The Generation and Evolution of Turbulence During Cluster Formation

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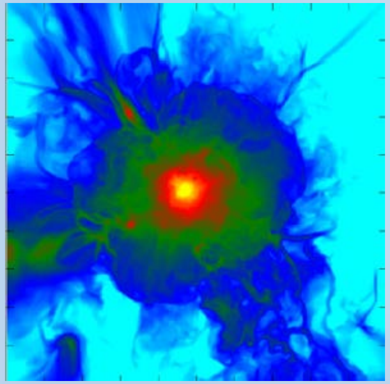
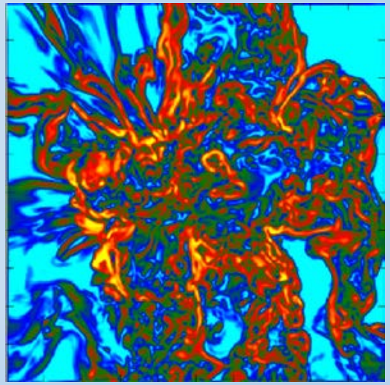




Set of 27 Cluster Formation Simulations Designed to Study Origins & Evolution of ICM Turbulence

(--Work in Progress– RESULTS PRELIMINARY--)

✓Generation of turbulence: how, where, when:
✓In this talk will focus on vorticity as a metric for solenoidal turbulence

- Fixed, nested grids (ENZO 2.1) (3 levels):
- 64 Mpc^3 = (45 Mpc/h)^3 box; 400^3 root grid
  Inner 32 Mpc^3 box; 400^3 grid
  Cluster-centered 6.4 Mpc^3 **uniform at Δx = 20 kpc (to beyond ~R_{vir})**
  Full outputs saved $\Delta t \approx$ 100 Myr near z = 0
- WMAP-7 cosmology from z = 30, adiabatic +'reionization'
- Zoomed in on merging, post merger & relaxed systems
  M_v ~ several x$10^{14}M_{sun}$
  R_{vir} ~ 2 – 2.5 Mpc

Evolution of Vorticity

Recall: Vorticity measures an 'inverse eddy time'

$$\vec{\omega} = \nabla \times \vec{u} \sim \frac{u_\ell}{\ell} \sim \frac{1}{t_{eddy,\ell}}$$

In K41 turbulence: $\omega_\ell \propto \ell^{-2/3}$

From Navier-Stokes:

Baroclinic Source Term

$$\frac{\partial \vec{\omega}}{\partial t} = \nabla \times (\vec{u} \times \vec{\omega}) + \frac{1}{\rho} \nabla \ln(\rho) \times \nabla P - \nu \nabla^2 \vec{\omega},$$

Conservative

or

$$\frac{\partial \vec{\omega}}{\partial t} = -\nabla \cdot (\vec{u}\vec{\omega}) + (\vec{\omega} \cdot \nabla)\vec{u} + \frac{1}{\rho} \nabla \ln(\rho) \times \nabla P - \nu \nabla^2 \vec{\omega},$$

Dissipation (numerical here)

Stretch Amplification

Two Ways to Seed Vorticity

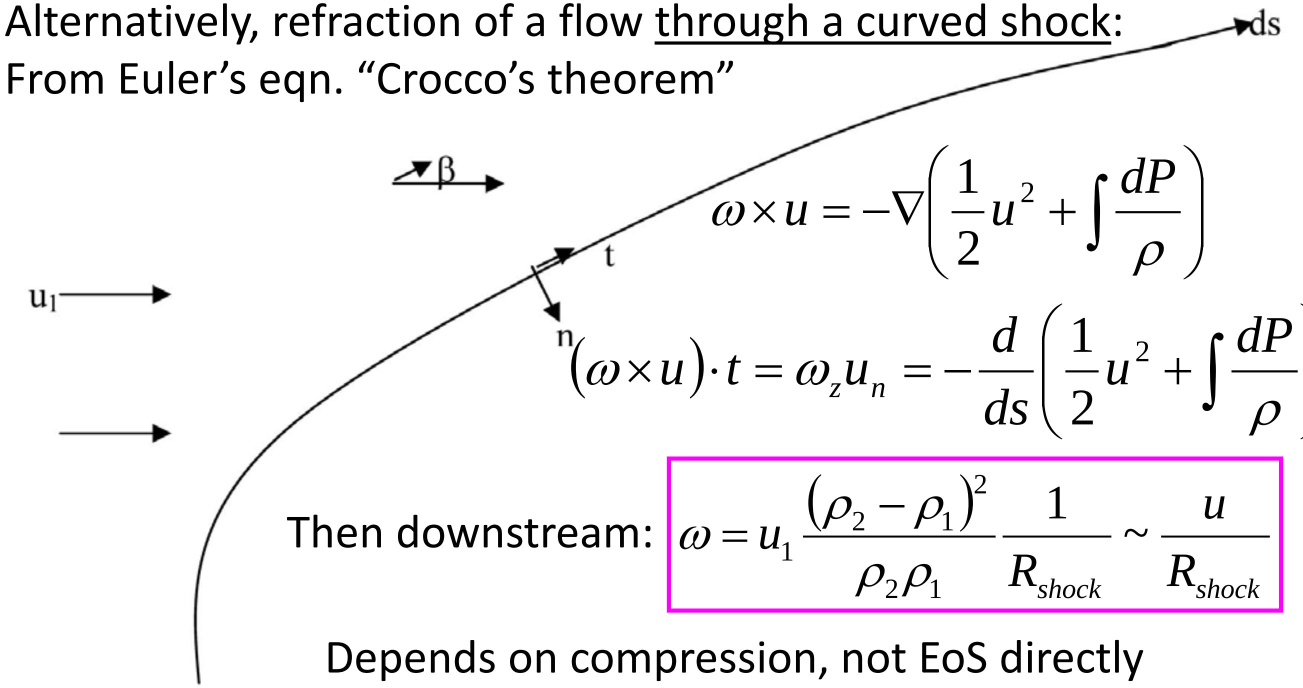
a) Baroclinic flows (e.g., downstream of curved shocks, merging, etc) create vorticity. From Euler's equation:

$$\frac{d\omega}{dt} = \cdots \frac{1}{\rho} \quad \frac{1}{\rho}$$

Vanishes if P=P(ρ) (e.g., isothermal)

$$S = \frac{P}{\rho^{5/3}}$$

b) Alternatively, refraction of a flow through a curved shock:
From Euler's eqn. "Crocco's theorem"



$$\omega \times u = -\nabla\left(\frac{1}{2}u^2 + \int \frac{dP}{\rho}\right)$$

$$(\omega \times u)\cdot t = \omega_z u_n = -\frac{d}{ds}\left(\frac{1}{2}u^2 + \int \frac{dP}{\rho}\right)$$

Then downstream: $\boxed{\omega = u_1 \frac{(\rho_2 - \rho_1)^2}{\rho_2 \rho_1} \frac{1}{R_{shock}} \sim \frac{u}{R_{shock}}}$

