

The Niels Bohr  
International Academy



**APCTP**  
Asia Pacific Center for Theoretical Physics

# Testing Isotropy in the Local Universe

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S. Appleby, R. Battye, A. Moss (PRD 81 (2011) 081301)  
S. Appleby, A. Shafieloo (JCAP 1403 (2014) 007)  
S. Appleby, A. Shafieloo (arXiv:1405.4595)  
S. Appleby, A. Shafieloo ... (in preparation)

# Overview

- 1 Testing Isotropy using the CMB
  - Methodology
  - Results
- Testing Isotropy using Supernova
  - Methodology
  - Results
- Testing Isotropy using Large Scale Structure (LSS)
  - Methodology
  - Results
- Conclusions

# CMB

- Late time acceleration indicates new physics at the present horizon time  $\sim c/H_0$
- 
- WMAP and Planck both indicate the existence of a number of anomalies at large scales.
- An ellipsoidal Universe will induce a quadrupole in the CMB power spectrum (Campanelli et al. 06)
- Could we explain the observed low value with dark energy?
- The late time acceleration is typically attributed to a fluid with an isotropic pressure tensor
- What if the late time acceleration is anisotropic?

# CMB

- The simplest relaxation of the isotropic assumption is to take a diagonal pressure tensor with three equations of state, which we decompose as

$$P_i{}^j = \rho_{\text{de}} [w\delta_i{}^j + \Delta w_i{}^j] ,$$

- The metric is generalized from FRW to a Bianchi I spacetime

$$ds^2 = a^2 \left( -d\eta^2 + \gamma_{ij}(\eta) dx^i dx^j \right) ,$$

- Expansion equations

$$\begin{aligned} 3\mathcal{H}^2 &= 8\pi G a^2 \rho_{\text{tot}} + \frac{1}{2} \sigma^2 , & \sigma_{ij} &= \frac{1}{2} \frac{d}{d\eta} \gamma_{ij} \\ \rho_{\text{de}}' &= -3\mathcal{H}(1+w)\rho_{\text{de}} - \sigma_j{}^i \Delta w_i{}^j \rho_{\text{de}} , \\ \sigma_i{}^{j'} &= -2\mathcal{H}\sigma_i{}^j + 8\pi G a^2 \Delta w_i{}^j \rho_{\text{de}} , \end{aligned}$$

# CMB

- The effect of the shear on the Temperature anisotropy

$$\frac{\Delta T}{T}(\hat{n}) = - \int_{\eta_{\text{rec}}}^{\eta_0} \sigma_{ij} \hat{n}^i \hat{n}^j d\eta ,$$

- Linearizing in the anisotropic component

$$\frac{\Delta T}{T}(\hat{n}) = -\Delta w_{ij} \hat{n}^i \hat{n}^j J(\Omega_{\text{m}}, w) ,$$

$$J(\Omega_{\text{m}}, w) = 3(1 - \Omega_{\text{m}}) \int_{a_{\text{rec}}}^1 \frac{da}{a^4 E(a)} \int_0^a \frac{db}{b^{1+3w} E(b)} ,$$

$$\Delta w_{ij} = \text{diag}(\Delta w_1, \Delta w_2, -(\Delta w_1 + \Delta w_2))$$

$$\frac{\Delta T}{T}(\hat{n}) = -J(\Omega_{\text{m}}, w) \left( \Delta w_1 \sin^2 \theta \cos^2 \phi \right.$$

$$\left. + \Delta w_2 \sin^2 \theta \sin^2 \phi - (\Delta w_1 + \Delta w_2) \cos^2 \theta \right) .$$

