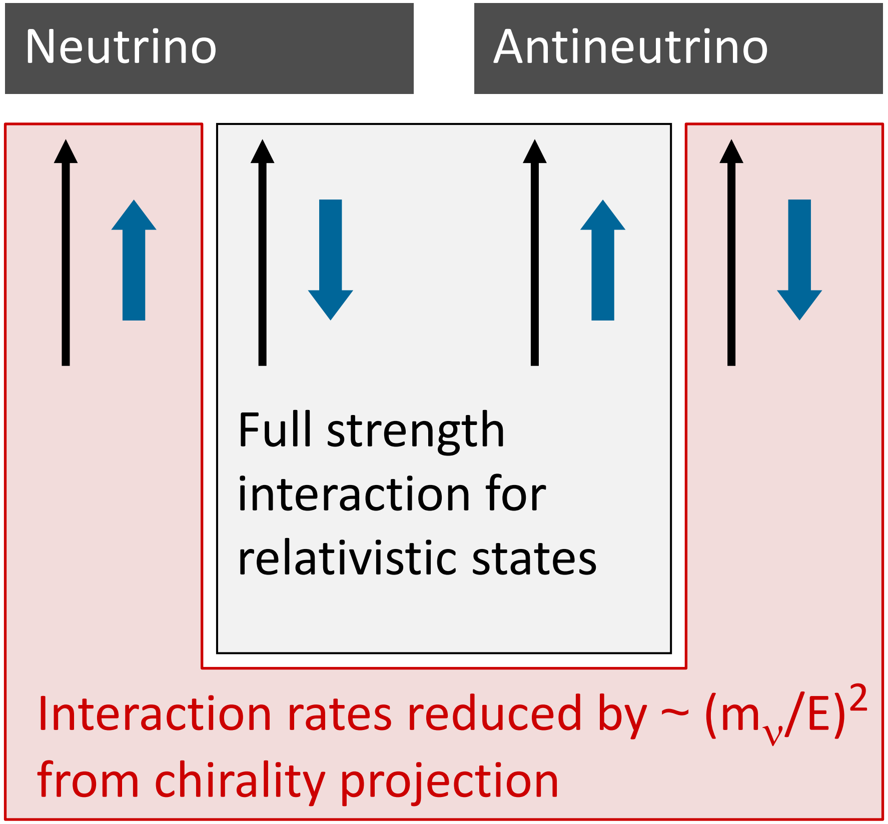
Right-handed
neutrino with
small mass
$E \gg m_\nu$

Rate smaller by
$\sim (m_\nu/E)^2$



Neutrinos with Mass: Dirac vs. Majorana

Dirac neutrinos are like charged leptons
2 spin states
2 “charge” states



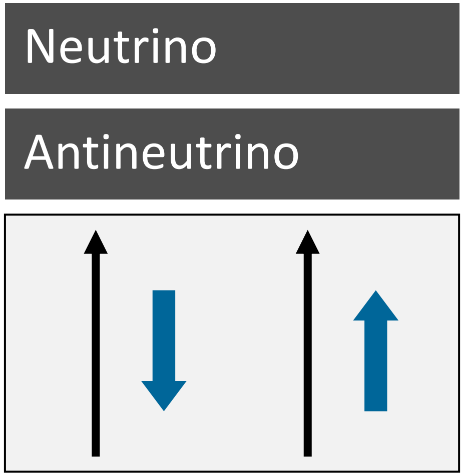
Neutrino

Antineutrino

Full strength interaction for relativistic states

Interaction rates reduced by ~ $(m_\nu/E)^2$ from chirality projection

Majorana neutrinos
2 spin states



Neutrino

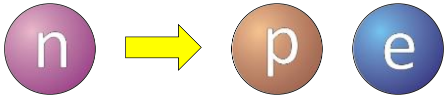
Antineutrino

Neutrino can produce wrong-sign charged lepton:

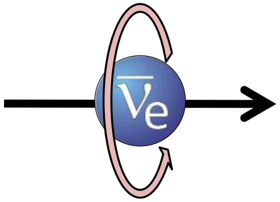
Lepton-number violation

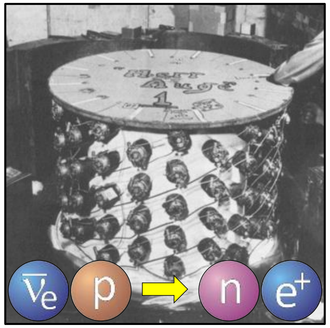
Role of Neutrino Helicity (Handedness)

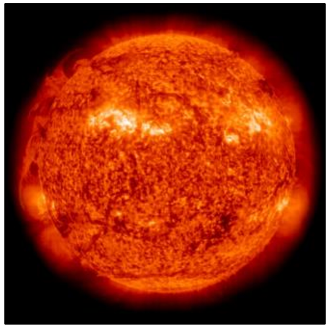
Basic production process
in reactors



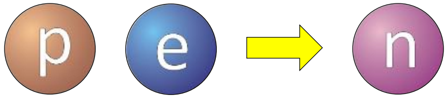
Anti-neutrinos always
right-handed helicity



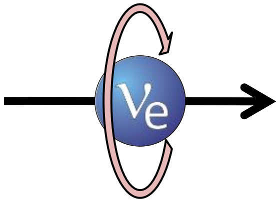


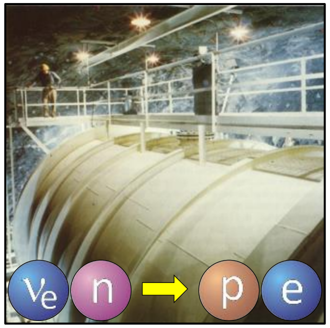


Basic production process
in the Sun



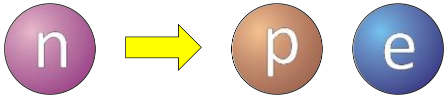
Neutrinos always
left-handed helicity





Role of Neutrino Helicity (Handedness)

Basic production process
in reactors



Anti-neutrinos always
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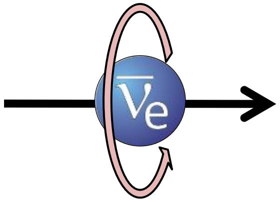
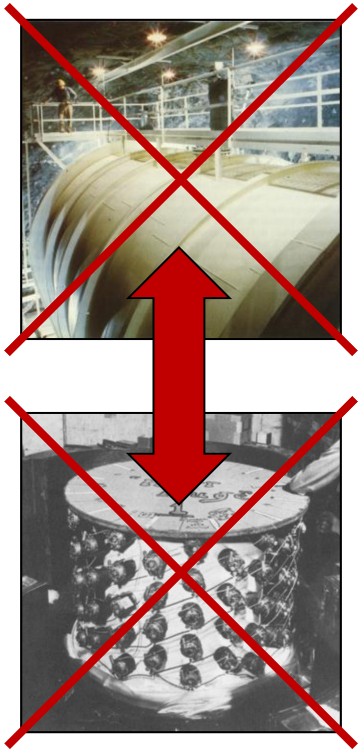
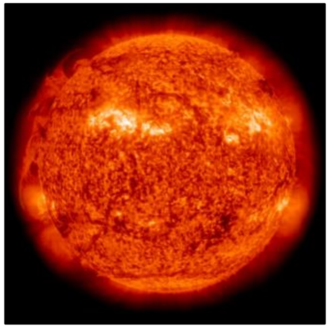
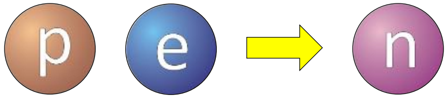


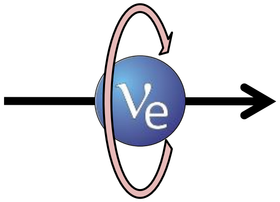




Basic production process
in the Sun

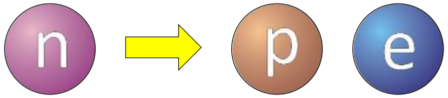


Neutrinos always
left-handed helicity

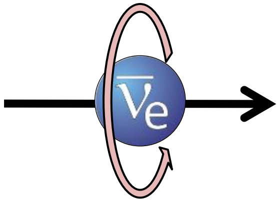


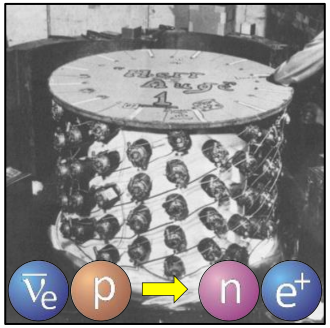
Role of Neutrino Helicity (Handedness)

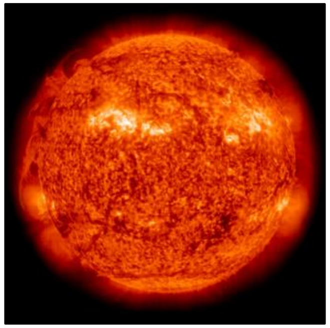
Basic production process in reactors



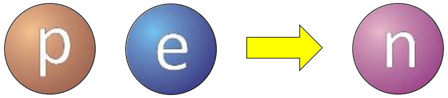
Anti-neutrinos always right-handed helicity



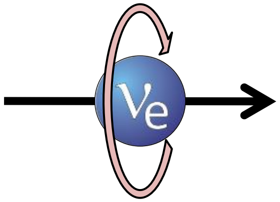


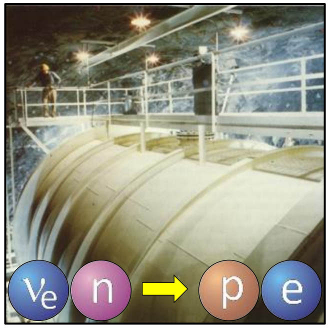


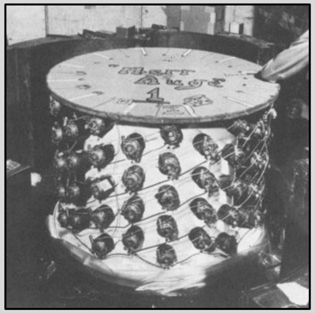
Basic production process in the Sun



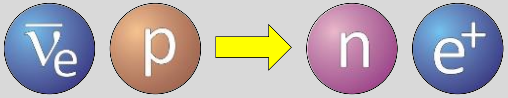
Neutrinos always left-handed helicity

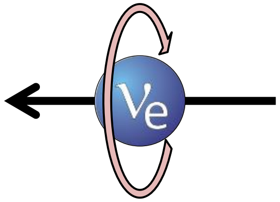






Cowan & Reines $\overline{\nu}_e$ detector in fast-moving rocket, overtakes small-mass solar ν_e





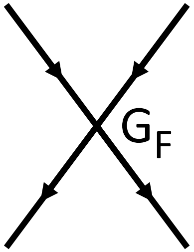
Majorana neutrinos:
Helicity flip → anti-neutrino
$\nu_e/\overline{\nu}_e$ property depends on Lorentz frame

Neutrino Thermal Equilibrium for Dirac Neutrinos

Neutrino reaction rate

Examples for neutrino processes

$$e^+ + e^- \leftrightarrow \overline{\nu} + \nu$$
$$\overline{\nu} + \nu \leftrightarrow \overline{\nu} + \nu$$
$$\nu + e^\pm \leftrightarrow \nu + e^\pm$$



Dimensional analysis of reaction rate in a thermal medium for T ≪ $m_{W,Z}$

$$\Gamma \sim G_F^2 T^5 \times (m_\nu/T)^2$$

Cosmic expansion rate

Friedmann equation (flat universe)

$$H^2 = \frac{8\pi}{3}\frac{\rho}{m_{Pl}^2} \qquad \left(G_N = \frac{1}{m_{Pl}^2}\right)$$

Radiation dominates

$$\rho \sim T^4$$

Expansion rate

$$H \sim \frac{T^2}{m_{Pl}}$$

Condition for thermal equilibrium: $\Gamma > H$

$$m_\nu > G_F^{-1}(m_{Pl}T)^{-1/2}$$

To avoid excessive dilution at quark-hadron phase transition: $T < T_{QCD} \sim 170\ \mathrm{MeV}$

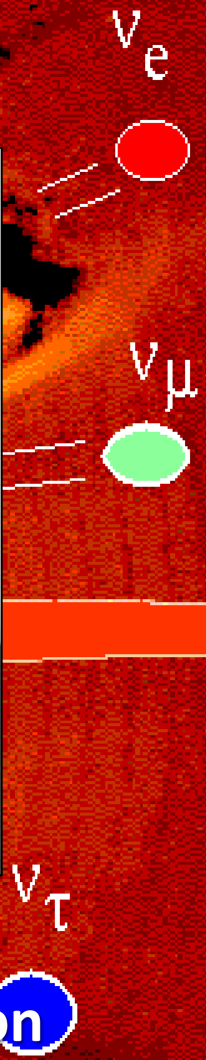
$$m_\nu > 60\ \mathrm{keV}$$

For eV-scale Dirac neutrinos, r.h. helicity degrees not thermally excited

Dirac vs Majorana Neutrinos

Paul Dirac

Ettore Majorana



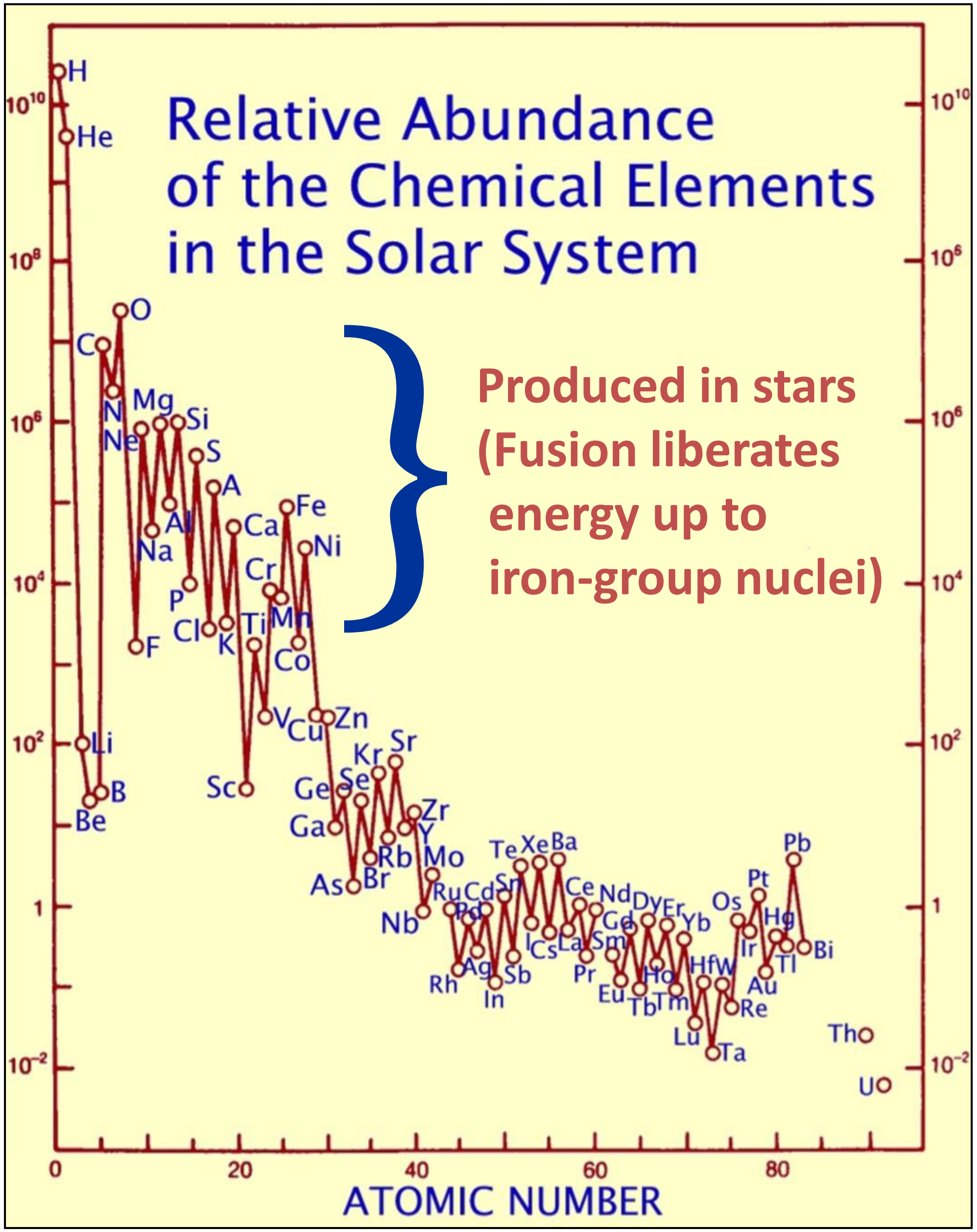
4 states per flavor
Twice the radiation density?

2 states per flavor
Standard assumption in cosmology

Not thermalized

Big Bang Nucleosynthesis

Origin of Elements

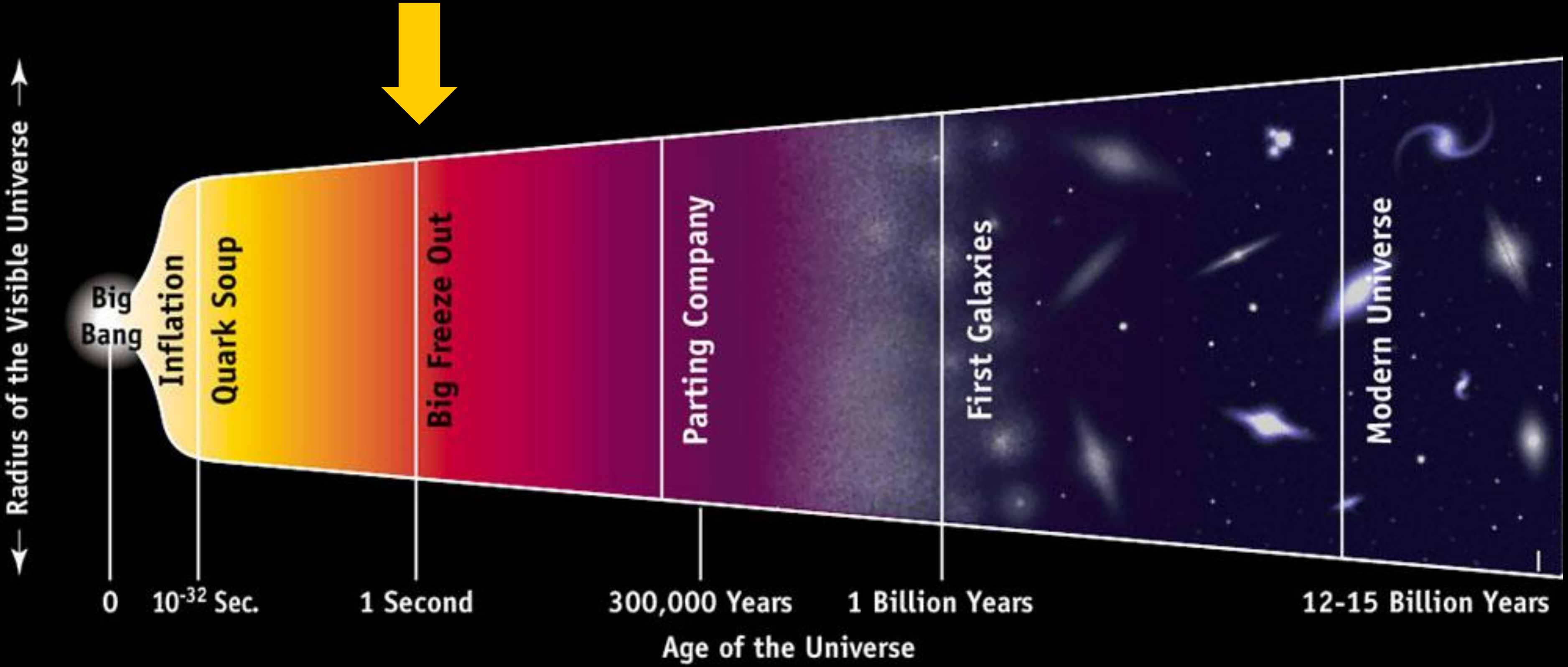


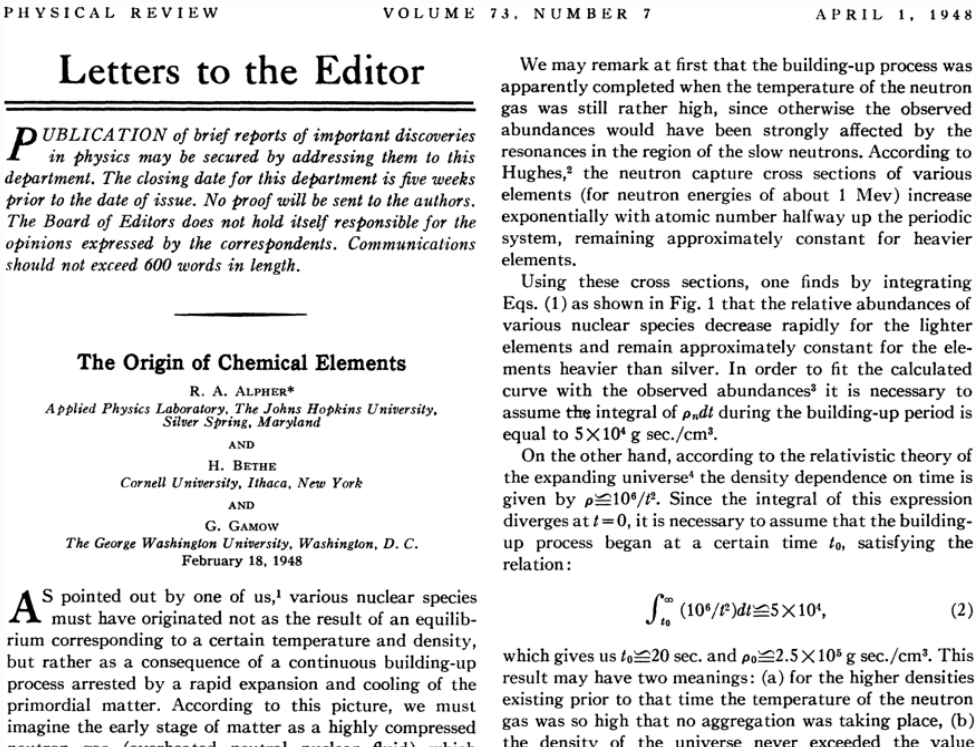
Produced in stars
(Fusion liberates
energy up to
iron-group nuclei)

- Mass fraction of helium ~ 25% everywhere in the universe
- Most of it not produced in stars (far too little star light from liberated energy)
- Big-bang nucleosynthesis (BBN) is a pillar of modern cosmology
- Neutrinos play a crucial role

Where, When and What?

Epoch:	1 sec – 3 minutes
Temperature:	1 MeV – 30 keV
Constituents:	Photons, neutrinos, e^+e^-
Baryon density:	$0.07\ \mathrm{g\ cm^{-3}}$ (at 1 sec)





PHYSICAL REVIEW VOLUME 73, NUMBER 7 APRIL 1, 1948

Letters to the Editor

PUBLICATION of brief reports of important discoveries in physics may be secured by addressing them to this department. The closing date for this department is five weeks prior to the date of issue. No proof will be sent to the authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents. Communications should not exceed 600 words in length.

The Origin of Chemical Elements

R. A. ALPHER*
Applied Physics Laboratory, The Johns Hopkins University, Silver Spring, Maryland
AND
H. BETHE
Cornell University, Ithaca, New York
AND
G. GAMOW
The George Washington University, Washington, D. C.
February 18, 1948

AS pointed out by one of us,[1] various nuclear species must have originated not as the result of an equilibrium corresponding to a certain temperature and density, but rather as a consequence of a continuous building-up process arrested by a rapid expansion and cooling of the primordial matter. According to this picture, we must imagine the early stage of matter as a highly compressed

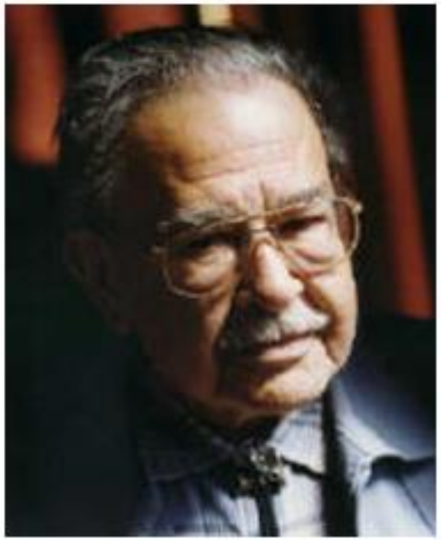
We may remark at first that the building-up process was apparently completed when the temperature of the neutron gas was still rather high, since otherwise the observed abundances would have been strongly affected by the resonances in the region of the slow neutrons. According to Hughes,[2] the neutron capture cross sections of various elements (for neutron energies of about 1 Mev) increase exponentially with atomic number halfway up the periodic system, remaining approximately constant for heavier elements.

Using these cross sections, one finds by integrating Eqs. (1) as shown in Fig. 1 that the relative abundances of various nuclear species decrease rapidly for the lighter elements and remain approximately constant for the elements heavier than silver. In order to fit the calculated curve with the observed abundances[3] it is necessary to assume ~~the~~ integral of $\rho_n dt$ during the building-up period is equal to 5×10^4 g sec./cm³.