Depends on compression, not EoS directly

Easier to Work with Scalar, Enstrophy (ω^2=ε)

Sources

Conservative

$$\frac{\partial \omega^2}{\partial t} = F_{adv} + F_{stretch} + F_{comp} + F_{baroc} + F_{diss},$$

Amplification

$$F_{adv} = -\nabla \cdot \vec{u}\omega^2 = -(\omega^2 \nabla \cdot \vec{u} + \vec{u} \cdot \nabla \omega^2),$$

$$F_{stretch} = -2\vec{\omega} \cdot (\vec{\omega} \cdot \nabla)\vec{u}$$

$$F_{comp} = -\omega^2 \nabla \cdot \vec{u} = \omega^2 \frac{d\rho}{dt},$$

$$\begin{aligned} F_{baroc} &= 2\frac{\vec{\omega}}{\rho} \cdot \nabla \ln(\rho) \times \nabla P \\ &= 2\mathcal{R}\, \vec{\omega} \cdot \nabla \ln(\rho) \times \nabla T \\ &= 2\rho^{2/3} \vec{\omega} \cdot \nabla \ln(\rho) \times \nabla S, \end{aligned}$$

where $\mathcal{R} = k_B/(\mu m_H)$

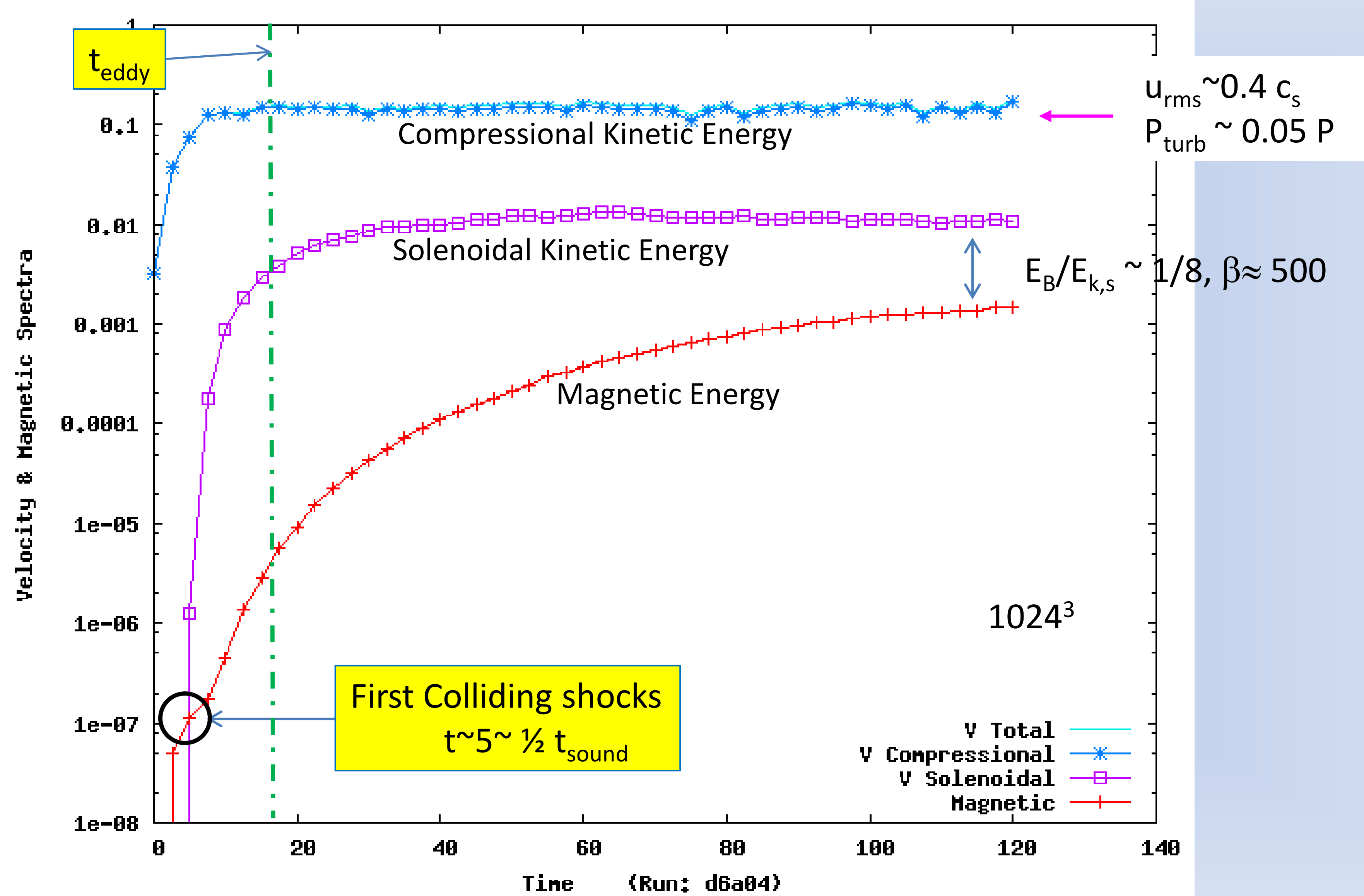
$$F_{diss} = -2\nu\vec{\omega} \cdot \nabla^2 \vec{\omega}$$

-These quantities can all be computed numerically from simulation data-

1) Methods Test Using Compressively Driven Isothermal Turbulence

- 3D periodic box (so net $F_{adv} = 0$)
  ($L_x = L_y = L_z = 1$) Eulerian, compressible, ideal (TVD-MHD)
- Isothermal, $\underline{c_s = 1}$ (so $\nabla\rho\times\nabla P = 0$; $F_{baroc} = 0$)
  $t_{sound} = L/\ c_s = 10$, $\langle\rho\rangle=1$, so $\langle P\rangle = 1$
- Random **Compressive** Driving; $\nabla\times\delta\mathbf{u} = \mathbf{0}$ (so no explicit vorticity added)
  peak ~ $L_d = 2/3\ L_x$, $k = 3/2$
  => $\underline{u_{RMS} \sim 1/2\ c_s}$, largest eddy time $t_{eddy} \sim L_d/u_{RMS} \sim 13\text{-}17$
- MHD with very weak (uniform) seed field, $\beta=10^6$, negligible early feedback

