Driven and Damped Acoustic Waves in Galaxy Clusters

Ellen Zweibel & Vladimir Mirnov
UW Madison

Questions About Acoustic Waves

- How are acoustic waves generated? *By piston motions or other sources of pressure fluctuations.*
- How do acoustic waves propagate? *Linear dispersion relation is straightforward to calculate.*
- How do acoustic waves heat the host medium? *The main subject of this talk.*

Dissipation Mechanisms

- Electron thermal conduction (largest by a factor scaling as $(m_i/m_e)^{1/2}$
- Ion viscosity
- Ion thermal conduction (40% of ion viscosity)
- Electron-ion thermal equilibration (important when electron thermal conduction is important but ion thermal conduction is not).
- Coefficients can be manipulated, e.g. magnetically suppressed.

System has 3 modes: acoustic, thermal relaxation, & T_e-T_i equilibration.

Which Species are Heated?

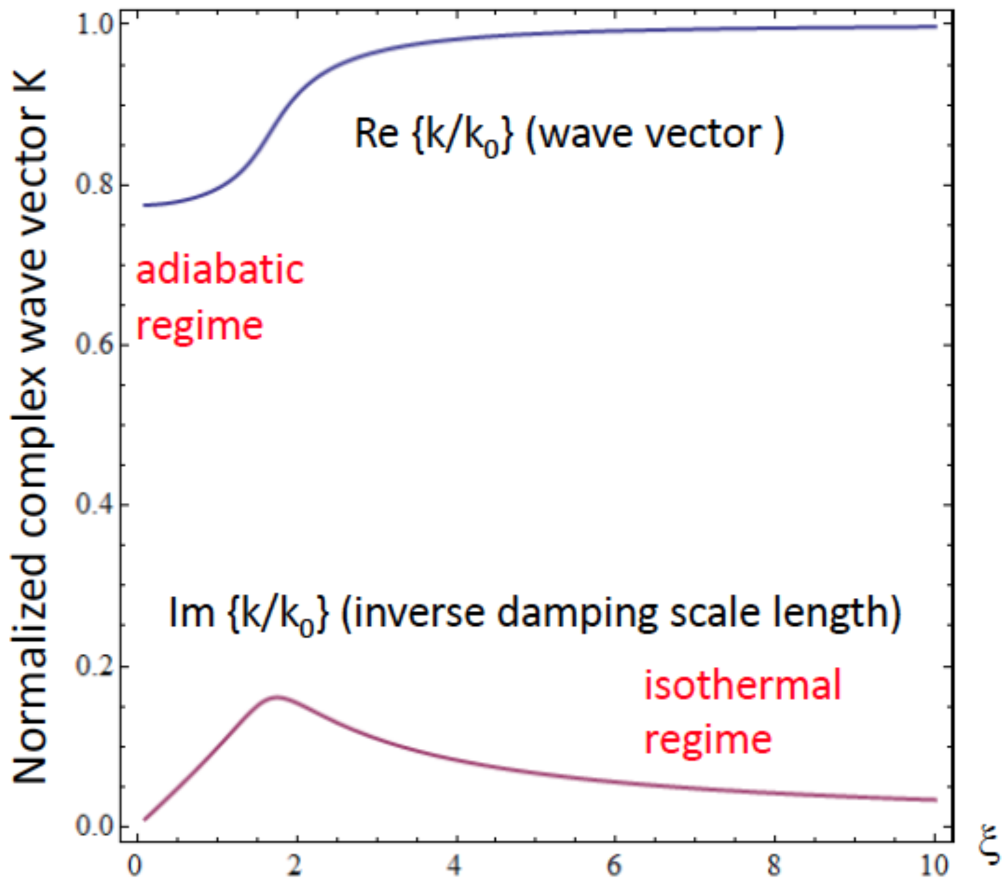
Electron & ion temperature equations:

$$\frac{3}{2}\frac{\partial T_{e1}}{\partial t} = -T\nabla\cdot\mathbf{u}_1 + \chi_e\nabla^2 T_{e1} - 3\frac{m}{M}\frac{(T_{e1}-T_{i1})}{\tau_e}$$

$$\frac{3}{2}\frac{\partial T_{i1}}{\partial t} = -T\nabla\cdot\mathbf{u}_1 + \chi_i\nabla^2 T_{i1} + 3\frac{m}{M}\frac{(T_{e1}-T_{i1})}{\tau_e}$$

$\tau_{ei} = \tau_e\, m_i/(4m_e) = 1.1\; 10^{11}\, T_7^{3/2}/n_e$ s.

