Analytical model for turbulence pressure in galaxy clusters & comparison to hydro simulations

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Why is an analytical model of turbulence pressure* needed?

* non-thermal pressure associated to random gas motions

~ 20% at r_{500c}

cf. Kaylea, Elena and Daisuke's talks

observations at cluster outskirts

SZ power spectrum,
HSE mass bias,
evolution of P&T&K profiles,
scaling relations...

Why is **an analytical** model of turbulence pressure needed?

fluid dynamics + cosmology

approximations

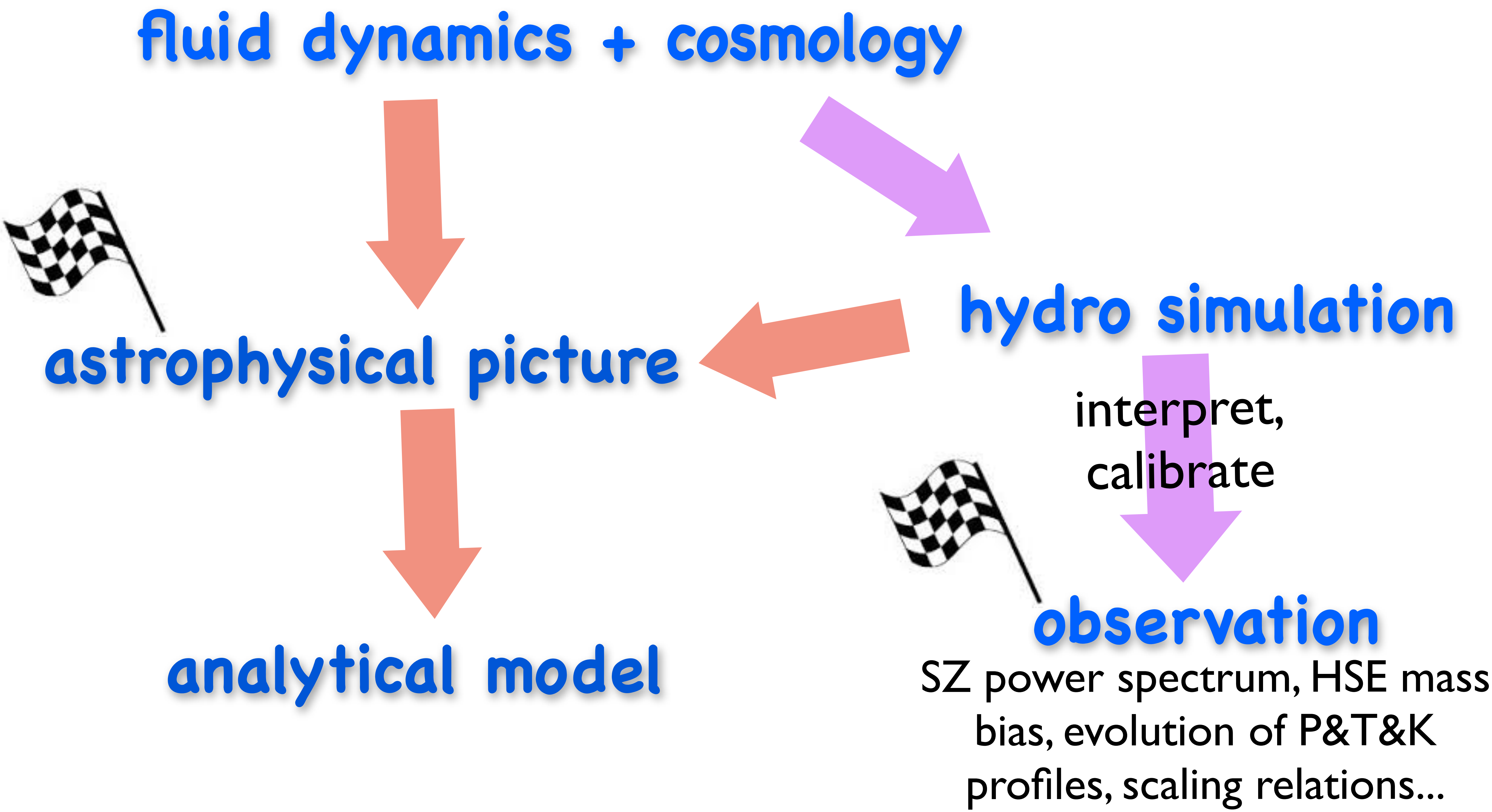
hydro simulation

interpret,
calibrate

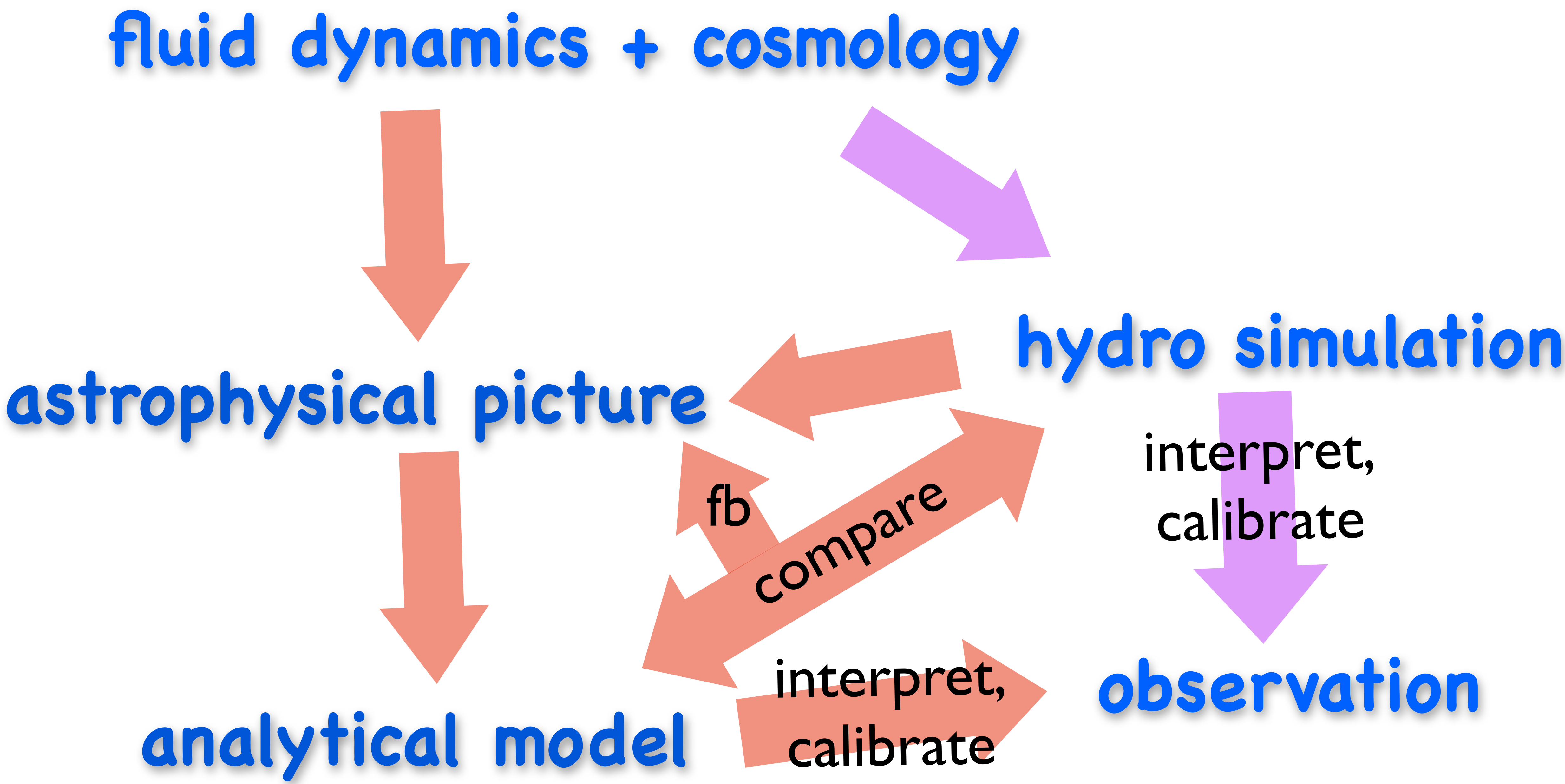
observation

SZ power spectrum, HSE mass bias, evolution of P&T&K profiles, scaling relations...

Why an analytical model is needed?



Why an analytical model is needed?



Why is an analytical model of turbulence pressure possible?

Why is an analytical model of turbulence pressure possible?

turbulence

mechanism of transport over k and x,
origin of intermittency, ...

Why an analytical model is possible?

turbulence

may be

intracluster turbulence pressure

Pressure ~ energy density

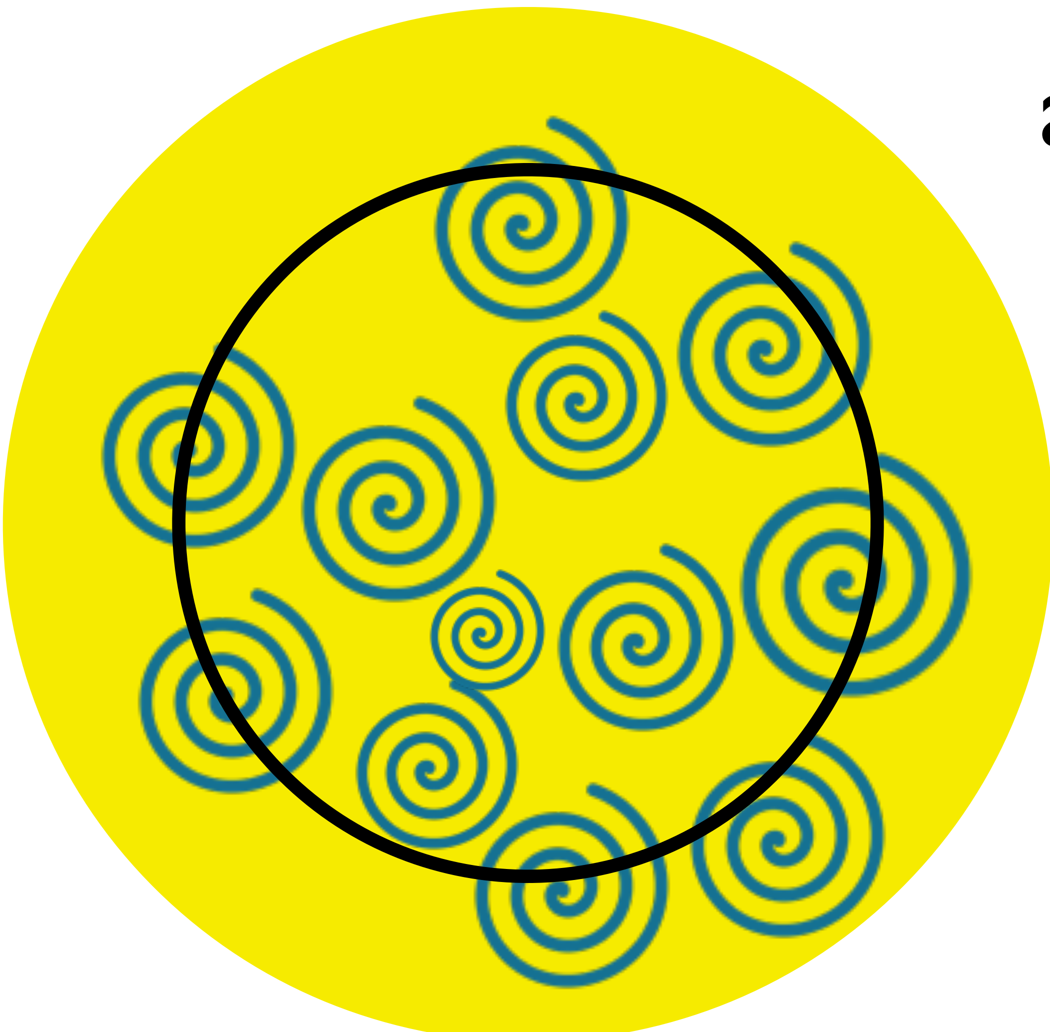
- injection & dissipation of random kinetic energy
- microphysics may not be critical

Analytical model for turbulence pressure

as a function of :

- ✓ Radius
- ✓ Mass
- ✓ Time

The model



at one Eularian radius r

$$\frac{d\sigma^2_{nth}}{dt} = -\frac{\sigma^2_{nth}}{t_d} + \eta \frac{d\sigma^2_{tot}}{dt}$$

$\sigma_{nth}^2 = P_{nth} / \rho_{gas}$

$\propto$ turbulence energy per unit mass

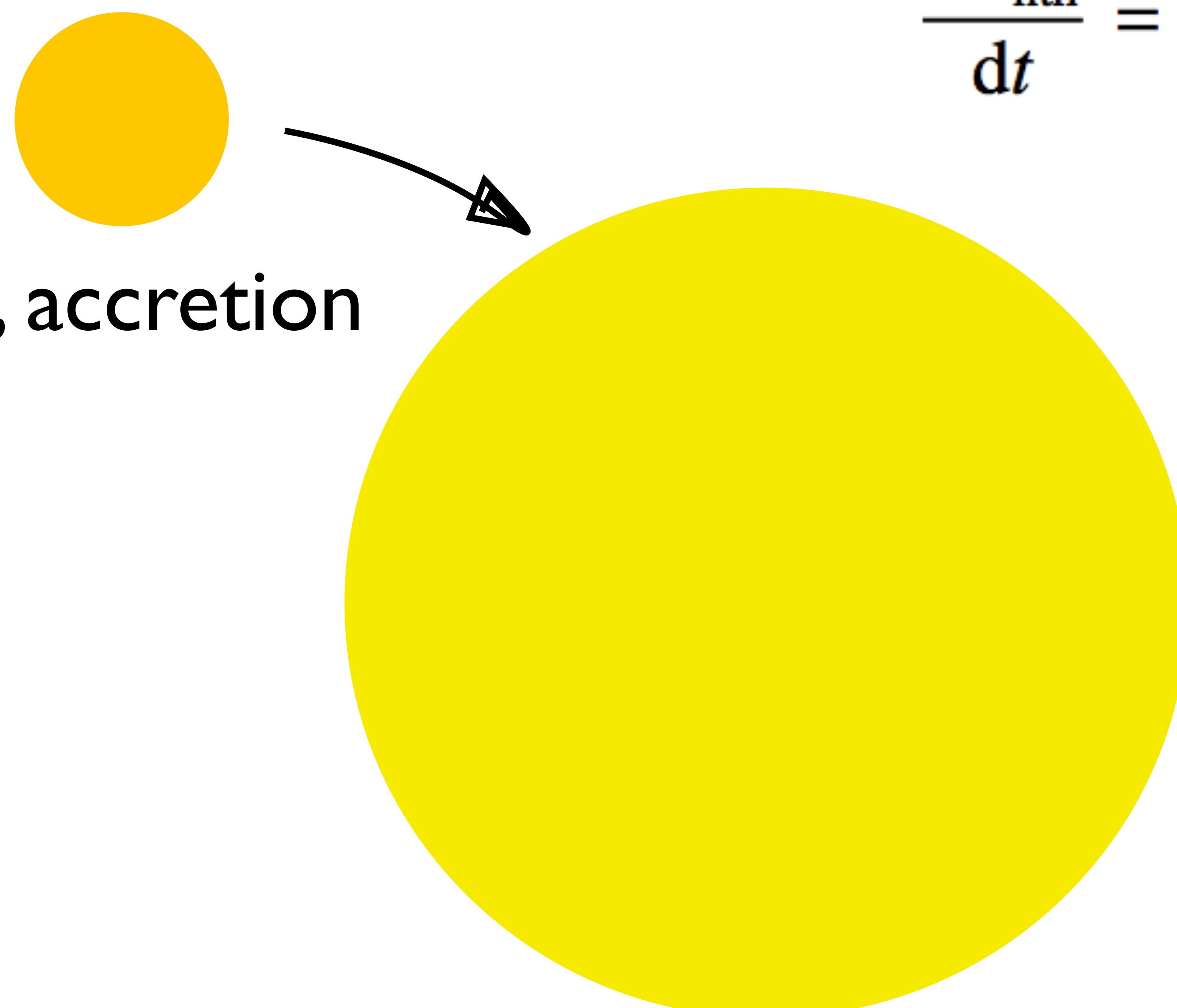
input: $\sigma_{tot}^2 = \sigma_{th}^2 + \sigma_{nth}^2 \quad \sim T$