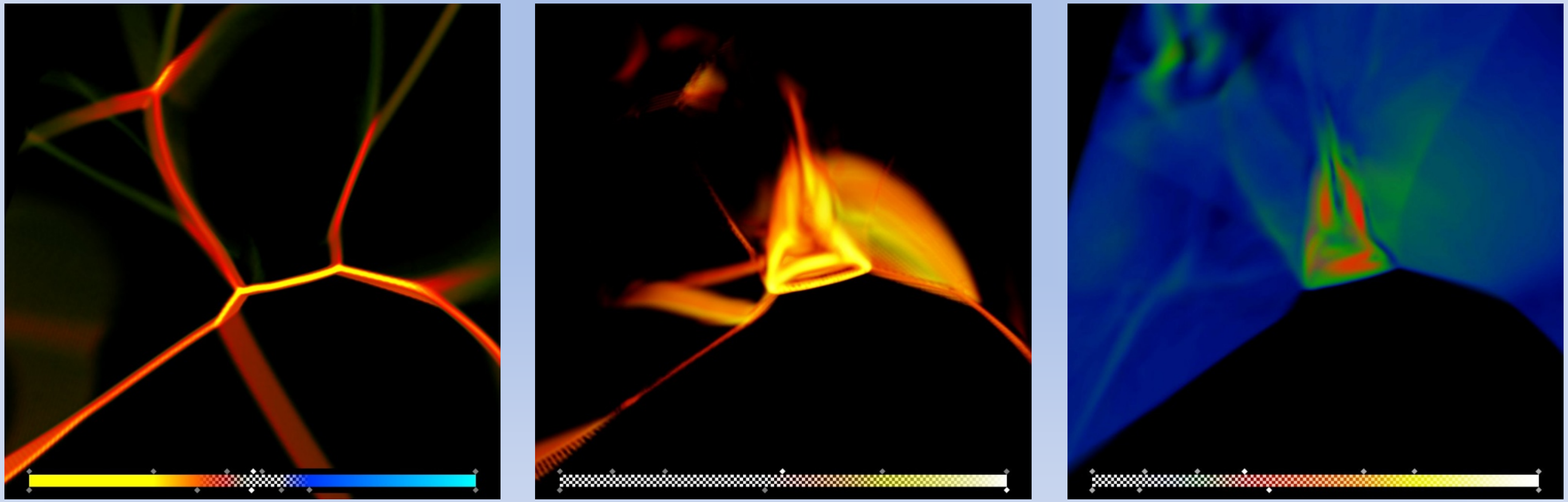
Turbulent Energy Evolution



Compressional Forcing Leads to Shocks:
Curved & Intersecting Shocks Create Vorticity via “Crocco’s Theorm”:

(Vorticity Amplifies Magnetic Field)

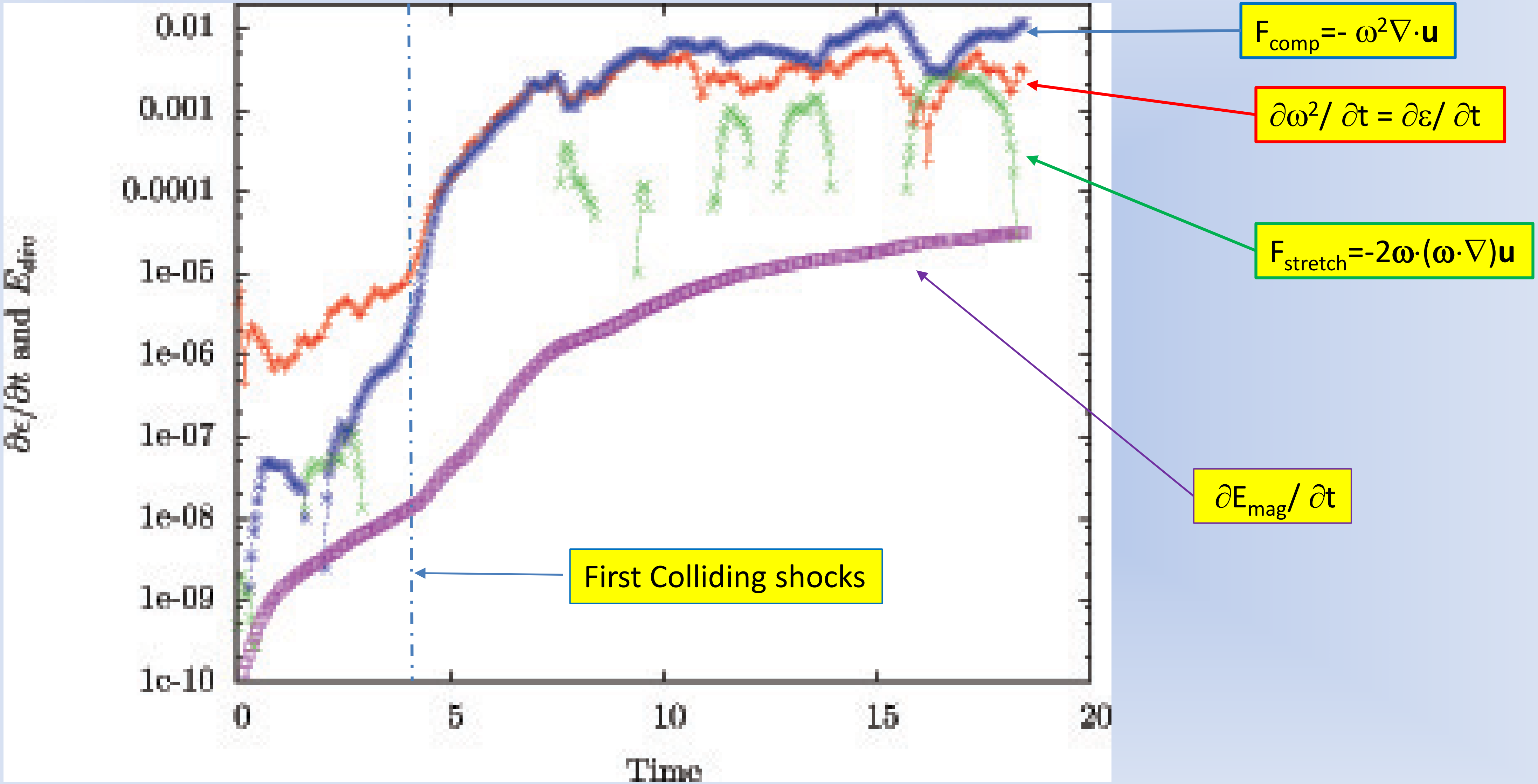
Intersecting shocks => Vorticity => Magnetic field amplification



Zoomed in slice at t = 5

512^3 simulation

Enstrophy Evolution Rates (Compressive Driving; $\nabla\times\delta u = 0$)



2) Cluster Formation Simulations

Two example clusters:
A) Cluster ‘actively merging’ until z~ 0 (it922)
B) Cluster ‘relaxing’ since z ~ 0.5 (it100)