On the other hand, according to the relativistic theory of the expanding universe[4] the density dependence on time is given by $\rho\cong10^6/t^2$. Since the integral of this expression diverges at $t=0$, it is necessary to assume that the building-up process began at a certain time t_0, satisfying the relation:

$$\int_{t_0}^{\infty}(10^6/t^2)dt\cong5\times10^4, \tag{2}$$

which gives us $t_0\cong20$ sec. and $\rho_0\cong2.5\times10^5$ g sec./cm³. This result may have two meanings: (a) for the higher densities existing prior to that time the temperature of the neutron gas was so high that no aggregation was taking place, (b) the density of the universe never exceeded the value

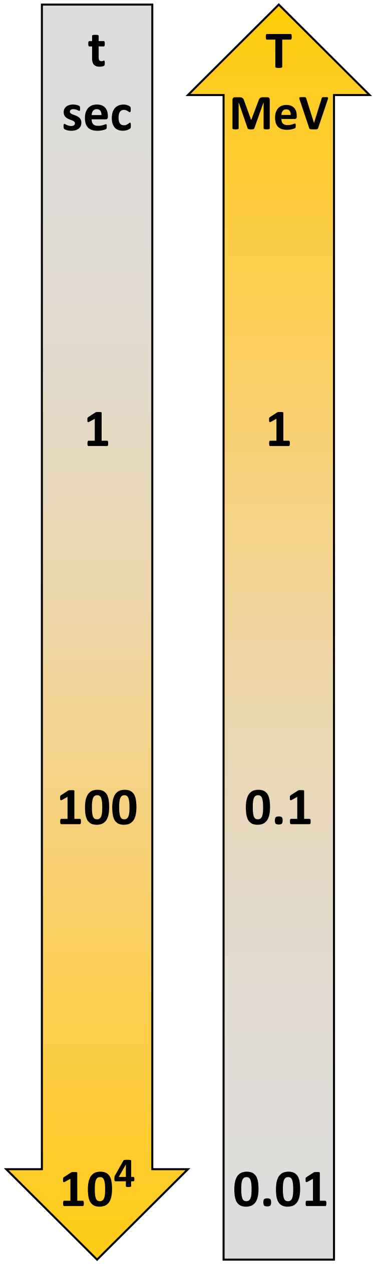


Ralph Alpher

Hans Bethe

George Gamow

Helium Synthesis – Three Easy Steps



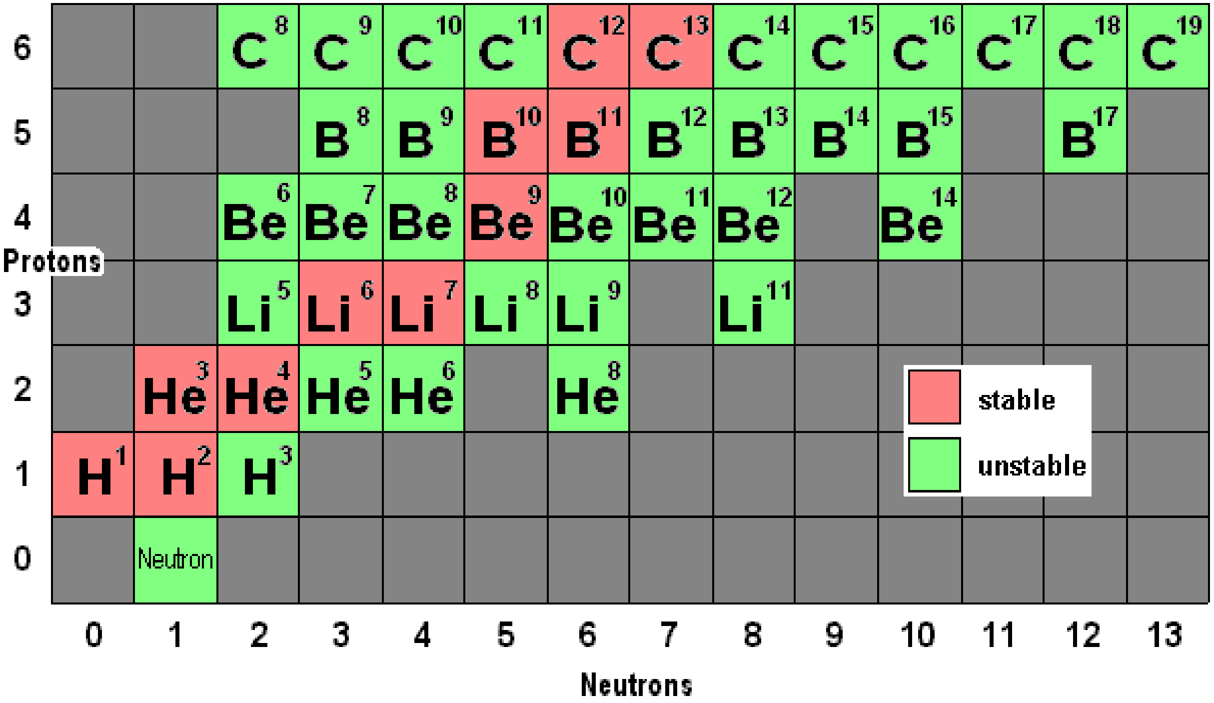
β equilibrium	$p + e^- \leftrightarrow n + \nu_e$ $p + \bar{\nu}_e \leftrightarrow n + e^+$	$\frac{n}{p} = \exp\left(-\frac{m_n - m_p}{T}\right) \approx 1$ $m_n - m_p = 1.293\ \mathrm{MeV}$
β freeze-out	$\frac{n}{p} \approx \frac{1}{6}$	β rates fall below expansion rate H
• Neutron decay • Nuclear statist. equilibrium	$n_n \propto \exp\left(-\frac{t}{\tau_n}\right)$ $\tau_n = 880\ \mathrm{sec}\ (?)$	$2\mathrm{p} + 2\mathrm{n} \leftrightarrow {}^4\mathrm{He} + \gamma$ Helium suppressed by large entropy
Neutrons freeze in helium	$\frac{n}{p} \approx \frac{1}{7}$	Helium mass fraction $Y_P \approx 25\%$
Thermonuclear reaction chains	Production and destruction of traces of D, ^{3}He, ^{6}Li, ^{7}Li, ^{7}Be	
All done		

Why do nuclei form so late?

- Thermal equilibrium → all nuclei present
- Binding energies much larger than MeV, so why are they still dissociated at weak-interaction freeze-out? Why not everything in iron?
- Basic answer: High-entropy environment with $\sim 10^9$ photons per baryon

$$Nn + Zp \leftrightarrow (A,Z) + \text{photons}$$

High-E tail of photon distribution enough to keep nuclei dissociated

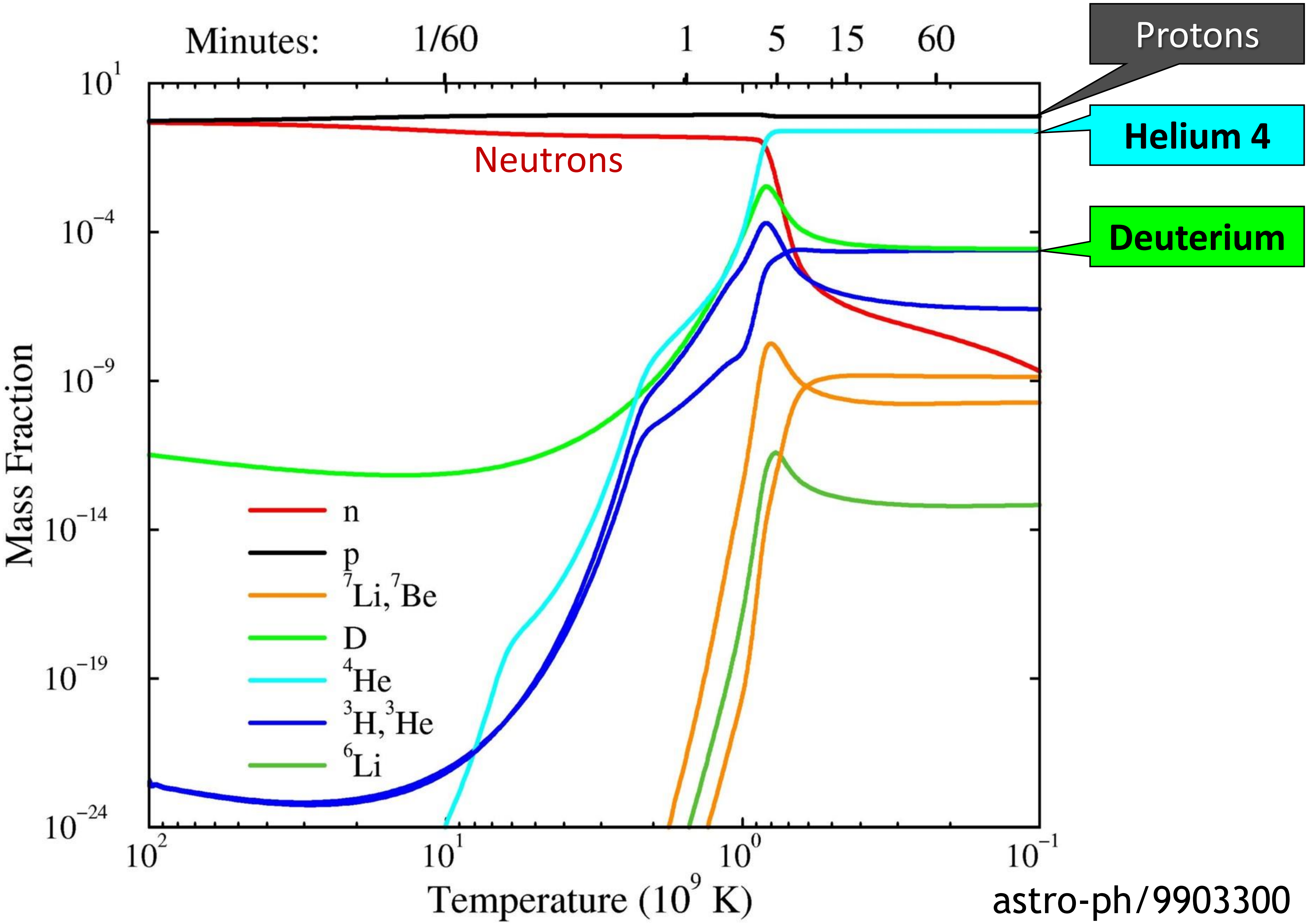


Protons \ Neutrons	0	1	2	3	4	5	6	7	8	9	10	11	12	13
6			C^{8} 🟩	C^{9} 🟩	C^{10} 🟩	C^{11} 🟩	C^{12} 🟥	C^{13} 🟥	C^{14} 🟩	C^{15} 🟩	C^{16} 🟩	C^{17} 🟩	C^{18} 🟩	C^{19} 🟩
5				B^{8} 🟩	B^{9} 🟩	B^{10} 🟥	B^{11} 🟥	B^{12} 🟩	B^{13} 🟩	B^{14} 🟩	B^{15} 🟩		B^{17} 🟩	
4			Be^{6} 🟩	Be^{7} 🟩	Be^{8} 🟩	Be^{9} 🟥	Be^{10} 🟩	Be^{11} 🟩	Be^{12} 🟩		Be^{14} 🟩			
3			Li^{5} 🟩	Li^{6} 🟥	Li^{7} 🟥	Li^{8} 🟩	Li^{9} 🟩		Li^{11} 🟩					
2		He^{3} 🟥	He^{4} 🟥	He^{5} 🟩	He^{6} 🟩		He^{8} 🟩							
1	H^{1} 🟥	H^{2} 🟥	H^{3} 🟩											
0		Neutron 🟩												

🟥 stable
🟩 unstable

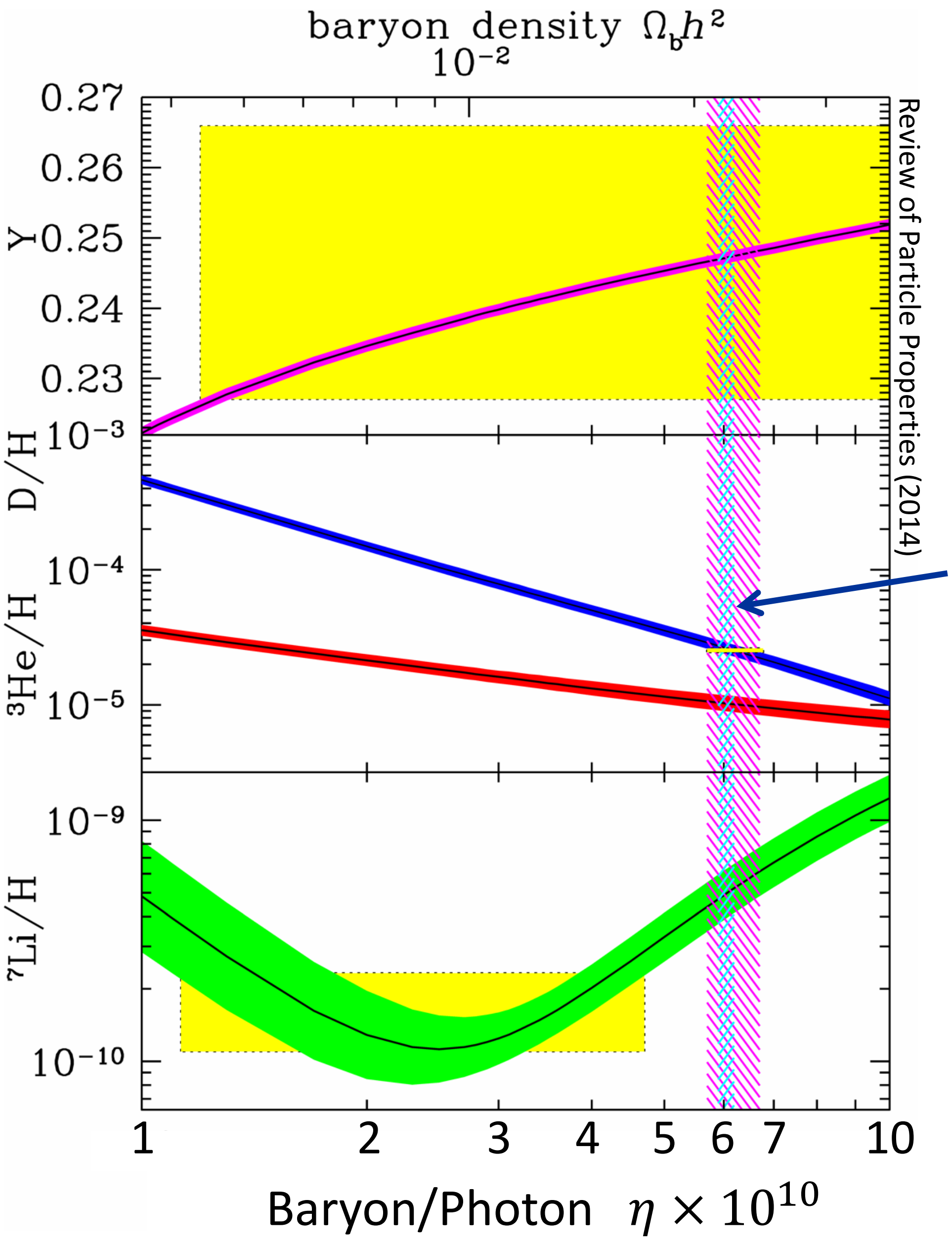
	B (MeV)	B/A (MeV)
D	2.23	1.1
^{3}H	6.92	2.3
^{3}He	7.72	2.6
^{4}He	28.30	7.1
^{6}Li	31.99	5.3
^{7}Li	39.25	5.6
^{7}Be	37.60	5.4
^{12}C	92.2	7.7

Formation of Light Elements



astro-ph/9903300

BBN Theory vs Observations



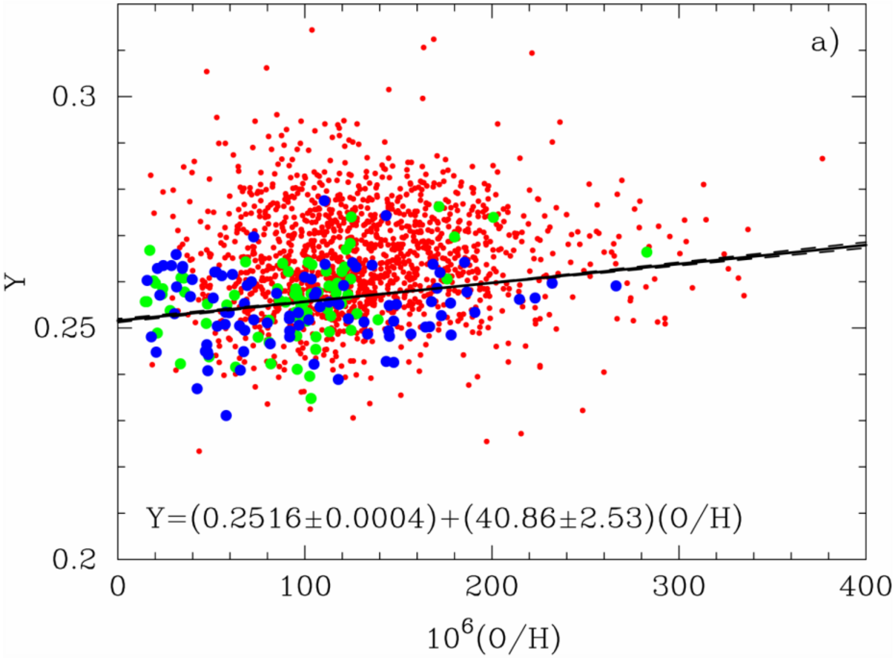
Review of Particle Properties (2014)

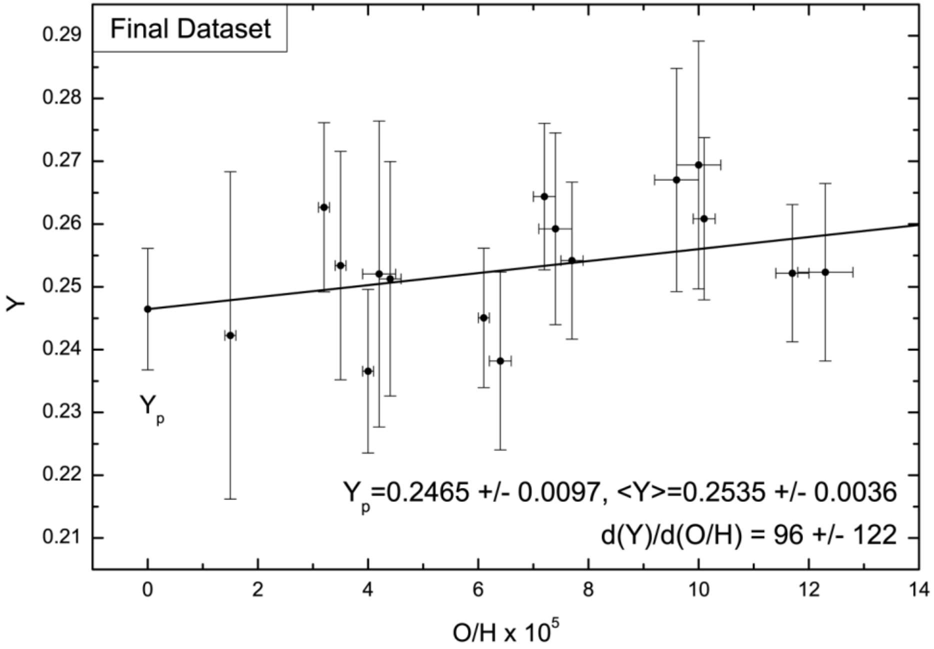
Precision cosmology

Helium Mass Fraction from HII Regions

Extrapolation to zero metalicity in many HII regions

Izotov, Stasinska & Guseva arXiv:1308.2100	Aver, Olive, Porter & Skillman arXiv:1309.0047



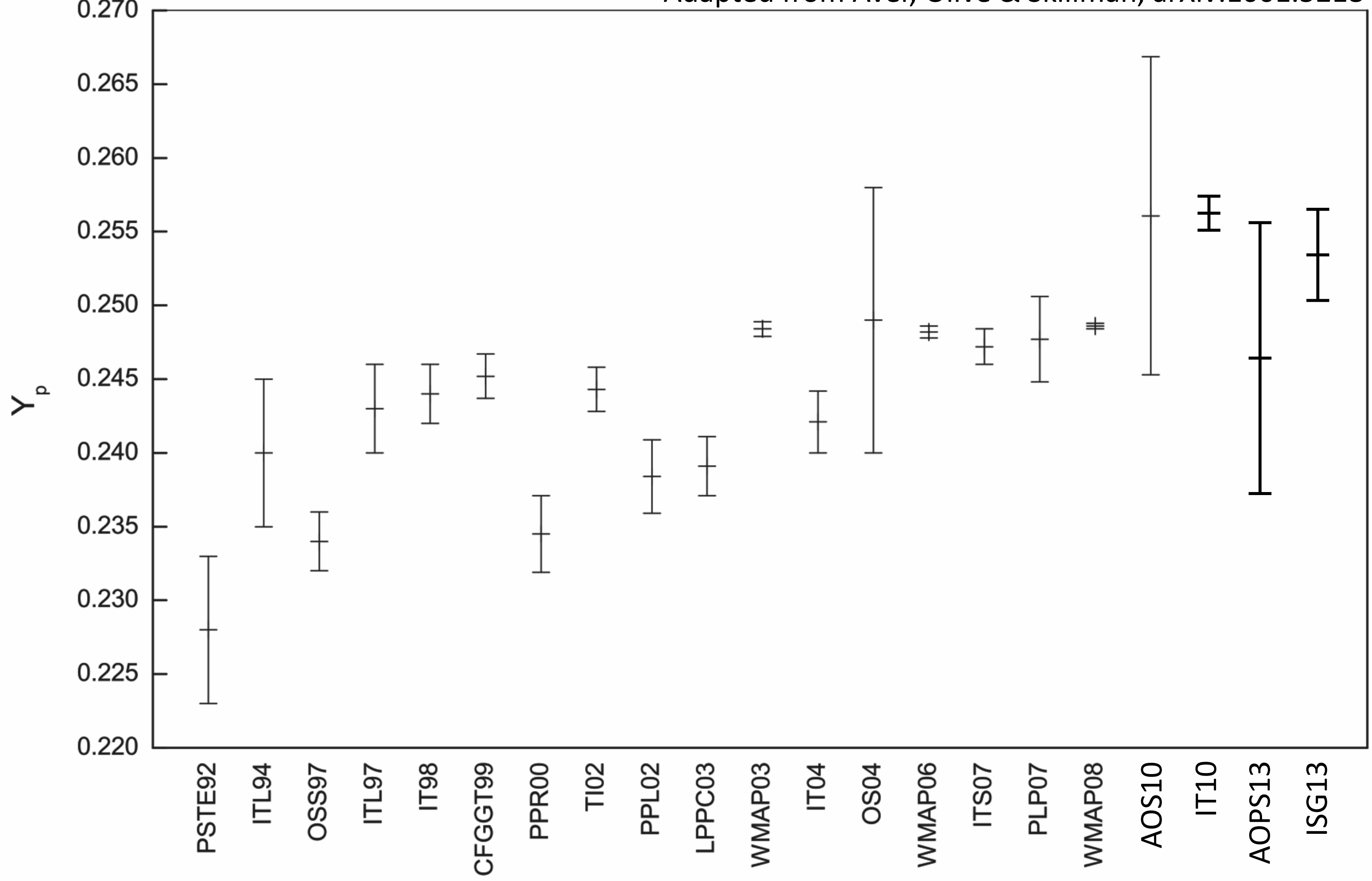


$$Y_\mathrm{P} = 0.254 \pm 0.003$$

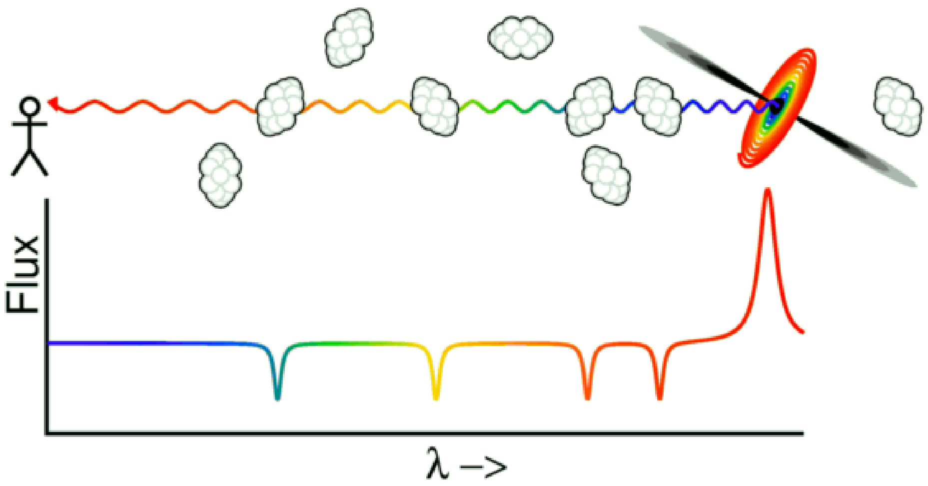
$$Y_\mathrm{P} = 0.2465 \pm 0.0097$$

Progression of Best-Fit Helium Abundance

Adapted from Aver, Olive & Skillman, arXiv:1001.5218

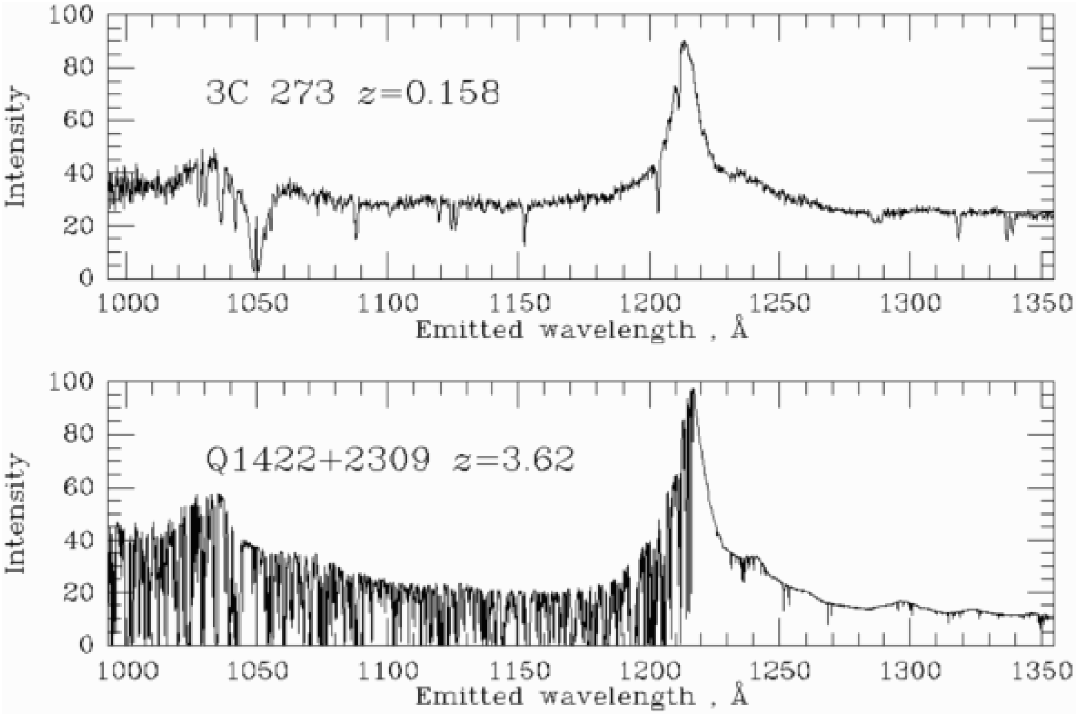


Lyman Alpha Forest



- Hydrogen clouds absorb from QSO continuum emission spectrum
- Absorption dips at Ly-α wavelengh corresponding to redshift