# CMB

It is clear that the dark energy anisotropy induces a quadrupole at linear order, with the following coefficients

$$a_{2,2} = a_{2,2}^{\text{I}} - \sqrt{\frac{2\pi}{15}} J(\Omega_{\text{m}}, w) (\Delta w_1 - \Delta w_2) ,$$

$$a_{2,1} = a_{2,1}^{\text{I}} ,$$

$$a_{2,0} = a_{2,0}^{\text{I}} + \sqrt{\frac{4\pi}{5}} J(\Omega_{\text{m}}, w) (\Delta w_1 + \Delta w_2) ,$$

$$a_{2,-1} = a_{2,-1}^{\text{I}} ,$$

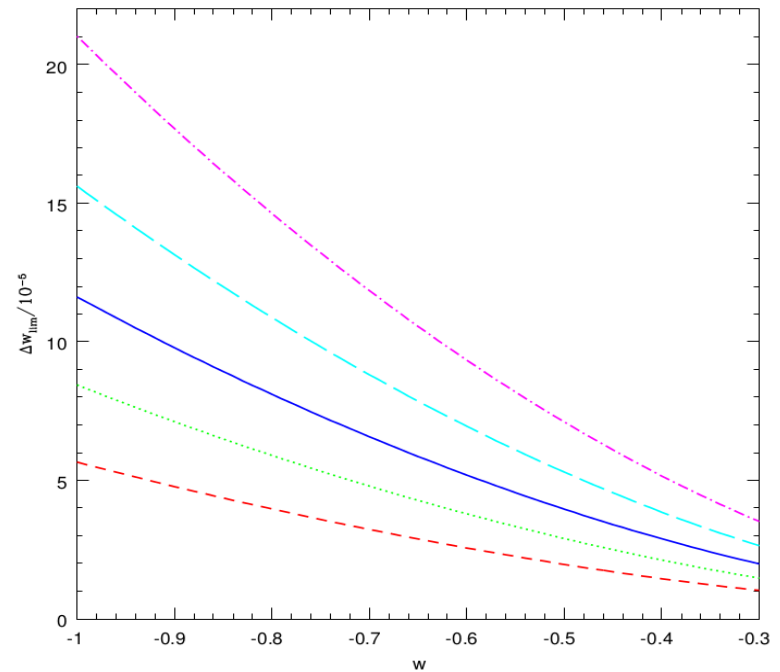
$$a_{2,-2} = a_{2,-2}^{\text{I}} - \sqrt{\frac{2\pi}{15}} J(\Omega_{\text{m}}, w) (\Delta w_1 - \Delta w_2) ,$$

$$C_2^{\text{A}} = \frac{8\pi}{75} [J(\Omega_{\text{m}}, w)]^2 (\Delta w)^2$$
$$(\Delta w)^2 = \Delta w_i^j \Delta w_j^i = 2(\Delta w_1^2 + \Delta w_2^2 + \Delta w_1 \Delta w_2)$$

# CMB

- The resulting constraint on the equation of state parameters is dependent upon the underlying cosmology.

$w$	$\Delta w$
-1	$1.1 \times 10^{-4}$
-2/3	$6.0 \times 10^{-5}$
-1/3	$2.2 \times 10^{-5}$



# SN Ia

- Cosmological anisotropy is severely constrained by the CMB (although we only focused on a simple parameterization)
- Other effects, such as large scale bulk motion, can also introduce an anisotropic signal into other data sets
- The local group is moving with velocity  $V \sim 400$  km/s in direction  $(b, l) = (30, 276)$
- Can we observe such local anisotropic signals in the data?
- Can we estimate at what redshift the bulk velocity remains coherent?
- Based on work by Colins et al (2010)



# SN Ia

- We search for bulk flow in the Supernova catalogs using a null test
- The algorithm:
  - Take a Supernova catalog, and find the best fit isotropic cosmological model

$$\chi^2 = \delta\mu^T \Sigma^{-1} \delta\mu$$

- We then construct the error weighted distance modulus residuals

$$q_i(z_i, \theta_i, \phi_i) = \frac{\mu_i(z_i, \theta_i, \phi_i) - \tilde{\mu}(z_i)}{\sigma_i}$$

