Non-Gaussianities & the Scale of UV Physics

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APCTP

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based on 1210.3020,1406.1947,1407.8268

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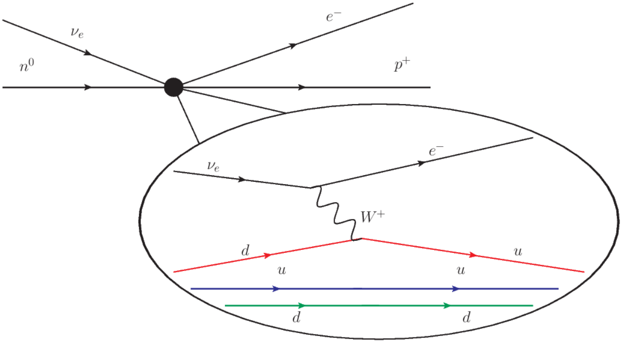
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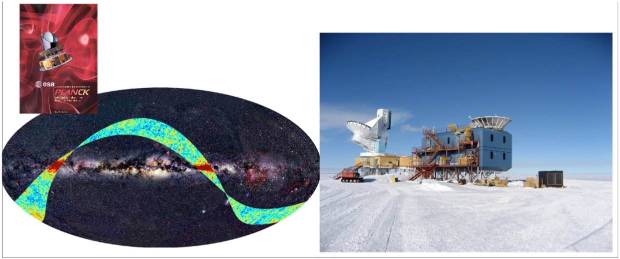
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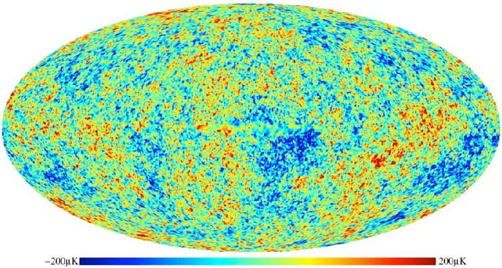
PLANCK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Outline

1. Effective Field Theory For Inflation
2. EFT of Weakly Coupled Models
   - Effective description of heavy physics
   - New physics regime
3. Interpretation of Cosmological Observables
   - Energy Scales of the EFT
4. Concluding Remarks

- General statement: Inflation = QFT on a time dependent gravitational background

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- We want to study perturbations of a scalar field following a time-dependent solution

$$\boxed{\phi(x,t) = \phi_0(t) + \delta\phi(x,t)}$$

Creminelli et al. '06, Cheung et al., Weinberg '08,

Senatore/Zaldarriaga '09

How to construct the EFT for the fluctuations $\delta\phi$?

Use every possible operator that respects the symmetries of the theory !

The set of such operators is easily identifiable in the unitary gauge:

$$\mathcal{L} = \frac{1}{2}M_{\rm Pl}^2 R - c(t)g^{00} - \Lambda(t) + \mathcal{L}^{(2)}(\delta g^{00}, \delta K_{\mu\nu}, \delta R_{\mu\nu\kappa\lambda}, \nabla_\mu; t)$$

Requirement of homogeneous background

$$\Longrightarrow c(t) = -\dot{H}M_{Pl}^2, \quad \Lambda(t) = (3H^2 + \dot{H})M_{Pl}^2$$

Now $\delta\varphi = \pi\dot{\phi}_0$ may appear via the Stuckelberg trick, i.e. as the Goldstone boson related to the breaking of time reparametrizations due to the time dependence of the background:

$$\boxed{\mathcal{L}^{(2)} \supset -M_{\rm Pl}^2 \dot{H}\left[\dot{\pi}^2 - \frac{(\partial\pi)^2}{a^2}\right] + 2M_2^4\dot{\pi}^2}$$

$$\boxed{\mathcal{L}^{(3)} \supset +2M_2^4\left[\dot{\pi}^3 - \dot{\pi}\frac{(\partial\pi)^2}{a^2}\right] - \frac{4}{3}M_3^4\dot{\pi}^3 + \cdots}$$

- $\star$ $c_s < 1 \Rightarrow f_{NL} > 0$,
- $\star$ Multiple contributions to the 3-point functions, etc...

Heavy "imprints" in the EFT ?