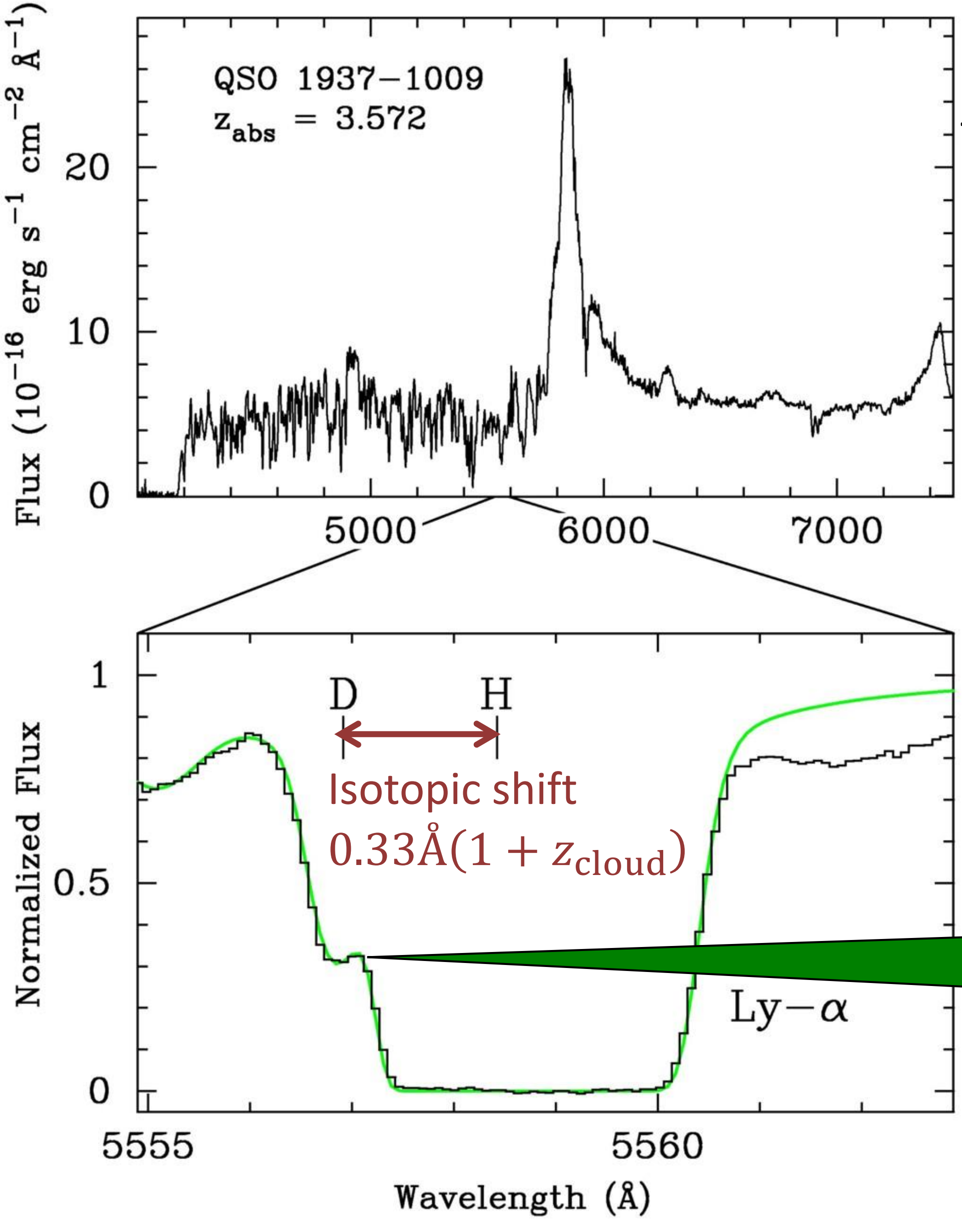
www.astro.ucla.edu/~wright/Lyman-alpha-forest.html



Examples for Lyman-α forest in low- and high-redshift quasars

http://www.astr.ua.edu/keel/agn/forest.gif

Measuring Primordial Deuterium



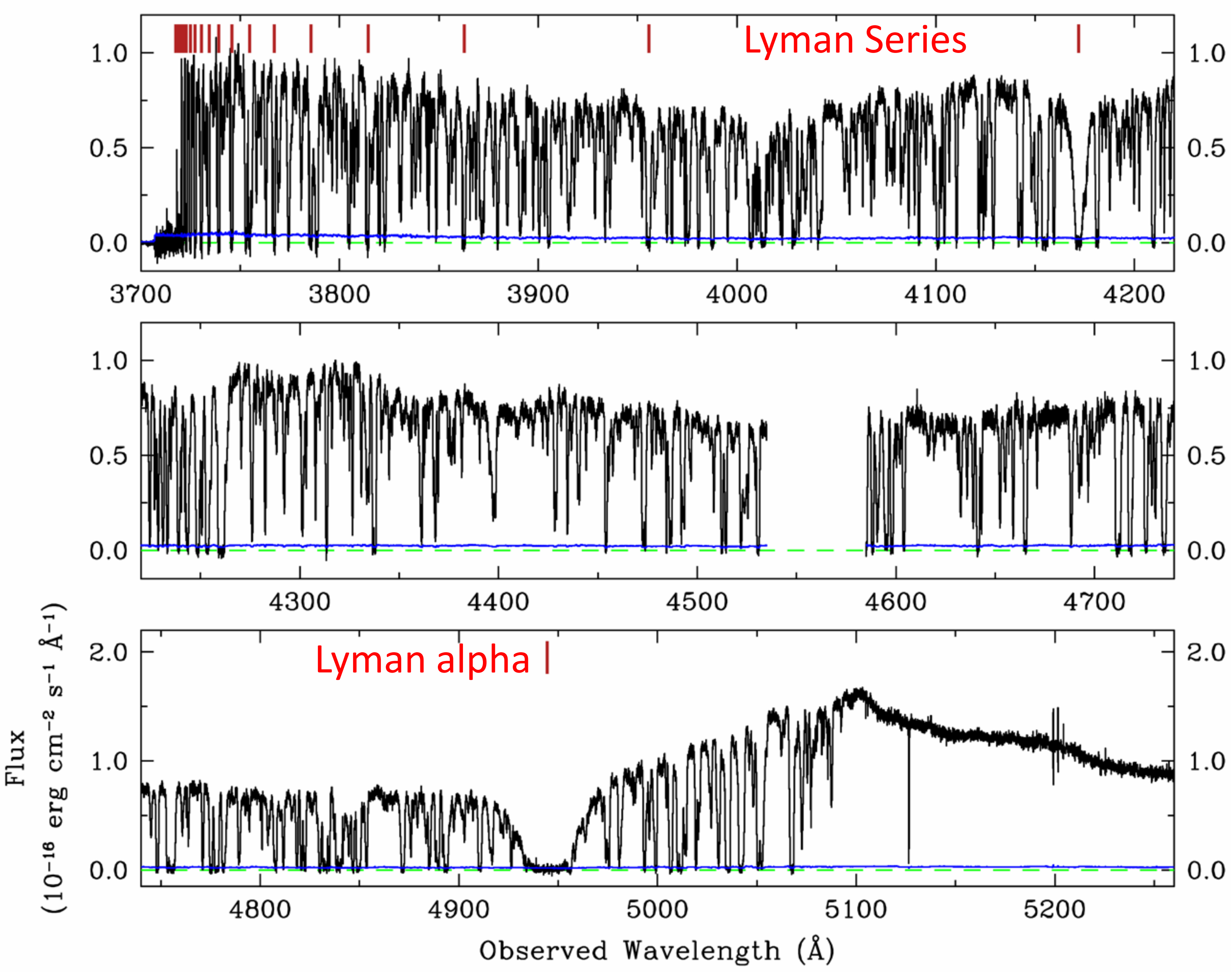
astro-ph/9903300

Hydrogen absorption spectrum of a background quasar in high-redshift hydrogen clouds

Isotopic shift
$0.33\text{Å}(1 + z_{\text{cloud}})$

Deuterium Lyman-α in the flank of the saturated hydrogen Lyman-α line

Lyman Absorption in QSO SDSS J1358+6522



arXiv:1308.3240

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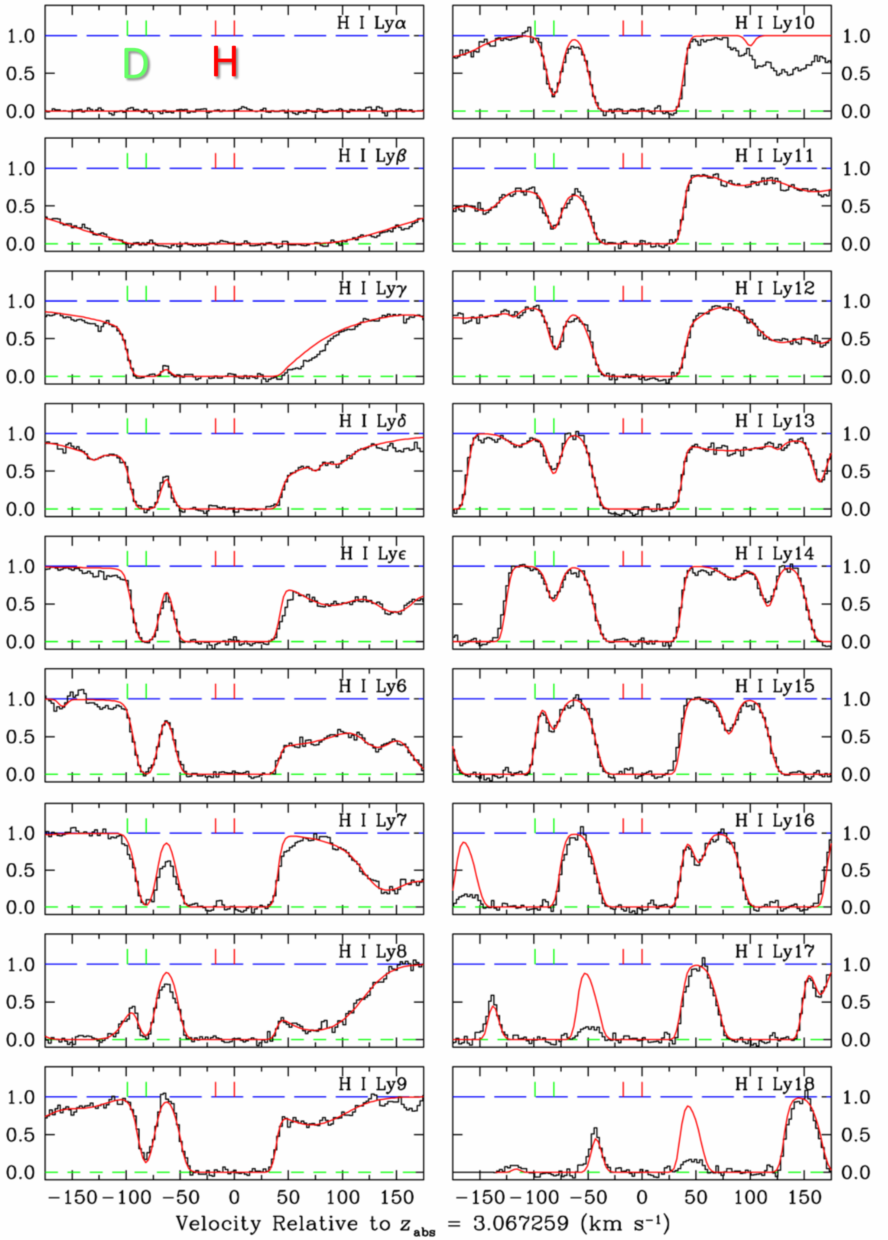
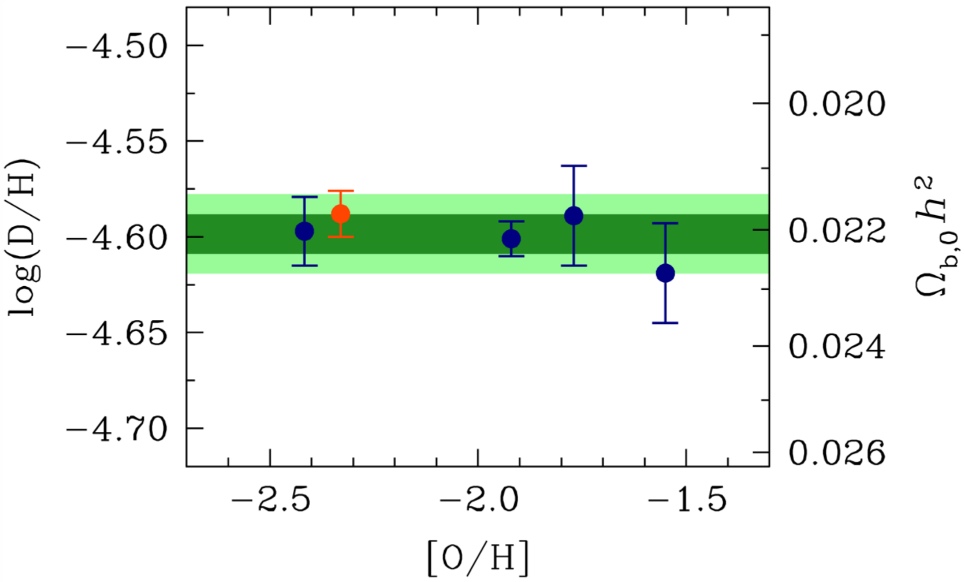




Deuterium determination

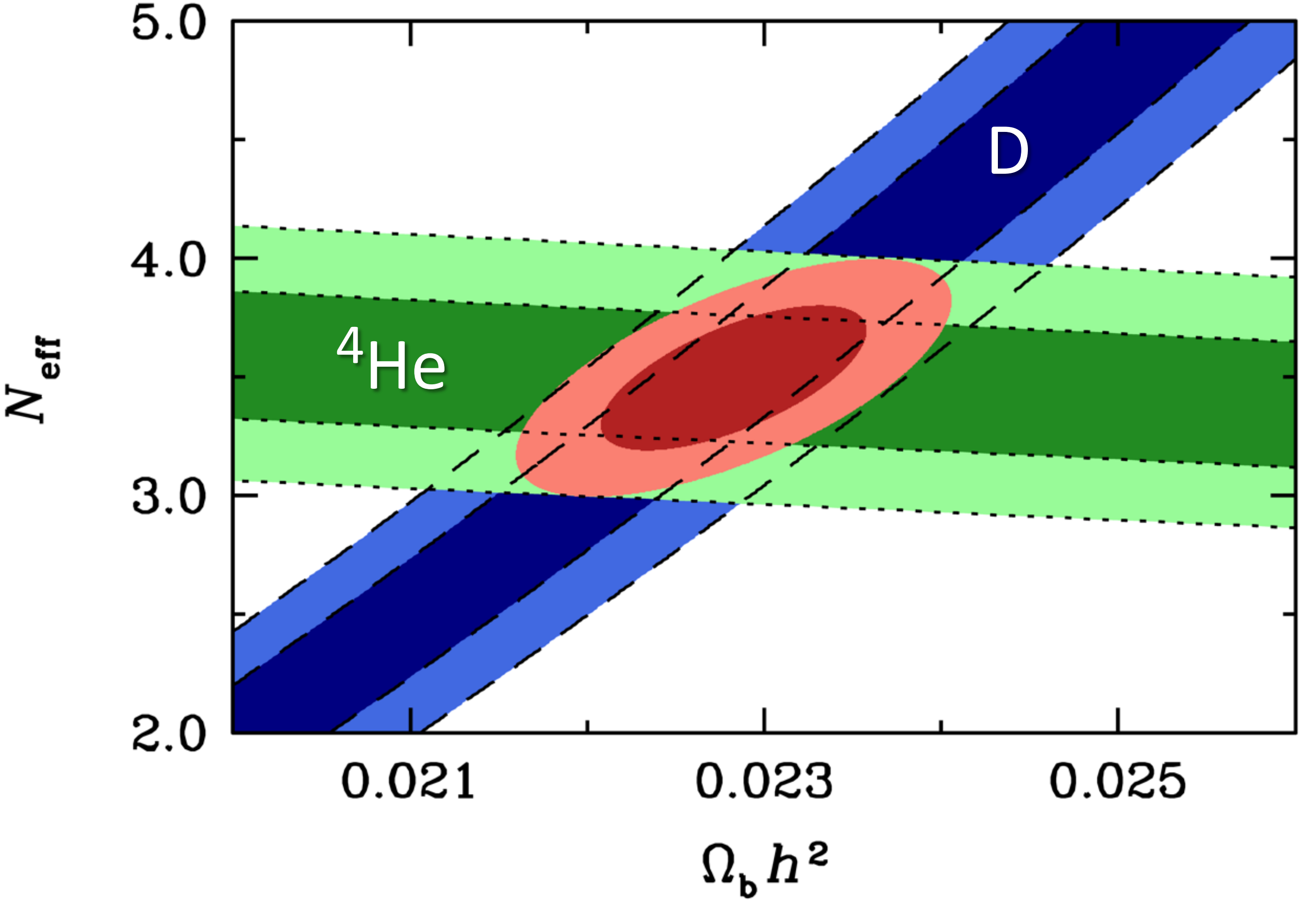
$$\left(\frac{\mathrm{D}}{\mathrm{H}}\right)_{\mathrm{P}} = (2.53 \pm 0.04) \times 10^{-5}$$

Corresponding with standard BBN to

$$\Omega_B h^2 = 0.02202 \pm 0.00046$$

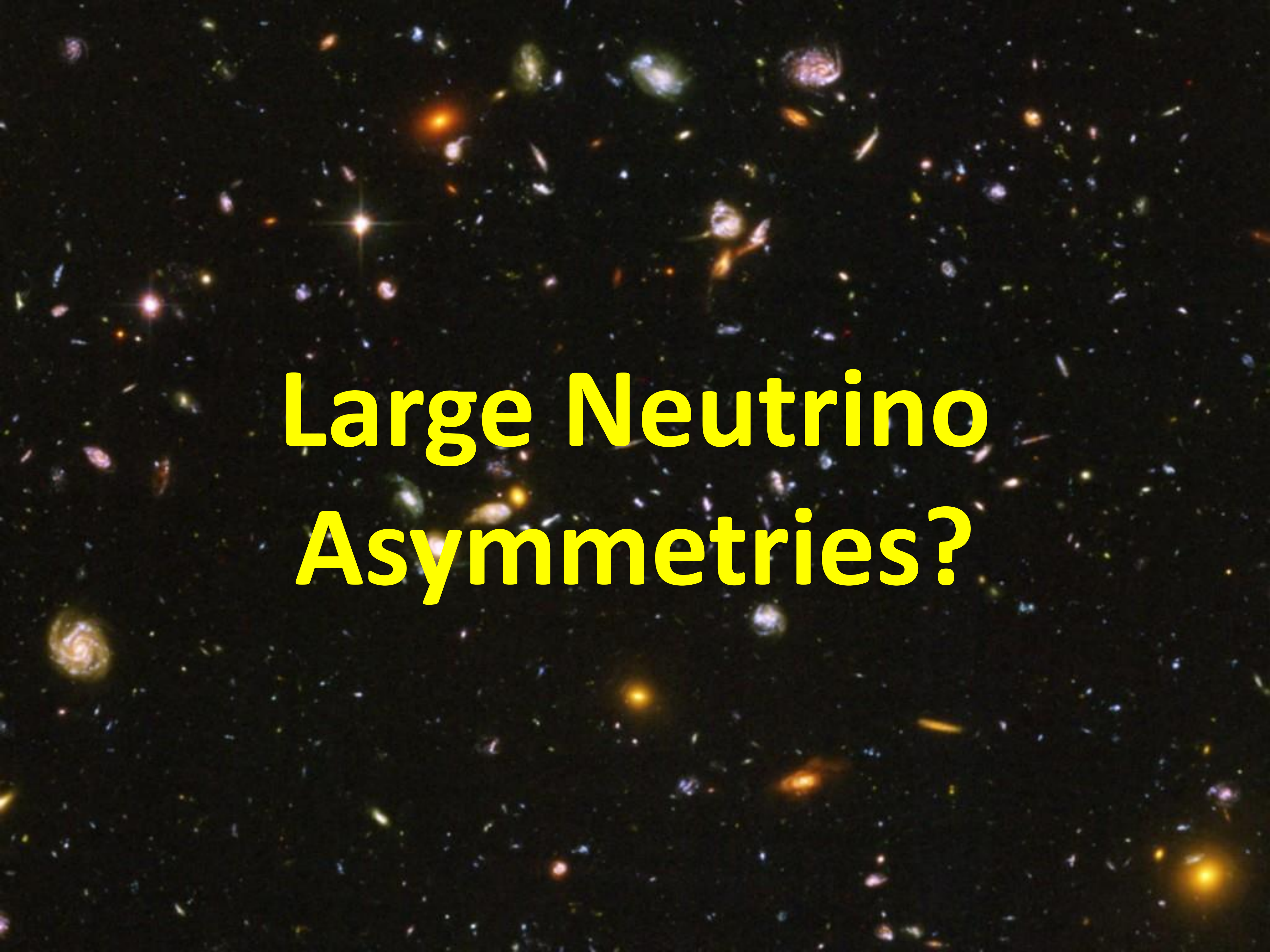
[Cooke et al., arXiv:1308.3240]

Baryon and Radiation Density from BBN



Cooke et al., arXiv:1308.3240

D abundance from Cook et al. (2013) and He-4 from Izotov et al. (2013)
BBN hint for extra radiation (evidence driven by He abundance)



Large Neutrino Asymmetries?

Thermal Neutrino Distribution

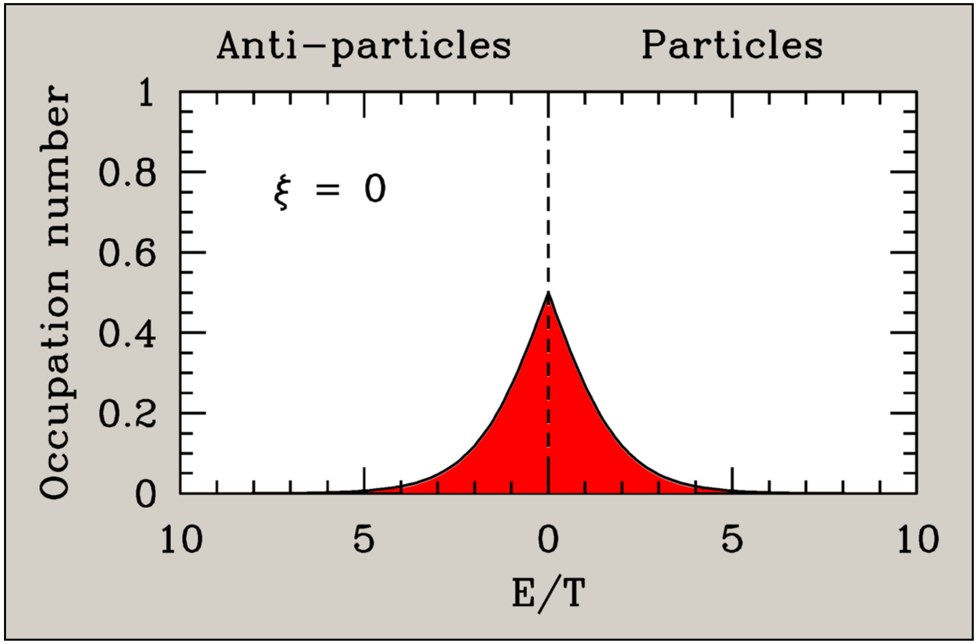
Fermi-Dirac distribution

- Temperature T
- Chemical potential μ

$\mu > 0$ Particles

$\mu < 0$ Anti-particles

$$f_{\boldsymbol{p}} = \frac{1}{e^{(E_{\boldsymbol{p}}-\mu)/T} + 1}$$



Degeneracy parameter $\xi = \frac{\mu}{T}$ Invariant under cosmic expansion

Number density

$$n_{\nu\bar{\nu}} = \int dE \, \frac{4\pi}{(2\pi)^3} \left(\frac{E^2}{1 + e^{E/T-\xi}} + \frac{E^2}{1 + e^{E/T+\xi}} \right)$$

$$= \frac{3}{2\pi^2} T^3 \left[\zeta_3 + \frac{2\ln(2)}{3} \xi^2 + \frac{\xi^4}{72} + \cdots \right]$$

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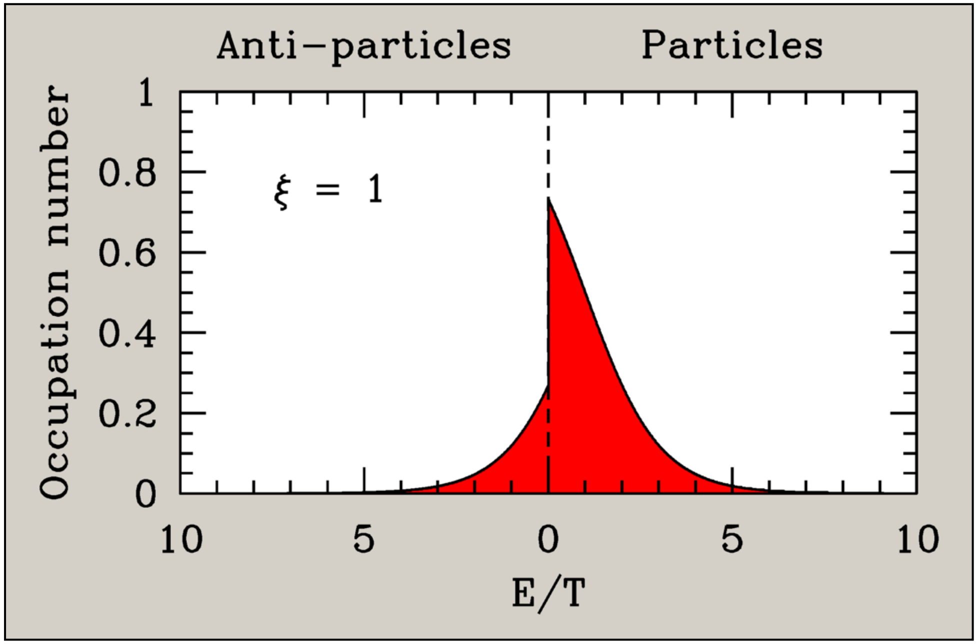