Both have M_v ~ several x10^{14} M_{sun}

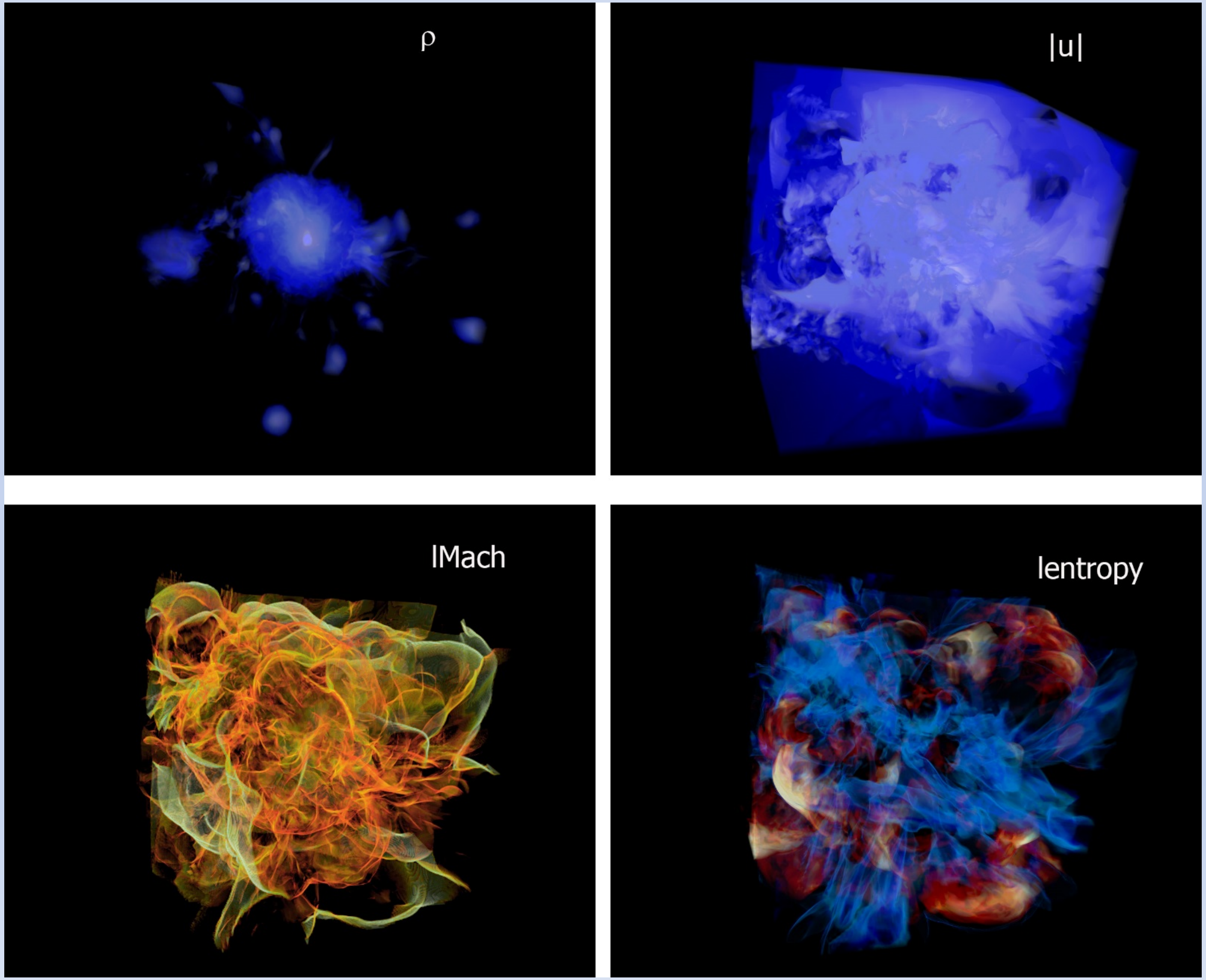
- Goal is to establish when, where and how cluster turbulence is generated, amplified and transported

- First look focuses on vorticity

Cluster A (it922) – Volume View of Dynamic State –z ~ 0

Log plasma density

Flow velocity



Shock Mach number

red = weak > 1.3
white = strong < 20

Log Entropy

(6.4 Mpc)^3 box

z = 0.074

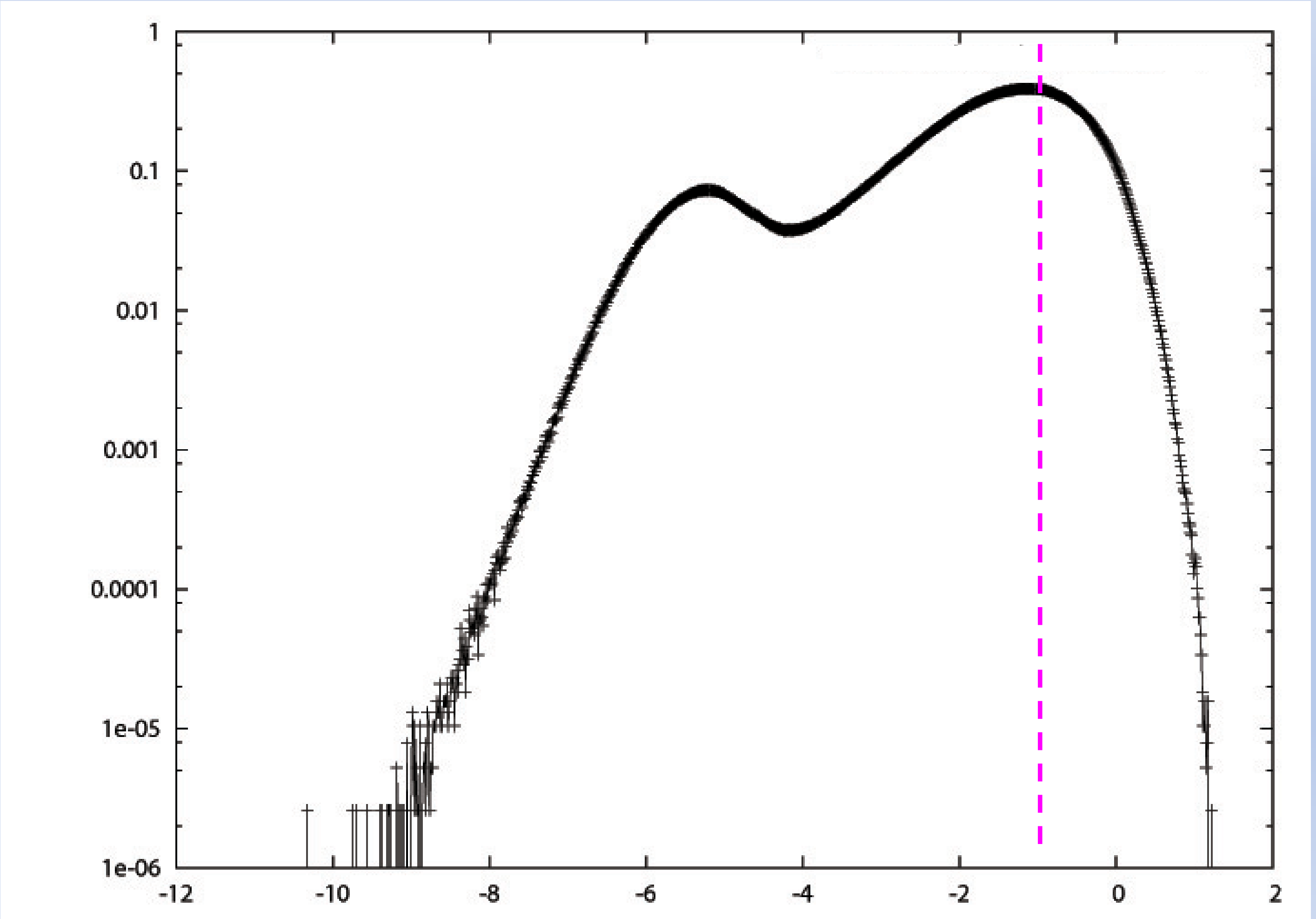
Cluster A (it922) –Density Evolution

(6.4 Mpc)^3 box

Cluster A (it922) –Enstrophy Evolution

(6.4 Mpc)^3 box

Cluster A (it922) –Enstrophy Distribution Function – z ~ 0



$P(\omega^2)$

$\mathrm{Log}(\omega^2)$

Note:
ω In Code units;
~ 1/(130 Myr)

