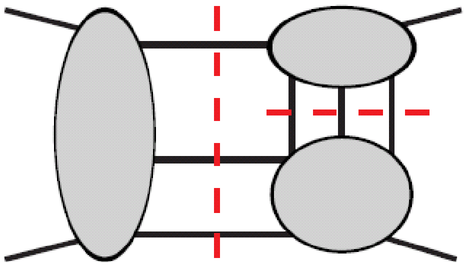
Enhanced Ultraviolet Cancellations in Supergravity

August 28, 2014

Copenhagen
Zvi Bern, UCLA

Recent papers with Scott Davies, Tristan Dennen, Yu-tin Huang, Sasha Smirnov and Volodya Smirnov.

Also earlier work with John Joseph Carrasco, Lance Dixon, Henrik Johansson and Radu Roiban.



Outline

1) A hidden structure in gauge and gravity amplitudes.

— a duality between color and kinematics.

— gravity from gauge theory.

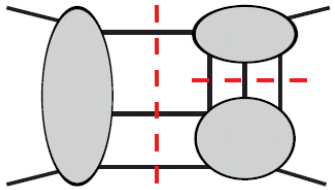
2) Review of ultraviolet properties of supergravity and standard arguments.

3) "Enhanced" UV cancellations supergravity. A new type of UV cancellations beyond the ones understood from standard symmetries.

4) Explicit calculations demonstrating enhanced UV cancellations in N = 4, 5 supergravity at 3, 4 loops

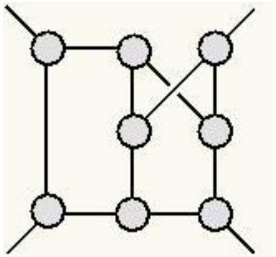
Our Basic Tools

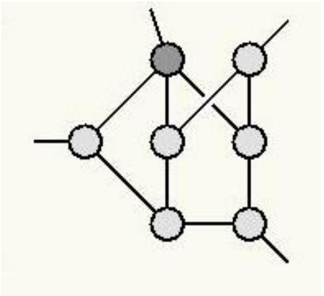
We have powerful tools for complete calculations including nonplanar contributions and for discovering new structures:



- **Unitarity Method.**

  ZB, Dixon, Dunbar, Kosower
  ZB, Carrasco, Johansson , Kosower





- **Duality between color and kinematics.**

  ZB, Carrasco and Johansson

- **Advanced loop integration technology.**

  Chetyrkin, Kataev and Tkachov; A.V. Smirnov; V. A. Smirnov, Vladimirov; Marcus, Sagnotti; Cazkon; etc

Many other tools and advances that I won't discuss here.
In this talk we will explain how above tools allow us to probe the UV properties of supergravity theories leading to some surprising results.

Duality Between Color and Kinematics

ZB, Carrasco, Johansson (BCJ)

See Johansson's talk

Consider five-point tree amplitude:

sum is over diagrams

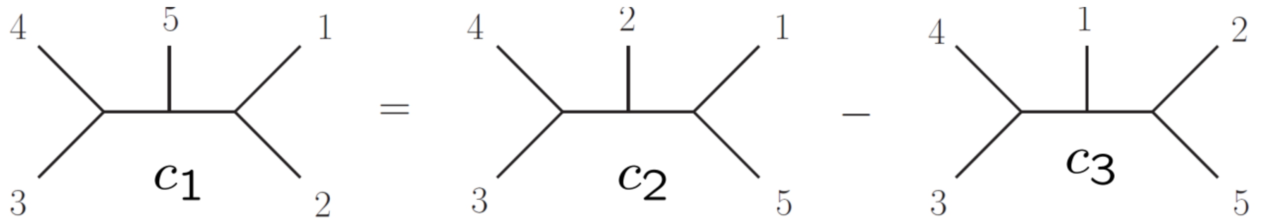
color factor

kinematic numerator factor

gauge theory

$$\mathcal{A}_5^{\text{tree}} = \sum_{i=1}^{15} \frac{c_i\, n_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

Feynman propagators



$$c_1 \equiv f^{a_3 a_4 b} f^{b a_5 c} f^{c a_1 a_2}, \quad c_2 \equiv f^{a_3 a_4 b} f^{b a_2 c} f^{c a_1 a_5}, \quad c_3 \equiv f^{a_3 a_4 b} f^{b a_1 c} f^{c a_2 a_5}$$

$$n_i \sim k_4 \cdot k_5\, k_2 \cdot \varepsilon_1\, \varepsilon_2 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5 + \cdots$$

$$\boxed{c_1 - c_2 + c_3 = 0 \Leftrightarrow n_1 - n_2 + n_3 = 0}$$

Claim: At n-points we can always find a rearrangement where color and kinematics satisfy the same algebraic constraint equations.

Nontrivial constraints on amplitudes in field theory and string theory

BCJ, Bjerrum-Bohr, Feng, Damgaard, Vanhove, ; Mafra, Stieberger, Schlotterer; Cachazo; Tye and Zhang; Feng, Huang, Jia; Chen, Du, Feng; Du, Feng, Fu; Naculich, Nastase, Schnitzer

BCJ

Gravity and Gauge Theory

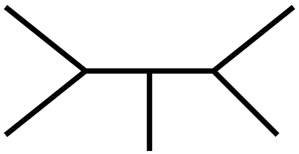
kinematic numerator color factor

gauge theory: $$\frac{1}{g^{n-2}}\mathcal{A}_n^{\text{tree}}(1,2,3,\ldots,n) = \sum_i \frac{n_i\, c_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

sum over diagrams with only 3 vertices

Assume we have:

$$c_1 + c_2 + c_3 = 0 \quad \Leftrightarrow \quad n_1 + n_2 + n_3 = 0$$



Then: $c_i \Rightarrow \tilde{n}_i$ **kinematic numerator of second gauge theory**

Proof: ZB, Dennen, Huang, Kiermaier

gravity: $$\boxed{-i\left(\frac{2}{\kappa}\right)^{(n-2)} \mathcal{M}_n^{\text{tree}}(1,2,\ldots,n) = \sum_i \frac{n_i\, \tilde{n}_i}{\prod_{\alpha_i} p_{\alpha_i}^2}}$$

Encodes KLT tree relations

Gravity numerators are a double copy of gauge-theory ones.

This works for ordinary Einstein gravity and susy versions.

Cries out for a unified description of the sort given by string theory!

Gravity From Gauge Theory

BCJ

$$-i\left(\frac{2}{\kappa}\right)^{(n-2)} \mathcal{M}_n^{\text{tree}}(1,2,\ldots,n) = \sum_i \frac{n_i\,\tilde{n}_i}{\prod_{\alpha_i} p^2_{\alpha_i}}$$

	n	$\tilde{n}$
***N* = 8 sugra:**	**(*N* = 4 sYM)$\times$**	**(*N* = 4 sYM)**
***N* = 5 sugra:**	**(*N* = 4 sYM)$\times$**	**(*N* = 1 sYM)**
***N* = 4 sugra:**	**(*N* = 4 sYM)$\times$**	**(*N* = 0 sYM)**

Spectrum controlled by simple tensor product of YM theories.
Recent papers show more sophisticated lower-susy cases.

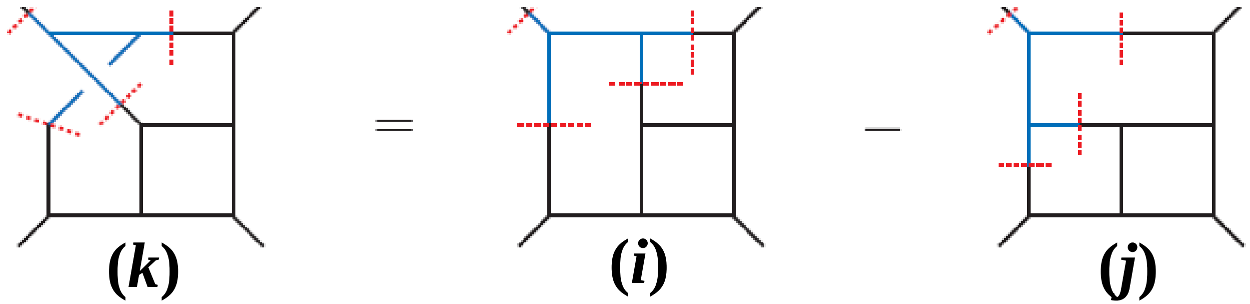
Anastasiou, Bornsten, Duff; Duff, Hughs, Nagy; Johansson and Ochirov; Carrasco, Chiodaroli, Günaydin and Roiban; ZB, Davies, Dennen, Huang and Nohle; Nohle; Chiodaroli, Günaydin, Johansson, Roiban.