depth of gravitational potential

Generation of intracluster turbulence

during structure formation

$$\frac{\mathrm{d}\sigma_{\mathrm{nth}}^2}{\mathrm{d}t} = -\frac{\sigma_{\mathrm{nth}}^2}{t_{\mathrm{d}}} + \eta \frac{\mathrm{d}\sigma_{\mathrm{tot}}^2}{\mathrm{d}t}$$



merger, accretion

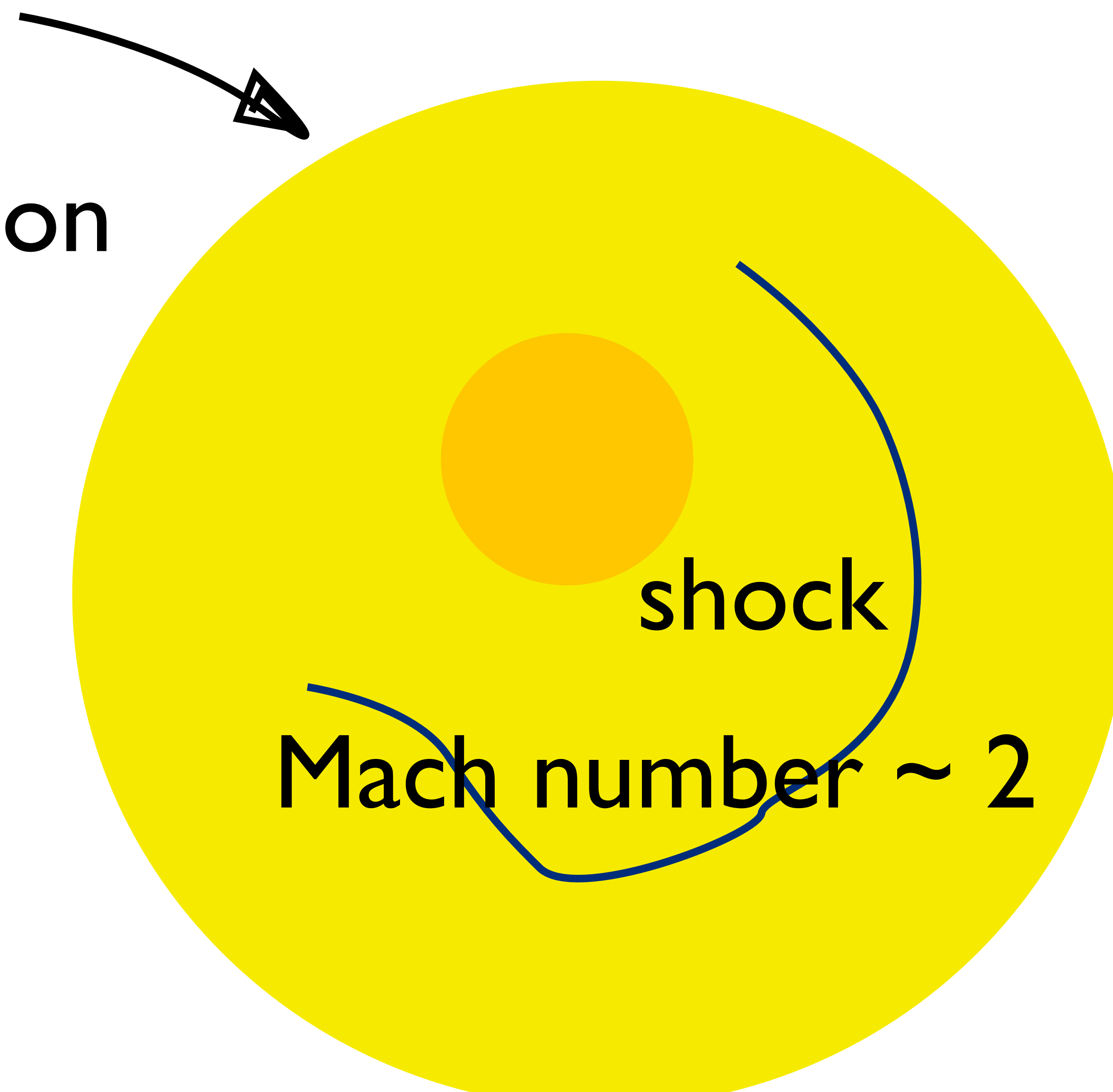
as the cluster grows in mass

turbulence from MTI (Parrish+12, McCourt+13) & feedback neglected here

Generation of intracluster turbulence

$$\frac{d\sigma^2_{\rm nth}}{dt} = -\frac{\sigma^2_{\rm nth}}{t_{\rm d}} + \eta \frac{d\sigma^2_{\rm tot}}{dt}$$

merger, accretion



$v^2/2 \sim -\Phi$

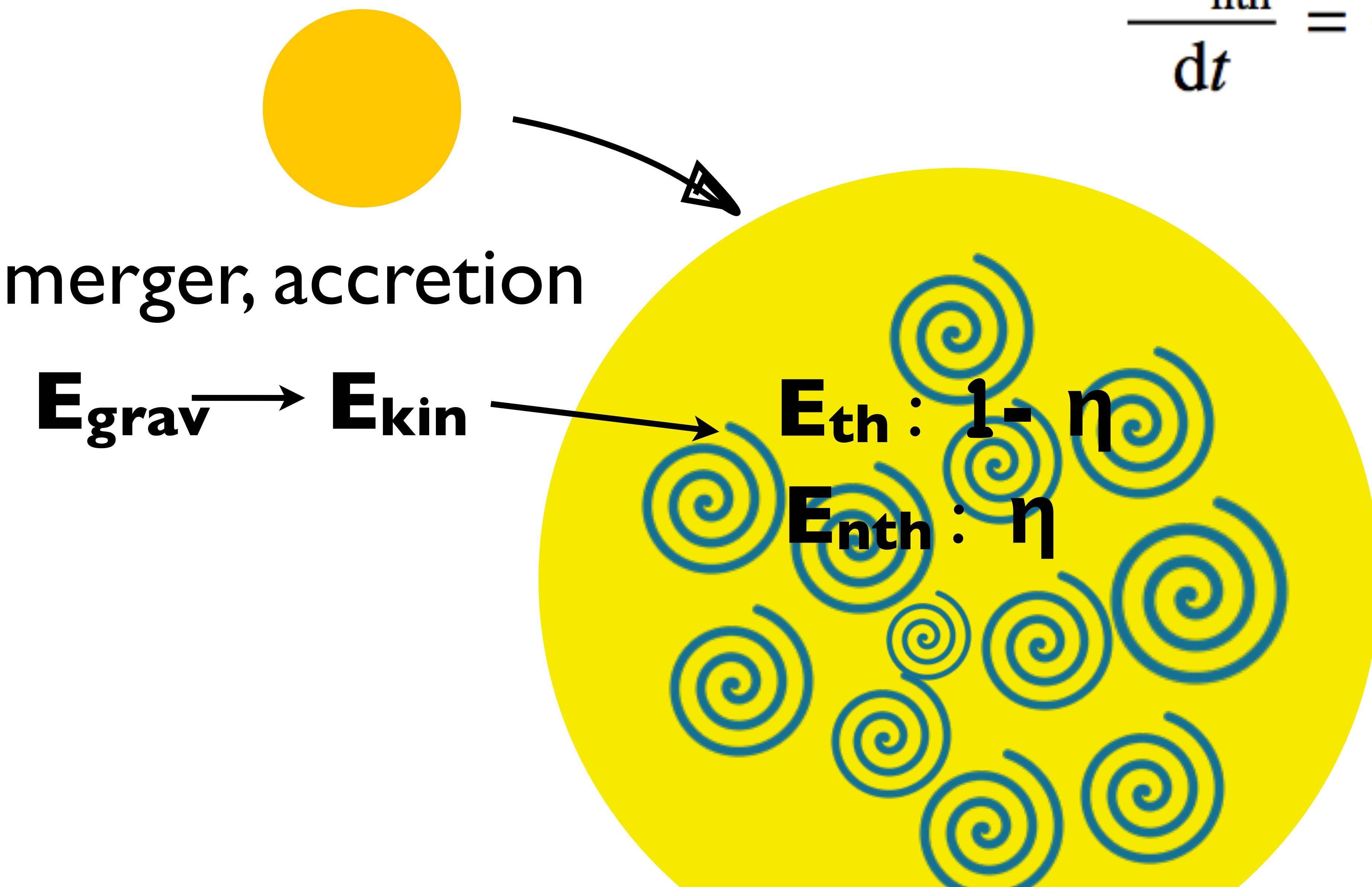
$\gamma c_s^2 \sim -\Phi$

$v \gtrsim cs$

weak shock

Generation of intracluster turbulence

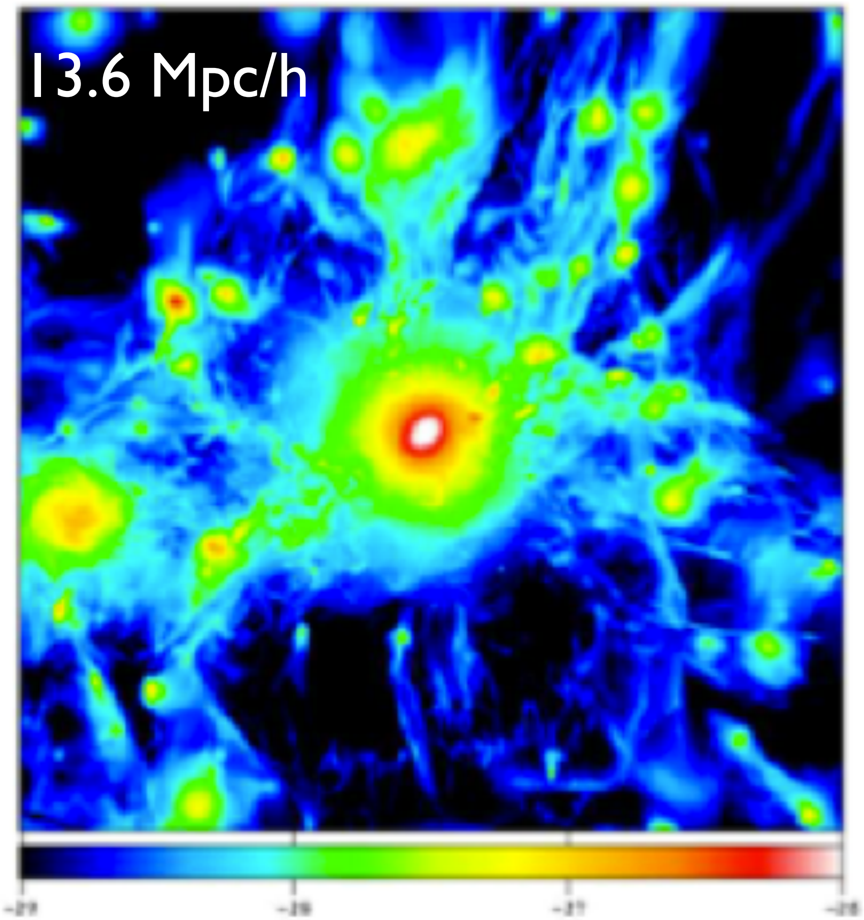
$$\frac{d\sigma^2_{\rm nth}}{dt} = -\frac{\sigma^2_{\rm nth}}{t_{\rm d}} + \eta\,\frac{d\sigma^2_{\rm tot}}{dt}$$



