Neutrinos

in Astrophysics and Cosmology

Cosmological Neutrinos 1

Georg G. Raffelt
Max-Planck-Institut für Physik, München, Germany


Luminous Matter
(mostly galaxies)
Gravitating mass
5.1 GeV/m^3
"Baryonic matter" (mostly dark)
0.25 GeV/m^3
Cosmic microwave photons
(T = 2.75 K)
4.11×10^8/m^3
"Dark matter"
1.3 GeV/m^3
Neutrinos+Antineutrinos
per flavor 1.12×10^8/m^3

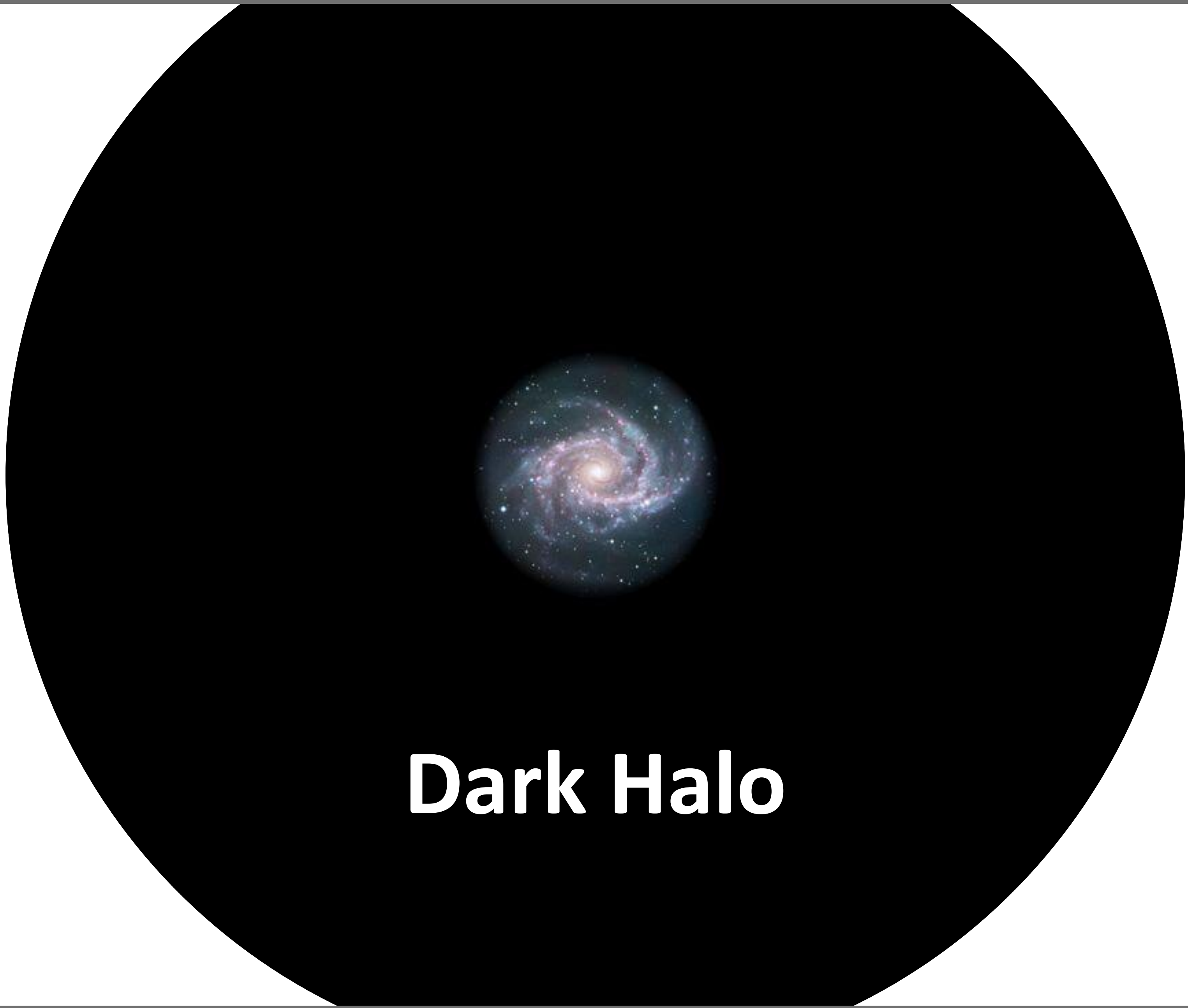
Structure of Spiral Galaxies

Spiral Galaxy NGC 2997

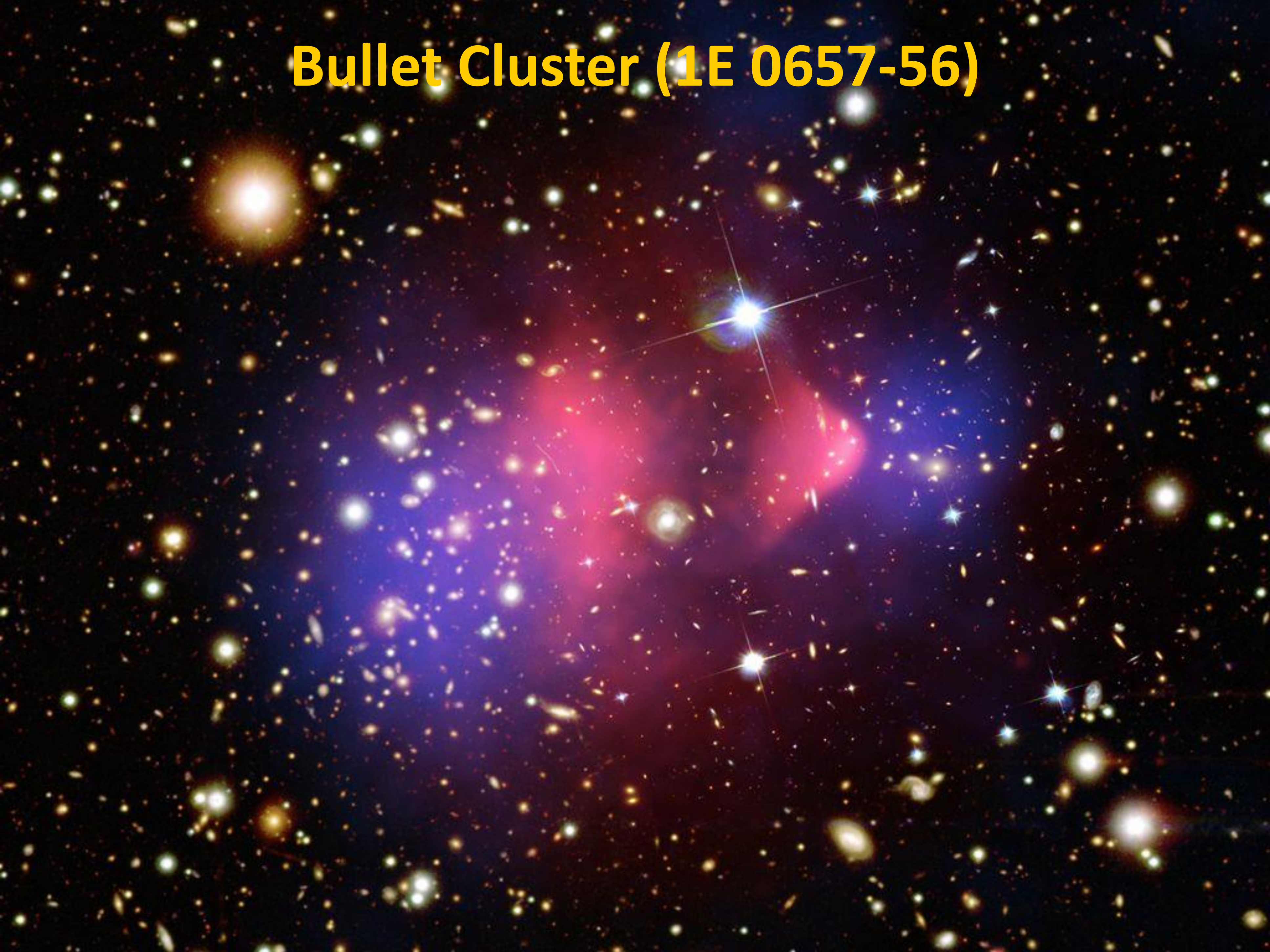
Spiral Galaxy NGC 891

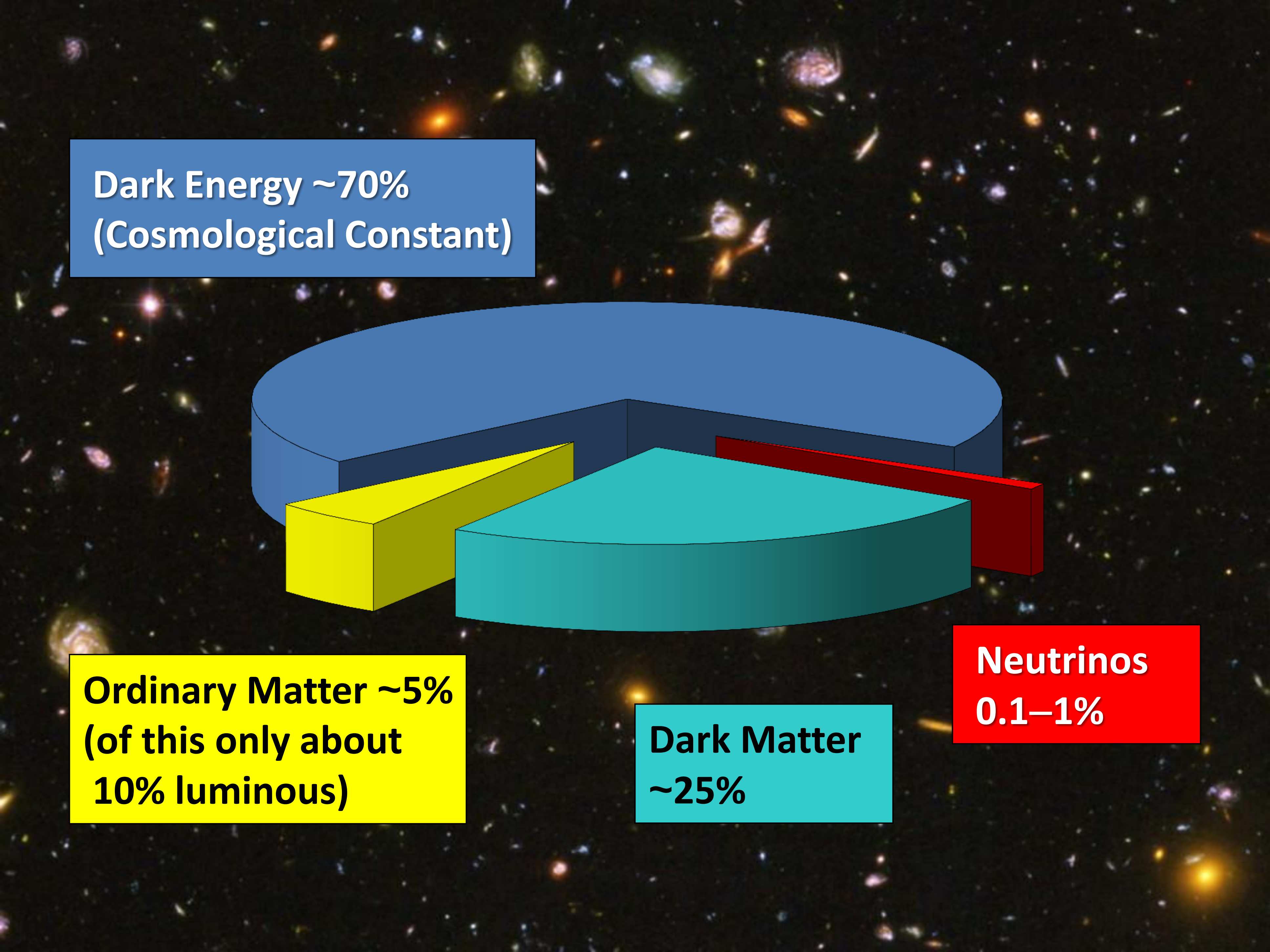
Structure of a Spiral Galaxy

Structure of a Spiral Galaxy



Bullet Cluster (1E 0657-56)




Dark Energy ~70%
(Cosmological Constant)
Ordinary Matter ~5%
(of this only about
10% luminous)
Dark Matter
~25%
Neutrinos
0.1–1%

- How Many Neutrinos?
  (Dark radiation/sterile neutrinos?)

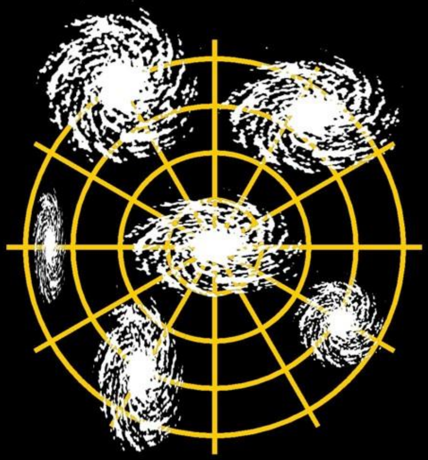
- Absolute mass determination and limits

- Big Bang Nucleosynthesis – BBN
  (Origin of light elements)

- Leptogenesis
  (Origin of Matter Abundance)

Some Basics of Cosmology

Expanding Universe and the Big Bang



Hubble's law

$v_{expansion} = H_0 \times \text{Distance}$

Hubble's constant

$H_0 = 67.3 \pm 1.2\ \text{km s}^{-1}\ \text{Mpc}^{-1}$

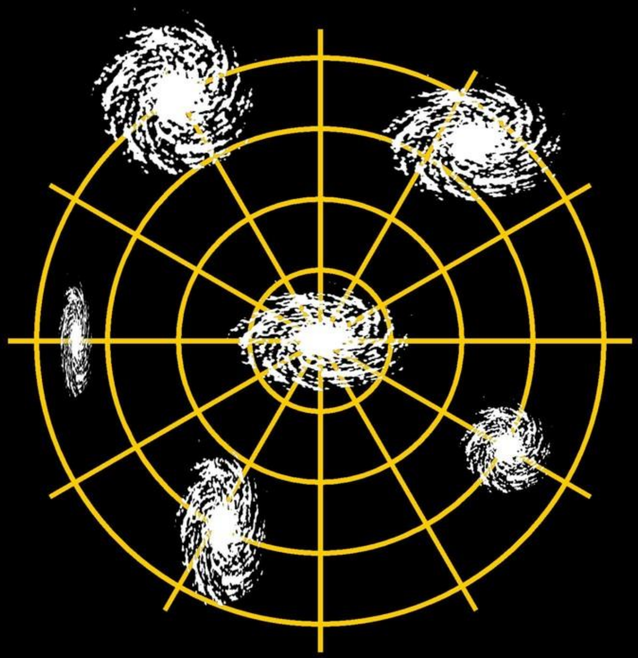
$1\ \text{Mpc} = 3.26 \times 10^6\ \text{lyr}$

$= 3.08 \times 10^{24}\ \text{cm}$

Expansion age of the universe

$t_0 \approx H_0^{-1} \approx 14 \times 10^9\ \text{years}$

Expanding Universe and the Big Bang



Hubble's law

$v_{expansion} = H_0 \times \text{Distance}$

Hubble's constant

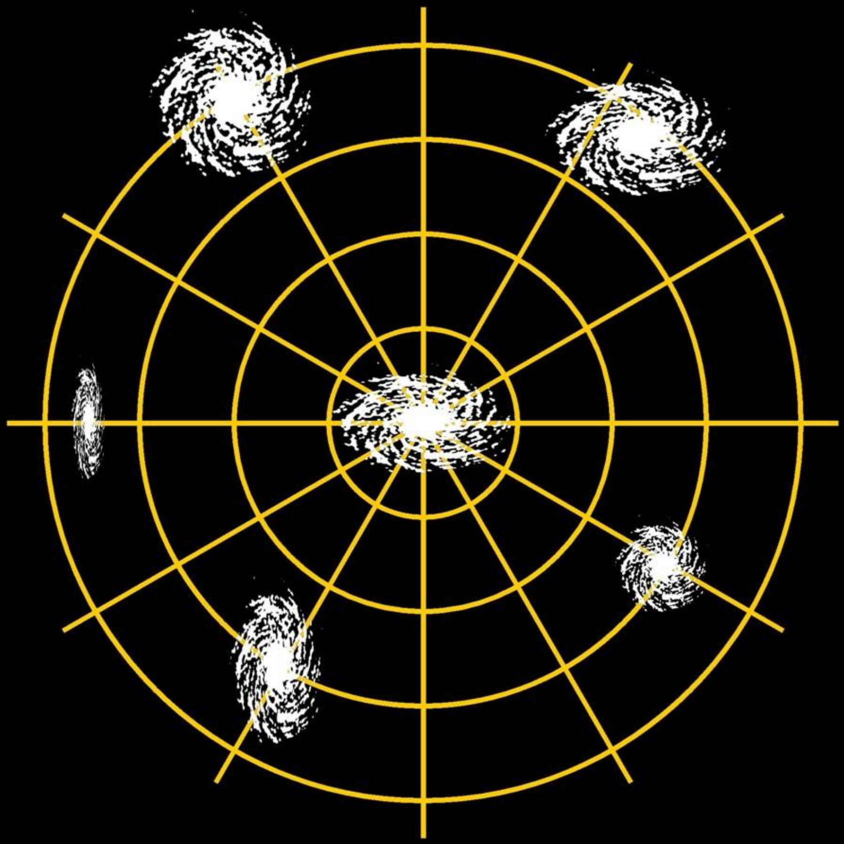
$H_0 = 67.3 \pm 1.2\ \text{km s}^{-1}\ \text{Mpc}^{-1}$

$1\ \text{Mpc} = 3.26 \times 10^6\ \text{lyr}$
$= 3.08 \times 10^{24}\ \text{cm}$

Expansion age of the universe

$t_0 \approx H_0^{-1} \approx 14 \times 10^9\ \text{years}$

Expanding Universe and the Big Bang



Hubble's law

$v_{expansion} = H_0 \times \text{Distance}$

Hubble's constant

$H_0 = 67.3 \pm 1.2\ \text{km s}^{-1}\ \text{Mpc}^{-1}$

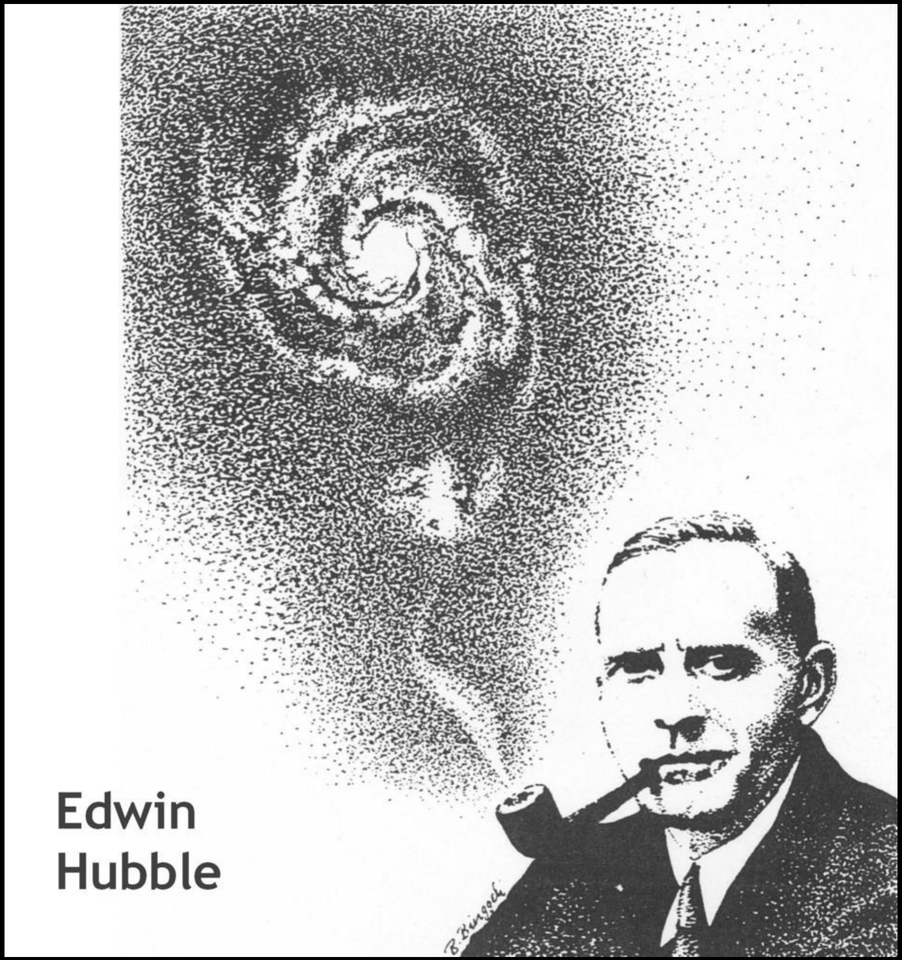
$1\ \text{Mpc} = 3.26 \times 10^6\ \text{lyr}$