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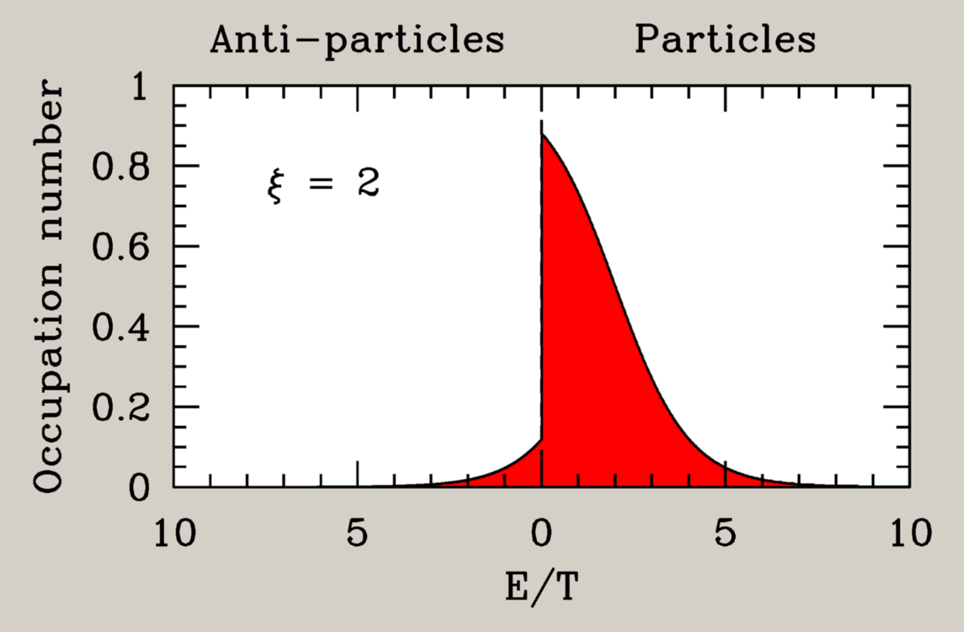

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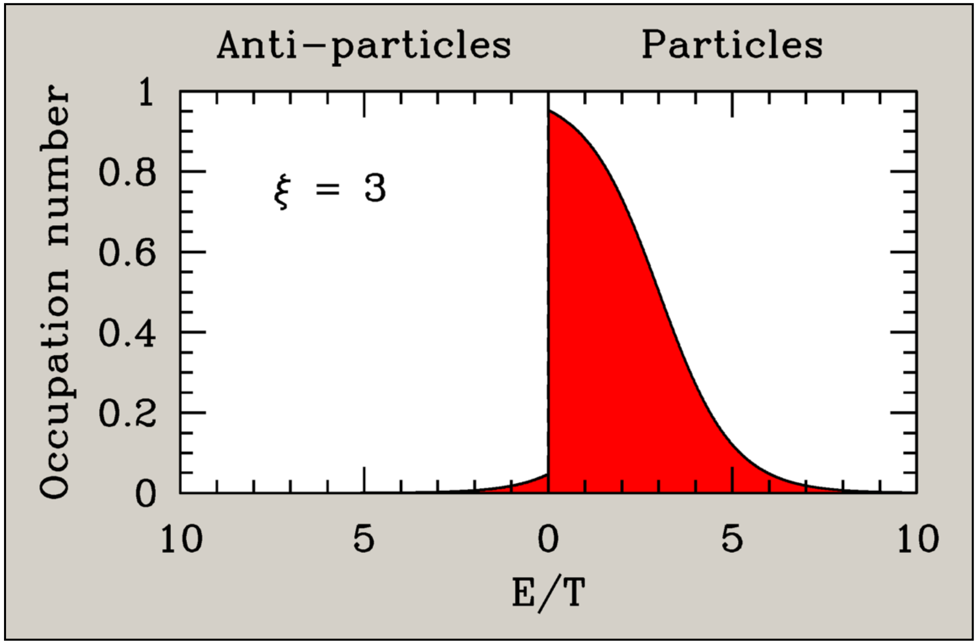

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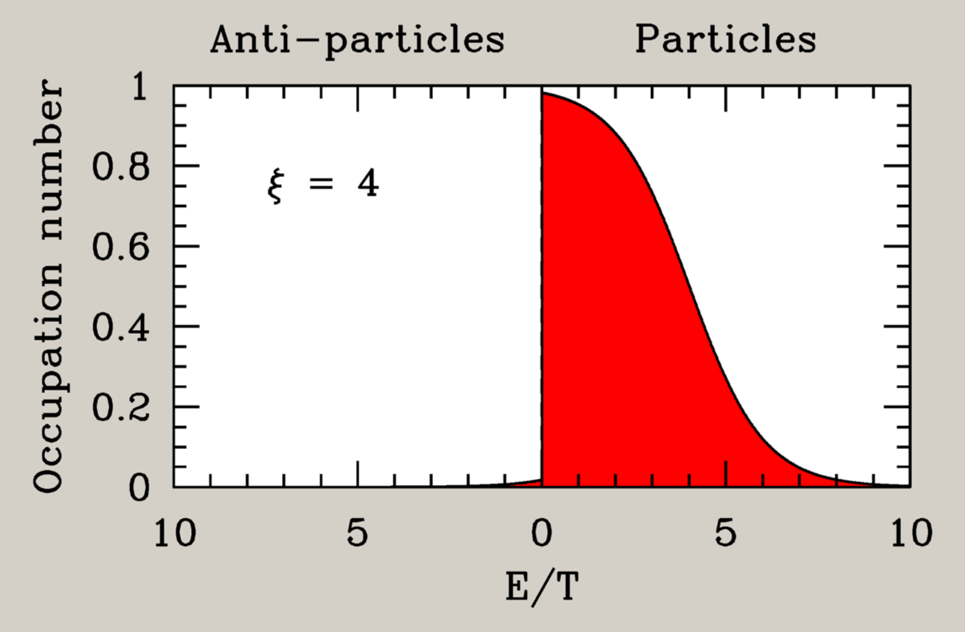

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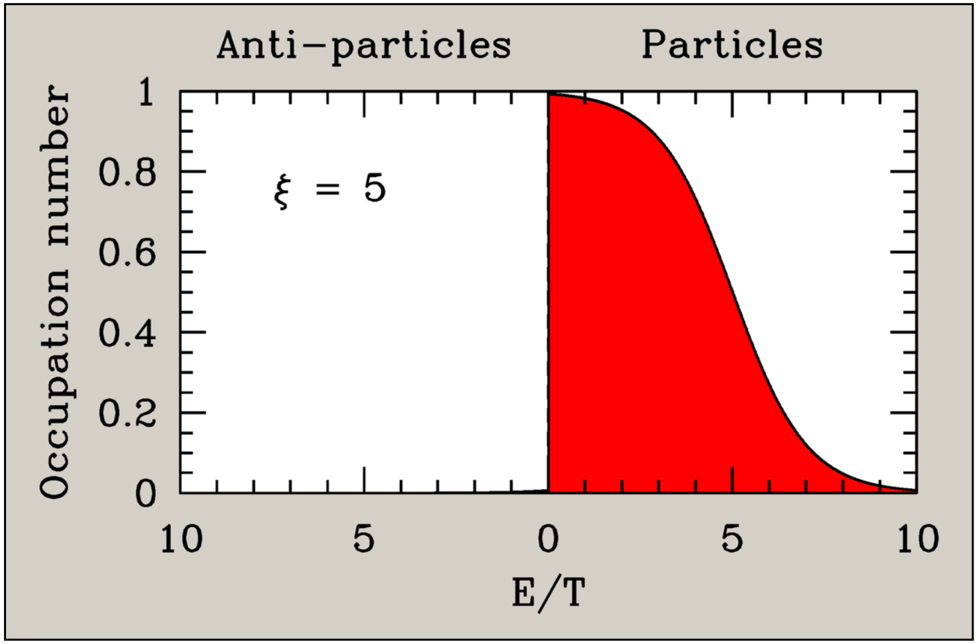

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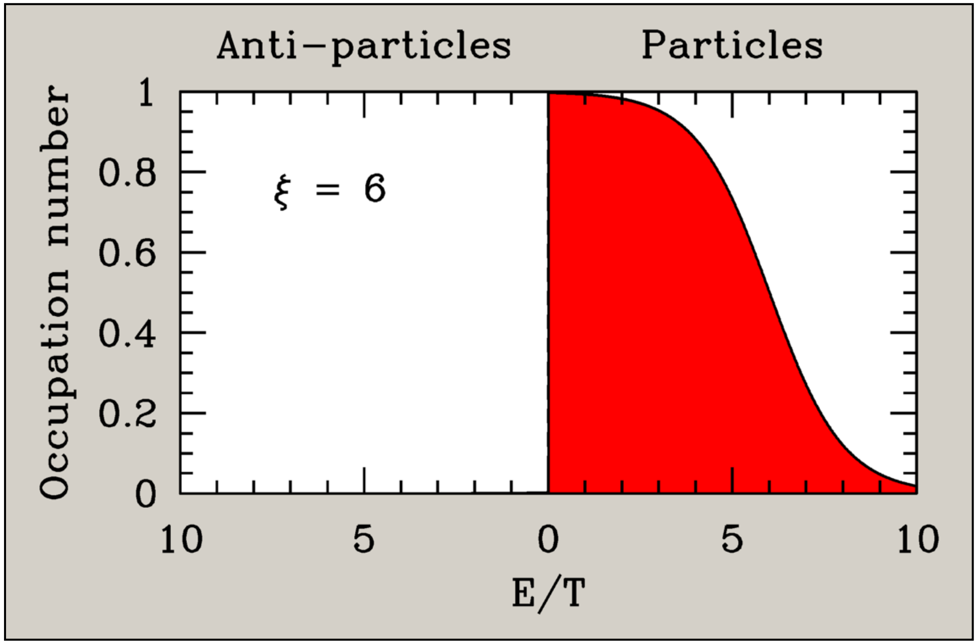

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BBN and Neutrino Chemical Potentials

Expansion rate effect (all flavors)

Energy density in one neutrino flavor with degeneracy parameter $\xi = \eta/T$

$$\rho_{\nu\bar{\nu}} = \frac{7\pi^2}{120}\, T_\nu^4 \left[1 + \underbrace{\frac{30}{7}\left(\frac{\xi}{\pi}\right)^2 + \frac{15}{7}\left(\frac{\xi}{\pi}\right)^4}_{\Delta N_{\rm eff}}\right]$$

Beta equilibrium effect for electron flavor

$n + \nu_e \leftrightarrow p + e^-$

Helium abundance essentially fixed by n/p ratio at beta freeze-out

$$\frac{n}{p} = e^{-(m_n - m_p)/T_{\rm F} - \xi_{\nu_e}}$$

Effect on helium equivalent to

$$\Delta N_{\rm eff} \sim -18\, \xi_{\nu_e}$$

- ν_e beta effect can compensate expansion-rate effect of $\nu_{\mu,\tau}$
- Naively, no significant BBN limit on neutrino number density
- However, flavor oscillations equalize chemical potentials before BBN

Chemical Potentials and Flavor Oscillations

Flavor mixing
(neutrino oscillations)

Flavor lepton numbers
not conserved

Only one common neutrino
chemical potential

Stringent ξ_{ν_e} limit

applies to all flavors

$|\xi_{\nu_{e,\mu,\tau}}| < 0.07$

Extra neutrino density
$\Delta N_{\mathrm{eff}} < 0.0064$

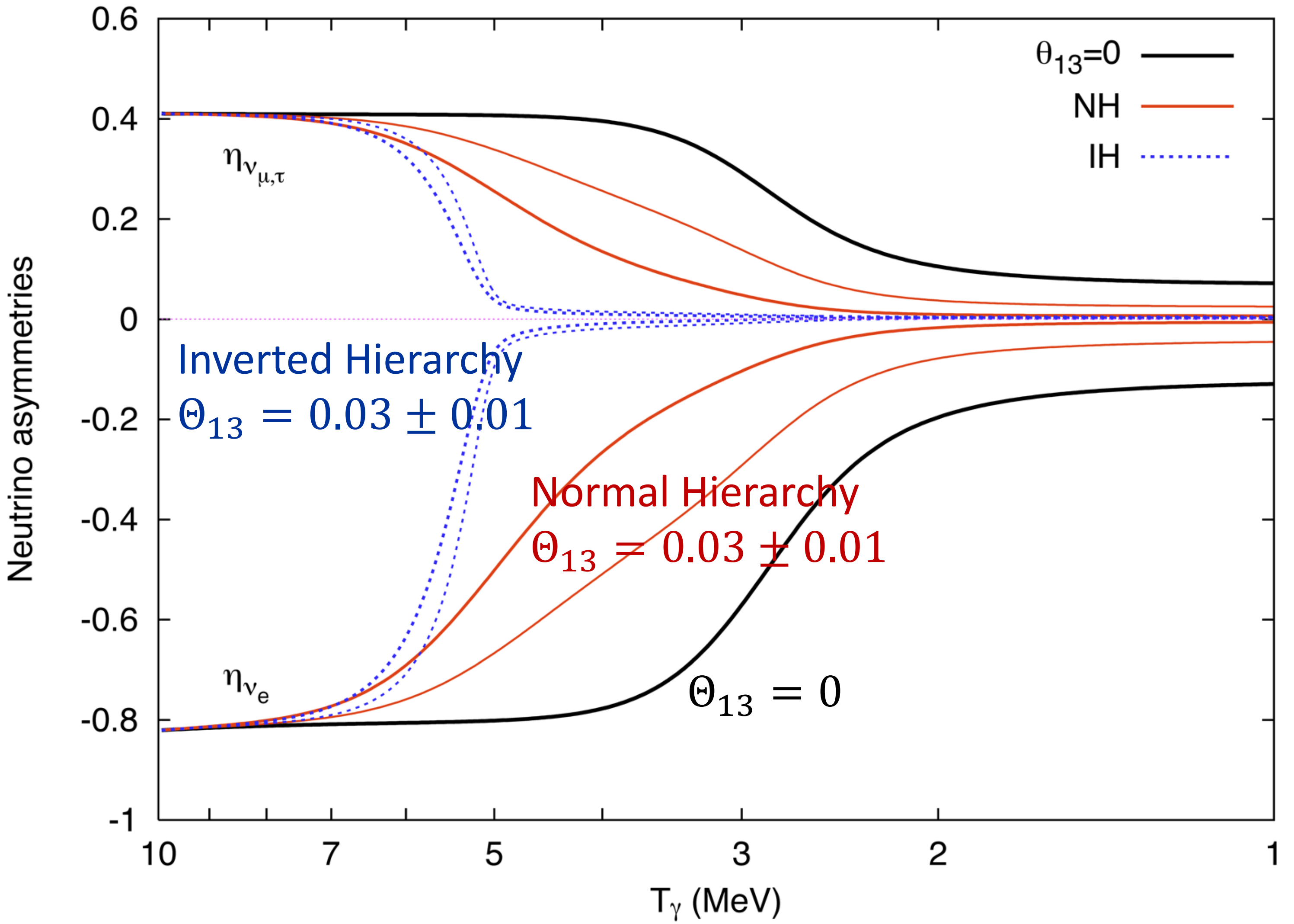
Cosmic neutrino density
close to standard value

Flavor equilibrium before n/p freeze out assured because no mixing angle small

Our knowledge of the cosmic neutrino density depends on measured oscillation parameters!

arXiv:hep-ph/0012056 , hep-ph/0201287,
astro-ph/0203442, hep-ph/0203180,
arXiv:0808.3137, 1011.0916, 1110.4335

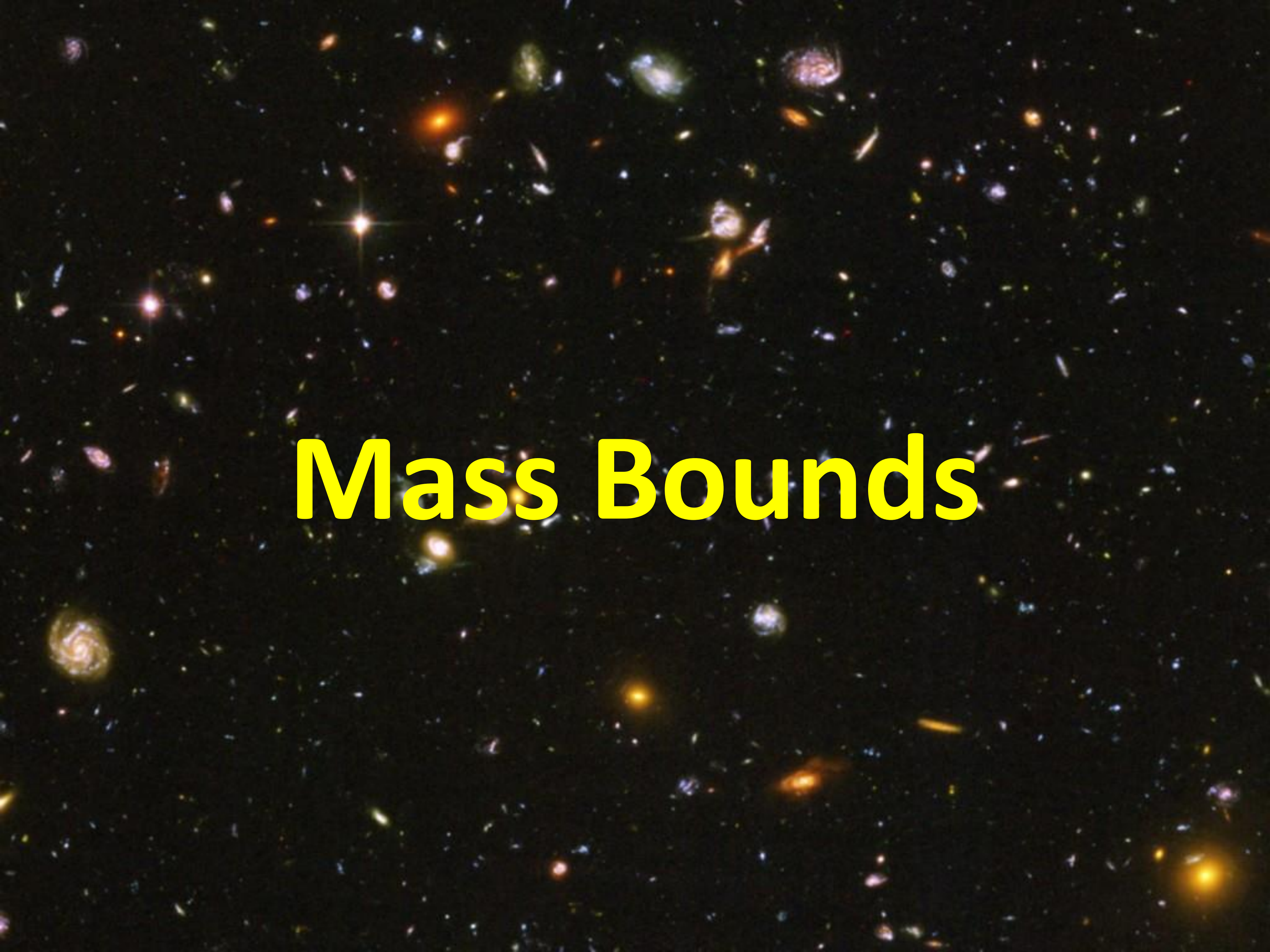
Flavor Conversion before BBN (Θ_{13} not small)



Mangano, Miele, Pastor, Pisanti & Sarikas, arXiv:1110.4335

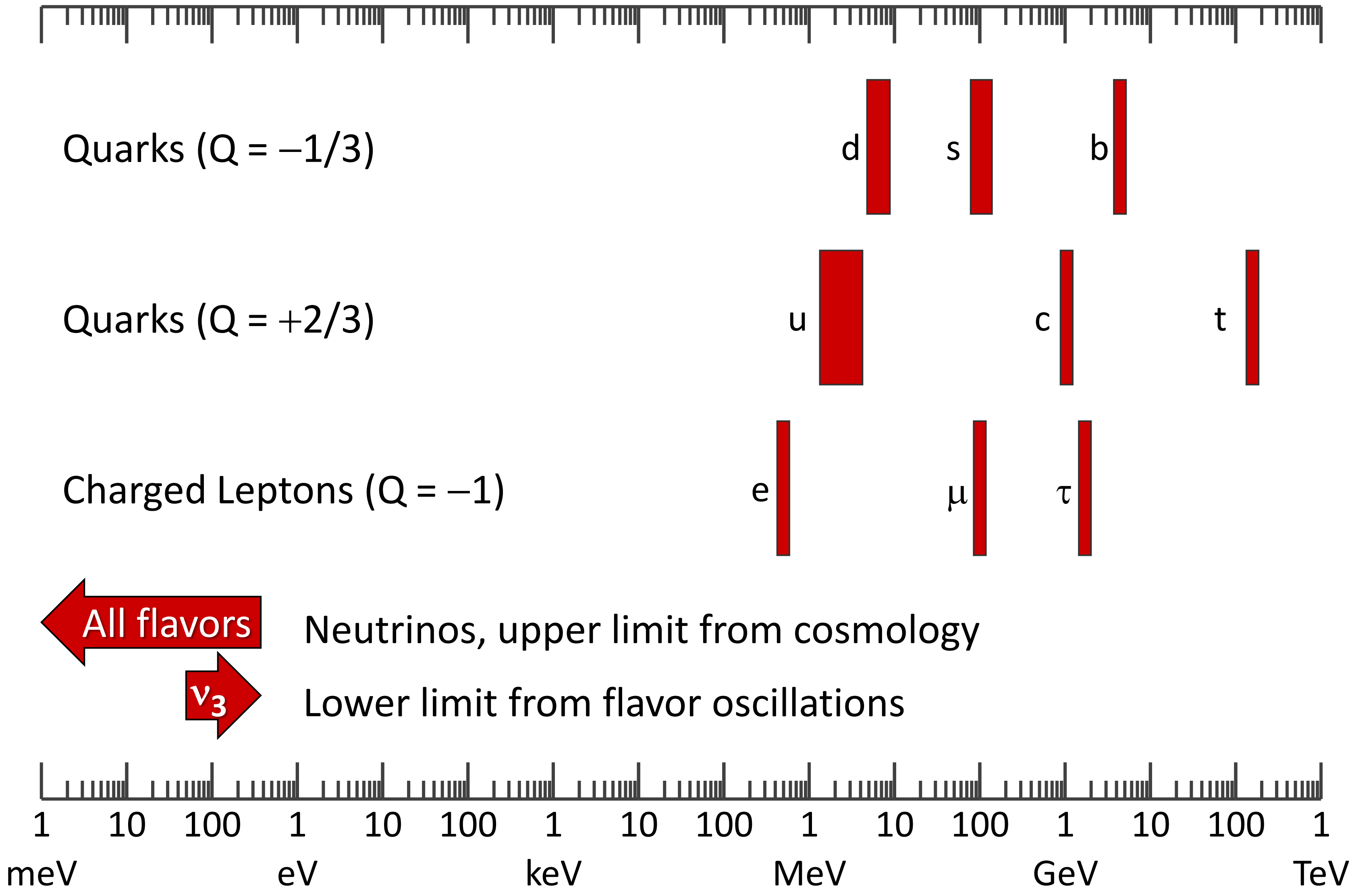
BBN Summary

- BBN accounts well for He and D abundance
- Lithium-7 problem unresolved
- Weak hint for extra radiation
- Large neutrino asymmetries not possible
- Cosmic neutrino background exists with roughly the predicted abundance