See Johansson's talk for general constructions and new developments.

BCJ

Gravity integrands are free!

Ideas generalize to loops:



color factor

$$c_k = c_i - c_j$$

$$n_k = n_i - n_j$$

kinematic numerator

If you have a set of duality satisfying numerators.

To get:

gauge theory $\longrightarrow$ gravity theory

simply take

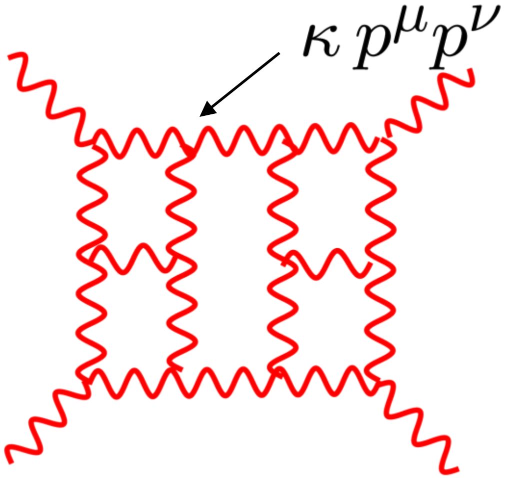
color factor $\longrightarrow$ kinematic numerator

$$c_k \longrightarrow n_k$$

Gravity loop integrands are trivial to obtain once we have gauge theory in a form where duality works.

Is a UV finite field theory of gravity possible?

$\kappa = \sqrt{32\pi G_N}$ ← Dimensionful coupling



Gravity: $$\int \prod_{i=1}^{L} \frac{dp_i^D}{(2\pi)^D} \frac{(\kappa\, p_j^\mu p_j^\nu)\cdots}{\text{propagators}}$$

Gauge theory: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{(g\, p_j^\nu)\cdots}{\text{propagators}}$$

- **Extra powers of loop momenta in numerator means integrals are badly behaved in the UV.**
- **Much more sophisticated power counting in supersymmetric theories but this is the basic idea.**

Reasons to focus on extended supergravity, especially *N* = 8:

- **With more susy expect better UV properties.** Cremmer and Julia
- **High symmetry implies simplicity.**

UV Finiteness of *N* = 8 Supergravity?

If *N* = 8 supergravity is perturbatively finite it would imply a new symmetry or non-trivial dynamical mechanism. No known symmetry can render a *D* = 4 gravity theory finite.

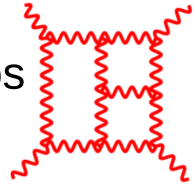
The discovery of such a mechanism would have a fundamental impact on our understanding of gravity.

Of course, perturbative finiteness is not the only issue for consistent gravity: Nonperturbative completions? High-energy behavior of theory? Realistic models?

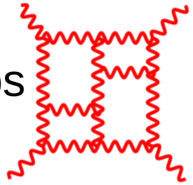
Consensus opinion for the late 1970's and early 1980's:
All supergravity theories would diverge by three loops and therefore are not viable as fundamental theories.

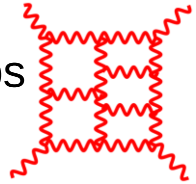
Feynman Diagrams for Gravity

SUPPOSE WE WANT TO CHECK IF CONSENSUS OPINION IS TRUE

3 loops  $\sim 10^{20}$ TERMS

No surprise it has never been calculated via Feynman diagrams.

4 loops  $\sim 10^{26}$ TERMS

5 loops  $\sim 10^{31}$ TERMS

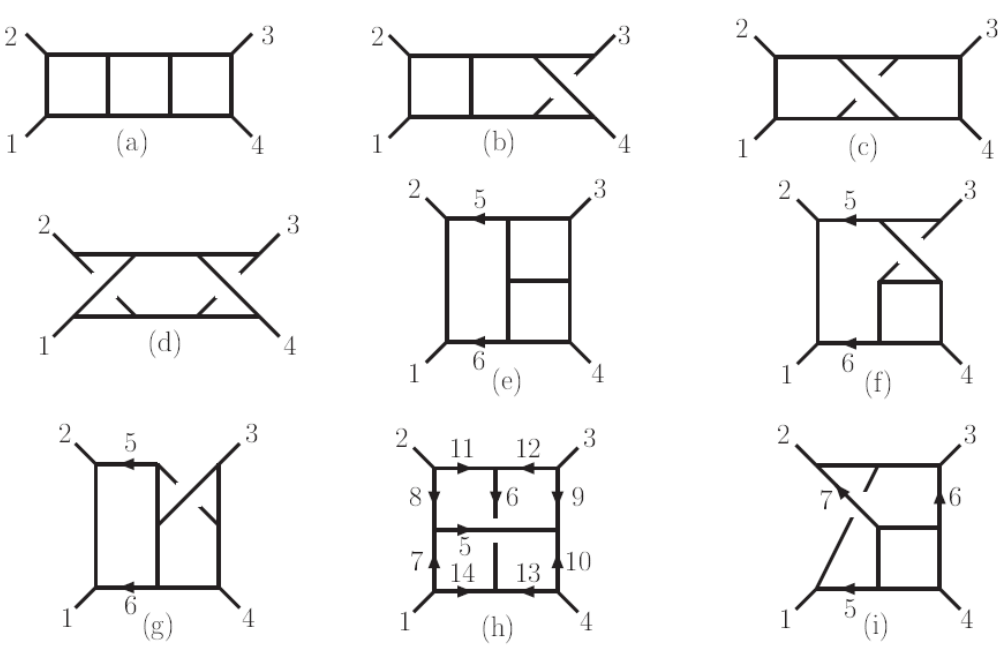
More terms than atoms in your brain!

- **Calculations to settle this seemed utterly hopeless!**
- **Seemed destined for dustbin of undecidable questions.**

Complete Three-Loop Result

Analysis of unitarity cuts shows highly nontrivial all-loop cancellations. ZB, Dixon and Roiban (2006); ZB, Carrasco, Forde, Ita, Johansson (2007)

To test completeness of cancellations, we decided to directly calculate potential three-loop divergence.



ZB, Carrasco, Dixon, Johansson, Kosower, Roiban (2007)

Three loops is not only ultraviolet finite it is "superfinite"— finite for *D* < 6.

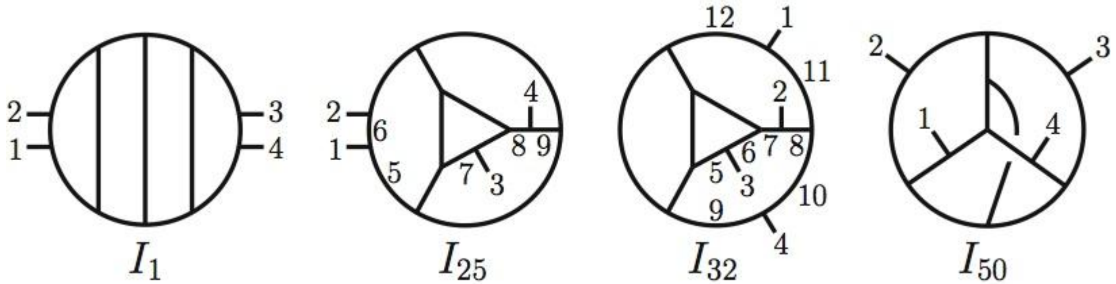
It is very finite!

Obtained via on-shell unitarity method.

Four-Loop $N = 8$ Supergravity Amplitude Construction

ZB, Carrasco, Dixon, Johansson, Roiban (2009)

Get 85 distinct diagrams or integrals.



$$M_4^{\text{4-loop}} = \left(\frac{\kappa}{2}\right)^{10} stu M_4^{\text{tree}} \sum_{S_4} \sum_{i=1}^{50} c_i I_i$$

Integral

symmetry factor

leg perms

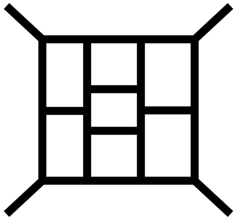
UV finite for $D <$ 11/2
It's very finite!