η: primarily determined by the Mach number

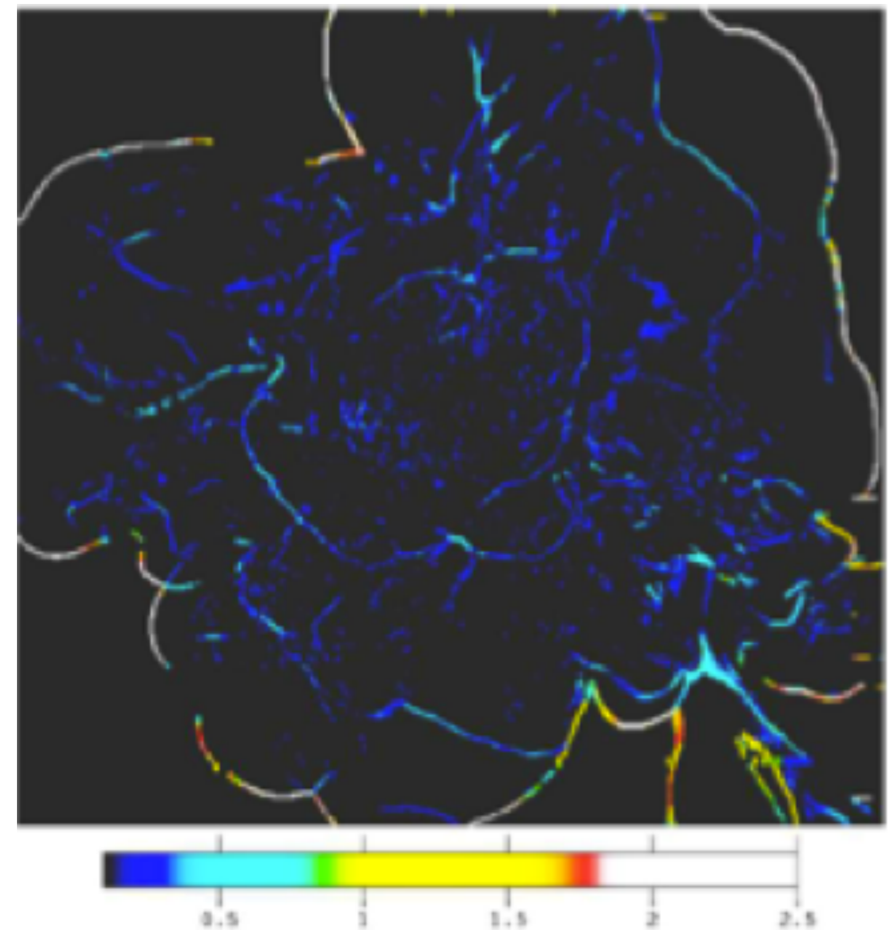
weak shock: η comparable to 1

Vazza et al 2010



1e-29 1e-26

gas density

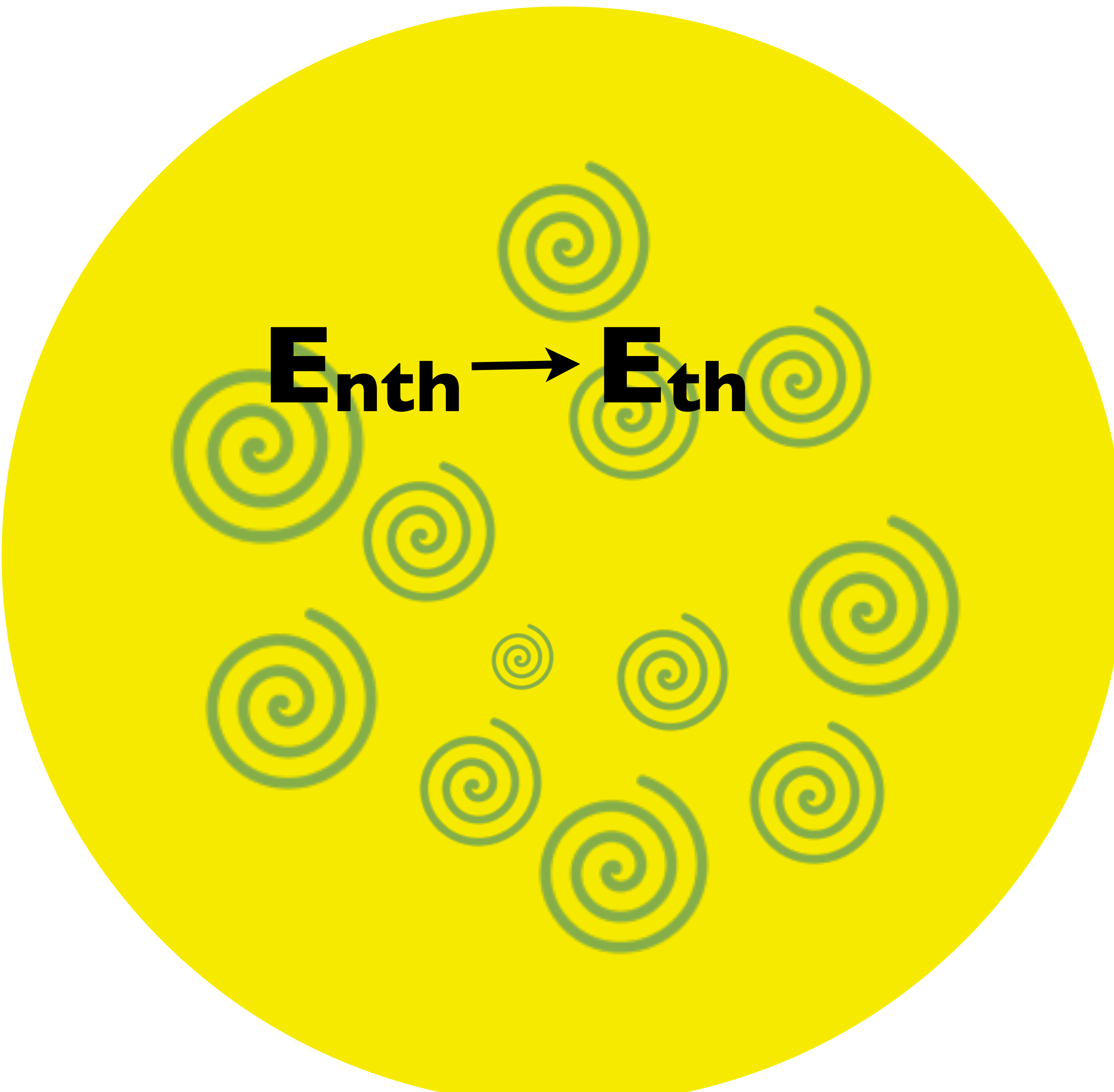


M=3 M=100

Mach number of shocked cells

see also Ryu+03, Pfrommer+06, Skillman+08, Vazza+09, 11, Schaal & Springel 14

Dissipation of intracluster turbulence



$$\frac{d\sigma_{nth}^2}{dt} = \boxed{-\frac{\sigma_{nth}^2}{t_d}} + \eta\,\frac{d\sigma_{tot}^2}{dt}$$

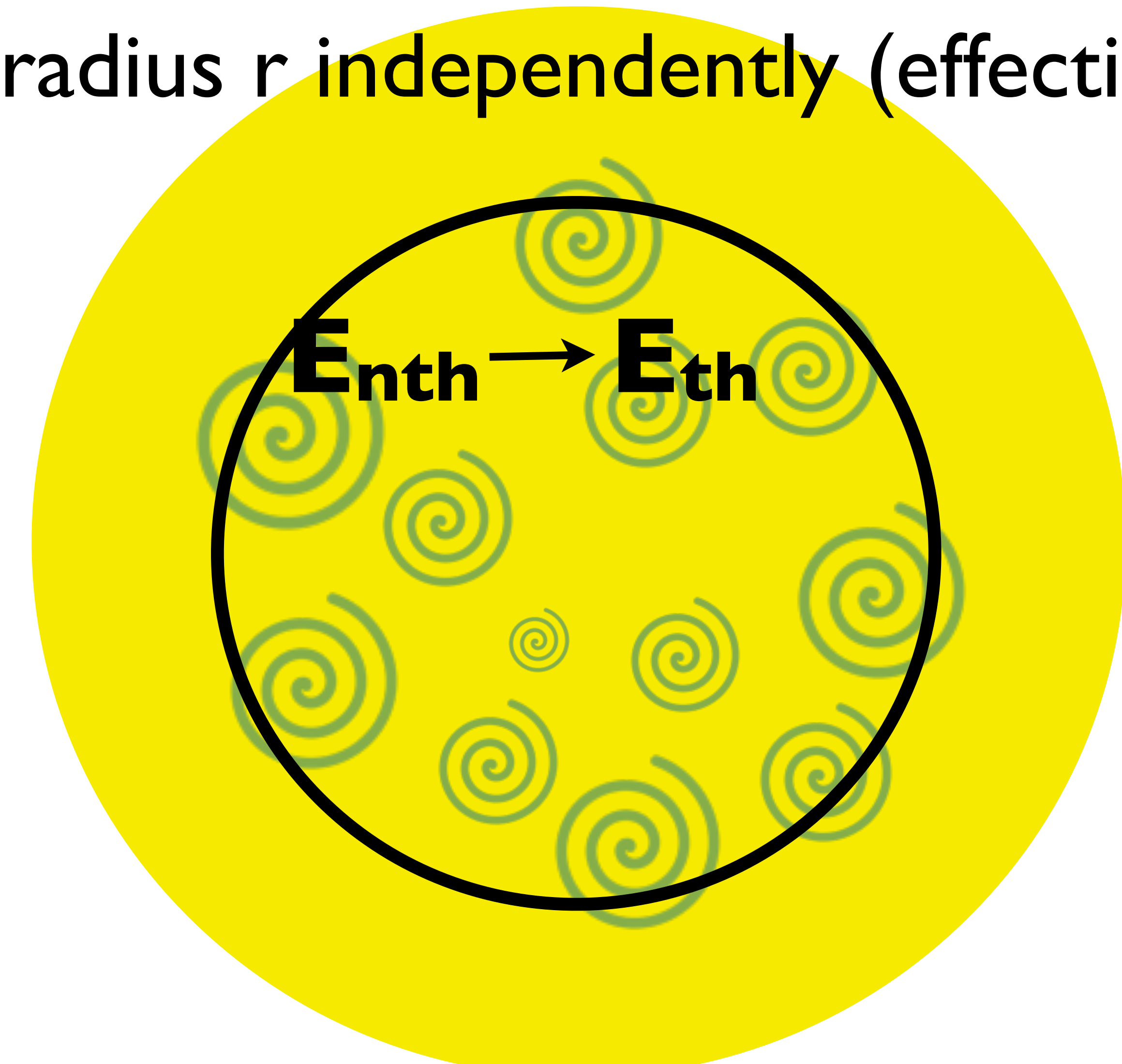
dissipation time scale t_d

$\propto$ eddy turn-over time of the largest eddies

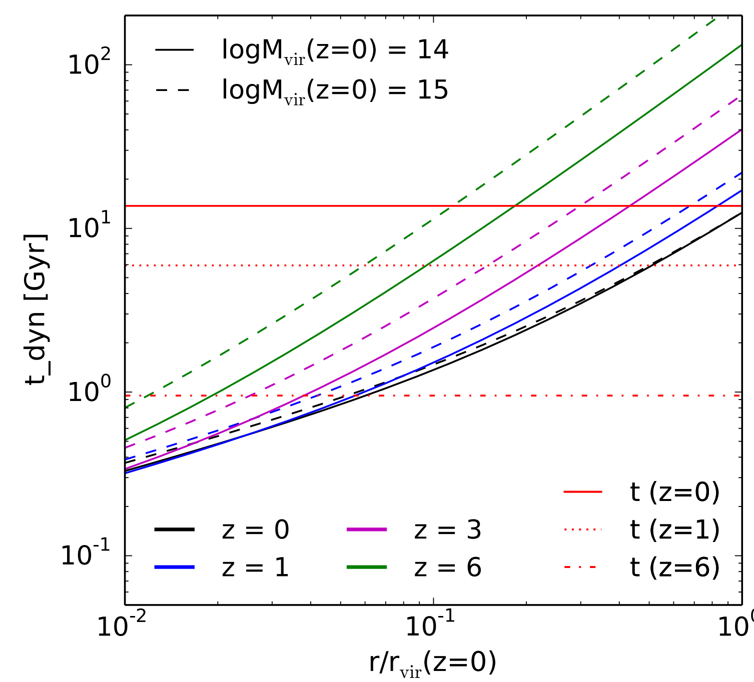
$\propto$ dynamical time t_{dyn}, $\quad t_d = \beta\, t_{dyn}/2$

Dissipation of intracluster turbulence

can be treated at each Eularian radius r independently (effectively)



$$\frac{d\sigma^2_{\rm nth}}{dt} = \boxed{-\frac{\sigma^2_{\rm nth}}{t_{\rm d}}} + \eta \frac{d\sigma^2_{\rm tot}}{dt}$$



t_{dyn} : steep function of r

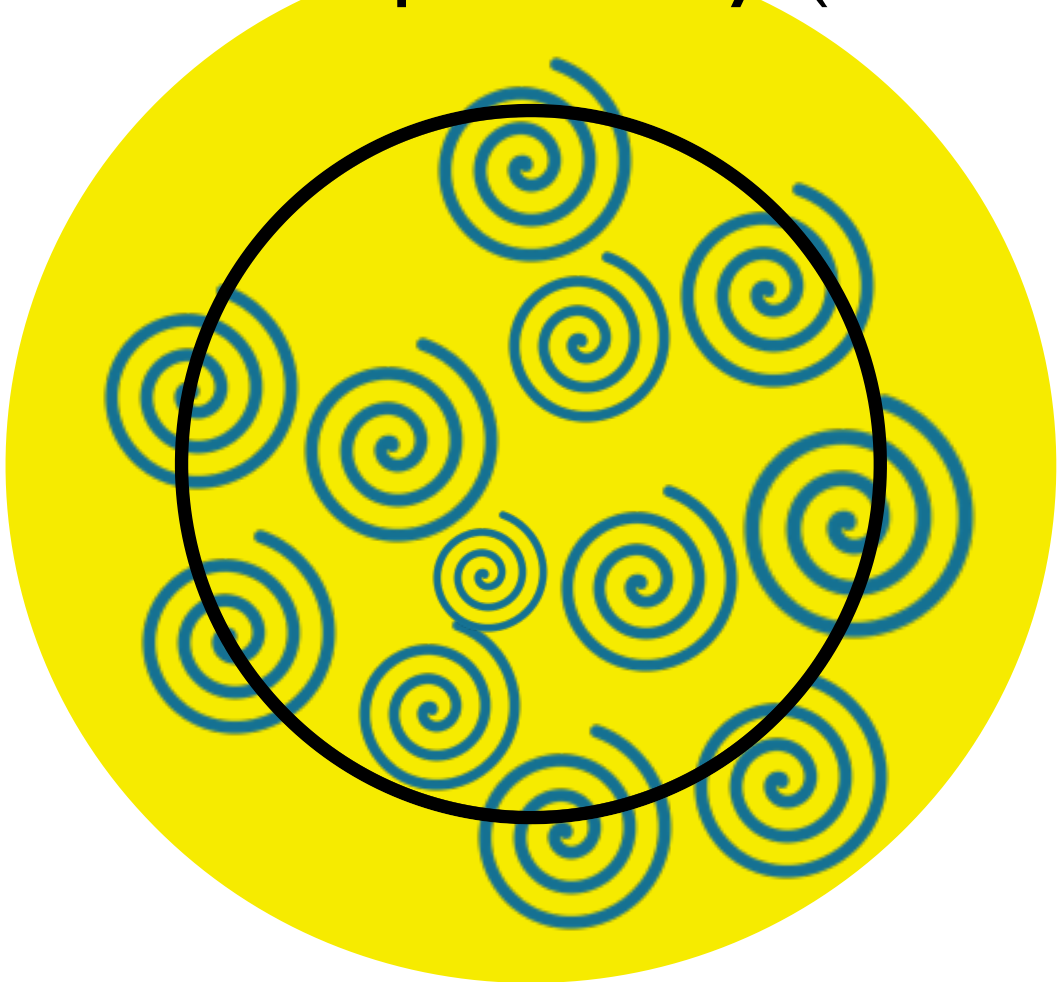
dissipation time scale t_d

$\propto$ eddy turn-over time of the largest eddies

$\propto$ dynamical time t_{dyn} , $\quad t_d = \beta\, t_{dyn} /2$

Turbulence injection, too,

can be treated at each Eularian radius r independently (effectively)



$$\frac{d\sigma_{\mathrm{nth}}^2}{dt} = -\frac{\sigma_{\mathrm{nth}}^2}{t_{\mathrm{d}}} + \eta \frac{d\sigma_{\mathrm{tot}}^2}{dt}$$

