Mimetic Cosmology

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• Mimetic Dark Matter

$$G_{\mu\nu} = 0 \Rightarrow g_{\mu\nu} = \eta_{\mu\nu}$$

Modification of Eistein equations

$$\begin{cases} G_{\mu\nu} - G\,\partial_\mu\varphi\,\partial_\nu\varphi = 0, \\ \partial_\mu\varphi\,\partial^\mu\varphi = 1 \end{cases}$$

$$\Downarrow$$

$$G\left(1 - (\partial\varphi)^2\right) = 0 \Rightarrow G \neq 0$$

???

In synch. coord. system $\varphi = t$ and $(G\,\partial^\mu\varphi)_{;\mu} = 0 \Rightarrow G \propto \frac{c(x^i)}{\sqrt{\gamma}} \propto a^{-3}$

- Action

- $$S = \int \left(-\frac{1}{2} R(g) + \mathcal{L}_m\right) \sqrt{-g}\, d^4x$$

$$g_{\mu\nu} = H \tilde{g}_{\mu\nu}, \quad H \equiv \tilde{g}^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi$$

$$S = \int \dots R(g(\tilde{g})) + \dots$$

variation with respect to g $\Longrightarrow$

$$G_{\mu\nu} - T_{\mu\nu} - (G - T) \partial_\mu \varphi \partial_\nu \varphi = 0$$

and

$$g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi = H^{-1} \tilde{g}^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi = 1$$

$$S = \int \Big[-\frac{1}{2} R(g) + \underbrace{\lambda \left(\partial\varphi^2 - 1 \right)}_{\text{dust}}$$

$$+ \underbrace{\lambda' \left(\partial_\mu V^\mu - 1 \right)}_{\text{cosm. const.}}$$

(Brunster, Henneaux)

$$+ \underbrace{F(\varphi^{int})\, \varphi}_{\text{to preserve dust from decay during inflation}} \Big] \sqrt{-g}\, d^4x$$

Generalization

$$S = \int \left(\dots + V(\varphi) \right) \sqrt{-g}\, d^4x$$

$$G_{\mu\nu} = T_{\mu\nu} + \underbrace{(G - T - 4V)\partial_\mu\varphi\,\partial_\nu\varphi + V g_{\mu\nu}}_{\tilde T_{\mu\nu}}$$

$$\partial_\mu\varphi\,\partial^\mu\varphi = 1$$

$$\tilde T_{\mu\nu} = (\tilde\varepsilon + \tilde p)\,\partial_\mu\varphi\,\partial_\nu\varphi - \tilde p\, g_{\mu\nu}$$

where

$$\tilde\varepsilon = G - T - 3V$$

$$\tilde p = -V$$

$$\tilde T_{\mu\nu}{}^{;\nu} = 0 \Rightarrow \nabla^\mu\left((\tilde\varepsilon - V)\partial_\mu\varphi\right) = -\frac{\partial V}{\partial\varphi}$$

Homogeneous expanding Universe

$$ds^2 = dt^2 - a^2(t)\,\delta_{ik}\,dx^i dx^k$$

$$(\partial\varphi)^2 = 1 \Rightarrow \varphi = \pm t + A,$$

• $$\tilde{p} = -V(\varphi) = -V(t)$$

$$\frac{1}{a^3}\left(a^3(\tilde{\varepsilon} - V)\right) = -\dot{V} \Rightarrow$$

•• $$\tilde{\varepsilon} = V - \frac{1}{a^3}\int a^3 \dot{V}\,dt = \frac{3}{a^3}\int a^2 V\,da$$

Given $V(\varphi) \Rightarrow \tilde{p} = -V(\varphi = t) \Rightarrow \tilde{\varepsilon}(t)$

$$H^2 = \frac{1}{3}\tilde{\varepsilon} = \frac{1}{a^3}\int a^2 V\, da$$

$$2\dot{H} + 3H^2 = V(t)$$

$y = a^{3/2}$

$$\ddot{y} - \frac{3}{4}V(t)\,y = 0$$

Given $a(t)$ we can find the needed potential

$$V(\varphi) = V(t) = \frac{4}{3}\,\frac{\left(a^{3/2}\right)^{\cdot\cdot}}{a^{3/2}}$$

Solutions

(1) $$V(\phi) = \frac{\alpha}{\varphi^2} = \frac{\alpha}{t^2},$$

$$\ddot{y} - \frac{3\alpha}{4t^2}\, y = 0,$$

$$a \equiv y^{2/3} = t^{\frac{1}{3}\left(1+\sqrt{1+3\alpha}\right)}\left(1 + A\, t^{-\sqrt{1+3\alpha}}\right)^{2/3}$$

$\alpha > -\frac{1}{3}$

$$w = \frac{\tilde{p}}{\tilde{\varepsilon}} = -3\alpha\left(1+\sqrt{1+3\alpha}\right)^{-2}$$

(a) $\alpha = -\frac{1}{3}$, $\quad \tilde{p} = +\tilde{\varepsilon}$,

(b) $\alpha = -\frac{1}{4}$. $\quad \tilde{p} = \frac{1}{3}\tilde{\varepsilon}$,

(c) $\alpha \ll 1 \qquad \tilde{p} \approx 0$.