Duality between color and kinematic discovered by doing this calculation.

Current Status of N = 8 Divergences

Consensus that in $N = 8$ supergravity trouble starts at 5 loops and by 7 loops we have valid UV counterterm in $D = 4$ under all known symmetries (suggesting divergences).

Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Bjornsson ; Bossard , Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger



For $N = 8$ sugra in $D = 4$:

- **All counterterms ruled out until 7 loops.**
- **D^8R^4 counterterm available at 7 loops under all known symmetries. Oddly, it is not a full superspace integral.**

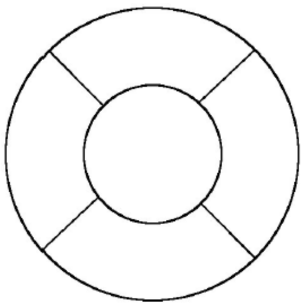
Bossard, Howe, Stelle and Vanhove

Based on this a reasonable person would conclude that $N = 8$ supergravity almost certainly diverges at 7 loops in $D = 4$.

Predictions of Ultraviolet Cancellations

Bjornsson and Green developed a first quantized form of Berkovits' pure-spinor formalism. mentioned in Green's talk

Key point: *all* supersymmetry cancellations are exposed.



They identify contributions that are poorly behaved. Only a miraculous cancellation can save us.

Poor UV behavior, unless new types of cancellations between diagrams exist that are "not consequences of supersymmetry in any conventional sense":

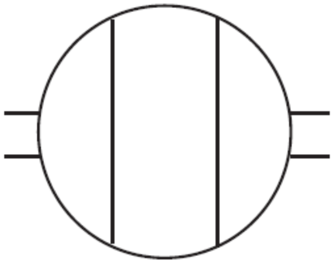
- **N = 8 sugra should diverge at 7 loops in D = 4.** David Gross' bet
- **N = 8 sugra should diverge at 5 loops in D = 24/5.** Kelly Stelle's bet

All other groups that looked at the question of symmetries agree. Looked like a safe bet that these divergences are present.

Maximal Cut Power Counting

Maximal cuts of diagrams poorly behaved:

$\mathcal{N} = 4$ sugra



already log divergent

$\mathcal{N} = 4$ sugra: pure YM x $\mathcal{N} = 4$ sYM

This diagram is log divergent

$\mathcal{N} = 8$ sugra should diverge at 7 loops in $D = 4$. **David Gross' bet**

$\mathcal{N} = 8$ sugra should diverge at 5 loops in $D = 24/5$ **Kelly Stelle's bet**

$\mathcal{N} = 4$ sugra should diverge at 3 loops in $D = 4$

$\mathcal{N} = 5$ sugra should diverge at 4 loops in $D = 4$

should have bet on these two cases.

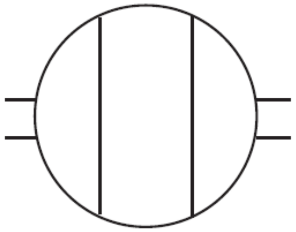
Although somewhat different, this is really equivalent to Bjornsson and Green's approach: Identify bad terms and count.

If the above full amplitudes are actually finite something new and nontrivial must be happening.

Enhanced UV Cancellations

Suppose there exists terms in a covariant diagrammatic representations with a worse power count than the amplitude as a whole, yet the terms cannot be removed.

***N* = 4 sugra**



By definition we then have "enhanced" cancellations.

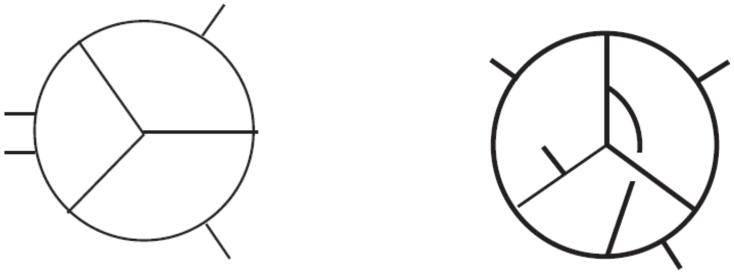
- The Bjornsson and Green power counting does *not* include enhanced cancellations.
- We can also define the enhanced cancellations as any cancellation beyond those identified by the Bjornsson and Green.
- Through four loops in *N* = 8 sugra, UV cancellations are *not* enhanced.
- Standard UV cancellations in susy gauge theory *not* enhanced.

Enhanced UV Cancellations

ZB, Davies, Dennen

Here we will prove that *enhanced cancellations* do in fact exist in *D* = 4 supergravity theories, contrary to consensus expectations.

We do so the old fashioned way: we calculate.



Why might we expect enhanced cancellations?

- **Certain unitarity cut show remarkable cancellations that have no right to be there by standard-symmetry arguments**

ZB, Dixon, Roiban

- **In a nontrivial example, duality between color and kinematics implies new cancellations.**

ZB, Davies, Dennen. Huang

Examples of Enhanced Cancellations?

Three nontrivial examples:

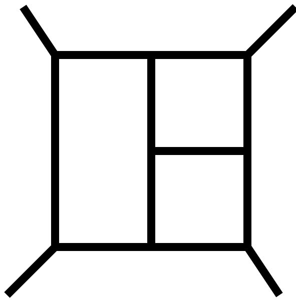
- **$N = 4$ supergravity in $D = 4$ at 3 loops.**
- **Half-maximal supergravity in $D = 5$ at 2 loops.**
- **$N = 5$ supergravity in $D = 4$ at 4 loops.**

Three-Loop $\mathcal{N} = 4$ Supergravity Construction

ZB, Davies, Dennen, Huang

$\mathcal{N} = 4$ sugra : ($\mathcal{N} = 4$ sYM) x ($\mathcal{N} = 0$ YM)

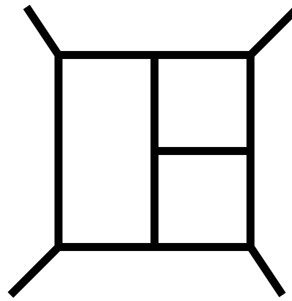
$\mathcal{N} = 4$ sYM



$\sim l \cdot k \, s^2 t A_4^{\text{tree}}$

BCJ representation

pure YM



$\sim (\varepsilon_i \cdot l)^4 \; l^4$

Feynman representation

$c_i \rightarrow n_i$

$\mathcal{N} = 4$ sugra diagrams linearly divergent

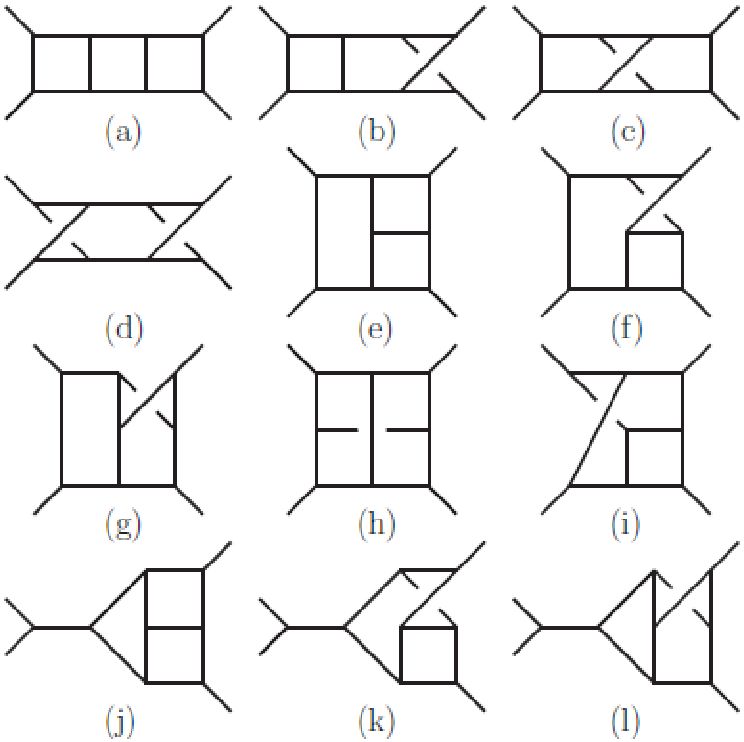
$$\int (d^D l)^3 \frac{k^7 l^9}{l^{20}}$$