$= 3.08 \times 10^{24}\ \text{cm}$

Expansion age of the universe

$t_0 \approx H_0^{-1} \approx 14 \times 10^9\ \text{years}$

Expanding Universe and the Big Bang



Hubble's law

$v_{expansion} = H_0 \times \text{Distance}$

Hubble's constant

$H_0 = 67.3 \pm 1.2\ \text{km s}^{-1}\ \text{Mpc}^{-1}$

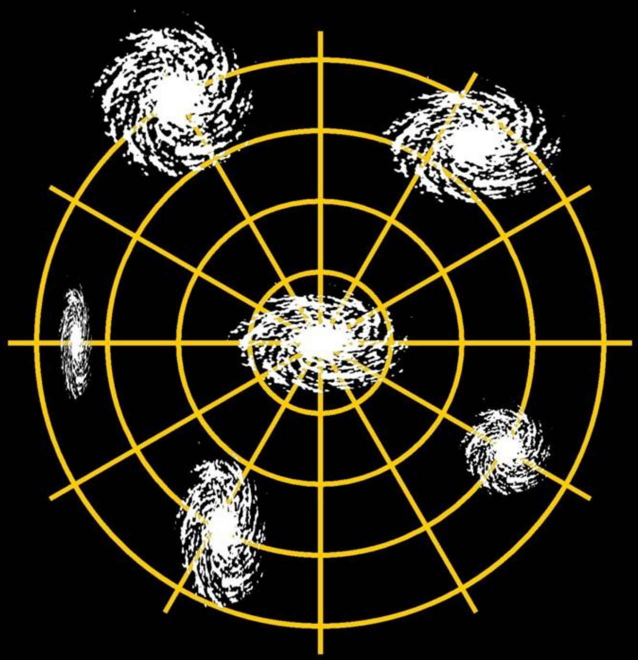
$1\ \text{Mpc} = 3.26 \times 10^6\ \text{lyr}$

$= 3.08 \times 10^{24}\ \text{cm}$

Expansion age of the universe

$t_0 \approx H_0^{-1} \approx 14 \times 10^9\ \text{years}$

Expanding Universe and the Big Bang



Hubble's law

$$v_{\text{expansion}} = H_0 \times \text{Distance}$$

Hubble's constant

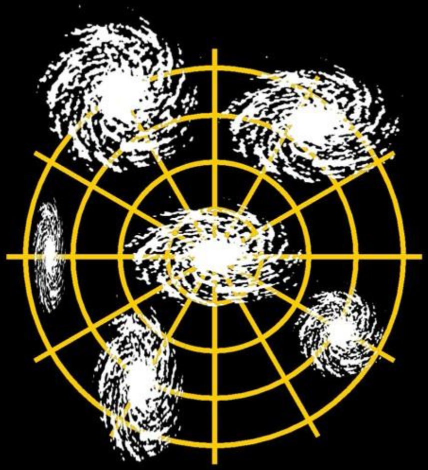
$$H_0 = 67.3 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$1 \text{ Mpc} = 3.26 \times 10^6 \text{ lyr}$$
$$= 3.08 \times 10^{24} \text{ cm}$$

Expansion age of the universe

$$t_0 \approx H_0^{-1} \approx 14 \times 10^9 \text{ years}$$

Expanding Universe and the Big Bang



Hubble's law

$v_{expansion} = H_0 \times \text{Distance}$

Hubble's constant

$H_0 = 67.3 \pm 1.2\ \text{km s}^{-1}\ \text{Mpc}^{-1}$

$1\ \text{Mpc} = 3.26 \times 10^6\ \text{lyr}$

$= 3.08 \times 10^{24}\ \text{cm}$

Expansion age of the universe

$t_0 \approx H_0^{-1} \approx 14 \times 10^9\ \text{years}$

Expanding Universe and the Big Bang

Hubble's law

$v_{\text{expansion}} = H_0 \times \text{Distance}$

Hubble's constant

$H_0 = 67.3 \pm 1.2\ \text{km s}^{-1}\ \text{Mpc}^{-1}$

$1\ \text{Mpc} = 3.26 \times 10^6\ \text{lyr}$

$= 3.08 \times 10^{24}\ \text{cm}$

Expansion age of the universe

$t_0 \approx H_0^{-1} \approx 14 \times 10^9\ \text{years}$

Expanding Universe and the Big Bang

Hubble's law

$v_{expansion} = H_0 \times \text{Distance}$

Hubble's constant

$H_0 = 67.3 \pm 1.2\ \text{km s}^{-1}\ \text{Mpc}^{-1}$

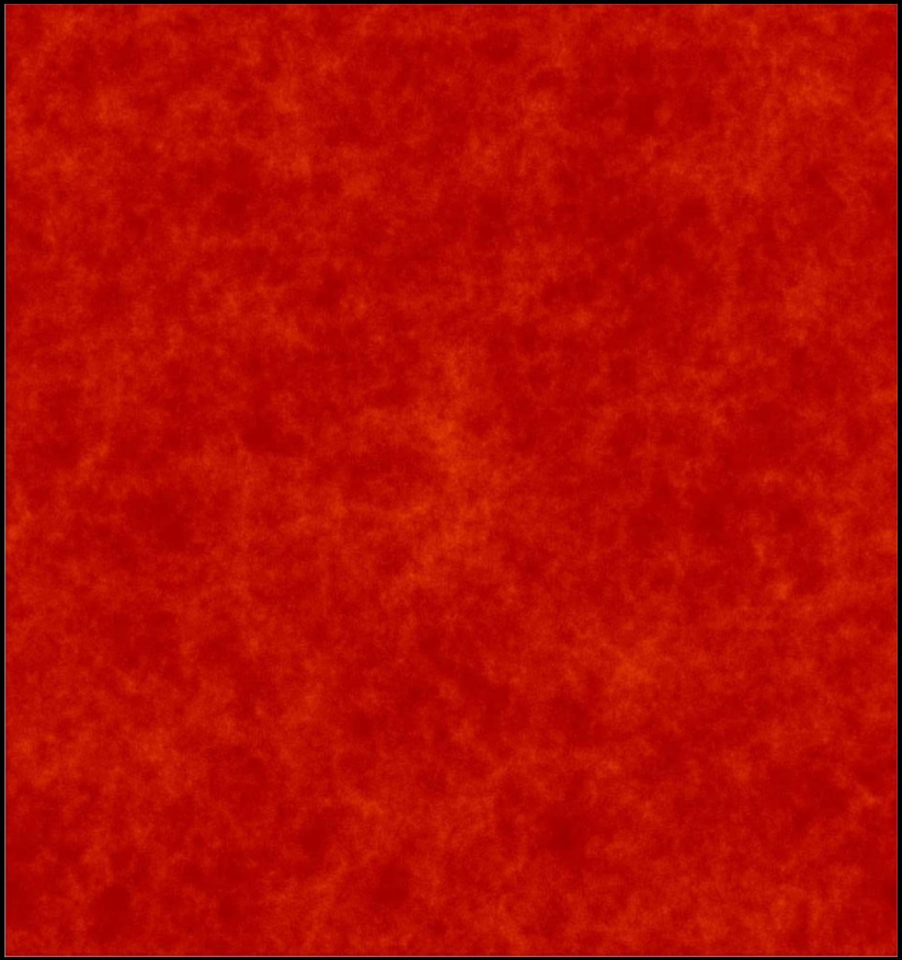
$1\ \text{Mpc} = 3.26 \times 10^6\ \text{lyr}$

$= 3.08 \times 10^{24}\ \text{cm}$

Expansion age of the universe

$t_0 \approx H_0^{-1} \approx 14 \times 10^9\ \text{years}$

Expanding Universe and the Big Bang



Hubble's law

$v_{expansion} = H_0 \times \text{Distance}$

Hubble's constant

$H_0 = 67.3 \pm 1.2\ \text{km s}^{-1}\ \text{Mpc}^{-1}$

$1\ \text{Mpc} = 3.26 \times 10^6\ \text{lyr}$

$= 3.08 \times 10^{24}\ \text{cm}$

Expansion age of the universe

$t_0 \approx H_0^{-1} \approx 14 \times 10^9\ \text{years}$

Expanding Universe and the Big Bang

- Photons
- Neutrinos
- Charged Leptons
- Quarks
- Gluons
- W- and Z-Bosons
- Higgs Particles
- Gravitons
- Dark-Matter Particles
- Topological defects
- …

Hubble's law

$v_{expansion} = H_0 \times \text{Distance}$

Hubble's constant

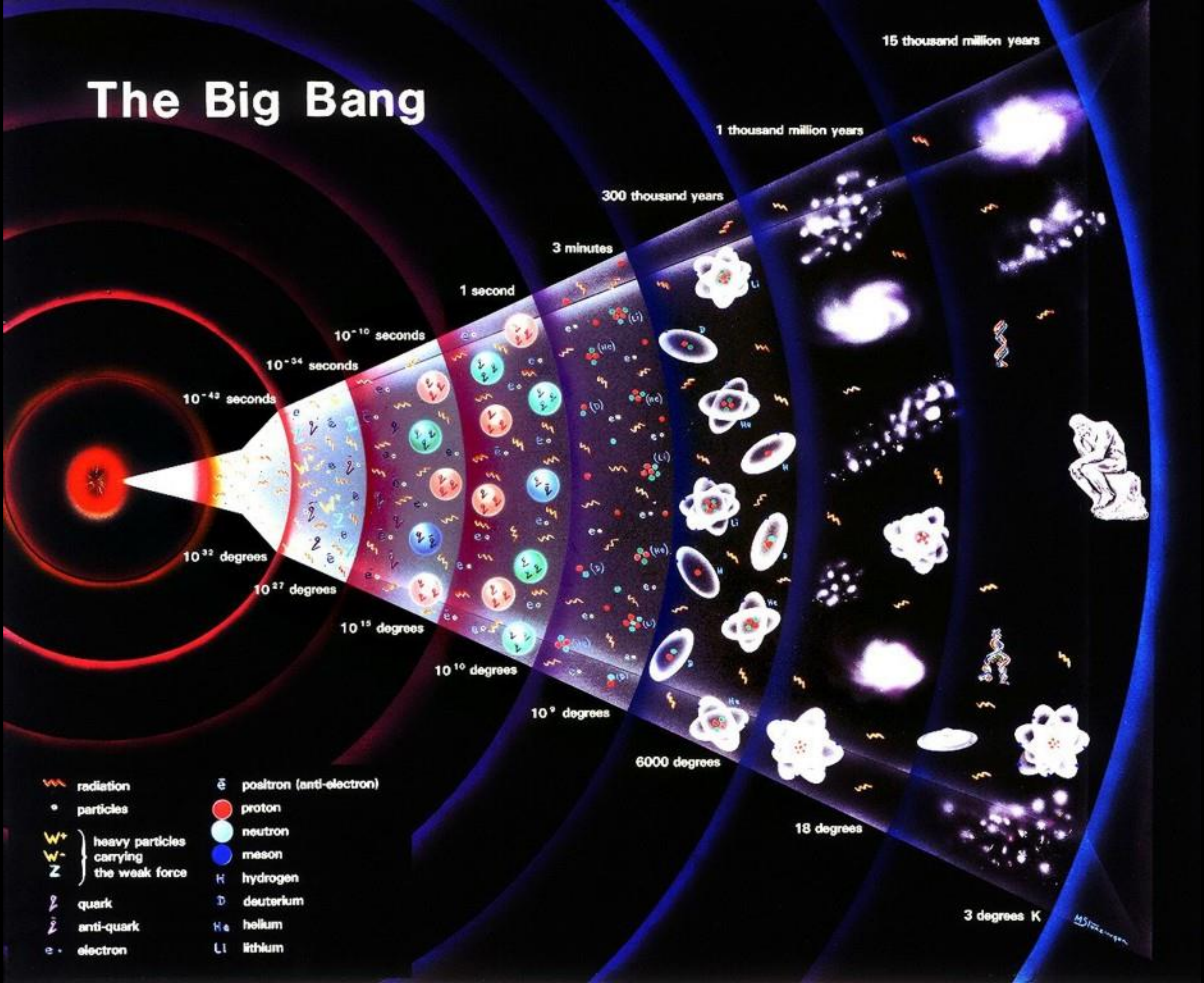
$H_0 = 67.3 \pm 1.2\ \text{km s}^{-1}\ \text{Mpc}^{-1}$

$1\ \text{Mpc} = 3.26 \times 10^6\ \text{lyr}$

$= 3.08 \times 10^{24}\ \text{cm}$

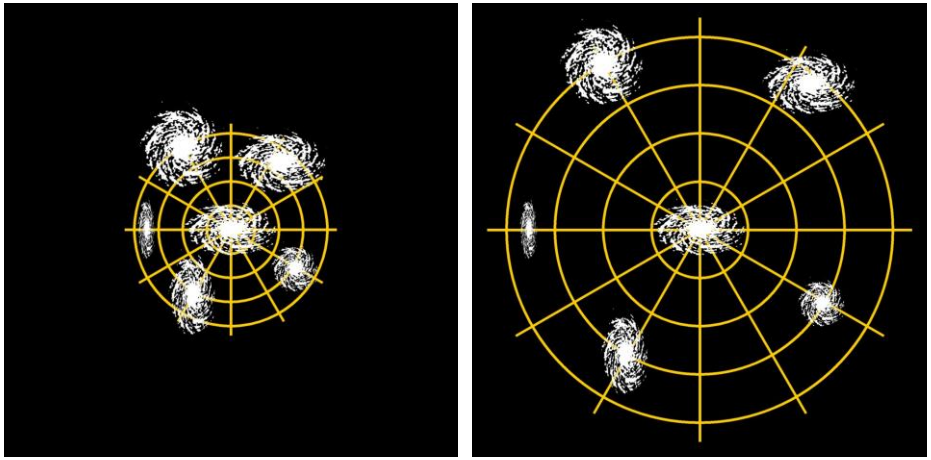
Expansion age of the universe

$t_0 \approx H_0^{-1} \approx 14 \times 10^9\ \text{years}$


The Big Bang
15 thousand million years
1 thousand million years
300 thousand years
3 minutes
1 second
10^{-10} seconds
10^{-34} seconds
10^{-43} seconds
10^{32} degrees
10^{27} degrees
10^{15} degrees
10^{10} degrees
10^{9} degrees
6000 degrees
18 degrees
3 degrees K
radiation
particles
W+ W- Z heavy particles carrying the weak force
quark
anti-quark
electron
positron (anti-electron)
proton
neutron
meson
hydrogen
deuterium
helium
lithium

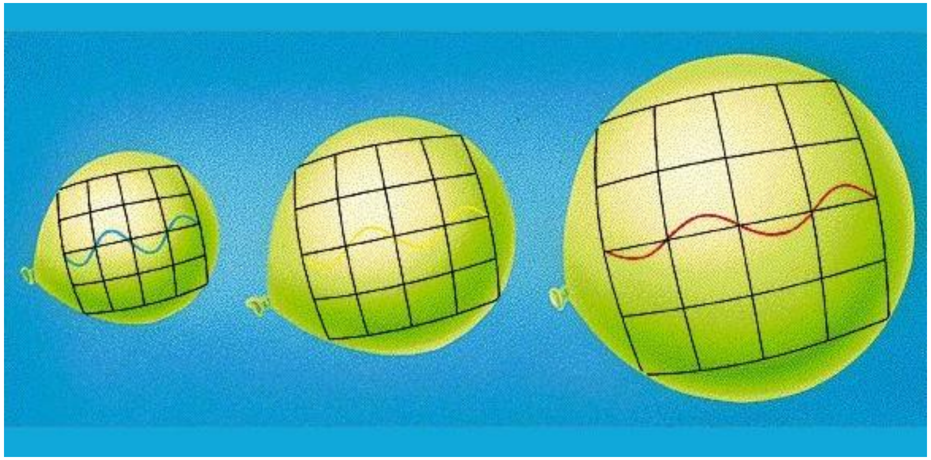
Cosmic Expansion

Cosmic Scale Factor



- Space between galaxies grows
- Galaxies (stars, people) stay the same (dominated by local gravity or by electromagnetic forces)
- Cosmic scale factor today: $a = 1$

Cosmic Redshift



- Wavelength of light is “stretched”
- Suffers redshift $z + 1 = \frac{\lambda_{\rm today}}{\lambda_{\rm then}}$
- Redshift today: $z = 0$

$$z + 1 = \frac{\lambda_{\rm today}}{\lambda_{\rm then}} = \frac{a_{\rm today}}{a_{\rm then}}$$

Friedman-Robertson-Walker-Lemaître Cosmology

- **On scales ≳ 100 Mpc, space is maximally symmetric (homogeneous & isotropic)**
- **The corresponding Robertson-Walker metric is**

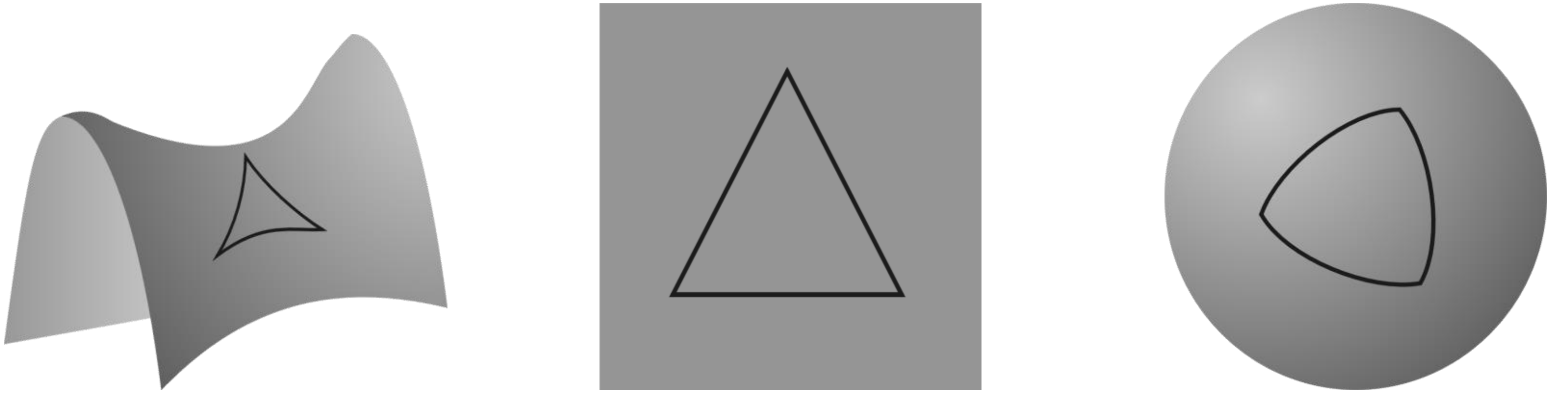
$$ds^2 = dt^2 - a^2(\mathrm{t})\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