# SN Ia

- We then construct the 'Q' function at regular points on the sphere

$$Q(\theta, \phi) = \sum_{i=1}^N q_i(z_i, \theta_i, \phi_i) W(\theta, \phi, \theta_i, \phi_i)$$

$$W(\theta, \phi, \theta_i, \phi_i) = \frac{1}{\sqrt{2\pi}\delta} \exp \left[ -\frac{L(\theta, \phi, \theta_i, \phi_i)^2}{2\delta^2} \right]$$

$$L(\theta, \phi, \theta_i, \phi_i) = 2 \arcsin \frac{R}{2}$$

$$R = \left( [\cos(\theta_i) \cos(\phi_i) - \cos(\theta) \cos(\phi)]^2 + [\cos(\theta_i) \sin(\phi_i) - \cos(\theta) \sin(\phi)]^2 + [\sin(\theta_i) - \sin(\theta)]^2 \right)^{1/2}$$

- We find the maximum dipole of this function on the sphere
- A large value of  $\Delta Q$  (dipole) indicates a large deviation from Isotropy in the catalog
- However, within the context of standard cosmology we do not expect this function to be zero.

# SN Ia

- We create  $N=1000$  realisations of the data, in which we keep the data points at fixed positions on the sky. The distance modulus is estimated (for now...) according to

$$\mu_i = \tilde{\mu}(z_i, \Omega_{m0}) + \delta\mu_{G,i}$$

- For each realisation we obtain the largest  $\Delta Q$  dipole value, and construct an empirical PDF from which we estimate the significance of the data value
- Our null hypothesis is that the Supernova catalogs are consistent with an isotropic, zero bulk velocity expansion rate
- We are basically testing the consistency of the catalogs to be used for cosmological parameter estimation(?)

# Results - SNIa

- We perform our analysis using five redshift bins
- The number of SNIa used in each survey and in each redshift bin are exhibited

Catalog	$0.015 \leq z < 0.025$	$0.025 \leq z < 0.035$	$0.035 \leq z < 0.045$	$0.045 \leq z < 0.06$	$0.06 \leq z < 0.1$
Union 2.1	61	51	15	17	19
Constitution	53	40	11	12	8
LOSS	76	64	23	17	19
Combined	98	67	22	27	12

# Results - SNIa

$\Delta z$	Catalog	$b_{\max}$	$\ell_{\max}$	$p$	$\delta$	$\Delta z$	Catalog	$b_{\max}$	$\ell_{\max}$	$p$	$\delta$
$0.015 \leq z < 0.025$	Union 2.1	54°	256°	0.059	0.65	$0.015 < z \leq 0.025$	Union 2.1	54°	256°	0.059	0.65
	Const (SALT II)	20°	284°	0.624	1.55		Const (SALT II)	20°	284°	0.624	0.65
	Const (MLCS 17)	70°	248°	0.249	0.35		Const (MLCS 17)	70°	248°	0.249	0.35
	LOSS	32°	263°	0.328	0.50		LOSS	32°	263°	0.328	0.50
	Combined	41°	259°	0.038	0.50		Combined	41°	259°	0.038	0.50
$0.025 \leq z < 0.035$	Union 2.1	-27°	288°	0.632	1.10	$0.015 < z \leq 0.035$	Union 2.1	32°	274°	0.139	0.80
	Const (SALT II)	45°	317°	0.201	0.65		Const (SALT II)	40°	295°	0.192	0.65
	Const (MLCS 17)	47°	313°	0.154	0.50		Const (MLCS 17)	54°	284°	0.126	0.65
	LOSS	38°	320°	0.147	1.40		LOSS	34°	274°	0.065	0.50
	Combined	61°	349°	0.229	1.10		Combined	40°	277°	0.026	0.50
$0.035 \leq z < 0.045$	Union 2.1	40°	328°	0.154	0.65	$0.015 < z \leq 0.045$	Union 2.1	36°	288°	0.049	0.65
	Const (SALT II)	25°	306°	0.649	1.40		Const (SALT II)	38°	298°	0.110	0.65
	Const (MLCS 17)	40°	320°	0.143	0.65		Const (MLCS 17)	50°	295°	0.045	0.50
	LOSS	-27°	292°	0.521	0.95		LOSS	30°	277°	0.043	0.50
	Combined	50°	345°	0.219	0.35		Combined	39°	280°	0.020	0.50
$0.045 \leq z < 0.06$	Union 2.1	-45°	54°	0.402	1.25	$0.015 < z \leq 0.06$	Union 2.1	31°	299°	0.112	0.35
	Const (SALT II)	-61°	68°	0.393	0.35		Const (SALT II)	22°	310°	0.207	1.25
	Const (MLCS 17)	-56°	68°	0.040	1.55		Const (MLCS 17)	45°	306°	0.308	0.65
	LOSS	48°	50°	0.712	0.65		LOSS	32°	278°	0.053	0.50
	Combined	30°	83°	0.495	0.35		Combined	38°	270°	0.025	0.50
$0.06 \leq z < 0.1$	Union 2.1	-2°	53°	0.415	1.10	$0.015 < z \leq 0.1$	Union 2.1	30°	298°	0.150	0.35
	Const (SALT II)	56°	317°	0.495	0.35		Const (SALT II)	36°	302°	0.145	0.50
	Const (MLCS 17)	-4°	65°	0.331	1.40		Const (MLCS 17)	45°	309°	0.346	0.50
	LOSS	57°	346°	0.234	0.35		LOSS	35°	281°	0.058	0.50
	Combined	-47°	61°	0.216	0.35		Combined	36°	280°	0.068	0.50