Mass Bounds

Fermion Mass Spectrum



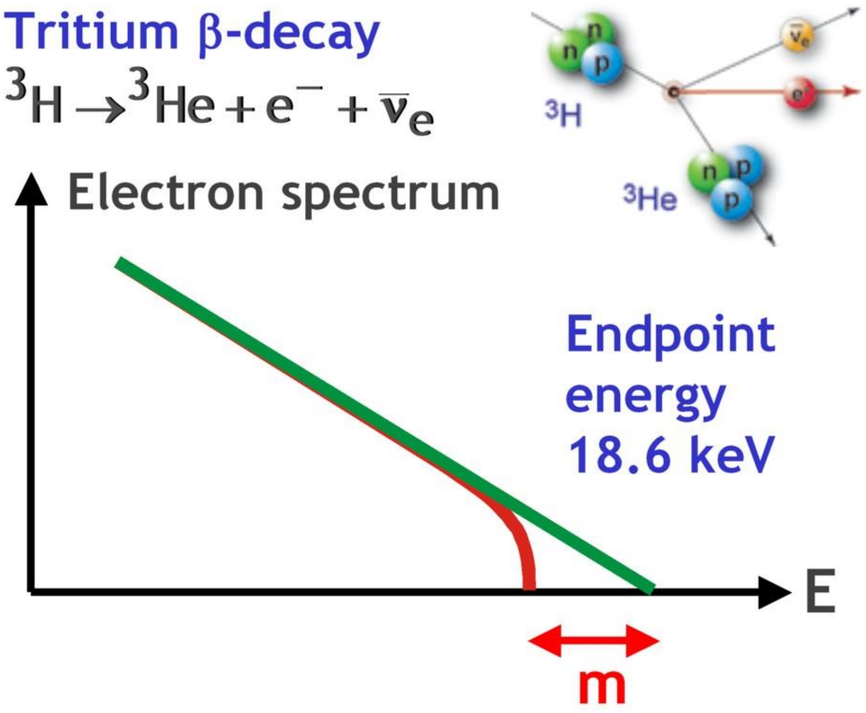
Neutrinos, upper limit from cosmology

Lower limit from flavor oscillations

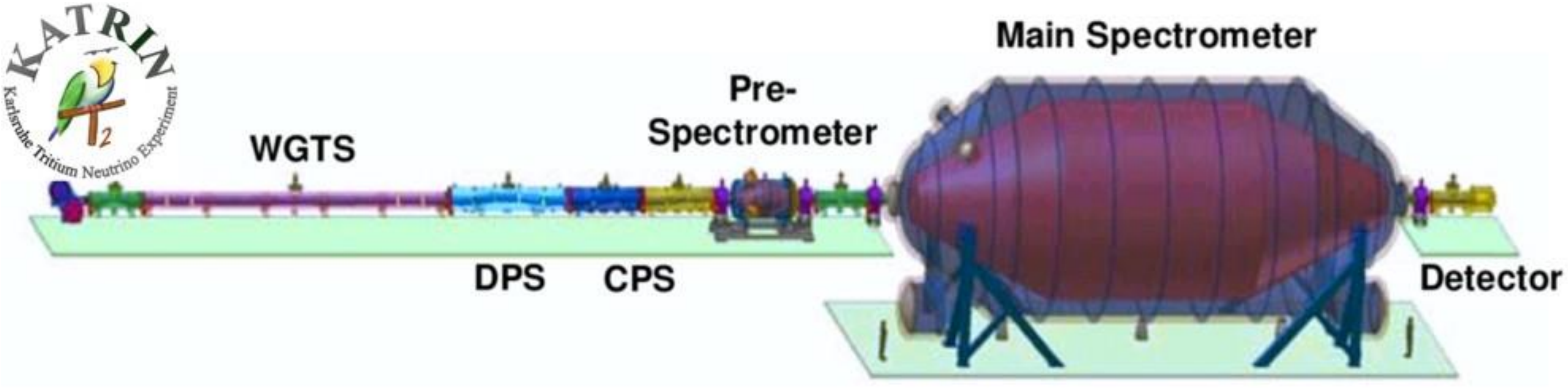
"Weighing" Neutrinos with KATRIN

Tritium β-decay

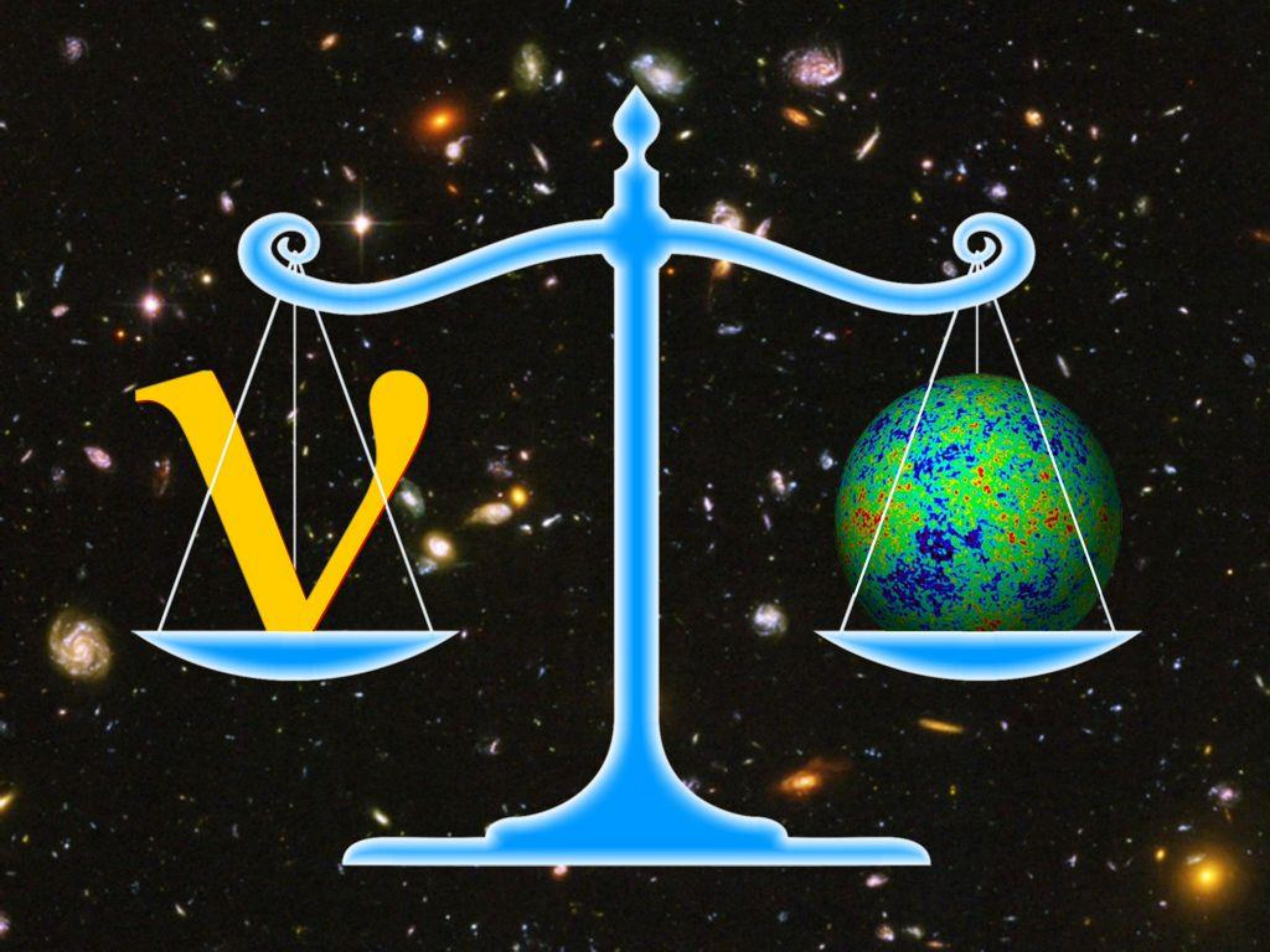
$^{3}H \rightarrow {}^{3}He + e^{-} + \overline{\nu}_{e}$



- Sensitive to common mass scale m for all flavors because of small mass differences from oscillations
- Best limit from Mainz und Troitsk
  m < 2.2 eV (95% CL)
- KATRIN can reach 0.2 eV
- Under construction
- Data taking to begin 2015/16
- http://www.katrin.kit.edu



“KATRIN Coming” (25 Nov 2006)


ν

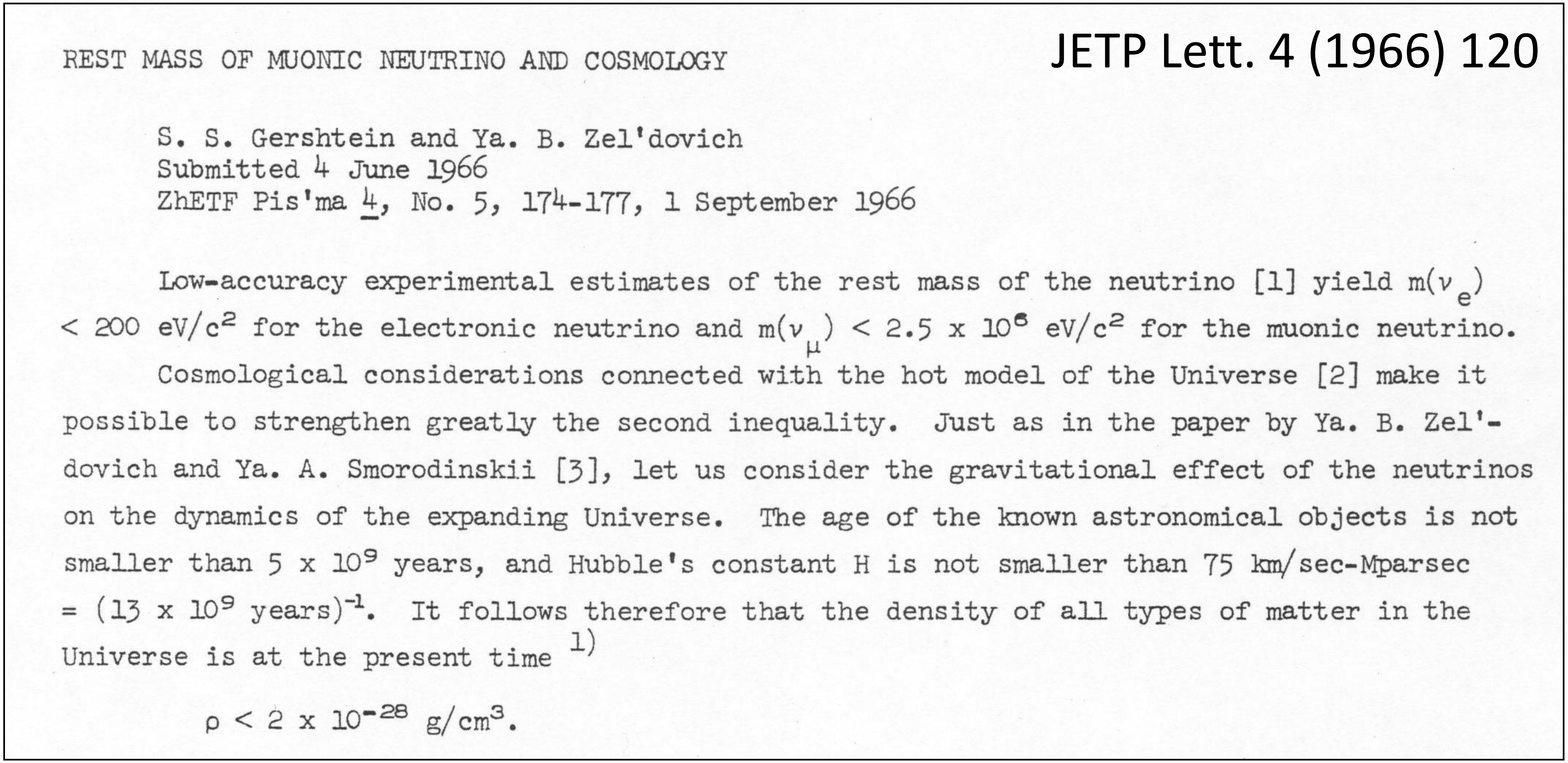
Cosmological Limit on Neutrino Masses

Cosmic neutrino “sea” ~112 cm^{-3} neutrinos + anti-neutrinos per flavor

$$\Omega_\nu h^2 = \sum \frac{m_\nu}{93\ \text{eV}} < 0.23$$

$$\sum m_\nu \lesssim 20\ \text{eV}$$

For all
stable flavors



JETP Lett. 4 (1966) 120

REST MASS OF MUONIC NEUTRINO AND COSMOLOGY

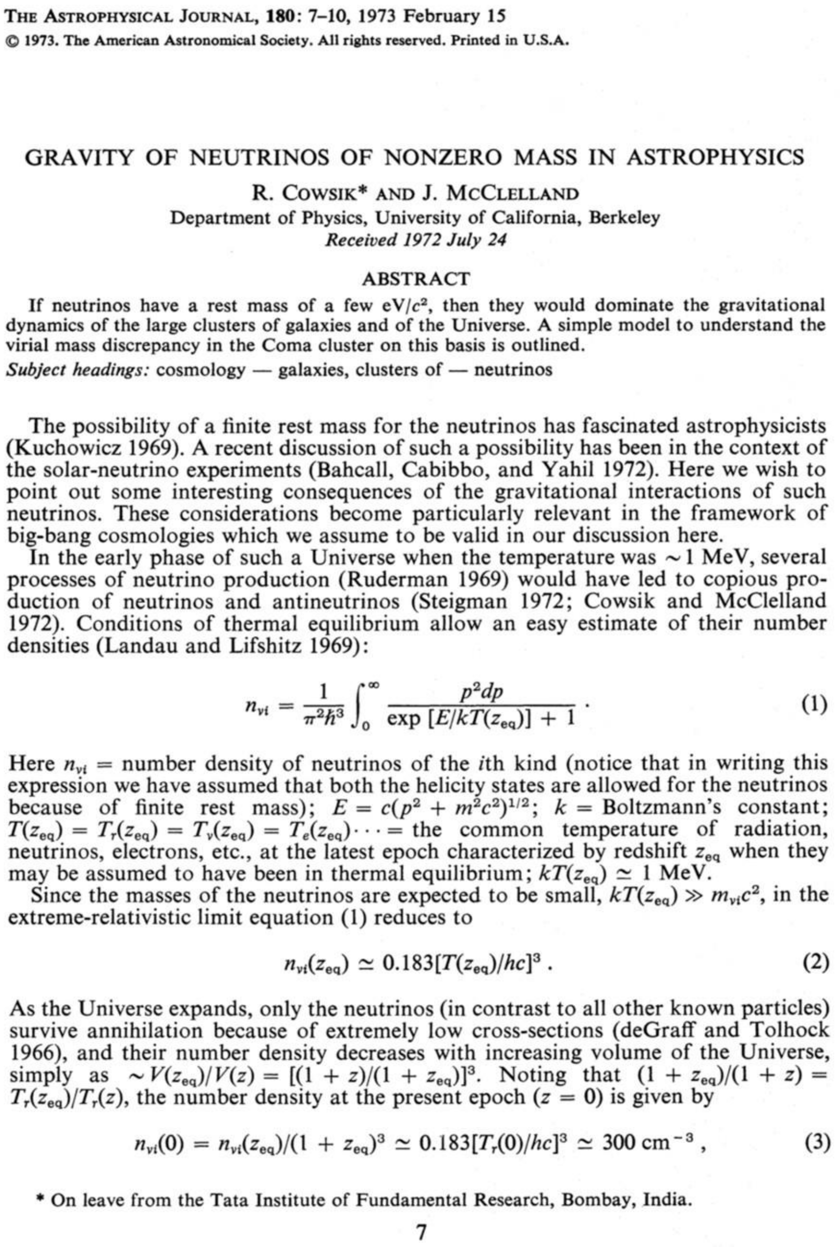
S. S. Gershtein and Ya. B. Zel'dovich
Submitted 4 June 1966
ZhETF Pis'ma 4, No. 5, 174-177, 1 September 1966

Low-accuracy experimental estimates of the rest mass of the neutrino [1] yield $m(\nu_e) < 200$ eV/c^2 for the electronic neutrino and $m(\nu_\mu) < 2.5 \times 10^6$ eV/c^2 for the muonic neutrino.

Cosmological considerations connected with the hot model of the Universe [2] make it possible to strengthen greatly the second inequality. Just as in the paper by Ya. B. Zel'dovich and Ya. A. Smorodinskii [3], let us consider the gravitational effect of the neutrinos on the dynamics of the expanding Universe. The age of the known astronomical objects is not smaller than 5×10^9 years, and Hubble's constant H is not smaller than 75 km/sec-Mparsec $= (13 \times 10^9\ \text{years})^{-1}$. It follows therefore that the density of all types of matter in the Universe is at the present time 1)

$$\rho < 2 \times 10^{-28}\ \text{g/cm}^3.$$

Weakly Interacting Particles as Dark Matter



The Astrophysical Journal, **180**: 7–10, 1973 February 15

GRAVITY OF NEUTRINOS OF NONZERO MASS IN ASTROPHYSICS

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Department of Physics, University of California, Berkeley

Received 1972 July 24

ABSTRACT

If neutrinos have a rest mass of a few eV/c^2, then they would dominate the gravitational dynamics of the large clusters of galaxies and of the Universe. A simple model to understand the virial mass discrepancy in the Coma cluster on this basis is outlined.

Subject headings: cosmology — galaxies, clusters of — neutrinos

The possibility of a finite rest mass for the neutrinos has fascinated astrophysicists (Kuchowicz 1969). A recent discussion of such a possibility has been in the context of the solar-neutrino experiments (Bahcall, Cabibbo, and Yahil 1972). Here we wish to point out some interesting consequences of the gravitational interactions of such neutrinos. These considerations become particularly relevant in the framework of big-bang cosmologies which we assume to be valid in our discussion here.

In the early phase of such a Universe when the temperature was ~1 MeV, several processes of neutrino production (Ruderman 1969) would have led to copious production of neutrinos and antineutrinos (Steigman 1972; Cowsik and McClelland 1972). Conditions of thermal equilibrium allow an easy estimate of their number densities (Landau and Lifshitz 1969):

$$n_{\nu i} = \frac{1}{\pi^2 \hbar^3} \int_0^\infty \frac{p^2 dp}{\exp\left[E/kT(z_{\mathrm{eq}})\right] + 1} \,. \tag{1}$$

Here $n_{\nu i}$ = number density of neutrinos of the ith kind (notice that in writing this expression we have assumed that both the helicity states are allowed for the neutrinos because of finite rest mass); $E = c(p^2 + m^2c^2)^{1/2}$; k = Boltzmann's constant; $T(z_{\mathrm{eq}}) = T_r(z_{\mathrm{eq}}) = T_\nu(z_{\mathrm{eq}}) = T_e(z_{\mathrm{eq}}) \cdots =$ the common temperature of radiation, neutrinos, electrons, etc., at the latest epoch characterized by redshift z_{eq} when they may be assumed to have been in thermal equilibrium; $kT(z_{\mathrm{eq}}) \simeq 1$ MeV.

Since the masses of the neutrinos are expected to be small, $kT(z_{\mathrm{eq}}) \gg m_{\nu i}c^2$, in the extreme-relativistic limit equation (1) reduces to

$$n_{\nu i}(z_{\mathrm{eq}}) \simeq 0.183[T(z_{\mathrm{eq}})/hc]^3 \,. \tag{2}$$

As the Universe expands, only the neutrinos (in contrast to all other known particles) survive annihilation because of extremely low cross-sections (deGraff and Tolhock 1966), and their number density decreases with increasing volume of the Universe, simply as $\sim V(z_{\mathrm{eq}})/V(z) = [(1 + z)/(1 + z_{\mathrm{eq}})]^3$. Noting that $(1 + z_{\mathrm{eq}})/(1 + z) = T_r(z_{\mathrm{eq}})/T_r(z)$, the number density at the present epoch ($z = 0$) is given by

$$n_{\nu i}(0) = n_{\nu i}(z_{\mathrm{eq}})/(1 + z_{\mathrm{eq}})^3 \simeq 0.183[T_r(0)/hc]^3 \simeq 300\ \mathrm{cm}^{-3} \,, \tag{3}$$

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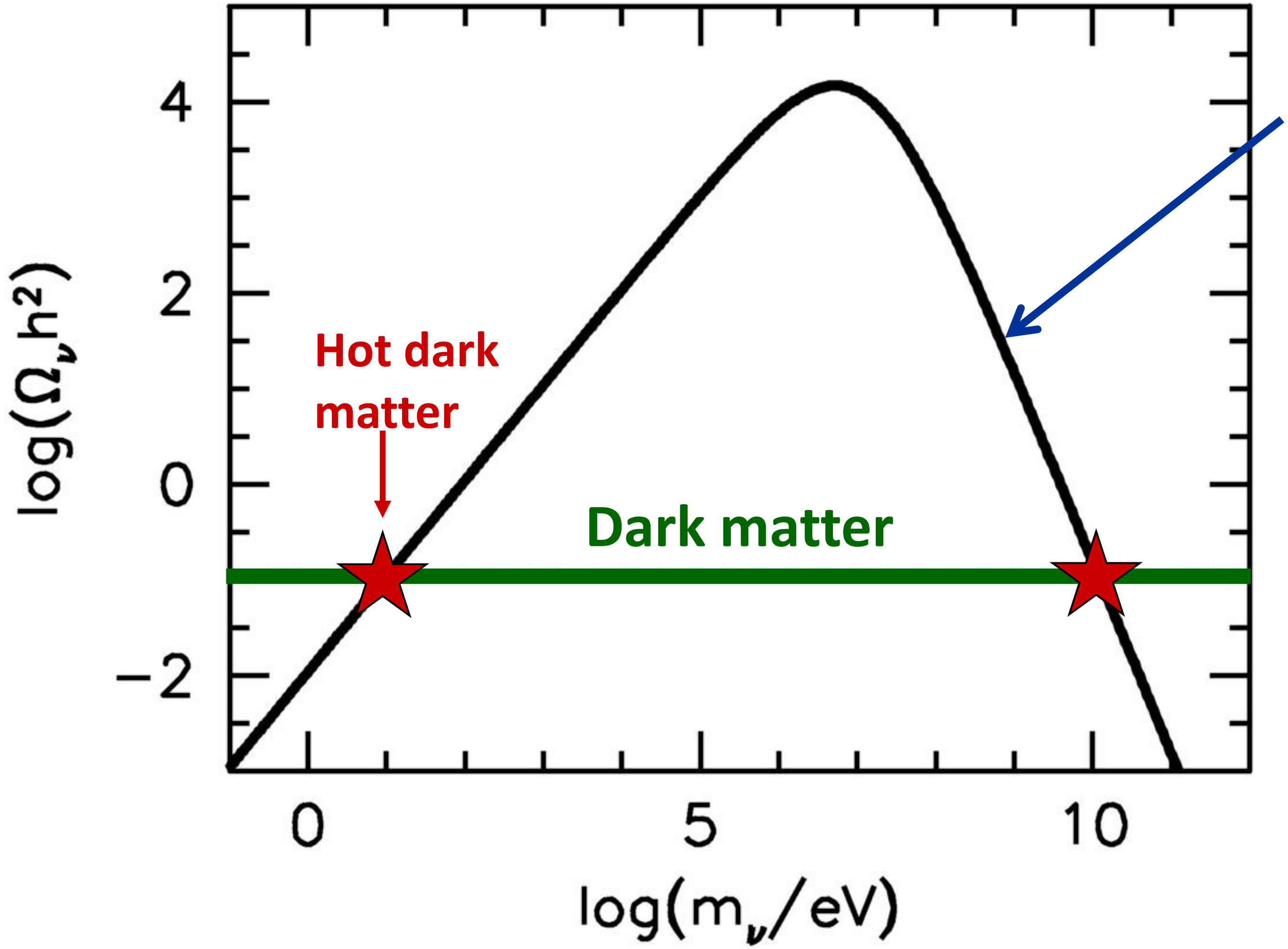
* On leave from the Tata Institute of Fundamental Research, Bombay, India.

- Almost 40 years ago, beginnings of the idea of weakly interacting particles (neutrinos) as dark matter
- Massive neutrinos are no longer a good candidate (hot dark matter)
- However, the idea of weakly interacting massive particles (WIMPs) as dark matter is now standard

Lee Weinberg Curve

Cosmic matter fraction as a function of neutrino mass



- For $m_\nu \gtrsim 1$ MeV neutrinos freeze out nonrelativistically
- Density suppressed by annihilation before freeze-out

Weakly interacting massive particles (WIMPs) possible as cold dark matter

What is wrong with neutrino dark matter?

Galactic Phase Space ("Tremaine-Gunn-Limit")

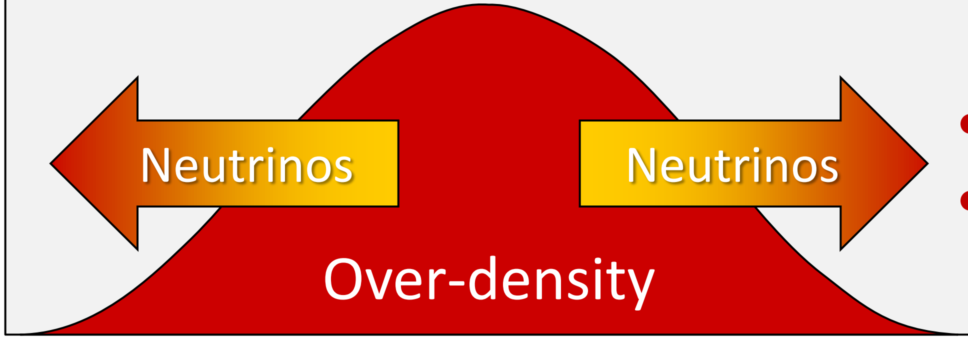
Maximum mass density of a degenerate Fermi gas

$$\rho_{\max} = m_\nu \underbrace{\frac{p_{\max}^3}{3\pi^2}}_{n_{\max}} = \frac{m_\nu \left(m_\nu v_{\text{escape}}\right)^3}{3\pi^2}$$

Spiral galaxies
m_ν > 20–40 eV
Dwarf galaxies
m_ν > 100–200 eV

Neutrino Free Streaming (Collisionless Phase Mixing)

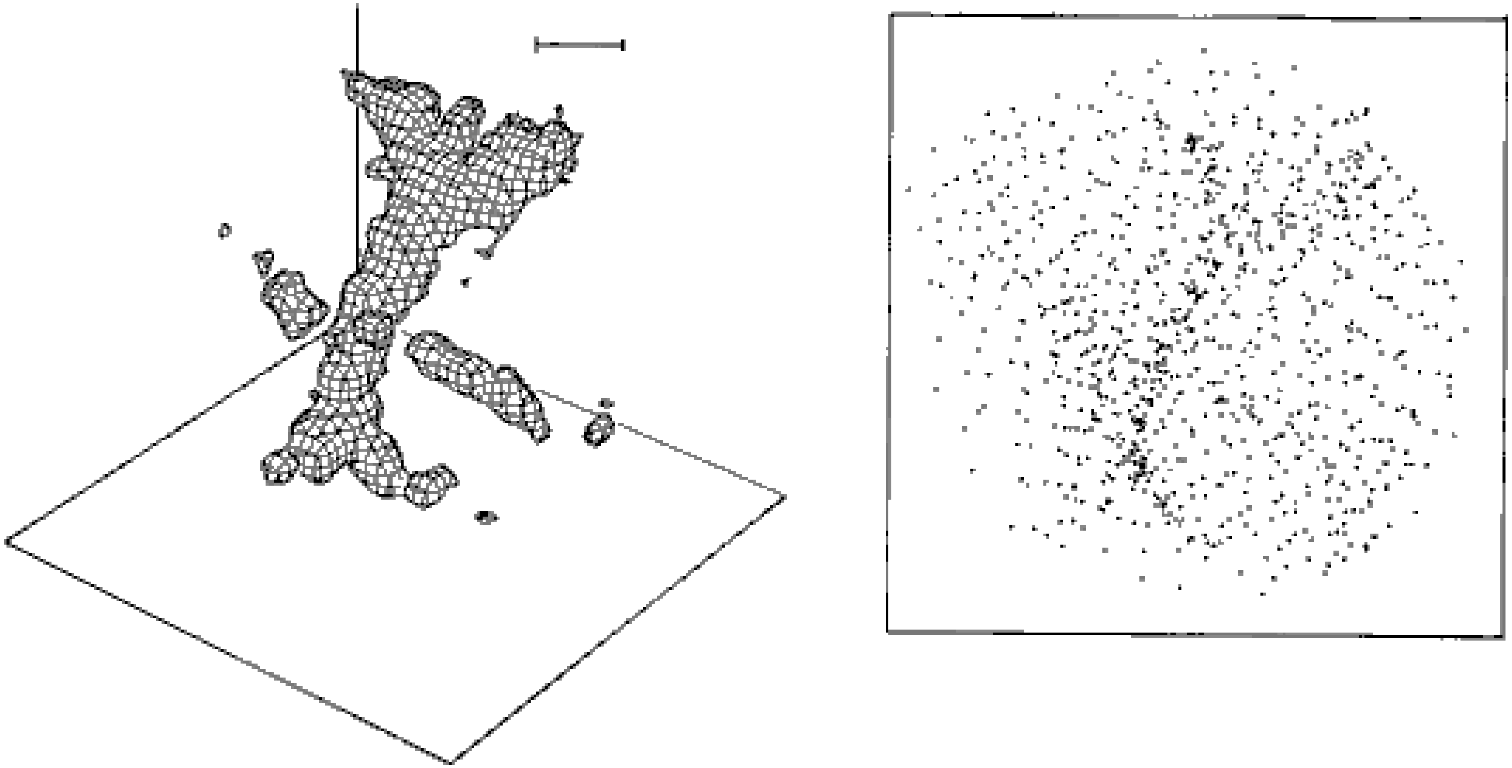
- At T < 1 MeV neutrino scattering in early universe is ineffective
- Stream freely until non-relativistic
- Wash out density contrasts on small scales