Main idea:

self-interactions in the IR appear due to mediation of massive particle states in the UV

Baumann/Green '11

Gwyn/Palma/Sakellariadou/SS '12

In other words

$$\boxed{M_n \to M_n \frac{\mathcal{M}^2}{\mathcal{M}^2 - \Box}}$$

Two EFT expansions:

$$\frac{1}{\mathcal{M}^2-\Box} = \frac{1}{\mathcal{M}^2-\nabla} - \frac{\partial_t^2}{(\mathcal{M}^2-\nabla)^2}$$

Gwyn/Palma/Sakellariadou/SS

or

$$\frac{1}{\mathcal{M}^2-\Box} = \frac{1}{\mathcal{M}^2}\left(1+\frac{\Box}{\mathcal{M}^2}\right)$$

Gong/Seo/SS

leading to different momentum/time dependence of the 3-point integrals.

EFT from integration of massive fields

$$\mathcal{L}_{\mathcal{F}} = \dot{\mathcal{F}}^2 - (\nabla\mathcal{F})^2 - \mathcal{M}^2\mathcal{F}^2 - \alpha\mathcal{F}\delta g^{00}(\pi) - \beta\mathcal{F}^2\delta g^{00}(\pi) - \gamma\mathcal{F}^3\delta g^{00}(\pi)$$

By restricting ourselves to low energies we can integrate out $\mathcal{F}$.

EOM: $\mathcal{F} = \frac{\alpha}{\mathcal{M}^2-\nabla^2}\left[\delta g^{00}\frac{\beta}{\mathcal{M}^2-\nabla^2}\right]\delta g^{00}$

EFT from integration of massive fields

In general the resulting effective Lagrangian reads:

$$\boxed{\mathcal{L} = -M_{\rm Pl}^2 a^3 \dot{H}\left[\dot{\pi}\left(1 + \frac{2M_2^4}{M_{\rm Pl}^2|\dot{H}|}\frac{\mathcal{M}^2}{\mathcal{M}^2 - \tilde{\nabla}^2}\right)\dot{\pi} - (\tilde{\nabla}\pi)^2\right] + \mathcal{O}(\pi^3)}$$

Recall ~~Lorentz~~ so the system may find itself in a non-relativistic regime.

Low energy condition :

$$\boxed{\omega^2 < \mathcal{M}^2 + p^2 \quad \Longrightarrow \quad \omega < \mathcal{M}/c_{\rm s} \equiv \Lambda_{\rm UV}}$$

where $\frac{1}{c_{\rm s}^2} = 1 + \frac{2M_2^4}{M_{\rm Pl}^2|\dot{H}|}$ the speed of sound.

Non-locality and ghosts

Higher derivative theories: Ostrogradsky instability.
EFT is not such a case. Eliezer/Woodard '89, Sousa/Woodard '03

Pole structure: Biswas/Mazumdar/Siegel '06, Barnaby/Kamran '08

$$\boxed{D(p^2) \propto \frac{1}{\Gamma(p^2)}, \qquad \Gamma(p^2) = p^2 - \omega^2 - \frac{2\mathcal{M}^2\omega^2/c_{\rm s}^2}{\mathcal{M}^2+p^2-\omega^2}}$$

Poles: $\omega_+^2(p) \sim \Lambda_{\rm UV}^2 + \mathcal{O}(p^2)$,

$$\omega_-^2(p) = c_{\rm s}^2 p^2 + \frac{(1-c_{\rm s}^2)^2}{\mathcal{M}^2 c_{\rm s}^{-2}} p^4 + \mathcal{O}(p^6)$$

ω_+^2 has a negative residue! $\boxed{\text{no ghosts} \Longrightarrow \omega \ll \Lambda_{\rm UV}}$

Dispersion relation

There is an important scale hidden in the dispersion relation!

$$\boxed{\omega^2(p) = c_s^2 p^2 + \frac{(1-c_s^2)}{\mathcal{M}^2 c_s^{-2}} p^4}$$