# Results - SNIa

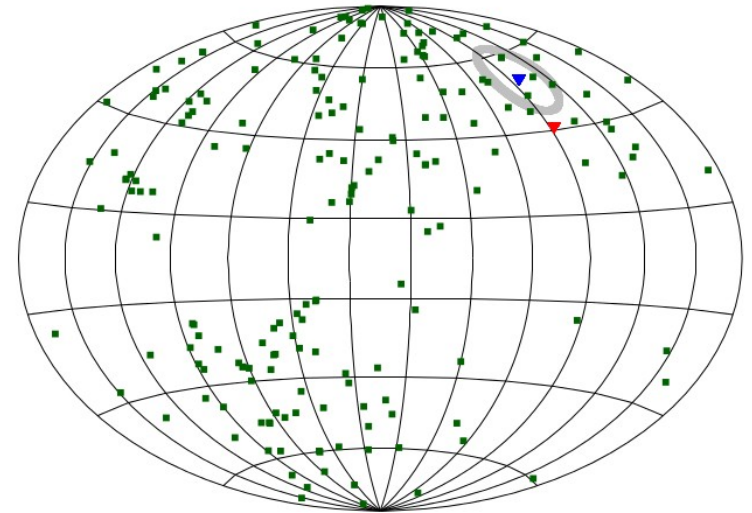
What is the effect of correlations?

$$\Phi^T \Sigma \Phi = \Lambda$$

$\Delta z$	$p_{\text{diag}}$	$p_{\text{full}}$
$0.015 \leq z < 0.025$	0.038	0.067
$0.015 \leq z < 0.035$	0.026	0.077
$0.015 \leq z < 0.045$	0.020	0.074
$0.015 \leq z < 0.060$	0.025	0.079
$0.015 \leq z < 0.100$	0.068	0.124