- **Ultraviolet divergences are obtained by series expanding small external momentum (or large loop momentum).**
- **Introduce mass regulator for IR divergences.**
- **In general, subdivergences must be subtracted.**

Vladimirov; Marcus and Sagnotti

The N = 4 Supergravity UV Cancellation

ZB, Davies, Dennen, Huang



Graph	$(\text{divergence})/(\langle 12\rangle^2[34]^2 st A^{\text{tree}}(\frac{\kappa}{2})^8)$
(a)-(d)	0
(e)	$\frac{263}{768}\frac{1}{\epsilon^3} + \frac{205}{27648}\frac{1}{\epsilon^2} + \left(-\frac{5551}{768}\zeta_3 + \frac{326317}{110592}\right)\frac{1}{\epsilon}$
(f)	$-\frac{175}{2304}\frac{1}{\epsilon^3} - \frac{1}{4}\frac{1}{\epsilon^2} + \left(\frac{593}{288}\zeta_3 - \frac{217571}{165888}\right)\frac{1}{\epsilon}$
(g)	$-\frac{11}{36}\frac{1}{\epsilon^3} + \frac{2057}{6912}\frac{1}{\epsilon^2} + \left(\frac{10769}{2304}\zeta_3 - \frac{226201}{165888}\right)\frac{1}{\epsilon}$
(h)	$-\frac{3}{32}\frac{1}{\epsilon^3} - \frac{41}{1536}\frac{1}{\epsilon^2} + \left(\frac{3227}{2304}\zeta_3 - \frac{3329}{18432}\right)\frac{1}{\epsilon}$
(i)	$\frac{17}{128}\frac{1}{\epsilon^3} - \frac{29}{1024}\frac{1}{\epsilon^2} + \left(-\frac{2087}{2304}\zeta_3 - \frac{10495}{110592}\right)\frac{1}{\epsilon}$
(j)	$-\frac{15}{32}\frac{1}{\epsilon^3} + \frac{9}{64}\frac{1}{\epsilon^2} + \left(\frac{101}{12}\zeta_3 - \frac{3227}{1152}\right)\frac{1}{\epsilon}$
(k)	$\frac{5}{64}\frac{1}{\epsilon^3} + \frac{89}{1152}\frac{1}{\epsilon^2} + \left(-\frac{377}{144}\zeta_3 + \frac{287}{432}\right)\frac{1}{\epsilon}$
(l)	$\frac{25}{64}\frac{1}{\epsilon^3} - \frac{251}{1152}\frac{1}{\epsilon^2} + \left(-\frac{835}{144}\zeta_3 + \frac{7385}{3456}\right)\frac{1}{\epsilon}$

Spinor helicity used to clean up table, but calculation for all states

***All* three-loop divergences and subdivergences cancel completely!**

4-point 3-loop N = 4 sugra UV finite contrary to expectations

A pity we didn't bet on this theory

Tourkine and Vanhove understood this result by extrapolating from two-loop heterotic string amplitudes.

Explanations?

Key Question:

Is there an ordinary symmetry explanation for this?
Or is something extraordinary happening?

Bossard, Howe and Stelle (2013) showed that 3 loop finiteness of *N* =4 sugra can be explained by ordinary superspace + duality symmetries, *assuming* a 16 supercharge off-shell superspace exists.

$$\int d^4x d^{16}\theta \, \frac{1}{\epsilon} \mathcal{L}$$

More θs implies more derivatives in operators

If true, there is a perfectly good "ordinary" symmetry explanation.

Does this superspace exist in *D* = 5 or *D* = 4?

Not easy to construct: A non-Lorentz covariant harmonic superspace .

Explanations?

Prediction of superspace: If you add $N = 4$ vector multiplets, amplitude should develop no new 2, 3 loop divergences.

Bossard, Howe and Stelle (2013)

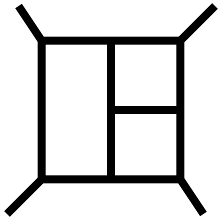
Note that $N = 4$ supergravity with matter already diverges at one loop!

Fischler (1979)

Prediction motivated us to check cases with vector multiplets.

ZB, Davies, Dennen (2013)

Four vector multiplet amplitude diverges at 2, 3 loops!



$$\mathcal{M}^{(3)}(1_{\mathrm{H}}, 2_{\mathrm{H}}, 3_{\mathrm{H}}, 4_{\mathrm{H}})\big|_{D=4\,\mathrm{div.}} = 0\,,$$

external graviton multiplets

$$\mathcal{M}^{(3)}(1_{\mathrm{H}}, 2_{\mathrm{H}}, 3_{\mathrm{V}}, 4_{\mathrm{V}})\big|_{D=4\,\mathrm{div.}} = 0\,,$$

UV divergence

$$\mathcal{M}^{(3)}(1_{\mathrm{V}}, 2_{\mathrm{V}}, 3_{\mathrm{V}}, 4_{\mathrm{V}})\big|_{D=4\,\mathrm{div.}} = -\frac{1}{(4\pi)^6}\left(\frac{\kappa}{2}\right)^8 (s^2+t^2+u^2)\,st A^{(0)}_{Q=16} \times \frac{(D_s-2)^2}{4}\left(\frac{D_s-2}{2\epsilon^3} - \frac{1}{\epsilon^2} + \frac{1}{\epsilon}\right)$$

$n_{\mathrm{V}} = D_s - 4$

matter multiplet

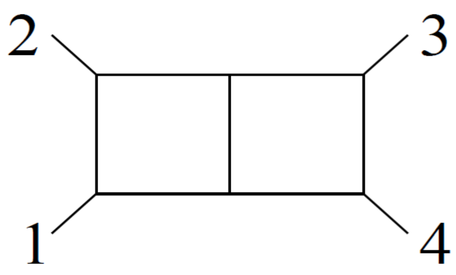
Similar story in $D = 5$

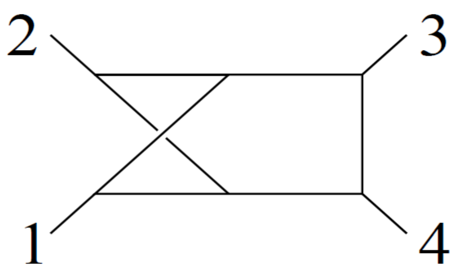
Adding vector multiplets causes new divergences both at 2, 3 loops.
Conclusion: currently no viable standard-symmetry understanding.

What is the new magic?

To analyze we need a simpler example: Half-maximal supergravity in *D* = 5 at 2 loop.

Similar to *N* = 4, *D* = 4 sugra at 3 loops, except that it is much simpler.





One-Loop Warmup in Half-Maximal Sugra

ZB, Boucher-Veronneau ,Johansson
ZB, Davies, Dennen, Huang

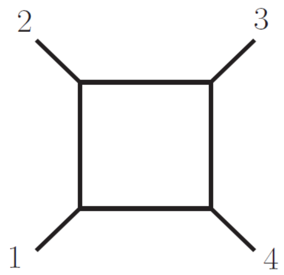
Generic color decomposition:

Dixon, Del Duca, Maltoni

color factor

$$\mathcal{A}_Q^{(1)} = ig^4\left[c_{1234}^{(1)} A_Q^{(1)}(1,2,3,4) + c_{1342}^{(1)} A_Q^{(1)}(1,3,4,2) + c_{1423}^{(1)} A_Q^{(1)}(1,4,2,3)\right]$$

Q = # supercharges **Q = 0 is pure non-susy YM**



$c_{1234}^{(1)}$ **is color factor of this box diagram**

$$s = (k_1 + k_2)^2$$
$$t = (k_2 + k_3)^2$$

To get Q + 16 supercharge supergravity take 2nd copy *N* = 4 sYM

***N* = 4 sYM numerators very simple: independent of loop momentum**

$$n_{1234} = n_{1342} = n_{1423} = st A_{Q=16}^{\text{tree}}(1,2,3,4) \qquad c_{1234}^{(1)} \to n_{1234}$$

$$\mathcal{M}_{Q+16}^{(1)} = i\left(\frac{\kappa}{2}\right)^4 st A_{Q=16}^{\text{tree}}(1,2,3,4)\left[A_Q^{(1)}(1,2,3,4) + A_Q^{(1)}(1,3,4,2) + A_Q^{(1)}(1,4,2,3)\right]$$