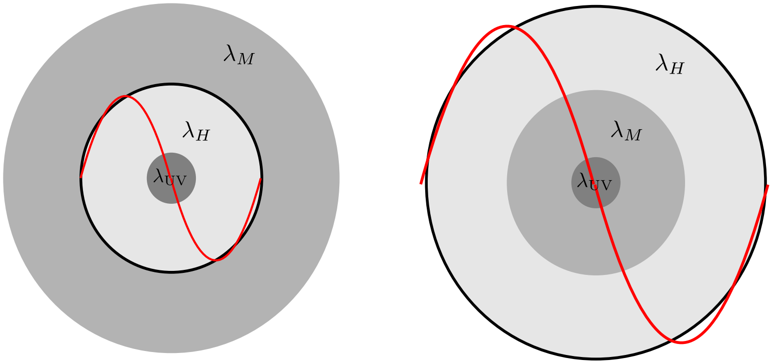
$$p \ll \mathcal{M} \qquad\qquad p \gg \mathcal{M}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\boxed{\omega \simeq c_s p} \qquad \mathcal{M} c_s \equiv \Lambda_{\rm new} \ll \Lambda_{\rm UV} \qquad \boxed{\omega \simeq \frac{p^2}{\Lambda_{\rm UV}} + \frac{1}{2}\Lambda_{\rm new}}$$

Light mode propagates in a medium $\rightarrow c_s \ll 1$.

Phonon excitations vs particle excitations



Left panel: mode freezes within the dispersive medium.
Right panel: mode freezes outside the effective medium.
λ_M sets the characteristic scale of the medium.

Inflationary Observables

For horizon crossing in the new phys. regime $H > \Lambda_{\rm new}$,

$$p^2 \to \Lambda_{\rm UV} H, \qquad \partial_t \to H$$

$$\pi_k(\tau) = \frac{H}{\sqrt{2M_{\rm Pl}^2\epsilon}}\sqrt{\frac{\pi}{8}}\frac{k}{\Lambda_{\rm UV}}(-\tau)^{5/2}H_{5/4}^{(1)}(x), \quad x \equiv \frac{H}{2\Lambda_{\rm UV}}k^2\tau^2$$

$$\boxed{\mathcal{P}_\zeta \propto \frac{H^2}{M_{\rm Pl}^2\epsilon}\sqrt{\frac{\Lambda_{\rm UV}}{H}}, \qquad r \propto \epsilon\sqrt{\frac{H}{\Lambda_{\rm UV}}}, \qquad f_{\rm NL} \sim \frac{\Lambda_{\rm UV}}{H}}$$

as compared to $\quad \mathcal{P}_\zeta \propto \frac{H^2}{M_{\rm Pl}^2\epsilon c_{\rm s}}, \qquad r \propto \epsilon c_{\rm s}, \qquad f_{\rm NL} \sim \frac{1}{c_{\rm s}^2}$

Speed of sound replaced by the ratio $\sqrt{H/\Lambda_{\rm UV}} \equiv v_{\rm ph}|_{\omega=H}$

Weakly coupled inflation

Scattering of four scalars → loss of unitarity → strong coupling scale

$$\mathcal{L}_{\rm int} = \frac{(1-c_s^2)}{16 M_{\rm Pl}^2 \epsilon H^2} (\nabla \pi_n)^2 \frac{\mathcal{M}^2 c_s^{-2}}{\mathcal{M}^2 - \nabla^2} (\nabla \pi_n)^2$$

$$\mathcal{A}(p_1, p_2 \to p_3, p_4) = 16\pi \left(\frac{\partial \omega}{\partial p} \frac{\omega^2}{p^2} \right) \sum_\ell (2\ell + 1) P_\ell(\cos\theta) a_\ell$$

optical theorem: $a_\ell + a_\ell^* \leq 1$

$$\boxed{\Lambda_{\rm s.c.} \sim \Lambda_{\rm s.b.} \sim \Lambda_{\rm UV}}$$

Low derivative EFT: $\Lambda_{\rm s.c.} \sim c_s^{5/4} (M_{\rm Pl}^2 |\dot{H}|)^{1/4}$, $\Lambda_{\rm s.b.} \sim c_s M_{\rm Pl}^2 |\dot{H}|$

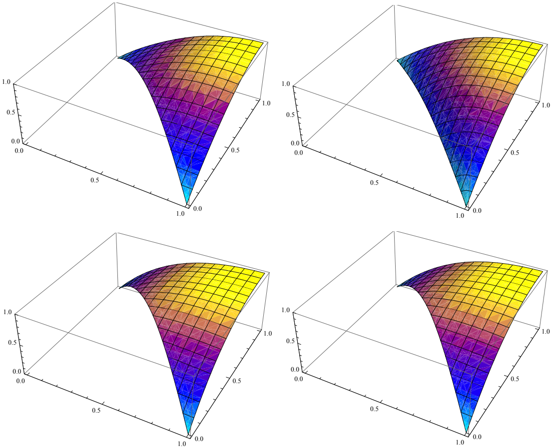
3-pt correlators

The main interactions leading to new effects are due to M_2^4 and M_3^4 and are given by