So ω^2 ~ 1 corresponds
to t_{eddy} ~ 130 Myr

While ω^2 ~0.1 corresponds
to t_{eddy} ~ 400 Myr

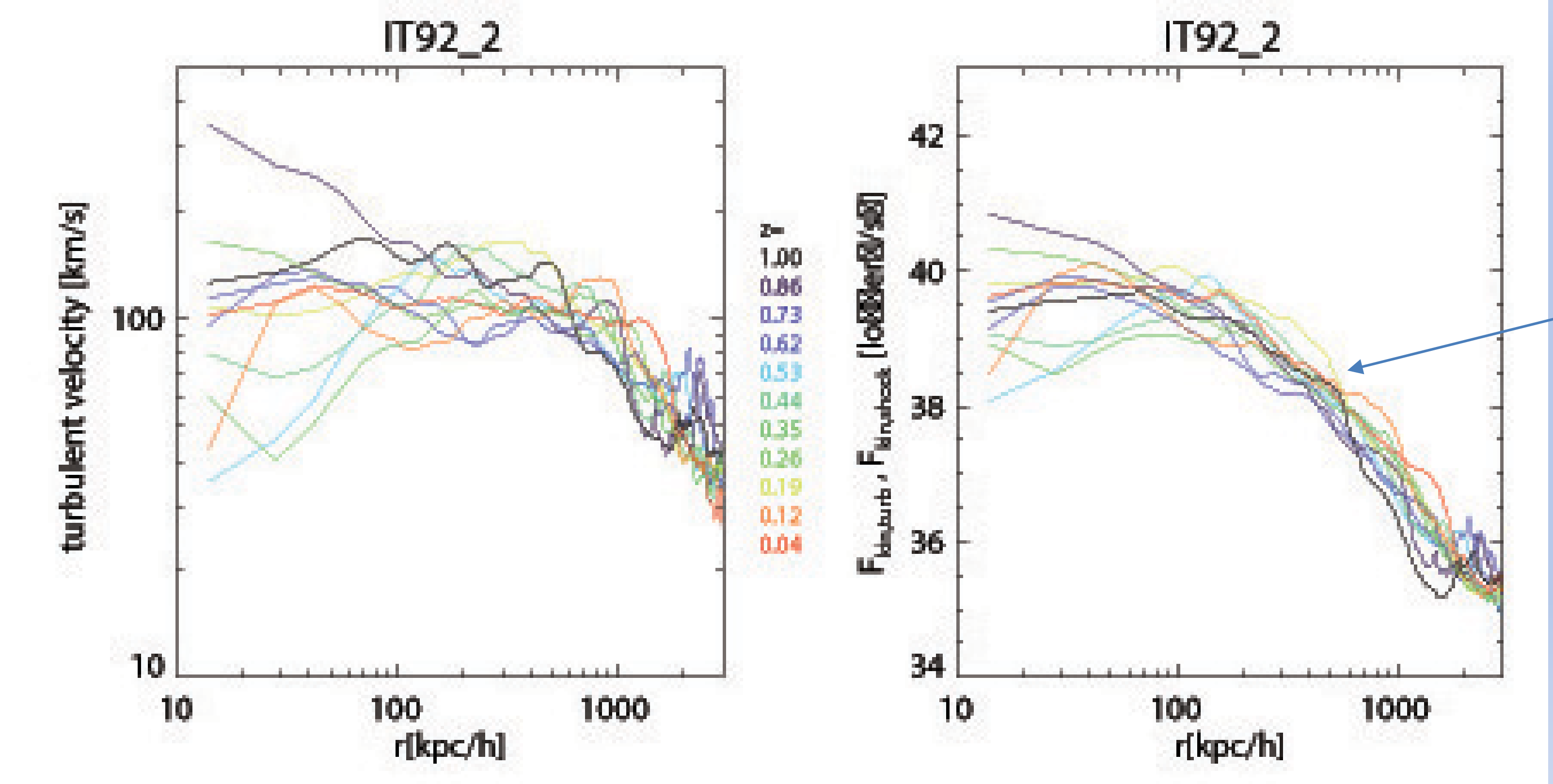
Or $u_\ell \sim \omega\, \ell \sim$ 100 km/sec
With ℓ ~ 40 kpc

(6.4 Mpc)^3 box

cf Miniati 2014
very similar result

Cluster A (it922) –Turbulence Distribution Evolution: Using Vazza + 2012 Multi-scale Turbulence Filter

Consistency check
With enstrophy anal.