Dispersion Relation

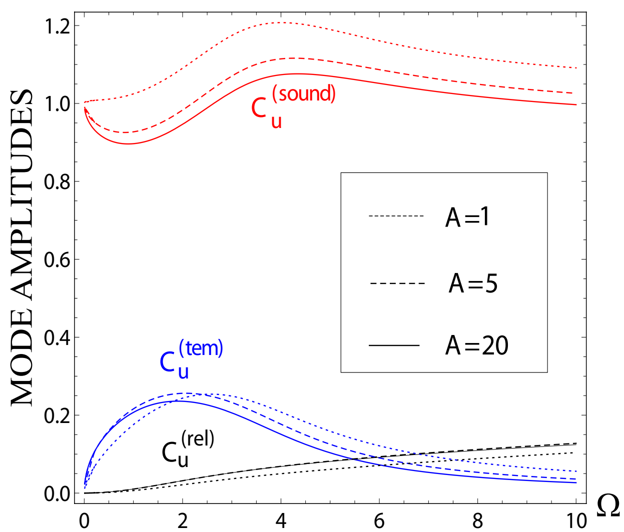


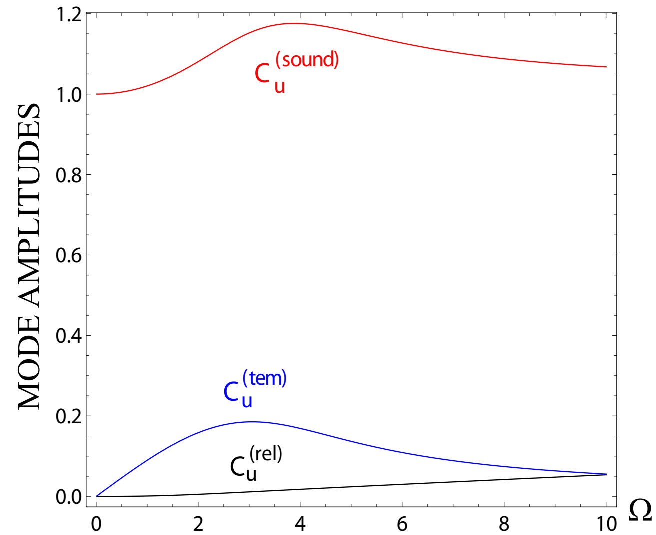
Single fluid, thermal conduction only

dimensionless thermal conduction

Amplitudes

Most of the power from an oscillating source goes into the acoustic mode.





Amplitudes of driven modes for zero temperature perturbation (left) or zero heat flux perturbation (right), as functions of scaled frequency $\Omega=\omega\tau_e(m_i/m_e)$ and driver size parameter $A=\omega a/v_i$. Results in zero heat flux case are insensitive to A. We estimate A ~ 10 – 20 for 10^7yr period waves from a 10 kpc source.

Energy of an Acoustic Wave

Assume single fluid with dissipation by thermal conduction only *(easily generalized)*.