Clock time of co-moving observer

Cosmic scale factor

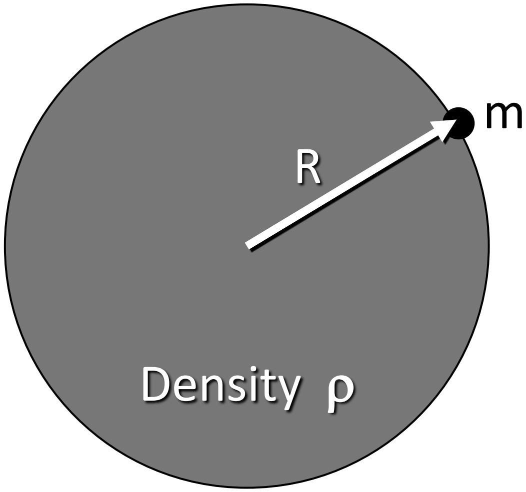
Curvature $k = 0, \pm 1$

Co-moving spherical coordinates r is dimensionless



Friedman Equation: Newtonian Derivation

- **Birkhoff's theorem:**
  Spherical symmetry implies that only the mass interior to a radius R is relevant for the motion of a test mass m at R



- **Energy conservation** $V_{\rm pot} + V_{\rm kin} = \text{const}$

$$-\frac{G_{\rm N}\frac{4\pi}{3}R^3\rho\, m}{R} + \frac{1}{2}\dot{R}^2 m = \text{const}$$

$$\Longrightarrow \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G_{\rm N}\rho + \frac{\text{const}}{R^2}$$

- **Rescale** $R = a\,R_C$ **with cosmic scale factor** a **and** R_C **radius of curvature today**

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G_{\rm N}\rho - \frac{k}{a^2 R_C^2}$$

Friedman Equation

with $k = 0, \pm 1$

Critical Density and Density Parameter

- Evolution of the cosmic scale factor $a(t)$ is governed by the **Friedman Equation**

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G_\mathrm{N}\rho - \frac{k}{a^2 R_C^2}$$

- In a flat universe ($k = 0$), the relationship between H and ρ is unique

$$\rho_\mathrm{crit} = \frac{3H^2}{8\pi G_\mathrm{N}} = \frac{3}{8\pi}(H m_\mathrm{Pl})^2$$ **critical density**

- Cosmic density always expressed in terms of density parameters

$$\Omega = \frac{\rho}{\rho_\mathrm{crit}} = \frac{8\pi G_\mathrm{N}\rho}{3H^2}$$

- With the present-day Hubble parameter $H_0 = 67.3\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ we have

$$\rho_\mathrm{crit} = 8.51 \times 10^{-30}\mathrm{g\ cm^{-3}} = 5\ \mathrm{GeV\ m^{-3}} = (2.5\ \mathrm{meV})^4$$

Most of this in the form of “dark energy”

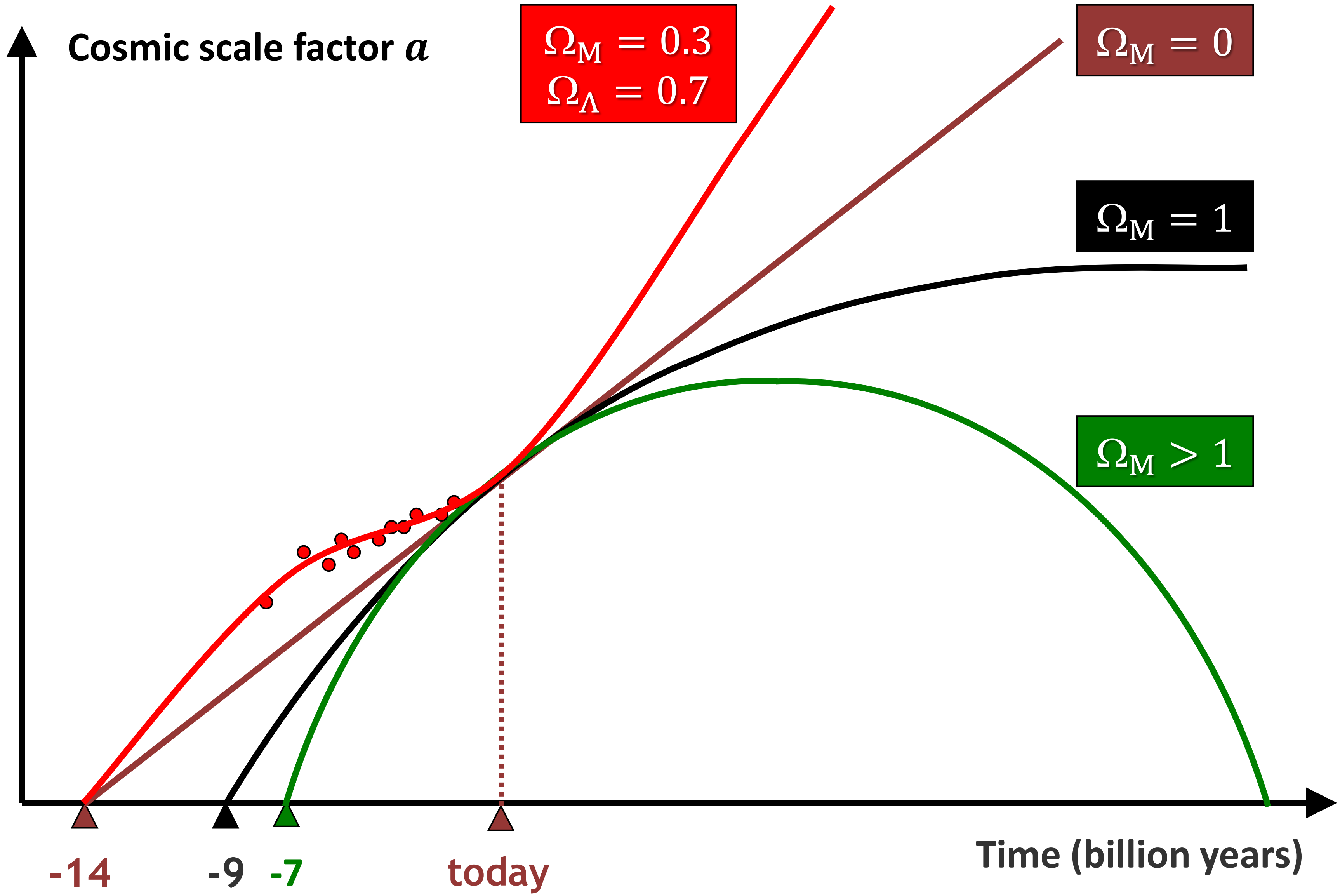
Generic Solutions of Friedman Equation

	Equation of state	Behavior of energy-density under cosmic expansion		Evolution of cosmic scale factor
Radiation	$p = \frac{\rho}{3}$	$\rho \propto a^{-4}$	Dilution of radiation and redshift of energy	$a(t) \propto t^{1/2}$
Matter	$p = 0$	$\rho \propto a^{-3}$	Dilution of matter	$a(t) \propto t^{2/3}$
Vacuum energy	$p = -\rho$	$\rho = \mathrm{const}$	Vacuum energy not diluted by expansion	$a(t) \propto e^{\sqrt{\Lambda/3}\,t}$ $\Lambda = 8\pi G_\mathrm{N}\rho_\mathrm{vac}$

Energy-momentum tensor of a perfect fluid with density ρ and pressure p

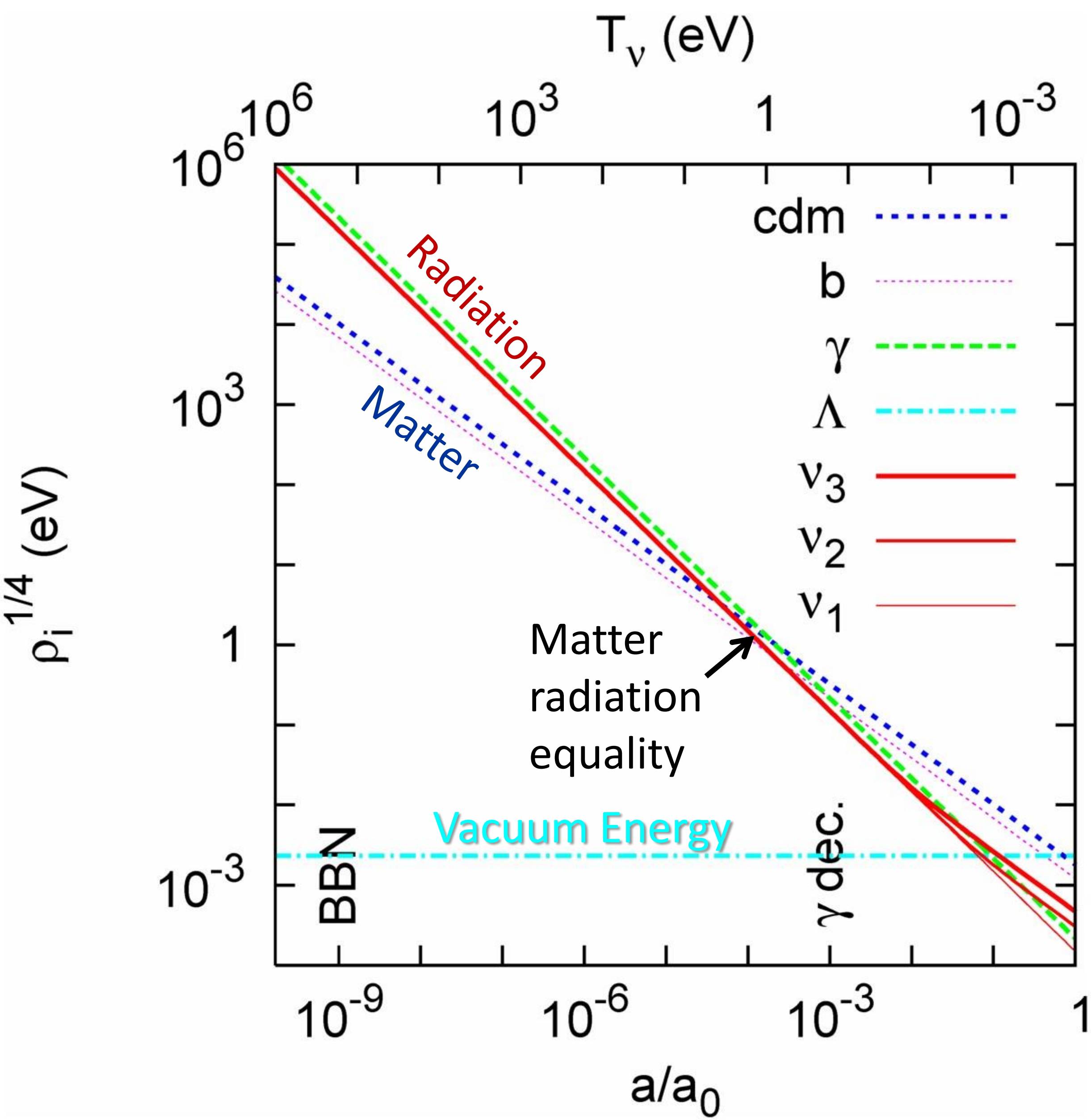
$$T^{\mu\nu} = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix} \qquad T^{\mu\nu}_\mathrm{vac} = \rho\, g^{\mu\nu} \begin{pmatrix} \rho & & & \\ & -\rho & & \\ & & -\rho & \\ & & & -\rho \end{pmatrix}$$