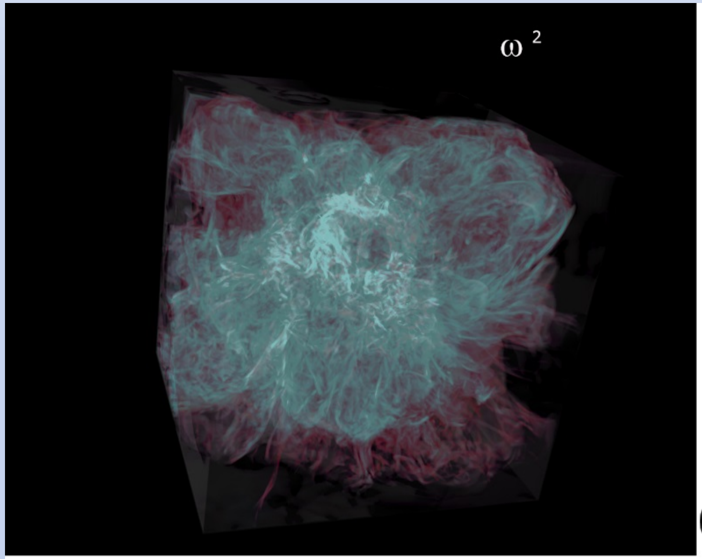
Turbulent cascade energy flux

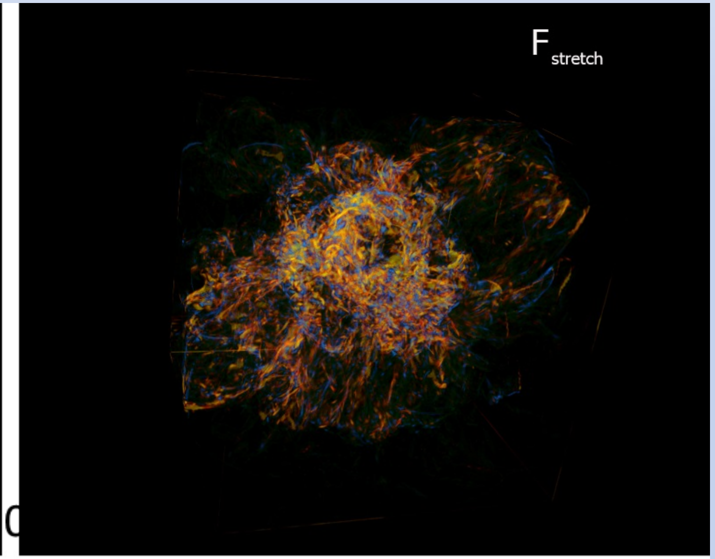
$$F_{turb} = \frac{1}{2}\rho U^3 L^2$$

$$\omega_\ell \sim 0.3 \left(\frac{2F_{turb,39}}{\rho_{-28}}\right)^{1/3} \left(\frac{\ell}{L}\right) \ell_{40kpc}^{-5/3}$$

Cluster A (it922) –Enstrophy Volume View -z ~ 0

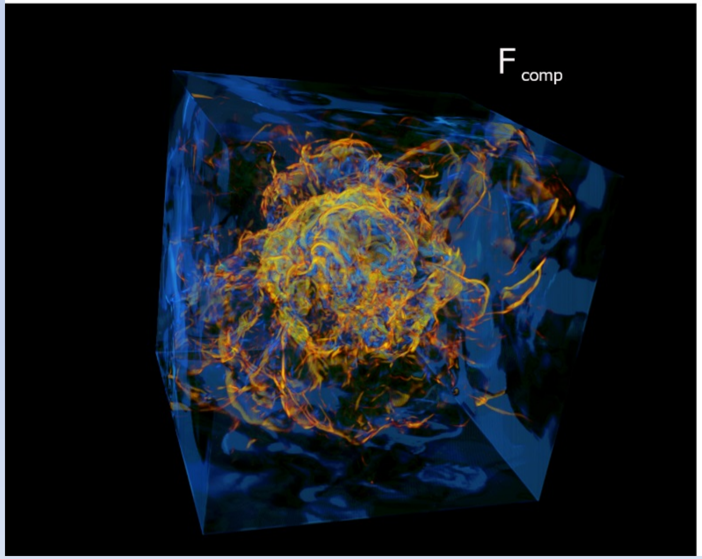
Log Enstrophy

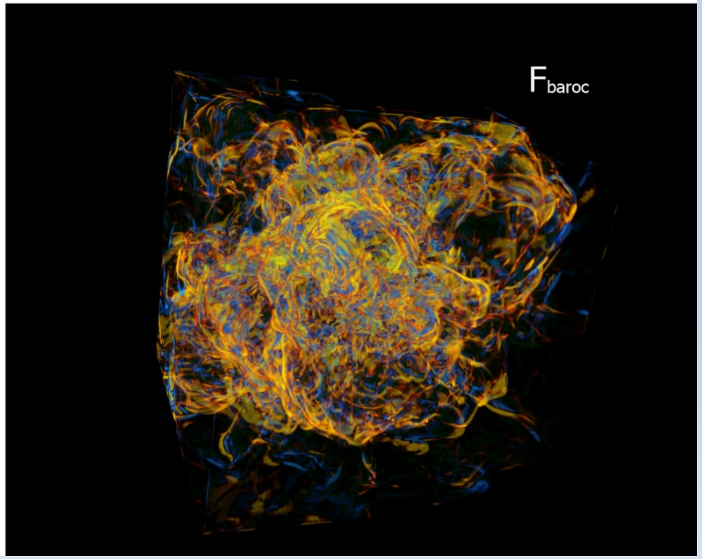




Enstro. “Stretching rate”

Enstr. “Compression rate”