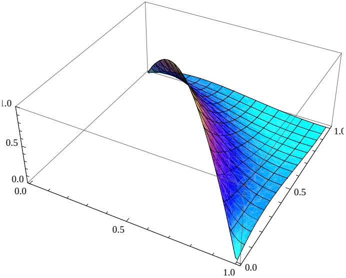
$$
\begin{aligned}
\mathcal{L}_{I}^{(3)} &= M_{\mathrm{Pl}}^2 a^3 |\dot{H}| \dot{\pi}^2 \Sigma(\nabla^2)\dot{\pi} \\
\mathcal{L}_{II_1}^{(3)} &= -M_{\mathrm{Pl}}^2 a^3 |\dot{H}| (\tilde{\nabla}\pi)^2 \Sigma(\nabla^2)\dot{\pi} \\
\mathcal{L}_{II_2}^{(3)} &= -M_{\mathrm{Pl}}^2 a^3 |\dot{H}| \frac{2M_3^4 c_{\mathrm{s}}^2}{3M_2^4}\, \dot{\pi}\Sigma(\nabla^2)\left(\dot{\pi}\Sigma(\nabla^2)\dot{\pi}\right) \\
\mathcal{L}_{III}^{(3)} &= M_{\mathrm{Pl}}^2 a^3 |\dot{H}| \frac{2M_2^2 \tilde{M}_3 c_{\mathrm{s}}^4}{3M_3^3} \left(\Sigma(\nabla^2)\dot{\pi}\right)^3
\end{aligned}
$$

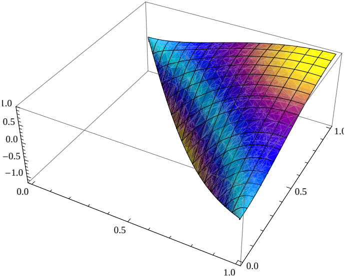
where $\Sigma(\tilde{\nabla}^2) \to -\frac{\Lambda_{\mathrm{UV}}^2}{\tilde{\nabla}^2}$

leading to



as well as folded and orthogonal shapes, arising from linear combinations of the four basic shapes





The non linearity parameters read

$$f_{\rm NL}^{\rm equil}(v_{\rm ph},\tilde{c}_3,\tilde{d}_3) = 0.0157 + 1.8961 v_{\rm ph}^{-2} + 0.0128\tilde{c}_3 v_{\rm ph}^{-2} + 0.0167\tilde{d}_3 v_{\rm ph}^{-4},$$

$$f_{\rm NL}^{\rm ortho}(v_{\rm ph},\tilde{c}_3,\tilde{d}_3) = 0.0005 + 0.1719 v_{\rm ph}^{-2} - 0.0004\tilde{c}_3 v_{\rm ph}^{-2} - 0.0003\tilde{d}_3 v_{\rm ph}^{-4},$$

$$f_{\rm NL}^{\rm flat}(v_{\rm ph},\tilde{c}_3,\tilde{d}_3) = 0.0028 + 0.3182 v_{\rm ph}^{-2} + 0.0024\tilde{c}_3 v_{\rm ph}^{-2} + 0.0031\tilde{d}_3 v_{\rm ph}^{-4},$$

which can be inverted to yield

$$\frac{\Lambda_{\rm UV}}{H} = -0.0009 + 38.4502 f_{\rm NL}^{\rm equil} - 29.577 f_{\rm NL}^{\rm ortho} - 209.997 f_{\rm NL}^{\rm flat},$$

$$\tilde{c}_3 \frac{\Lambda_{\rm UV}}{H} = 3.5240 + 46461.8 f_{\rm NL}^{\rm equil} - 41701.4 f_{\rm NL}^{\rm ortho} - 254330 f_{\rm NL}^{\rm flat},$$

$$\tilde{d}_3 \frac{\Lambda_{\rm UV}^2}{H^2} = -3.54037 - 39917.2 f_{\rm NL}^{\rm equil} + 35320.9 f_{\rm NL}^{\rm ortho} + 218778 f_{\rm NL}^{\rm flat}.$$

where $\tilde{c}_3, \tilde{d}_3$ dimensionless combinations of the couplings $M_2, M_3, \tilde{M}_3$.