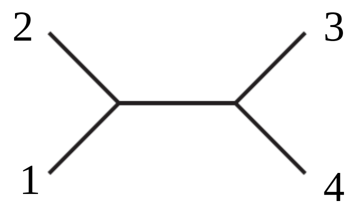
One-loop divergences in pure YM

ZB, Davies, Dennen, Huang

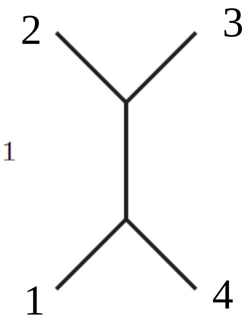
Go to a basis of color factors

Three independent color tensors

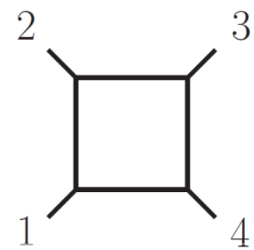
$$b_1^{(0)} = \tilde{f}^{a_1 a_2 b} \tilde{f}^{b a_3 a_4}$$



$$b_2^{(0)} = \tilde{f}^{a_2 a_3 b} \tilde{f}^{b a_4 a_1}$$



$$b_1^{(1)} \equiv c_{1234}^{(1)} = \tilde{f}^{a_1 b_2 b_1} \tilde{f}^{a_2 b_3 b_2} \tilde{f}^{a_3 b_4 b_3} \tilde{f}^{a_4 b_1 b_4}$$



All other color factors expressible in terms of these three:

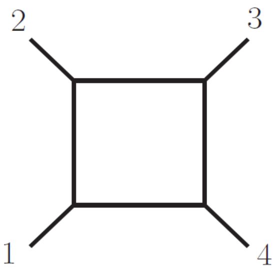
one-loop color tensor

$$\mathcal{A}_Q^{(1)} = i g^4 \Big[b_1^{(1)} \Big(A_Q^{(1)}(1,2,3,4) + A_Q^{(1)}(1,3,4,2) + A_Q^{(1)}(1,4,2,3) \Big) - \frac{1}{2} C_A b_1^{(0)} A_Q^{(1)}(1,3,4,2) - \frac{1}{2} C_A b_2^{(0)} A_Q^{(1)}(1,4,2,3) \Big]$$

tree color tensor

$C_A = 2\, N_c$ for $SU(N_c)$

One-loop divergences in pure YM



In a basis of color factors:

one-loop color tensor

$$\mathcal{A}_Q^{(1)} = ig^4\Big[b_1^{(1)}\Big(A_Q^{(1)}(1,2,3,4) + A_Q^{(1)}(1,3,4,2) + A_Q^{(1)}(1,4,2,3)\Big)$$
$$-\frac{1}{2}C_A b_1^{(0)} A_Q^{(1)}(1,3,4,2) - \frac{1}{2}C_A b_2^{(0)} A_Q^{(1)}(1,4,2,3)\Big]$$

tree color tensor

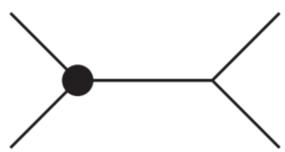
***Q* supercharges (mainly interested in *Q* = 0)**

$D = 4$: F^2 is only allowed counterterm by renormalizability

1-loop color tensor *not* allowed.

$D = 6$: F^3 counterterm: 1-loop color tensor again *not* allowed.

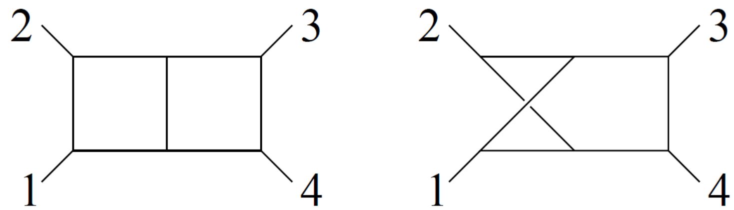
$$F^3 = f^{abc} F^{a\mu}_{\nu} F^{b\nu}_{\sigma} F^{c\sigma}_{\mu}$$



$$A_Q^{(1)}(1,2,3,4) + A_Q^{(2)}(1,3,4,2) + A_Q^{(1)}(1,4,2,3)\Big|_{D=4,6 \text{ div.}} = 0$$

$$\boxed{M_{Q+16}^{(1)}(1,2,3,4)\Big|_{D=4,6 \text{ div.}} = 0}$$

Two Loop Half Maximal Sugra in $D = 5$



ZB, Davies, Dennen, Huang

$$\mathcal{A}_Q^{(2)} = -g^6 \Big[c^{\rm P}_{1234} A_Q^{\rm P}(1,2,3,4) + c^{\rm P}_{3421} A_Q^{\rm P}(3,4,2,1) + c^{\rm NP}_{1234} A_Q^{\rm NP}(1,2,3,4) + c^{\rm NP}_{3421} A_Q^{\rm NP}(3,4,2,1) + \text{ cyclic} \Big]$$

$D = 5$ F^3 counterterm: 1,2-loop color tensors forbidden!

1) **Go to color basis.**
2) **Demand no forbidden color tensors in pure YM divergence.**
3) **Replace color factors with kinematic numerators.**

gravity $$\mathcal{M}^{(2)}_{16+Q}(1,2,3,4)\Big|_{D=5\,\text{div.}} = 0$$

Half-maximal supergravity four-point divergence vanishes because forbidden color tensor cancels in pure YM theory.
Note: this cancellation is mysterious from standard symmetries.

Two Loop $D = 5$ UV Magic

ZB, Davies, Dennen, Huang

At least for 2 loops in $D = 5$ we have identified the source of unexpected UV cancellations in half-maximal supergravity:

It is the *same* magic found by 't Hooft and Veltman 40 years ago preventing forbidden divergences appearing in ordinary non-susy gauge theory!

- **Explains the $D = 5$ two-loop half-maximal sugra case, which remains mysterious from standard supergravity viewpoint.**
- **Higher-loop cases, unfortunately, much more complicated**

Half-maximal supergravity at $L = 2$, $D = 5$ or $L = 3$, $D = 4$ are first potential divergences so we want to go beyond these.

Four-loop $N = 4$ Supergravity Divergences

ZB, Davies, Dennen, Smirnov, Smirnov

To make a deeper probe we calculated four-loop divergence in $N = 4$ supergravity.
Same methods as used at three loops.
Industrial strength software needed: FIRE5 and C++

$N = 4$ sugra: ($N = 4$ sYM) x ($N = 0$ YM)