- Neutrinos are "Hot Dark Matter"
- Ruled out by structure formation

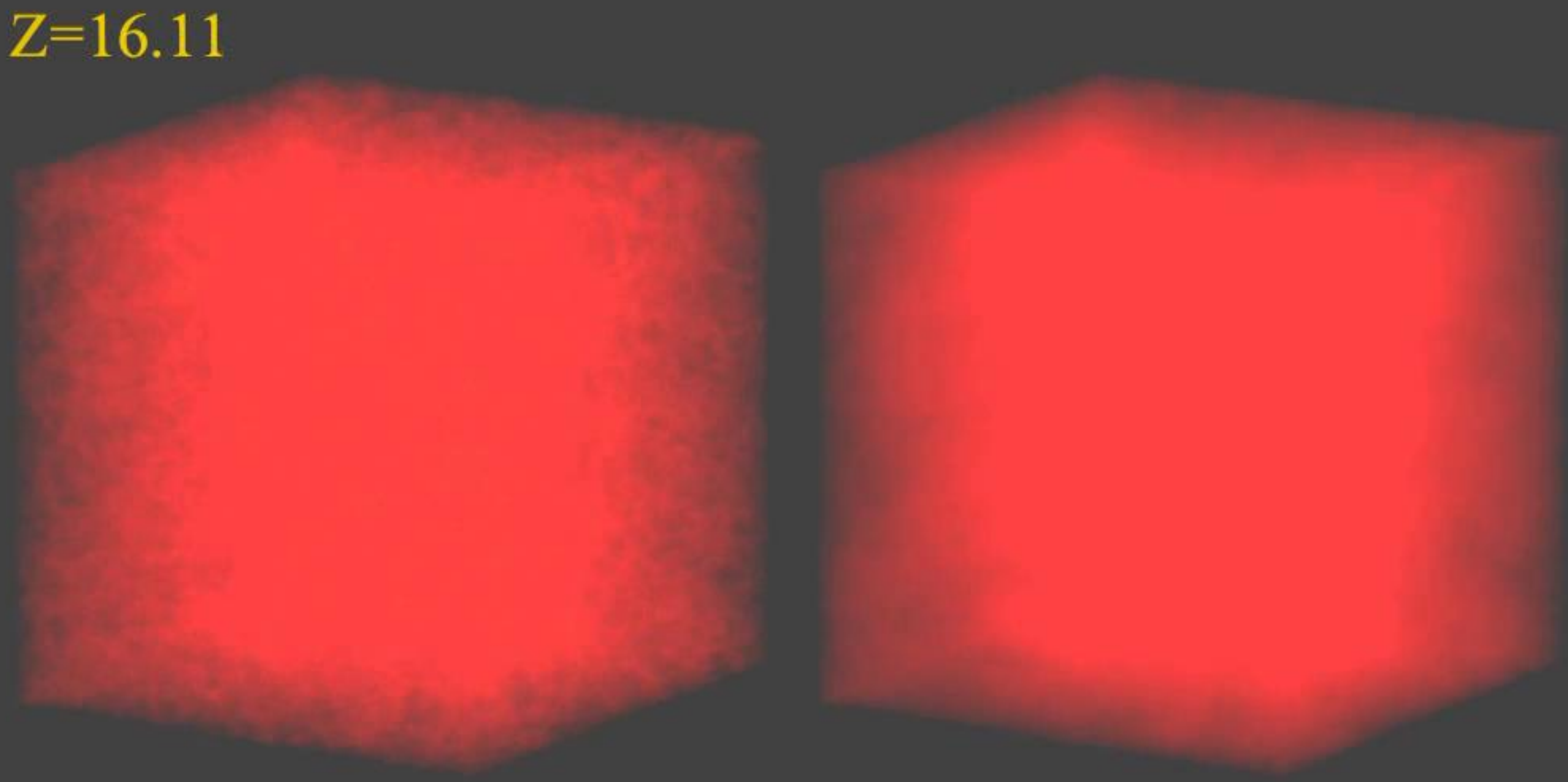
Hot dark matter ruled out in 1983

1000 particle simulation by Frenk, White & Davis, ApJ 271 (1983) 417



The coherence length of the neutrino distribution […] is too large to be consistent with the observed clustering scale of galaxies […] The conventional neutrino-dominated picture appears to be ruled out. White, Frenk & Davis, ApJ 274 (1983) L1.

Structure Formation with Hot Dark Matter



Standard ΛCDM Model

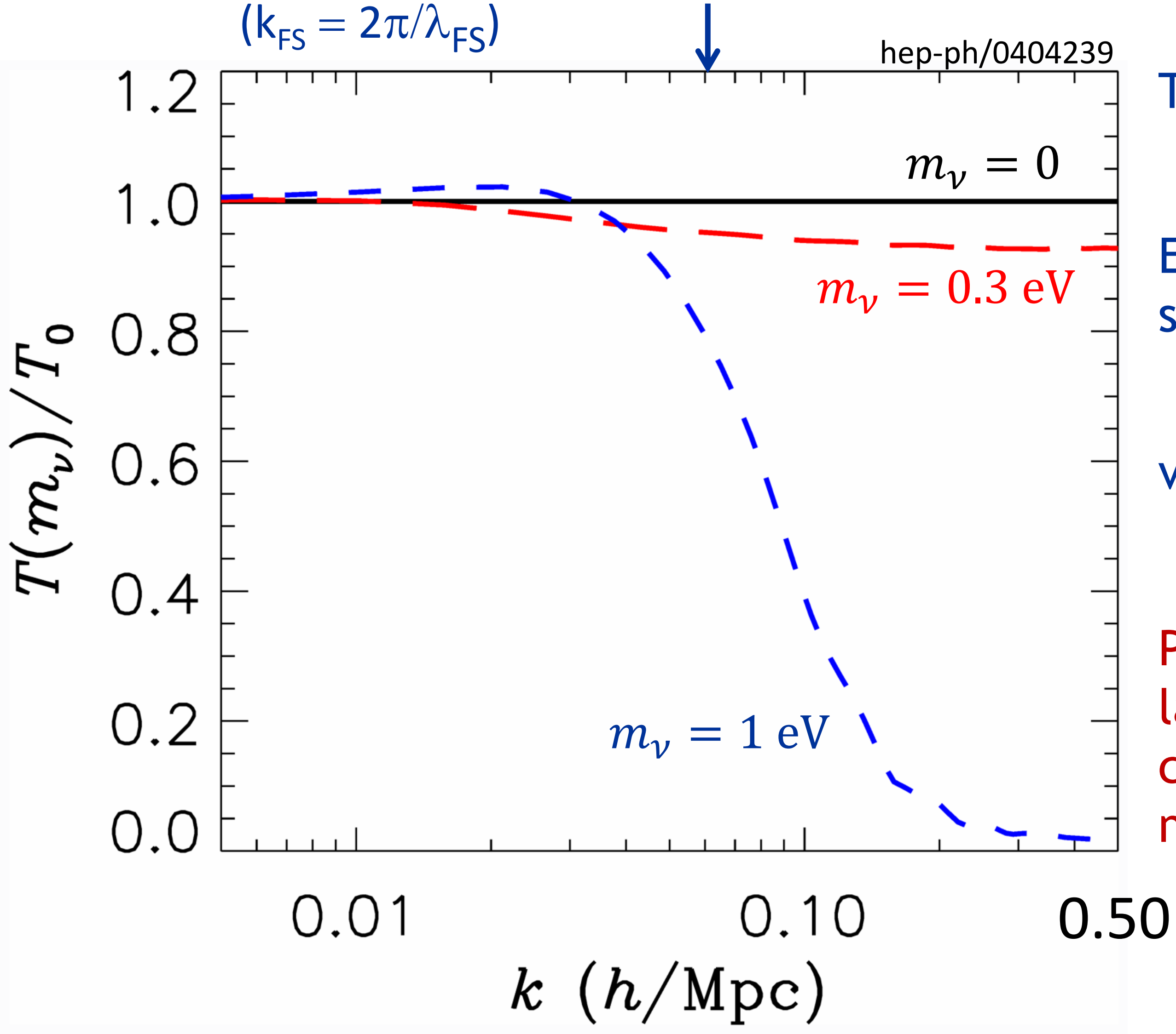
Neutrinos with $\Sigma m_\nu = 6.9$ eV

Structure fromation simulated with Gadget code
Cube size 256 Mpc at zero redshift
Troels Haugbølle, http://users-phys.au.dk/haugboel

Neutrino Free Streaming: Transfer Function

Power suppression for $\lambda_{FS} \gtrsim 100$ Mpc/h
($k_{FS} = 2\pi/\lambda_{FS}$)

hep-ph/0404239



Transfer function

$P(k) = T(k)\, P_0(k)$

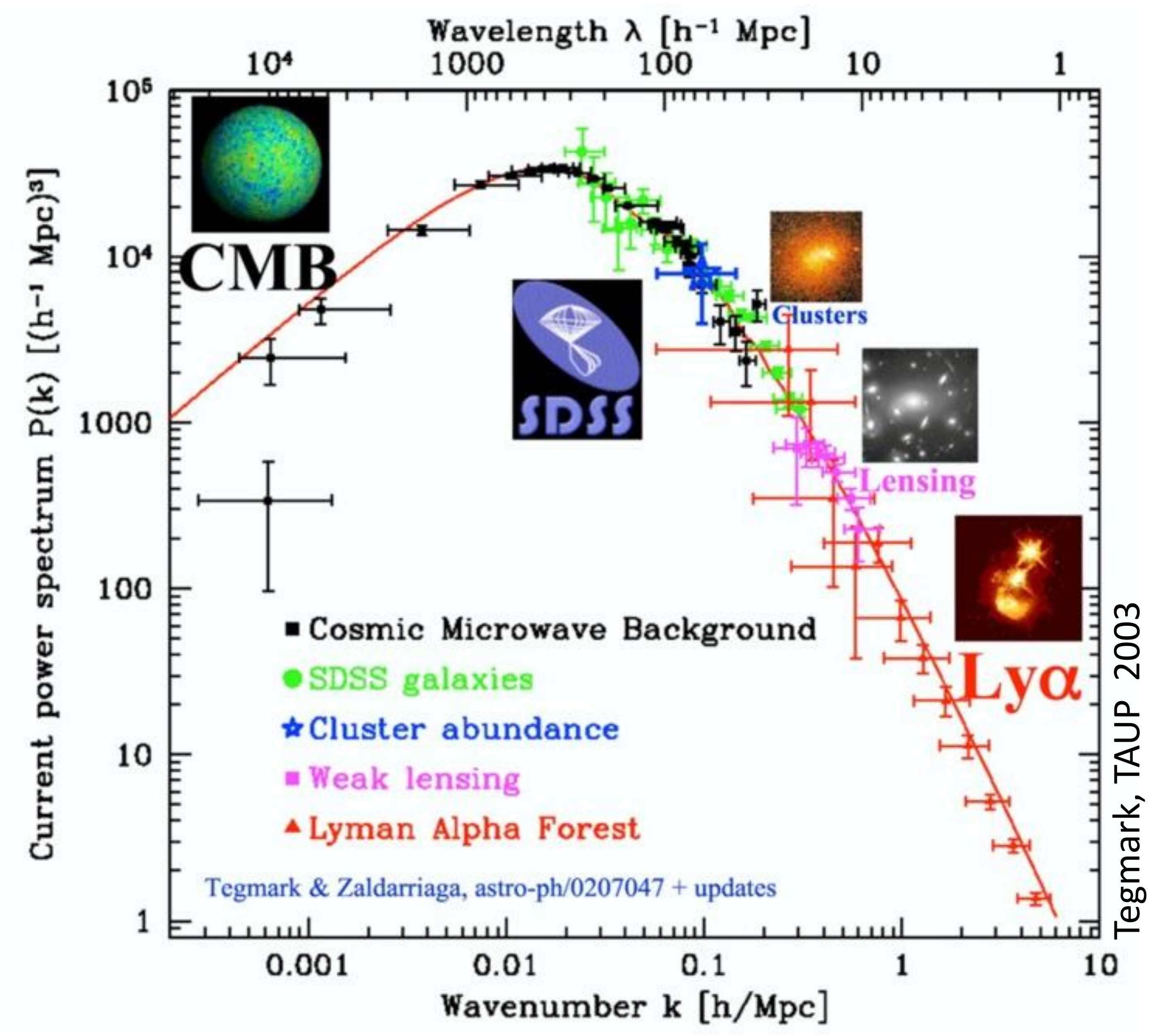
Effect of neutrino free streaming on small scales

$T(k) = 1 - 8\, \Omega_\nu/\Omega_M$

valid for $8\Omega_\nu/\Omega_M \ll 1$

Power suppression much larger (factor 8) than corresponds to neutrino mass fraction!

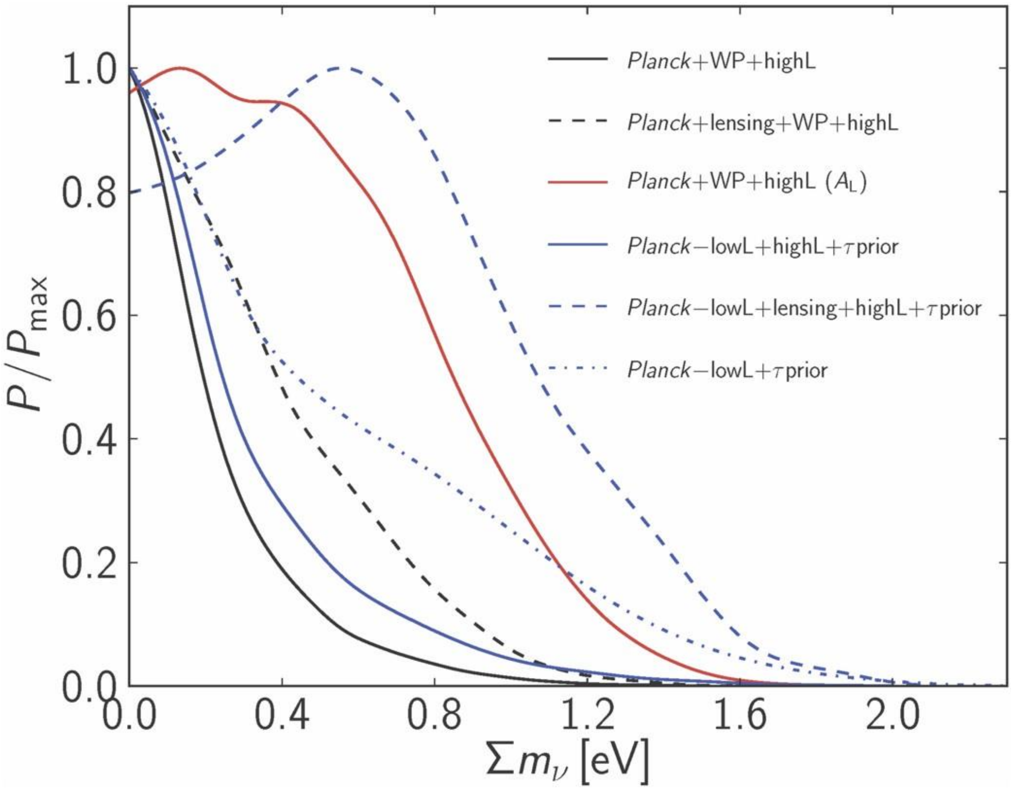
Power Spectrum of Cosmic Density Fluctuations



Tegmark, TAUP 2003

Neutrino Mass Limits Post Planck (2013)

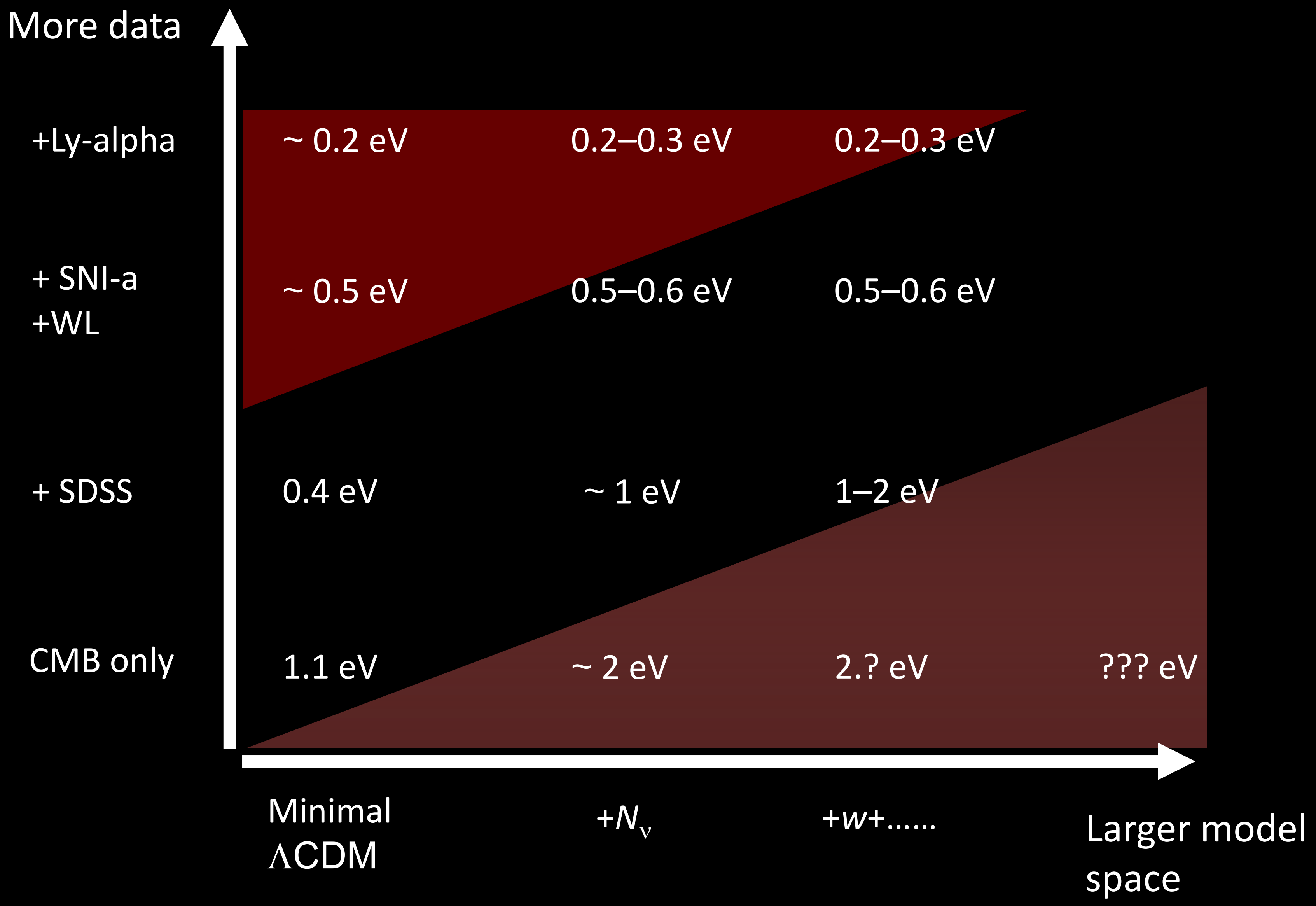
Depends on used data sets
Many different analyses in the literature



Planck alone: $\Sigma m_\nu < 1.08$ eV (95% CL)
CMB + BAO limit: $\Sigma m_\nu < 0.23$ eV (95% CL)

Ade et al. (Planck Collaboration), arXiv:1303.5076

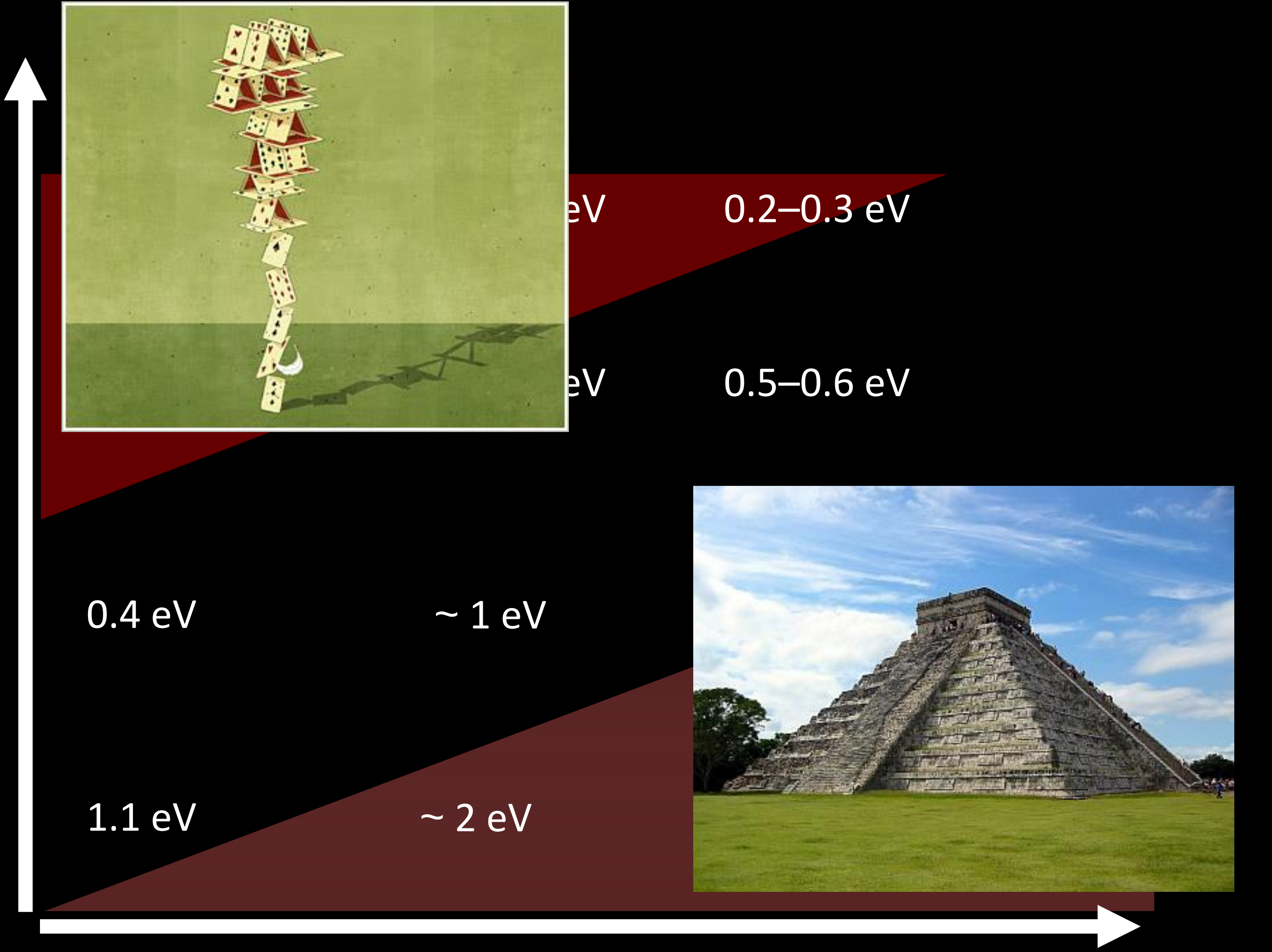
Neutrino Mass from Cosmology (Hannestad)



More data

	Minimal ΛCDM	+N_ν	+*w*+……	Larger model space
+Ly-alpha	~ 0.2 eV	0.2–0.3 eV	0.2–0.3 eV	
+ SNI-a +WL	~ 0.5 eV	0.5–0.6 eV	0.5–0.6 eV	
+ SDSS	0.4 eV	~ 1 eV	1–2 eV	
CMB only	1.1 eV	~ 2 eV	2.? eV	??? eV

Neutrino Mass from Cosmology (Hannestad)