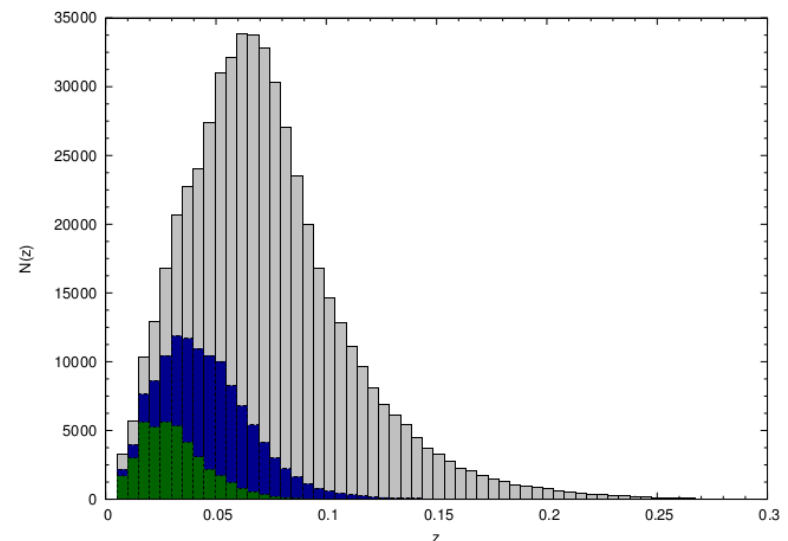
# SN Ia

- We are testing for coherent bulk flows in the Supernova catalogs.
- Care must be taken when we attempt to reconstruct the direction of maximal anisotropy
- The direction that the method picks up might be biased by the inhomogeneous distribution of data on the sky
- One must test the ability of the method to reproduce the 'correct' anisotropic signal



# Galaxy Distribution

- Can we estimate the direction of the bulk flow using the local galaxy distribution?
- We adopt the 2MASS galaxy sample, only limited spectroscopic redshifts known (2MPZ – Peacock et al. 2013)
- Our method – decompose the sky into  $N=192$  equal area pixels (according to the HEALPIX scheme)
- Search for differences in the galaxy distributions amongst the different pixels
- We use the shape of the luminosity function as a measure of the anisotropy between any two patches
- We take successive magnitude cuts





# Galaxy Distribution

- The luminosity function is the number of a galaxies binned in absolute magnitude, which we must infer from their observed apparent magnitudes and redshifts

$$M_i = m_i - 5 \log_{10} [d_l(z_i)] - 25 + K(z_i, K_{s,i}, J_i) + E(z)$$

$$K(z, K_s, J) = \sum_{k,j} a_{k,j} z^k (J - K_s)^j$$

- The luminosity function can be estimated in multiple ways

- Using a parametric form – Schechter function

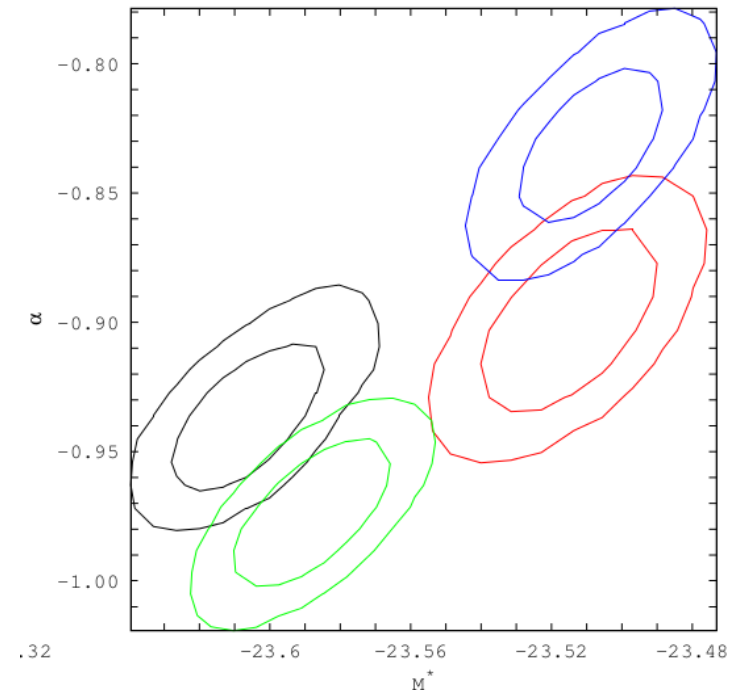
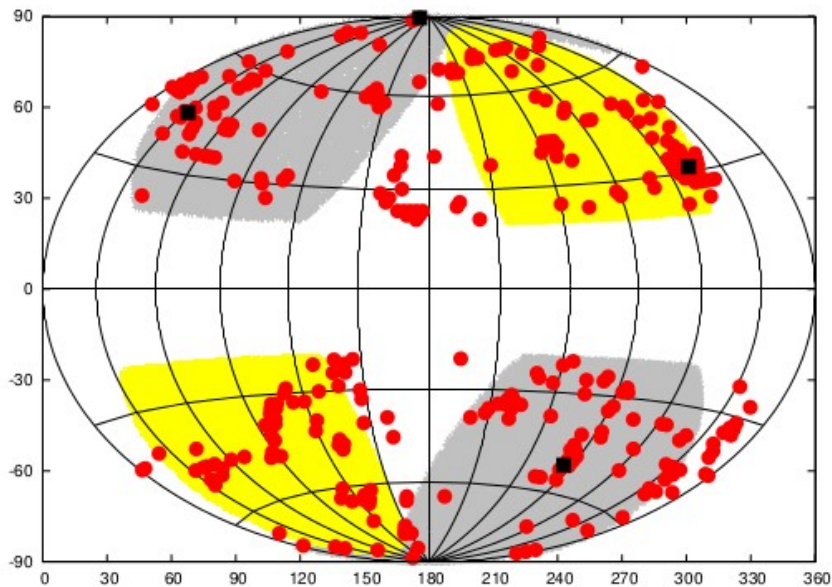
$$\phi(M) dM = 0.4 \ln[10] \phi^* 10^{0.4(1+\alpha)(M-M^*)} \exp \left[ -10^{0.4(M-M^*)} \right] dM$$