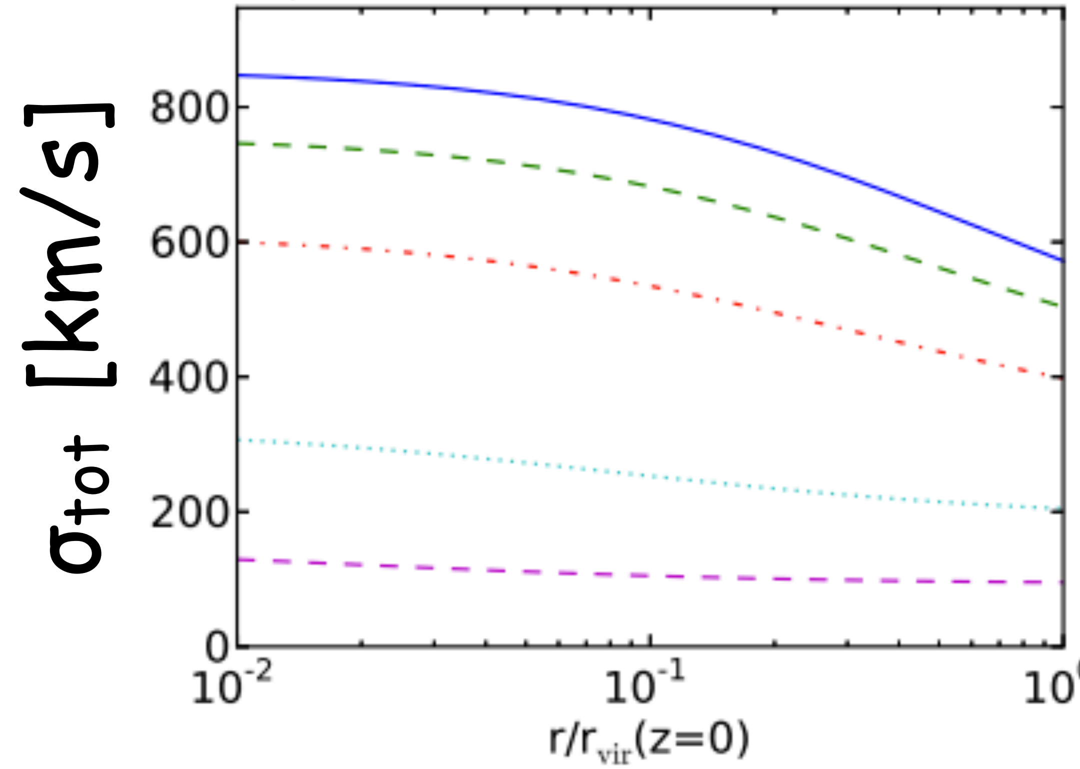
- radial distribution of injected energy: keep self-similarity

$$\Delta\sigma_{\mathrm{tot}}^2 \propto \sigma_{\mathrm{tot}}^2$$

Tracing σ_{nth}^2 along mass growth history of galaxy clusters

INPUT ~ $T^{1/2}$

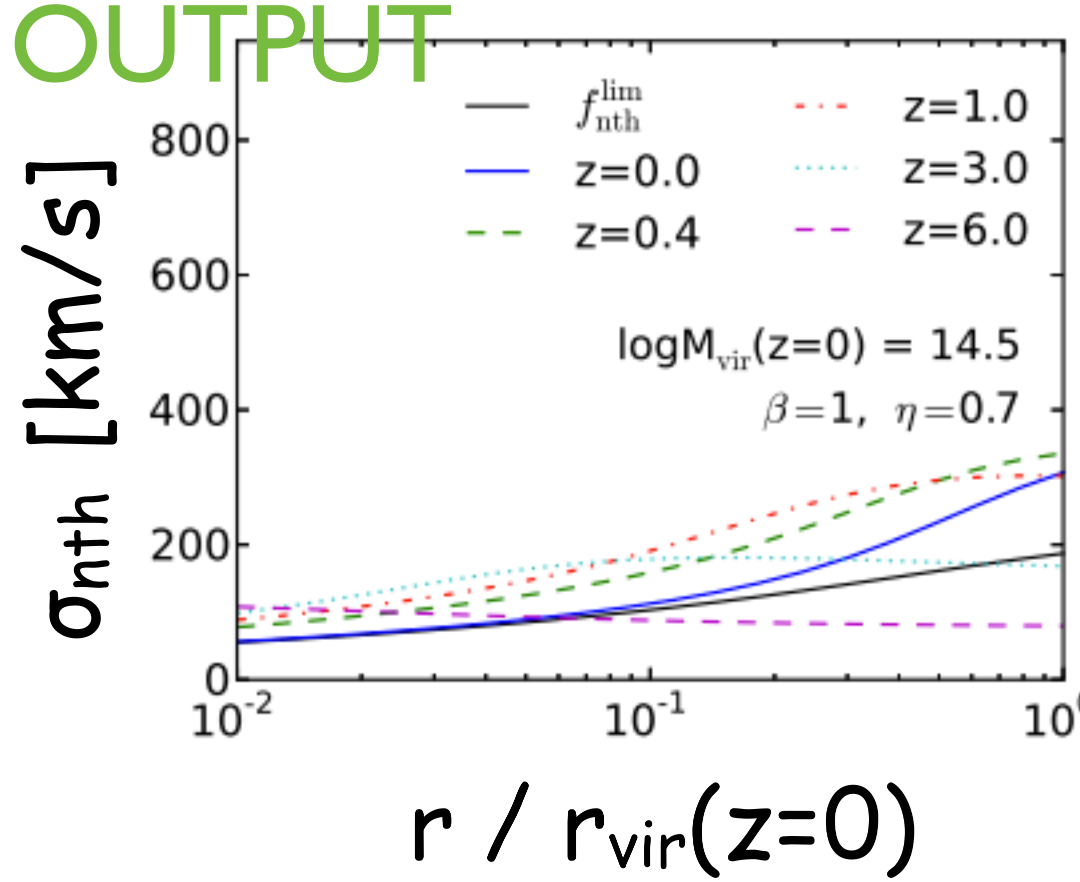
using average mass growth history in LanCDM (**Zhao+09**) and **Komatsu & Seljak 2001** to give $d\sigma_{tot}/dt$ analytically



$$\frac{d\sigma_{\rm nth}^2}{dt} = -\frac{\sigma_{\rm nth}^2}{t_{\rm d}} + \eta\,\frac{d\sigma_{\rm tot}^2}{dt}$$

OUTPUT

modeled σ_{nth} evolution



Tracing σ_{nth}^2 along mass growth history of galaxy clusters

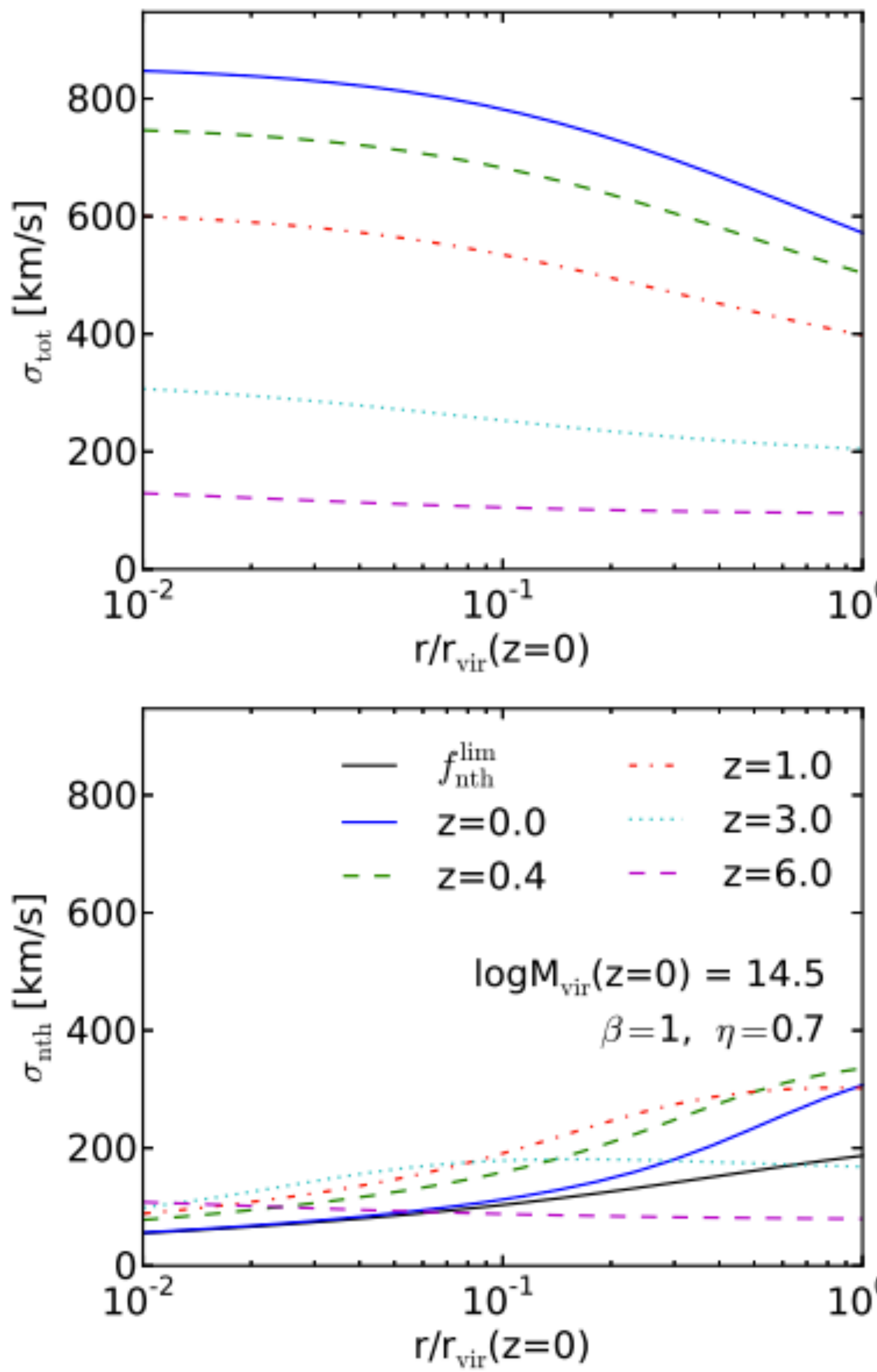
competition bet. injection & dissipation

time scales:

turbulence dissipation: $t_d = \beta\, t_{dyn}/2$;

cluster mass growth: $t_{growth} = \dfrac{\sigma_{tot}^2}{d\sigma_{tot}^2/dt}$;

$$\frac{d\sigma_{nth}^2}{dt} = -\frac{\sigma_{nth}^2}{t_d} + \eta\,\frac{d\sigma_{tot}^2}{dt}$$



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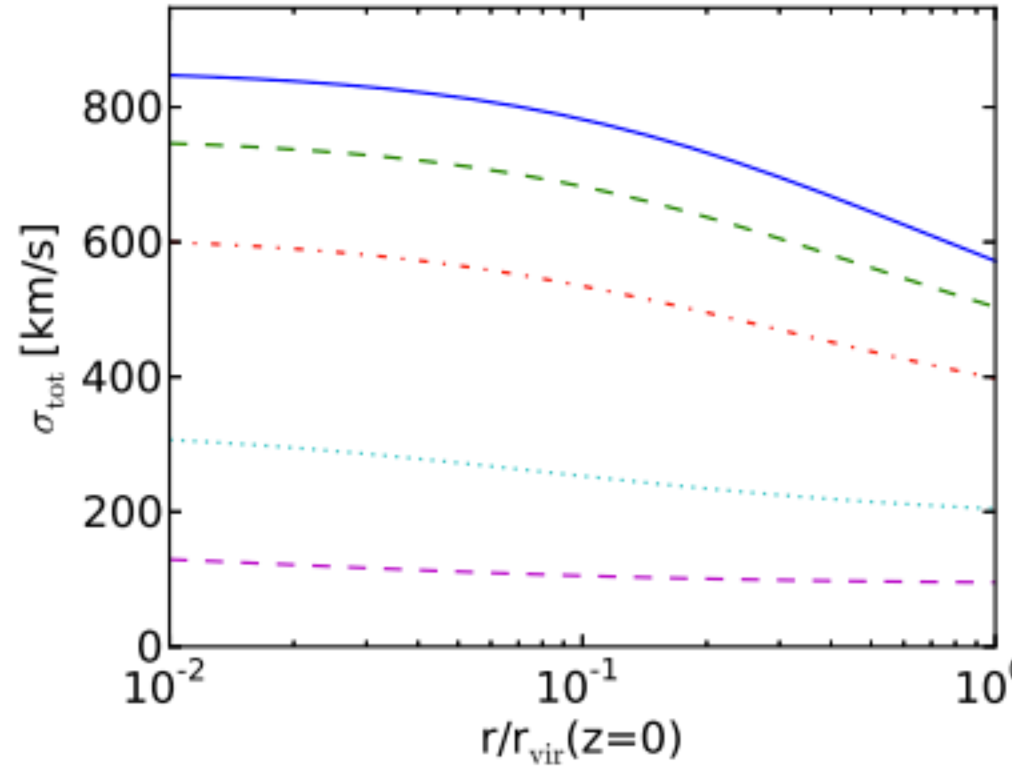
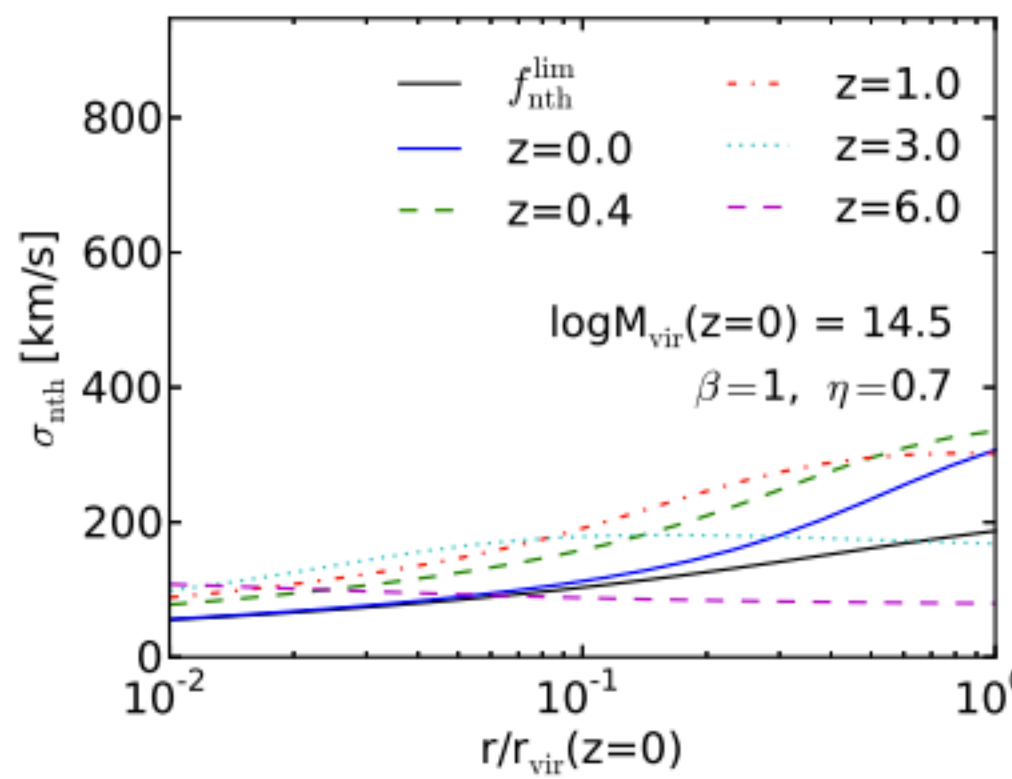
$$\frac{d\sigma^2_{nth}/dt}{d\sigma^2_{tot}/dt} = \eta - \frac{\sigma^2_{nth}}{\sigma^2_{tot}}\frac{t_{growth}}{t_d}$$

attractor of f_{nth} at $\dfrac{d\sigma^2_{nth}/dt}{d\sigma^2_{tot}/dt} = \dfrac{\sigma^2_{nth}}{\sigma^2_{tot}}$

$$f^{lim}_{nth} = \eta\frac{t_d}{t_d + t_{growth}}$$

non-thermal fraction $f_{nth} \doteq P_{nth}/P_{tot} = \sigma_{nth}^2/\sigma_{tot}^2$