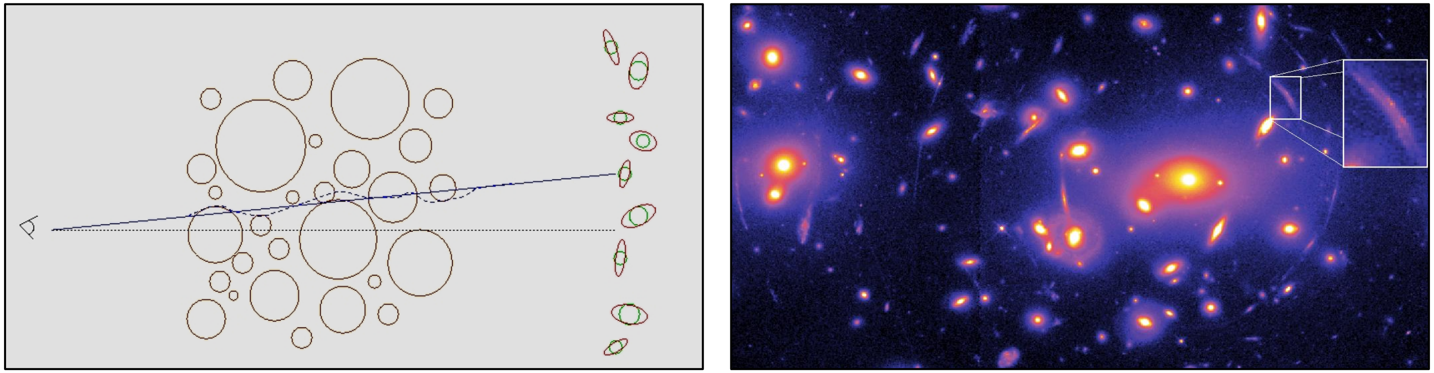


More data

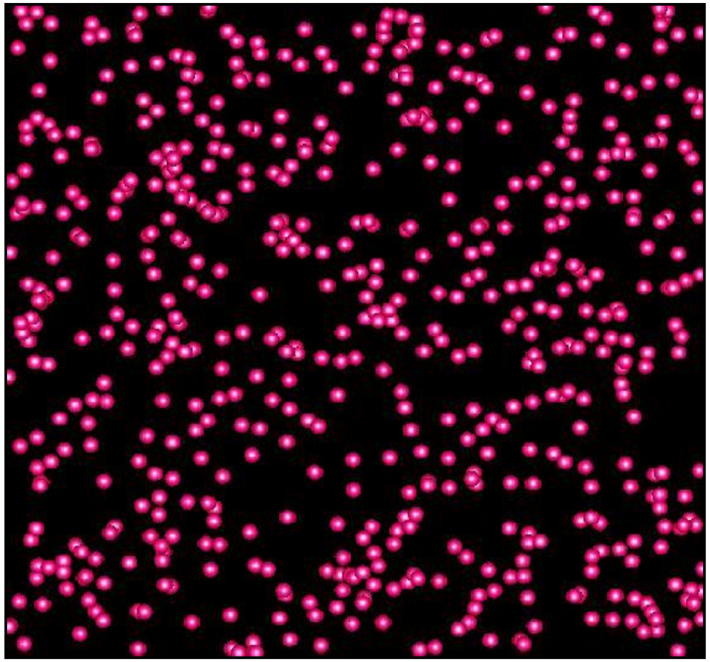
	Minimal ΛCDM	$+N_\nu$	$+w+$……
+Ly-alpha		eV	0.2–0.3 eV
+ SNI-a +WL		eV	0.5–0.6 eV
+ SDSS	0.4 eV	~ 1 eV	
CMB only	1.1 eV	~ 2 eV	

Larger model space

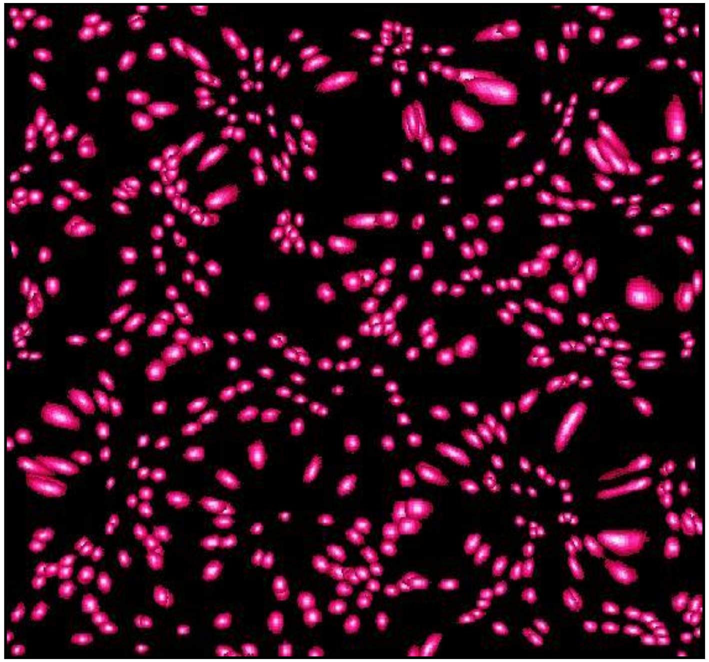
Weak Lensing – A Powerful Probe for the Future



Distortion of background images by foreground matter



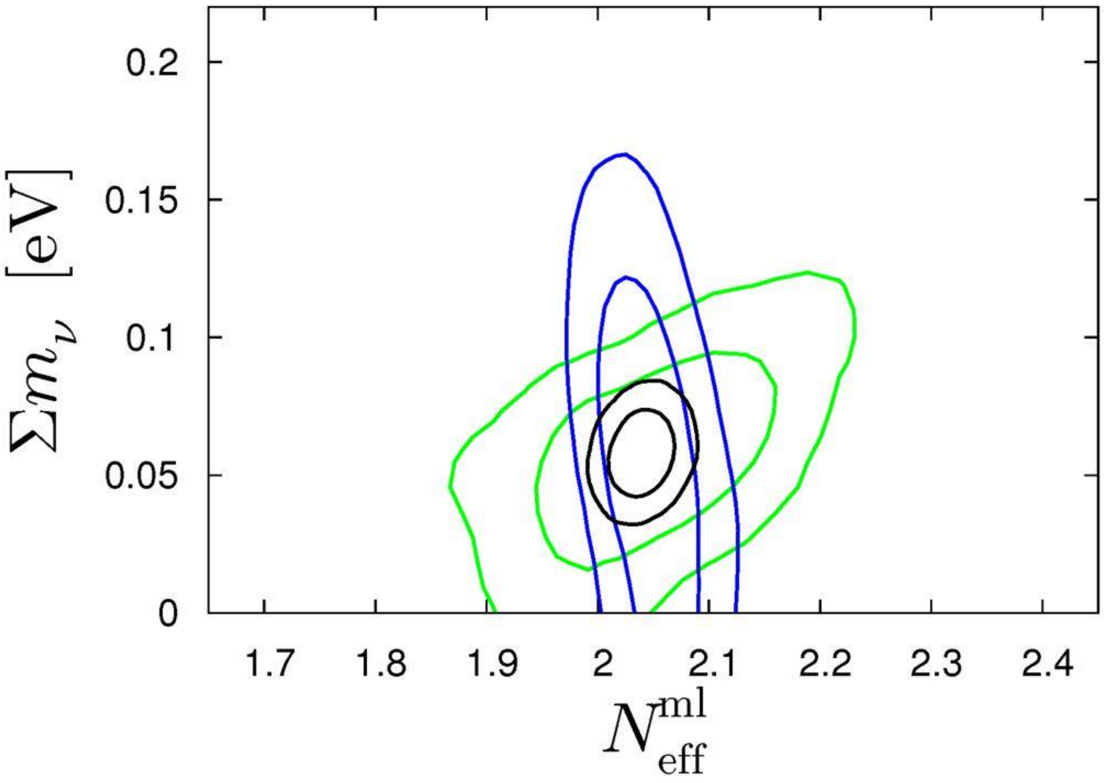
Unlensed

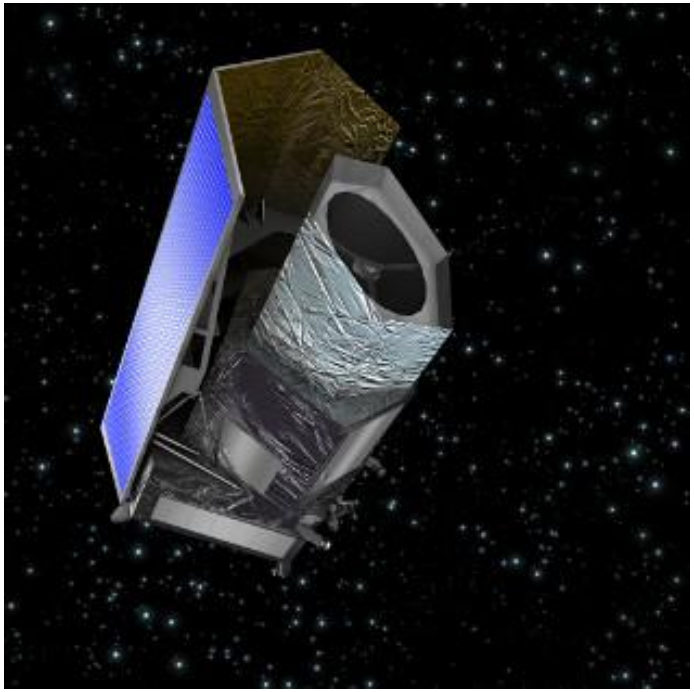


Lensed

Future Cosmological Neutrino Mass Sensitivity

Pin down the neutrino mass in the sky!

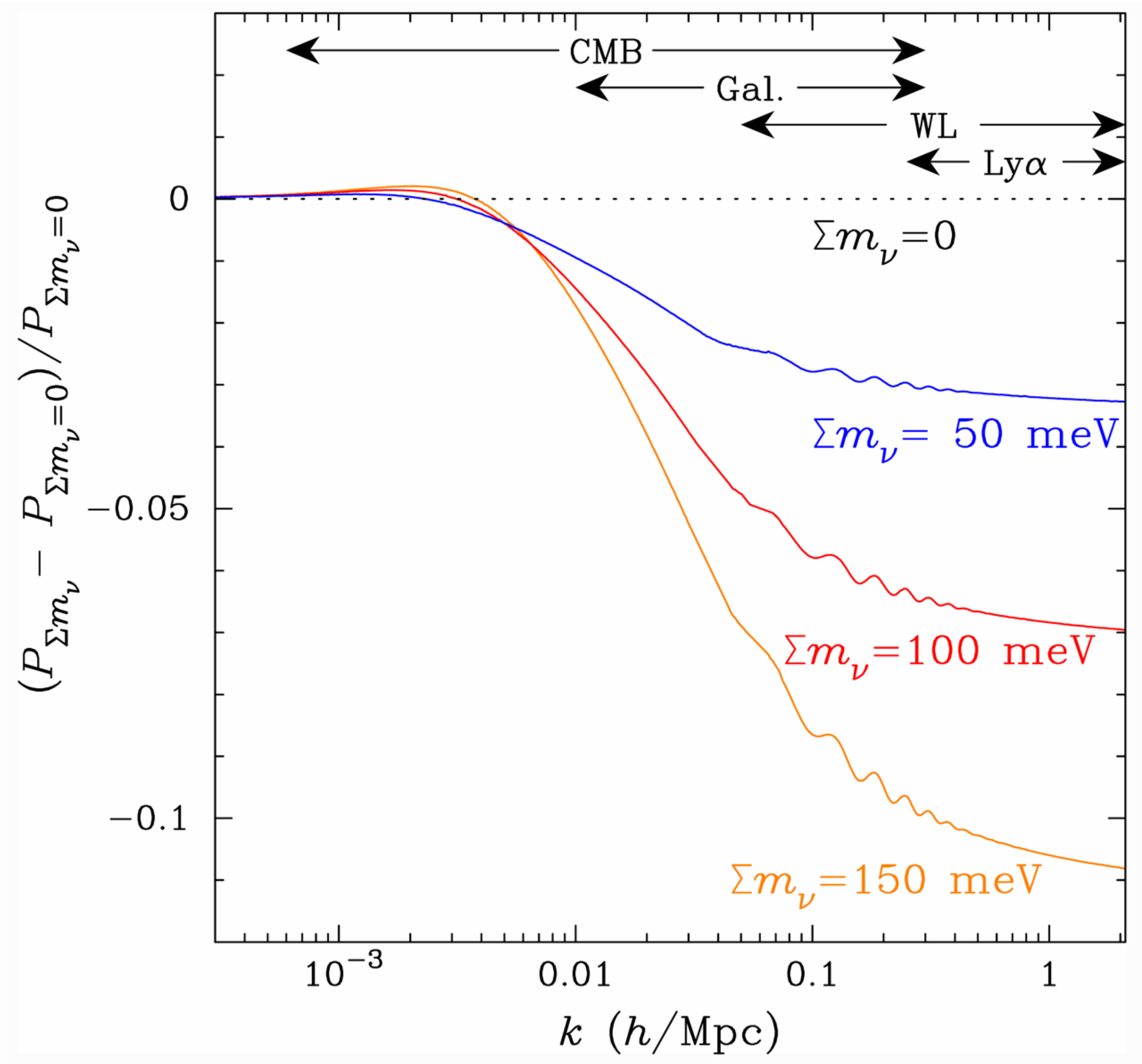




ESA’s Euclid satellite to be launched in 2020
Precision measurement of the universe out to redshift of 2

Basse, Bjælde, Hamann, Hannestad & Wong, arXiv:1304.2321:
Dark energy and neutrino constraints from a future EUCLID-like survey

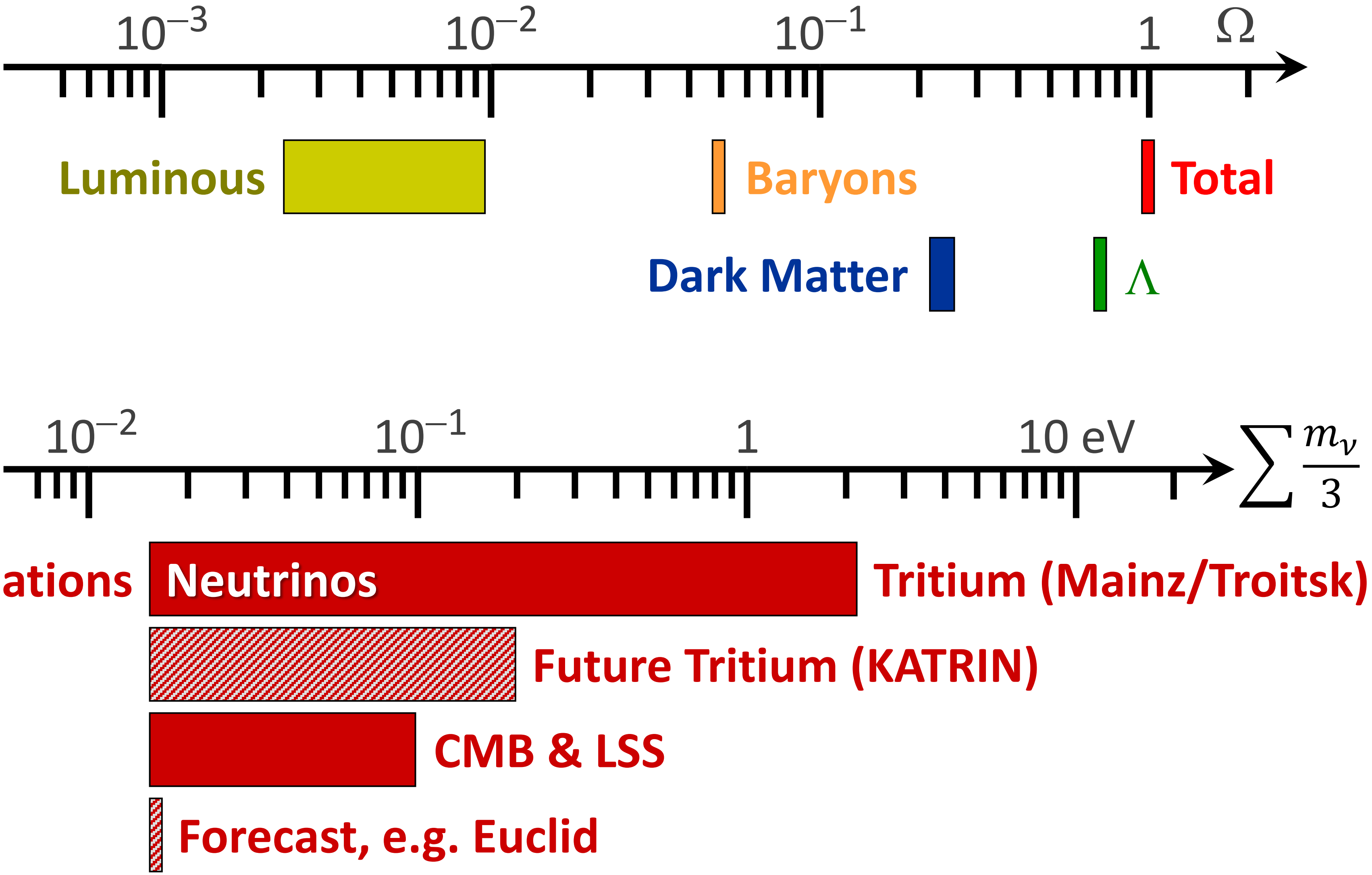
Transfer Function with Massive Neutrinos



Measuring few-percent suppression requires structure-formation theory beyond linear order!

Community Planning Study: Snowmass 2013, arXiv:1309.5383

Mass-Energy-Inventory of the Universe



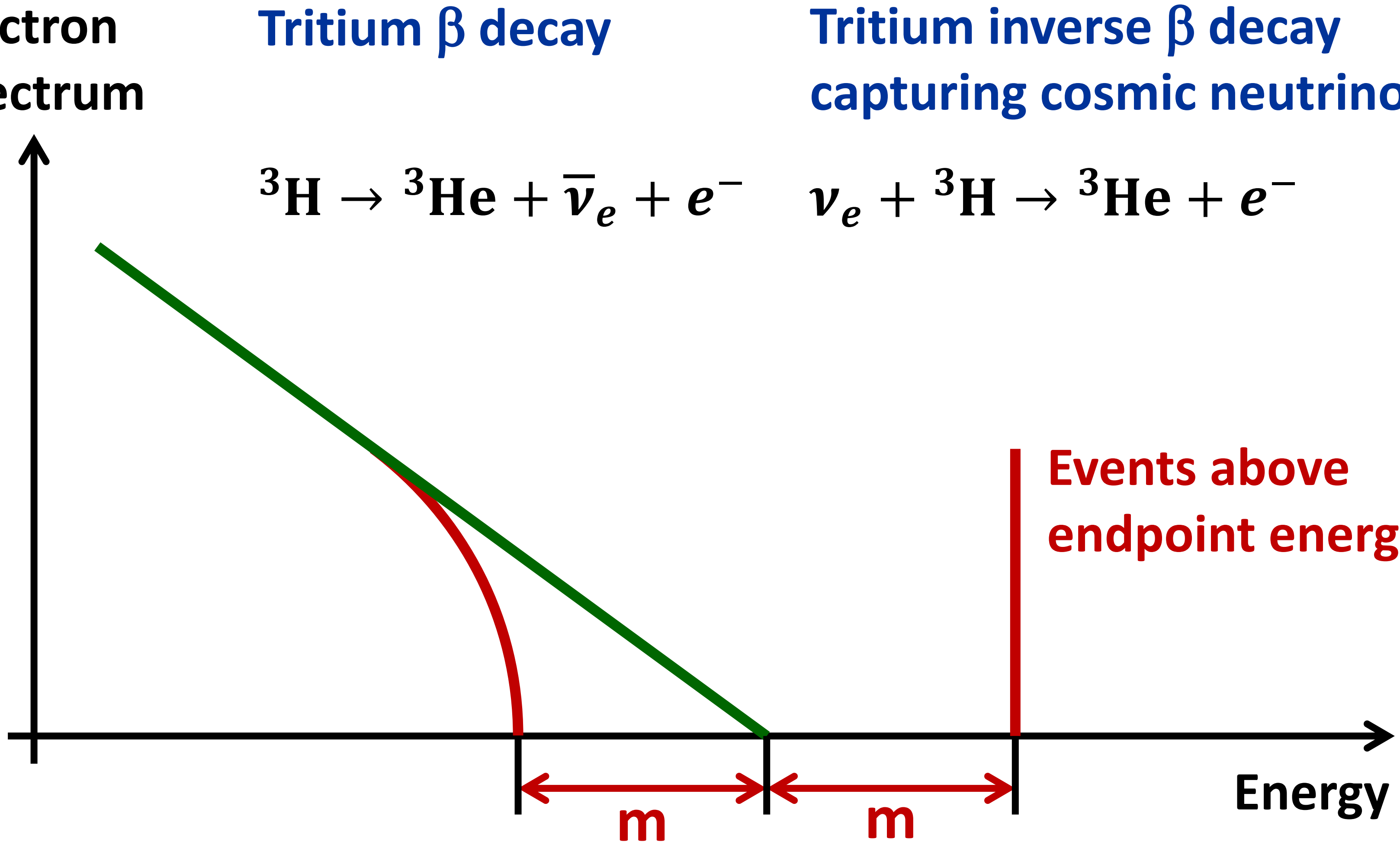
Cosmic Neutrino Capture in Tritium Beta Decay

Tritium β decay

$$^{3}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + \bar{\nu}_e + e^-$$

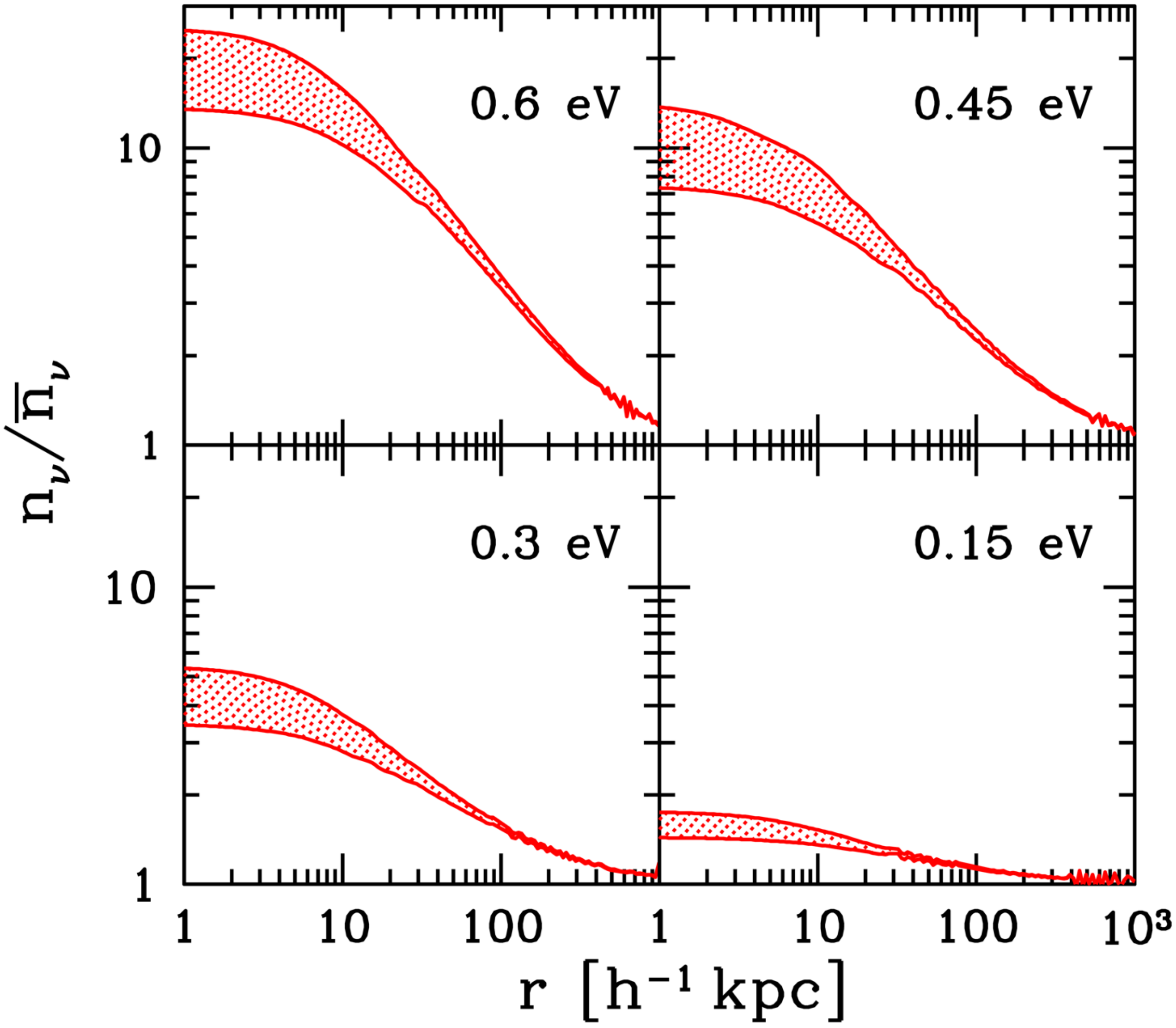
Tritium inverse β decay capturing cosmic neutrinos

$$\nu_e + {}^{3}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + e^-$$



Ptolemy project in Princeton (Chris Tully et al.)