Expansion of Different Cosmological Models



Adapted from Bruno Leibundgut

Evolution of Cosmic Density Components



Assumed neutrino masses

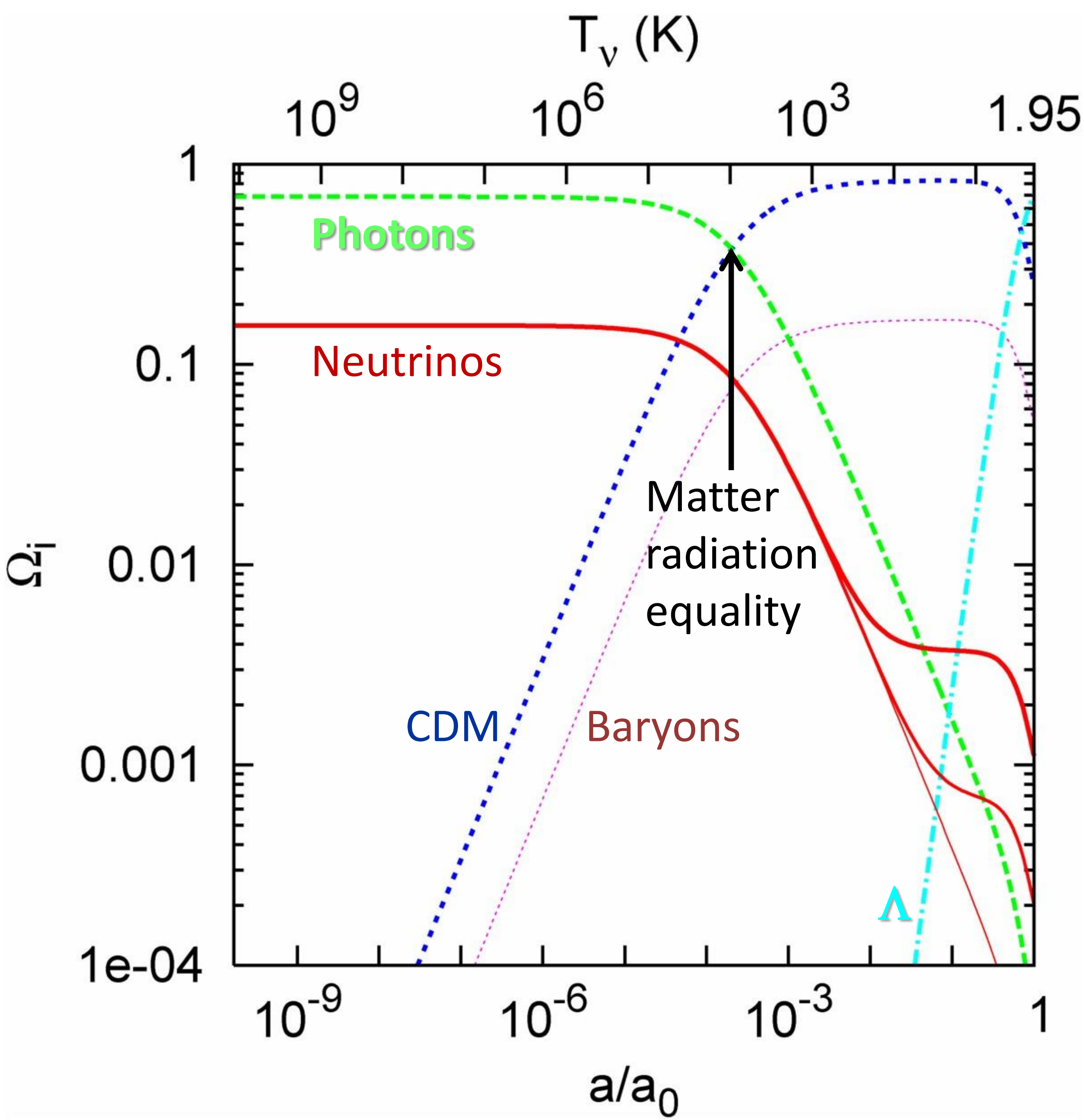
m_3 = 50 meV

m_2 = 9 meV

m_1 = 0

Lesgourgues & Pastor
astro-ph/0603494

Evolution of Cosmic Density Components



Assumed neutrino masses

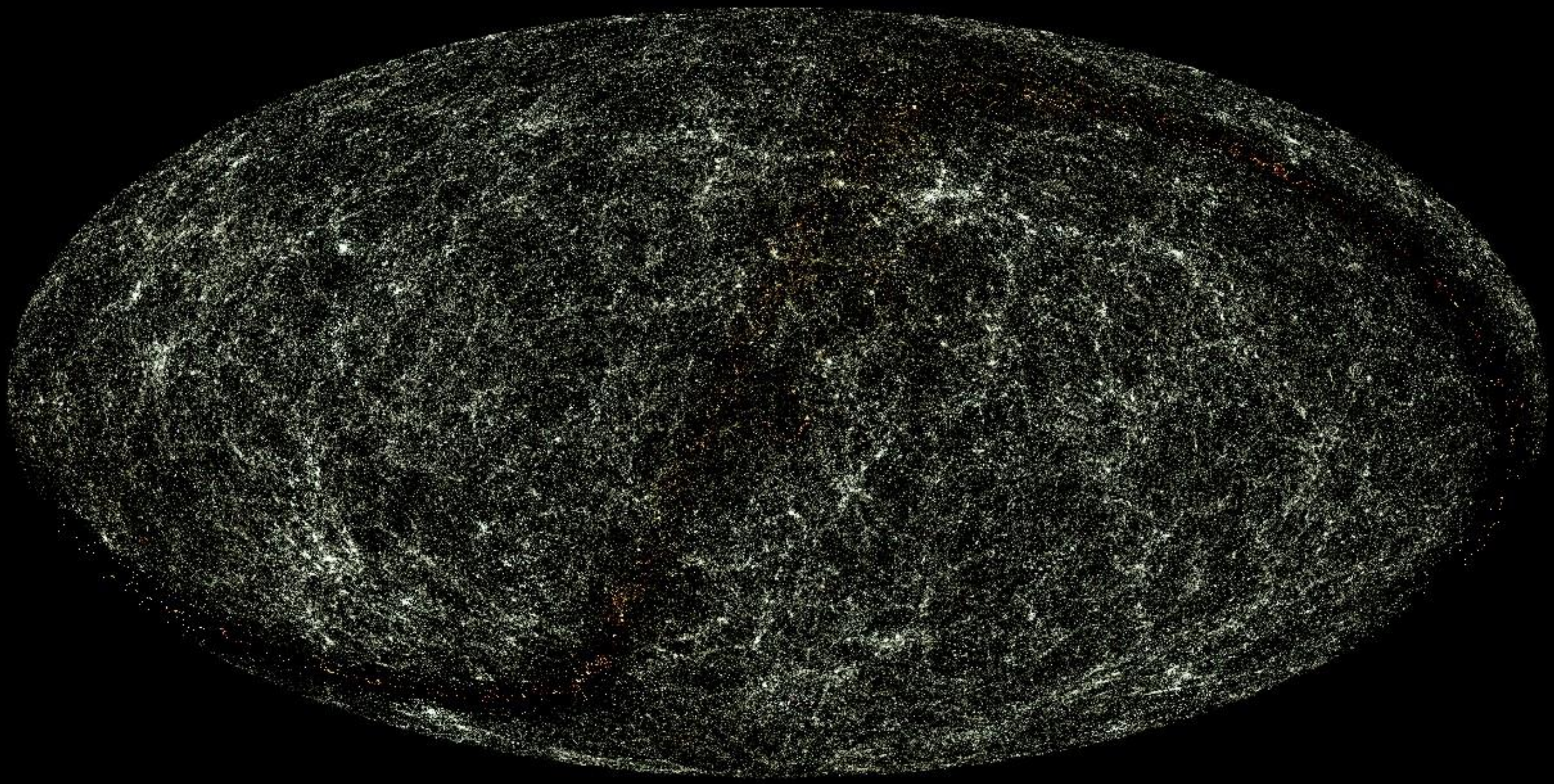
m_3 = 50 meV

m_2 = 9 meV

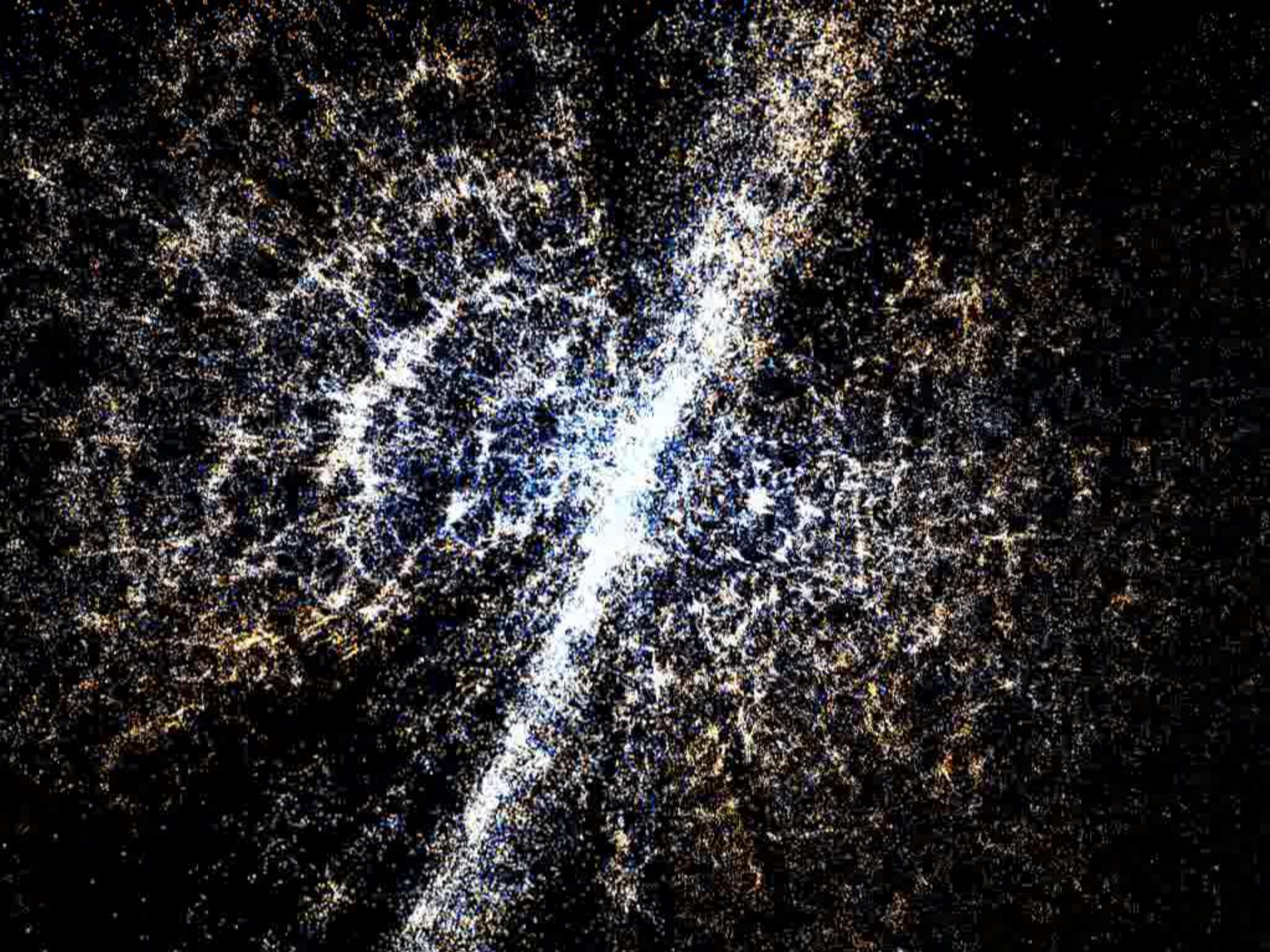
m_1 = 0

Lesgourgues & Pastor
astro-ph/0603494

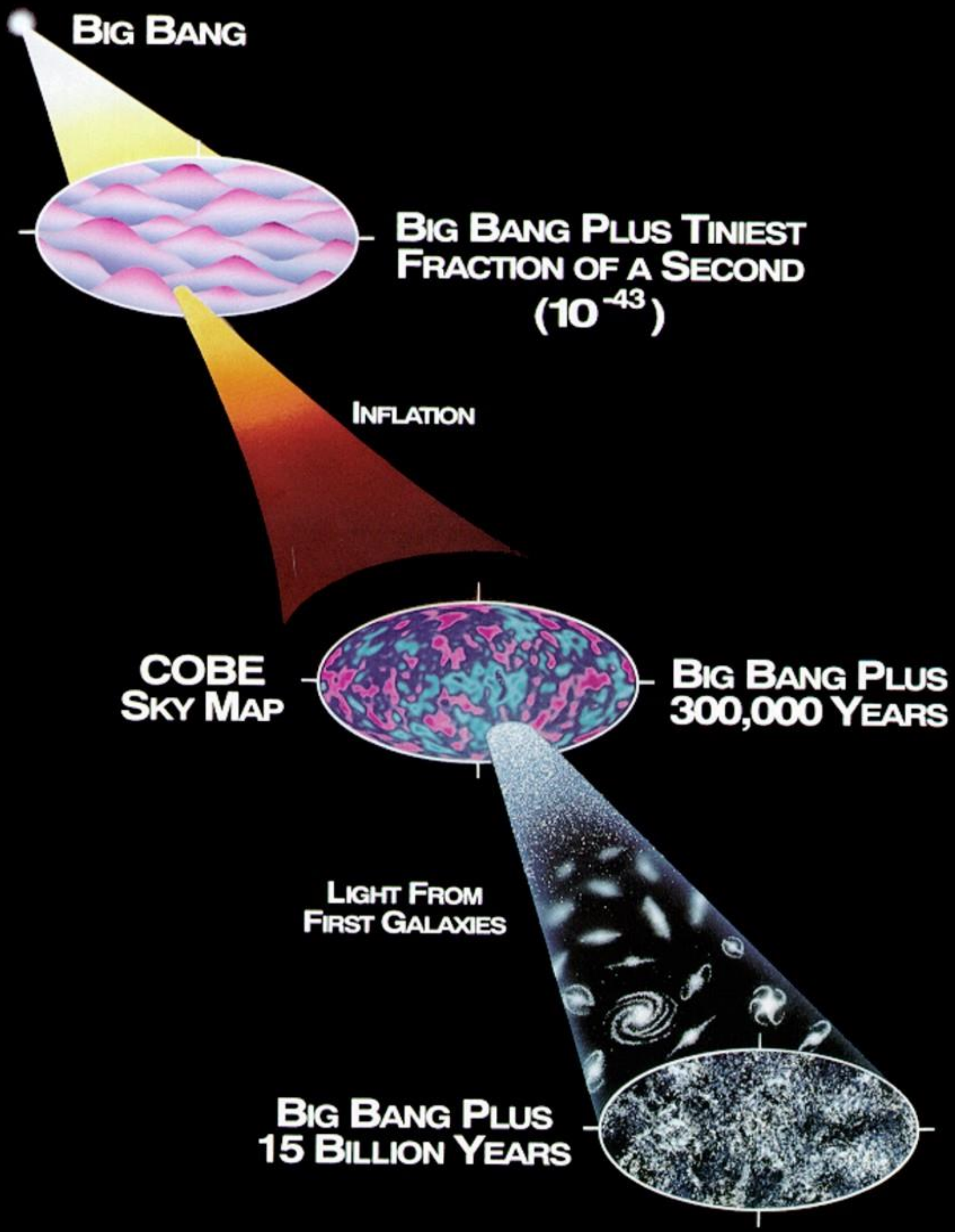
Sky Map of Galaxies (XMASS XSC)



http://spider.ipac.caltech.edu/staff/jarrett/2mass/XSC/jarrett_allsky.html



Structure Formation in the Universe



Early phase of exponential expansion (Inflationary epoch)

Zero-point fluctuations of quantum fields are stretched and frozen

Structure grows by gravitational instability

Cosmic density fluctuations are frozen quantum fluctuations

Power Spectrum of Density Fluctuations

Field of density fluctuations of matter (e.g. dark matter)

$$\delta(\boldsymbol{x}) = \frac{\delta\rho(\boldsymbol{x})}{\bar{\rho}}$$

Fourier transform of density field

$$\delta_{\boldsymbol{k}} = \int d^3\boldsymbol{x}\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\delta(\boldsymbol{x})$$

Power spectrum is essentially the square of the Fourier transform ($\hat{\delta}$ is δ-function)

$$\langle\delta_{\boldsymbol{k}}\delta_{\boldsymbol{k}'}\rangle = (2\pi)^3\hat{\delta}(\boldsymbol{k}-\boldsymbol{k}')P(\boldsymbol{k})$$

Power spectrum is Fourier transform of two-point correlation function ($x = x_2 - x_1$)

$$\xi(\boldsymbol{x}) = \langle\delta(\boldsymbol{x}_2)\delta(\boldsymbol{x}_1)\rangle = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} P(\boldsymbol{k}) = \int \frac{d\Omega}{4\pi}\frac{dk}{k}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \underbrace{\frac{k^3 P(\boldsymbol{k})}{2\pi^2}}_{\Delta^2(\boldsymbol{k})}$$

Gaussian random field (phases of δ_k uncorrelated) is fully characterized by

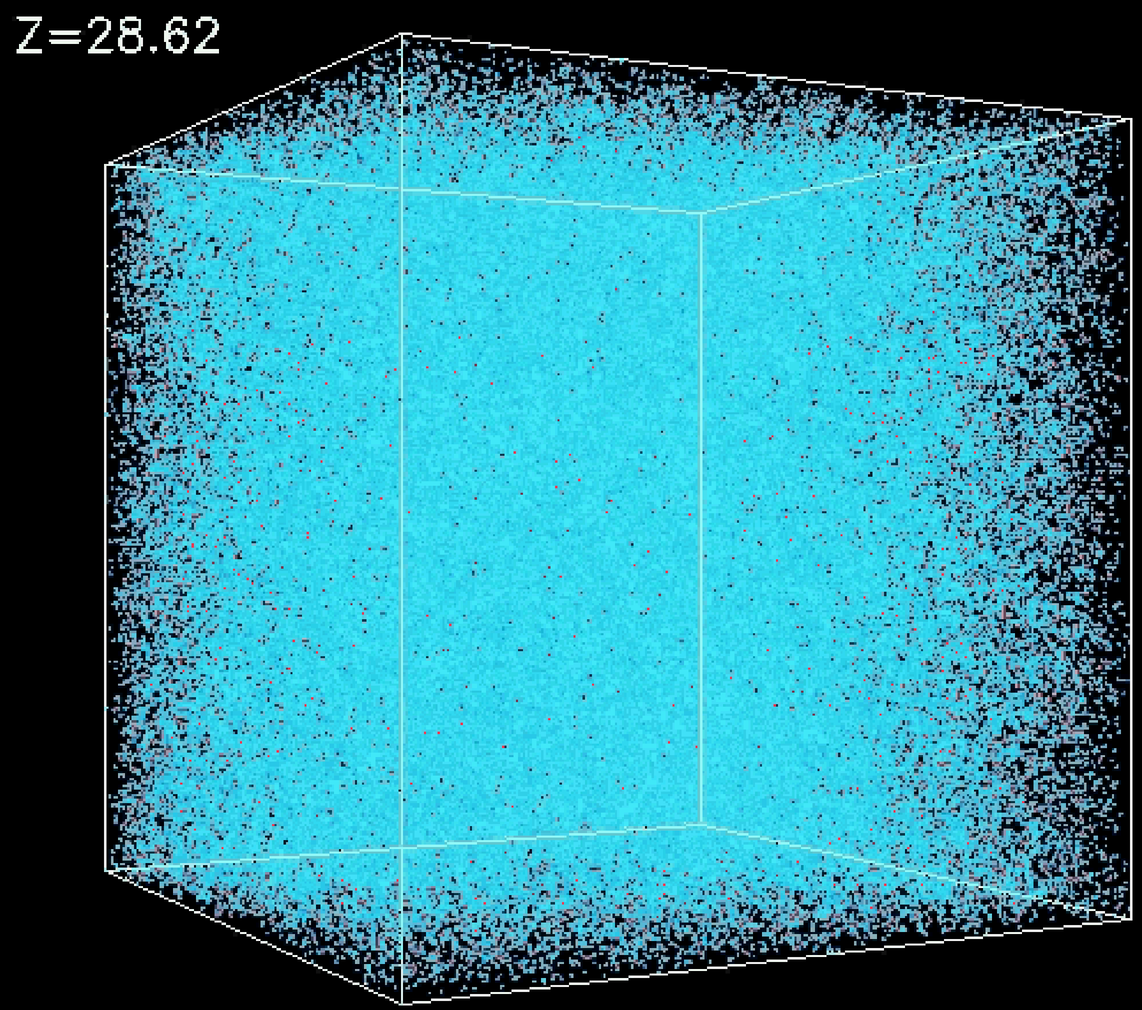
$P(k) = |\delta_k|^2$ Power Spectrum

or equivalently by

$$\Delta(k) = \left(\frac{k^3 P(k)}{2\pi^2}\right)^{1/2} = \frac{k^{3/2}|\delta_k|}{\sqrt{2}\,\pi}$$

No “non-Gaussianities” in cosmological precision data (Planck CMBR results)

Structure Formation by Gravitational Instability



Gravitational Growth of Density Perturbations

The dynamical evolution of small perturbations

$$\delta(x) = \frac{\delta\rho(x)}{\bar{\rho}} \ll 1$$

is independent for each Fourier mode δ_k (linear regime)

- For pressureless, nonrelativistic matter (cold dark matter) naively expect exponential growth by gravitational instability
- But only power-law growth in expanding universe (competition between expansion and gravitational instability)

	Sub-horizon $\lambda \ll H^{-1}$	Super-horizon $\lambda \gg H^{-1}$
Radiation dominates $a \propto t^{1/2}$	$\delta_k \propto \text{const}$	$\delta_k \propto a^2 \propto t$
Matter dominates $a \propto t^{2/3}$	$\delta_k \propto a \propto t^{2/3}$	