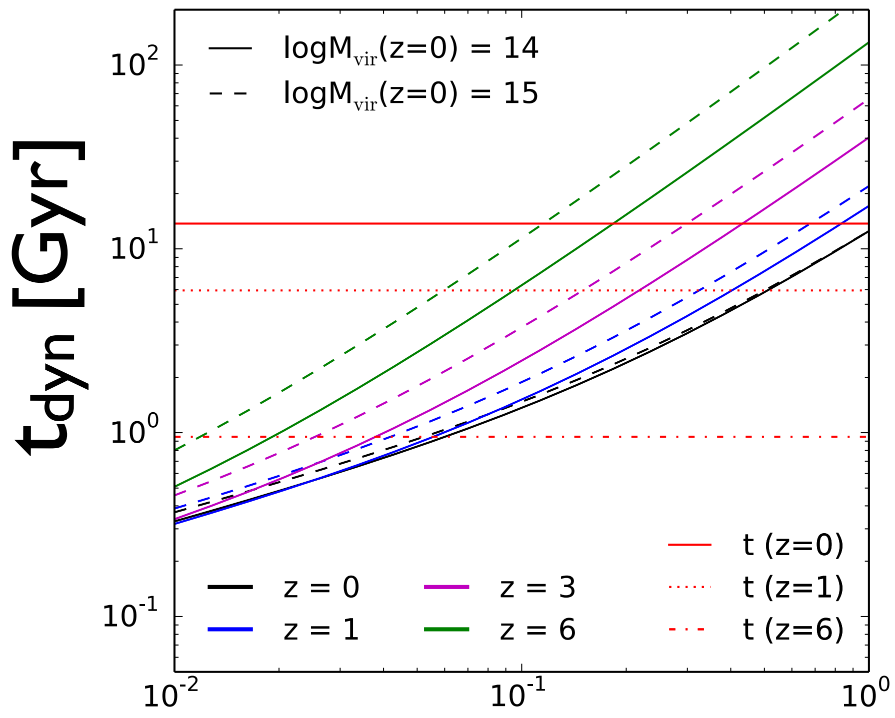
$f_{nth} \Rightarrow f_{nth}^{lim}$ when

$\Delta t > \min(t_d, t_{growth}) \times$ a few

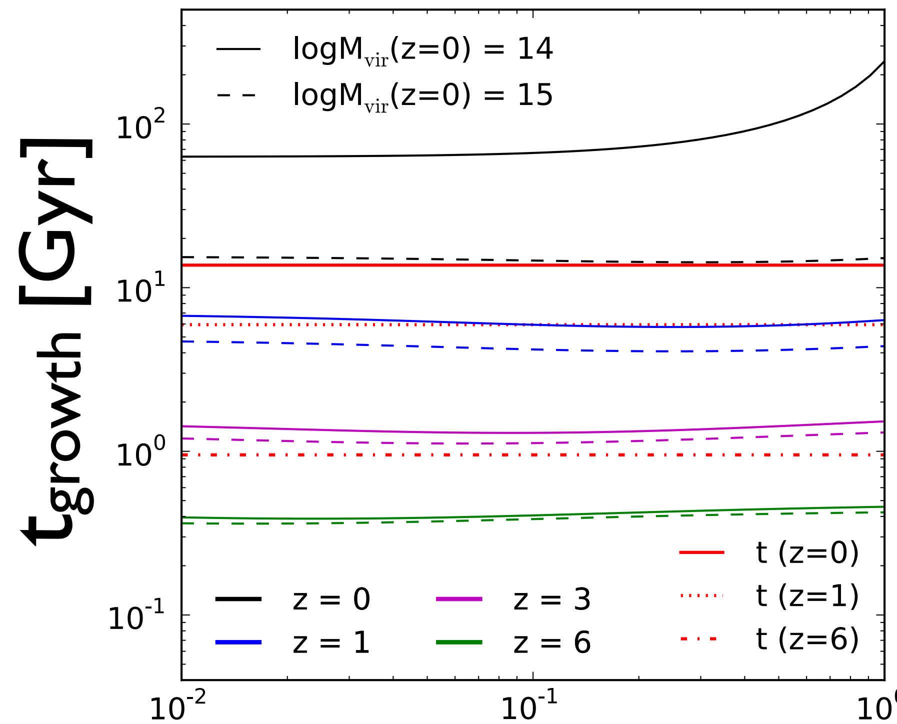
time scales: dependencies:

$t_d \longrightarrow r$

$t_{growth} \longrightarrow M, \ z$



$r \ / \ r_{vir}(z=0)$



$r \ / \ r_{vir}(z=0)$

prediction: $f_{nth} = P_{nth}/P_{tot}$ increases with r (slower dissipation), M & z (faster growth), or equivalently, r & accretion rate (hence HSE mass bias, too)

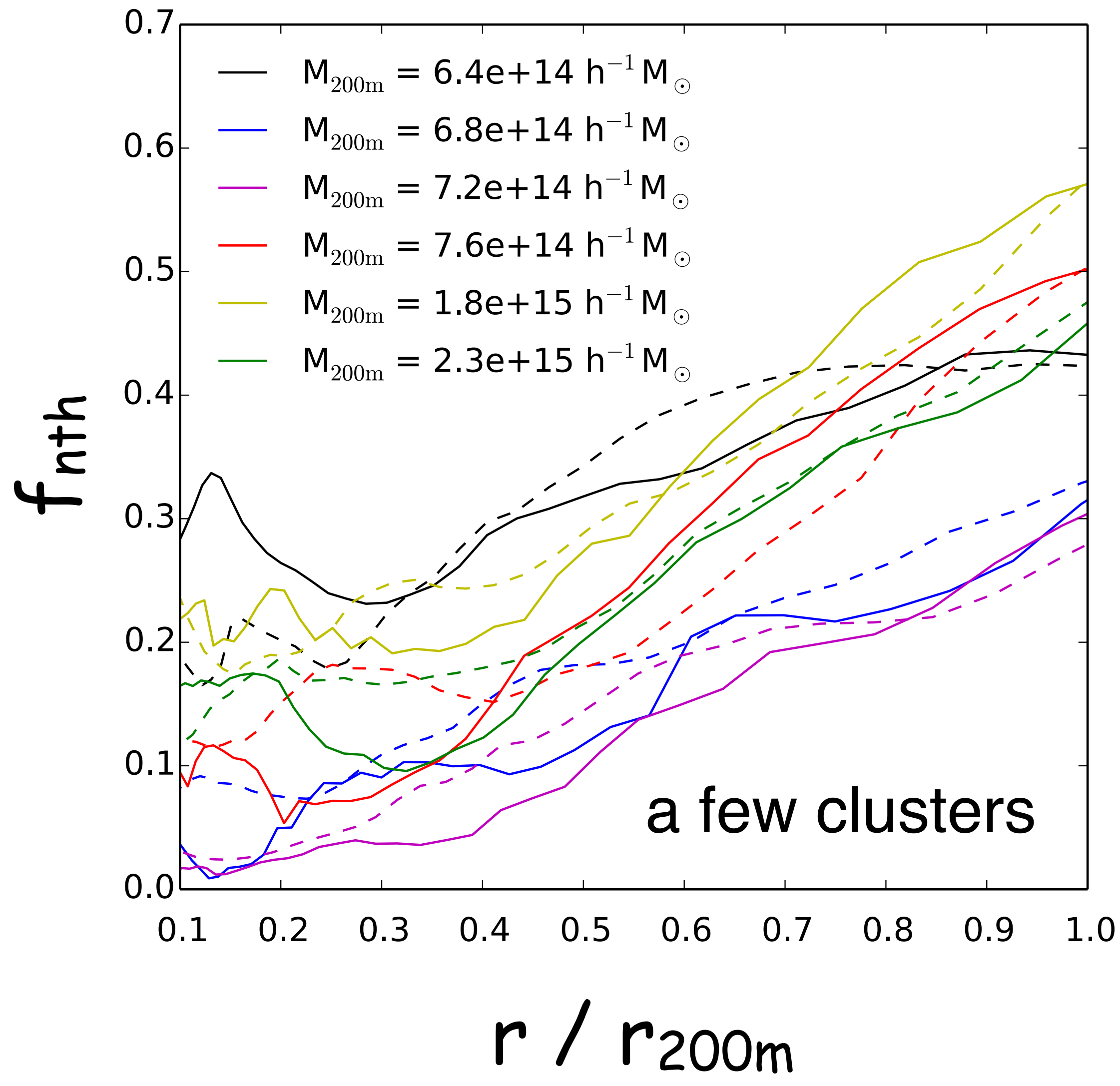
z & M dependencies strongly reduced when scale with r200m. cf. Kaylea's talk

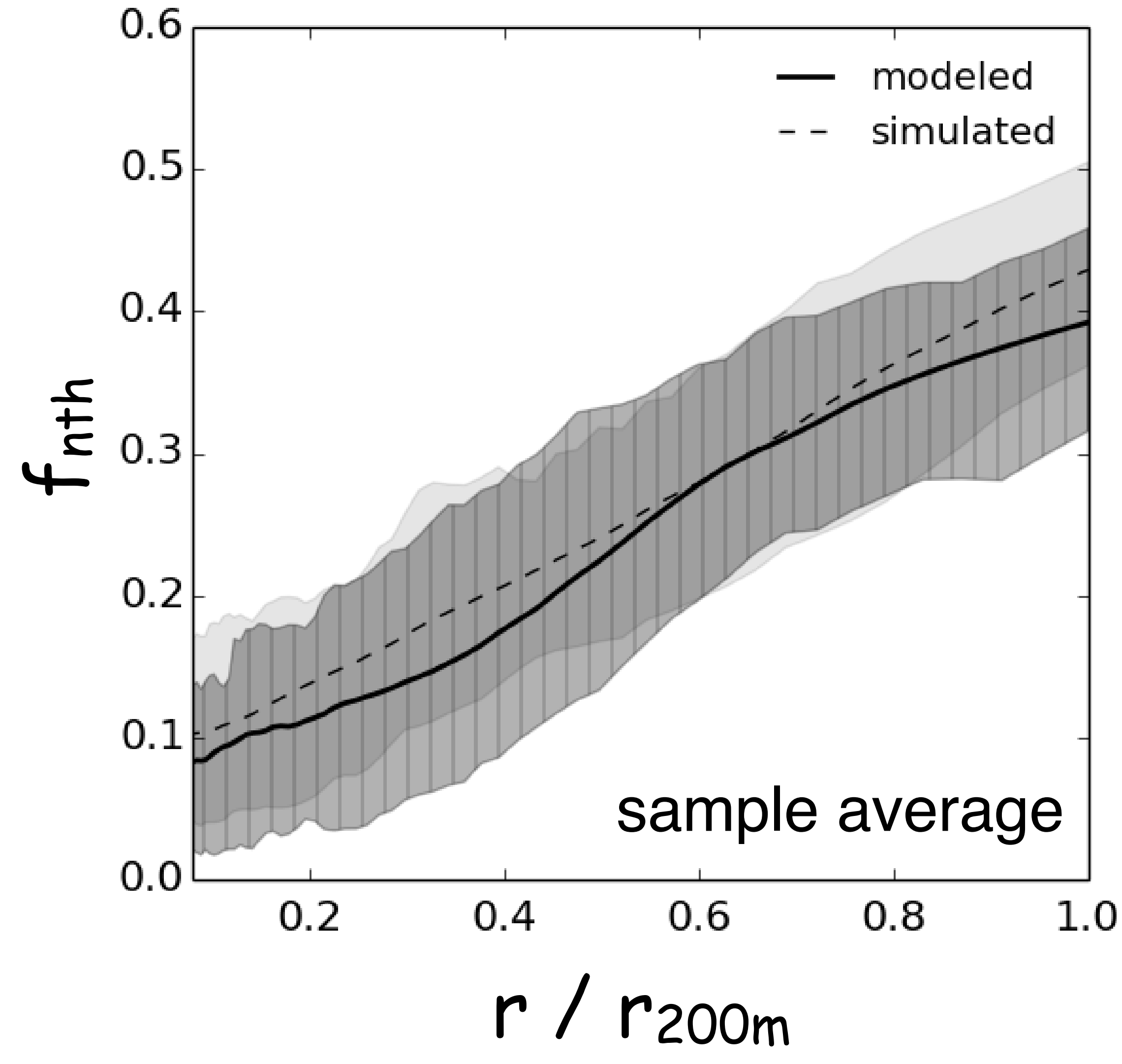
Non-thermal fraction vs simulations

a mass-limited sample of 65 simulated clusters at z=0

Omega500 simulation (Nelson et al 2014)