Enstro. Baroclinic Source

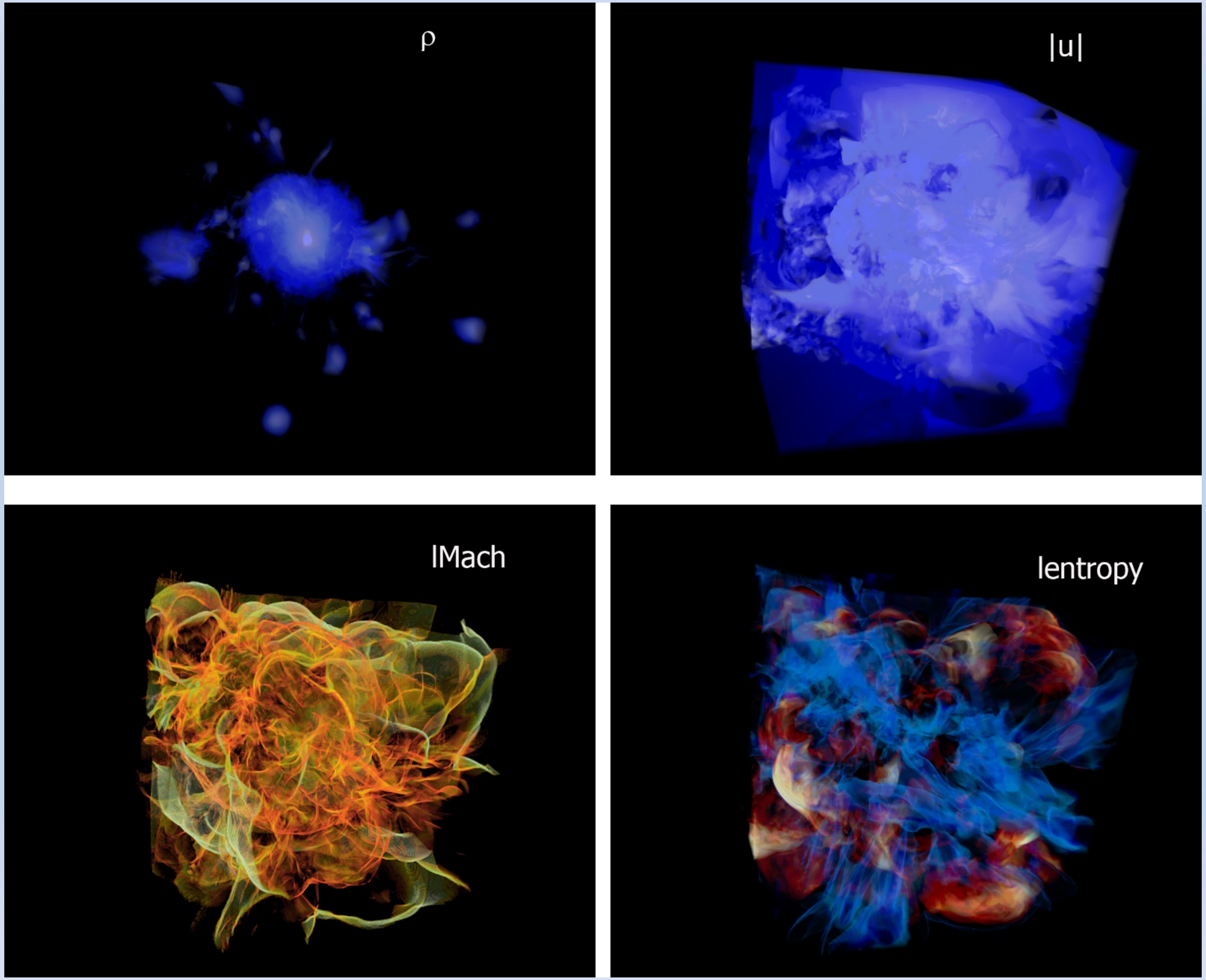
(6.4 Mpc)^3 box

z = 0.074

Cluster A (it922) – Volume View of Dynamic State –z ~ 0

Log plasma density

Flow velocity



Shock Mach number

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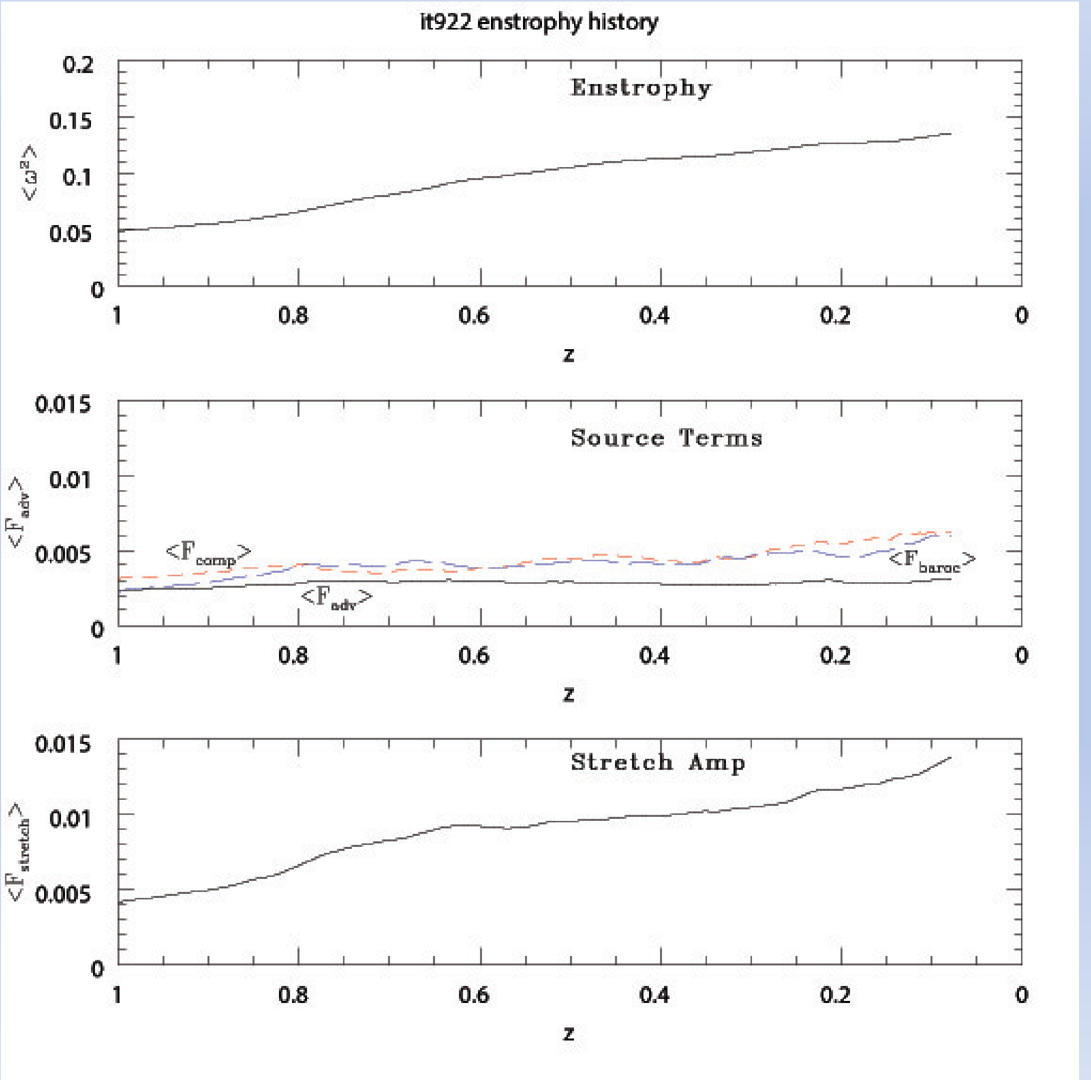
white = strong < 20