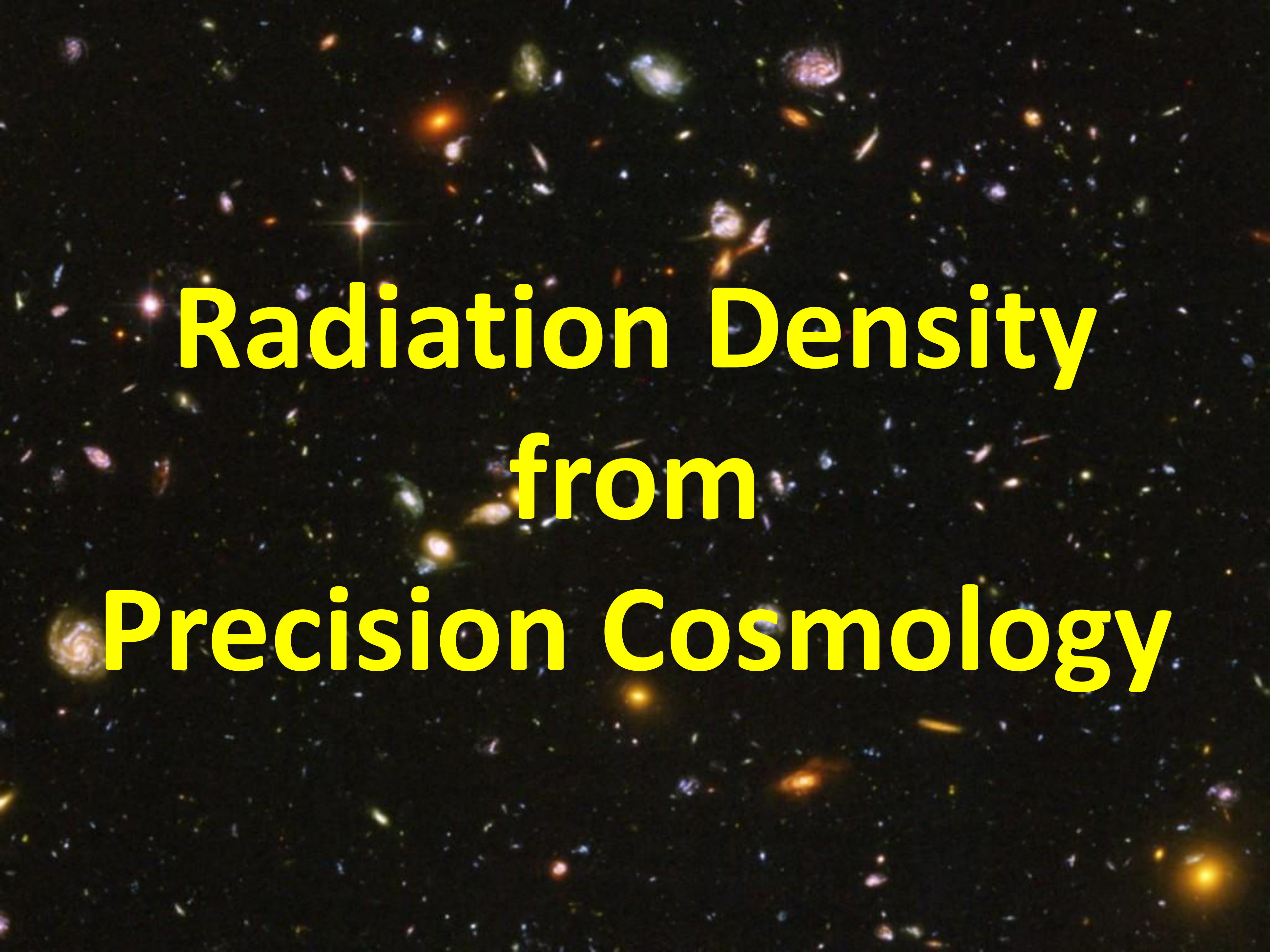
Neutrino Clustering in the Galaxy



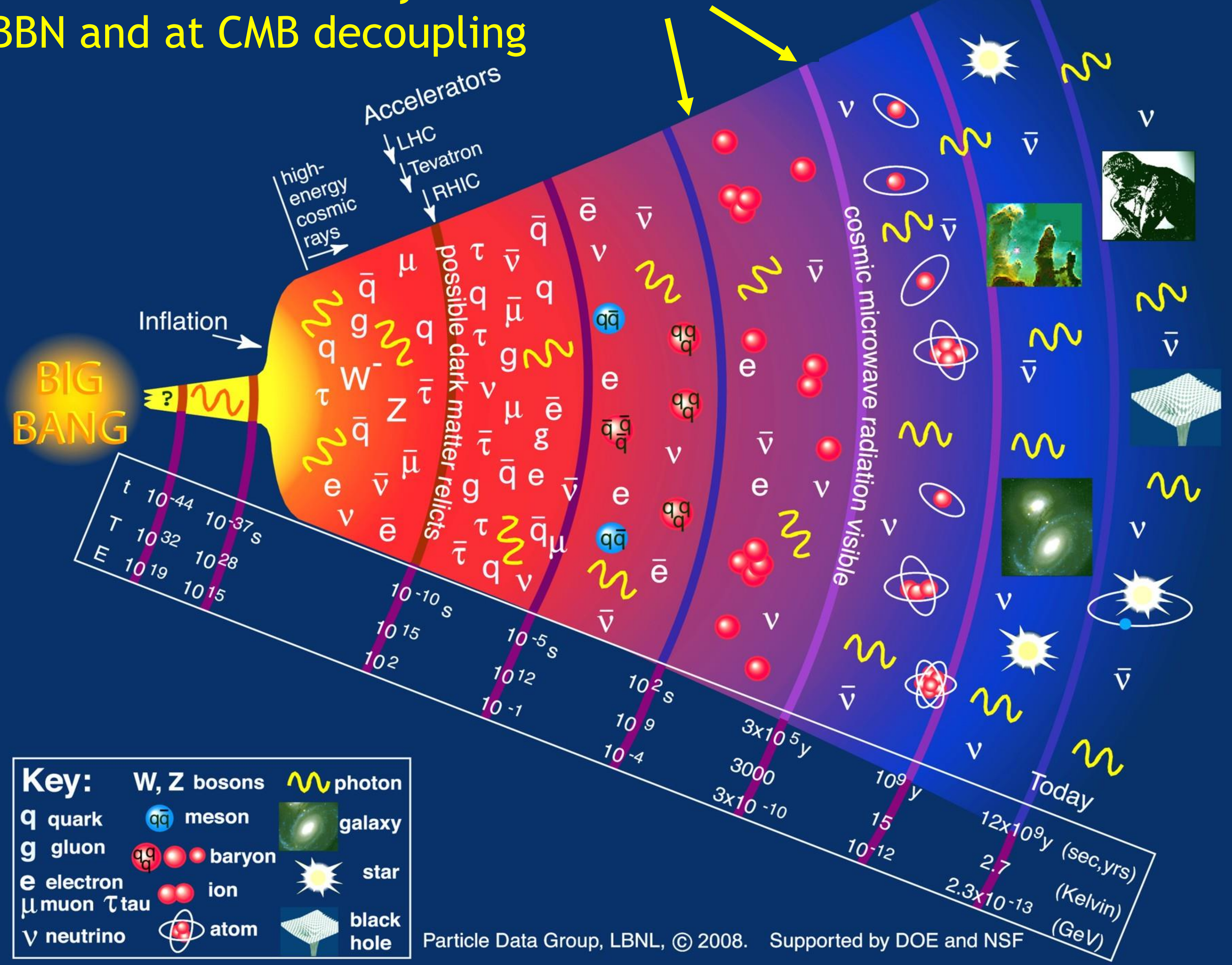
Excess nu density in the galaxy above cosmic mean of 56 cm^{-3}/flavor/spin

Ringwald & Wong, arXiv:hep-ph/0408241

Radiation Density from Precision Cosmology



Cosmic radiation density can be measured at BBN and at CMB decoupling



Particle Data Group, LBNL, © 2008. Supported by DOE and NSF

Power Spectrum of CMB Temperature Fluctuations

Sky map of CMBR temperature fluctuations

$$\Delta(\theta,\varphi)=\frac{T(\theta,\varphi)-\langle T\rangle}{\langle T\rangle}$$

Multipole expansion

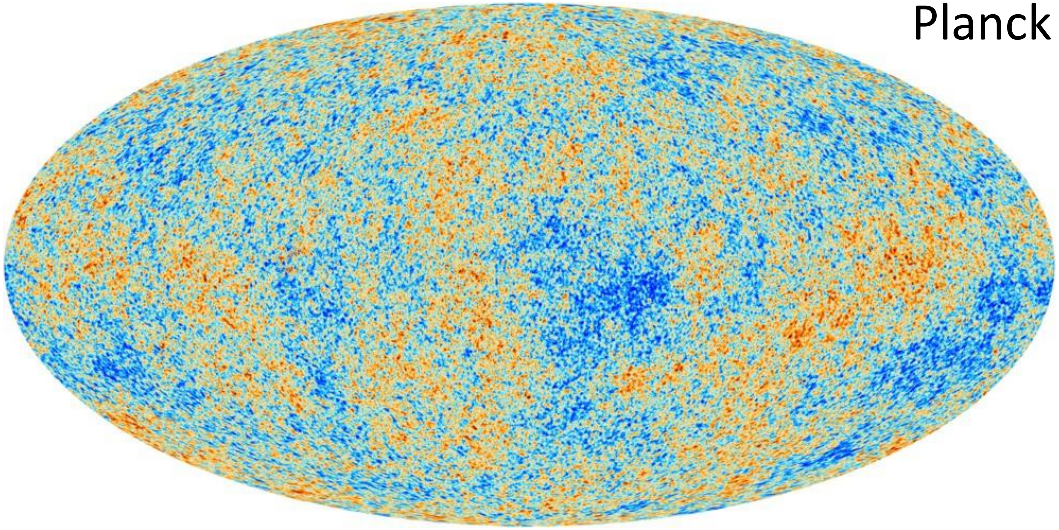
$$\Delta(\theta,\varphi)=\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}a_{\ell m}Y_{\ell m}(\theta,\varphi)$$

Angular power spectrum

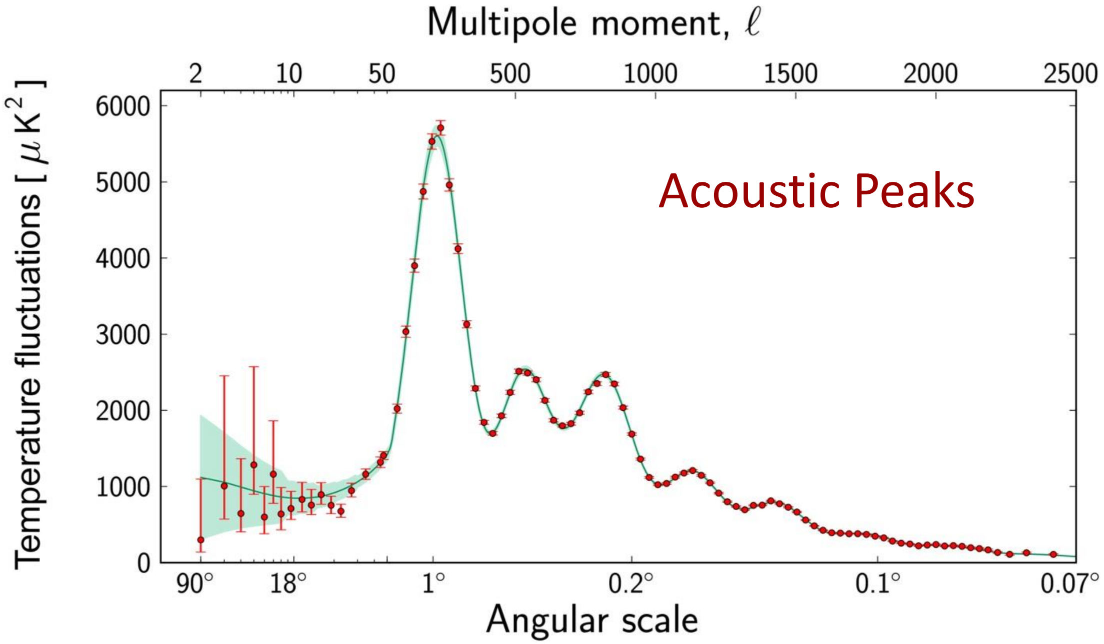
$$C_\ell=\langle a_{\ell m}^{*}a_{\ell m}\rangle$$

$$=\frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell}a_{\ell m}^{*}a_{\ell m}$$

Provides "acoustic peaks" and a wealth of cosmological information

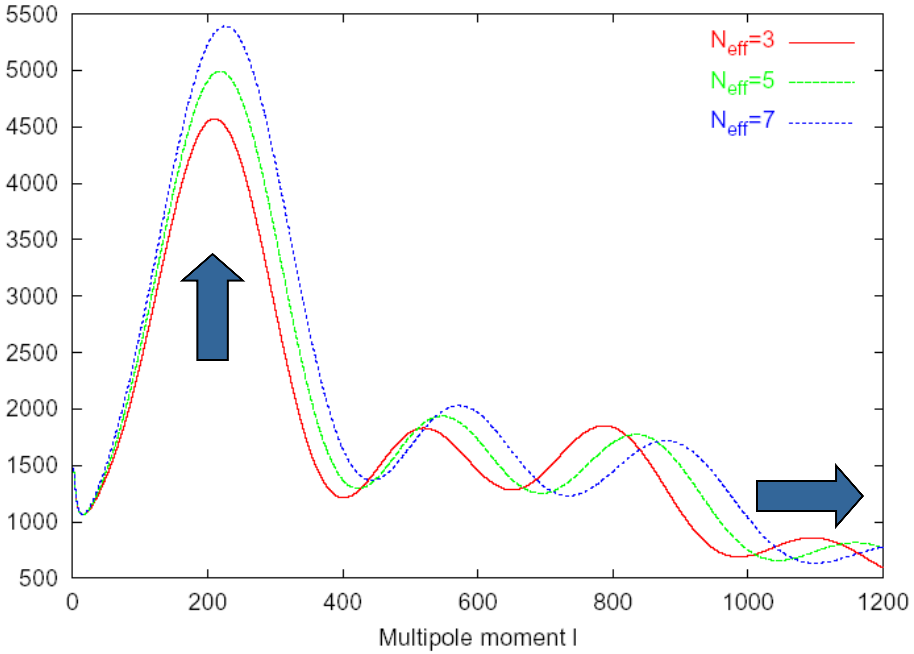


Planck

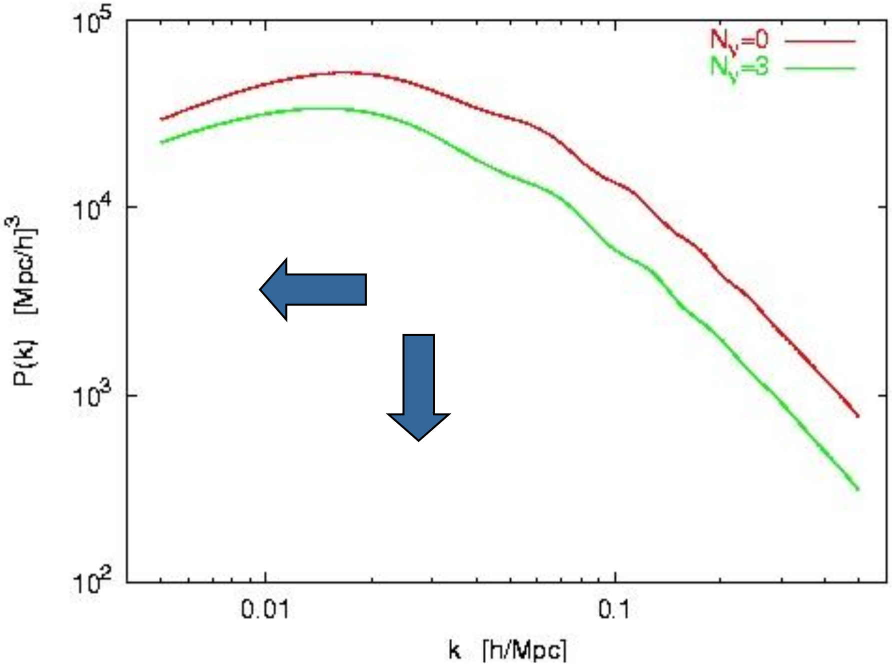


Impact of extra radiation

Redshift of matter-radiation equality modified by N_{eff}



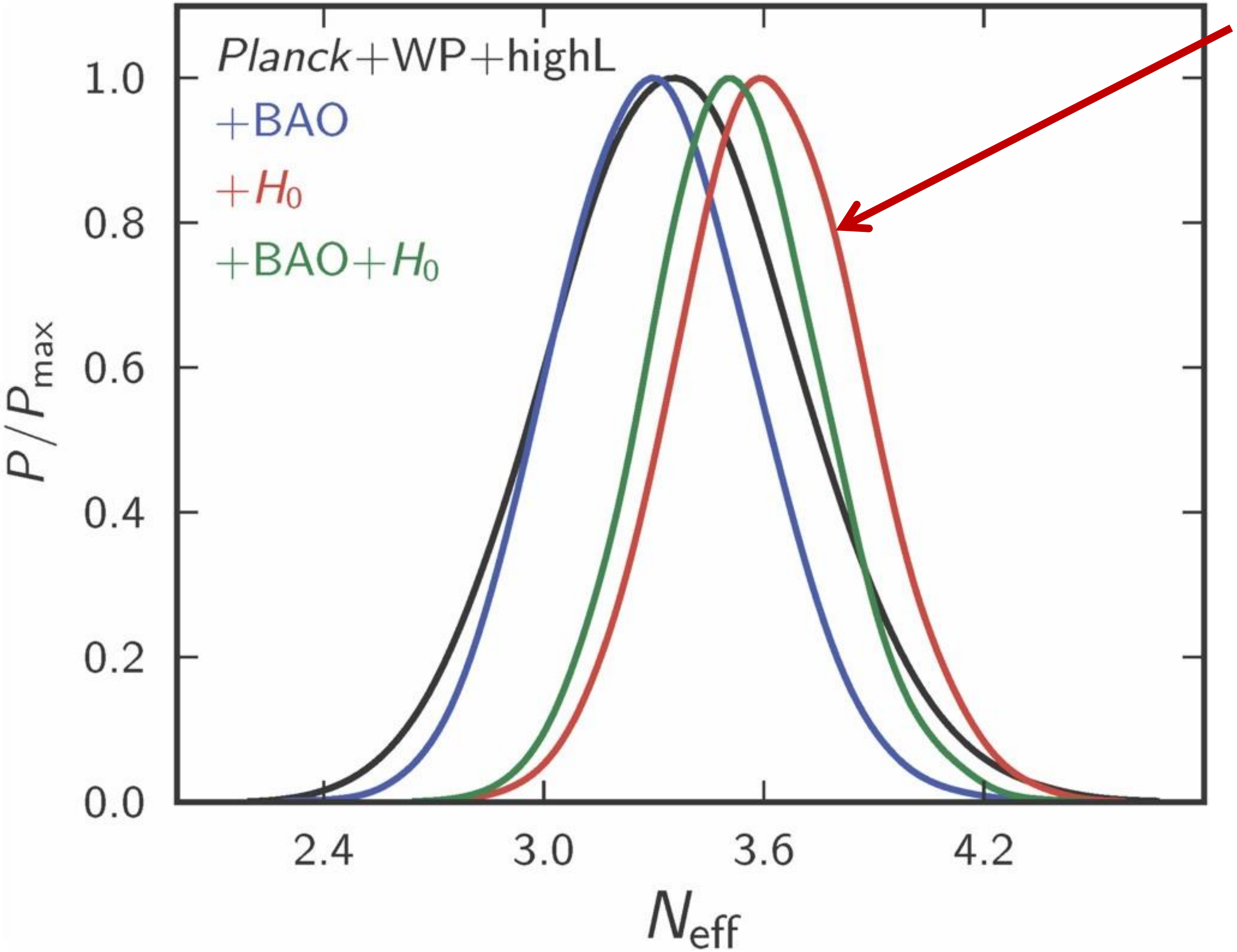
CMB angular power spectrum



Matter power spectrum

Hint for "Dark Radiation" in the Universe?

Depending on used data sets, indication for extra radiation at CMB decoupling
Same as pre-Planck situation based on WMAP-9 etc.

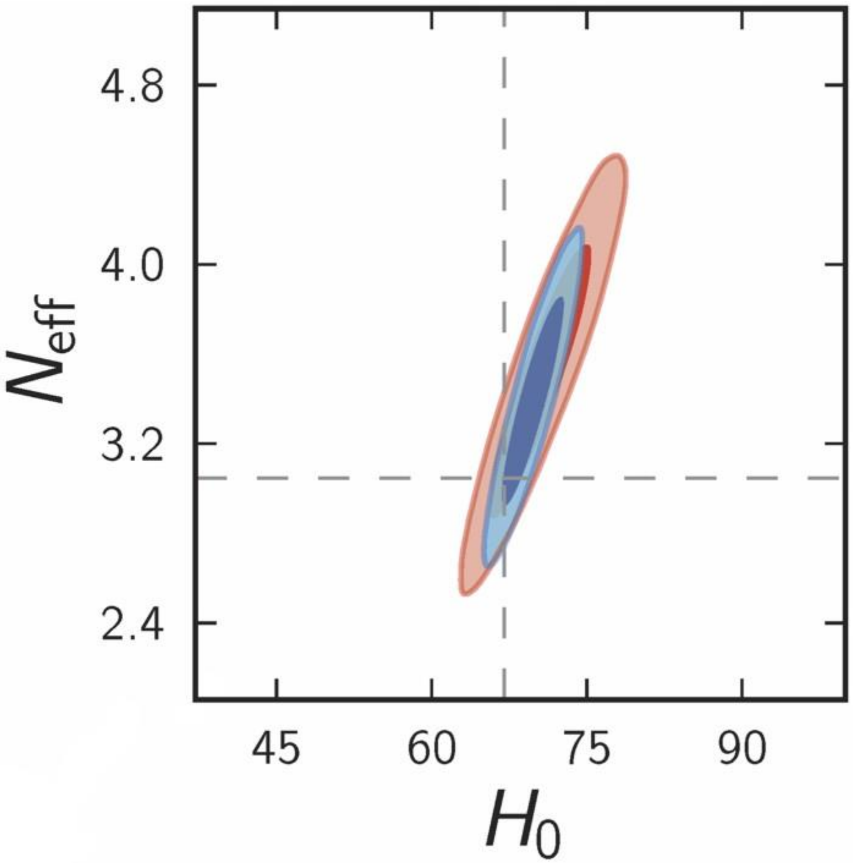


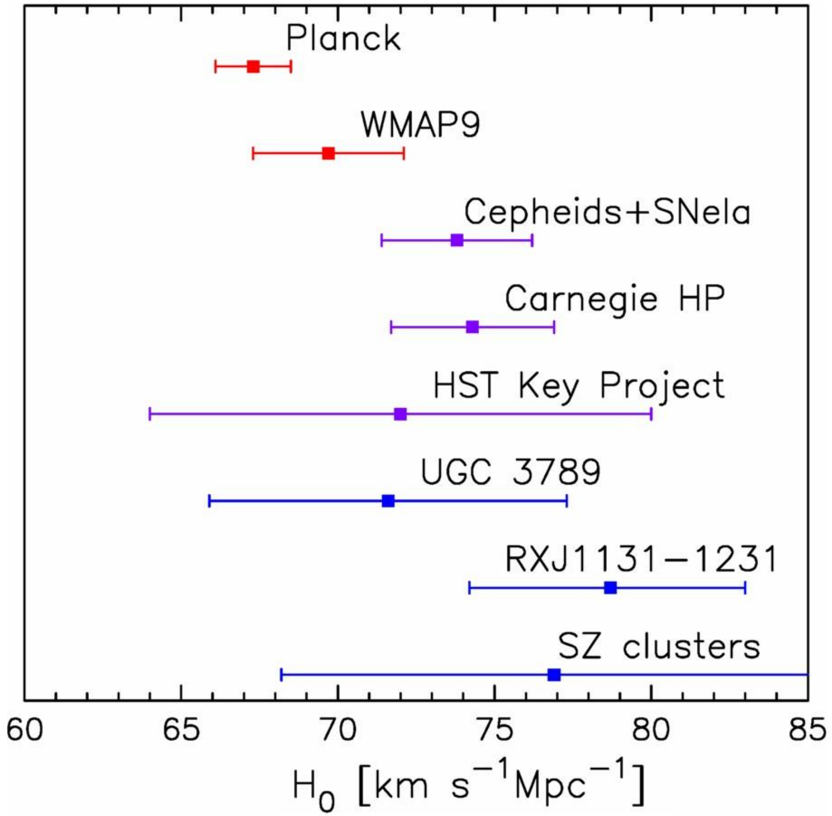
Including prior on H_0 provides > 2σ "evidence" for extra radiation at CMB decoupling

Ade et al. (Planck Collaboration), arXiv:1303.5076

Degeneracy between N_{eff} and H_0

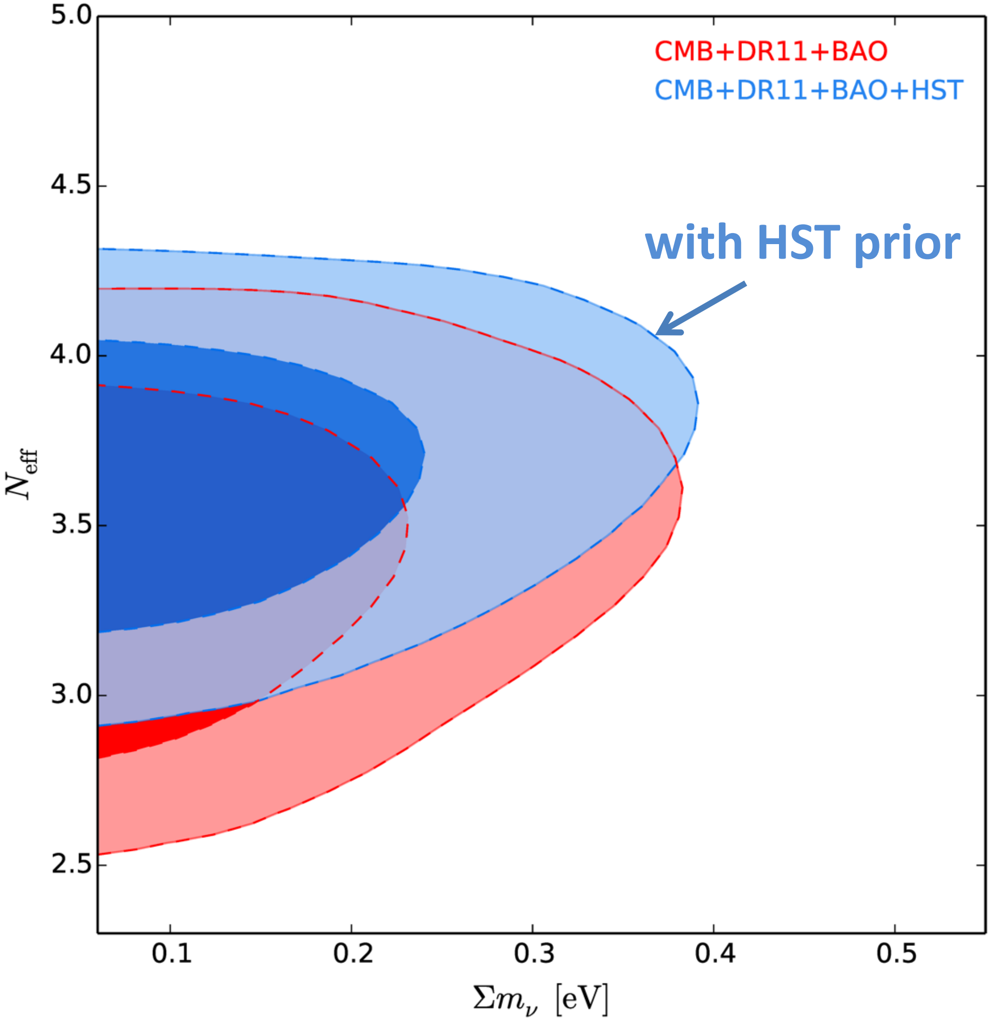
Extra radiation relaxes tension between H_0 determinations





Ade et al. (Planck Collaboration), arXiv:1303.5076

Neutrino Mass and N_{eff} Limits



Giusarma, Di Valentino, Lattanzi, Melchiorri & Mena, arXiv:1403.4852