use $\sigma_{tot}(r,t)$ from simulation to model the non-thermal fraction





reproduce the variation among clusters

both mean & scatter match

note: not all relaxed

Shi, Komatsu, Nelson, Nagai, in prep

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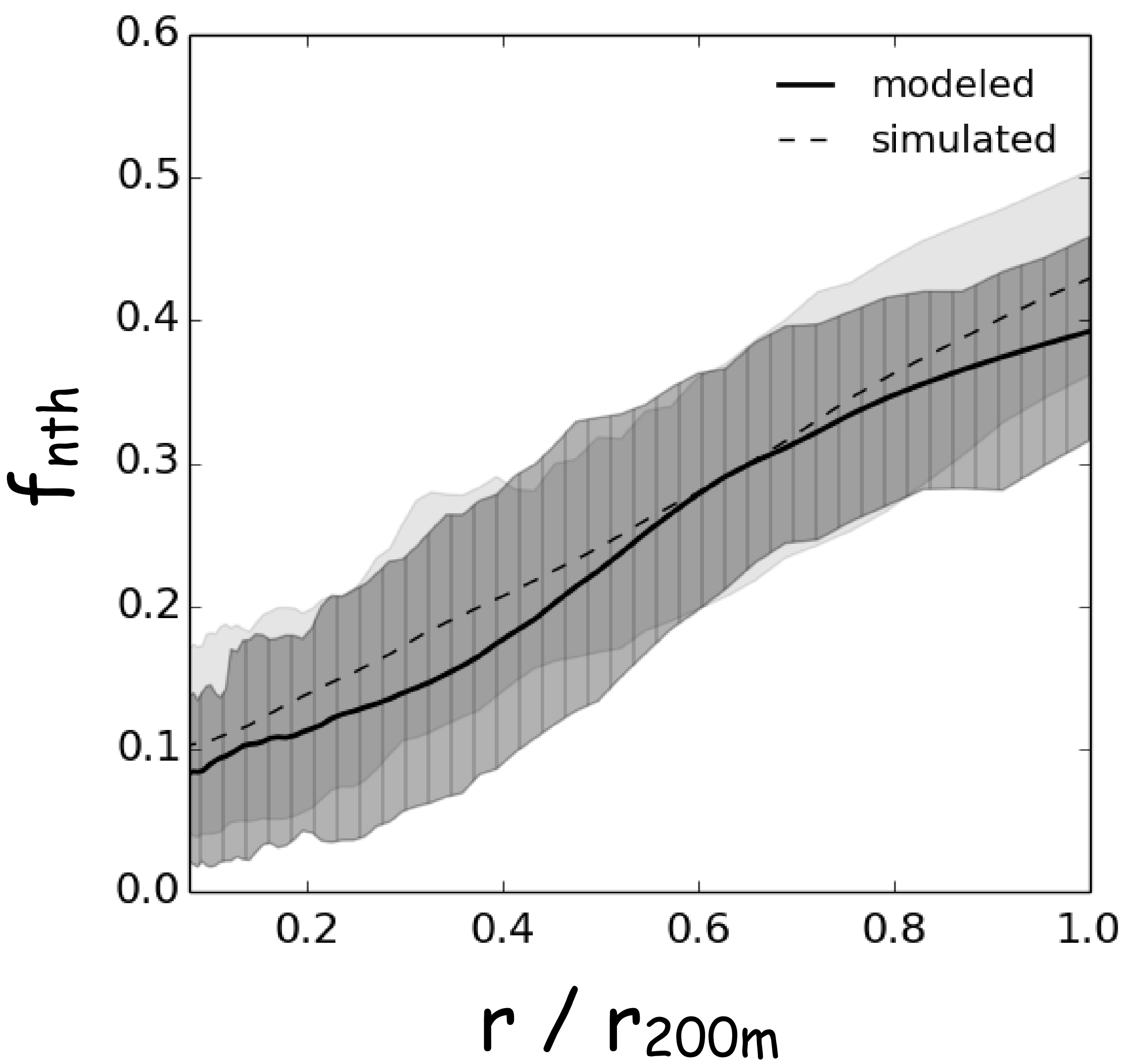

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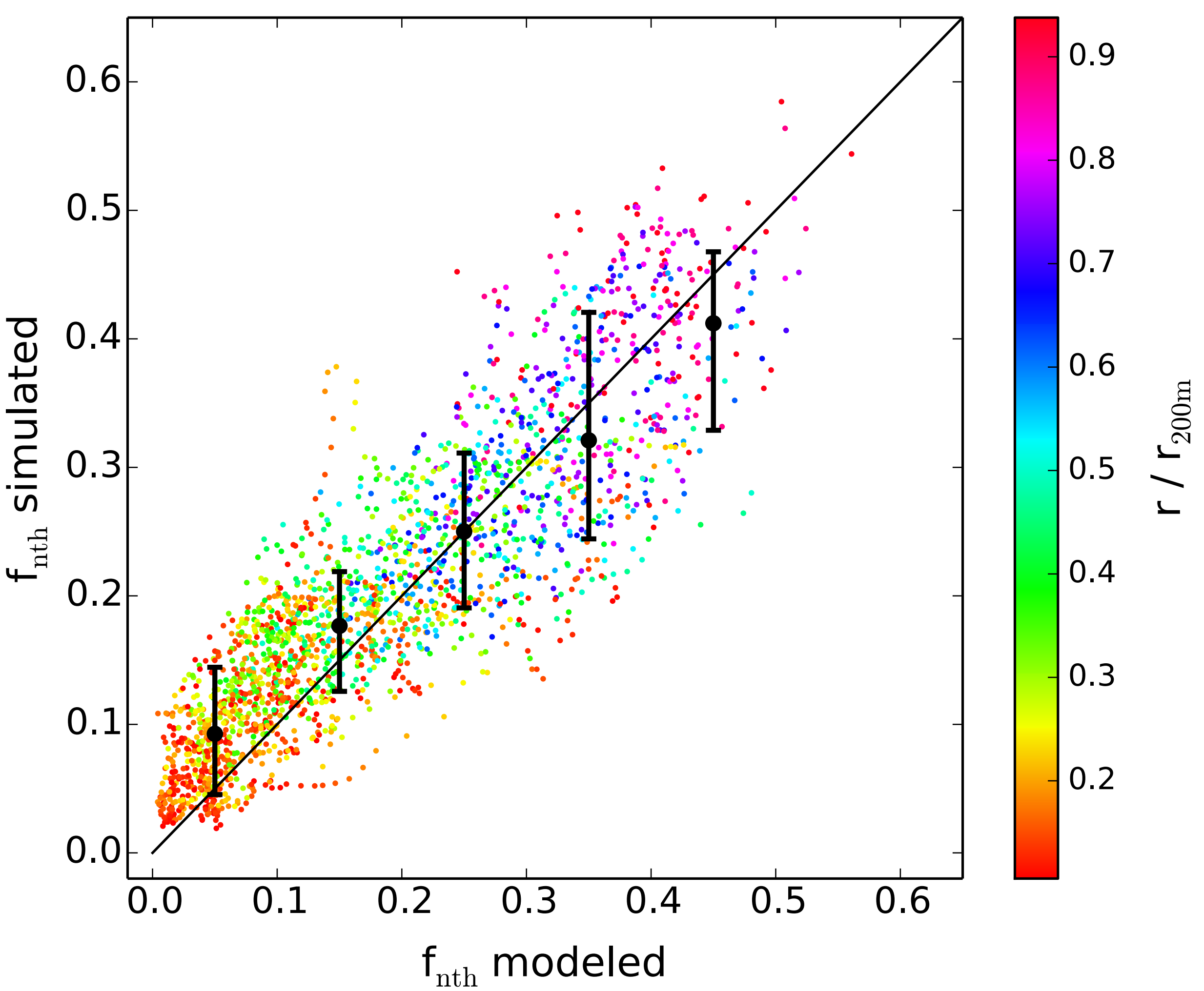
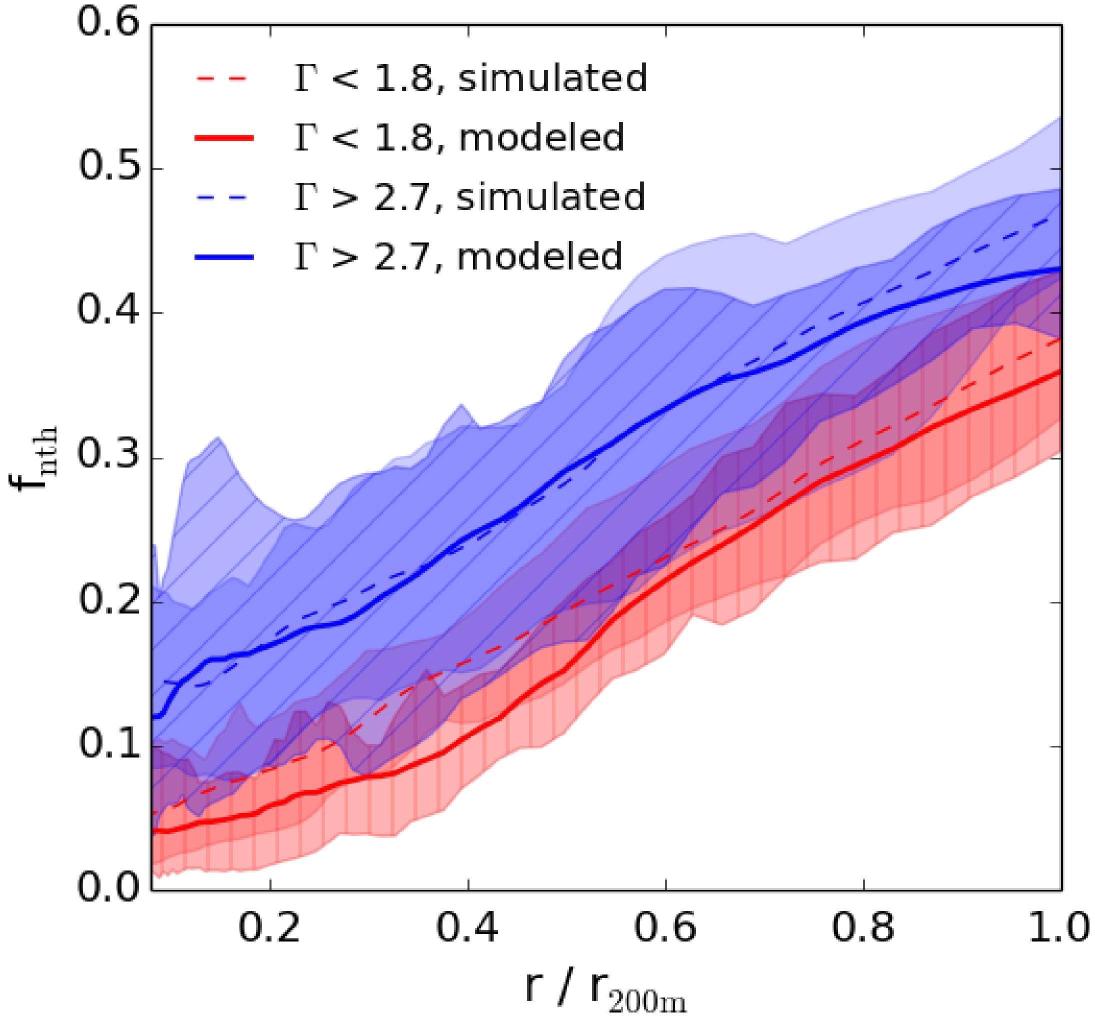


each color point: a radial bin of one cluster

Shi, Komatsu, Nelson, Nagai, in prep

Effect of recent accretion history / dynamical state



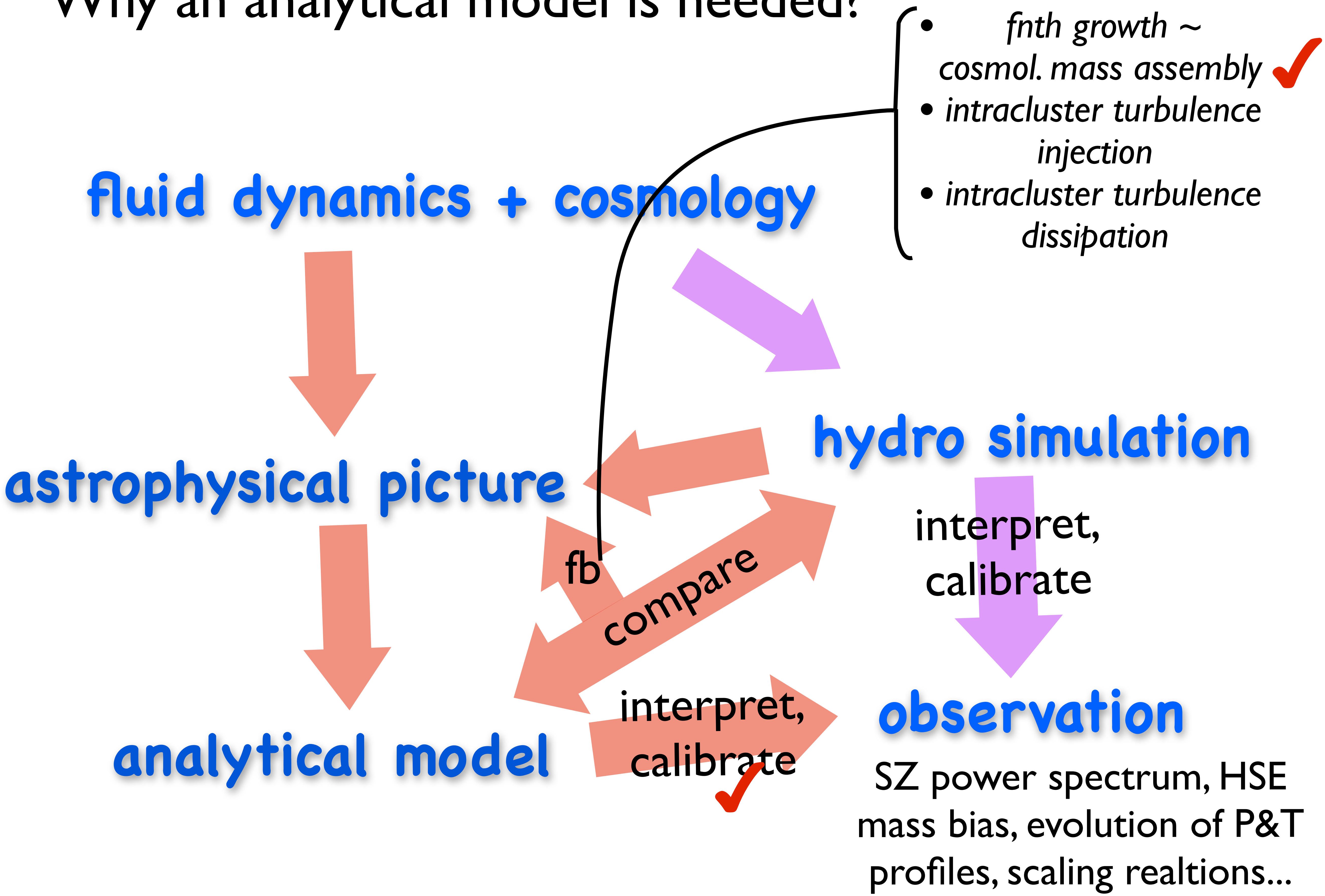
2 sub-samples at z=0

high mass growth since z=0.5

low mass growth since z=0.5

difference in f_{nth} successfully reproduced;
confirms the relation bet. f_{nth} & accretion history

Why an analytical model is needed?