Energy conservation law:

$$\frac{\partial}{\partial t}\left(\frac{\rho u^2}{2}+\frac{p}{\gamma-1}\right)=-\nabla\cdot\left[\rho\mathbf{u}\frac{u^2}{2}+\frac{\gamma p}{\gamma-1}\mathbf{u}-\kappa\nabla T\right] \quad \rightarrow \quad \frac{\partial E}{\partial t}=-\nabla\cdot\mathbf{J}$$

To extract acoustic part, subtract mass flux

$$\mathbf{J}_a=\mathbf{J}-\frac{\gamma T_0}{m(\gamma-1)}\rho\mathbf{u}$$

And define acoustic energy density $E_a = E - E_{na}$

Energy of an Acoustic Wave

Assume the acoustic part obeys its own conservation law

$$\frac{\partial E_a}{\partial t} = -\nabla \cdot \mathbf{J}_a$$

& vanishes in the unperturbed state, leading to

$$E_a = \frac{\rho u^2}{2} + \frac{p - p_0}{\gamma - 1} - \frac{\gamma T_0}{m(\gamma - 1)}(\rho - \rho_0)$$

Want to expand E_a & $\boldsymbol{J}_a$ to second order & time average.

Acoustic Energy Flux

Using our expression for E_a we can rewrite $\boldsymbol{J_a}$ as

$$\mathbf{J}_a = (p - p_0)\mathbf{u} + \mathbf{u}E_a$$

For *nearly adiabatic waves*, the 2nd term is 3rd order in wave amplitude & can be neglected. In general, 2nd term is related to the entropy perturbation.

$$\overline{\mathbf{J}_a} = \frac{\gamma p_0}{m(\gamma - 1)}\overline{\mathbf{u}_1 T_1} - \kappa\nabla\overline{T_2}$$

Entropy Production

Evolution eqn. for entropy density $S=\rho s$ (q is heat flux)

$$T\frac{\partial S}{\partial t} = -T\nabla\cdot\left(\mathbf{u}S - \frac{\mathbf{q}}{T}\right) + \underbrace{\frac{\kappa(\nabla T)^2}{T}}_{Q^{(\text{positive})}}$$

- Integration over volume gives global growth of entropy
- For nearly adiabatic waves, $Q^{(\text{positive})}$ dominates over flux
- If conduction is large, flux term is pointwise important

Contribution from Flux

$$Q^{(flux)} = -T\,\overline{\nabla \cdot \left(\rho \mathbf{u}_2 s + \mathbf{u}_1 \rho_1 s + \mathbf{u}_1 s_1 \rho + \frac{\kappa \nabla T_2}{T} - \frac{\kappa \nabla T_1^2}{2T^2}\right)}$$