Density contrast grows linearly with scale factor

Processed Power Spectrum in CDM Scenario

Primordial spectrum usually assumed to be a power law

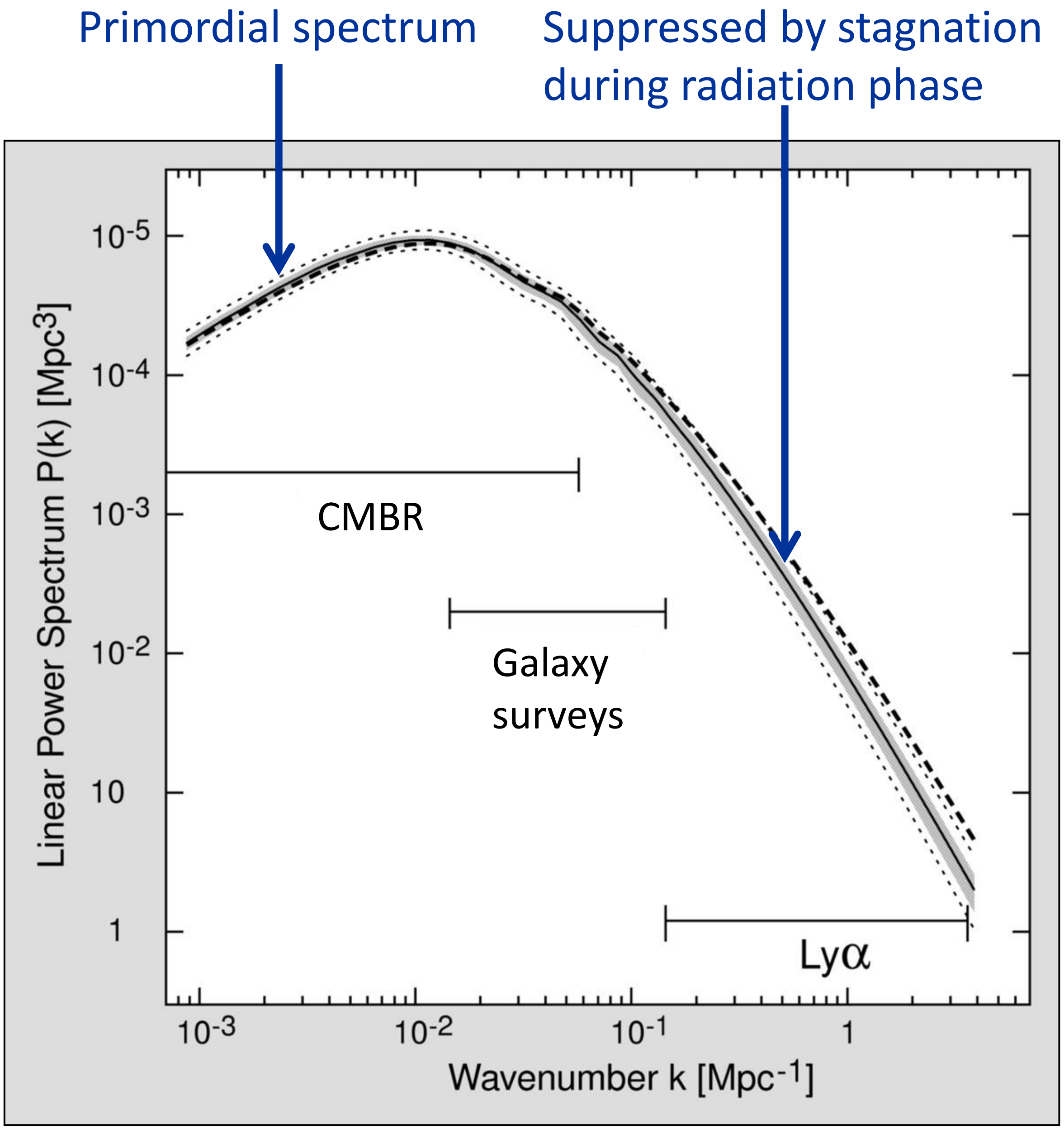
$$P(k) = |\delta_k|^2 \propto k^{n_s}$$

Harrison-Zeldovich spectrum ("flat") has $n_s = 1$

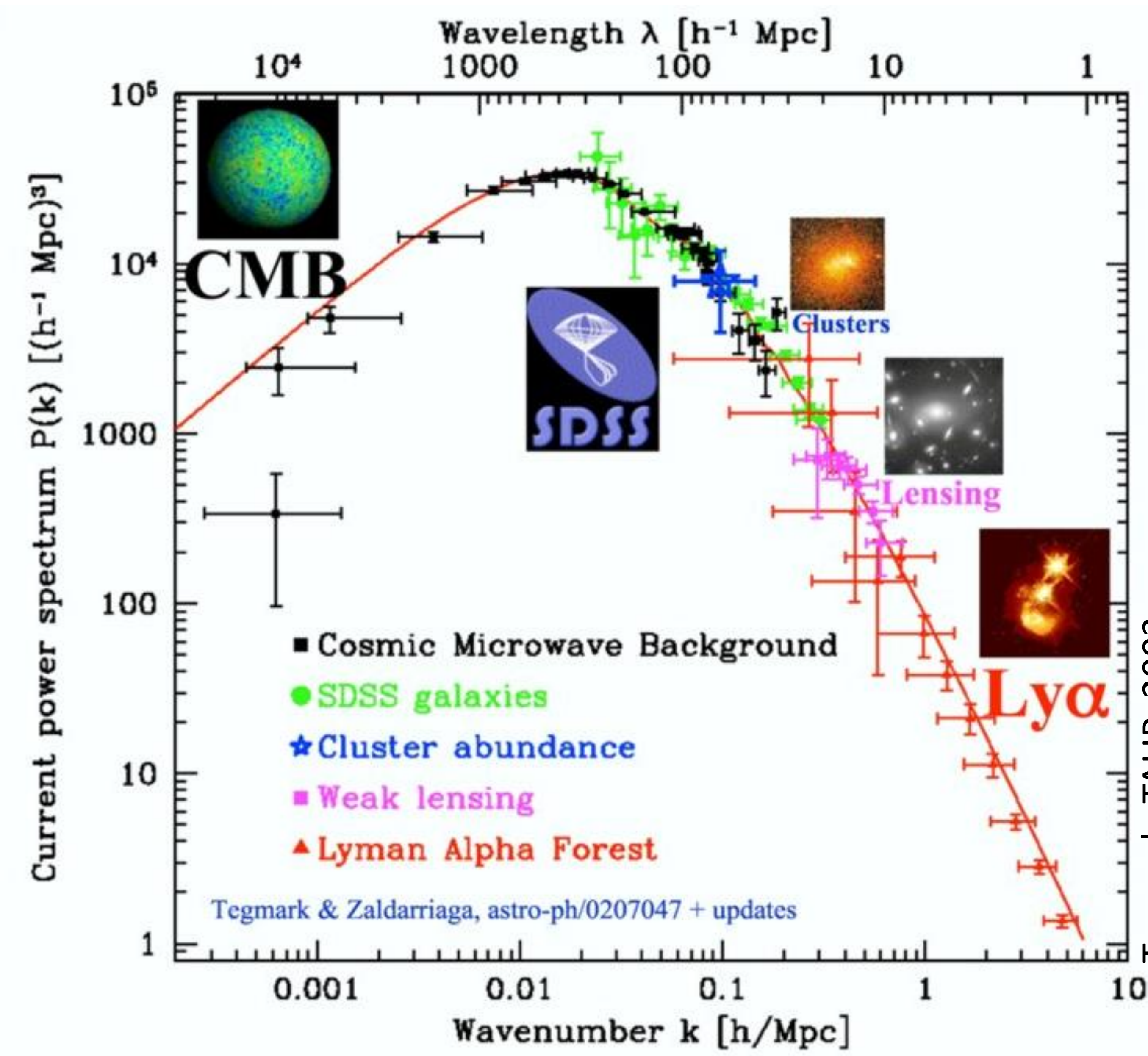
Precision cosmology provides

$$n_s = 0.960 \pm 0.007$$

in spectacular agreement with simplest theories of inflation

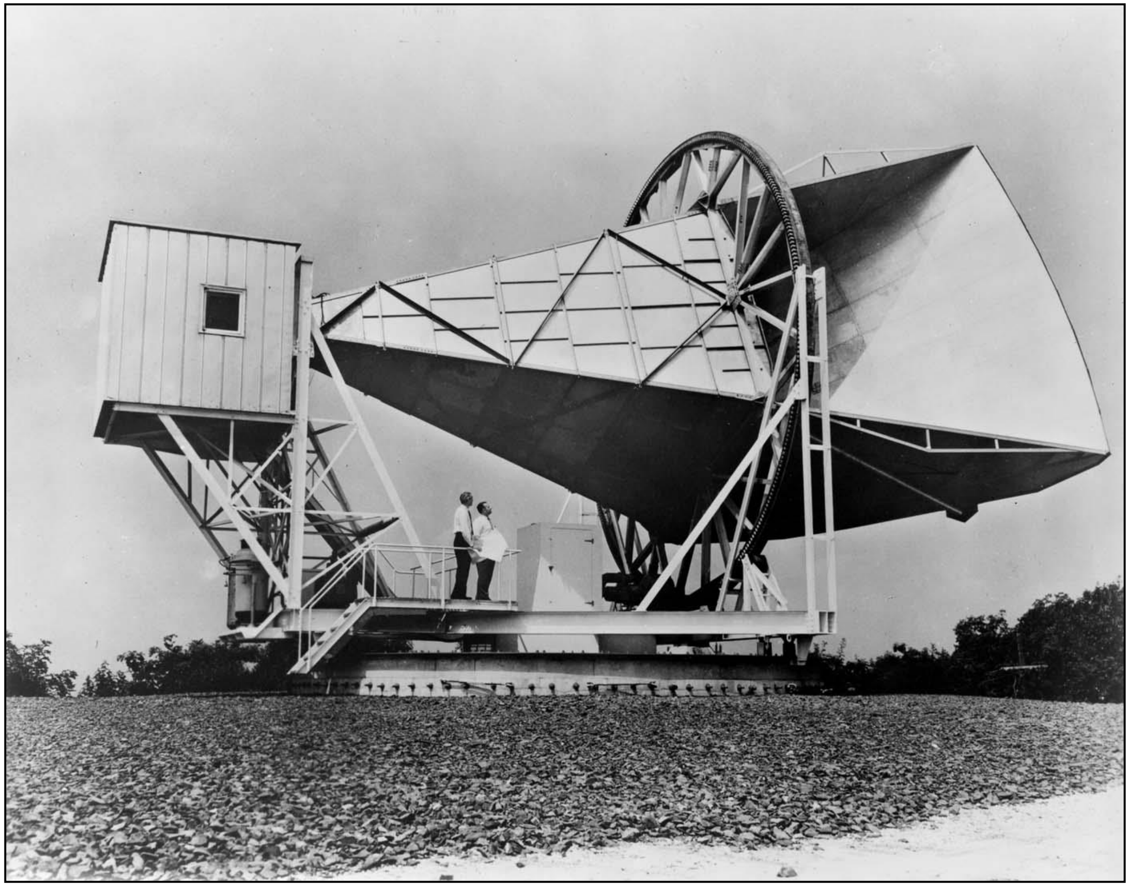


Power Spectrum of Cosmic Density Fluctuations



Tegmark, TAUP 2003

Discovery of the Cosmic Microwave Background Radiation



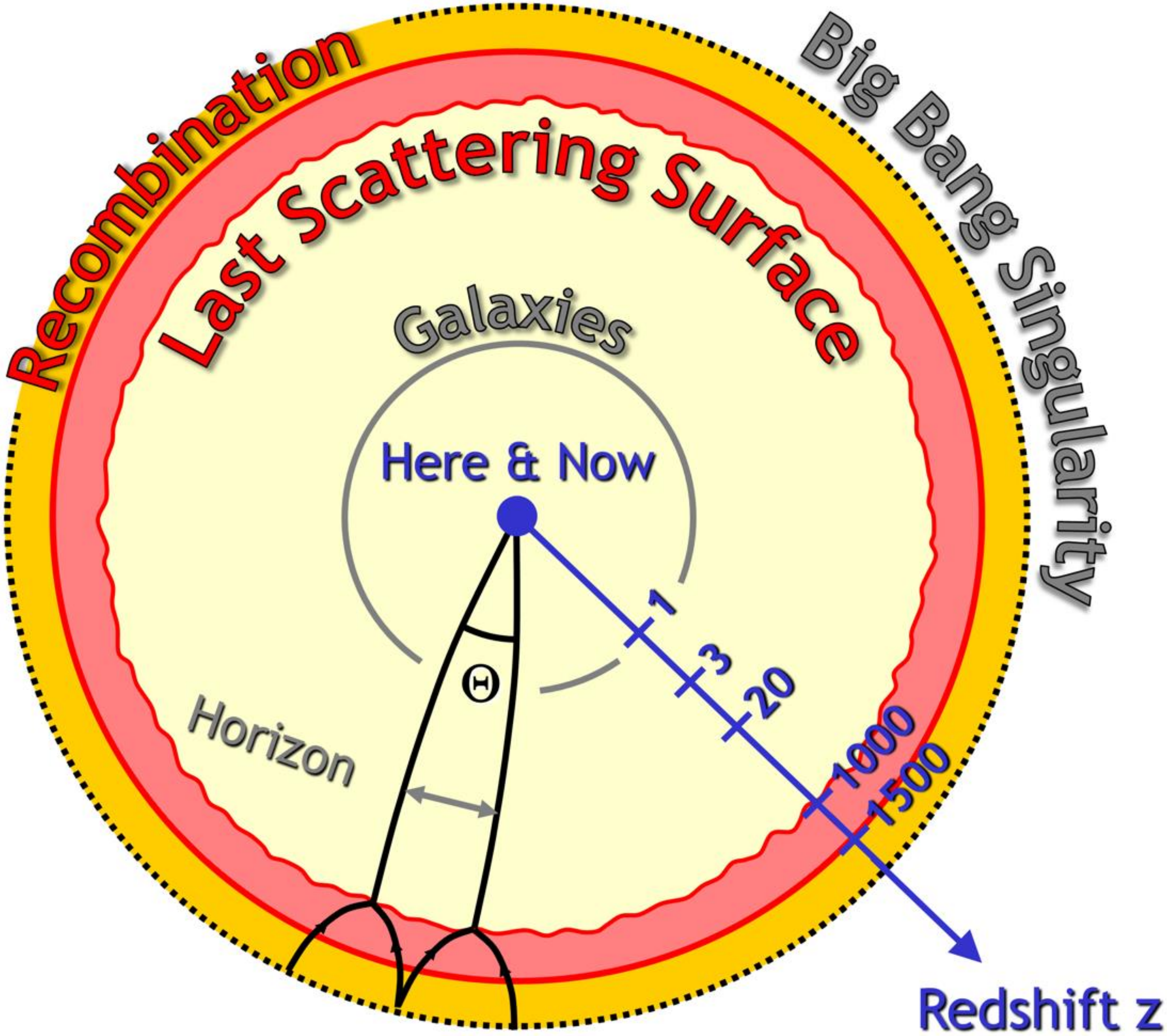
Robert W. Wilson
Born 1936

Arno A. Penzias
Born 1933

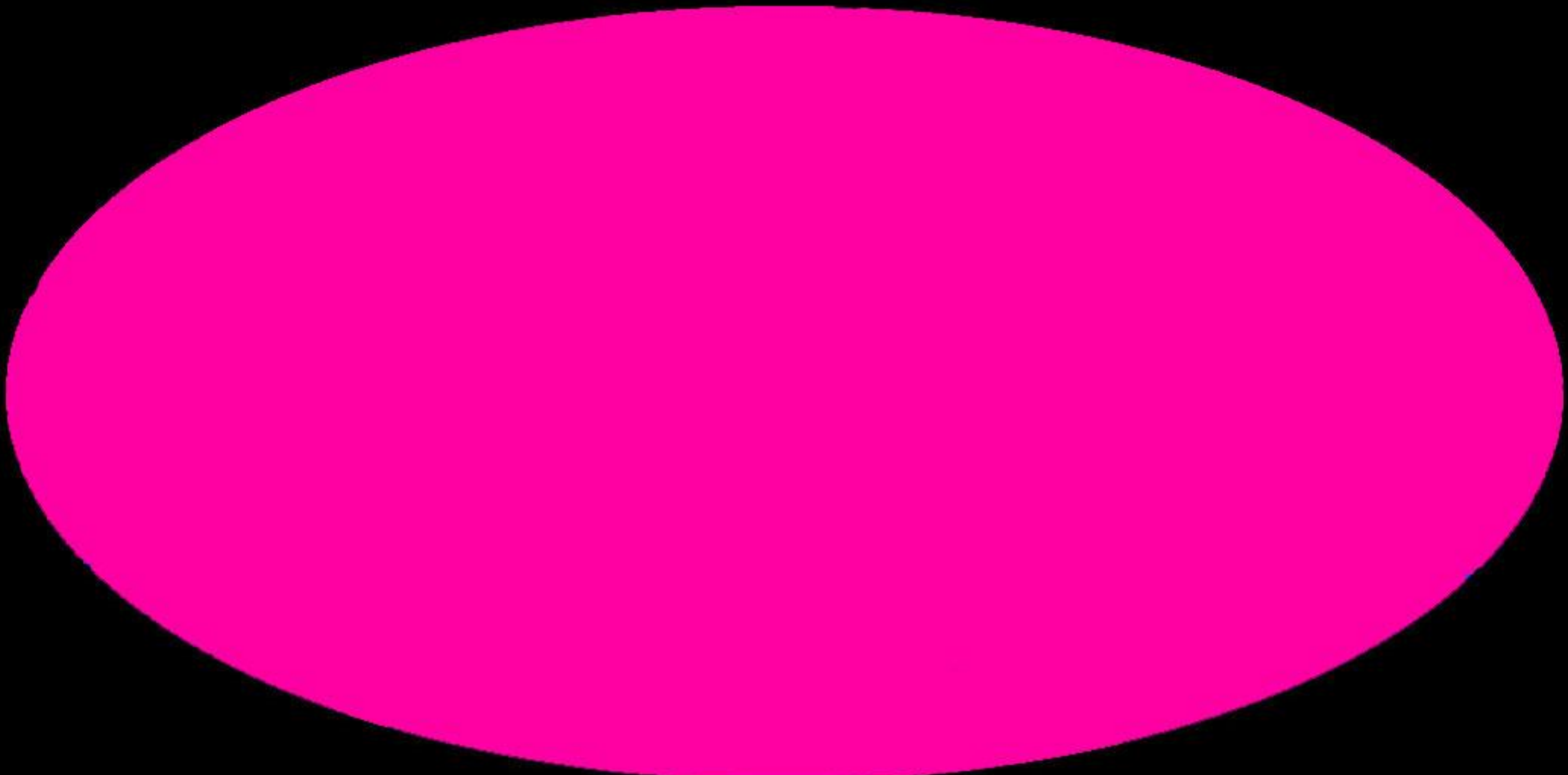
Discovery of 2.7 Kelvin
Cosmic microwave background radiation
by Penzias and Wilson in 1965
(Nobel Prize 1978)

Beginning of “big-bang cosmology”

Last Scattering Surface

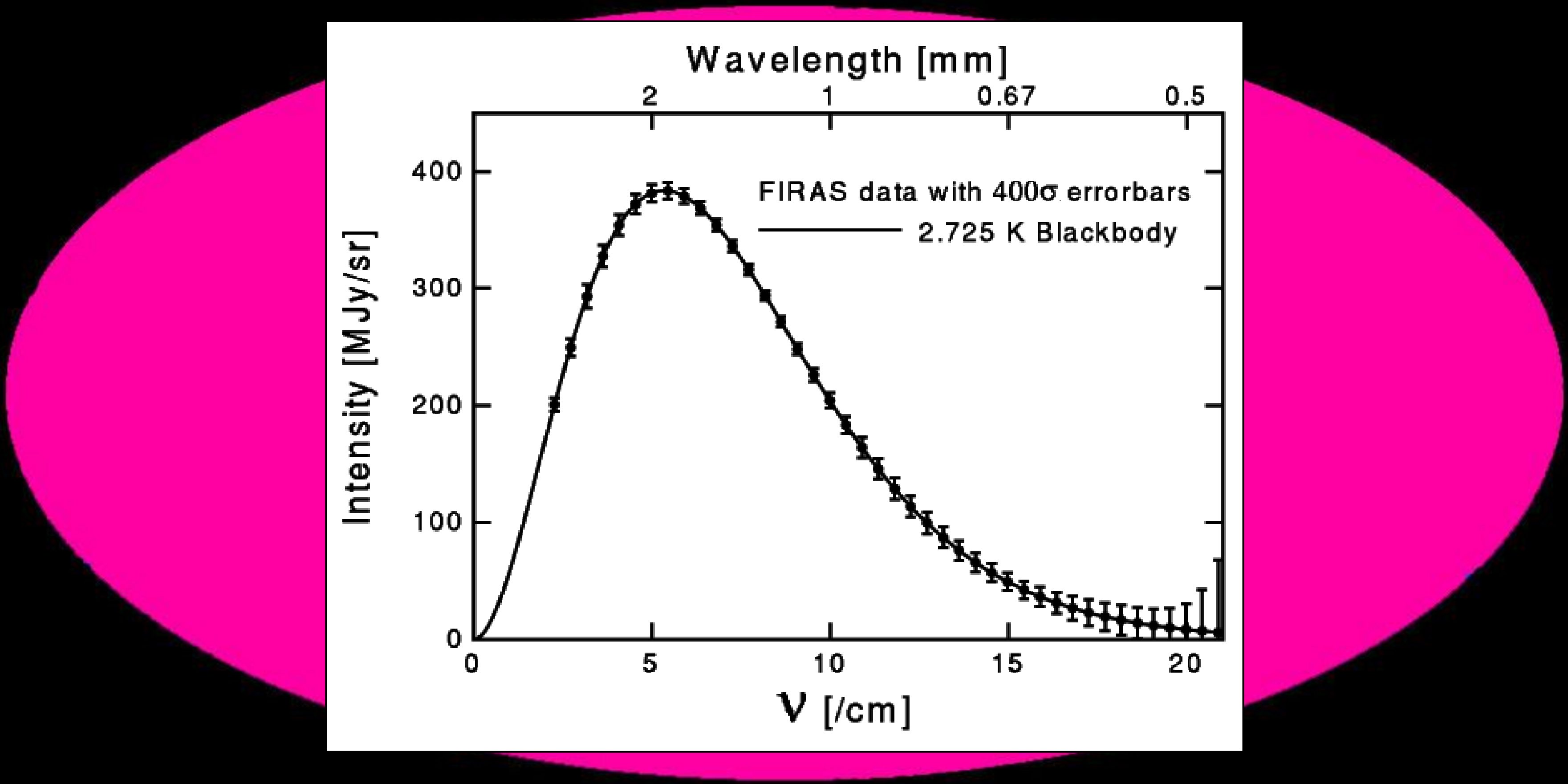


COBE Temperature Map of the Microwave Background



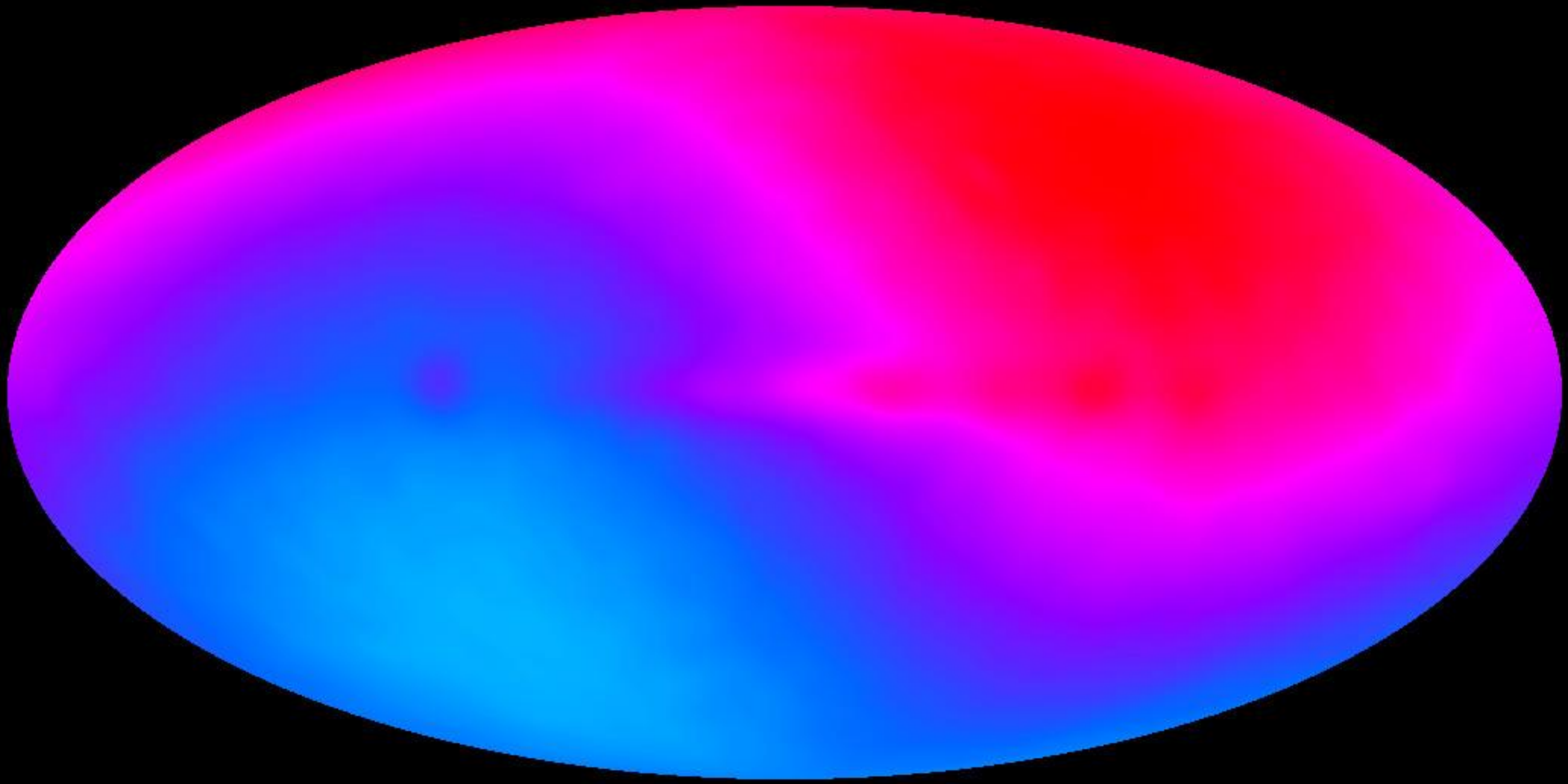
T = 2.725 K (uniform on the sky)