Input from PLANCK/BICEP $\rightarrow$ Bounds on $\Lambda_{\rm UV}, \tilde{c}_3, \tilde{d}_3$

Conclusions

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Conclusions

- Including UV heavy modes corresponds to a derivative expansion of the standard EFT formalism
- Non-trivial change in the dispersion relation at horizon exit leading to novel interpretations for the cosmological observables: constraints on the scale of UV physics
- Heavy fields consistent with PLANCK/BICEP, e.g. assuming $\Lambda_{\rm UV} \sim \Lambda_{\rm GUT}$ requires $f_{\rm NL} = \mathcal{O}(1)$
- Shapes are highly degenerate with the ones obtained in absence of heavy fields

Thank you !

$$\begin{pmatrix} f_{\mathrm{NL}}^{\mathrm{equil}} \\ f_{\mathrm{NL}}^{\mathrm{ortho}} \\ f_{\mathrm{NL}}^{\mathrm{flat}} \end{pmatrix} = \begin{pmatrix} \frac{S_I * S_{\mathrm{equil}}}{S_{\mathrm{equil}} * S_{\mathrm{equil}}} & \frac{S_{II} * S_{\mathrm{equil}}}{S_{\mathrm{equil}} * S_{\mathrm{equil}}} & \frac{S_{III} * S_{\mathrm{equil}}}{S_{\mathrm{equil}} * S_{\mathrm{equil}}} & \frac{S_{II'} * S_{\mathrm{equil}}}{S_{\mathrm{equil}} * S_{\mathrm{equil}}} \\ \frac{S_I * S_{\mathrm{ortho}}}{S_{\mathrm{ortho}} * S_{\mathrm{ortho}}} & \frac{S_{II} * S_{\mathrm{ortho}}}{S_{\mathrm{ortho}} * S_{\mathrm{ortho}}} & \frac{S_{III} * S_{\mathrm{ortho}}}{S_{\mathrm{ortho}} * S_{\mathrm{ortho}}} & \frac{S_{II'} * S_{\mathrm{ortho}}}{S_{\mathrm{ortho}} * S_{\mathrm{ortho}}} \\ \frac{S_I * S_{\mathrm{flat}}}{S_{\mathrm{flat}} * S_{\mathrm{flat}}} & \frac{S_{II} * S_{\mathrm{flat}}}{S_{\mathrm{flat}} * S_{\mathrm{flat}}} & \frac{S_{III} * S_{\mathrm{flat}}}{S_{\mathrm{flat}} * S_{\mathrm{flat}}} & \frac{S_{II'} * S_{\mathrm{flat}}}{S_{\mathrm{flat}} * S_{\mathrm{flat}}} \end{pmatrix} \begin{pmatrix} f_{\mathrm{NL}}^{I} \\ f_{\mathrm{NL}}^{II} \\ f_{\mathrm{NL}}^{III} \\ f_{\mathrm{NL}}^{II'} \end{pmatrix}$$

Using the templates

$$S_{\mathrm{equil}}(x_1,x_2,x_3) = 6\left(-\frac{1}{x_1^3x_2^3} - \frac{1}{x_1^3x_3^3} - \frac{1}{x_2^3x_3^3} - \frac{2}{x_1^2x_2^2x_3^2} + \frac{1}{x_1x_2^2x_3^3} + 5\ \mathrm{perm}\right)$$

$$S_{\mathrm{ortho}}(x_1,x_2,x_3) = 6\left(-\frac{3}{x_1^3x_2^3} - \frac{3}{x_1^3x_3^3} - \frac{3}{x_2^3x_3^3} - \frac{8}{x_1^2x_2^2x_3^2} + \frac{3}{x_1x_2^2x_3^3} + 5\ \mathrm{perm}\right)$$

$$S_{\mathrm{flat}}(x_1,x_2,x_3) = 6\left(\frac{1}{x_1^3x_2^3} + \frac{1}{x_1^3x_3^3} + \frac{1}{x_2^3x_3^3} + \frac{3}{x_1^2x_2^2x_3^2} - \frac{1}{x_1x_2^2x_3^3} + 5\ \mathrm{perm}\right)$$