$N = 4$ sYM

$$\sim (l \cdot k)^2 \, s^2 t A_4^{\text{tree}}$$

BCJ representation

pure YM

$$\sim (\varepsilon_i \cdot l)^4 \, l^6$$

Feynman representation

$N = 4$ sugra diagrams quadratically divergent

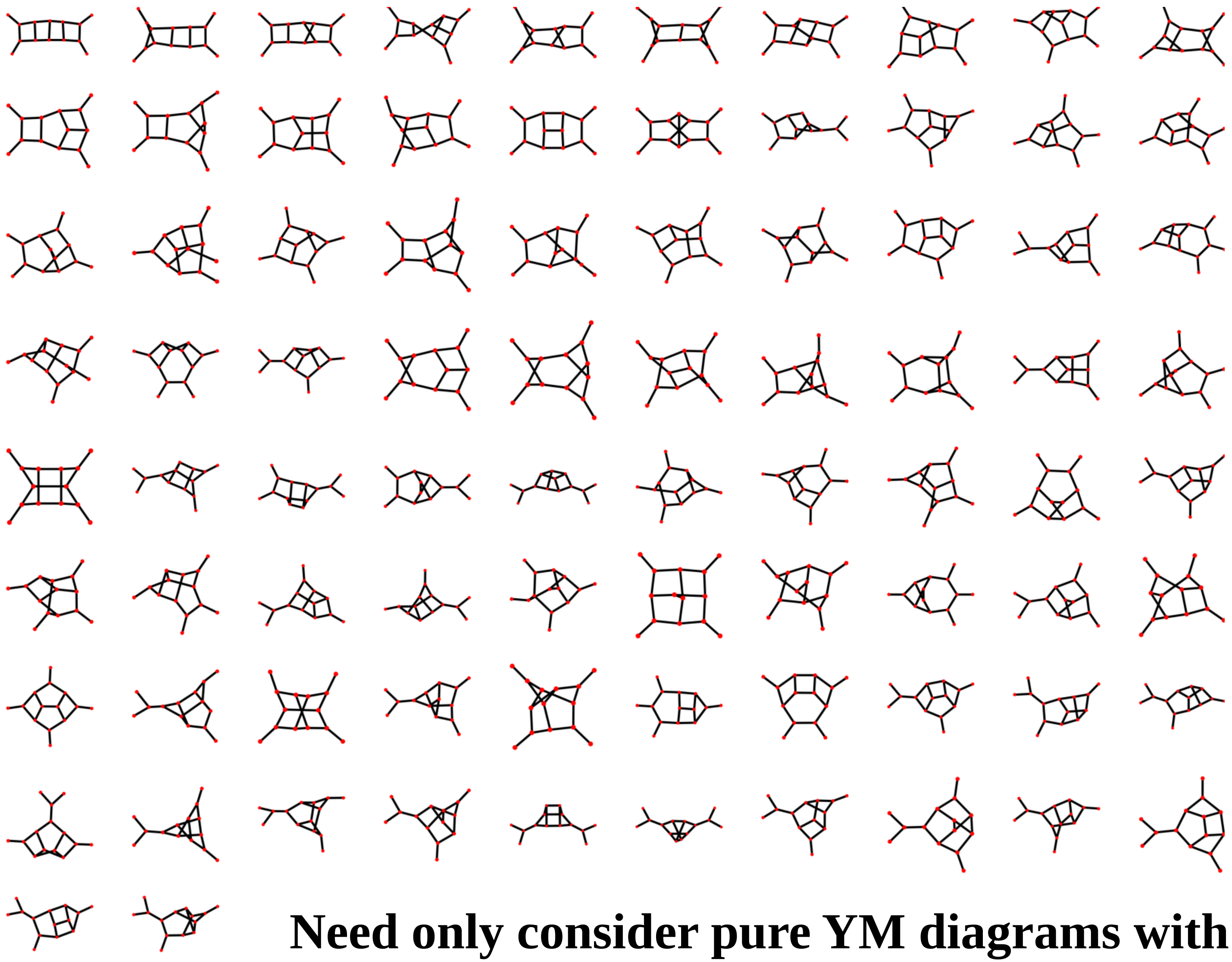
$$\int (d^D l)^4 \frac{k^8 l^{12}}{l^{26}}$$

$D^2 R^4$ counterterm

82 nonvanishing diagram types using $N = 4$ sYM BCJ form.

82 nonvanishing numerators in BCJ representation

ZB, Carrasco, Dixon, Johansson, Roiban ($\mathcal{N}$ = 4 sYM)



Need only consider pure YM diagrams with color factors that match these.

The 4 loop Divergence of *N* = 4 Supergravity

ZB, Davies, Dennen, Smirnov, Smirnov

Similar to three loops except industrial level: C++ and FIRE5

Pure *N* = 4 supergravity is divergent at 4 loops with divergence

dim. reg. UV pole

Result is for Siegel dimensional reduction.

$$\boxed{\mathcal{M}^{4\text{-loop}}\Big|_{\text{div.}} = \frac{1}{(4\pi)^8}\frac{1}{\epsilon}\left(\frac{\kappa}{2}\right)^{10}\frac{1}{144}(1-264\zeta_3)\,\mathcal{T}}$$

$$\mathcal{T} = st A^{\text{tree}}_{\mathcal{N}=4}\left(\mathcal{O}_1 - 28\mathcal{O}_2 - 6\mathcal{O}_3\right)$$

$$\mathcal{O}_1 = \sum_{\mathcal{S}_4} (D_\alpha F_{1\mu\nu})\,(D^\alpha F_2^{\mu\nu})\,F_{3\rho\sigma}F_4^{\rho\sigma}$$

$$\mathcal{O}_2 = \sum_{\mathcal{S}_4} (D_\alpha F_{1\mu\nu})\,(D^\alpha F_2^{\nu\sigma})\,F_{3\sigma\rho}F_4^{\rho\mu}$$

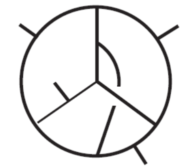
$$\mathcal{O}_3 = \sum_{\mathcal{S}_4} (D_\alpha F_{1\mu\nu})\,(D_\beta F_2^{\mu\nu})\,F_{3\sigma}{}^{\alpha}F_4^{\sigma\beta}$$

$$s = (k_1+k_2)^2$$
$$t = (k_2+k_3)^2$$

$$F_j^{\mu\nu} \equiv i(k_j^\mu\varepsilon_j^\nu - k_j^\nu\varepsilon_j^\mu)\,,$$
$$D^\alpha F_j^{\mu\nu} \equiv -k_j^\alpha(k_j^\mu\varepsilon_j^\nu - k_j^\nu\varepsilon_j^\mu)$$

Valid for all nonvanishing 4-point amplitudes of pure *N* = 4 sugra

Some Peculiar Properties



Linear combinations to expose D = 4 helicity structure

Refers to helicities of pure YM component

$$\mathcal{O}^{--++} = \mathcal{O}_1 - 4\mathcal{O}_2$$
$$\mathcal{O}^{-+++} = \mathcal{O}_1 - 4\mathcal{O}_3$$
$$\mathcal{O}^{++++} = \mathcal{O}_2$$

$$\mathcal{O}^{--++} = 4s^2t \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,1\rangle},$$
$$\mathcal{O}^{-+++} = -12s^2t^2 \frac{[2\,4]^2}{[1\,2]\langle 2\,3\rangle\langle 3\,4\rangle[4\,1]},$$
$$\mathcal{O}^{++++} = 3st(s+t)\frac{[1\,2]\,[3\,4]}{\langle 1\,2\rangle\langle 3\,4\rangle},$$

The latter two configurations would vanish if the $U(1)$ symmetry were not anomalous.

See Carrasco, Kallosh, Tseytlin and Roiban

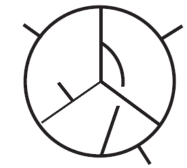
All three independent configurations have similar divergence!

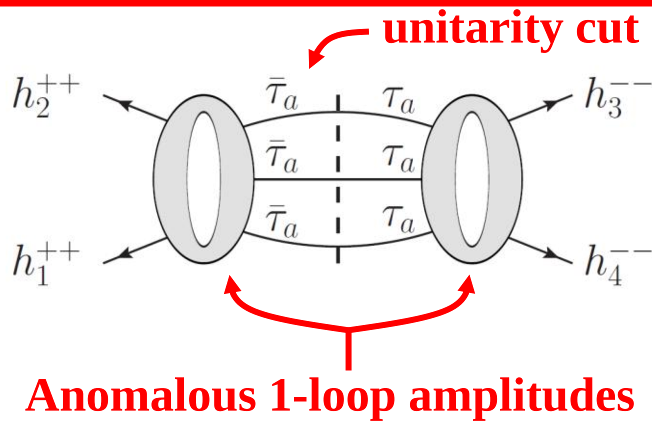
Very peculiar because the nonanomalous sector should have a very different analytic structure. Not related by any supersymmetry Ward identities.

For anomalous sectors:

- ***D* = 4 generalized cuts decomposing into tree amplitudes vanish.**
- **At one-loop anomalous sectors purely rational functions, no logs**
- **Anomaly is ε/ε (UV divergence suppressed by ε).**

Relation to $U(1)$ Anomaly





Anomalous sector feeds poor UV behavior into non-anomalous sector

Figure from arXiv:1303.6219
Carrasco, Kallosh, Tseytlin and Roiban

- **As pointed out by Carrasco, Kallosh Roiban, Tseytlin the anomalous amplitudes are poorly behaved and contribute to a 4-loop UV divergence (unless somehow canceled as they are at 3 loops).**
- **Via the anomaly it is easy to understand why all three sectors can have similar divergence structure.**
- **The dependence of the divergence on vector multiplets matches anomaly.**

$$\mathcal{M}^{4\text{-loop}}_{n_V}\Big|_{\text{div.}} = \frac{1}{(4\pi)^8}\left(\frac{\kappa}{2}\right)^{10}\frac{\boxed{n_V+2}}{2304}\left[\frac{6(n_V+2)n_V}{\epsilon^2} + \frac{(n_V+2)(3n_V+4)-96(22-n_V)\zeta_3}{\epsilon}\right]\mathcal{T}$$

anomaly has exactly this factor

n_V is number vector multiplets

Bottom line: The divergence looks specific to *N* = 4 sugra and likely due to an anomaly. Won't be present in *N* $\geqslant$ 5 sugra.