Evolution of intracluster turbulence: *towards a better picture*

1d model

$$\frac{d\sigma^2_{nth}}{dt} = -\frac{\sigma^2_{nth}}{t_d} + \eta \frac{d\sigma^2_{tot}}{dt}$$

$t_d = \beta\, t_{dyn}/2$:

1-1 relationship between l_0/v_0 & r

→ exponential decay

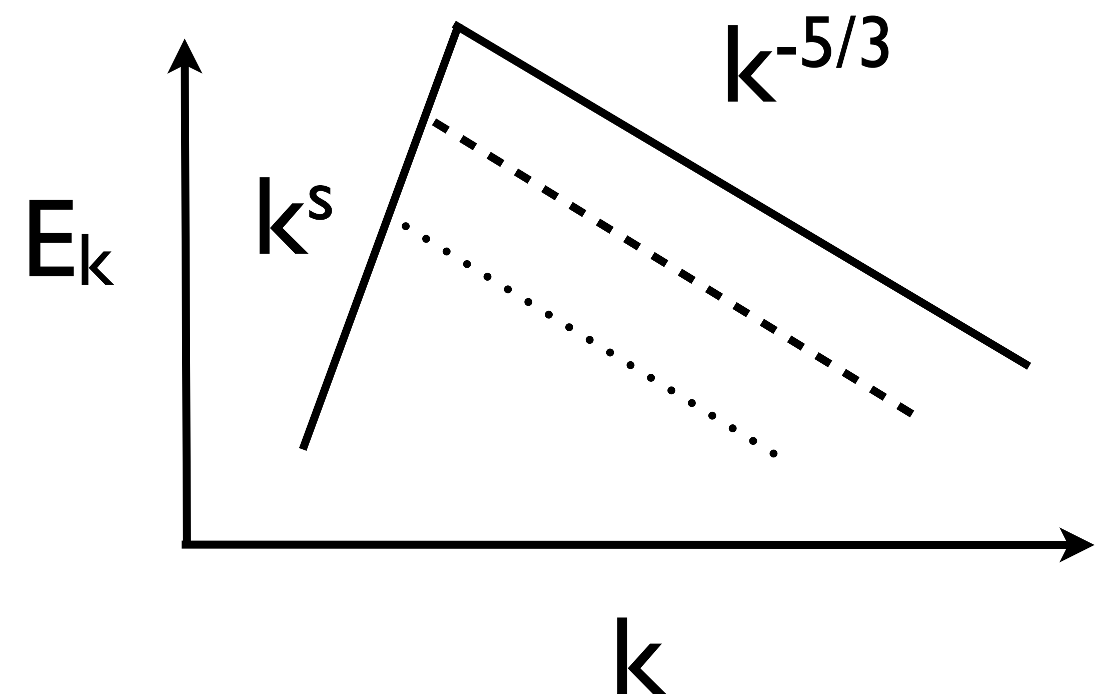
(affect where $t_d \ll t_{growth}$ i.e. at small radii)

1+1d model

$$\frac{d\sigma^2_{nth}}{dt}(r, k)$$

link to magnetic field, cosmic rays

... towards a unified picture of non-thermal phenomena? *need more detailed knowledge abt intracluster turbulence injection & cascade*



$$E \sim t^{-2(s+1)/(s+3)}$$

$$l_0 \sim t^{2/(s+3)}$$

Subramanian+06

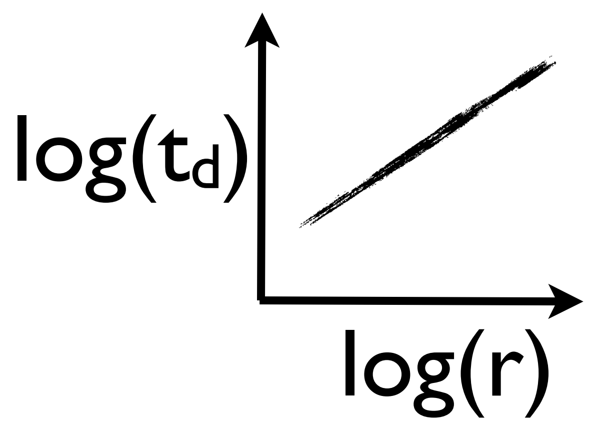
power law decay

(Landau & Lifshitz 59, Frisch95)

+ multiple injections

Conclusions

physical motivated 1d model for non-thermal pressure without free parameters

* 

$\Rightarrow$ larger f_{nth} at large r

* higher t_{growth} rate at higher z and M $\Rightarrow$ larger f_{nth} at high z and M

* current form can already be used to interpret data @ cluster outskirts, and effectively correct for turbulence-induced HSE mass bias

* $\sigma^2_{nth}(r) \longrightarrow \sigma^2_{nth}(r, k)$

* values of η and β from dedicated simulations?

+

Neglected in this model

- acceleration/rotation/streaming motion

  can contribute at the outskirts of some clusters

- magnetic field pressure

  μG field detected in few cases. origin & universality & P_B uncertain

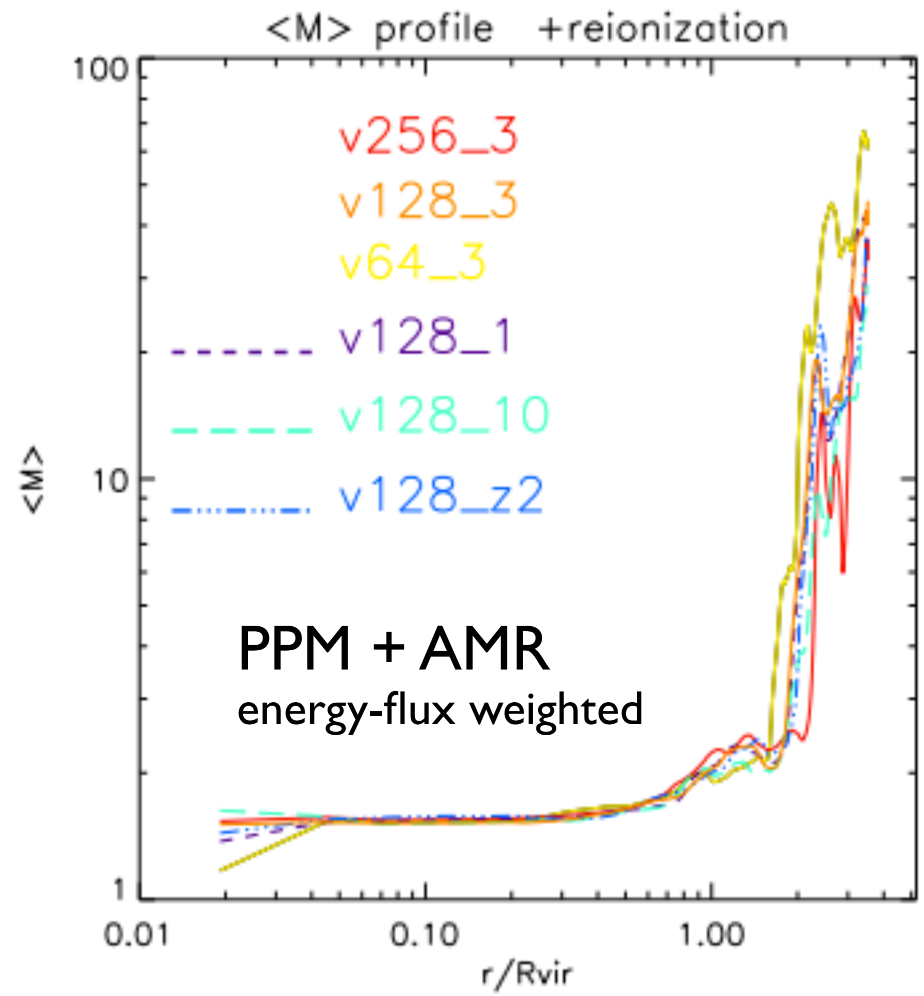
- cosmic ray pressure

  γ-ray signature of cosmic ray ions not yet detected

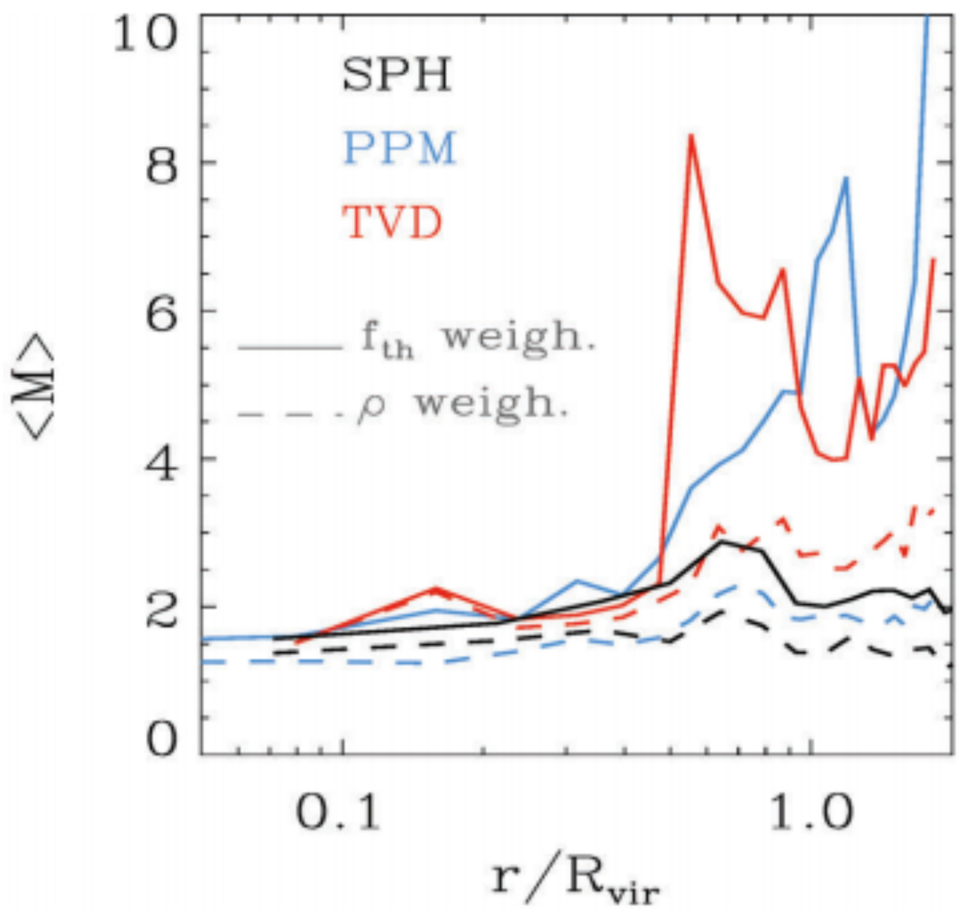
- possible radius & redshift dependencies of η and β