Log Entropy

(6.4 Mpc)^3 box

z = 0.074

Cluster A (it922) History of Integrated Enstrophy Development since z = 1



Same range

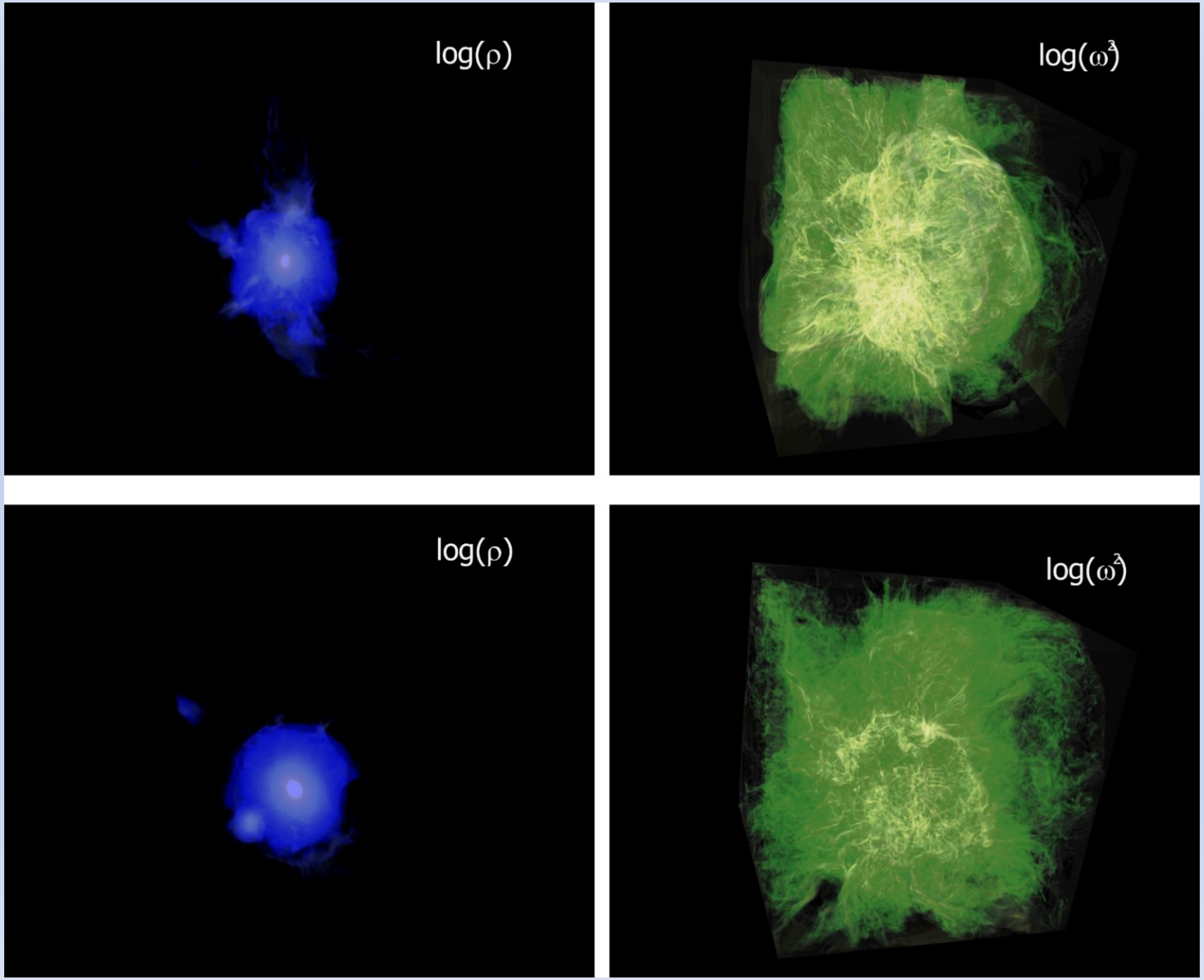
Enstrophy evolution

Baroclinic & Compressive Sources & Net flux rate into volume

Amplification rate

Amplification timescale: $\omega^2 / (\partial\omega^2/\partial t)$ ~ 1 Gyr

Cluster B (it100) Relaxing Since z ~ 0.5–Two Snapshots



Actively Merging

z= 0.9

Log plasma density

Log enstrophy

Relaxing

z= 0

$(6.4\ \mathrm{Mpc})^3$ box

Cluster B (it100) Density Evolution

$(6.4\ \mathrm{Mpc})^3$ box

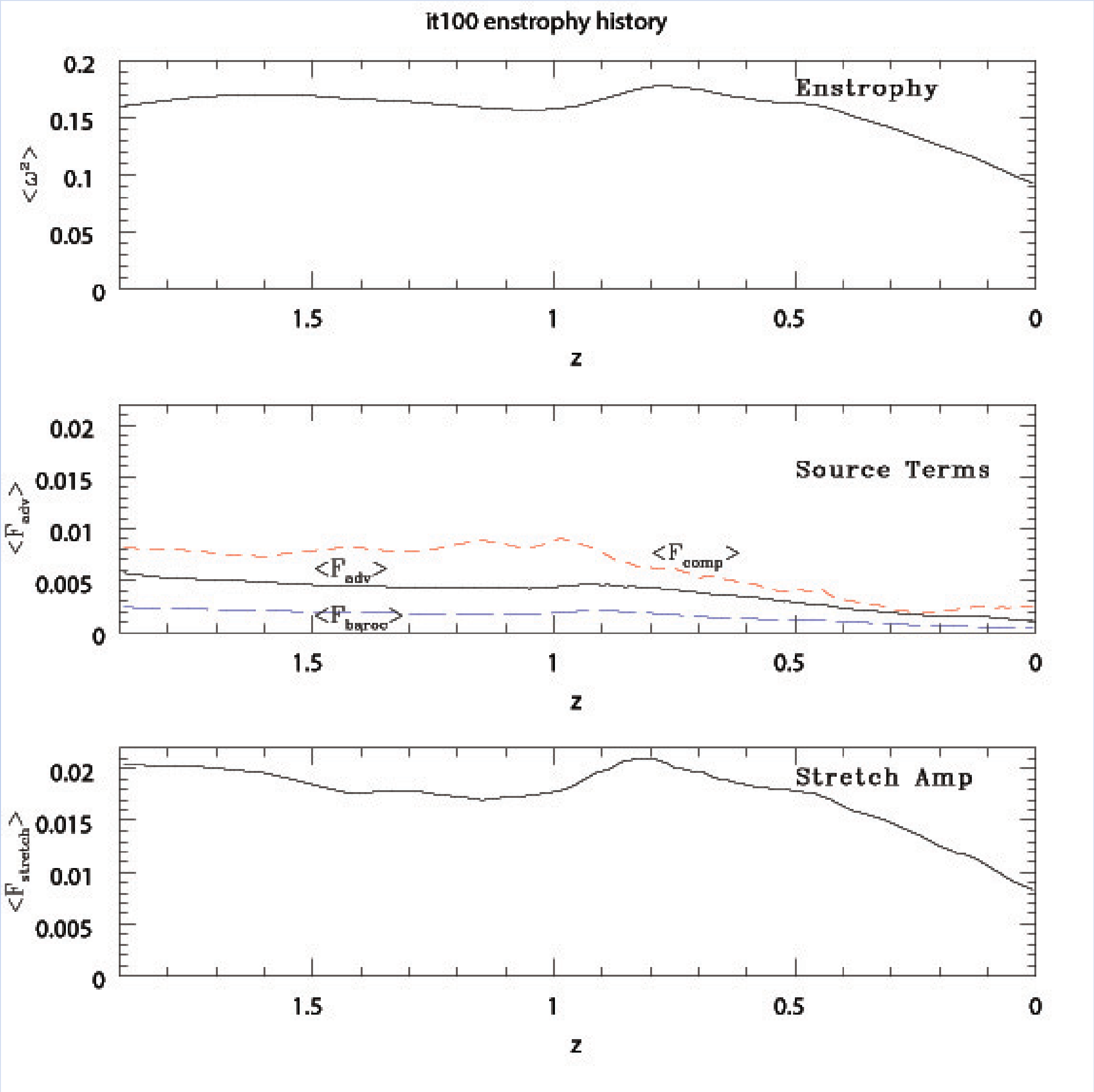
Cluster B (it100) Enstrophy Evolution

(6.4 Mpc)^3 box

Cluster B (it100) History of Integrated Enstrophy Development since z = 1.9

Note: Enstrophy decays for z < 0.8

Dissipation dominates decreased seed and amplification rates



Enstrophy evolution

Baroclinic & Compressive Sources & Net flux rate into volume

Amplification rate

Summary & Conclusions

- Solenoidal turbulence (vorticity) is seeded during cluster formation broadly by (at least) 3 comparable sources:
  - Directly by refraction at curved and (especially) intersecting cluster shocks
  - Baroclinicity in flows downstream of shocks (other baroclinicity generators likely; e.g., AGNs, cooling, etc)
  - Influx from accretion

- Spatial and temporal distributions of seeding are complex (& under investigation)

- Amplification by stretching is effective on Gyr timescales; further evolution comes from advection & dissipation

Tak!