② $V = \alpha\varphi^n = \alpha t^n$ $(n \neq -2)$

$$a \propto t^{1/3}\, Z^{2/3}_{\frac{1}{n+2}}\left(i \frac{\sqrt{-3\alpha}}{n+2}\, t^{\frac{n+2}{2}}\right)$$

- $n < -2$ (V decays faster than $1/\varphi^2$)

  $a \propto t^{2/3} \equiv$ dust

- $n > -2$
  - $\alpha < 0$ $(\hat{p} > 0)$ we have an oscillating singular Universe
  - $\alpha > 0$ $(\hat{p} < 0)$ we have inflation

$n = 0 \Rightarrow a \propto \exp(\dots t) \equiv$ de Sitter

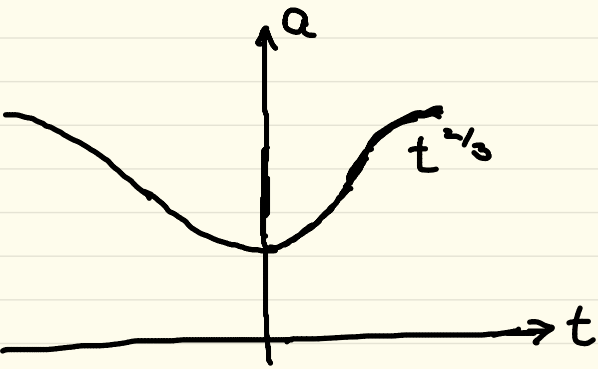
$n = 2$ $(V \propto \varphi^2) \Rightarrow a \propto t^{-1/3} \exp\left(\sqrt{\frac{\alpha}{12}}\, t^2\right)$

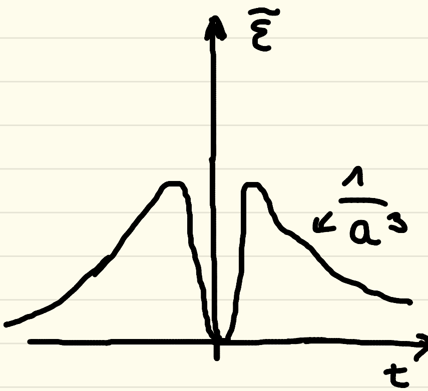
$V = \frac{\alpha\varphi^2}{e^{\varphi}+1}$ gives inflation with graceful exit

③ Bouncing Universe

$$V = \frac{4}{3}\frac{1}{(1+\varphi^2)^2}$$

$$a \propto (1+t^2)^{1/3}$$





$$\tilde{\varepsilon} + \tilde{p} = \frac{4}{3}\frac{t^2-1}{(1+t^2)^2}$$

$\tilde{\varepsilon} + \tilde{p} > 0$ for $|t| > 1$

$\tilde{\varepsilon} + \tilde{p} < 0$ for $|t| < 1$

Perturbations

$$\varphi = t + \delta\varphi$$

$$ds^2 = (1+2\Phi)dt^2 - (1-2\Phi)a^2\delta_{ik}dx^i dx^k$$

$$(\partial\varphi)^2 = 1 \Longrightarrow \delta\dot{\varphi} = \Phi$$

0-i $$\dot{\Phi} + H\Phi = \frac{1}{2}(\tilde{\varepsilon} + \tilde{p})\delta\varphi$$

$$\delta\ddot{\varphi} + H\delta\dot{\varphi} + \dot{H}\delta\varphi = 0,$$

$\delta\varphi = A\frac{1}{a}\int a\,dt$ and $\Phi = A\left(1 - \frac{H}{a}\int a\,dt\right)$

as for longwave pert.

$\tilde{p} \neq 0$ but nevertheless $c_s^2 = 0$ for pert.

$$S = \int \ldots + \lambda\left(\partial\varphi^2 - 1\right) + \frac{1}{2}\gamma\left(\Box\varphi\right)^2$$

no extra degrees of freedom!

- Background

$$2\dot{H} + 3H^2 = \frac{2}{2-3\gamma}V$$

- - Perturbations

$$\ddot{\delta\varphi} + H\dot{\delta\varphi} - \frac{c_s^2}{a^2}\Delta\delta\varphi + \dot{H}\delta\varphi = 0,$$

where

$$c_s^2 = \frac{\gamma}{2-3\gamma}$$

After inflation

$$\Phi_\lambda \sim \sqrt{\frac{c_s}{\gamma}}\, H_{c_s k \sim Ha}$$

$$\frac{T}{S} \simeq \begin{cases} c_s^{1/2} & \text{for} \quad c_s \ll 1, \\ c_s^{-1/2} & \text{for} \quad c_s \gg 1 \end{cases}$$