If anything, this suggests *N* = 8 sugra UV finite at 8 loops.

$N = 5$ supergravity at Four Loops

ZB, Davies and Dennen

No anomaly in $N = 5$ sugra so expect no divergences

$N = 5$ sugra: ($N = 4$ sYM) x ($N = 1$ sYM)

Again crucial help from Fire5 and (Smirnov)^2

$N = 4$ sYM

$N = 1$ sYM

Had we made susy cancellation manifest we would have expected log divergence

Straightforward following what we did in $N = 4$ sugra.

$N = 5$ supergravity has no D^2R^4 divergence at four loops.

This is another example analogous to 7 loops in $N = 8$ sugra.

A pity we did not bet on this one as well!

$\mathcal{N} = 5$ supergravity at Four Loops

ZB, Davies and Dennen (to appear)

graphs	(divergence) $\times\, u/(-i/(4\pi)^8\langle 12\rangle^2[34]^2 st A^{\rm tree}(\frac{\kappa}{2})^{10})$
1–30	$\frac{1}{\epsilon^4}\Big[\frac{7358585}{7962624}s^2+\frac{2561447}{2654208}st-\frac{872683}{1990656}t^2\Big]+\frac{1}{\epsilon^3}\Big[\frac{75972559}{35389440}s^2+\frac{240984061}{26542080}st+\frac{1302037}{1310720}t^2\Big]+\frac{1}{\epsilon^2}\Big[\zeta_3\big(-\frac{369234283}{11059200}s^2-\frac{257792411}{4915200}st-\frac{101847769}{14745600}t^2\big)+\zeta_2\big(\frac{7358585}{3981312}s^2+\frac{2561447}{1327104}st-\frac{872683}{995328}t^2\big)-\mathrm{S2}\big(\frac{1223621}{49152}s^2+\frac{46816475}{442368}st+\frac{2639903}{221184}t^2\big)+\frac{206093335871}{11466178560}s^2+\frac{320983191023}{3822059520}st+\frac{53309416589}{2866544640}t^2\Big]+\frac{1}{\epsilon}\Big[\zeta_5\big(-\frac{84777347}{368640}s^2+\frac{382194721}{1474560}st+\frac{417476581}{1474560}t^2\big)-\zeta_4\big(\frac{3062401}{2457600}s^2+\frac{3881051}{3276800}st-\frac{112081813}{29491200}t^2\big)+\zeta_3\big(\frac{28162691399797}{53747712000}s^2+\frac{19354492750651}{35831808000}st-\frac{22092683352811}{107495424000}t^2\big)-\zeta_2\big(\frac{70861961}{17694720}s^2+\frac{227180689}{13271040}st+\frac{105727243}{53084160}t^2\big)+\mathrm{T1ep}\big(-\frac{1223621}{663552}s^2-\frac{46816475}{5971968}st-\frac{2639903}{2985984}t^2\big)-\mathrm{S2}\big(\frac{11916028151}{5898240}s^2+\frac{72637733971}{13271040}st+\frac{17223563447}{53084160}t^2\big)+\mathrm{D6}\big(-\frac{9001177}{552960}s^2-\frac{264491}{10240}st-\frac{2610157}{552960}t^2\big)+\frac{110945914744727}{1146617856000}s^2+\frac{16989492195991}{127401984000}st-\frac{21362122998269}{573308928000}t^2\Big]$
31–60	$\frac{1}{\epsilon^4}\Big[-\frac{5502451}{2654208}s^2-\frac{3675877}{884736}st+\frac{11269}{497664}t^2\Big]+\frac{1}{\epsilon^3}\Big[\frac{38102993}{26542080}s^2-\frac{291607201}{106168320}st-\frac{565798829}{318504960}t^2\Big]+\frac{1}{\epsilon^2}\Big[\zeta_3\big(\frac{108955183}{2211840}s^2+\frac{653019571}{8847360}st+\frac{9453043}{1769472}t^2\big)+\zeta_2\big(-\frac{5502451}{1327104}s^2-\frac{3675877}{442368}st+\frac{11269}{248832}t^2\big)+\mathrm{S2}\big(\frac{16797481}{1327104}s^2+\frac{1172969}{16384}st+\frac{978427}{82944}t^2\big)-\frac{304243754383}{19110297600}s^2-\frac{2032063711381}{19110297600}st-\frac{257798086613}{7166361600}t^2\Big]+\frac{1}{\epsilon}\Big[\zeta_5\big(\frac{33327659}{122880}s^2+\frac{13276219}{24576}st+\frac{22251887}{184320}t^2\big)+\zeta_4\big(\frac{12299887}{1474560}s^2+\frac{258056147}{5898240}st+\frac{46913759}{5898240}t^2\big)+\zeta_3\big(-\frac{26846001990157}{42998169600}s^2-\frac{337106527201}{265420800}st-\frac{5298324906787}{42998169600}t^2\big)+\zeta_2\big(\frac{282283789}{39813120}s^2+\frac{975199319}{53084160}st+\frac{60394451}{159252480}t^2\big)+\mathrm{T1ep}\big(\frac{16797481}{17915904}s^2+\frac{1172969}{221184}st+\frac{978427}{1119744}t^2\big)+\mathrm{S2}\big(\frac{10516980893}{4976640}s^2+\frac{389045625329}{53084160}st+\frac{216032337589}{159252480}t^2\big)+\mathrm{D6}\big(\frac{503413}{23040}s^2+\frac{12342607}{552960}st+\frac{3661}{184320}t^2\big)-\frac{166777358259461}{1146617856000}s^2-\frac{565137511429117}{1146617856000}st-\frac{21629055712141}{191102976000}t^2\Big]$
61–82	$\frac{1}{\epsilon^4}\Big[\frac{285899}{248832}s^2+\frac{1058273}{331776}st+\frac{275869}{663552}t^2\Big]+\frac{1}{\epsilon^3}\Big[-\frac{380329649}{106168320}s^2-\frac{74703227}{11796480}st+\frac{124701919}{159252480}t^2\Big]+\frac{1}{\epsilon^2}\Big[\zeta_3\big(-\frac{1371419}{86400}s^2-\frac{236241539}{11059200}st+\frac{4326077}{2764800}t^2\big)+\zeta_2\big(\frac{285899}{124416}s^2+\frac{1058273}{165888}st+\frac{275869}{331776}t^2\big)+\mathrm{S2}\big(\frac{8120143}{663552}s^2+\frac{1893289}{55296}st+\frac{92293}{663552}t^2\big)-\frac{58867708103}{28665446400}s^2+\frac{71191292711}{3185049600}st+\frac{83016363427}{4777574400}t^2\Big]+\frac{1}{\epsilon}\Big[\zeta_5\big(-\frac{1520563}{36864}s^2-\frac{1178767861}{1474560}st-\frac{595491677}{1474560}t^2\big)-\zeta_4\big(\frac{6539029}{921600}s^2+\frac{313837819}{7372800}st+\frac{21665663}{1843200}t^2\big)+\zeta_3\big(\frac{20790944575597}{214990848000}s^2+\frac{6505876281371}{8957952000}st+\frac{70676991239557}{214990848000}t^2\big)+\zeta_2\big(-\frac{491377507}{159252480}s^2-\frac{66476563}{53084160}st+\frac{128393639}{79626240}t^2\big)+\mathrm{T1ep}\big(\frac{8120143}{8957952}s^2+\frac{1893289}{746496}st+\frac{92293}{8957952}t^2\big)+\mathrm{S2}\big(-\frac{14810628499}{159252480}s^2-\frac{19698937889}{10616832}st-\frac{10272602953}{9953280}t^2\big)+\mathrm{D6}\big(-\frac{616147}{110592}s^2+\frac{1939907}{552960}st+\frac{1299587}{276480}t^2\big)+\frac{9307894793789}{191102976000}s^2+\frac{206124003456599}{573308928000}st+\frac{21562322533673}{143327232000}t^2\Big]$