$$p_i = P(M_i | z_i) = \frac{\phi(M_i)}{\int_{M_{\min}(z_i)}^{M_{\max}(z_i)} \phi(M') dM'} \quad \mathcal{L} = \prod_{i=1}^{N_{\text{gal}}} p_i$$

- In a model independent manner (for example by binning) – Efstathiou et al.

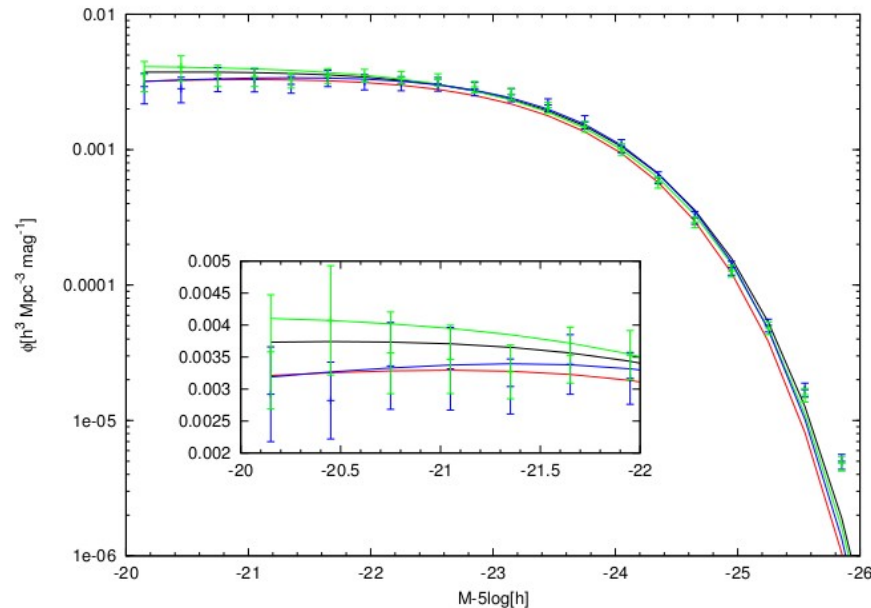
# Galaxy Distribution

- The Schechter function exhibits some anisotropy in the sky – if we decompose into four large patches and consider a magnitude cut of  $K_s < 13.5\text{mag}$  -



# Galaxy Distribution

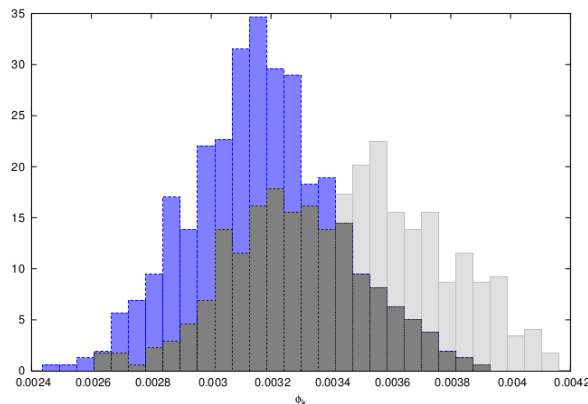
- The Schechter function exhibits some evidence of asymmetry on the sky...
- However if we fit the Schechter function to the binned estimator of the luminosity function, we find that within the error bars associated with photometric redshifts, there is no evidence for anisotropy.