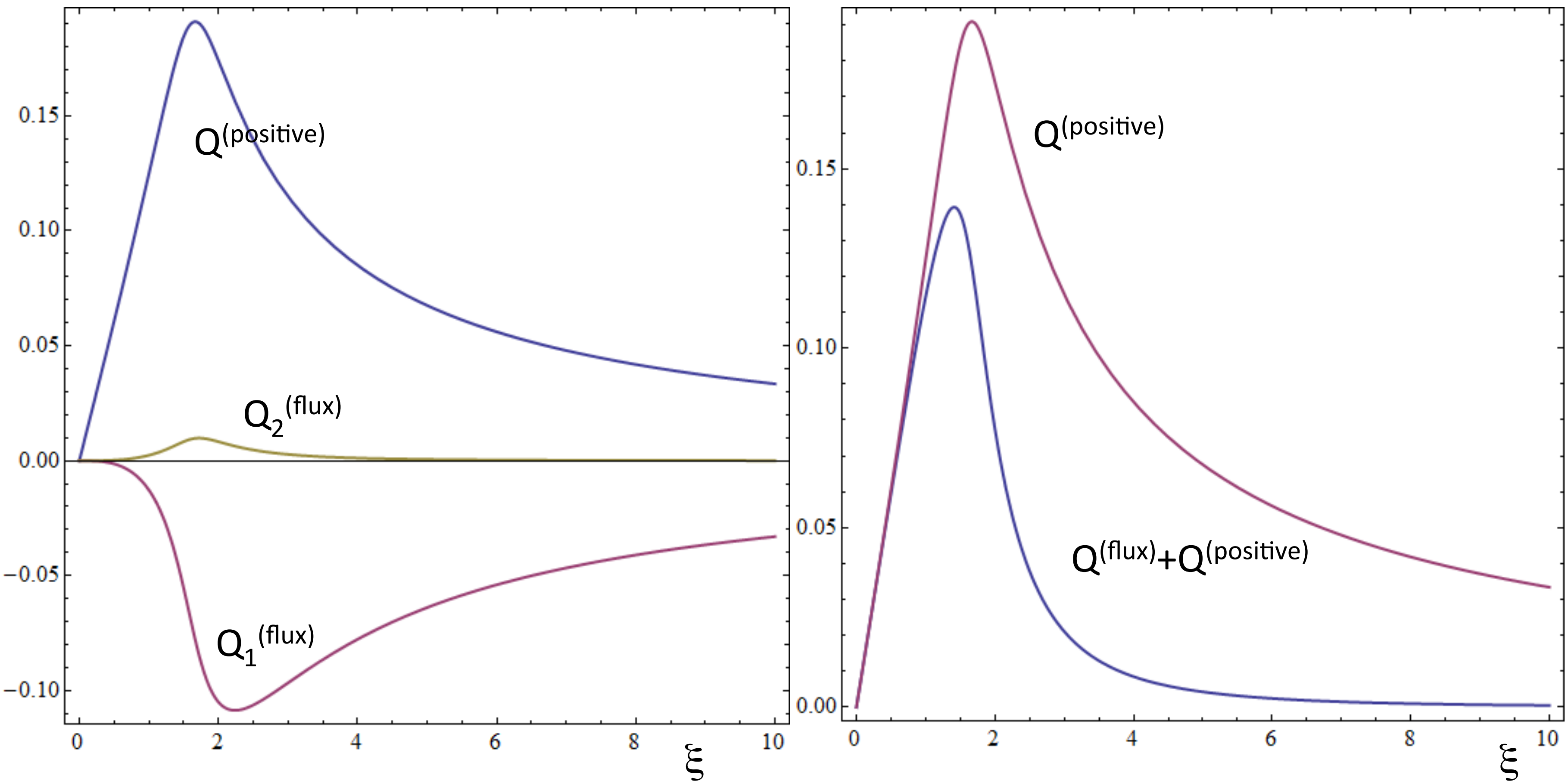
$Q_1^{(flux)}$ $Q_2^{(flux)}$

Assumes κ is independent of *T*, but can be generalized.