graphs	(divergence) $\times\, u/(-i/(4\pi)^8\langle 12\rangle^2[34]^2 st A^{\rm tree}(\frac{\kappa}{2})^{10})$
1–30	$\frac{1}{\epsilon^4}\Big[\frac{1052159}{995328}s^2+\frac{509789}{331776}st-\frac{121001}{497664}t^2\Big]+\frac{1}{\epsilon^3}\Big[\frac{9042569}{1474560}s^2+\frac{34360945}{1327104}st+\frac{73518401}{13271040}t^2\Big]+\frac{1}{\epsilon^2}\Big[\zeta_3\big(-\frac{11443919}{2764800}s^2+\frac{32520079}{552960}st+\frac{5836531}{230400}t^2\big)+\zeta_2\big(\frac{1052159}{497664}s^2+\frac{509789}{165888}st-\frac{121001}{248832}t^2\big)-\mathrm{S2}\big(\frac{637991}{6144}s^2+\frac{10978729}{27648}st+\frac{5080825}{55296}t^2\big)+\big(\frac{270806866183}{7166361600}s^2+\frac{89848068067}{597196800}st+\frac{218093645149}{7166361600}t^2\big)\Big]+\frac{1}{\epsilon}\Big[\zeta_5\big(\frac{100843}{360}s^2+\frac{17118043}{30720}st-\frac{30266471}{92160}t^2\big)+\zeta_4\big(\frac{11435323}{614400}s^2+\frac{232002227}{1843200}st+\frac{22211783}{460800}t^2\big)+\zeta_3\big(\frac{223300432349}{3359232000}s^2-\frac{178732984847}{716636160}st+\frac{951659436383}{53747712000}t^2\big)-\zeta_2\big(\frac{5492357}{245760}s^2+\frac{53468887}{663552}st+\frac{129714599}{6635520}t^2\big)+\mathrm{T1ep}\big(-\frac{637991}{82944}s^2-\frac{10978729}{373248}st-\frac{5080825}{746496}t^2\big)+\mathrm{S2}\big(-\frac{5700088747}{3686400}s^2-\frac{69470348491}{16588800}st-\frac{713512871}{6635520}t^2\big)+\mathrm{D6}\big(-\frac{357421}{43200}s^2-\frac{2891743}{230400}st-\frac{470219}{138240}t^2\big)-\frac{3571506237341}{28665446400}s^2-\frac{1611591325291}{5971968000}st+\frac{2301084608777}{143327232000}t^2\Big]$
31–60	$\frac{1}{\epsilon^4}\Big[-\frac{150715}{82944}s^2-\frac{668333}{221184}st-\frac{7213}{1990656}t^2\Big]+\frac{1}{\epsilon^3}\Big[-\frac{68021833}{13271040}s^2-\frac{36852103}{1327104}st-\frac{298377299}{39813120}t^2\Big]+\frac{1}{\epsilon^2}\Big[\zeta_3\big(-\frac{36448033}{2764800}s^2-\frac{455889533}{2764800}st-\frac{82059281}{1382400}t^2\big)+\zeta_2\big(-\frac{150715}{41472}s^2-\frac{668333}{110592}st-\frac{7213}{995328}t^2\big)+\mathrm{S2}\big(\frac{13910839}{165888}s^2+\frac{1340033}{4096}st+\frac{26303855}{331776}t^2\big)-\frac{68286245653}{2388787200}s^2-\frac{20649690431}{119439360}st-\frac{351701043553}{7166361600}t^2\Big]+\frac{1}{\epsilon}\Big[\zeta_5\big(-\frac{2362679}{9216}s^2-\frac{178668311}{92160}st-\frac{1268313}{10240}t^2\big)+\zeta_4\big(-\frac{124344121}{1843200}s^2-\frac{491722333}{1843200}st-\frac{68141309}{921600}t^2\big)-\zeta_3\big(\frac{630084012997}{53747712000}s^2-\frac{1250670277213}{663552000}st-\frac{6913218302303}{13436928000}t^2\big)+\zeta_2\big(\frac{352368061}{19906560}s^2+\frac{35509679}{663552}st+\frac{227699801}{19906560}t^2\big)+\mathrm{T1ep}\big(\frac{13910839}{2239488}s^2+\frac{1340033}{55296}st+\frac{26303855}{4478976}t^2\big)+\mathrm{S2}\big(\frac{188312318729}{99532800}s^2+\frac{110749829741}{16588800}st+\frac{5056299197}{3981312}t^2\big)+\mathrm{D6}\big(\frac{1220779}{76800}s^2+\frac{44791}{6912}st-\frac{1159831}{230400}t^2\big)+\frac{2755666297013}{28665446400}s^2+\frac{5622513975899}{35831808000}st-\frac{196197363193}{1769472000}t^2\Big]$
61–82	$\frac{1}{\epsilon^4}\Big[\frac{756421}{995328}s^2+\frac{985421}{663552}st+\frac{163739}{663552}t^2\Big]+\frac{1}{\epsilon^3}\Big[-\frac{1670161}{1658880}s^2+\frac{415193}{221184}st+\frac{4863881}{2488320}t^2\Big]+\frac{1}{\epsilon^2}\Big[\zeta_3\big(\frac{110861}{6400}s^2+\frac{16293841}{153600}st+\frac{9408019}{276480}t^2\big)+\zeta_2\big(\frac{756421}{497664}s^2+\frac{985421}{331776}st+\frac{163739}{331776}t^2\big)+\mathrm{S2}\big(\frac{1657459}{82944}s^2+\frac{7734025}{110592}st+\frac{4181095}{331776}t^2\big)-\frac{8243516153}{895795200}s^2+\frac{558349337}{24883200}st+\frac{11133949867}{597196800}t^2\Big]+\frac{1}{\epsilon}\Big[\zeta_5\big(-\frac{1094509}{46080}s^2+\frac{63657091}{46080}st+\frac{5210161}{11520}t^2\big)+\zeta_4\big(\frac{11254769}{230400}s^2+\frac{129860053}{921600}st+\frac{23717743}{921600}t^2\big)-\zeta_3\big(\frac{2745647960587}{53747712000}s^2+\frac{3654260151947}{2239488000}st+\frac{5720906529119}{10749542400}t^2\big)+\zeta_2\big(\frac{11564107}{2488320}s^2+\frac{2244901}{82944}st+\frac{40360999}{4976640}t^2\big)+\mathrm{T1ep}\big(\frac{1657459}{1119744}s^2+\frac{7734025}{1492992}st+\frac{4181095}{4478976}t^2\big)+\mathrm{S2}\big(-\frac{420043}{1215}s^2-\frac{825589625}{331776}st-\frac{5785239343}{4976640}t^2\big)+\mathrm{D6}\big(-\frac{210731}{27648}s^2+\frac{4196129}{691200}st+\frac{1457647}{172800}t^2\big)+\frac{33976742047}{1194393600}s^2+\frac{4046536311847}{35831808000}st+\frac{212357840779}{2239488000}t^2\Big]$

Adds up to zero: no divergence. Enhanced cancellations!

Enhanced Cancellations

Many of you are saying: "There has to be a better way"

Yes, take it as a challenge. These are enhanced cancellations so standard arguments will *not* work.

As we have been arguing for years, a new class of nontrivial cancellations must exist in supergravity theories. We now have explicit examples:

- **Enhanced cancellations in $N = 4$ sugra at 3 loops.**
- **Enhanced cancellations in $N = 5$ sugra at 4 loops.**

Future Directions

- We need to find five- and higher-loop BCJ representations.

- Now that we examples of enhanced cancellations we need to understand the general all-loop consequences.

- Anomalies ruin finiteness properties. Needs further study.

- Role of BCJ in enhanced cancellations. To go beyond the two loop case discussed here, need much better control over loop integration.

- Study theories with even fewer supersymmetries.

See Henrik Johansson's talk

Summary

- A duality conjectured between color and kinematics. When manifest, it trivially gives us (super) gravity loop integrands.
- At sufficiently high loop orders in any supergravity theory covariant diagrammatic representations have divergences:
  - — Bjornsson and Green pure spinor formalism.
  - — maximal cut power counting.
- Phenomenon of "enhanced cancellations": Bjornsson and Green divergences cancel. Proven in examples by direct computation.
- For half-maximal supergravity in D = 5, 2 loops we know precisely the origin of the enhanced UV cancellations: it is *standard magic* that restricts counterterms of nonsusy YM.

- Key problem is to develop better methods for finding BCJ representations. Five and higher loops awaits us.

We can expect many more surprises as we probe perturbative supergravity theories using modern tools.