  should not be significant compared to dependencies of the modeled f_{nth}

Radius dependence of the Mach number



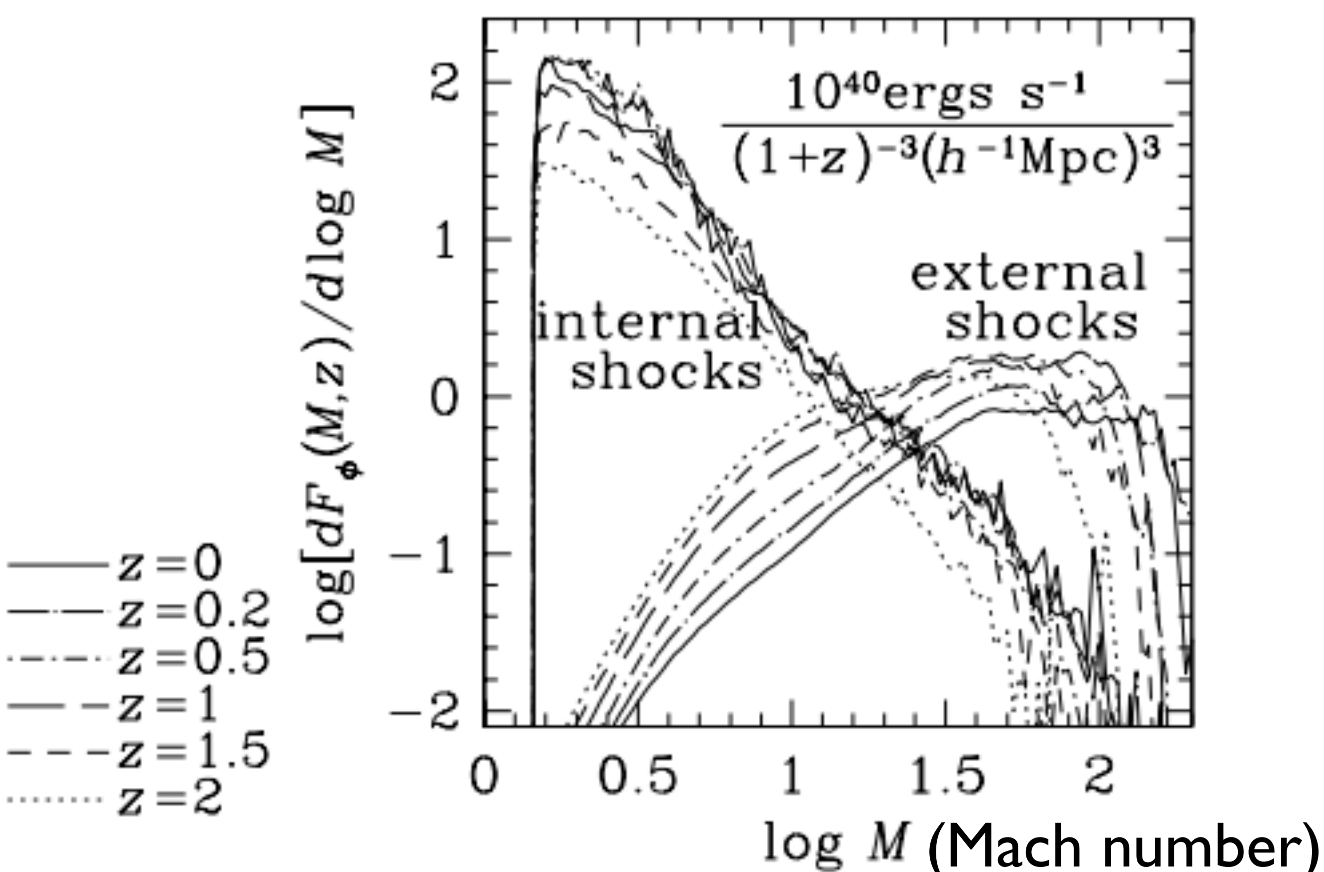
Vazza+09



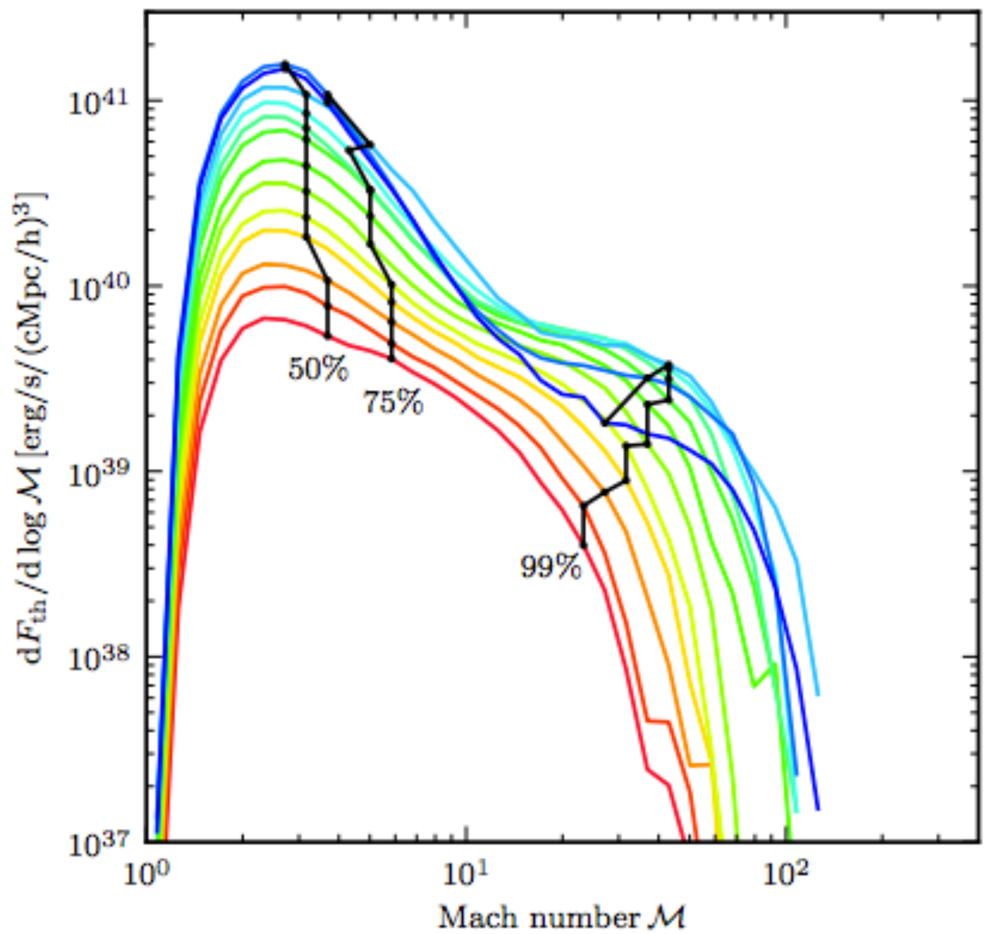
Vazza+11

Low Mach number internal shocks process more kinetic energy

kinetic energy flux



Ryu+03



Schaal & Springel 14

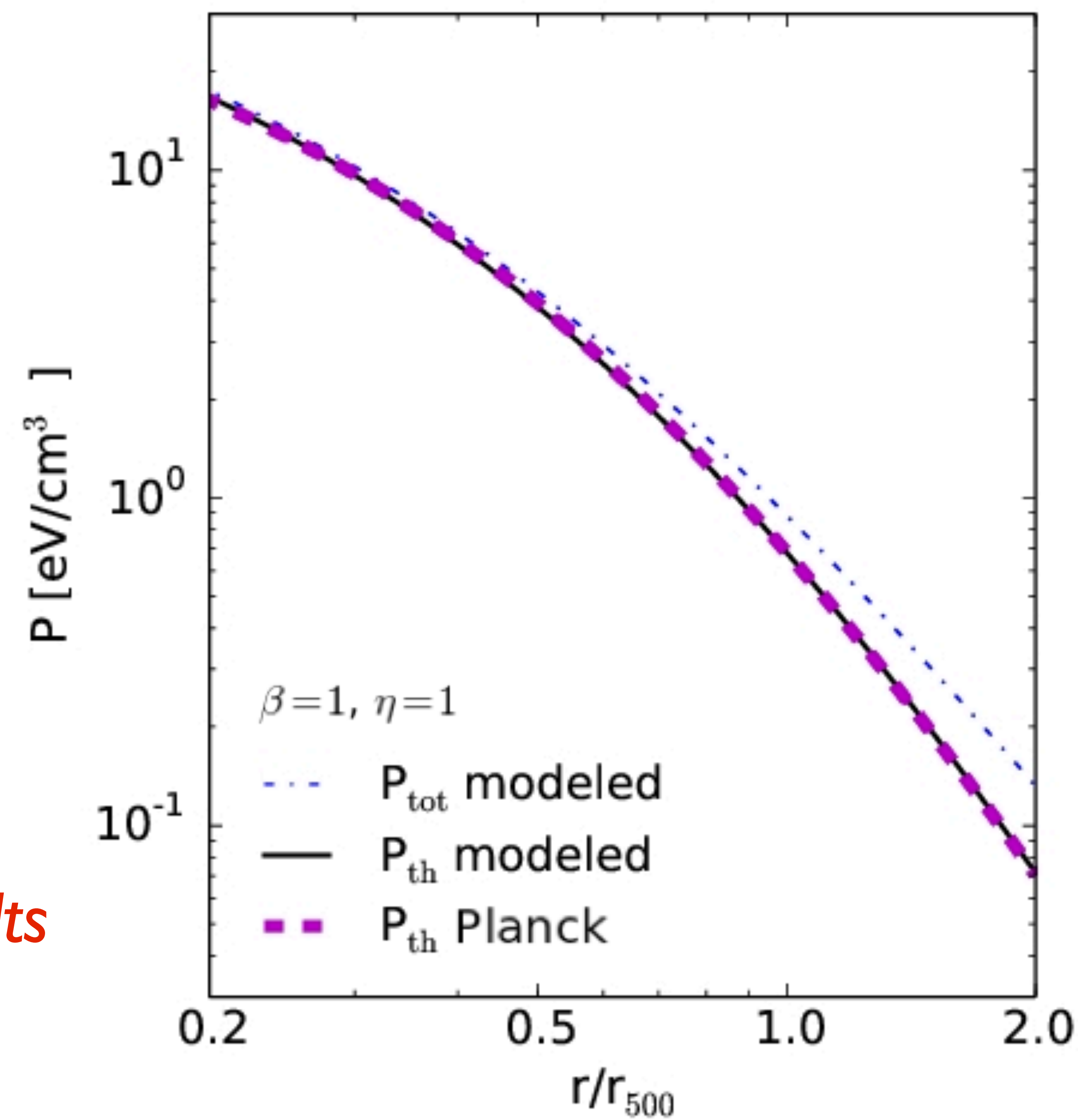
Thermal pressure vs observations

- Planck team 2013:
  SZ (joint fit with X-ray)

- Ptot model:
  Komatsu & Seljack 2001

- Pth model:
  Ptot - Pnth model

consistent with Planck results



Total pressure profile

$$\frac{1}{\rho_{\rm gas}}\frac{{\rm d}P_{\rm tot}}{{\rm d}r} = -\frac{{\rm d}\Phi(r)}{{\rm d}r}$$

NFW

hydrostatic equilibrium

polytropic EoS: $P_{\rm tot} \propto \rho_{\rm g}^{\Gamma}$

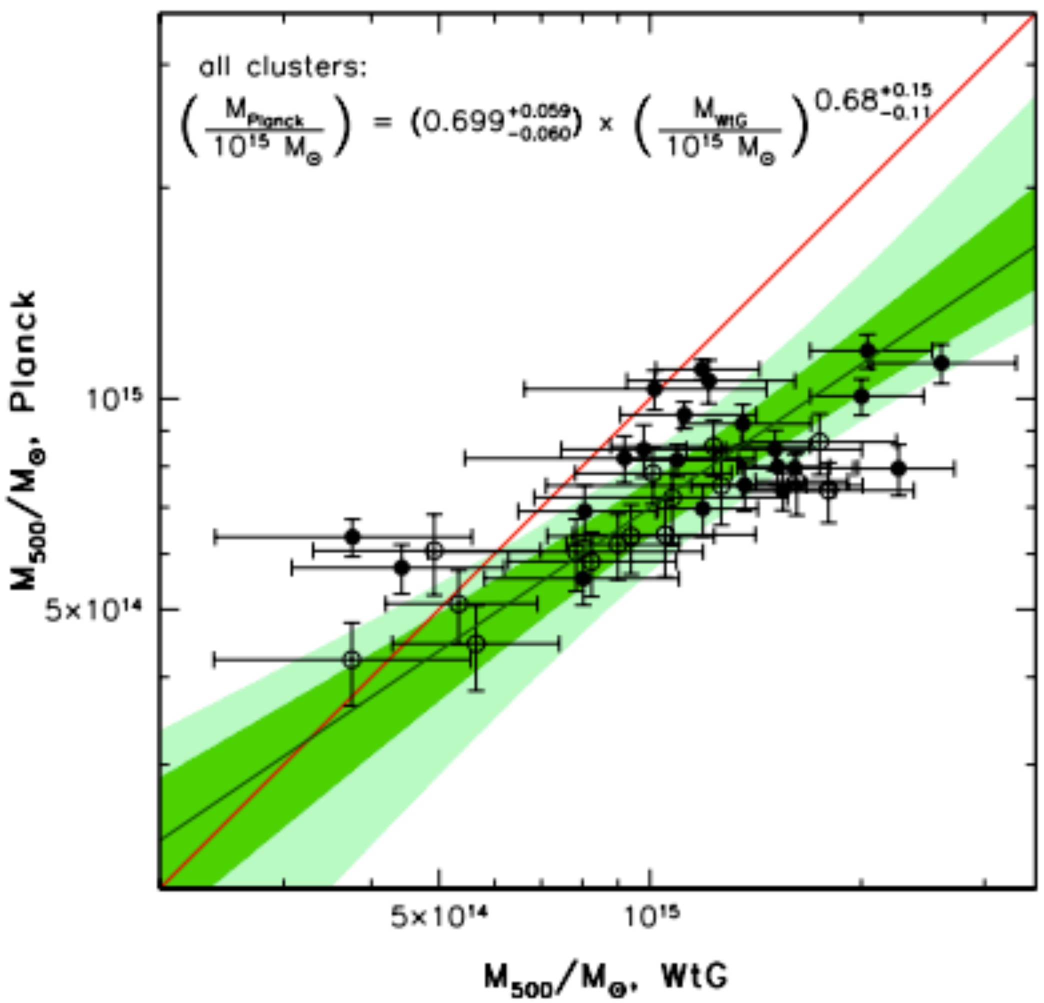
additional assumption: gas density profile traces the dark matter density profile in outer parts of the haloes

Komatsu & Seljak 01

P_{nth} in observations

Bias between X-ray mass/ SZ mass & lensing mass
(e.g. Allen98, Mahdavi+08, Zhang+10, von der Linden+14, Israel+14)

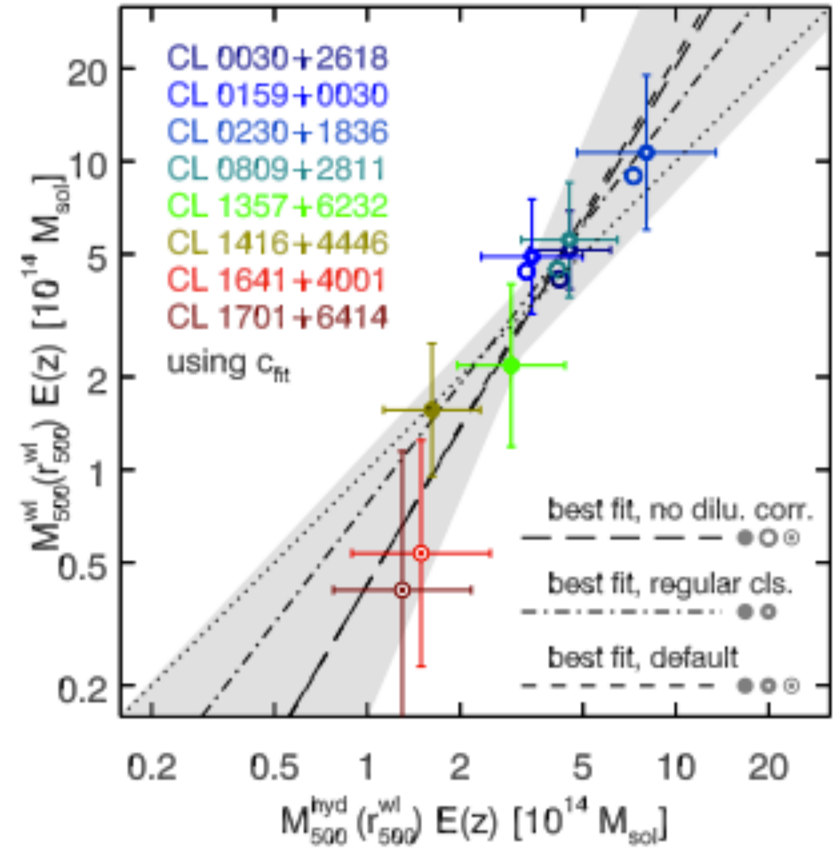
SZ mass

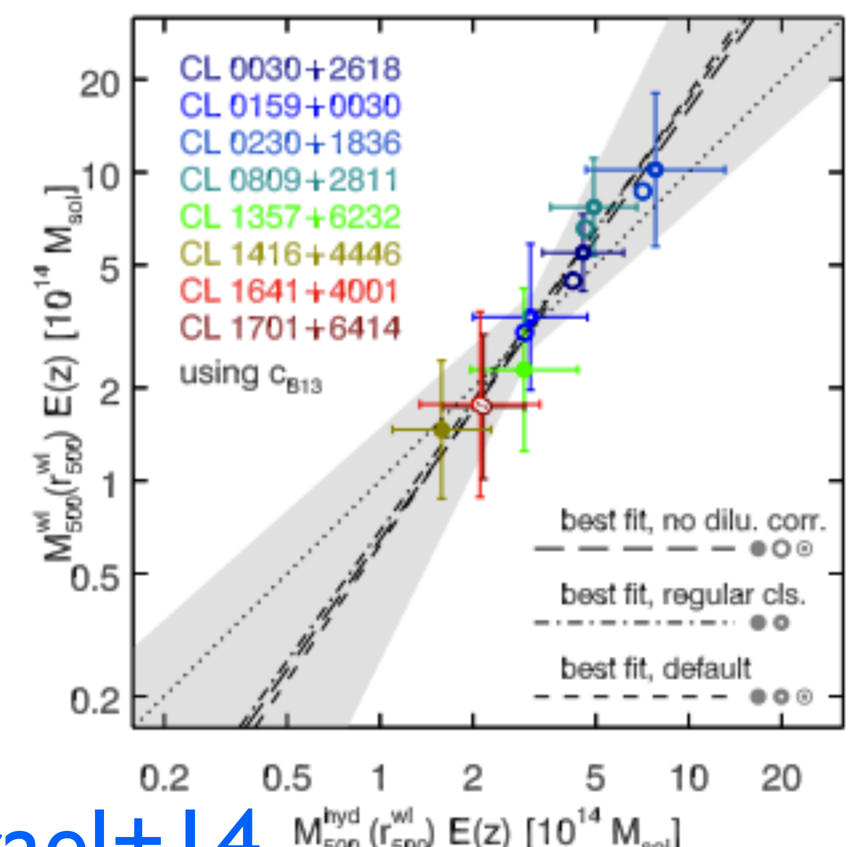


weak lensing mass

von der Linden+14

weak lensing mass





Israel+14

X-ray mass