COBE Temperature Map of the Microwave Background



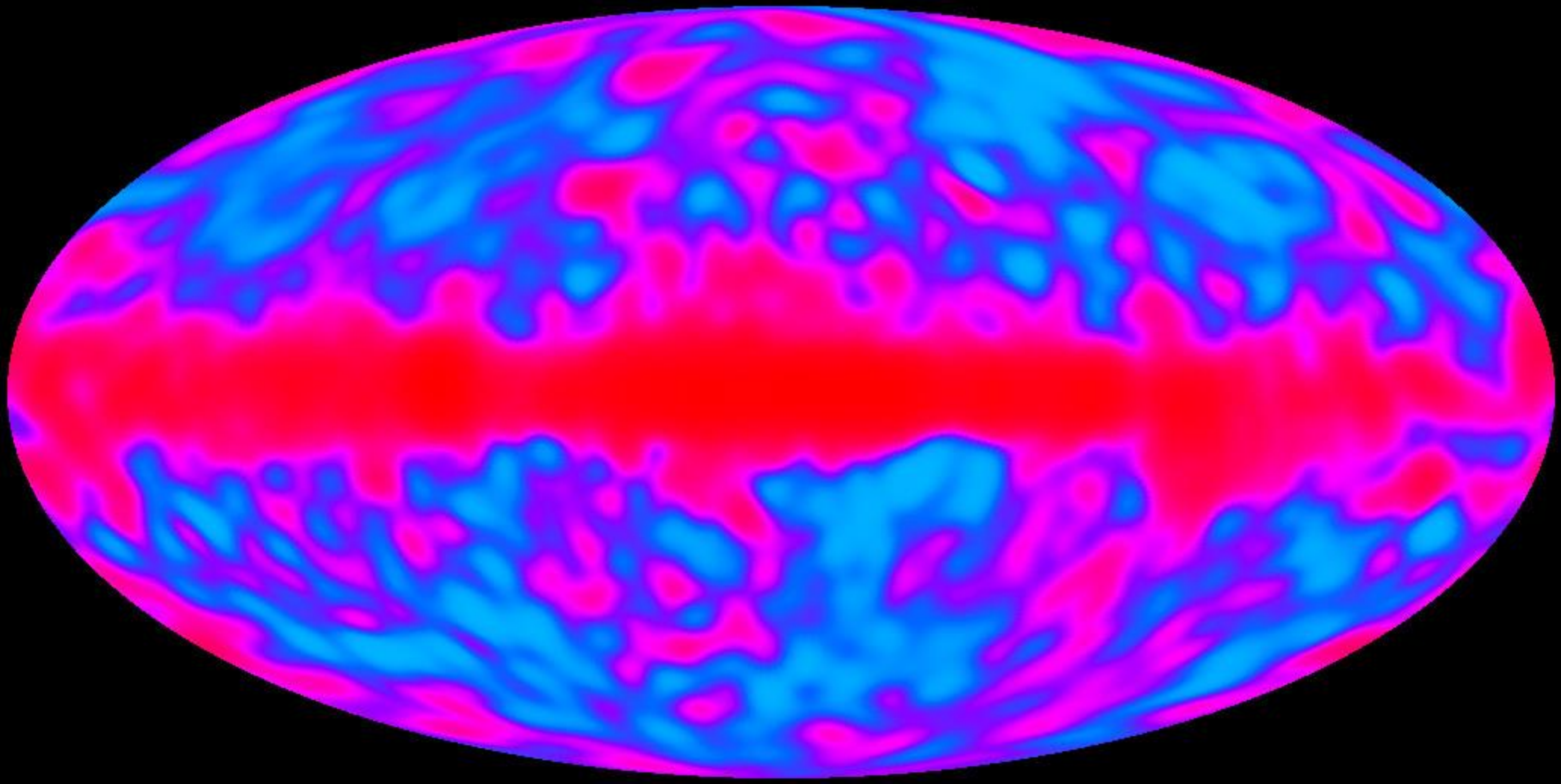
T = 2.725 K (uniform on the sky)

COBE Temperature Map of the Microwave Background



Dynamical range $\Delta T = 3.353$ mK ($\Delta T/T \approx 10^{-3}$)
Dipole temperature distribution from Doppler effect caused by our motion relative to the cosmic frame

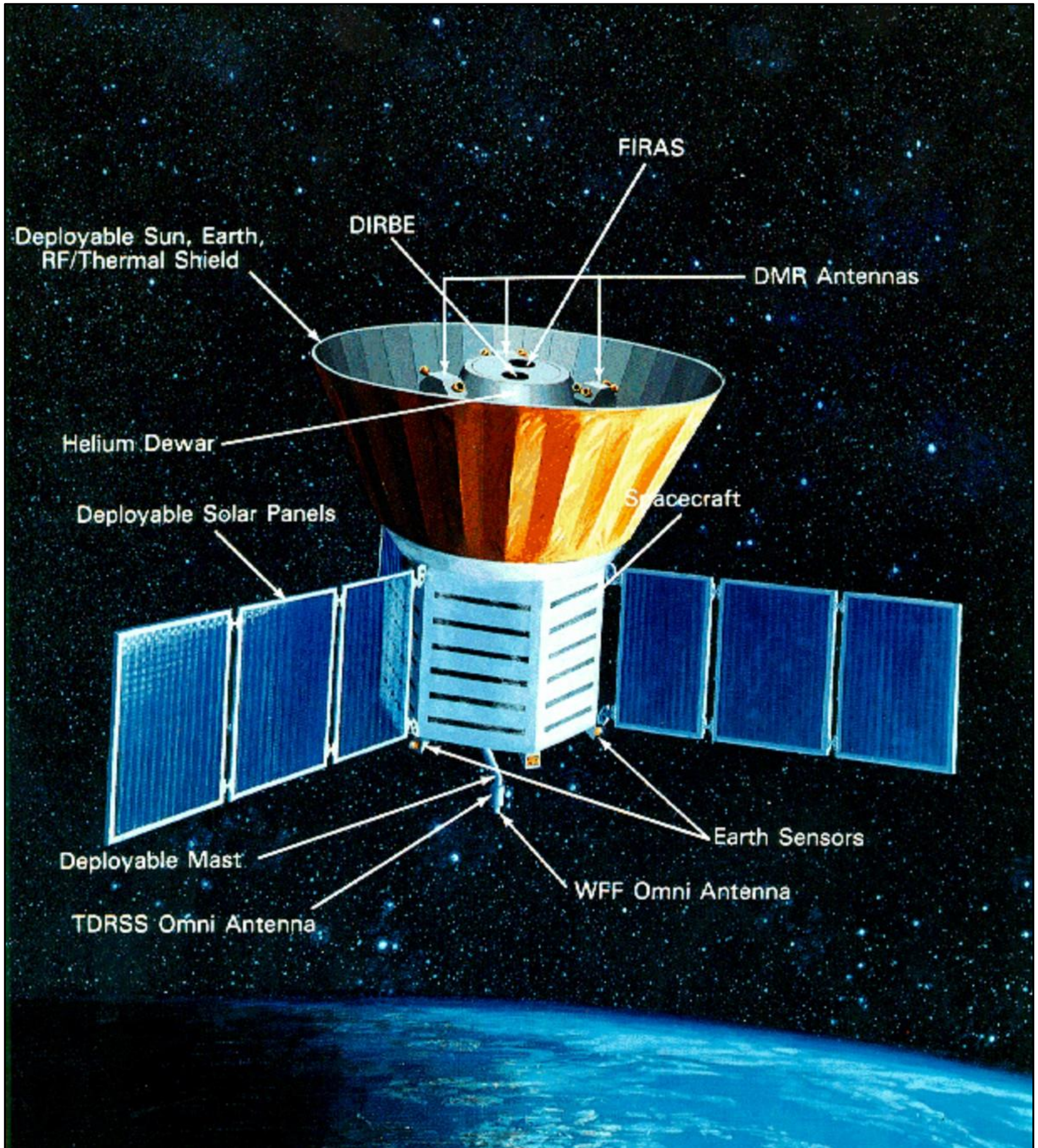
COBE Temperature Map of the Microwave Background

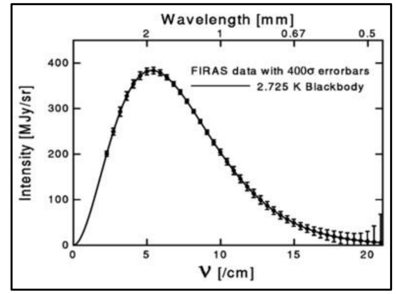


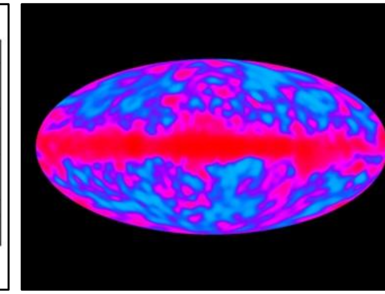
Dynamical range $\Delta T = 18\ \mu K$ ($\Delta T/T \approx 10^{-5}$)

Primordial temperature fluctuations

COBE Satellite



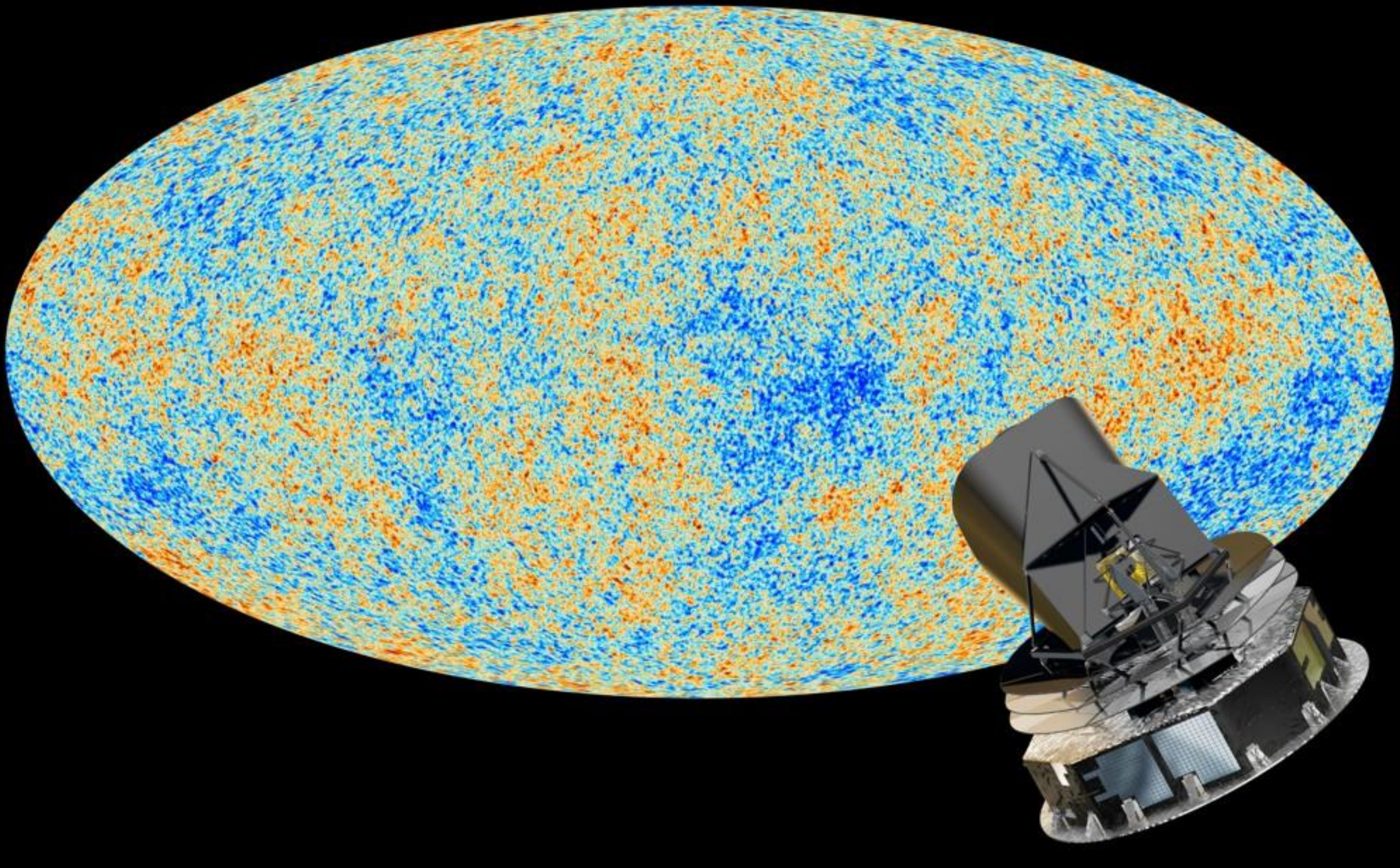




Nobel Prize 2006

John C. Mather Born 1946	George F. Smoot Born 1945

Cosmic Microwave Background (Planck 2013)



Power Spectrum of CMB Temperature Fluctuations

Sky map of CMBR temperature fluctuations

$$\Delta(\theta,\varphi)=\frac{T(\theta,\varphi)-\langle T\rangle}{\langle T\rangle}$$

Multipole expansion

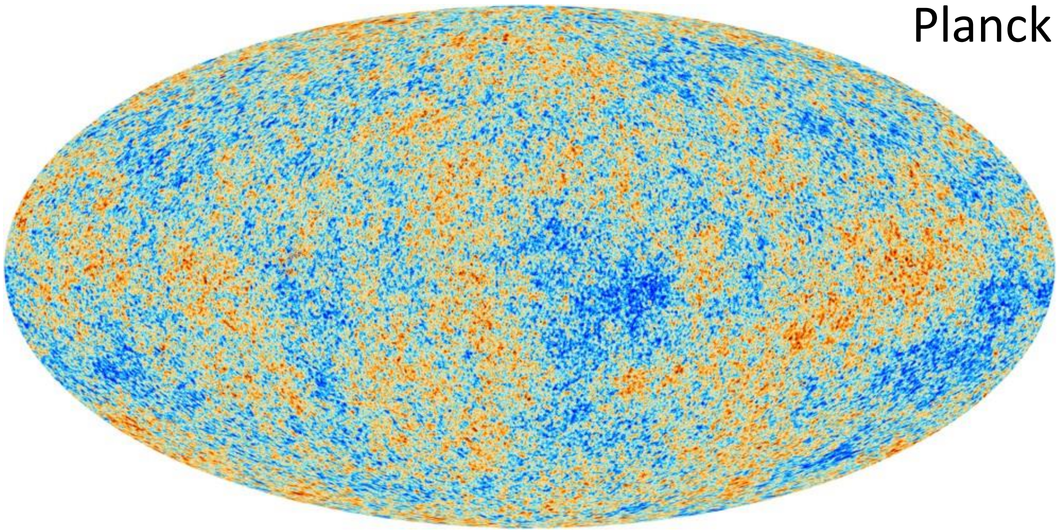
$$\Delta(\theta,\varphi)=\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}a_{\ell m}Y_{\ell m}(\theta,\varphi)$$