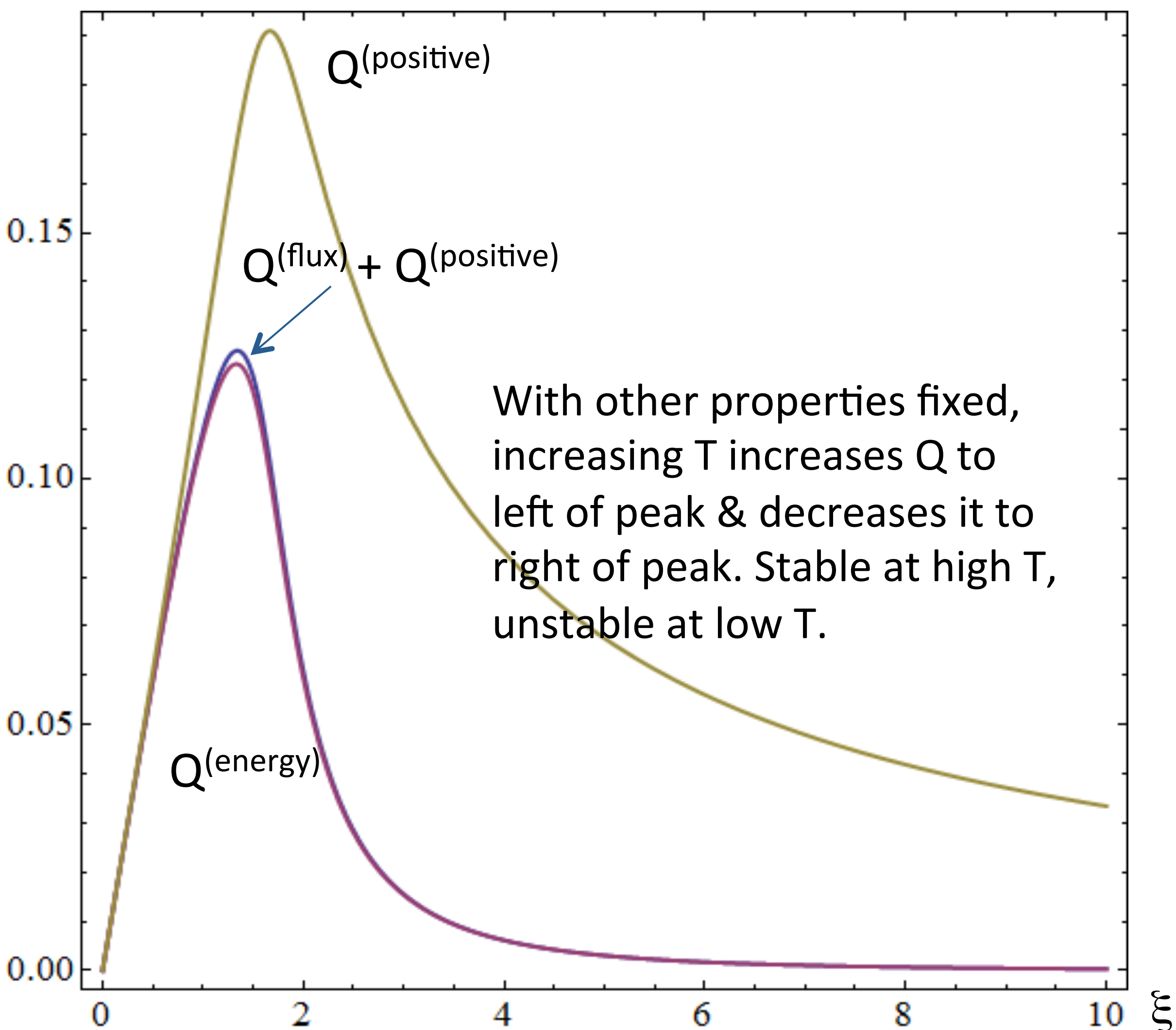
Heating is Reduced by the Entropy Flux

At large thermal conduction, $\xi = \chi\, \omega / c_T^2 \sim 1$, acoustic oscillations become non-isentropic

Heat production rate is reduced by outgoing entropy flux ($Q^{(flux)} < 0$)



Energy and Entropy Approaches Agree



Issues:

(a) small difference of the peak values?

(b) contribution from $\nabla^2 T_2$ term?

Pointwise entropy production exceeds the actual heating rate.

Left to Do

- Follow waves as they propagate & dissipate in a model cluster.

- How do heating & cooling compare?

- Can the waves persist over several wavelengths, as apparently observed?

Collisionless Regime

- Ion mean free path $\lambda_i \sim .14\ (T_7^2/n_{cgs})$pc
- Waves with wavenumber k damp through ion Landau resonance in less than one wavelength if $k\lambda_i > 0.4$.
- Waves generated by infall or propagating into outskirts should be collisionless & should damp on the ions.

Effects Not Considered

- Magnetic forces (effect on transport already parameterized).
- Gravity waves: Could the putative acoustic waves be gravity waves? *Thermal damping is weaker because buoyancy driven*.
- Cosmic rays:
  - Increased phase speed mitigates ion Landau damping problem.
  - Diffusive damping?
  - Drury instability?
- Radiative Cooling

Nonlinear Effects

- Are acoustic waves linearly damped near their source or do they drive a turbulent cascade?
  - cascade rate $\omega(u/c_s)$ vs $k_i c_{s;}$
  - figure of merit is $(k_r/k_i)(u/c_s)$
- Is dissipation in a turbulent cascade volumetrically smooth?
  - Observational constraints on an intermittent heating mechanism?

Overarching Questions

- How is ICM plasma stirred?
- How are disturbances dissipated?
  - How far is energy transported?
  - Is the heating stable?
  - What species are heated?