# Galaxy Distribution

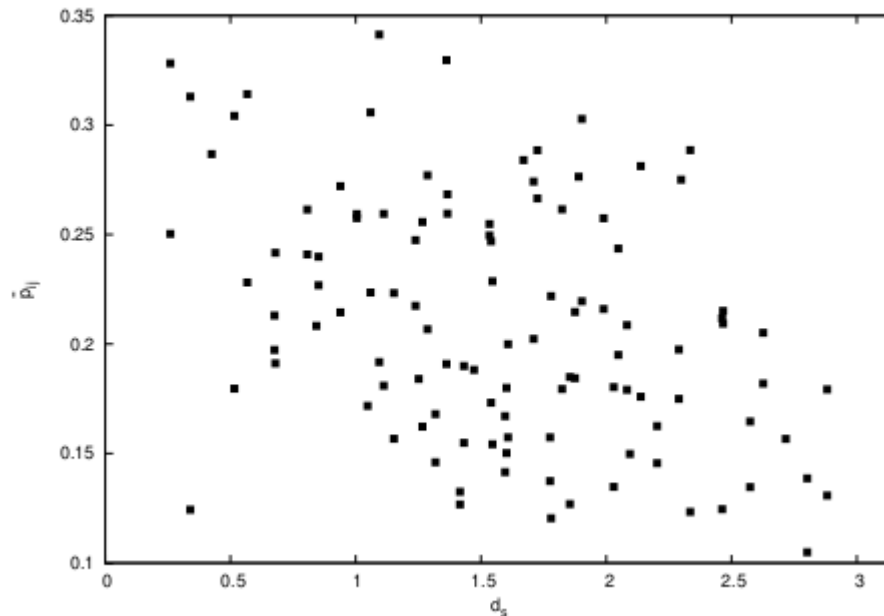
- Returning to our small,  $N=192$  equal area patches. We estimate the Luminosity function in each patch and calculate the 'difference' between each.
- The primary source of error in the luminosity function lies in the photometric redshift uncertainties estimated for the higher redshift sample.
- We construct  $N=1000$  realisations of the 2MASS galaxy sample in each patch, estimating the spectroscopic redshifts from an empirical PDF when not known.
- Finally, in each bin we calculate

$$r_j = \frac{\sum_{i=1}^{N_{\text{pix}}} (\bar{p}_{ij} - \tilde{p}_{ij})(d_{s,ij} - \bar{d}_{s,ij})}{\sqrt{\sum_{i=1}^{N_{\text{pix}}} (\bar{p}_{ij} - \tilde{p}_{ij})^2} \sqrt{\sum_{i=1}^{N_{\text{pix}}} (d_{s,ij} - \bar{d}_{s,ij})^2}}$$



# Galaxy Distribution

- The pixel which possesses the highest correlation coefficient is located at  $(b,l)=(30,315)$  in galactic coordinates
- The direction is broadly in agreement with other findings of a dipole in the galaxy distribution (Huterer et al, 2010).



# Conclusions

- There are multiple ways of testing our data for the existence of anisotropic signals, I have focused on three.
- The CMB places extremely tight constraints on the simplest Bianchi I anisotropic dark energy model. More complicated scenario's, in which the anisotropy manifests itself at low redshift, might alleviate this concern.
- There is mild evidence of a bulk flow in both the SN and large scale structure data.
- Photometric redshift information is not sufficient to make any statement regarding the local bulk flow
- To make a more definitive statement, we require spectroscopic redshift information