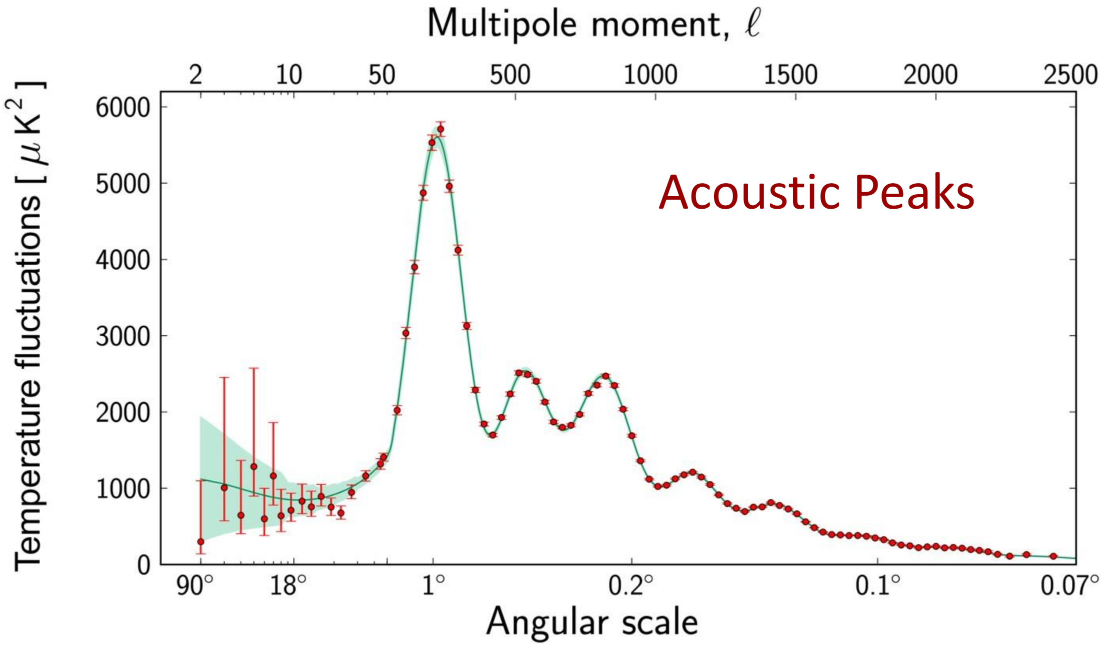
Angular power spectrum

$$C_\ell=\langle a_{\ell m}^{*}a_{\ell m}\rangle$$

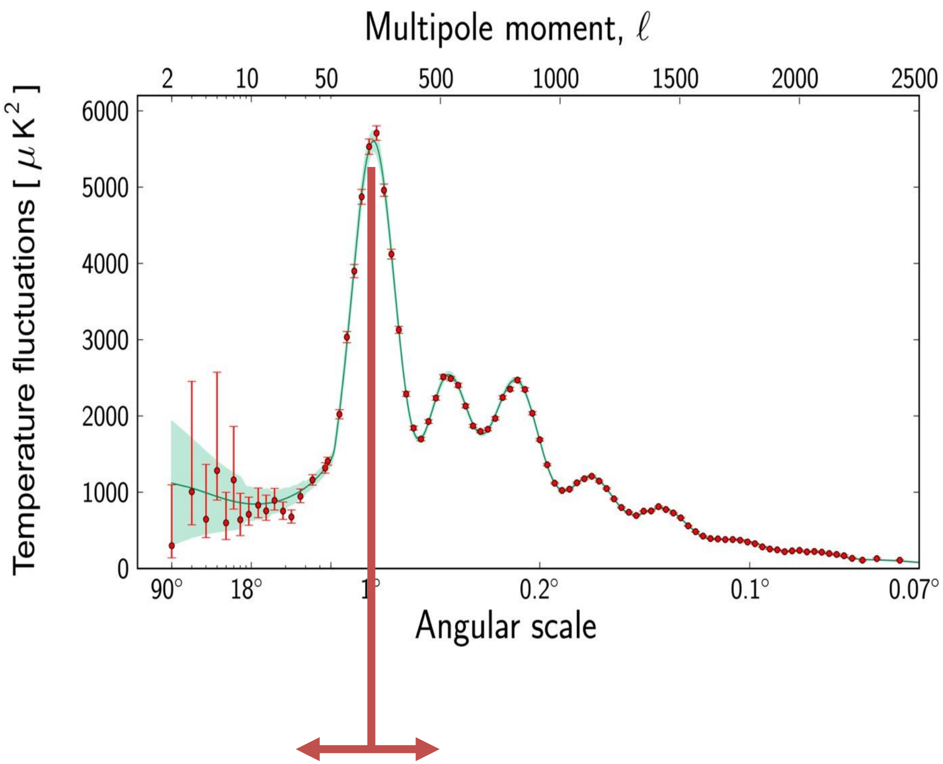
$$=\frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell}a_{\ell m}^{*}a_{\ell m}$$

Provides "acoustic peaks" and a wealth of cosmological information





Flat Universe from CMBR Angular Fluctuations

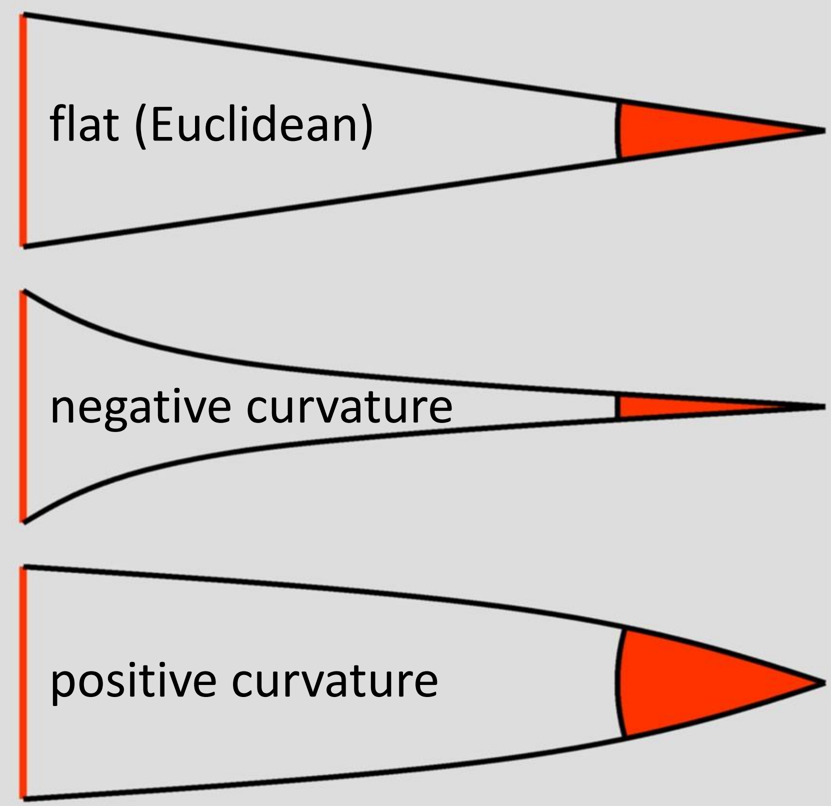


$\Omega_{\rm tot} = 0.963^{+0.043}_{-0.049}$ (CMB alone)

$\Omega_{\rm tot} = 0.9995^{+0.0065}_{-0.0066}$ (All data)

(95% ranges from Planck, arXiv:1303.5076)

Triangulation with acoustic peak

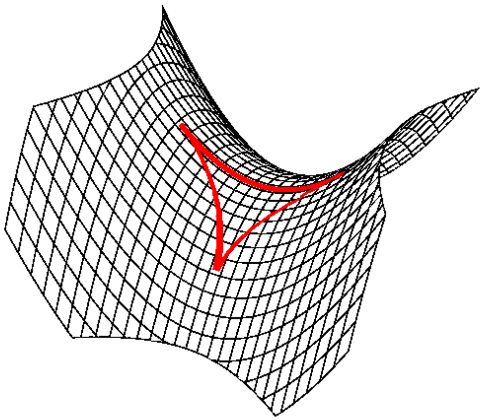


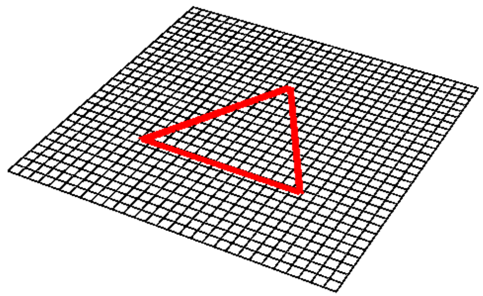
Known physical size of acoustic peak at decoupling ($z \approx 1100$)

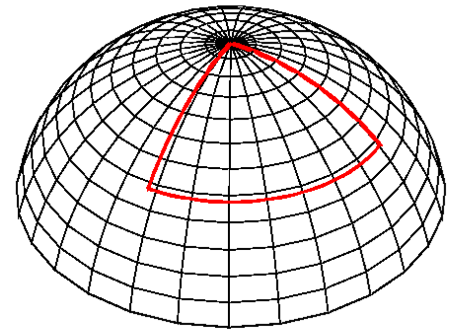
Measured angular size today ($z = 0$)

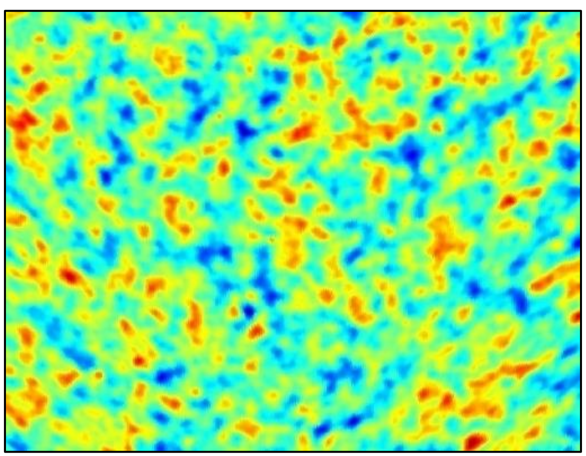
Geometry of the Universe and Angular Scales

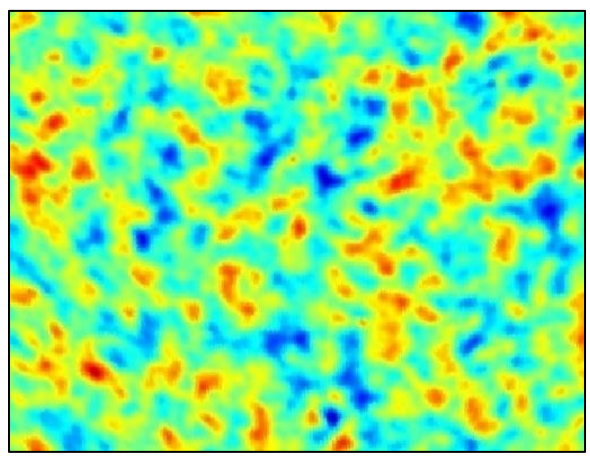
Open Universe	Flat Universe	Closed Universe
$\Omega < 1$	$\Omega = 1$	$\Omega > 1$

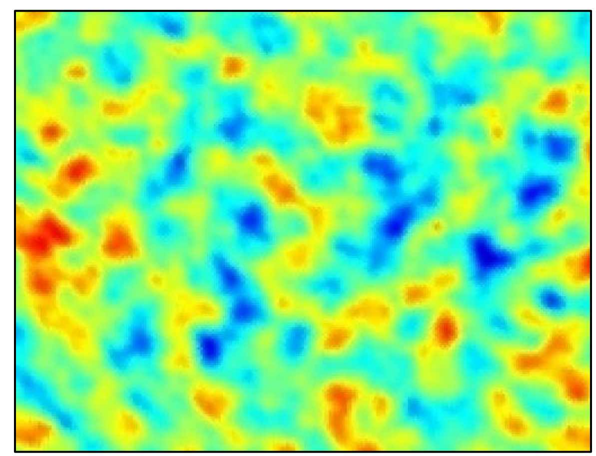


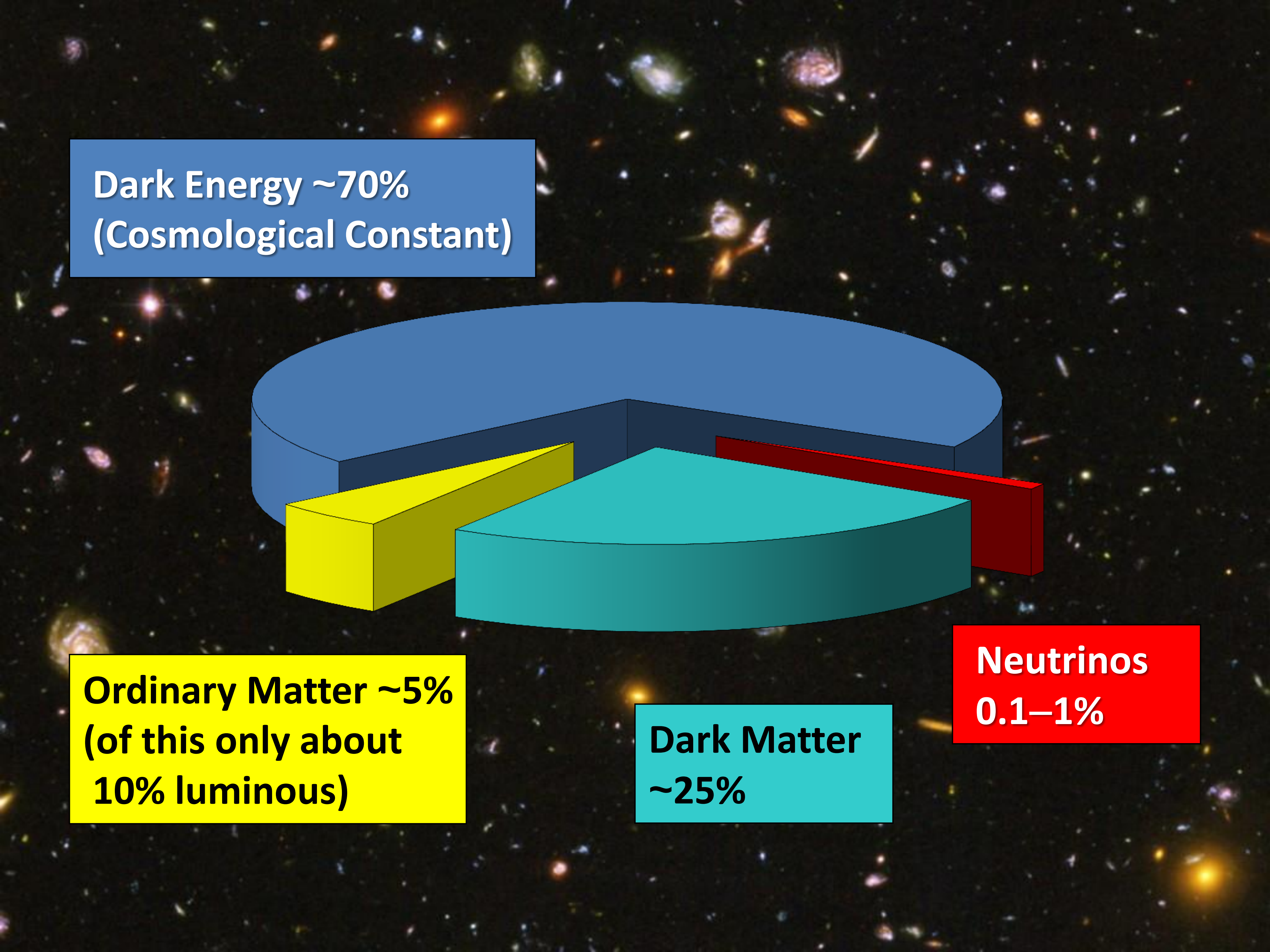




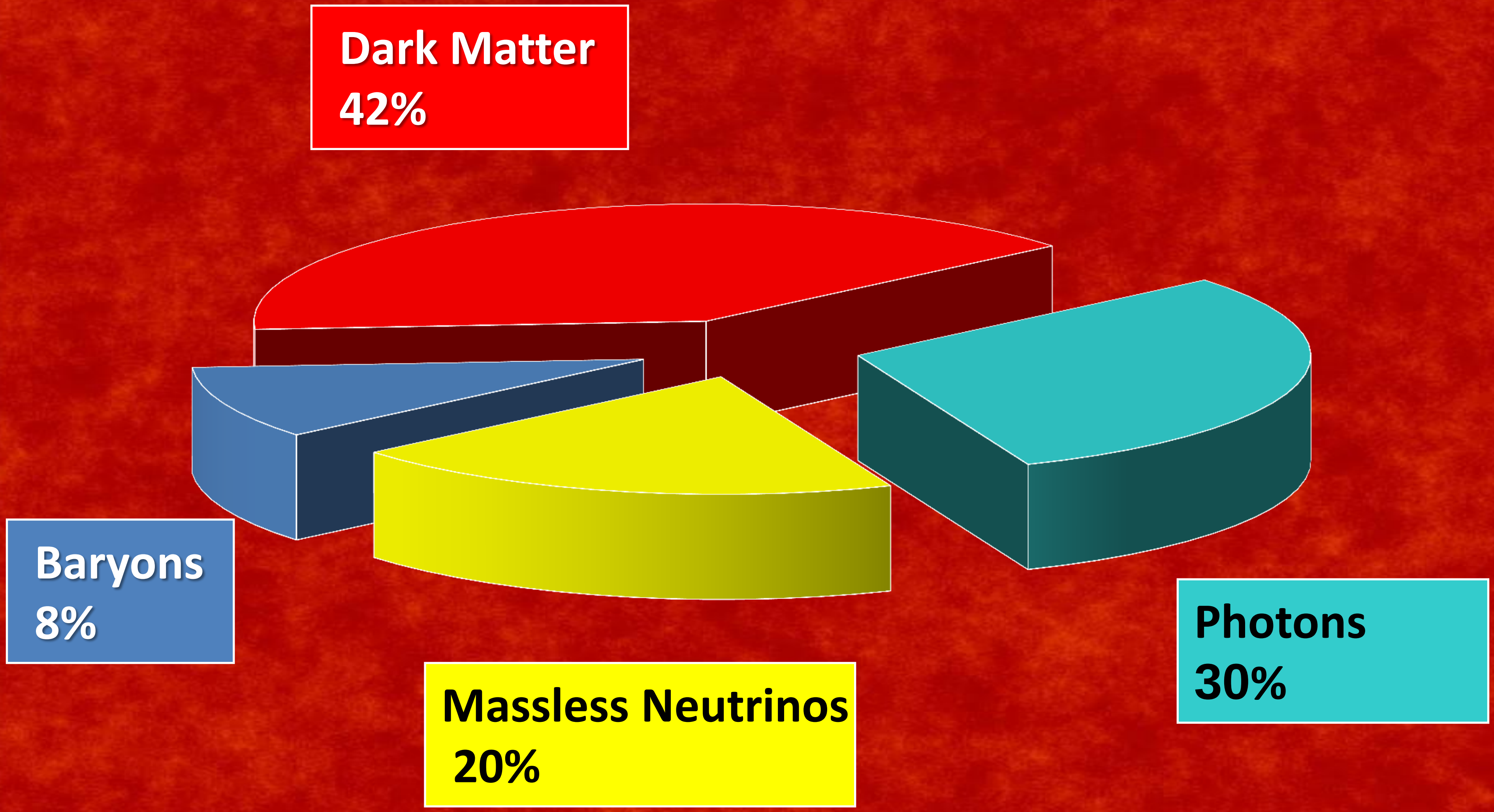




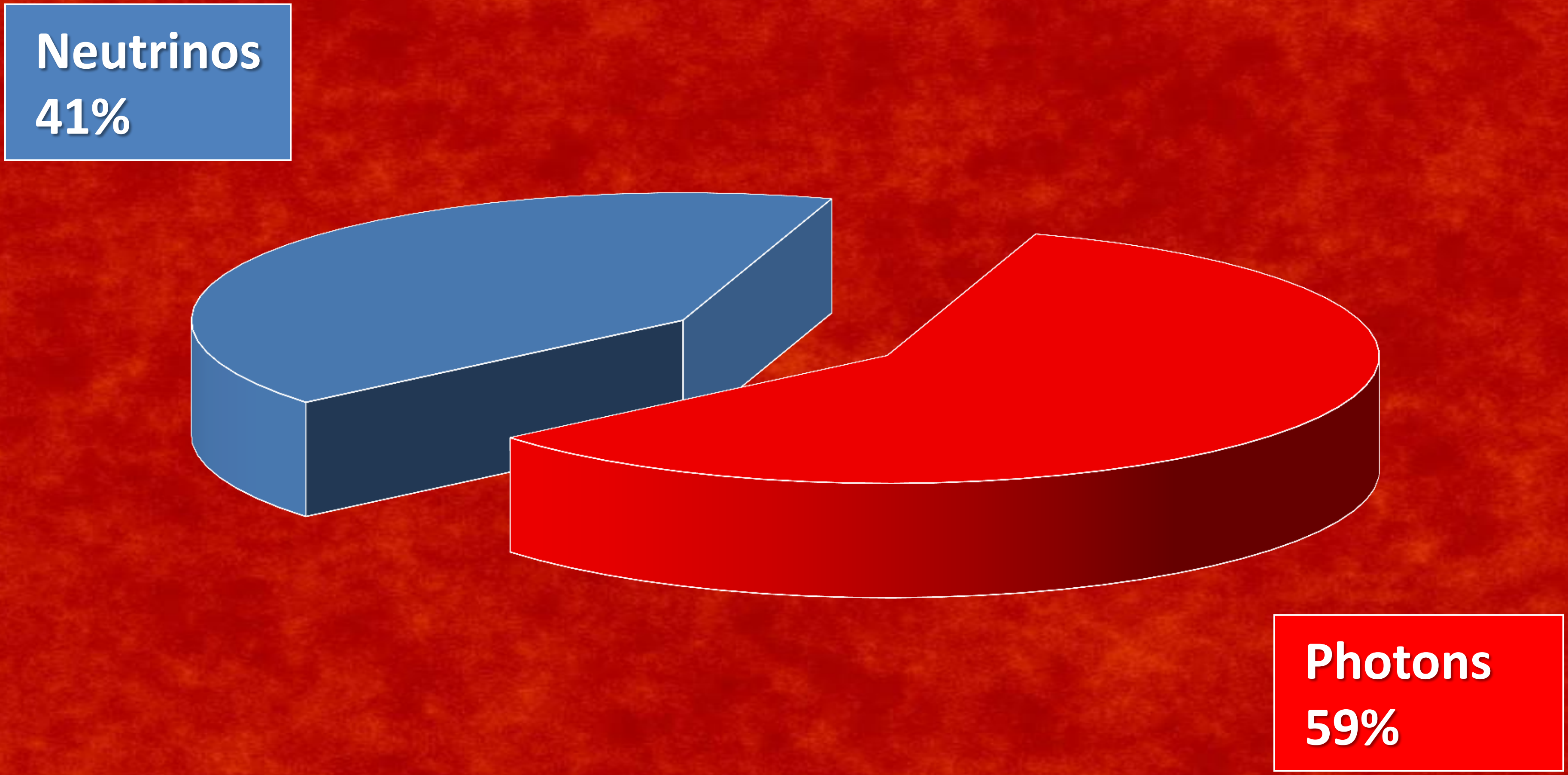



Dark Energy ~70%
(Cosmological Constant)
Ordinary Matter ~5%
(of this only about
10% luminous)
Dark Matter
~25%
Neutrinos
0.1–1%

Matter-Radiation Equality (Redshift 3400)

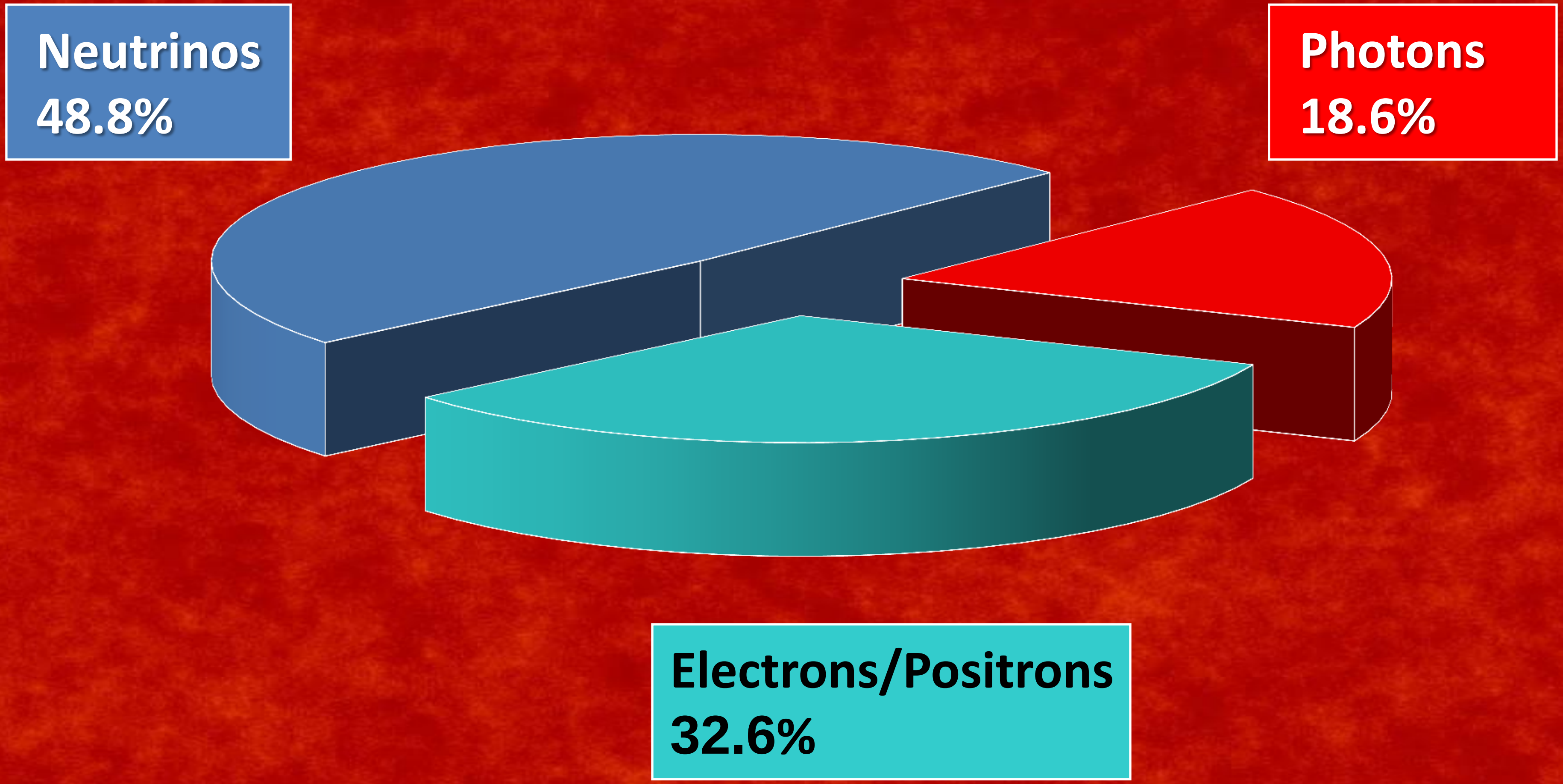


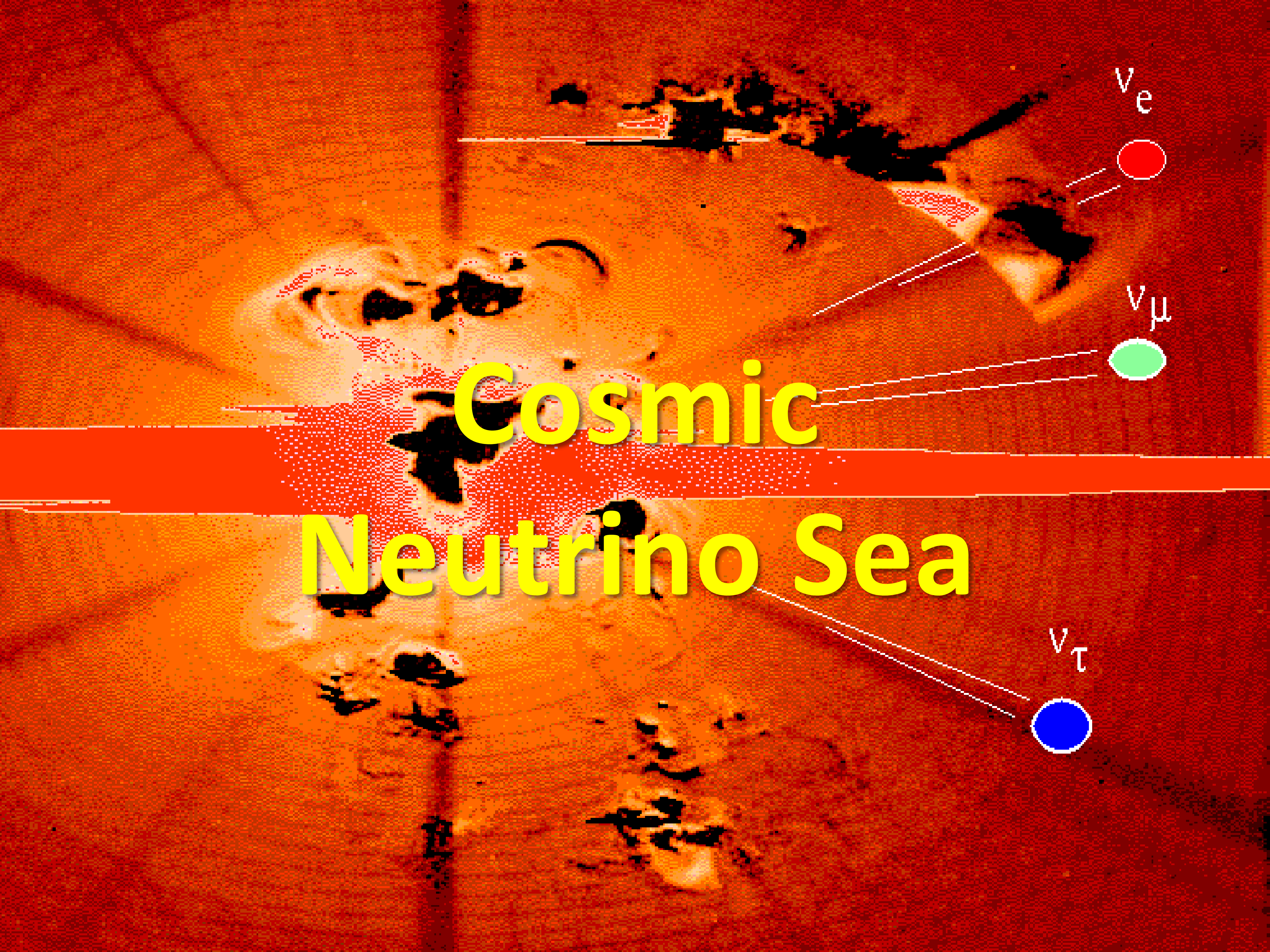
After Electron-Positron Annihilation (T = 100 keV)



Relevant for Big Bang Nucleosynthesis (BBN)

Before Electron-Positron Annihilation (T = 1 MeV)



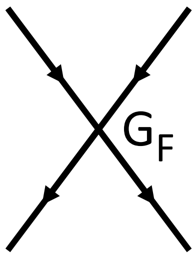

Cosmic
Neutrino Sea
ν_e
ν_μ
ν_τ

Neutrino Thermal Equilibrium

Neutrino reaction rate

Examples for neutrino processes

$$e^+ + e^- \leftrightarrow \overline{\nu} + \nu$$
$$\overline{\nu} + \nu \leftrightarrow \overline{\nu} + \nu$$
$$\nu + e^\pm \leftrightarrow \nu + e^\pm$$



Dimensional analysis of reaction rate in a thermal medium for $T \ll m_{W,Z}$

$$\Gamma \sim G_F^2 T^5$$

Cosmic expansion rate

Friedmann equation (flat universe)

$$H^2 = \frac{8\pi}{3}\frac{\rho}{m_{Pl}^2} \qquad \left(G_N = \frac{1}{m_{Pl}^2}\right)$$

Radiation dominates

$$\rho \sim T^4$$

Expansion rate

$$H \sim \frac{T^2}{m_{Pl}}$$

Condition for thermal equilibrium: $\Gamma > H$

$$T > \left(m_{Pl} G_F^2\right)^{-1/3} \sim [10^{19}\mathrm{GeV}\,(10^{-5}\mathrm{GeV}^{-2})^2]^{-1/3} = 1\ \mathrm{MeV}$$

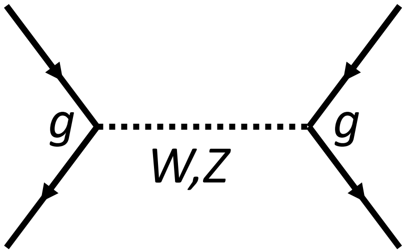
Neutrinos are in thermal equilibrium for $T \gtrsim 1$ MeV corresponding to $t \lesssim 1$ sec

Neutrino Thermal Equilibrium

Neutrino reaction rate

Examples for neutrino processes

$$e^+ + e^- \leftrightarrow \bar{\nu} + \nu$$
$$\bar{\nu} + \nu \leftrightarrow \bar{\nu} + \nu$$
$$\nu + e^\pm \leftrightarrow \nu + e^\pm$$



Dimensional analysis of reaction rate in a thermal medium for $T \gg m_{W,Z}$

$$\Gamma \sim (g^2/4\pi)^2\, T$$

Cosmic expansion rate

Friedmann equation (flat universe)

$$H^2 = \frac{8\pi}{3}\frac{\rho}{m_{Pl}^2} \qquad \left(G_N = \frac{1}{m_{Pl}^2}\right)$$

Radiation dominates

$$\rho \sim T^4$$

Expansion rate

$$H \sim \frac{T^2}{m_{Pl}}$$

Condition for thermal equilibrium: $\Gamma > H$

$$T < (g^2/4\pi)^2\, m_{Pl} \approx 10^{16}\,\mathrm{GeV} \approx \Lambda_{GUT}$$

It depends on very early cosmic history when neutrinos first enter equilibrium, presumably at reheating after inflation

Thermal Radiations

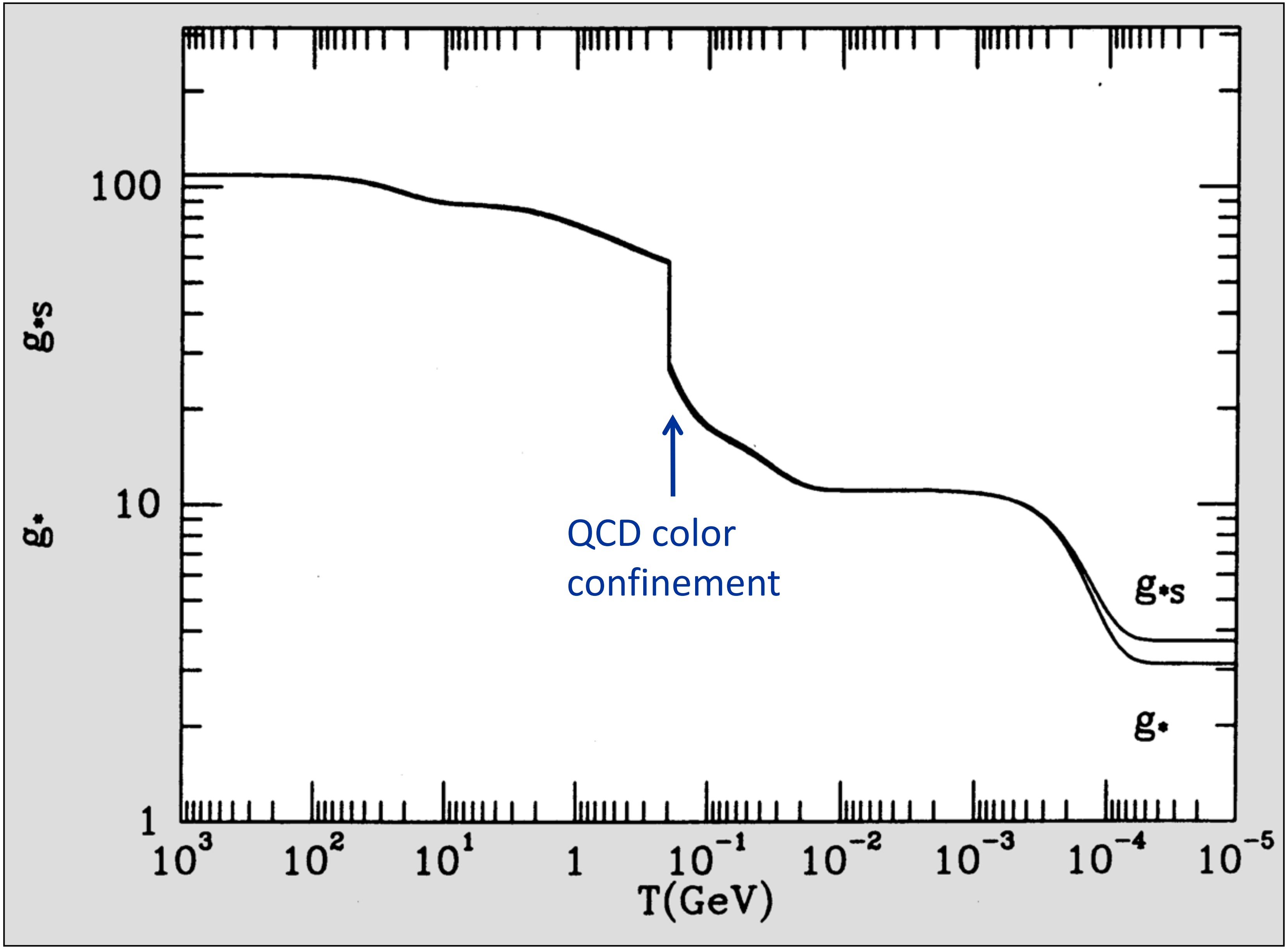
	General	Bosons	Fermions
Number density n	$g\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{e^{E_{\boldsymbol{p}}/T}\pm 1}$	$g_B\frac{\zeta_3}{\pi^2}T^3$	$\frac{3}{4}g_F\frac{\zeta_3}{\pi^2}T^3$
Energy density ρ	$g\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{E_{\boldsymbol{p}}}{e^{E_{\boldsymbol{p}}/T}\pm 1}$	$g_B\frac{\pi^2}{30}T^4$	$\frac{7}{8}g_F\frac{\pi^2}{30}T^4$
Pressure P	$\frac{\rho}{3}$		
Entropy density s	$\frac{\rho+P}{T}=\frac{4}{3}\frac{\rho}{T}$	$g_B\frac{2\pi^2}{45}T^3$	$\frac{7}{8}g_F\frac{2\pi^2}{45}T^3$

Riemann Zeta Function
$\zeta_3 = 1.2020569\ldots$

Thermal Degrees of Freedom

Mass threshold		Particles	g_B	g_F	g_*
	low	γ, 3ν	2	6	(7.25)
m_e	0.5 MeV	$e^\pm$	2	10	10.75
m_μ	105 MeV	$\mu^\pm$	2	14	14.25
m_π	135 MeV	π^0, $\pi^\pm$	5	14	17.25
Λ_{QCD}	~170 MeV	u, d, s, gluons	18	50	61.75
$m_{c,\tau}$	2 GeV	c, τ	18	66	75.75
m_b	6 GeV	$b^\pm$	18	78	86.25
$m_{W,Z}$	90 GeV	Z^0, $W^\pm$	27	78	92.25
m_H	126 GeV	Higgs	28	78	93.25
m_t	170 GeV	t	28	90	106.75
Λ_{SUSY}	~ 1 TeV ?	SUSY particles	118	118	213.50

Thermal Degrees of Freedom in the Early Universe



Present-Day Neutrino Density

Neutrino decoupling (freeze out)	$H \sim \Gamma$ $T \approx 2.4$ MeV (electron flavor) $T \approx 3.7$ MeV (other flavors)
Redshift of Fermi-Dirac distribution (“nothing changes at freeze-out”)	$\frac{dn_{\nu\bar{\nu}}}{dE} = \frac{1}{\pi^2}\frac{E^2}{e^{E/T}+1}$ Temperature scales with redshift $T_\nu = T_\gamma \propto (z+1)$
Electron-positron annihilation beginning at T ≈ m_e = 0.511 MeV	• QED plasma is “strongly” coupled • Stays in thermal equilibrium (adiabatic process) • Entropy of e^+e^- transfered to photons $\underbrace{g_*}_{2+\frac{7}{8}4=\frac{11}{2}} T_\gamma^3\Big\vert_{\text{before}} = \underbrace{g_*}_{2} T_\gamma^3\Big\vert_{\text{after}} \Bigg\} T_\gamma^3\Big\vert_{\text{before}} = \mathbf{\frac{4}{11}} T_\gamma^3\Big\vert_{\text{after}}$
Redshift of neutrino and photon thermal distributions so that today we have	$n_{\nu\bar{\nu}}(1\ \text{flavor}) = \frac{4}{11}\times\frac{3}{4}\times n_\gamma = \frac{3}{11}n_\gamma \approx 112\ \text{cm}^{-3}$ $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \approx 1.95\ \text{K}$ for massless neutrinos